Baryon spectrum and structure, nucleon Compton scattering

Gernot Eichmann

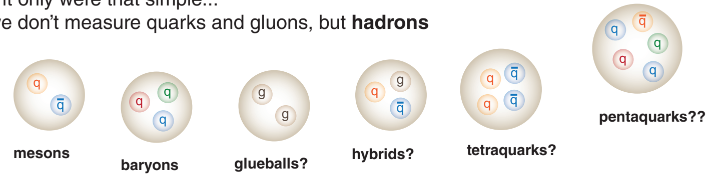
INT Workshop „Tomography of Hadrons and Nuclei at Jefferson Lab“

August 30, 2017
Seattle, WA, USA

Why?

QCD Lagrangian: $\mathcal{L} = \bar{\psi}\,(\partial\!\!\!/ + ig A\!\!\!/ + m)\,\psi + \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a}$

- if it only were that simple...
  we don't measure quarks and gluons, but **hadrons**

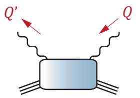


- origin of **mass generation** and **confinement?**

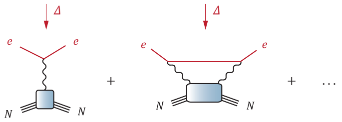
	u	d	s	c	b	t
Current mass [GeV]	**0.003**	**0.005**	**0.1**	**1**	**4**	**175**
„Constituent“ mass [GeV]	**0.35**	**0.35**	**0.5**	**1.5**	**4.5**	**175**

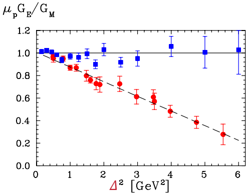
- need to understand **spectrum and interactions!**

Compton scattering



- **Two-photon corrections to form factors:**
  can explain difference between Rosenbluth and polarization transfer measurements
  Guichon, Vanderhaeghen, PRL 91 (2003)

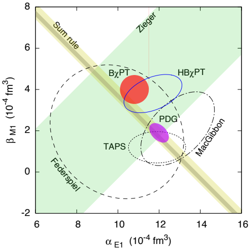




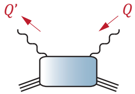
Arrington, Blunden, Melnitchouk
Prog. Part. Nucl. Phys. 66 (2011)

- **Proton radius puzzle:**
  can TPE explain discrepancy between e & μ measurements? So far: probably not, but . . .
  Antonigni et al., Ann Phys 331 (2013), Pohl et al., Ann Rev Nucl Part Sci 63 (2013), Carlson, Prog. Part. Nucl. Phys. 82 (2015)

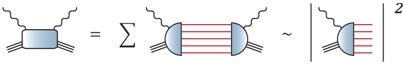
- **Nucleon polarizabilities:**
  efforts from ChPT & dispersion relations
  Hagelstein, Miskimen, Pascalutsa, Prog. Part. Nucl. Phys. 88 (2016)

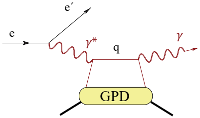


Compton scattering



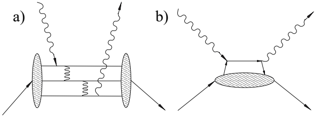
- **Forward limit:**
  determined by photoabsorption cross section and nucleon structure functions





- **Virtual CS:** generalized polarizabilities, DVCS: factorization & handbag dominance, extraction of GPDs

- **Real CS:** dominant quark-level mechanism in WACS?

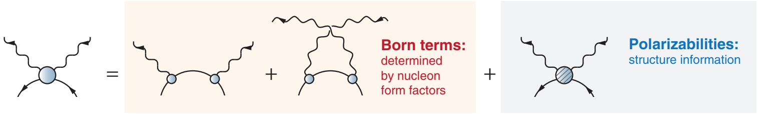


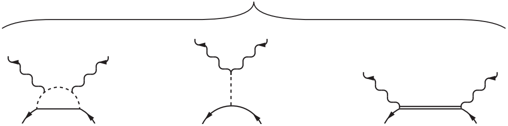
Hamilton et al., PRL 94 (2005)

- **Timelike CS:**
  $p\bar{p}$ annihilation @ PANDA

Compton scattering ...

Compton amplitude = sum of **Born terms + 1PI structure part:**

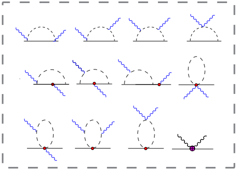




„Pion cloud“ (ChPT)

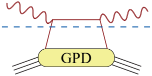
t-channel meson exchange $(\pi, \sigma, a_1, \ldots)$

s/u-channel nucleon resonances $(\Delta, N^*, \ldots)$



Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

but also:

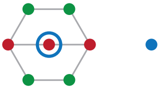


$\Rightarrow$ is there a common underlying **quark-level description?**

The hadron zoo

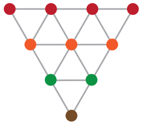
Mesons

0^{-+}	0^{++}	1^{-+}	1^{--}	1^{++}	1^{+-}	2^{-+}	2^{++}	3^{--}
$\pi(140)$ $\pi(1300)$ $\pi(1800)$	$a_0(980)$ $a_0(1450)$ $a_0(1950)$	$\pi_1(1400)$ $\pi_1(1600)$	$\rho(770)$ $\rho(1450)$ $\rho(1570)$ $\rho(1700)$ $\rho(1900)$	$a_1(1260)$ $a_1(1420)$ $a_1(1640)$	$b_1(1235)$	$\pi_2(1670)$ $\pi_2(1880)$	$a_2(1320)$ $a_2(1700)$	$\rho_3(1690)$ $\rho_3(1990)$
$K(494)$ $K(1460)$ $K(1830)$	$K_0^*(800)$ $K_0^*(1430)$ $K_0^*(1950)$		$K^*(892)$ $K^*(1410)$ $K^*(1680)$	$K_1(1400)$ $K_1(1650)$	$K_1(1270)$	$K_2(1580)$ $K_2(1770)$ $K_2(1820)$	$K_2^*(1430)$ $K_2^*(1980)$	$K_3^*(1780)$
$\eta(548)$ $\eta'(958)$ $\eta(1295)$ $\eta(1405)$ $\eta(1475)$ $\eta(1760)$	$f_0(500)$ $f_0(980)$ $f_0(1370)$ $f_0(1500)$ $f_0(1710)$		$\omega(782)$ $\phi(1020)$ $\omega(1420)$ $\omega(1650)$ $\phi(1680)$	$f_1(1285)$ $f_1(1420)$ $f_1(1510)$	$h_1(1170)$ $h_1(1380)$ $h_1(1595)$	$\eta_2(1645)$ $\eta_2(1870)$	$f_2(1270)$ $f_2(1430)$ $f_2'(1525)$ $f_2(1565)$ $f_2(1640)$ $f_2(1810)$ $f_2(1910)$ $f_2(1950)$	$\omega_3(1670)$ $\phi_3(1850)$

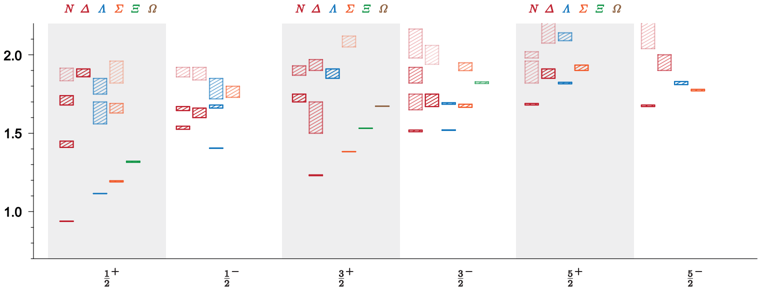


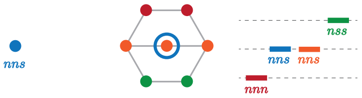
Baryons

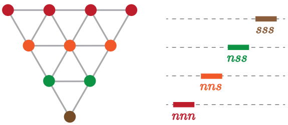
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



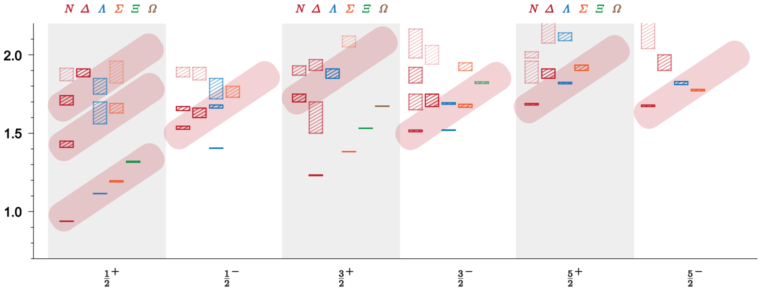
The hadron zoo

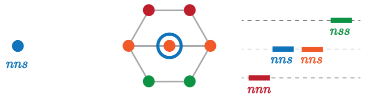


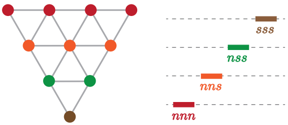




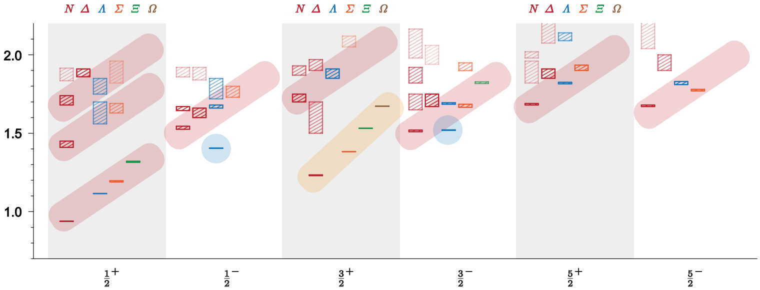
The hadron zoo

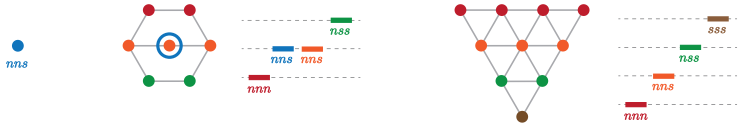




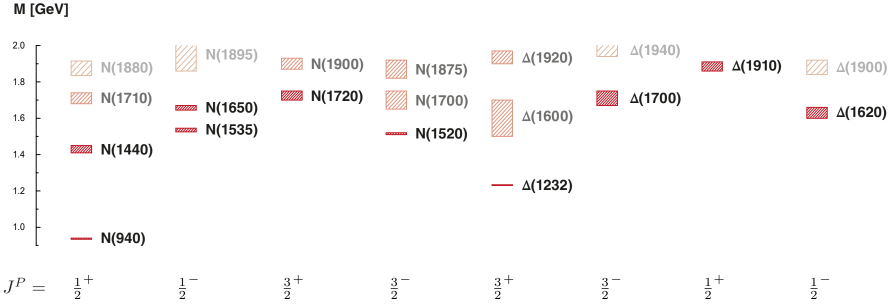


The hadron zoo

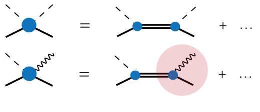




Light baryons



- Extraction of resonances?



