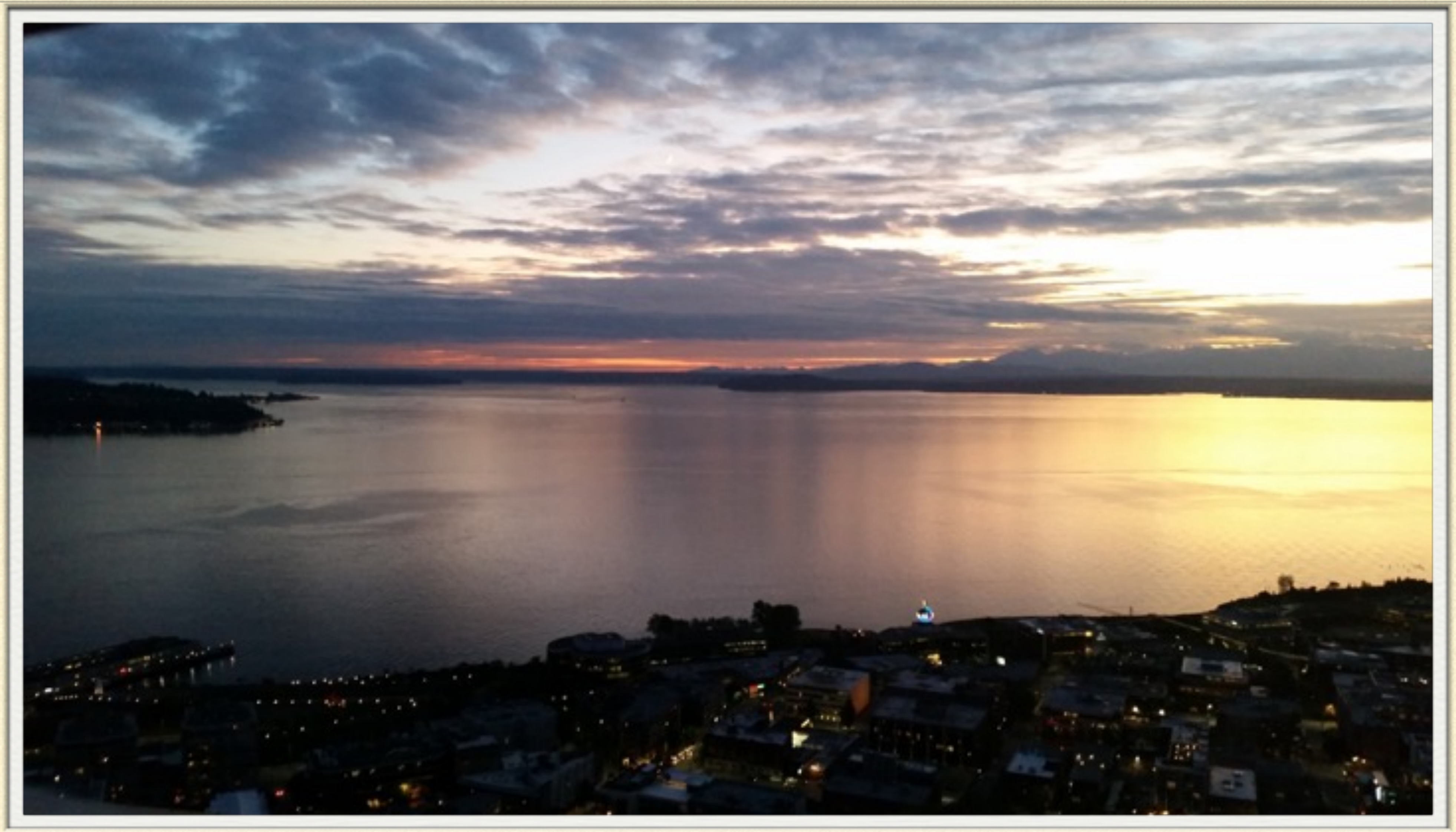


INT workshop 2017 - week 5

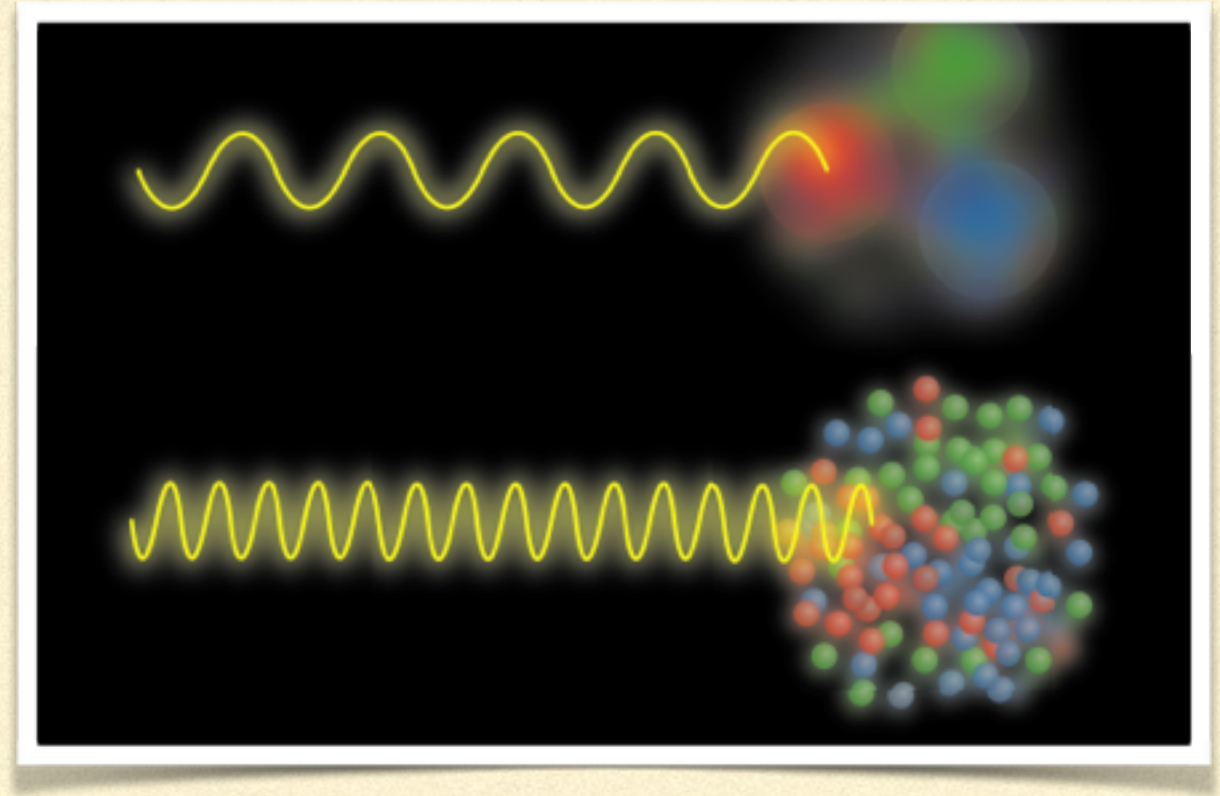
Seattle, 24-29 September 2017

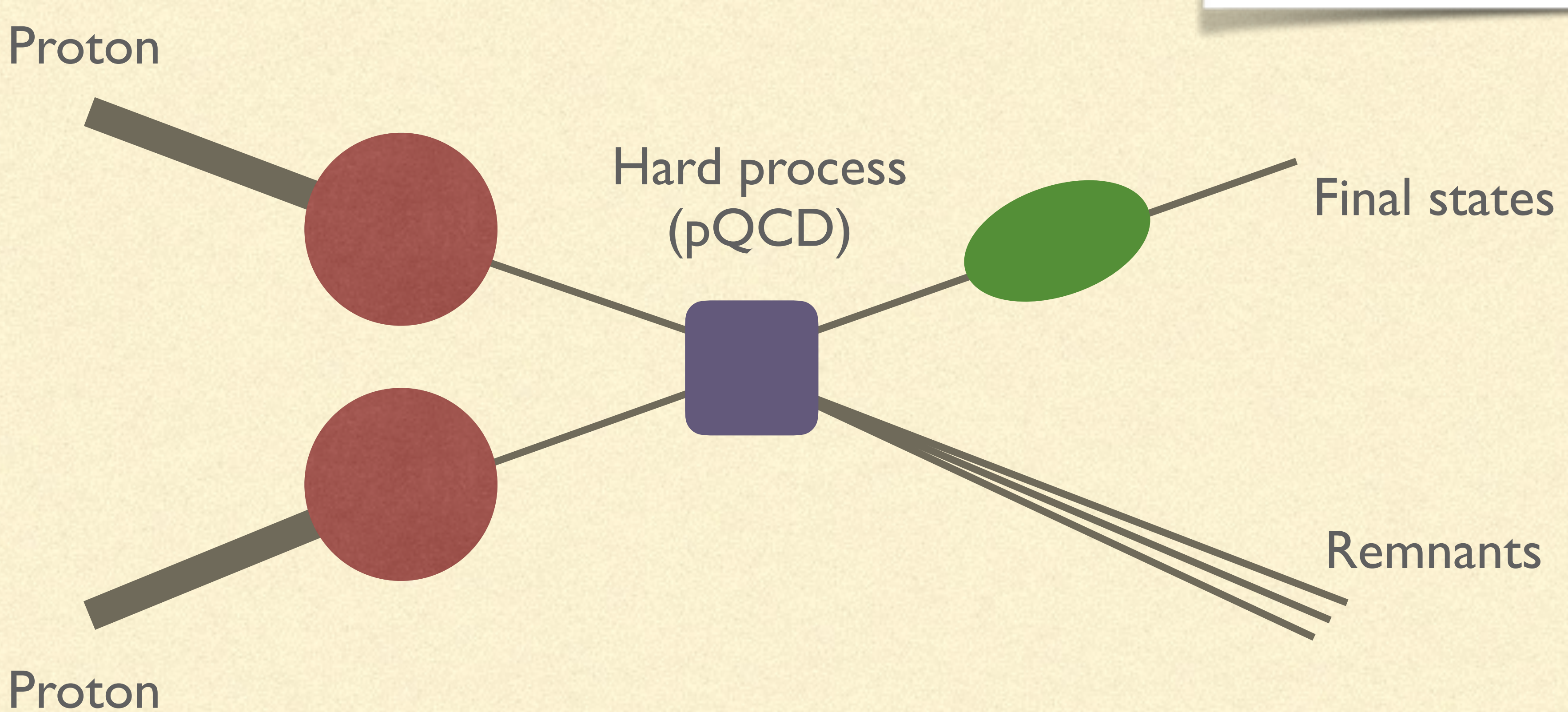
GLUON TMDs AND POSITIVITY BOUNDS FOR POLARIZED TARGETS