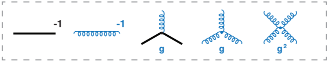
- **Gluon exchange** vs. flavor dependence?
- Nature of **Roper?**
- qqq vs. **quark-diquark?**

- “Quark core” vs. **chiral dynamics?**
- **Hybrid baryons?**

QCD

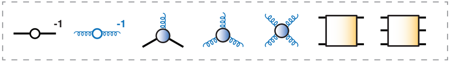
QCD's classical action:

$$S = \int d^4x \left[\bar{\psi} \left(\not{\partial} + ig\not{A} + m \right) \psi + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

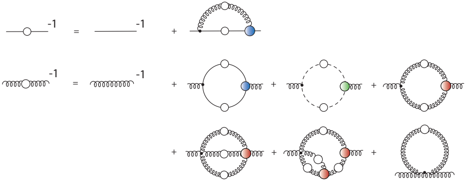
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} = e^{-\Gamma}$$



DSEs = quantum equations of motion:
derived from path integral, relate n-point functions



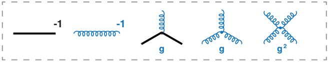
- infinitely many coupled equations
- reproduce perturbation theory, but **nonperturbative!**
- systematic truncations: neglect higher n-point functions to obtain **closed system**

Reviews:
Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
Alkofer, von Smekal, Phys. Rept. 353 (2001)
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602 [hep-ph]

QCD

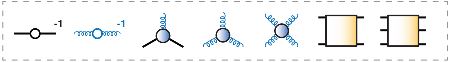
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m\right)\psi + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a\right]$$

$=$ 

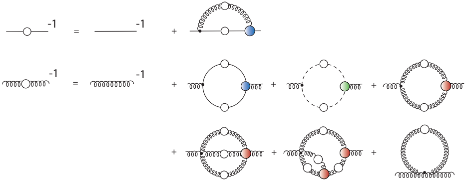
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



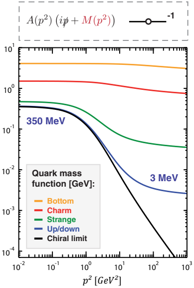
DSEs = quantum equations of motion:

derived from path integral, relate n-point functions



Quark propagator:

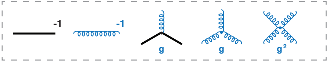
DCSB generates 'constituent-quark masses'



QCD

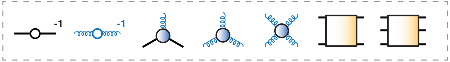
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m \right) \psi + \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

$=$ 

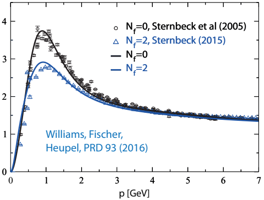
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A] \, e^{-S} = e^{-\Gamma}$$



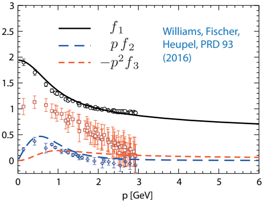
- **Gluon propagator**

$$\frac{D(p^2)}{p^2} \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)$$ 



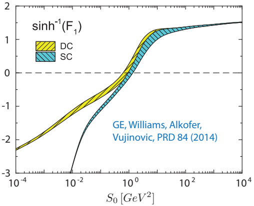
- **Quark-gluon vertex**

$$f_1 \gamma^\mu + f_2 \, i p^\mu + f_3 \, p^\mu p\!\!\!/ + \dots$$ 



- **Three-gluon vertex**

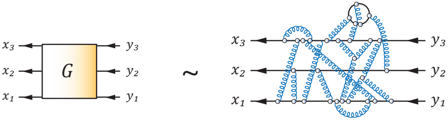
$$F_1 \left[\delta^{\mu\nu} (p_1 - p_2)^\rho + \delta^{\nu\rho} (p_2 - p_3)^\mu + \delta^{\rho\mu} (p_3 - p_1)^\nu \right] + \dots$$ 



Agreement between lattice, DSE & FRG within reach!

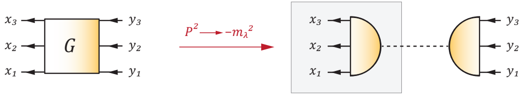
Hadrons?

- Simplest n-point function that encodes information on **baryons:** quark 6-point correlator
  $\langle\, \psi_\alpha(x_1)\, \psi_\beta(x_2)\, \psi_\gamma(x_3)\, \bar{\psi}_\rho(y_1)\, \bar{\psi}_\sigma(y_2)\, \bar{\psi}_\tau(y_3)\, \rangle$



Hadrons?

- Simplest n-point function that encodes information on **baryons:** quark 6-point correlator
  $\langle\, \psi_\alpha(x_1)\, \psi_\beta(x_2)\, \psi_\gamma(x_3)\, \bar\psi_\rho(y_1)\, \bar\psi_\sigma(y_2)\, \bar\psi_\tau(y_3)\, \rangle$



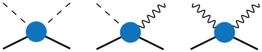
⇒ extract **gauge-invariant** baryon poles from gauge-fixed 6-quark function

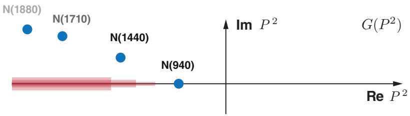
Bethe-Salpeter wave function:
residue at pole, contains all information about baryon

- Spectral decomposition:

$$\sum_\lambda |\lambda\rangle\langle\lambda| \quad \to \quad \sum_\lambda \frac{\cdots}{P^2+m_i^2}$$

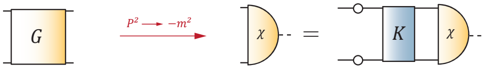
⇒ Same singularity structure as in



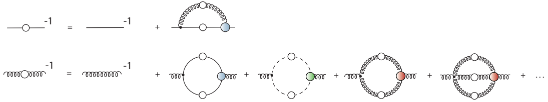


DSEs & BSEs

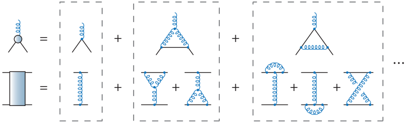
- Homogeneous **Bethe-Salpeter equation** for BS wave function:



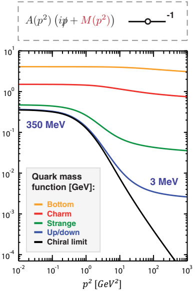
- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**



- Kernel can be derived in accordance with **chiral symmetry:**

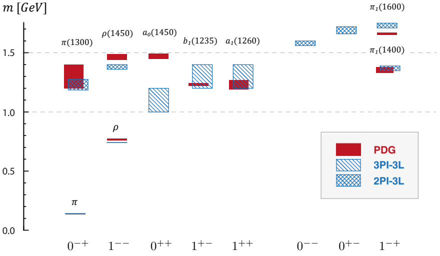


- **Quark propagator**



Dynamical chiral symmetry breaking generates 'constituent-quark masses'

DSEs & BSEs

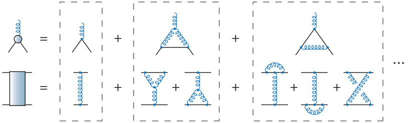


Light meson spectrum
beyond rainbow-ladder:

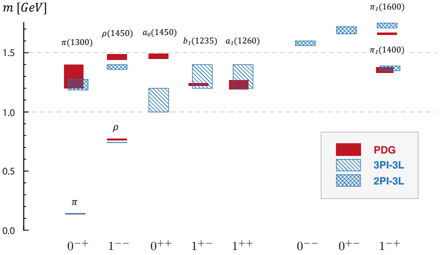
Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- Kernel can be derived in accordance with **chiral symmetry:**



DSEs & BSEs

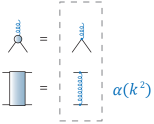


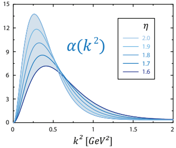
Light meson spectrum
beyond rainbow-ladder:

Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- Kernel can be derived in accordance with **chiral symmetry:**





Rainbow-ladder:
effective gluon exchange

$$\alpha(k^2) = \alpha_{\text{IR}}\left(\tfrac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\text{UV}}(k^2)$$

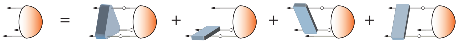
adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

Baryons

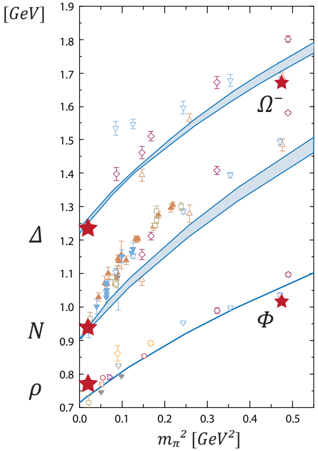
Covariant Faddeev equation for baryons:

GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

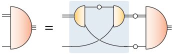
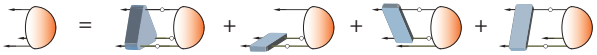


- 3-gluon diagram vanishes ⇒ **3-body effects small?**
- 2-body kernels same as for mesons,
  no further approximations: $M_N = 0.94\,\text{GeV}$
- **Relativistic bound states** carry OAM:
  64 (128) tensors for nucleon (Δ)
- Octet & decuplet baryons, pion cloud effects,
  first steps beyond rainbow-ladder
- **Baryon form factors:**
  nucleon and Δ FFs, $N\to\Delta\gamma$ transition, ...

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

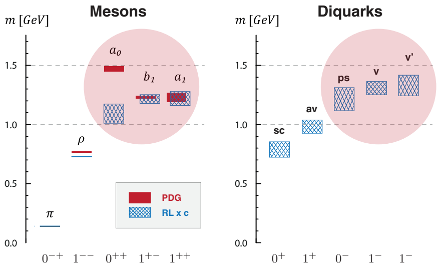


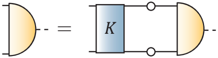
DSE / Faddeev landscape $N \to N^*\gamma$

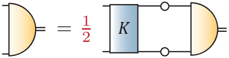
	Quark-diquark 			Three-quark 		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	…
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	√		
Roper	√	√	√	√	…	
$N \to N^*\gamma$	√	√	…	…		
$N^*(1535), \dots$	…	…	√	√	…	
$N \to N^*\gamma$	…	…	…	…		
	Roberts, Bashir, Segovia, Chen, Wilson, Lu, …	Oettel, Alkofer, Roberts, Cloet, Segovia, …	GE, Alkofer, Nicmorus, …	GE, Sanchis-Alepuz, Fischer, Alkofer, Williams, …		

The role of diquarks

Mesons and 'diquarks' closely related:
after taking traces, only factor 1/2 remains
⇒ **diquarks 'less bound' than mesons**







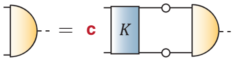
Pseudoscalar & vector mesons ⇔ **Scalar & axialvector diquarks**
already good in rainbow-ladder — sufficient for nucleon and Δ

Scalar & axialvector mesons ⇔ **Pseudoscalar & vector diquarks**
too light, repulsion beyond RL — important for remaining channels

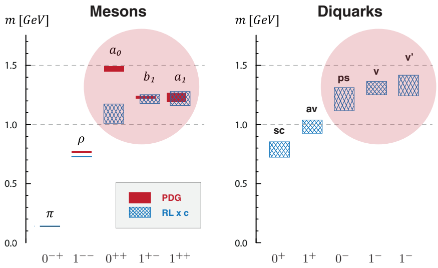
The role of diquarks

Simulate **beyond-RL effects:**

Insert factor **0 < c < 1** in 'bad' meson and diquark channels ⇒ increases masses, adjusted in meson sector (ρ–a_1 splitting)

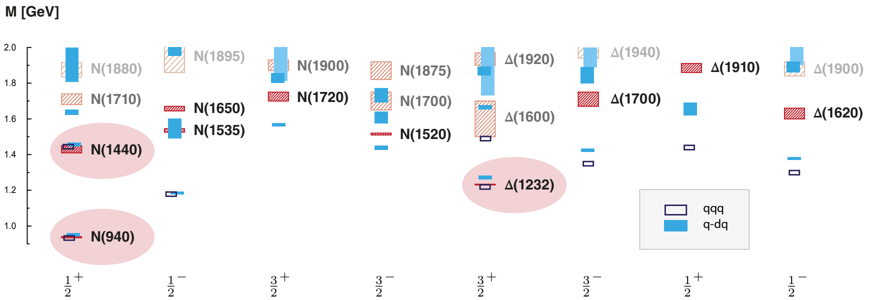


⇒ **reduces strength of ps + v diquarks**



Baryon spectrum I

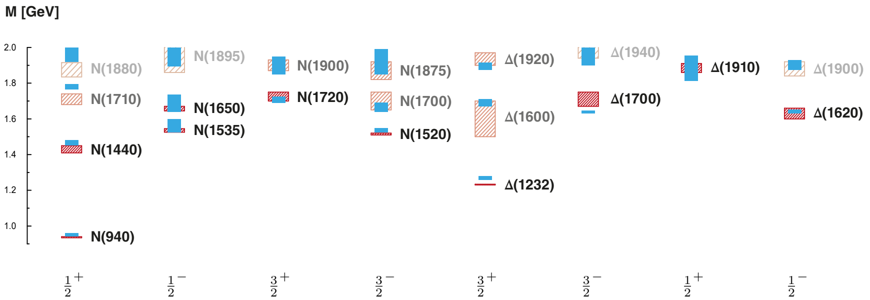
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



- **qqq** and **q-dq** agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

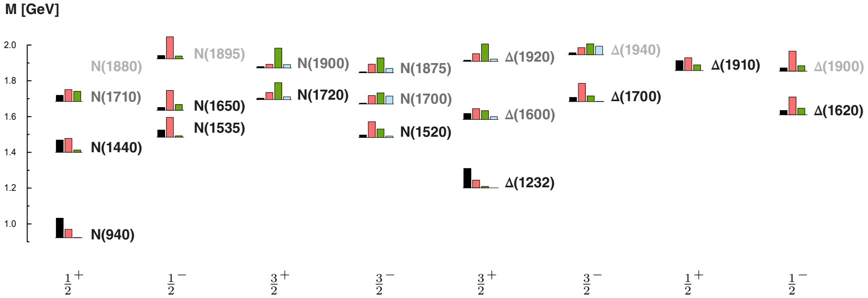


- **Quantitative agreement with experiment**
- N($\frac{1}{2}^+$) and Δ($\frac{3}{2}^+$) depend on sc + av diquarks; remaining ones "polluted" by ps + v diquarks
- Correct level ordering between **Roper and N(1535)**

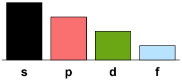
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, FBS 58 (2017)

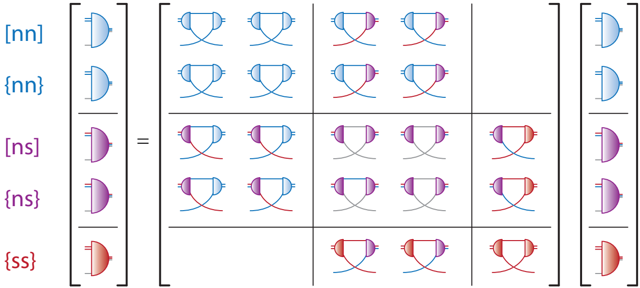


Orbital angular momentum content:

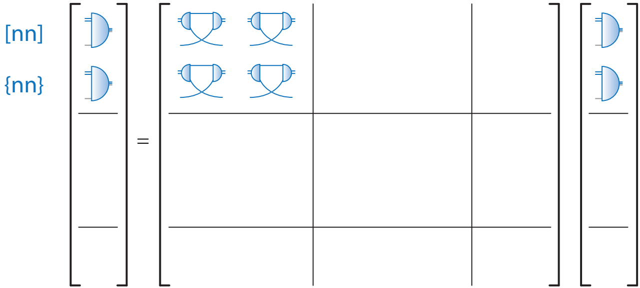


- in nonrelativistic quark model:
  N, Δ ~ **s waves,** negative-parity states ~ **p waves,** etc.
- Here: 'quark-model forbidden' contributions are always present, e.g. **Roper: dominated by p waves ⇒ relativity is important!**

Strange baryons

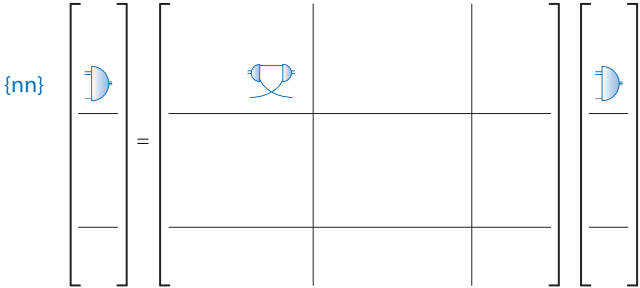


Strange baryons



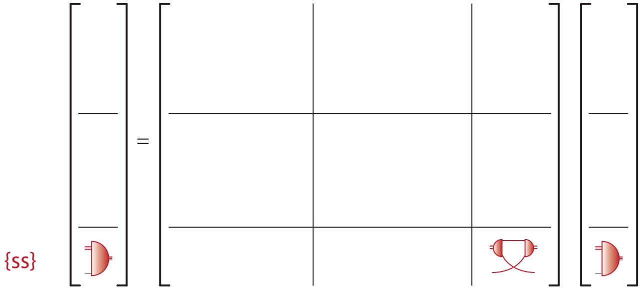
Nucleon

Strange baryons



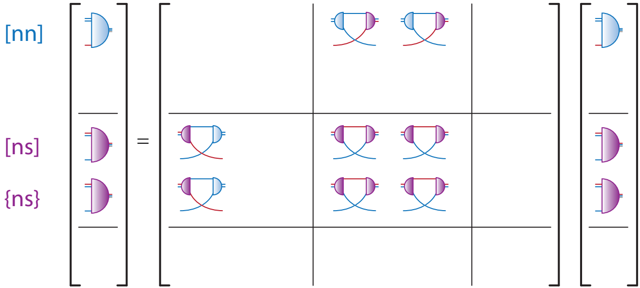
Delta

Strange baryons



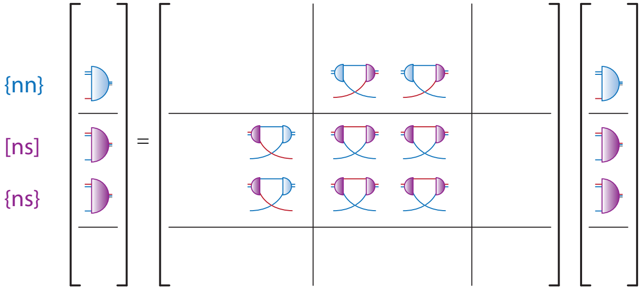
Omega

Strange baryons



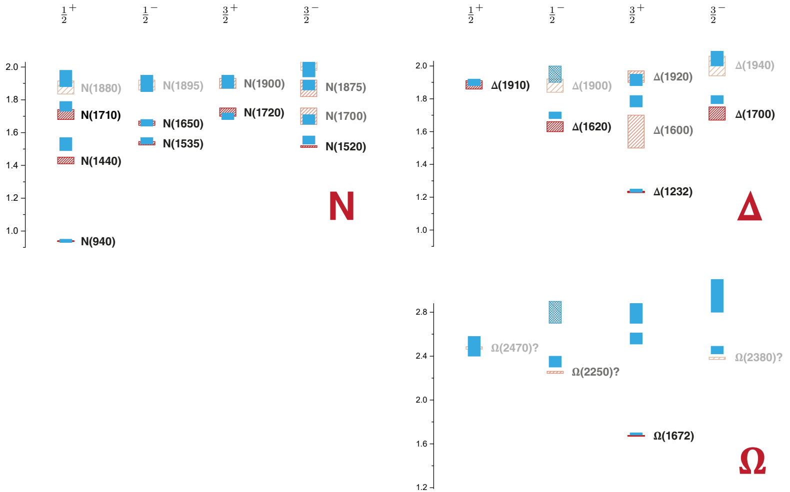
Lambda

Strange baryons

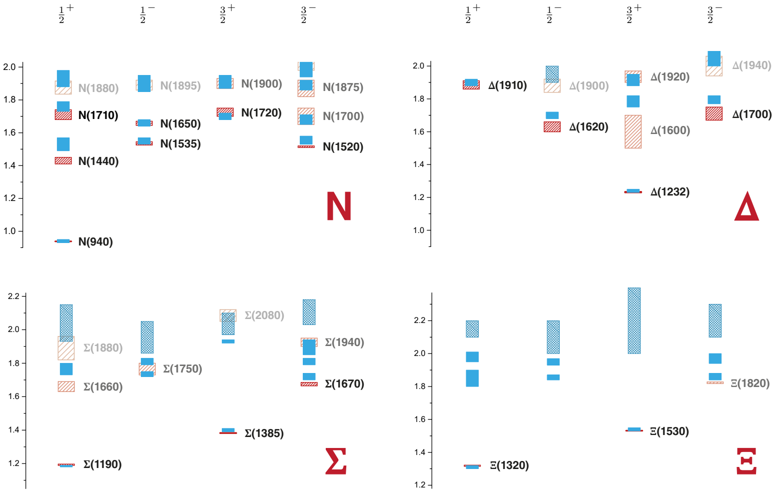


Sigma

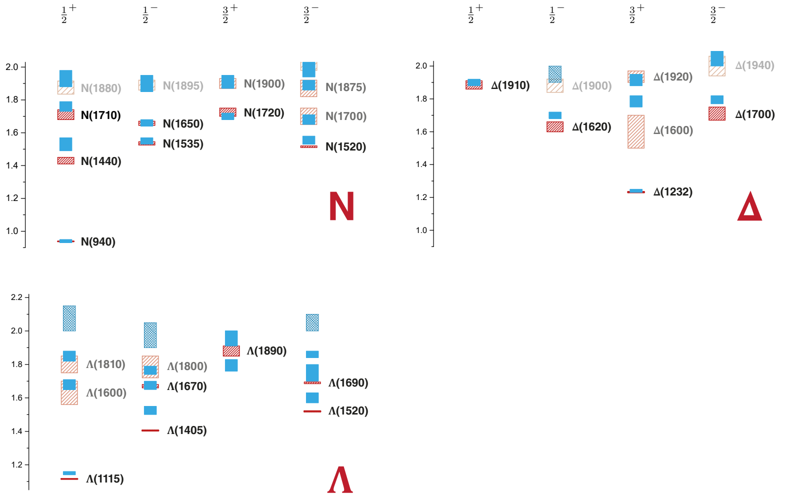
Strange baryons



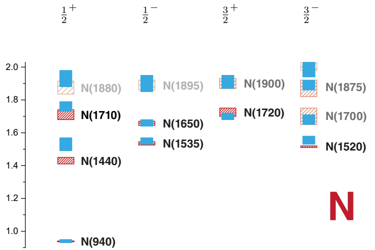
Strange baryons

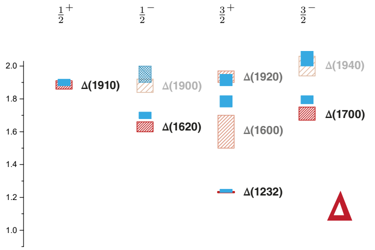


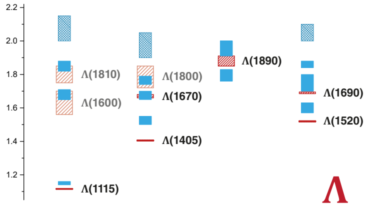
Strange baryons



Strange baryons







- Strange baryons similar to **light baryons:**

  $\boldsymbol{\Omega} \rightarrow \boldsymbol{\Delta}$

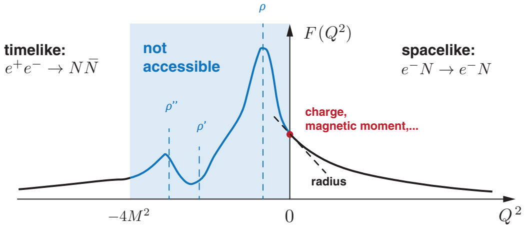
  $\boldsymbol{\Sigma}, \boldsymbol{\Xi} \rightarrow \mathbf{N} + \boldsymbol{\Delta}$ → rich spectrum!

  $\boldsymbol{\Lambda} \rightarrow \mathbf{N}$ + singlets

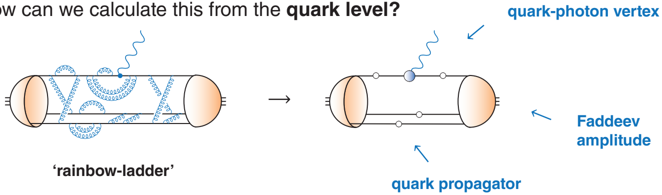
- Roper, **Δ**(1600), **Λ**(1405), **Λ**(1520): levels are there, but additional dynamics?
- **Structure information?** OAM, decays, form factors!

Form factors

Sketch of a generic electromagnetic form factor:

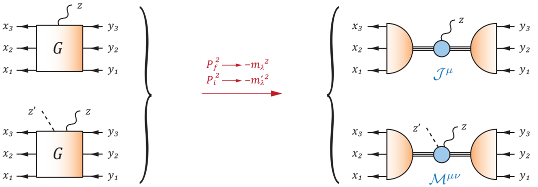


How can we calculate this from the **quark level?**



Matrix elements

Insert **spectral decomposition** in $\langle \cdots \psi(x_1) \cdots \bar{\psi}(y_1) \cdots j^\mu(z) \cdots \rangle$



"Gauging of equations":
Kvinikhidze, Blankleider, PRC 60 (1999)

GE, Fischer, PRD 85 (2012)
GE, Fischer, PRD 87 (2013)

Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

Relate **G** to elementary propagators, vertices and kernels:

$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

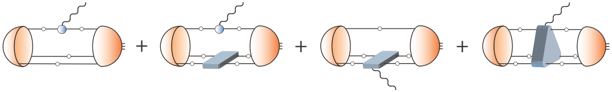
$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Current matrix element:

- impulse approximation + coupling to kernels
- **gauge invariance** is automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel

$\mathcal{J}^\mu$ =



Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

Relate **G** to elementary propagators, vertices and kernels:

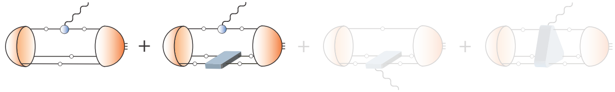
$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Current matrix element:

- impulse approximation + coupling to kernels
- **gauge invariance** is automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel

$\mathcal{J}^\mu$ = 

Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

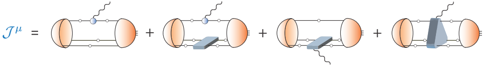
Relate **G** to elementary propagators, vertices and kernels:

$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Form factors

Nucleon em. form factors
from three-quark equation
GE, PRD 84 (2011)

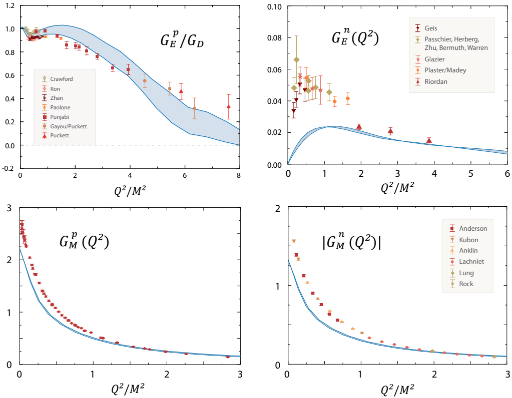


- **Timelike vector-meson poles** generated in quark-photon vertex
- **"Quark core without pion-cloud"**
- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

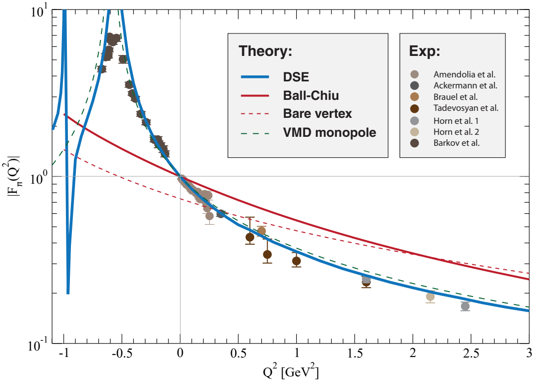
Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

- $\pi \to \gamma\gamma^*$ transition: vm. poles modify asymptotic scaling!