Sabrina Cotogno, T. van Daal, P. J. Mulders

arXiv:1709.07827

Probing the hadron structure





$$d\sigma \sim \text{TMD PDFs} \otimes \sigma_{\text{HARD}} \otimes (\text{final-states})$$

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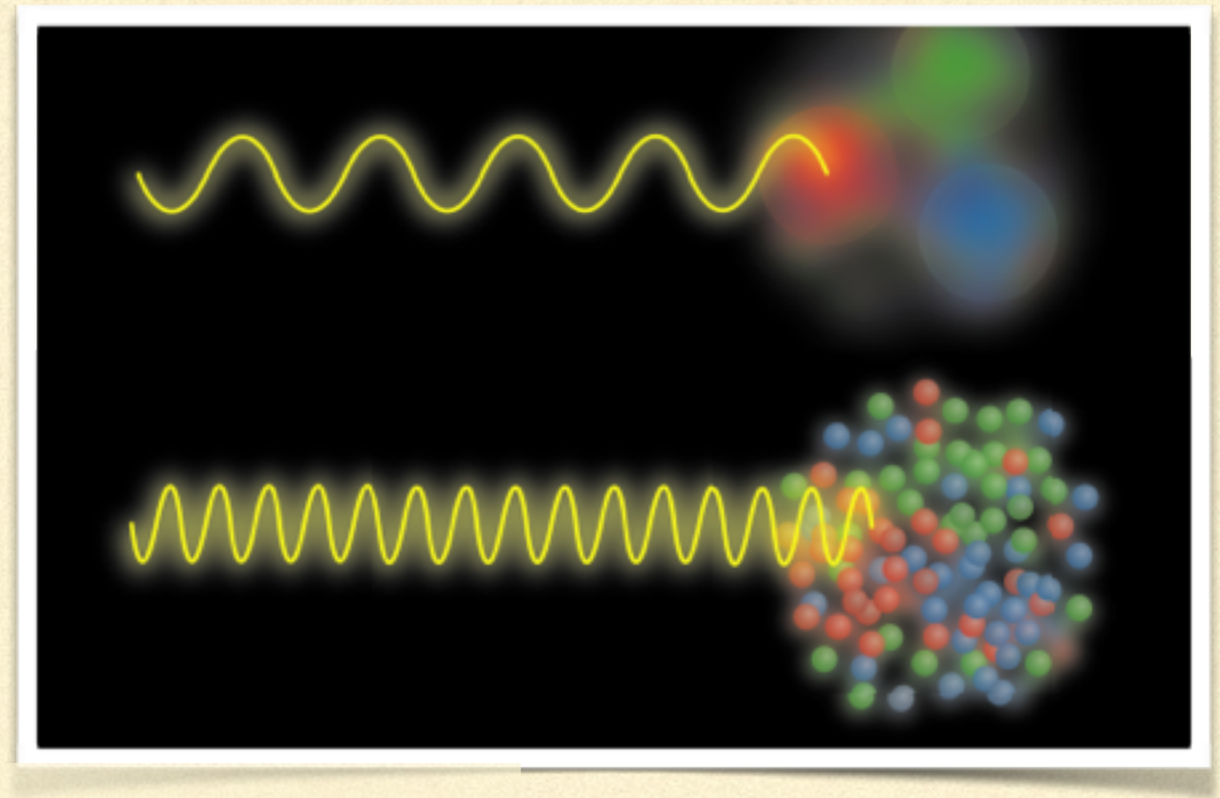

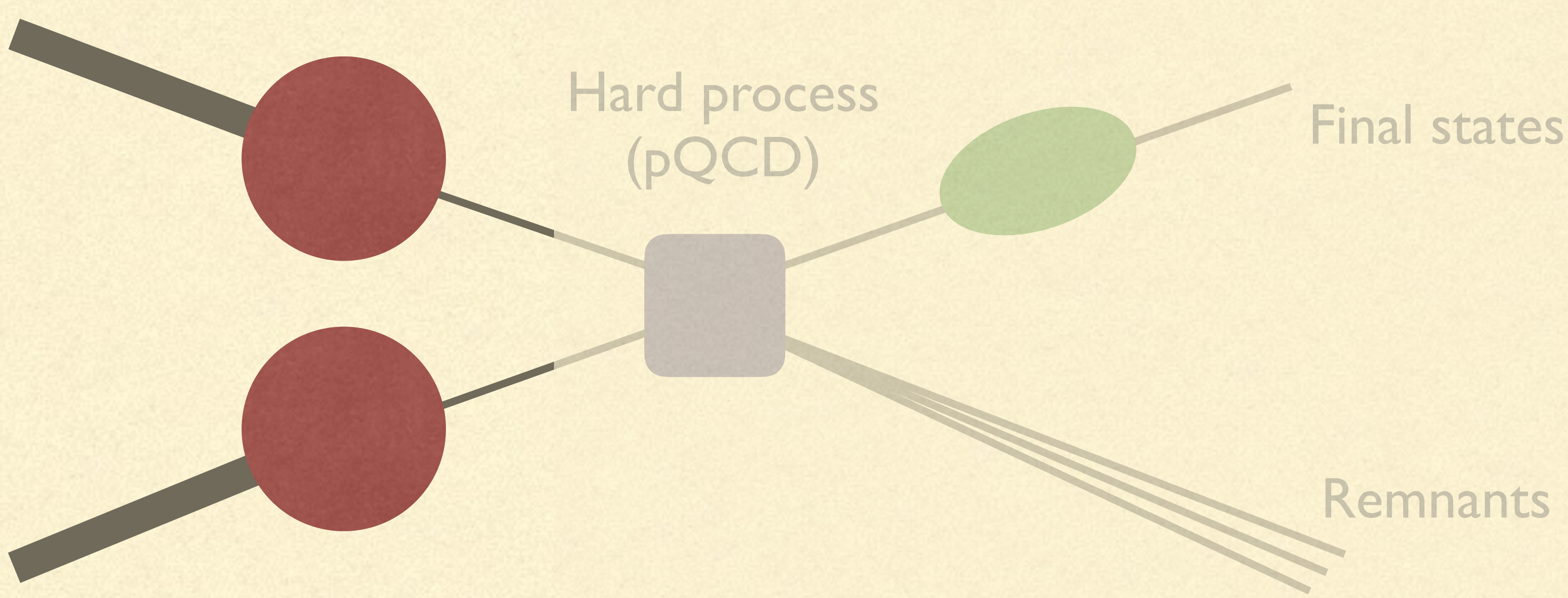

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Transverse momentum dependent (light-front) correlation function for quarks and gluons

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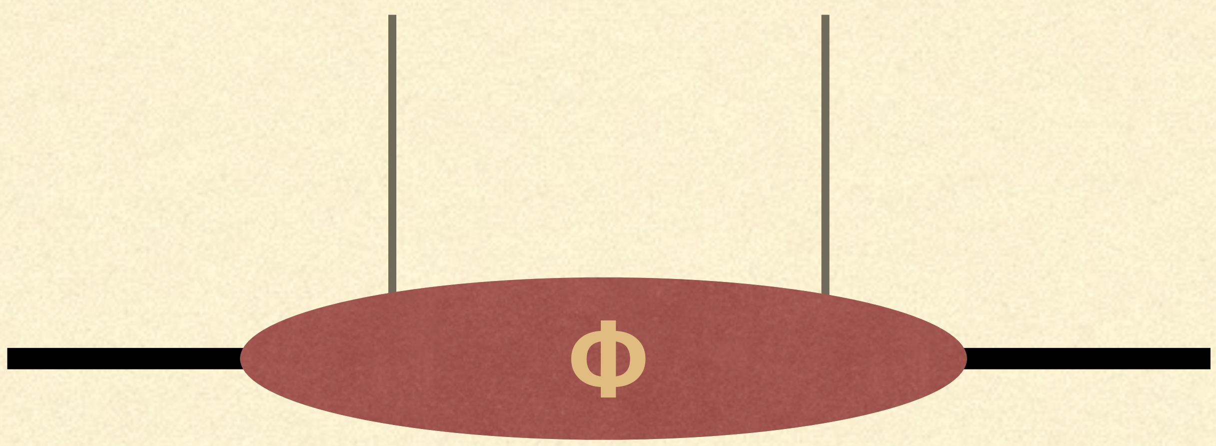
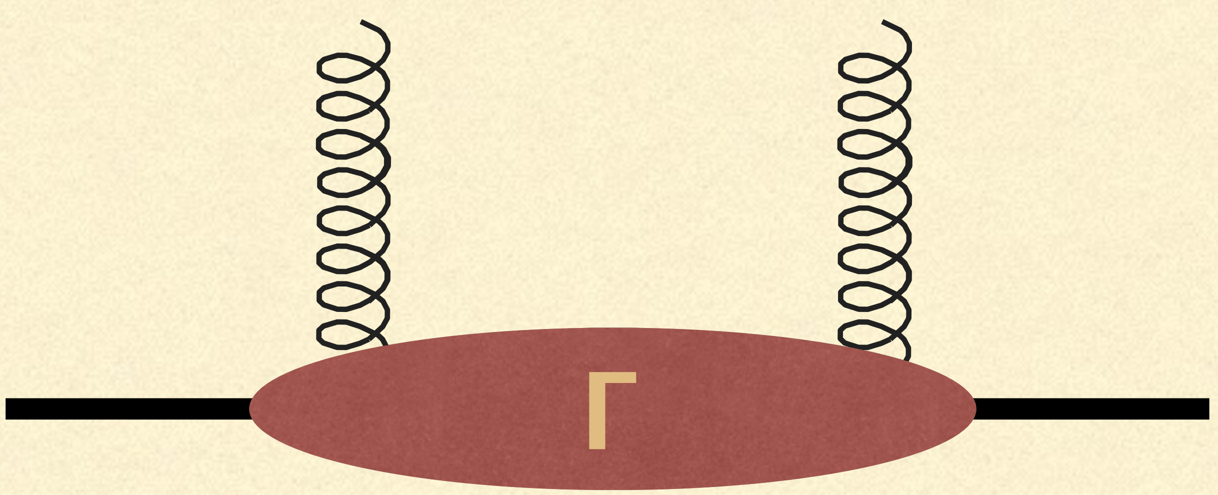
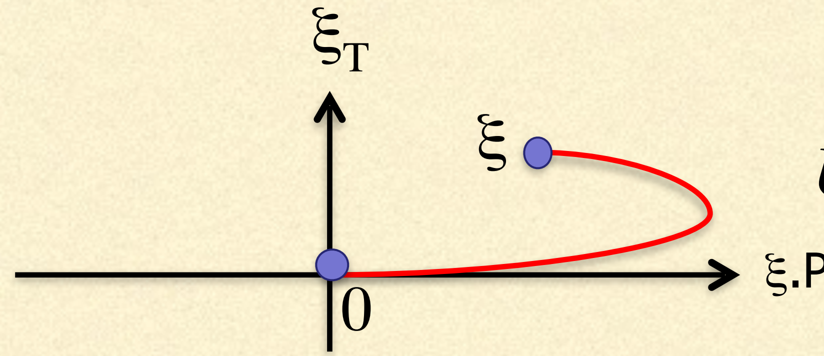
$$\Gamma^{[U,U']\,\mu\nu}(x,k_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\Big|_{\xi\cdot n=0}$$