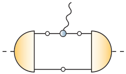
GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]



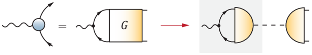
Pion form factor



A. Krassnigg (Schladming 2010),
Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

- Form factor from 

- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!



$\Rightarrow \Gamma^\mu =$ **Ball-Chiu** (em. gauge invariance)

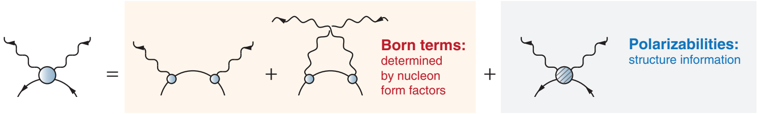
$+$ **Transverse part** (vm. poles & dominance)

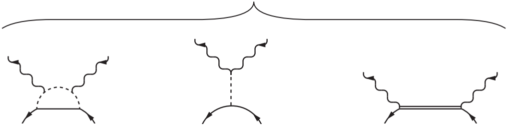
- Form factor at large Q^2
  Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)

- Include **pion cloud** effects:
  GE, Fischer, Kubrak, Williams, in preparation

Compton scattering ...

Compton amplitude = sum of **Born terms + 1PI structure part:**

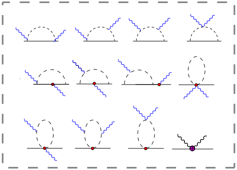




„Pion cloud“
(ChPT)

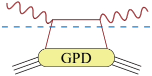
t-channel
meson exchange
$(\pi, \sigma, a_1, \ldots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \ldots)$



Griesshammer, McGovern, Phillips, Feldman,
Prog. Part. Nucl. Phys. 67 (2012)

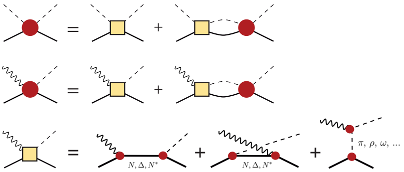
but also:



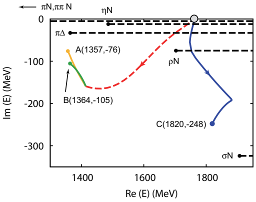
$\Rightarrow$ is there a common underlying
quark-level description?

Extracting resonances

Hadronic coupled-channel equations:



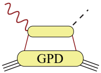
Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI, JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina, JPAC, . . .



Suzuki et al., PRL 104 (2010)

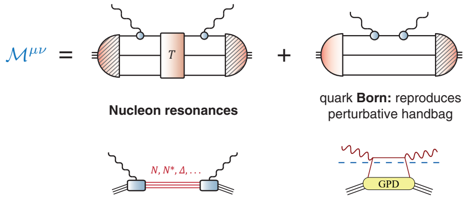
Microscopic effects?

What is an "offshell hadron"?



Matrix elements

Scattering amplitude:



Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

Relate **G** to elementary propagators, vertices and kernels:

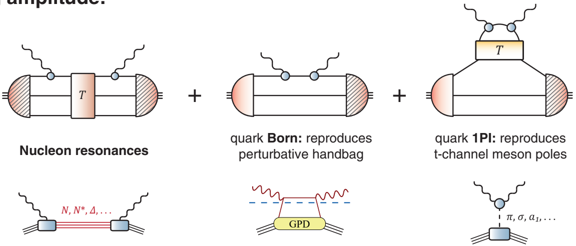
$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Scattering amplitude:

$\mathcal{M}^{\mu\nu} =$



Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

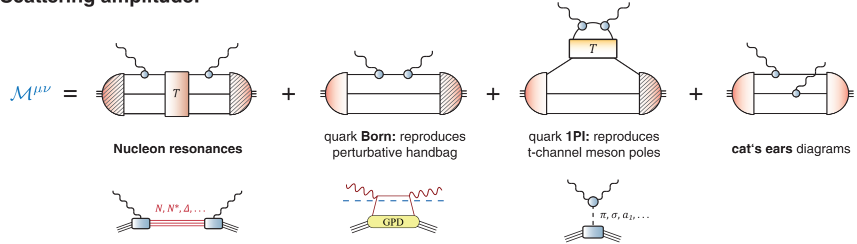
Relate **G** to elementary propagators, vertices and kernels:

$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Scattering amplitude:



Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

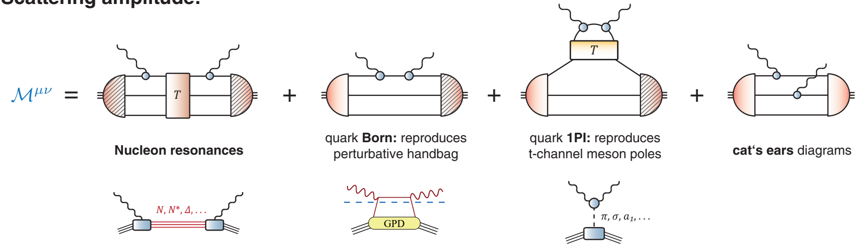
Relate **G** to elementary propagators, vertices and kernels:

$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Scattering amplitude:



Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \, (\mathbf{G}^{-1})^\mu \, \mathbf{\Psi}_i \qquad\qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[(\mathbf{G}^{-1})^{\{\mu} \, \mathbf{G} \, (\mathbf{G}^{-1})^{\nu\}} - (\mathbf{G}^{-1})^{\mu\nu} \right] \mathbf{\Psi}_i$$

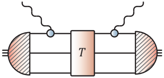
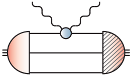
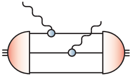
Relate **G** to elementary propagators, vertices and kernels:

$$(\mathbf{G}^{-1})^\mu = (\mathbf{G_0}^{-1})^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$(\mathbf{G}^{-1})^{\mu\nu} = (\mathbf{G_0}^{-1})^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

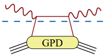
Scattering amplitude:

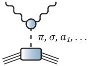
$\mathcal{M}^{\mu\nu}$ =  +  + 

Nucleon resonances

quark **Compton vertex:** reproduces
perturbative handbag & t-channel meson poles

cat‘s ears diagrams





Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

Relate **G** to elementary propagators, vertices and kernels:

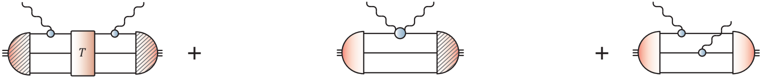
$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Scattering amplitude: GE, Fischer, PRD 85 (2012) & PRD 87 (2013)

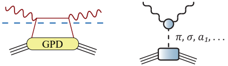
$\mathcal{M}^{\mu\nu} =$



Nucleon resonances

quark **Compton vertex:** reproduces perturbative handbag & t-channel meson poles

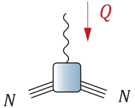
cat's ears diagrams



- **Poincaré covariance** and **crossing symmetry** are automatic
- **gauge invariance** and **chiral symmetry** are automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel
- **perturbative processes** are included
- **s, t, u channel poles** are generated dynamically, no need for "offshell hadrons"
- hadronic rescattering is implicit

Kinematics

Electromagnetic current:

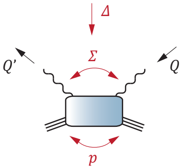


$$J^\mu(p,Q) = e\,\bar{u}(p_f)\,\Gamma^\mu(p,Q)\,u(p_i)$$

$$\Gamma^\mu = F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i}{4m}\,[\gamma^\mu, \not{Q}]$$

2 form factors (Dirac + Pauli),
1 kinematic variable Q^2

CS amplitude:



$$\mathcal{M}(p,Q,Q') = \frac{e^2}{m}\,\varepsilon^\mu(Q')\,\bar{u}(p_f)\,\Gamma^{\mu\nu}(p,Q,Q')\,u(p_i)\,\varepsilon^\nu(Q)$$

Tarrach, Nuovo Cim. A28 (1975),
GE, Ramalho, in preparation

18 Compton form factors (CFFs),
4 kinematic variables:

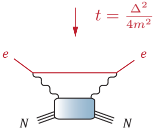
$$\eta_+ = \frac{Q^2+Q'^2}{2m^2}, \quad \eta_- = \frac{Q\cdot Q'}{m^2}, \quad \omega = \frac{Q^2-Q'^2}{2m^2}, \quad \lambda = -\frac{p\cdot Q}{m^2} = -\frac{p\cdot Q'}{m^2}$$

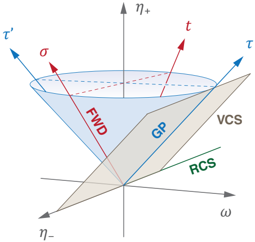
$$\Rightarrow \sum_{i=1}^{18} c_i(\eta_+,\eta_-,\omega,\lambda)\,\bar{u}(p_f)\,\tau_i^{\mu\nu}(p,Q,Q')\,u(p_i)$$

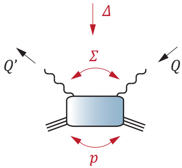
Kinematics

Kinematic domain for **two-photon exchange: cone around η_+**

$$t = \frac{\Delta^2}{4m^2} = \frac{\eta_+ - \eta_-}{2}, \quad \sigma = \frac{\Sigma^2}{m^2} = \frac{\eta_+ + \eta_-}{2}, \quad \tau = \frac{Q^2}{4m^2}, \quad \tau' = \frac{Q'^2}{4m^2}$$







$$\mathcal{M}(p,Q,Q') = \frac{e^2}{m}\, \varepsilon^\mu(Q')\, \underbrace{\bar{u}(p_f)\, \Gamma^{\mu\nu}(p,Q,Q')\, u(p_i)}\, \varepsilon^\nu(Q)$$

Tarrach, Nuovo Cim. A28 (1975),
GE, Ramalho, in preparation

18 Compton form factors (CFFs),
4 kinematic variables:

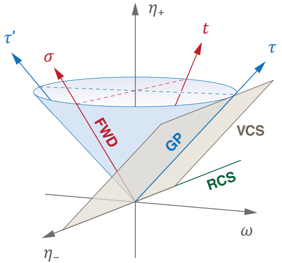
$$\eta_+ = \frac{Q^2 + Q'^2}{2m^2}, \quad \eta_- = \frac{Q \cdot Q'}{m^2}, \quad \omega = \frac{Q^2 - Q'^2}{2m^2}, \quad \lambda = -\frac{p\cdot Q}{m^2} = -\frac{p\cdot Q'}{m^2}$$

$$\Rightarrow \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda)\, \bar{u}(p_f)\, \tau_i^{\mu\nu}(p,Q,Q')\, u(p_i)$$

Forward CS

Forward limit: $\Delta^\mu = 0$ $\Rightarrow$ 2 variables: $\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}$, $\lambda = -\frac{p\cdot Q}{m^2}$, $\omega = 0$

$\Rightarrow$ 4 CFFs: $\bar{u}(p)\left(\frac{c_1}{m^4}\, t^{\mu\alpha}_{Qp}\, t^{\alpha\nu}_{pQ} + \frac{c_2}{m^2}\, t^{\mu\nu}_{QQ} + \frac{c_3}{m}\, i\varepsilon^{\mu\nu}_{Q\gamma} + \frac{c_4}{m^2}\,\lambda\left[t^{\mu\alpha}_{Q\gamma}, t^{\alpha\nu}_{\gamma Q}\right]\right) u(p)$



Forward CS

Forward limit: $\Delta^\mu = 0$ $\Rightarrow$ 2 variables: $\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}, \quad \lambda = -\frac{p\cdot Q}{m^2}, \quad \omega = 0$

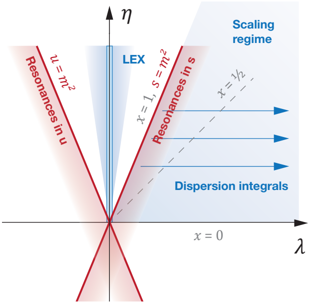
$\Rightarrow$ 4 CFFs: $\bar{u}(p)\left(\frac{c_1}{m^4}\, t^{\mu\alpha}_{Qp}\, t^{\alpha\nu}_{pQ} + \frac{c_2}{m^2}\, t^{\mu\nu}_{QQ} + \frac{c_3}{m}\, i\varepsilon^{\mu\nu}_{Q\gamma} + \frac{c_4}{m^2}\,\lambda\,[t^{\mu\alpha}_{Q\gamma}, t^{\alpha\nu}_{\gamma Q}]\right) u(p)$

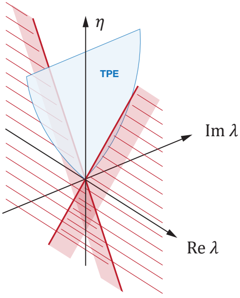
- Low-energy expansion:

$$c_1(\eta,\lambda) = c_1^{\text{Born}}(\eta,\lambda) + \alpha(\eta) + \beta(\eta) + \mathcal{O}(\lambda^2)$$
$$c_2(\eta,\lambda) = c_2^{\text{Born}}(\eta,\lambda) + \beta(\eta) + \mathcal{O}(\lambda^2)$$

- **Nucleon resonances** at $s, u > m^2$, $N\pi$ branch cuts for $s, u > (m+m_\pi)^2$
- **TPE region** → proton radius puzzle
- $\text{Im}\, c_i$ for $x = \eta/(2\lambda) \in [0,1]$ known from $N\gamma^* \to X$ cross section
- Use **dispersion relations** for rest:

$$c_i(\eta,\lambda) = \frac{1}{\pi}\int_{\lambda_{\text{el}}^2}^{\infty} d\lambda'^2\, \frac{\text{Im}\, c_i(\eta,\lambda')}{\lambda'^2 - \lambda^2 - i\epsilon}$$



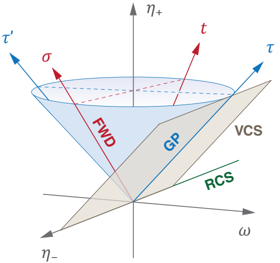


$\Rightarrow$ Baldin sum rule for $\alpha+\beta$, but β unconstrained (need subtracted DR)
$\Rightarrow$ ChPT + pQCD, but result much to small to explain discrepancy Birse, McGovern, EPJ A 48 (2012)

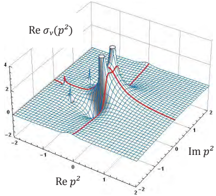
Forward CS

Forward limit: $\Delta^\mu = 0$ $\Rightarrow$ 2 variables: $\eta = \eta_+ = \eta_- = \frac{Q^2}{m^2}, \quad \lambda = -\frac{p\cdot Q}{m^2}, \quad \omega = 0$

$\Rightarrow$ 4 CFFs: $\bar{u}(p)\left(\frac{c_1}{m^4}\, t^{\mu\alpha}_{Qp}\, t^{\alpha\nu}_{pQ} + \frac{c_2}{m^2}\, t^{\mu\nu}_{QQ} + \frac{c_3}{m}\, i\varepsilon^{\mu\nu}_{Q\gamma} + \frac{c_4}{m^2}\,\lambda\left[t^{\mu\alpha}_{Q\gamma}, t^{\alpha\nu}_{\gamma Q}\right]\right) u(p)$

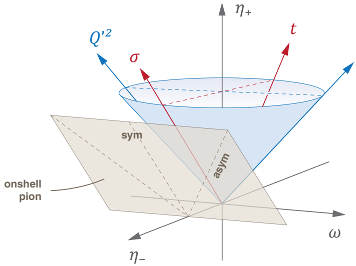


Singularity structure of quark propagator prevents **direct kinematic access** to all relevant regions ...



- if amplitudes free of kinematic singularities: **only phys. poles and cuts,** extrapolate from unphysical regions
- clean solution (expensive): **contour deformations**

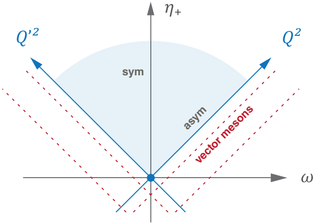
Pion transition form factor

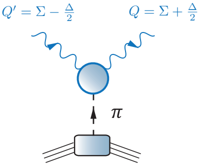


$$\eta_+ = \frac{Q^2+Q'^2}{2}$$

$$\omega = \frac{Q^2-Q'^2}{2}$$

$$\eta_- = Q\cdot Q'$$

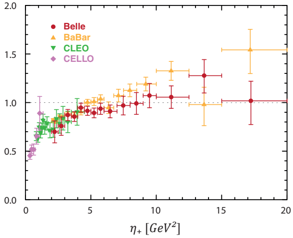




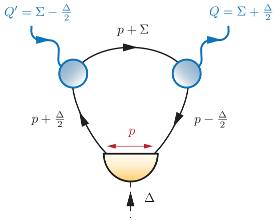
$$= e^2\,\frac{F(Q^2,Q'^2)}{4\pi^2 f_\pi}\,\varepsilon^{\mu\nu\alpha\beta}\,Q'^\alpha\,Q^\beta$$

- $F(0,0)=1$ in chiral limit
- $\dfrac{\eta_+ F(Q^2,Q'^2)}{4\pi^2 f_\pi^2} \xrightarrow{\eta_+\to\infty} \frac{2}{3}\dots 1(?)$

Lepage, Brodsky, PRD 22 (1980)



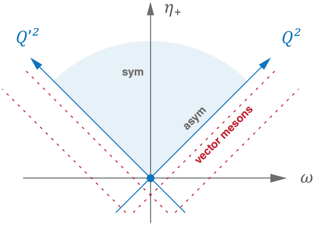
Pion transition form factor



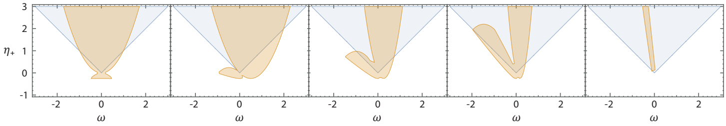
$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$

$$\omega = \frac{Q^2 - Q'^2}{2}$$

$$\eta_- = Q \cdot Q'$$



Quark singularities complicate matters: symmetric limit ok, but asymmetric limit only up to ~4 GeV^2 Maris, Tandy, PRC 65 (2002)

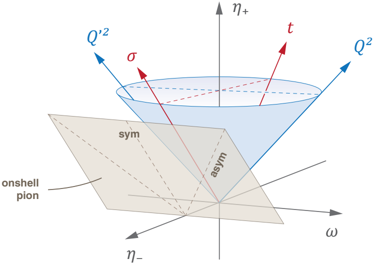


exploit Lorentz invariance to change frame

Weil, GE, Fischer, Williams, 1704.06046 [hep-ph]

Σ Breit frame ⟷ Δ rest frame

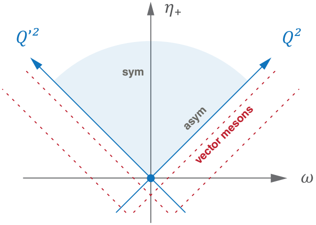
Pion transition form factor

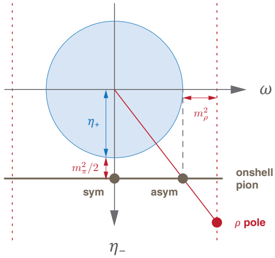


$$\eta_+ = \frac{Q^2+Q'^2}{2}$$

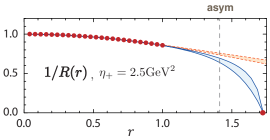
$$\omega = \frac{Q^2-Q'^2}{2}$$

$$\eta_- = Q \cdot Q'$$

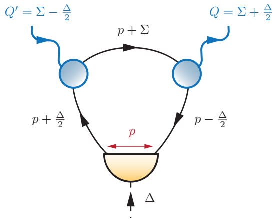




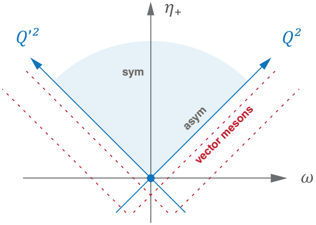
Idea:
- calculate FF inside cone
- interpolate to physical plane using VM pole as constraint
- can be done for arbitrary Q^2

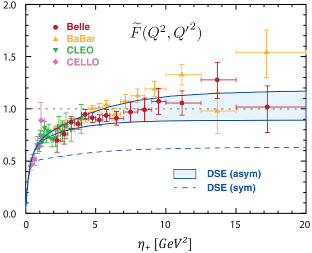


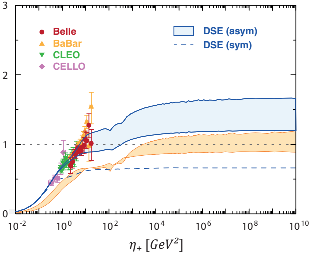
Pion transition form factor



$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$
$$\omega = \frac{Q^2 - Q'^2}{2}$$
$$\eta_- = Q \cdot Q'$$

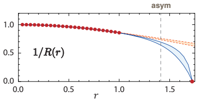




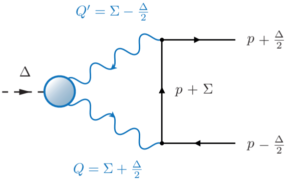


VM poles modify asymptotic scaling!

GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]



Rare pion decay $\pi^0 \to e^+e^-$



- After reanalysis of radiative corrections still 2σ discrepancy in branching ratio between exp and theory:

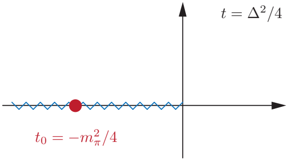
$6.87(36) \times 10^{-8}$ **KTeV Collab.:** Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)

$6.23(09) \times 10^{-8}$ Dorokhov, JETP Lett. 91 (2010), Masjuan, Sanchez-Puertas, 1504.07001

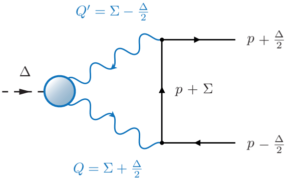
- Depends on **pion transition FF** as input: GE, Fischer, Weil, Williams, 1704.05774

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \, .$$

- cannot be calculated directly in Euclidean kinematics because of **photon and lepton poles**



Rare pion decay $\pi^0 \to e^+ e^-$



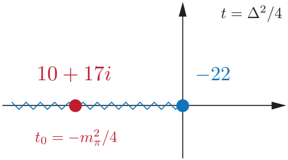
- After reanalysis of radiative corrections still 2σ discrepancy in branching ratio between exp and theory:

$6.87(36) \times 10^{-8}$ **KTeV Collab.:** Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)

$6.23(09) \times 10^{-8}$ Dorokhov, JETP Lett. 91 (2010), Masjuan, Sanchez-Puertas, 1504.07001

- Depends on **pion transition FF** as input: GE, Fischer, Weil, Williams, 1704.05774

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \,.$$

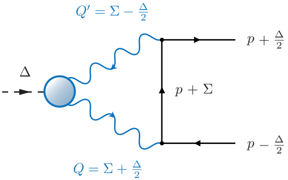


- cannot be calculated directly in Euclidean kinematics because of **photon and lepton poles**

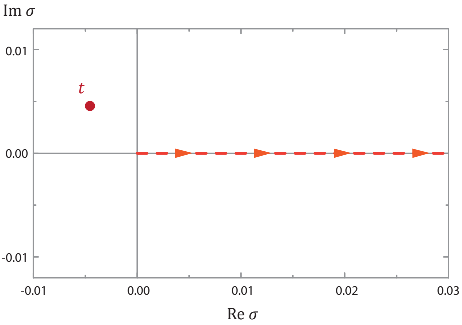
- workaround with dispersion relations:

$$\text{Im}\, \mathcal{A}^{\text{LO}}(t) = \frac{\pi \, \ln\gamma(t)}{2\beta(t)} \, F(0,0) \quad \Rightarrow \quad \text{Re}\, \mathcal{A}(t) = \mathcal{A}(0) + \frac{\ln^2\gamma(t) + \frac{1}{3}\pi^2 + 4\,\text{Li}_2(-\gamma(t))}{4\beta(t)}$$

Rare pion decay $\pi^0 \to e^+ e^-$



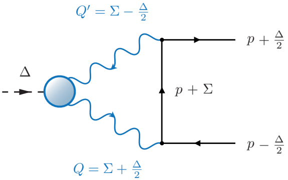
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \, .$$



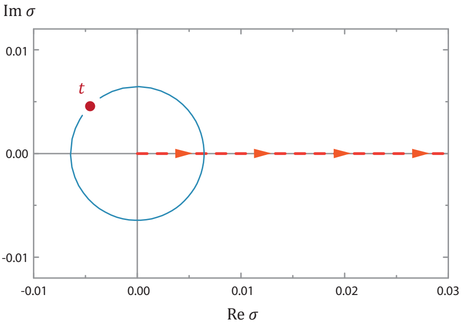
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$

Rare pion decay $\pi^0 \to e^+e^-$



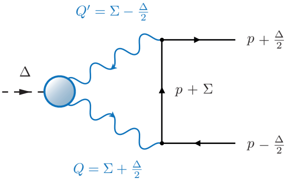
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \, .$$



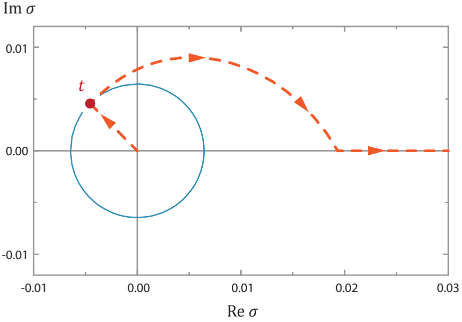
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut

Rare pion decay $\pi^0 \to e^+ e^-$



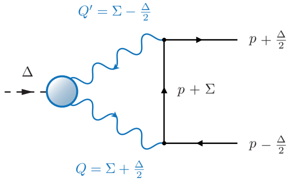
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \,.$$



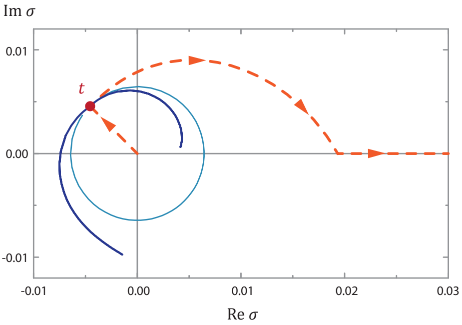
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at t

Rare pion decay $\pi^0 \to e^+e^-$



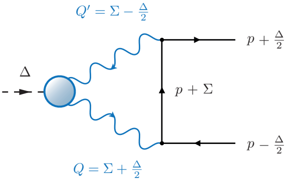
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \, .$$



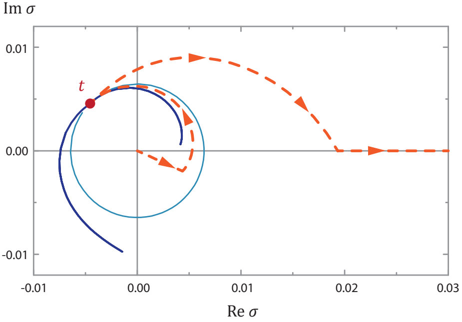
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at t
- but lepton cut does **not** open at t!