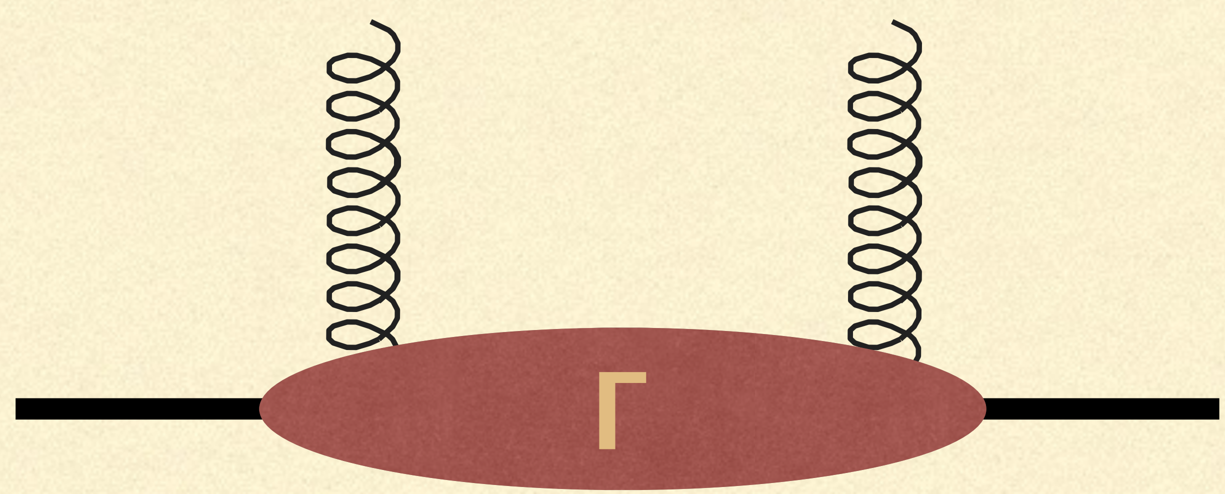


Transverse momentum dependent light-front correlation function for gluons



$$U(0,\xi) = \mathcal{P}\,\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

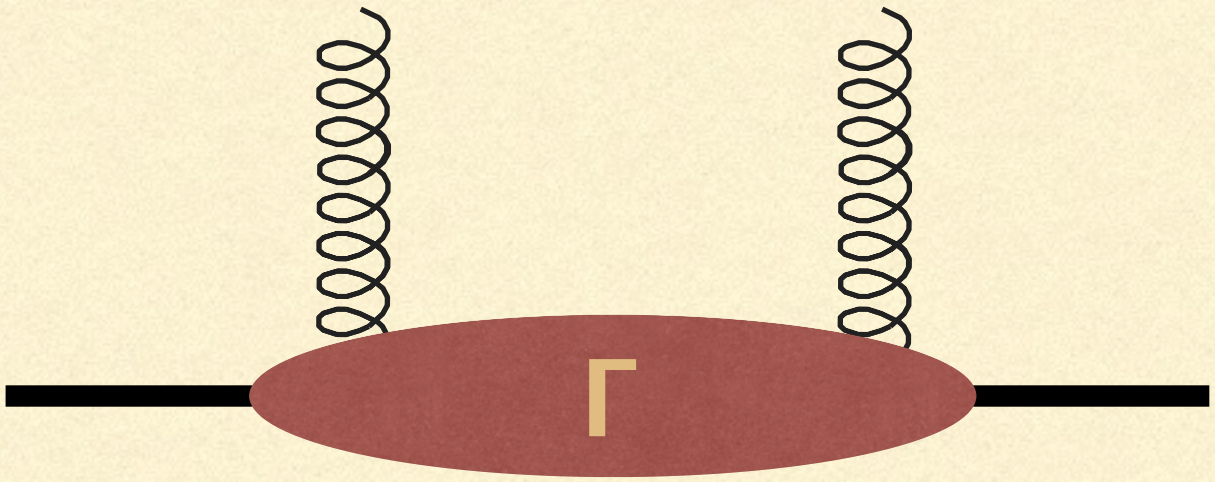
$$\Gamma^{[U,U']\,\mu\nu}(x,k_T;n) = \int \frac{d\,\xi\cdot P\, d^2\xi_T}{(2\pi)^3}\; e^{i\,k\cdot\xi}\, \langle P,S|\, F^{n\mu}(0)\, U_{[0,\xi]}\, F^{n\nu}(\xi)\, U'_{[\xi,0]}\, |P,S\rangle\big|_{\xi\cdot n=0}$$



Transverse momentum dependent light-front correlation function for gluons

$$\Gamma^{[U,U']\,\mathrm{ij}}(x,k_T;n) = \int \frac{d\,\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{i\,k\cdot\xi}\, \langle P,S|\, F^{n\,\mathrm{i}}(0)\, U_{[0,\xi]}\, F^{n\,\mathrm{j}}(\xi)\, U'_{[\xi,0]}\, |P,S\rangle\big|_{\xi\cdot n=0}$$

Process dependence!



on the gauge link structure:
[Efremov, Radyushkin, Qiu,Sterman,Collins,Brodsky, Hwang, Schmidt,Boer, Mulders Teryaev,… - various works]
process dependence, non universality and more:
[Belitsky, Ji, Yuan, 2003]
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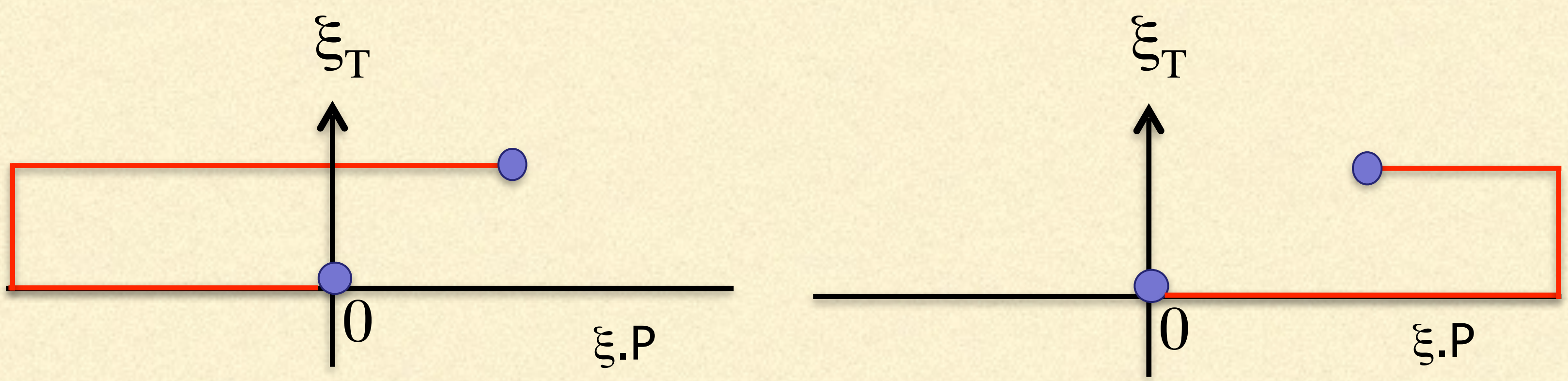

Leading twist contribution: μ and ν transverse
Gauge link structures: staple-like

$$\Gamma^{[U,U']\,ij}(x,k_T;n) = \int \frac{d\,\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S|\, F^{n\,i}(0)\, U_{[0,\xi]}\, F^{n\,j}(\xi)\, U'_{[\xi,0]}\, |P,S\rangle\big|_{\xi\cdot n=0}$$

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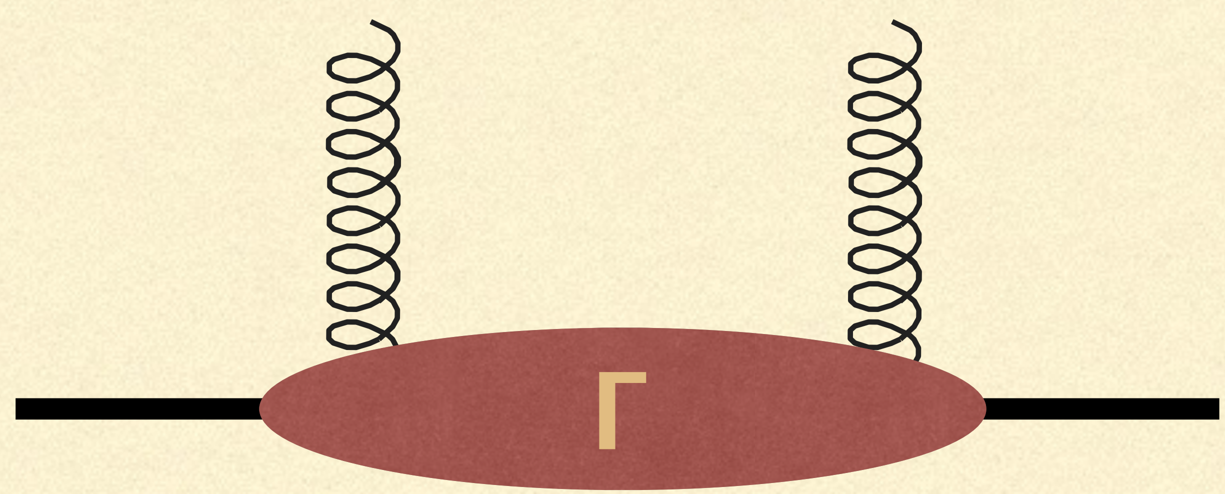