Rare pion decay $\pi^0 \to e^+ e^-$



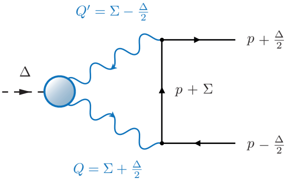
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \, .$$



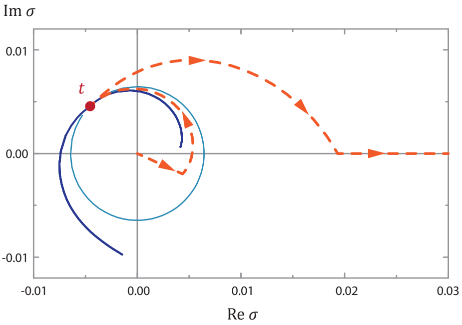
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

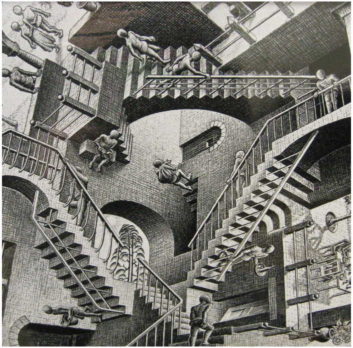
- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at t
- but lepton cut does **not** open at t!
- **deform contour** such that it never crosses any cut!

Rare pion decay $\pi^0 \to e^+e^-$

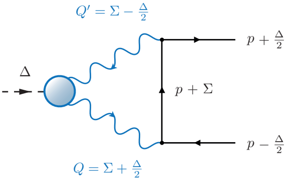


$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \, .$$

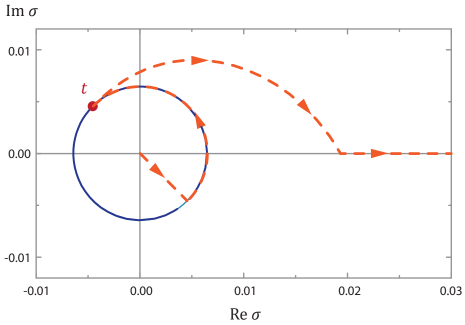




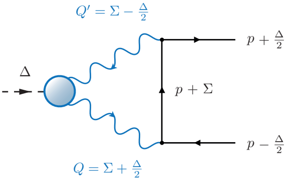
Rare pion decay $\pi^0 \to e^+ e^-$



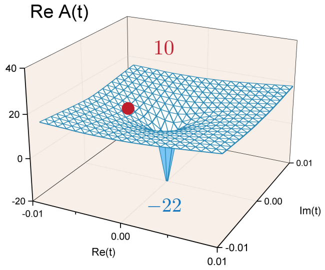
$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \, .$$

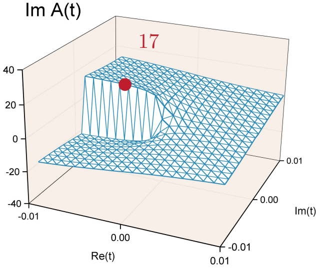


Rare pion decay $\pi^0 \to e^+e^-$



$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \,.$$



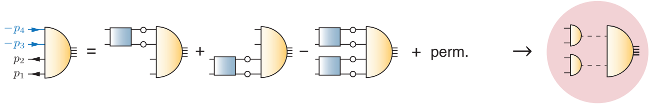


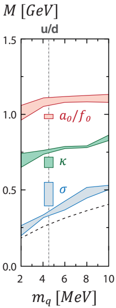
- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known
- Useful for treating **resonances!**

Weil, GE, Fischer, Williams, PRD 96 (2017)

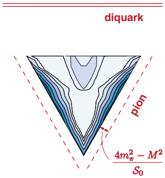
Tetraquarks

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)



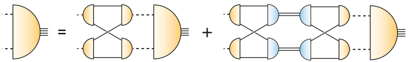


BSE dynamically generates **meson poles** in wave function:



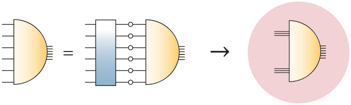
Four quarks rearrange to **"meson molecule"**

- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)



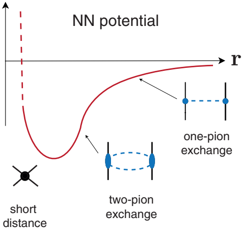
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

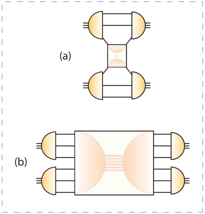


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



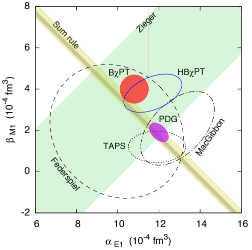
Weise, Nucl. Phys. A805 (2008)



- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Compton scattering

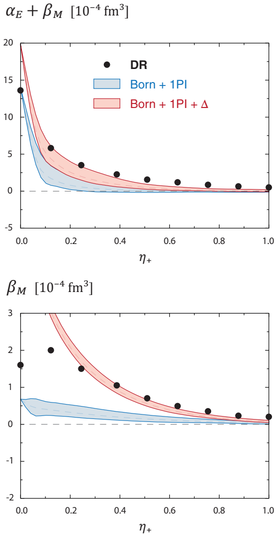
Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)



First DSE results:
GE, FBS 57 (2016)

- Quark Compton vertex (Born + 1PI) calculated, added Δ exchange
- compared to DRs
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E dominated by handbag, β_M by Δ contribution

⇒ **large "QCD background"!**

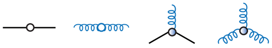


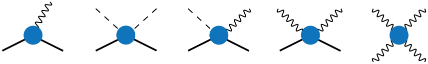
In total: polarizabilities ≈

Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly Δ)
+ pion cloud (at low η_+)?

Hadron physics with functional methods

Understand properties of **elementary n-point functions** ↔ Calculate hadronic **observables**: mass spectra, form factors, scattering amplitudes, . . .





- **QCD**
- **symmetries** intact (Poincare invariance & chiral symmetry important)
- access to all momentum scales & all quark masses
- compute mesons, baryons, tetraquarks, . . . **from same dynamics**

- **systematic** construction of truncations
- technical challenges: coupled integral equations, complex analysis, structure of 3-, 4-, ... point functions, **need lots of computational power!**

access to underlying nonperturbative dynamics!

Backup slides

QED

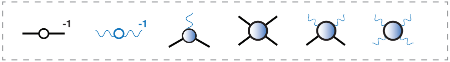
QED's classical action:

$$S = \int d^4x \left[\bar{\psi} \left(\partial\!\!\!/ + ig A\!\!\!/ + m \right) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A] \, e^{-S} = e^{-\Gamma}$$



QED

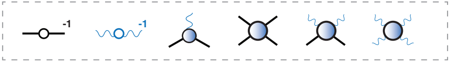
QED's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m \right) \psi + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

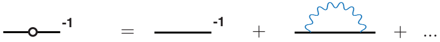
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$

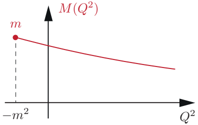


Perturbation theory: expand Green functions in powers of the coupling



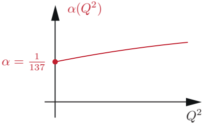
$A(p^2)\left(i p\!\!\!/ + M(p^2)\right)$ $\qquad$ $i p\!\!\!/ + m$

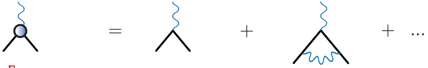
mass function



$D^{-1}(p^2)\left(p^2\,\delta^{\mu\nu} - p^\mu p^\nu\right)$ $\qquad$ $p^2\,\delta^{\mu\nu} - p^\mu p^\nu$

running coupling





$F_1 \gamma^\mu - \frac{F_2}{2m}\,\sigma^{\mu\nu} Q^\nu + \ldots$ $\qquad$ γ^μ

anomalous magnetic moment

$F_2(0) = \frac{\alpha}{2\pi}$

QED

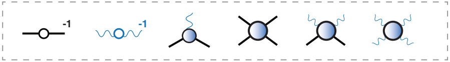
QED's classical action:

$$S \;=\; \int d^4x \left[\bar\psi \, (\partial\!\!\!/ + ig A\!\!\!/ + m) \, \psi + \tfrac{1}{4} \, F_{\mu\nu} \, F^{\mu\nu} \right]$$

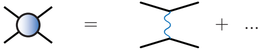
$=$

Quantum "effective action":

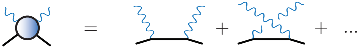
$$\int \mathcal{D}[\psi, \bar\psi, A] \, e^{-S} \;=\; e^{-\Gamma}$$



Perturbation theory: expand Green functions in powers of the coupling



Moller scattering



Compton scattering

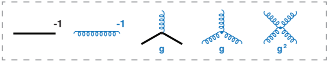
Light-by-light scattering

$\Longrightarrow$ **extremely precise theory predictions!**

QCD

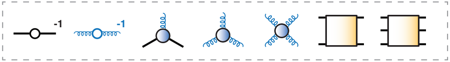
QCD's classical action:

$$S \;=\; \int d^4x\,\left[\bar{\psi}\,(\partial\!\!\!/ + ig A\!\!\!/ + m)\,\psi + \tfrac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a\right]$$

$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi,\bar{\psi},A]\,e^{-S} \;=\; e^{-\Gamma}$$

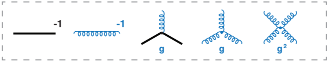


Perturbation theory: expand Green functions in powers of the coupling

QCD

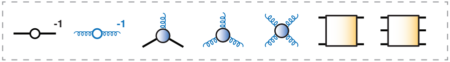
QCD's classical action:

$$S = \int d^4x\,\left[\bar\psi\,(\partial\!\!\!/ + igA\!\!\!/ + m)\,\psi + \tfrac{1}{4}\,F^{a}_{\mu\nu}\,F^{\mu\nu}_{a}\right]$$

$=$ 

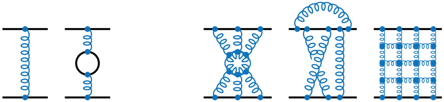
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} \;=\; e^{-\Gamma}$$



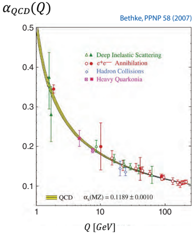
Perturbation theory: expand Green functions in powers of the coupling

But ... $\alpha(Q^2)$ becomes large at low momenta



dominant at **small distances**

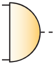
large distances: all these can contribute with same magnitude!