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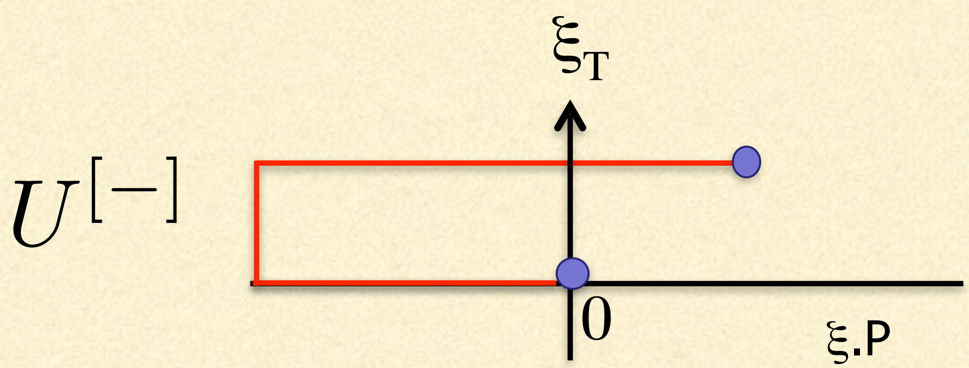
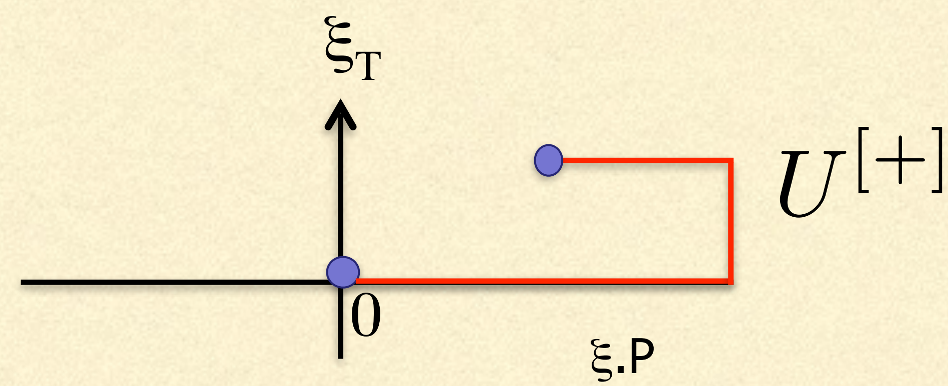
The Wilson loop correlator

$$\Gamma_0^{[\Box]}(\boldsymbol{k}_T) \equiv \int \frac{d^2\boldsymbol{\xi}_T}{(2\pi)^2} e^{-i\boldsymbol{k}_T\cdot\boldsymbol{\xi}_T} \langle P;S,T|\,\mathrm{Tr}_c\left(U^{[\Box]}\right)|P;S,T\rangle$$





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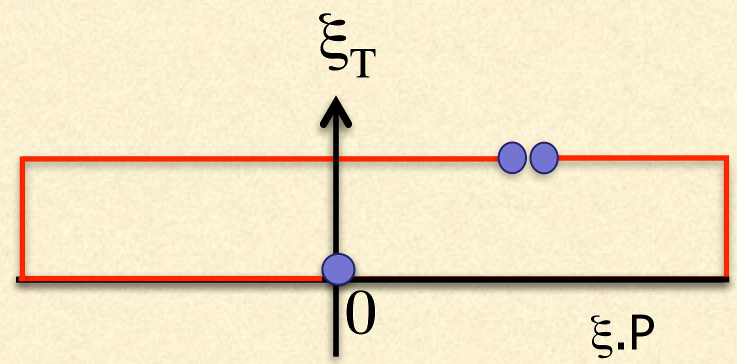

$$U^{[\Box]}$$

Wilson loop correlator

The correspondence at small-x

$$2\pi L\,\Gamma^{[+,-]ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} k_T^i k_T^j \Gamma_0^{[\square]}(\boldsymbol{k}_T)$$

Longitudinal dimension of the Wilson loop $L \equiv \int d\xi \cdot P = 2\pi\delta(0)$

Choice for the gauge link structure: dipole type operator.

[Dominguez,Marquet,Xiao,Yuan,2011]
[Boer,G.Echevarria,Mulders,Zhou,2015]
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Choice for the gauge link structure: dipole type operator.

- The correspondence is mathematically exact at x=0
- It allows for an estimate of the behavior of the gluon TMDs at small-x (see later)

[Dominguez,Marquet,Xiao,Yuan,2011]
[Boer,G.Echevarria,Mulders,Zhou,2015]
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TARGET SPIN

Polarized Hadrons

- Parent hadron momentum P ;
- Parton momentum $k^\mu = xP^\mu + k_T^\mu + (k \cdot P - xM^2)n^\mu$;
- n, light-like vector satisfying $P \cdot n = 1$;

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Vector polarization:

- Construction of the space-like spin vector satisfying $P \cdot S = 0$;

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$$S^\mu = S_L \frac{P^\mu}{M} + S_T^\mu - M S_L n^\mu$$

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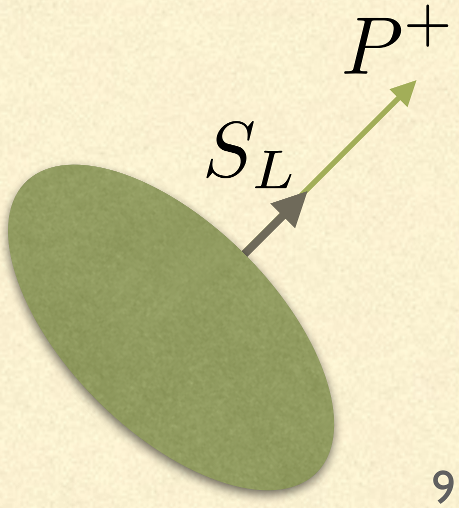
"Light-cone helicity"
(IMF → helicity of the particle)



- Simple interpretation in light-front formalism.

Tensor polarization (relevant for spin-1 and higher)

- Construction of the symmetric traceless spin tensor satisfying $P_\mu T^{\mu\nu} = 0\,;$

$$T^{\mu\nu} = \frac{1}{2}\left[\frac{2}{3}S_{LL}\,g_T^{\mu\nu} + \frac{4}{3}S_{LL}\frac{P^\mu P^\nu}{M^2} + \frac{S_{LT}^{\{\mu}P^{\nu\}}}{M} + S_{TT}^{\mu\nu} - \frac{4}{3}S_{LL}P^{\{\mu}n^{\nu\}} - M S_{LT}^{\{\mu}n^{\nu\}} + \frac{4}{3}M^2 S_{LL}\,n^\mu n^\nu\,\frac{P^\mu P^\nu}{M^2}\right]$$

- Five more spin components (representing combinations of probability of finding the system in a certain spin state → less simple to visualize)

[Bacchetta,PhD Thesis,2002]
[Leader, “Spin in Particle Physics”, 2001]

GLUON TMDs

	GLUON POLARIZATION			WILSON LOOP
TARGET SPIN	**Unpolarized**	**Circular**	**Linear**	
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T

The functions have dependence:

$$f, g, h \longrightarrow f(x, \boldsymbol{k}_T^2)$$

$$e \longrightarrow e(\boldsymbol{k}_T^2)$$

Spin 0 and 1/2
[Mulders,Rodrigues,2001]
[Meissner, Metz and Goeke,2007]

GLUON POLARIZATION WILSON LOOP

TARGET SPIN

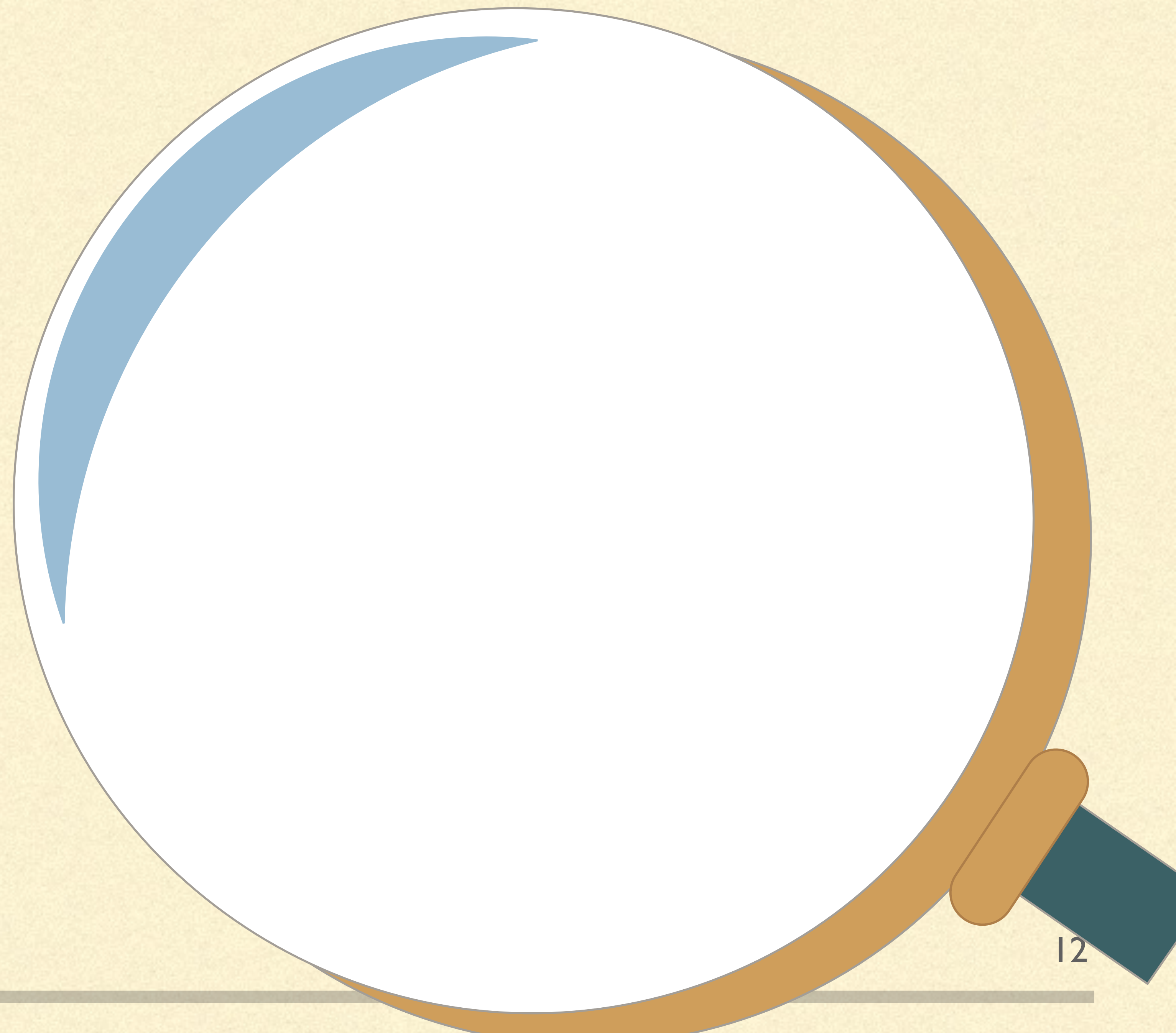
	Unpolarized	Circular	Linear	
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T

The functions have dependence:

$$f, g, h \quad\quad e$$

$$\downarrow \quad\quad \downarrow$$

$$f(x, \boldsymbol{k}_T^2) \quad\quad e(\boldsymbol{k}_T^2)$$



Spin 0 and 1/2
[Mulders,Rodrigues,2001]
[Meissner, Metz and Goeke,2007]