$\Longrightarrow$ need **non-perturbative methods!**

Bethe-Salpeter equations

- Example **pion:** quark-antiquark **bound state** ⇔ Goldstone boson of **DCSB**



$= \gamma_5 \left(f_1 + f_2 \not{P} + f_3\, q\cdot P \not{q} + f_4 [\not{q}, \not{P}] \right) \otimes$ Color $\otimes$ Flavor

most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

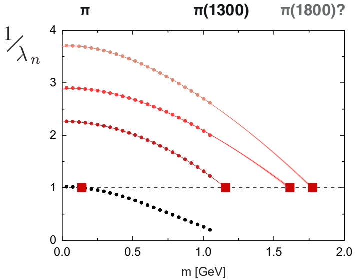
$f_i = f_i(q^2, q\cdot P, P^2 = -m^2)$ $\Longrightarrow$ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Homogeneous BSE becomes

$$f_i(q^2, z) = \int d^4q'\, K_{ij}(q^2, q'^2, z, z', q\cdot q')\, f_j(q'^2, z')$$

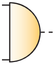
Eigenvalue spectrum of BS kernel:

$$K_{ij\,qq'\,zz'}\, f^{(n)}_{jq'z'} = \lambda_n(P^2)\, f^{(n)}_{iqz} \qquad \lambda_n \xrightarrow{P^2 \to -m_n^2} 1$$



Bethe-Salpeter equations

- Example **pion:** quark-antiquark **bound state** ⟺ Goldstone boson of **DCSB**



$$= \gamma_5 \left(f_1 + f_2 \not{P} + f_3\, q\cdot P\, \not{q} + f_4\, [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q\cdot P, P^2 = -m^2)$$

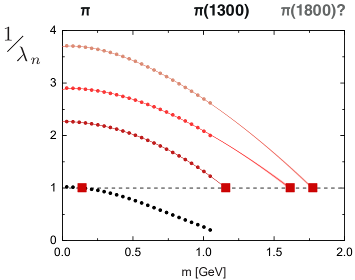
⟹ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Homogeneous BSE becomes

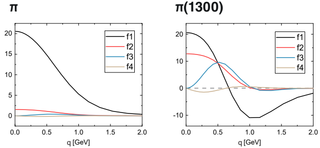
$$f_i(q^2, z) = \int d^4q'\, K_{ij}(q^2, q'^2, z, z', q\cdot q')\, f_j(q'^2, z')$$

Eigenvalue spectrum of BS kernel:

$$K_{ij\,qq'\,zz'}\, f^{(n)}_{jq'z'} = \lambda_n(P^2)\, f^{(n)}_{iqz} \qquad \lambda_n \xrightarrow{P^2 \to -m_n^2} 1$$

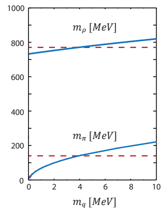


- Eigenvectors = BS amplitudes

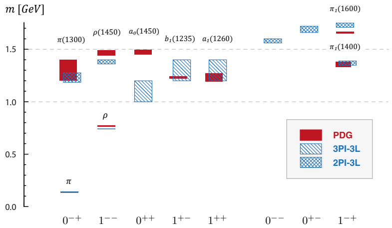


Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

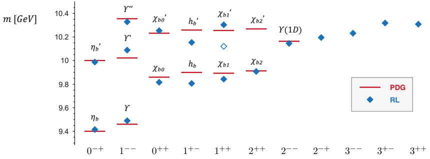


Williams, Fischer, Heupel, PRD 93 (2016)

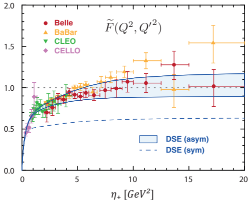
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Charmonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)

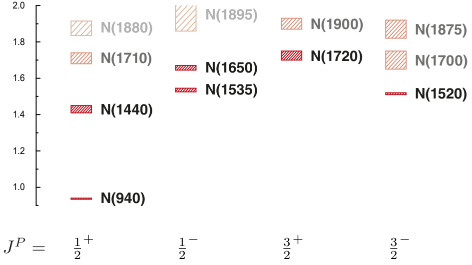


- **Pion transition form factor**

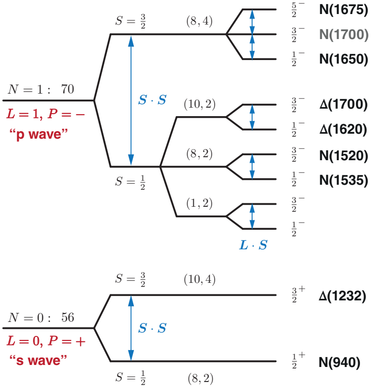


GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]

Light baryons

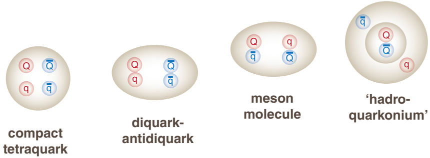


Nonrelativistic quark model:
$P = (-1)^L$

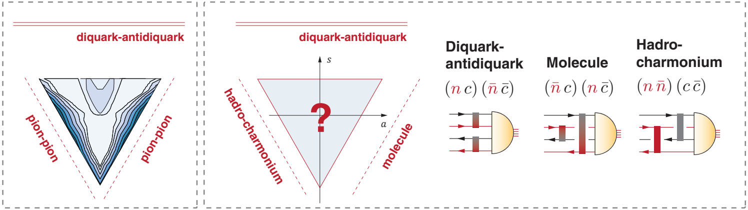


Tetraquarks in charm region?

- Can we **distinguish** different tetraquark configurations?

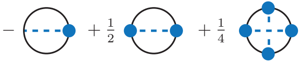


- **Four quarks** dynamically rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium; strengths determined by four-body BSE:



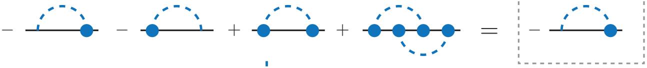
nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.

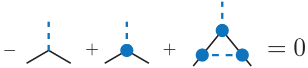
$\Gamma_2 =$


see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

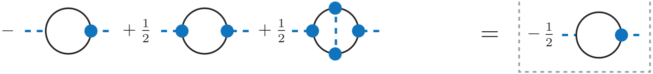
Self-energy:

$\Sigma = \frac{\delta \Gamma_2}{\delta D} =$


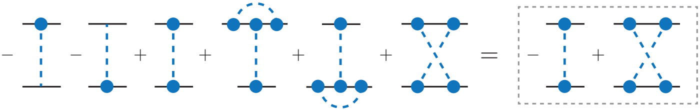
Vertex:

$\frac{\delta \Gamma_2}{\delta V} = 0 \quad \Rightarrow$


Vacuum polarization:

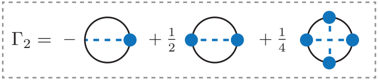
$\Sigma' = \frac{\delta \Gamma_2}{\delta D'} =$


BSE kernel:

$-K = \frac{\delta \Sigma}{\delta D} =$


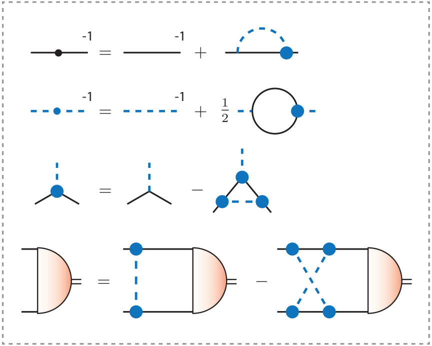
nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.



see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

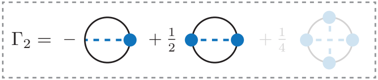
So we arrive at a closed system of equations:



- Crossed ladder
  cannot be added by hand,
  requires **vertex correction!**

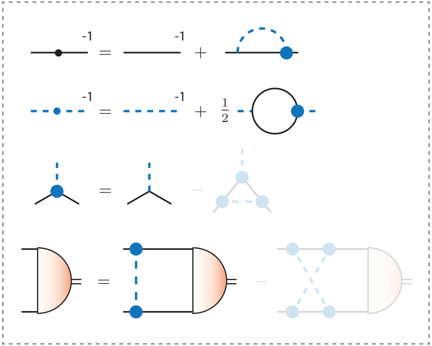
nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.



see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

So we arrive at a closed system of equations:

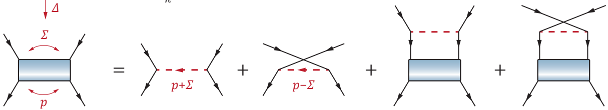


- Crossed ladder cannot be added by hand, requires **vertex correction!**
- without 3-loop term: **rainbow-ladder** with tree-level vertex ⇒ 2PI
- but still requires **DSE solutions** for propagators!
- Similar in QCD. nPI truncation guarantees chiral symmetry, massless pion in chiral limit, etc.

A toy model

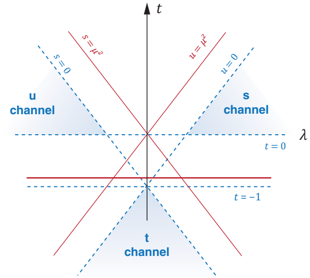
Scattering amplitude for two **massive scalar particles** (mass m) with **massive exchange particle** (mass μ):

$$T(p,\Sigma,\Delta) = K(p,\Sigma) + \int_k T(p,k,\Delta)\, D(k_+)\, D(k_-)\, K(k,\Sigma)$$



Onshell amplitude: Mandelstam plane

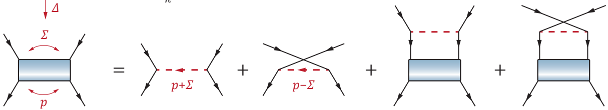
- $t = \frac{\Delta^2}{4m^2}, \quad \lambda = -\frac{p\cdot\Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel



A toy model

Scattering amplitude for two **massive scalar particles** (mass m) with **massive exchange particle** (mass μ):

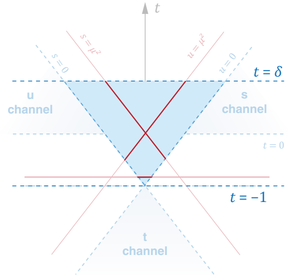
$$T(p,\Sigma,\Delta) = K(p,\Sigma) + \int_k T(p,k,\Delta)\, D(k_+)\, D(k_-)\, K(k,\Sigma)$$



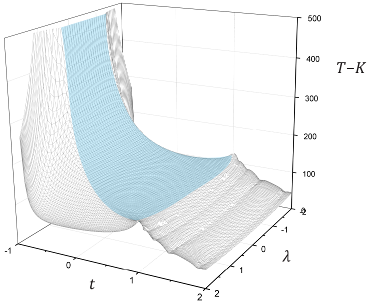
Onshell amplitude: Mandelstam plane

- $t = \frac{\Delta^2}{4m^2}, \quad \lambda = -\frac{p\cdot\Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
- **Poles** in propagators and exchange particle pose **restrictions:**

$$-1 < t < \delta, \quad |\lambda| < 1+t, \quad \delta = \frac{\mu^2}{m^2} - 1$$



A toy model

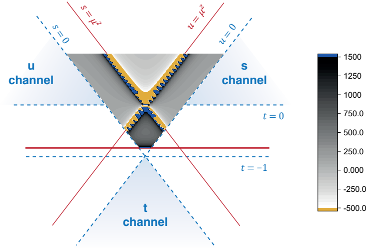


- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
- **Poles** in propagators and exchange particle pose **restrictions:**

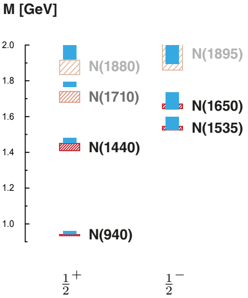
$$-1 < t < \delta, \quad |\lambda| < 1+t, \quad \delta = \frac{\mu^2}{m^2} - 1$$

Subtract Born terms to get rid of s- and u-channel poles (↔ 1PI part):

- rise is due to t-channel bound state
- outside blue region: naive integration over poles (wrong)
- scattering amplitude almost independent of λ!

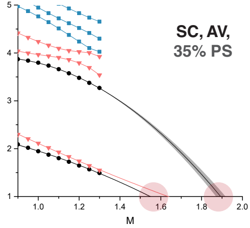


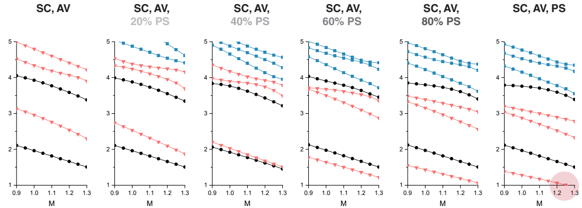
Baryon spectrum



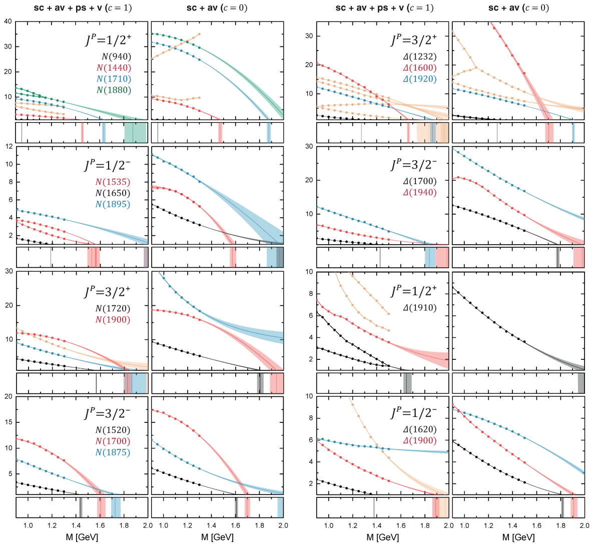
Level ordering between
Roper and N(1535):

dynamics of ps diquark produces
2 nearby states: **N(1535), N(1650)**





Eigenvalue spectra



GE, Fischer, Sanchis-Alepuz, 1607.05748

- $N(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ channels hardly affected by ps, v diquarks

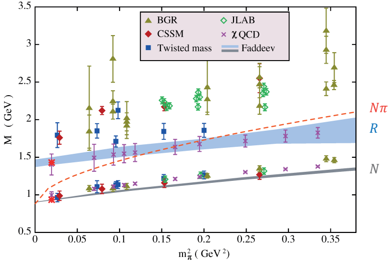
- all other channels:
  sc, av → masses too high
  sc, av, ps, v → masses too low

- not all eigenvalues extrapolate to masses below 2 GeV

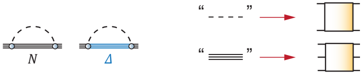
- some are complex conjugate (but imaginary parts small), some split into 2 real branches: numerical or truncation artifact?