GLUON POLARIZATION WILSON LOOP

TARGET SPIN	Unpolarized	Circular	Linear	
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T

The functions have dependence:

$f, g, h \rightarrow f(x, \boldsymbol{k}_T^2)$

$e \rightarrow e(\boldsymbol{k}_T^2)$

$UU: \quad g_T^{ij} f_1$

$LL: \quad \dfrac{i\epsilon_{T\alpha}^{\{i} k_T^{j\}\alpha}}{M^2} S_L h_{1L}^\perp$

$TU: \quad \dfrac{g_T^{ij}\, \epsilon^{k \cdot S_T}}{M} f_{1T}^\perp$

$TC: \quad \dfrac{i\epsilon_T^{ij} k_T \cdot S_T}{M} g_{1T}$

etc…

Spin 0 and 1/2
[Mulders,Rodrigues,2001]
[Meissner, Metz and Goeke,2007]

TARGET SPIN	GLUON POLARIZATION			WILSON LOOP
	Unpolarized	Circular	Linear	
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
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T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T

The functions have dependence:

$$f, g, h \;\longrightarrow\; f(x, \boldsymbol{k}_T^2) \qquad\qquad e \;\longrightarrow\; e(\boldsymbol{k}_T^2)$$

Notice: scales dependence on the TMDs is frozen (more comments later..)

$$UU: \quad g_T^{ij} f_1$$

$$LL: \quad \frac{i\epsilon_{T\alpha}^{\{i} k_T^{j\}\alpha}}{M^2} S_L h_{1L}^\perp$$

$$TU: \quad \frac{g_T^{ij} \epsilon^{k \cdot S_T}}{M} f_{1T}^\perp$$

$$TC: \quad \frac{i\epsilon_T^{ij} k_T \cdot S_T}{M} g_{1T}$$

etc…

Spin 0 and 1/2
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GLUON POLARIZATION WILSON LOOP

TARGET SPIN

	Unpolarized	Circular	Linear	Wilson loop
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$	e_{LL}
LT	f_{1LT}	g_{1LT}	$h_{1LT}, h_{1LT}^\perp$	e_{LT}
TT	f_{1TT}	g_{1TT}	$\boldsymbol{h_{1TT}}, h_{1TT}^\perp, h_{1TT}^{\perp\perp}$	e_{TT}

Spin 1
[Jaffe & Manohar,1989]
[Boer,C,vanDaal,Mulders,Signori,Zhou,2016]

GLUON POLARIZATION — WILSON LOOP

TARGET SPIN

	Unpolarized	Circular	Linear	Wilson loop
U	$\boldsymbol{f_1}$		$h_1^\perp$	e
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$	e_{LL}
LT	f_{1LT}	g_{1LT}	$h_{1LT}, h_{1LT}^\perp$	e_{LT}
TT	f_{1TT}	g_{1TT}	$\boldsymbol{h_{1TT}}, h_{1TT}^\perp, h_{1TT}^{\perp\perp}$	e_{TT}

$$LLU: \quad g_T^{ij} S_{LL} f_{1LL}$$

$$LTL: \quad \frac{S_{LT}^{\{i} k_T^{j\}}}{M} h_{1LT} + \frac{k_T^{ij\alpha}}{M^3} S_{LT\alpha} h_{1LT}^\perp$$

$$TTC: \quad i\epsilon^{ij} \frac{\epsilon_{T\gamma}^{\beta} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}$$

etc…

Spin 1
[Jaffe & Manohar,1989]
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GLUON POLARIZATION WILSON LOOP

TARGET SPIN

	Unpolarized	Circular	Linear	Wilson loop
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T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$	e_T
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$	e_{LL}
LT	f_{1LT}	g_{1LT}	$h_{1LT}, h_{1LT}^\perp$	e_{LT}
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Two more collinear functions

$$LLU: \quad g_T^{ij} S_{LL} f_{1LL}$$

$$LTL: \quad \frac{S_{LT}^{\{i} k_T^{j\}}}{M} h_{1LT} + \frac{k_T^{ij\alpha}}{M^3} S_{LT\alpha} h_{1LT}^\perp$$

$$TTC: \quad i\epsilon^{ij} \frac{\epsilon_{T\gamma}^{\beta} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}$$

etc…

Spin 1
[Jaffe & Manohar,1989]
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GLUON POLARIZATION | WILSON LOOP

TARGET SPIN

	Unpolarized	Circular	Linear	
U	$e^{(1)}$		e	e
L		$-$	$-$	
T	$e_T^{(1)}$	$-$	$e_T^{(1)}, -e_T$	e_T
LL	$e_{LL}^{(1)}$		e_{LL}	e_{LL}
LT	$\frac{1}{2}e_{LT}^{(1)}$	$-$	$\frac{1}{2}e_{LT}^{(1)}, -e_{LT}$	e_{LT}
TT	$\frac{1}{3}e_{TT}^{(1)}$	$-$	$e_{TT}^{(2)}, -\frac{2}{3}e_{TT}^{(1)}, e_{TT}$	e_{TT}

$1/x$ is understood in all the entries

$$e^{(n)} = \left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n e$$

[Dominguez,Marquet,Xiao,Yuan,2011]
[Boer,G.Echevarria,Mulders,Zhou,2015]
[Boer, Cotogno,vanDaal,Mulders, Signori,Zhou 2016]

TARGET SPIN	GLUON POLARIZATION: Unpolarized	Circular	Linear	WILSON LOOP
U	$e^{(1)}$		e	e
L		$-$	$-$	
T	$e_T^{(1)}$	$-$	$e_T^{(1)}, -e_T$	e_T
LL	$e_{LL}^{(1)}$		e_{LL}	e_{LL}
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The gluon TMD PDFs in the small-x limit:

TARGET SPIN	GLUON POLARIZATION: Unpolarized	Circular	Linear	WILSON LOOP
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L		$-$	$-$	
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The gluon TMD PDFs in the small-x limit:

- they vanish or become proportional to each other

TARGET SPIN	Unpolarized	Circular	Linear	
	GLUON POLARIZATION			WILSON LOOP
U	$e^{(1)}$		e	e
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The gluon TMD PDFs in the small-x limit:

- they vanish or become proportional to each other
- if not vanishing they are proportional to 1/x (modulo neglected logs)

GLUON POLARIZATION WILSON LOOP

TARGET SPIN

	Unpolarized	Circular	Linear	
U	$e^{(1)}$		e	e
L		$-$	$-$	
T	$e_T^{(1)}$	$-$	$e_T^{(1)}, -e_T$	e_T
LL	$e_{LL}^{(1)}$		e_{LL}	e_{LL}
LT	$\frac{1}{2}e_{LT}^{(1)}$	$-$	$\frac{1}{2}e_{LT}^{(1)}, -e_{LT}$	e_{LT}
TT	$\frac{1}{3}e_{TT}^{(1)}$	$-$	$e_{TT}^{(2)}, -\frac{2}{3}e_{TT}^{(1)}, e_{TT}$	e_{TT}

$1/x$ is understood in all the entries

$$e^{(n)} = \left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n e$$

[Dominguez,Marquet,Xiao,Yuan,2011]
[Boer,G.Echevarria,Mulders,Zhou,2015]
[Boer, Cotogno,vanDaal,Mulders, Signori,Zhou 2016]

The gluon TMD PDFs in the small-x limit:

- they vanish or become proportional to each other
- if not vanishing they are proportional to 1/x (modulo neglected logs)
- only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

POSITIVITY BOUNDS

Matrix representation of the gluon correlator

Quarks:
[Bacchetta,Boglione,Henneman, Mulders,2000]
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Gluons:
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Single out hadron spin:

$$\rho = \frac{1}{3}\left(I + \frac{3}{2} S^i \Sigma^i + 3 T^{ij} \Sigma^{ij}\right) .$$

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Gamma: matrix in the gluon polarization space

G: matrix in gluon $\otimes$ target spin space

Circular polarization bases $|\pm\rangle = \mp\frac{1}{\sqrt{2}}(|x\rangle \pm i|y\rangle);$

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Semi-positive definite matrix.
Allows for the interpretation of some functions as densities

Positivity Bounds on gluon TMDs

$$G^{ij}_{ss'}(\boldsymbol{k}_T) = \left(\begin{array}{ccc|ccc} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right)$$

Explicit form in arXiv:1709.07827

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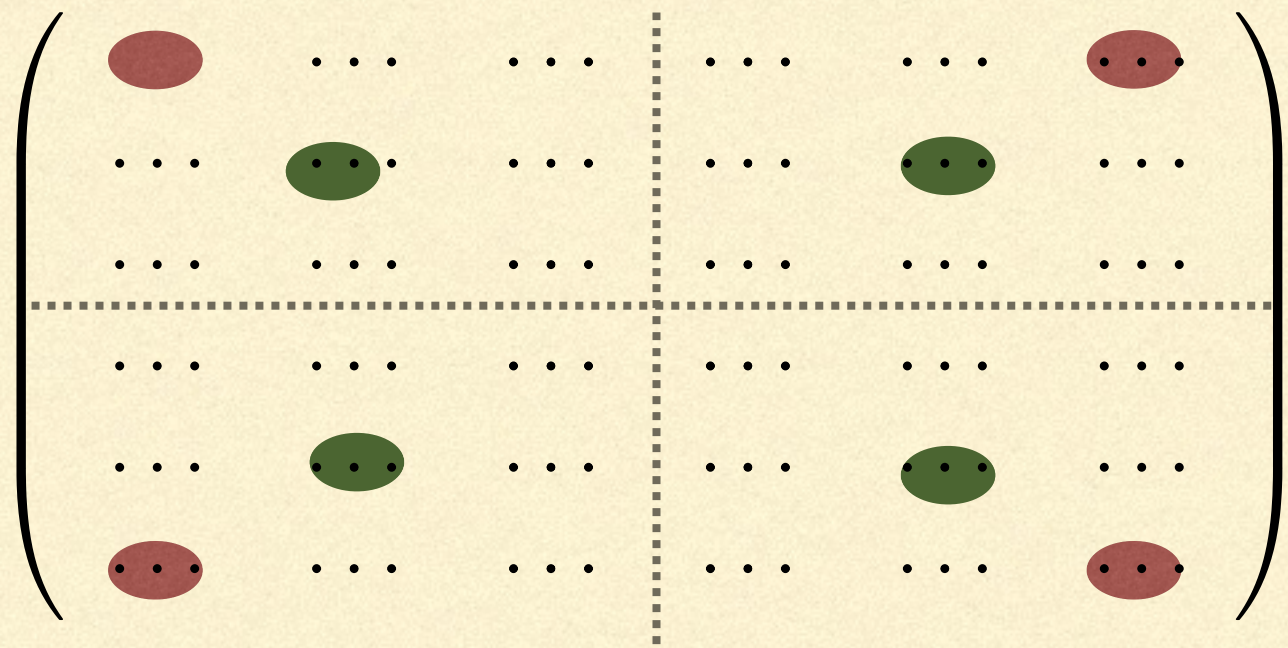

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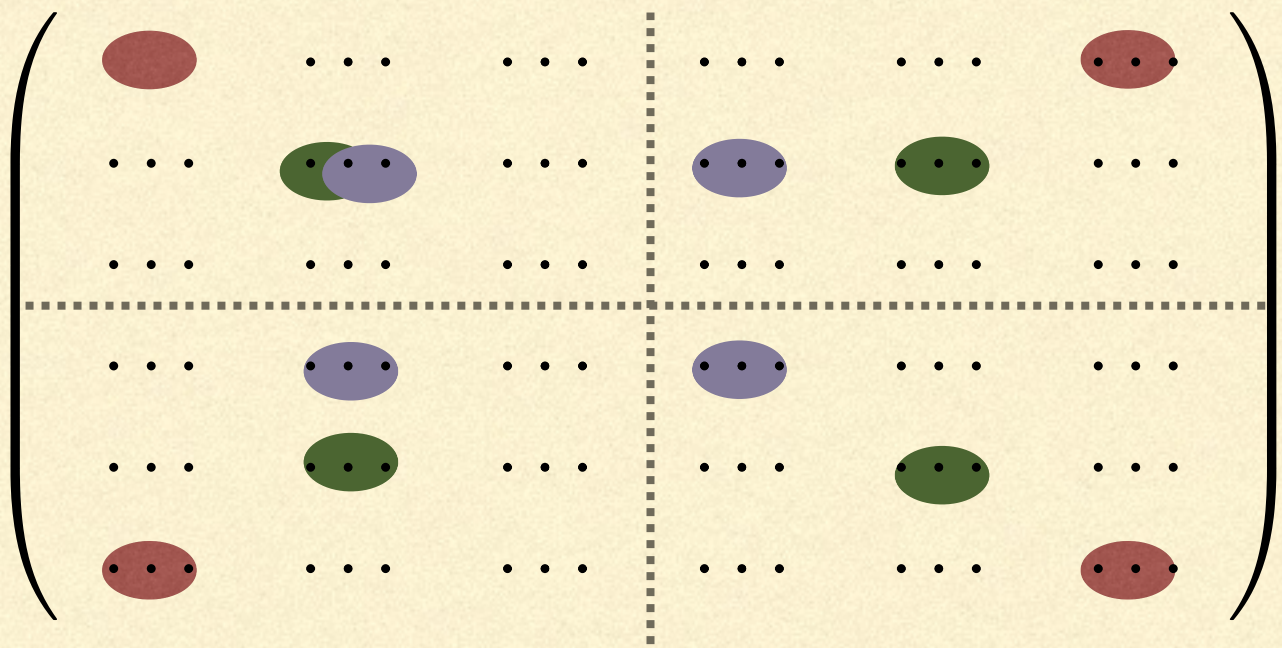

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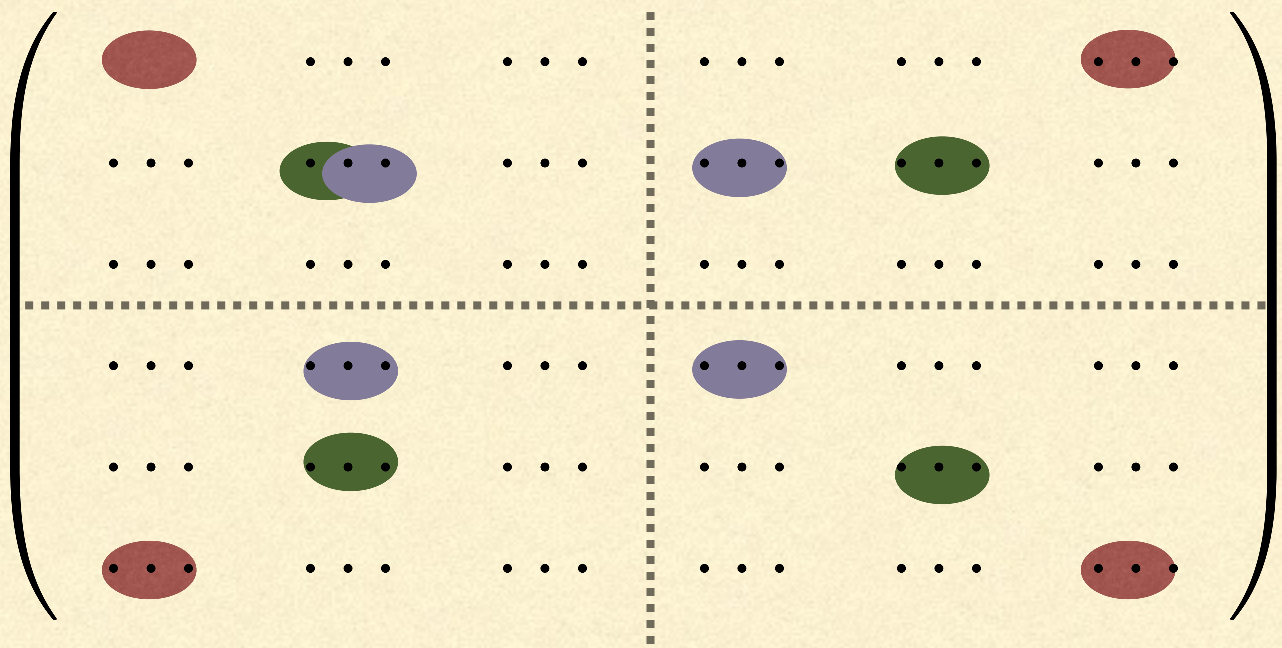

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… 6 more inequalities from the remaining 2x2 principal minors

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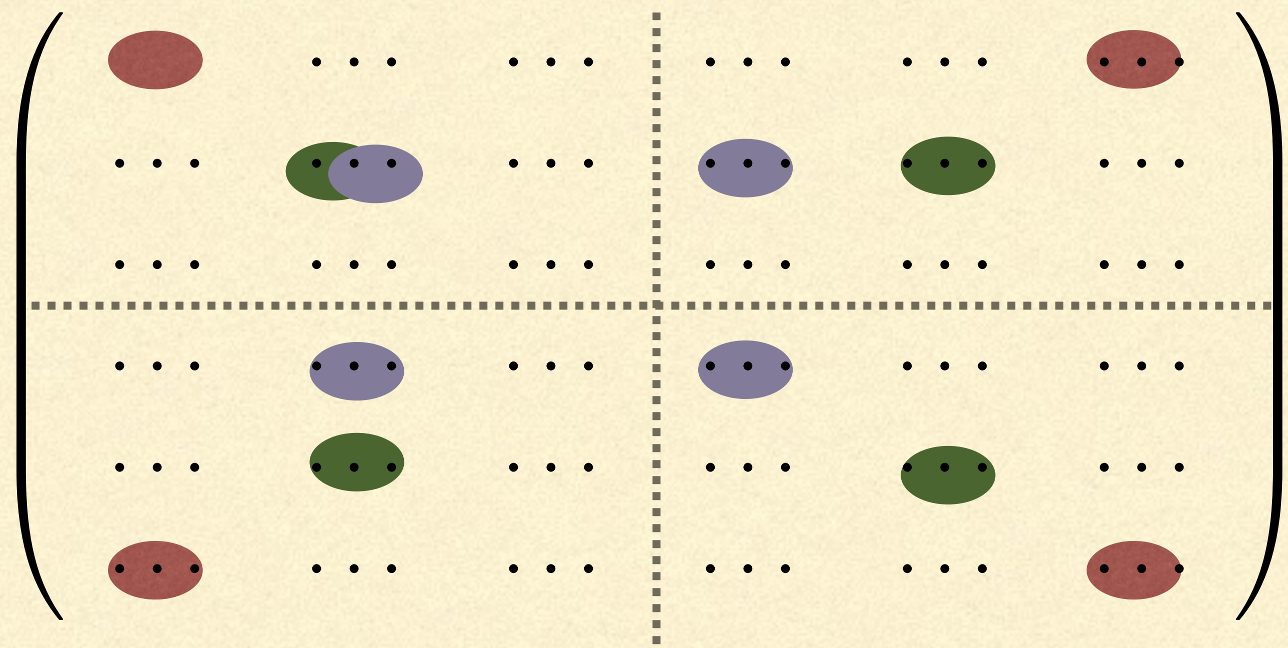

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Bounds can be sharpened!

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$$\Gamma_0^{[\square]} = \sum_{ss'} \rho_{ss'} G_{0ss'}^{[\square]}(\boldsymbol{k}_T)$$

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Two independent positivity bounds from the diagonalization of the 2x2 principal minors:

$$\frac{\boldsymbol{k}_T^2}{2M^2}\left(e_T^2 + e_{LT}^2\right) \leq \left(e - e_{LL}\right)\left(e + \frac{e_{LL}}{2}\right),$$

$$\frac{\boldsymbol{k}_T^2}{M^2}\left|e_{TT}\right| \leq e + \frac{e_{LL}}{2}.$$

Gluon TMDs and Wilson loop correlator: the bounds at small-x

$$\frac{\boldsymbol{k}_T^4}{2M^4}\,|h_{1TT}^{\perp\perp}| \leq f_1 + \frac{f_{1LL}}{2} - g_1$$

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The small-x limit on the gluon TMDs bounds gives the bounds on the Wilson loop!

"Mutatis mutandis": the spin-1/2 case

partially done in:
[Mulders,Rodrigues,2001]
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“Mutatis mutandis”: the spin-1/2 case

- 2x2 density matrix parametrized in terms of S and the Pauli matrices
- Matrix representation of the correlator in gluon$\otimes$target space:
  - 4x4 matrix for the gluon TMD correlation function;
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$$|g_1| \leq f_1,$$
$$\frac{\boldsymbol{k}_T^4}{4M^4}\left[(h_{1L}^{\perp})^2 + (h_1^{\perp})^2\right] \leq (f_1+g_1)(f_1-g_1),$$
$$\frac{|\boldsymbol{k}_T|}{M}\,|h_1| \leq f_1 + g_1,$$
$$\frac{|\boldsymbol{k}_T|^3}{2M^3}\,|h_{1T}^{\perp}| \leq f_1 - g_1,$$
$$\frac{\boldsymbol{k}_T^2}{M^2}\left[(f_{1T}^{\perp})^2 + g_{1T}^2\right] \leq (f_1+g_1)(f_1-g_1)$$

$\longrightarrow$ small-x

$$\frac{|\boldsymbol{k}_T|}{M}\,|e_T| \leq e$$

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Recent lattice calculations on the first moment of this bound

[Detmold,Shanahan,2016]

The collinear PDF $h_{1TT}(x)$

[Jaffee,Manohar,1989]

Volume 223, number 2 PHYSICS LETTERS B 8 June 1989

NUCLEAR GLUONOMETRY ☆

R.L. JAFFE and Aneesh MANOHAR

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received 24 March 1989

We identify a new leading twist structure function in QCD which can be measured in deep elastic scattering from polarized targets (such as nuclei) with spin $\geqslant 1$. The structure function measures a gluon distribution in the target and vanishes for a bound state of protons and neutrons, thereby providing a clear signature for exotic gluonic components in the target.