Resonances

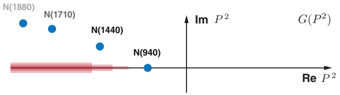
- **Current-mass evolution** of Roper:
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



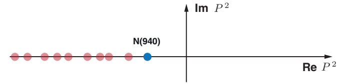
- **‘Pion cloud’** effects difficult to implement at **quark-gluon level:**

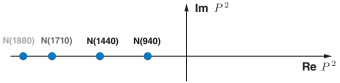


- Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



- **Lattice:** finite volume, **DSE** (so far): bound states



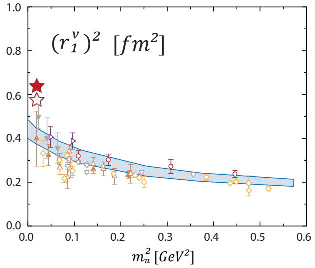


Resonance dynamics
shifts poles into complex plane,
but effects on real parts small?

Nucleon em. form factors

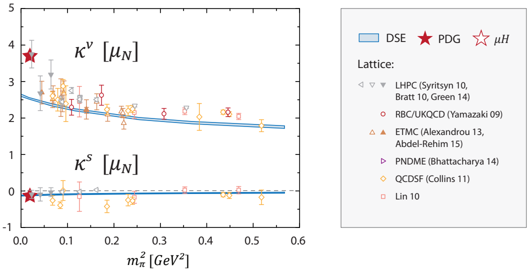
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

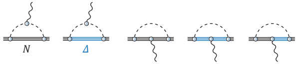


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.

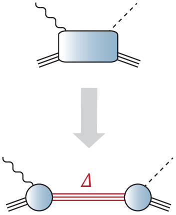


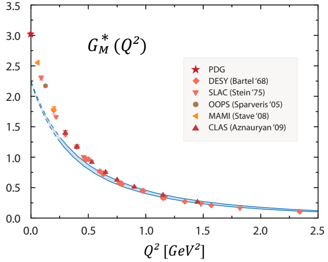
- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

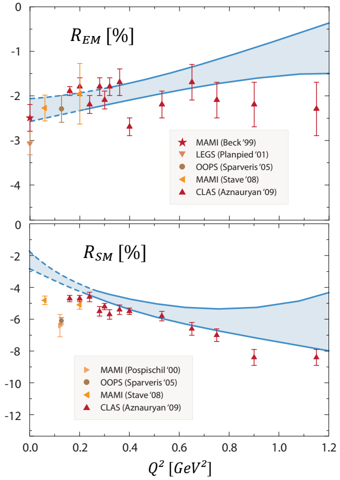
Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Nucleon-Δ-γ transition



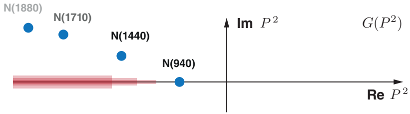




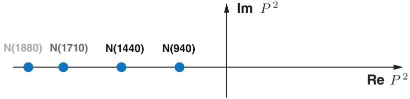
- **Magnetic dipole transition (G_M^*)** dominant:
  quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole ratios**
  small & negative, encode deformation.
  Reproduced without pion cloud: **OAM** from **p waves!**
  GE, Nicmorus, PRD 85 (2012)
- First three-body results similar
  Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)

Resonances?

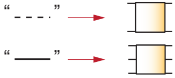
Branch cuts & widths generated by
meson-baryon interactions: Roper → $N\pi$, etc.



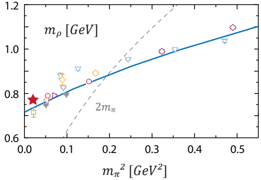
Without them: **bound states without widths**



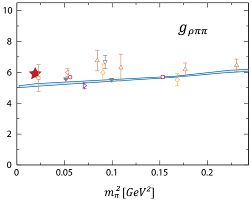
To generate resonances dynamically at **quark level:**
complicated topologies beyond rainbow-ladder



cf. ρ **meson:** bound state vs. resonance
below / above $\pi\pi$ threshold



resonance dynamics shifts pole into
complex plane, effect on real part small?



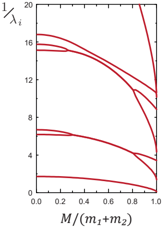
References:
see GE et al.,
PPNP 91 (2016)
1606.09602

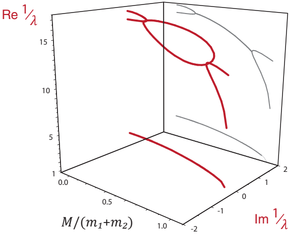
Complex eigenvalues?

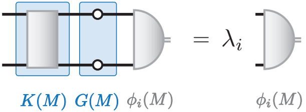
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\,L^\dagger L\,\phi_i = \lambda_i\,\phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

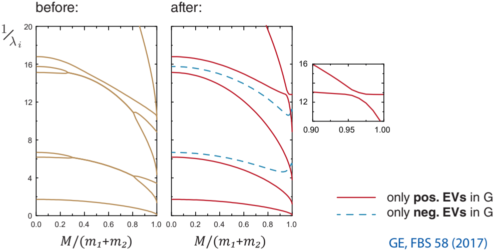
⇒ Hermitian problem with same EVs!

Complex eigenvalues?

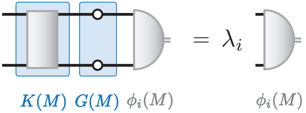
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)



GE, FBS 58 (2017)



K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\,L^\dagger L\,\phi_i = \lambda_i\,\phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

⇒ Hermitian problem with same EVs!

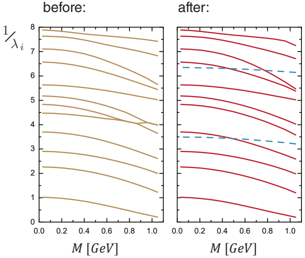
⇒ all EVs strictly **real**
⇒ level repulsion
⇒ “anomalous states” removed?

Complex eigenvalues?

Excited states: some EVs are complex conjugate?

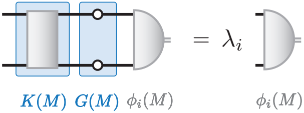
Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous**" **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)



Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)

only **pos. EVs** in G
only **neg. EVs** in G



K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

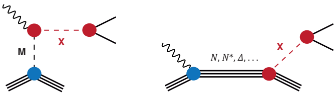
$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

⇒ Hermitian problem with same EVs!

⇒ all EVs strictly **real**
⇒ level repulsion
⇒ "anomalous states" removed?

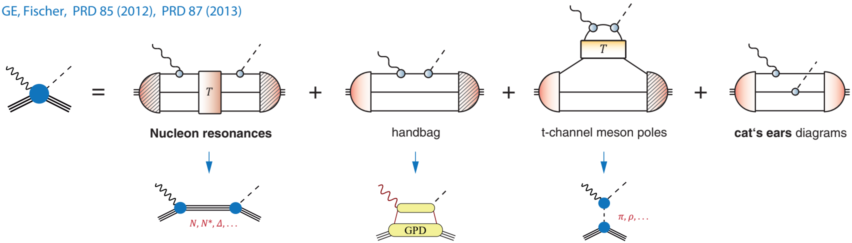
Extracting resonances

Photoproduction of **exotic mesons** at JLab/GlueX:



What if exotic mesons are **relativistic $q\bar{q}$ states?**
⇒ study with DSE/BSE!

Scattering amplitudes at quark-gluon level:
GE, Fischer, PRD 85 (2012), PRD 87 (2013)



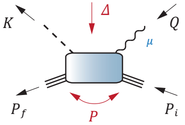
Meson electroproduction

3 independent variables ($\leftrightarrow$ s, t, u):

$$\tau = \frac{Q^2}{4m^2}\,, \qquad \eta = \frac{K\cdot Q}{m^2}\,, \qquad \lambda = -\frac{P\cdot Q}{m^2} = -\frac{P\cdot K}{m^2}$$

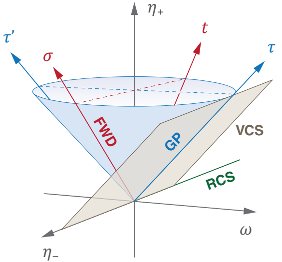
Amplitude depends on **6 Lorentz-invariant "FFs"**

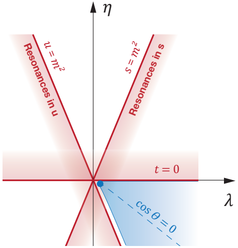
$$\mathcal{M}^\mu(P,K,Q) = \bar{u}(P_f)\left(\sum_{i=1}^{6} A_i(\tau,\eta,\lambda)\, M_i^\mu(P,K,Q)\right) u(P_i)$$



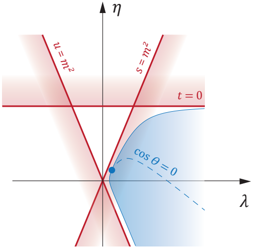
with appropriate tensor basis: no kinematic singularities
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

(in contrast to Dennery amplitudes)
Fubini, Nambu, Wataghin 1958, Dennery 1961



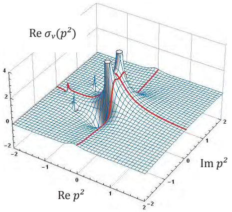


Photoproduction ($\tau = 0$)



Electroproduction ($\tau > 0$)

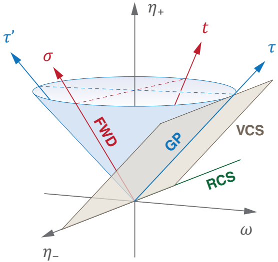
Meson electroproduction

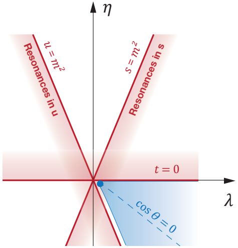


Singularity structure of quark propagator prevents **direct kinematic access** to all relevant regions

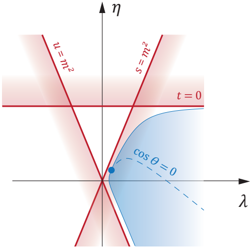
Strategies:

- if amplitudes free of kinematic singularities,
  only physical poles and cuts
  ⇒ extrapolate from unphysical regions (or offshell kinematics)
- clean solution (expensive): use **contour deformations**





Photoproduction $(\tau = 0)$



Electroproduction $(\tau > 0)$

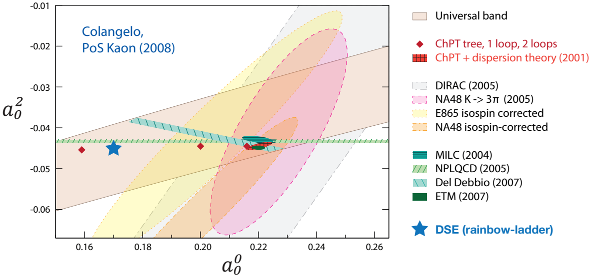
. . . and more

Scattering amplitudes from quark level:

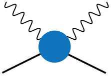
- ***ππ* scattering**

  Bicudo et al.,
  PRD 65 (2002),

  Cotanch, Maris,
  PRD 66 (2002)



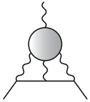
- **Nucleon Compton scattering**



  GE, Fischer, PRD 85 (2012) &
  PRD 87 (2013), GE, FBS 57 (2016)

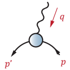
- **Hadronic light-by-light scattering**

  Goecke, Fischer, Williams, PLB 704 (2011),
  GE, Fischer, Heupel, PRD 92 (2015)



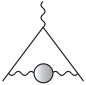
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

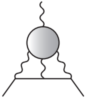


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



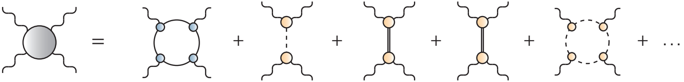
Hadronic light-by-light scattering

a_μ $[10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

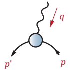
- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

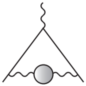
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

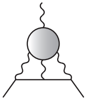


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



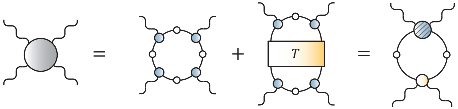
Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



quark Compton vertex

Born terms

- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)