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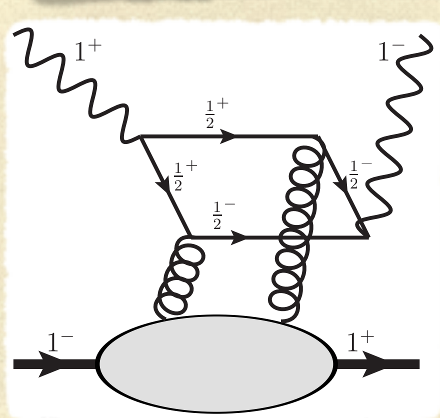

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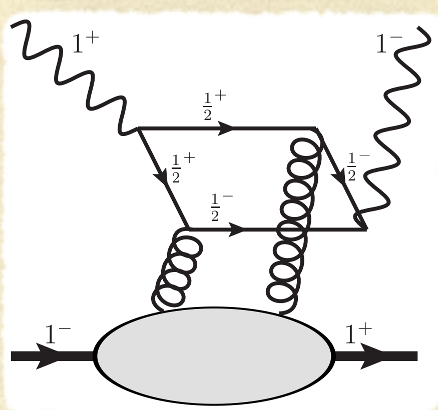

$\Delta(x)$ Double helicity flip structure function

$h_{1TT}(x)$ Probability to find linearly polarized gluons inside a transverse tensor polarized target

[Detmold,Shanahan,2016]

Experimental relevance at present facilities:

Possibilities at LHC:

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Spin-1 program at JLab:

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In the future…

Electron Ion Collider EIC

- Would allow to thoroughly study many gluon observables.

REMARKS ON POSITIVITY BOUNDS

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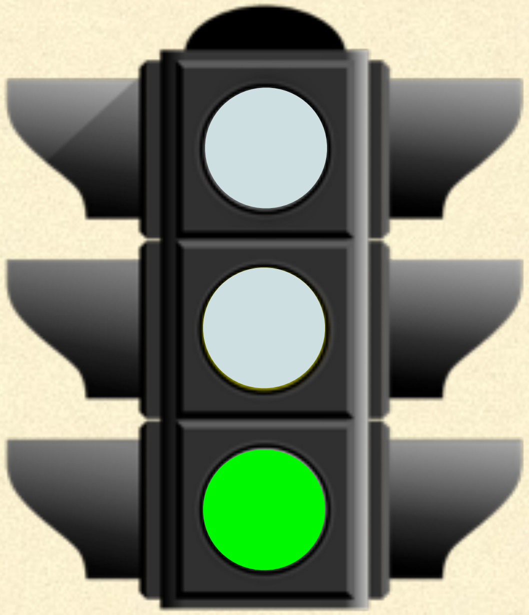

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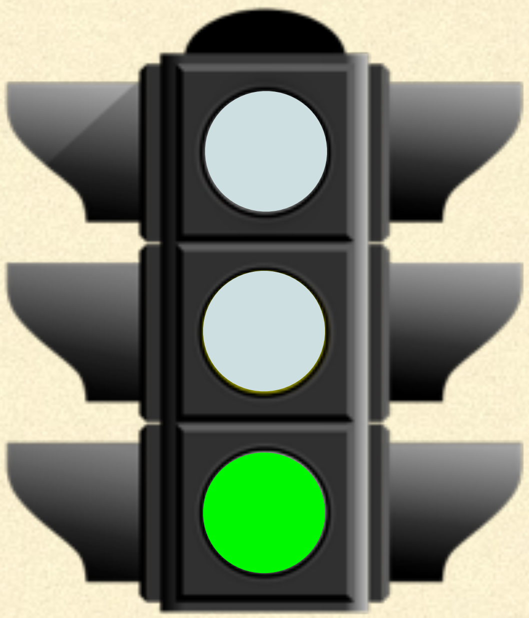

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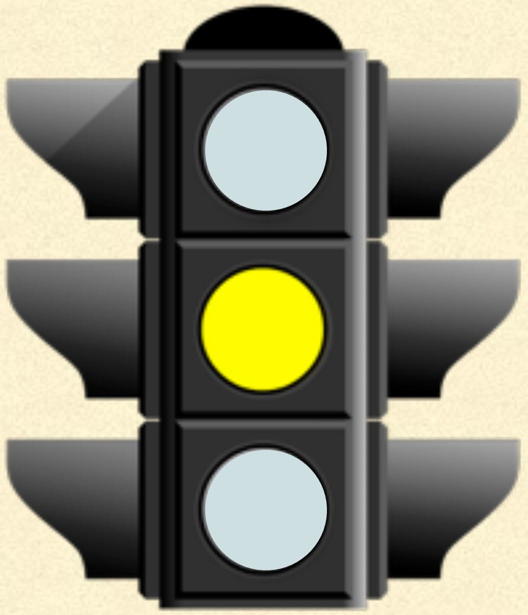
- QCD evolution effects:
  Multi-scale evolution: no studies have been performed on the (in)stability of the bounds after evolution.



Conclusions

- Gluon TMDs are fundamental tools to understand hadron internal structure (3D distribution of momentum);
- Their knowledge would reveal a lot about the internal dynamics of gluons in hadrons, which is at present almost unknown;
- When hadron polarization is included, the additional degrees of freedom could open up a wide range of new phenomena (signs of different types of parton-hadron correlations);
- "Exotic" gluonic effects within nuclei would also allow to study more thoroughly the binding between the constituents (which are not confined into separate nucleons);
- Positivity bounds can be used as model-independent tools to estimate magnitude of mainly unknown functions.
- Gluon distributions (PDFs and TMDs) are dominant is in the small-x limit: it is important that future facilities access this kinematical region.

For more information see arXiv: 1709.07827

Thank you!

Back up slides

Parametrization of the gluon correlator:

$$\Gamma^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{\frac{g_T^{ij}\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}k_T{\cdot}S_T}{M}\, g_{1T}^{[U]}(x,k_T^2) - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2)\right\}$$

$$\Gamma_{LL}^{ij[U]]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}S_{LL}\, f_{1LL}^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, S_{LL}h_{1LL}^{\perp[U]}(x,k_T^2)\right\}$$

$$\Gamma_{LT}^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\frac{k_T{\cdot}S_{LT}}{M} f_{1LT}^{[U]}(x,k_T^2) + i\epsilon_T^{ij}\frac{\epsilon_T^{S_{LT}k}}{M} g_{1LT}^{[U]}(x,k_T^2) + \frac{S_{LT}^{\{i}k_T^{j\}}}{M} h_{1LT}^{[U]}(x,k_T^2) + \frac{k_T^{ij\alpha}S_{LT\alpha}}{M^3} h_{1LT}^{\perp[U]}(x,k_T^2)\right\}$$

$$\Gamma_{TT}^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\frac{k_T^{\alpha\beta}S_{TT\alpha\beta}}{M^2}\, f_{1TT}^{[U]}(x,k_T^2) + i\epsilon_T^{ij}\frac{\epsilon_{T\,\gamma}^{\beta}k_T^{\gamma\alpha}S_{TT\alpha\beta}}{M^2}\, g_{1TT}^{[U]}(x,k_T^2) + S_{TT}^{ij}\, h_{1TT}^{[U]}(x,k_T^2) + \frac{S_{TT\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, h_{1TT}^{\perp[U]}(x,k_T^2) + \frac{k_T^{ij\alpha\beta}S_{TT\alpha\beta}}{M^4}\, h_{1TT}^{\perp\perp[U]}(x,k_T^2)\right\}$$

The gluon-gluon correlator at x=0

The gluon-gluon correlator at x=0

Choice for the gauge link structure: dipole type operator.

$$2\pi L\,\Gamma^{[+,-]ij}(0,\boldsymbol{k}_T) = \int \frac{d\xi\cdot P\, d^2\xi_T}{(2\pi)^3} e^{ik\cdot\xi} \langle P|F^{ni}(0)U^{[+]}_{[0,\xi]}F^{nj}(\xi)U^{[-]}_{[\xi,0]}|P\rangle\Big|_{\xi\cdot n=k\cdot n=0}$$

$$= k_T^i k_T^j \int \frac{d^2\xi}{(2\pi)^2} e^{ik_T\cdot\xi_T} \langle P|U^{[\square]}|P\rangle\Big|_{\xi\cdot n=0} = k_T^i k_T^j \Gamma_0^{[\square]}(\boldsymbol{k}_T)$$

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Wilson loop correlator

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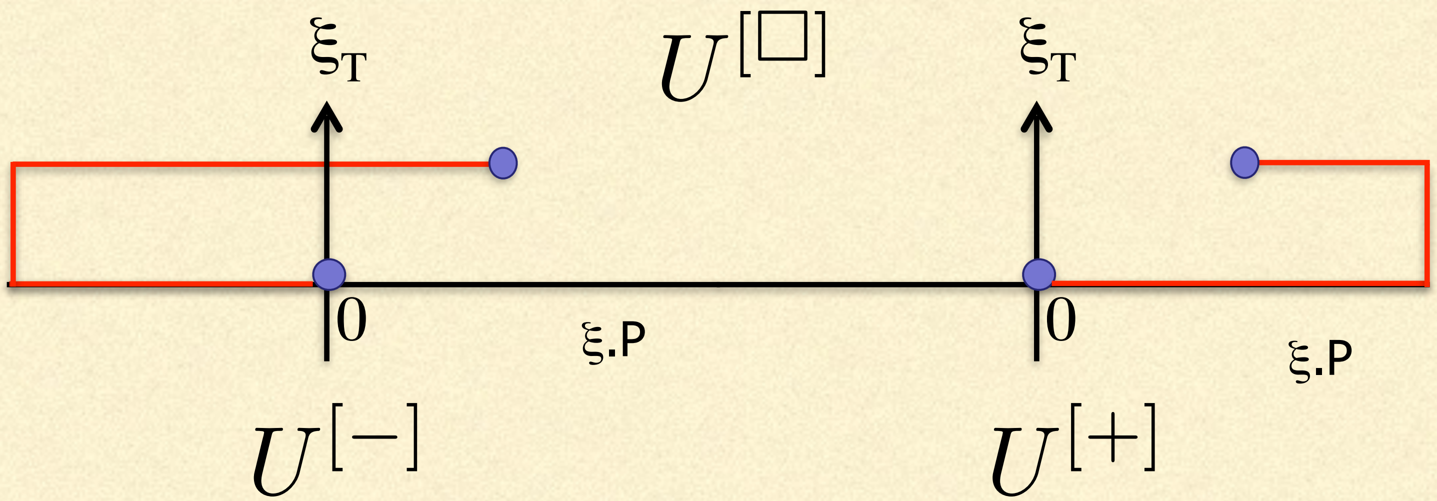

Wilson loop correlator

Longitudinal dimension of the Wilson loop

$$L \equiv \int d\xi\cdot P = 2\pi\delta(0)$$

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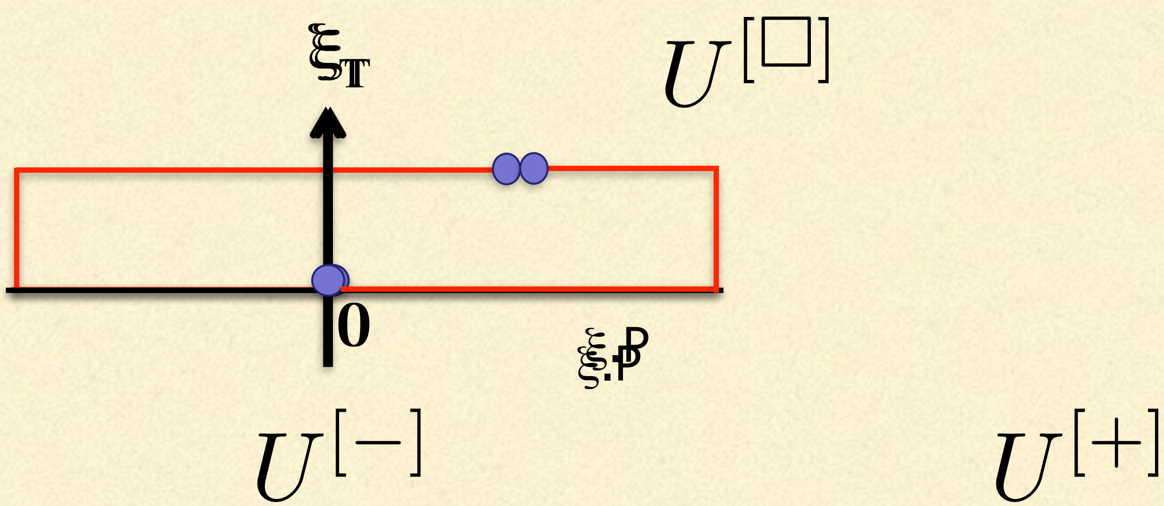

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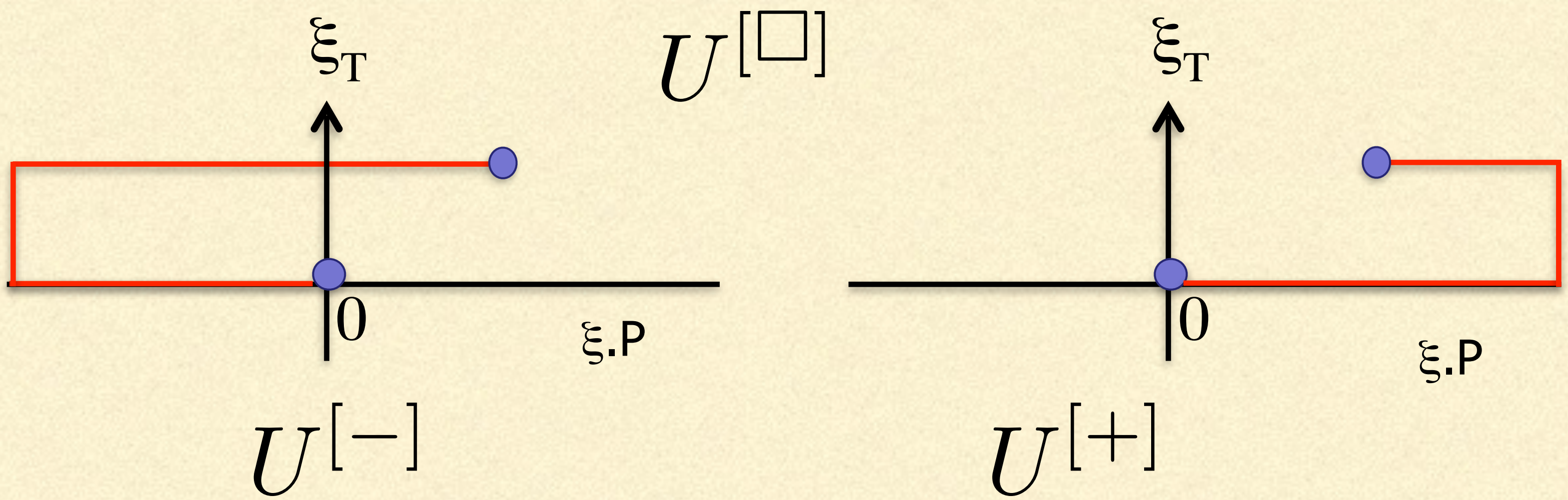

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QUARKS	PARTON SPIN: γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
TARGET SPIN: U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT} $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT} $h_{1TT}^\perp$

Hoodbhoy, Jaffe & Manohar, NP B312 (1988) 571: introduction of $f_{1LL} = b_1$

Bacchetta & M, PRD 62 (2000) 114004; h_{1LT} first introduced as T-odd PDF

X. Ji, PRD 49 (1994) 114; introduction of $H_{1LT} \equiv \hat{h}_{\bar{1}}$ (PFF)

GLUONS	GLUON POLARIZATION: $-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
TARGET SPIN: U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

PJM & Rodrigues, PR D63 (2001) 094021

Meissner, Metz and Goeke, PR D76 (2007) 034002

D Boer, S Cotogno, T van Daal, PJM, A Signori,Y Zhou, ArXiv 1607.01654

$$\rho = \frac{1}{3}\left(I + \frac{3}{2}S^i\Sigma^i + 3T^{ij}\Sigma^{ij}\right).$$

$$\rho = \begin{pmatrix} \frac{S_L}{2}+\frac{S_{LL}}{3}+\frac{1}{3} & \frac{S_{LTx}-iS_{LTy}}{2\sqrt{2}}+\frac{S_{Tx}-iS_{Ty}}{2\sqrt{2}} & \frac{1}{2}(S_{TTxx}-iS_{TTxy}) \\ \frac{S_{LTx}+iS_{LTy}}{2\sqrt{2}}+\frac{S_{Tx}+iS_{Ty}}{2\sqrt{2}} & \frac{1}{3}-\frac{2S_{LL}}{3} & \frac{-S_{LTx}+iS_{LTy}}{2\sqrt{2}}+\frac{S_{Tx}-iS_{Ty}}{2\sqrt{2}} \\ \frac{1}{2}(S_{TTxx}+iS_{TTxy}) & \frac{-S_{LTx}-iS_{LTy}}{2\sqrt{2}}+\frac{S_{Tx}+iS_{Ty}}{2\sqrt{2}} & -\frac{S_L}{2}+\frac{S_{LL}}{3}+\frac{1}{3} \end{pmatrix}$$

Positivity bounds: construction of the matrix

$$G = \frac{x}{2}\begin{pmatrix} A & B \\ B^\dagger & C \end{pmatrix}$$

C is the transformed of A under Parity

$$A = \begin{pmatrix} f_1 + \frac{f_{1LL}}{2} - g_1 & \frac{e^{-i\phi}k}{\sqrt{2}M}\left(f_{1LT} + i f_{1T}^{\perp} - g_{1T} - i g_{1LT} + h_{1LT}\right) & \frac{e^{-2i\phi}k^2}{M^2}\left(f_{1TT} + i g_{1TT} - h_{1TT}^{\perp}\right) \\ \frac{e^{i\phi}k}{\sqrt{2}M}\left(f_{1LT} - i f_{1T}^{\perp} - g_{1T} + i g_{1LT} + h_{1LT}\right) & f_1 - f_{1LL} & -\frac{e^{-i\phi}k}{\sqrt{2}M}\left(f_{1LT} - i f_{1T}^{\perp} + g_{1T} - i g_{1LT} + h_1 \right. \\ \frac{e^{2i\phi}k^2}{M^2}\left(f_{1TT} - i g_{1TT} - h_{1TT}^{\perp}\right) & -\frac{e^{i\phi}k}{\sqrt{2}M}\left(f_{1LT} + i f_{1T}^{\perp} + g_{1T} + i g_{1LT} + h_{1LT}\right) & f_1 + \frac{f_{1LL}}{2} + g_1 \end{pmatrix}$$

$$B = \begin{pmatrix} -\frac{e^{-2i\phi}k^2}{4M^2}\left(2h_1^{\perp} + h_{1LL}^{\perp} - 2i h_{1L}^{\perp}\right) & \frac{e^{-3i\phi}k^3}{2\sqrt{2}M^3}\left(h_{1LT}^{\perp} + i h_{1T}^{\perp}\right) & -\frac{e^{-4i\phi}k^4}{2M^4} h_{1TT}^{\perp\perp} \\ -\frac{e^{-i\phi}k}{\sqrt{2}M}\left(2h_{1LT} - i h_1\right) & -\frac{e^{-2i\phi}k^2}{2M^2}\left(h_1^{\perp} - h_{1LL}^{\perp}\right) & -\frac{e^{-3i\phi}k^3}{2\sqrt{2}M^3}\left(h_{1LT}^{\perp} - i h_{1T}^{\perp}\right) \\ -2h_{1TT} & \frac{e^{-i\phi}k}{\sqrt{2}M}\left(2h_{1LT} + i h_1\right) & -\frac{e^{-2i\phi}k^2}{4M^2}\left(2h_1^{\perp} + h_{1LL}^{\perp} + 2i h_{1L}^{\perp}\right) \end{pmatrix}$$

Positivity bounds: construction of the matrix

$$G_0^{[\square]} = \frac{\pi L}{M^2} \begin{pmatrix} e + \frac{e_{LL}}{2} & \frac{e^{-i\phi}k}{\sqrt{2}M}\left(e_{LT} + ie_T\right) & \frac{e^{-2i\phi}k^2}{M^2}\, e_{TT} \\ \frac{e^{i\phi}k}{\sqrt{2}M}\left(e_{LT} - ie_T\right) & e - e_{LL} & -\frac{e^{-i\phi}k}{\sqrt{2}M}\left(e_{LT} - ie_T\right) \\ \frac{e^{2i\phi}k^2}{M^2}\, e_{TT} & -\frac{e^{i\phi}k}{\sqrt{2}M}\left(e_{LT} + ie_T\right) & e + \frac{e_{LL}}{2} \end{pmatrix}$$