The proton spin and PDFs from lattice QCD

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INT Program

Spatial and Momentum Tomography of Hadrons and Nuclei

September 25, 2017

In collaboration with

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- F. Steffens[5]
- A. Vaquero[2]
- Ch. Wiese[5]

1. Univ. of Cyprus, 2. Cyprus Institute,
3. Adam Mickiewicz University, 4. Temple Univ. 5. DESY Zeuthen

LAYOUT OF THE TALK

A. Motivation

B. Introduction to LQCD

C. MainStream Hadron Structure

1. Axial charge
2. Quark momentum fraction
3. Gluon momentum fraction
4. Proton Spin

D. Novel directions in Hadron Structure

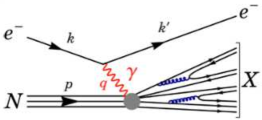
1. quasi-PDFs in Lattice QCD
2. Perturbative Renormalization
3. Non-perturbative Renormalization

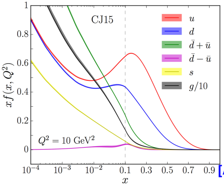
E. Discussion

A

Motivation

Probing Nucleon Structure





CJ15 PDFs

[A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- ★ **powerful tool to describe the structure of a nucleon**
- ★ **necessary for the analysis of Deep inelastic scattering (DIS) data**
- ★ **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar{P} = (p' + p)/2$, n: **light-cone vector (**$\bar{P}.n = 1$**)**, $\xi = -n \cdot \Delta/2$

PDFs on the Lattice

★ first principle calculations of PDFs are necessary

★ On the lattice: long history of moments of PDFs

$$f^n = \int_{-1}^{1} dx\, x^n f(x)$$

★ rely on OPE to reconstruct the PDFs (difficult task):

- signal-to-noise is bad for higher moments
- n $>$ 3: operator mixing (unavoidable!)

★ Alternative investigation of PDFs ?

Types:

- Unpolarized (vector current)
- Polarized (axial current)
- Transversity (tensor current)

PDFs on the Lattice

Novel direct approach: [X.Ji, arXiv:1305.1539]

★ compute quasi-PDF on the lattice

★ contact with physical PDFs in two steps:

1. Renormalization of quasi-PDFs in Lattice Regularization
2. Matching procedure (LaMET)

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Exploratory studies are maturing:

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],

[Y.-Q. Ma et al., arXiv:1412.2688], [C.Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575],

[J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018],

[C.Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],

[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],

[C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962],

[T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152]

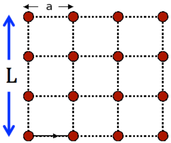
B

Introduction to LQCD

Lattice formulation of QCD

★ **Space-time discretization on a finite-sized 4-D lattice**

- **Quark fields on lattice points**
- **Gluons on links**



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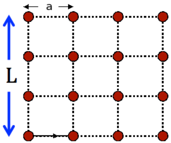

Technical Aspects

★ **Parameters (define cost of simulations):**

- **quark masses (aim at physical values)**
- **lattice spacing (ideally fine lattices)**
- **lattice size (need large volumes)**

★ **Discretization not unique:**

- **Wilson, Clover, Twisted Mass,**
- **Staggered, Overlap, Domain Wall**

Systematic uncertainties: Challenges & Progress

1. Cut-off Effects: finite lattice spacing

2. Finite Volume Effects

3. Contamination from other hadron states

4. Not simulating the physical world

5. Renormalization and mixing

Systematic uncertainties: Challenges & Progress

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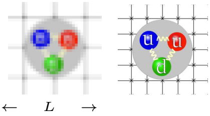
   - Continuum limit $a \to 0$
   - Simulations with fine lattices ($a < 0.1$ fm)
   - Improve actions, algorithmic improvements



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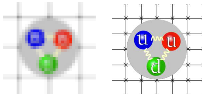

$\leftarrow \quad L \quad \rightarrow$

2. Finite Volume Effects
   - Infinite volume limit $L \to \infty$
   - Simulating hadrons in large volumes (Rule of thumb: $L\, m_\pi > 3.5$)
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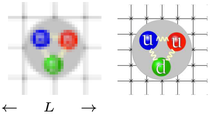

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- Infinite volume limit $L \to \infty$
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3 Contamination from other hadron states

- Various methods for extracting information from lattice data

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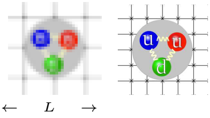
Systematic uncertainties: Challenges $\&$ Progress



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- **Continuum limit $a \to 0$**
- **Simulations with fine lattices ($a < 0.1$ fm)**
- **Improve actions, algorithmic improvements**

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3 **Contamination from other hadron states**

- **Various methods for extracting information from lattice data**

4 **Not simulating the physical world**

- **Chiral extrapolation**
- **Simulations at physical parameters are now feasible**

5 **Renormalization and mixing**

Systematic uncertainties: Challenges & Progress

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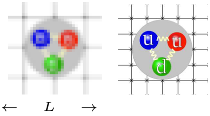

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4 Not simulating the physical world

- Chiral extrapolation
- Simulations at physical parameters are now feasible

5 Renormalization and mixing

- Subtraction of lattice artifacts, utilize perturbation theory

Lattice Parameters (in this work)

[C. Alexandrou et al. (ETMC), arXiv:1611.09163] [C. Alexandrou et al. (ETMC), arXiv:1611.03802]

Ensemble at the physical point:

- ★ $N_f=2$ Twisted Mass fermions with a clover term
- ★ Lattice size: $48^3\times 96$, $64^3\times 128$ (NEW)
- ★ Lattice spacing: $a\sim 0.094$
- ★ $m_\pi \sim 130$ MeV

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Measurements

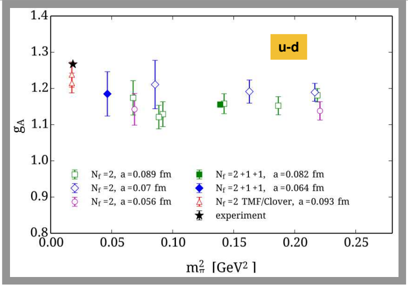
	Connected		Disconnected
	$L=48$	$L=64$	$L=48$
Statistics:	9,280	4,400	480,000
$\mathrm{T_{sink}}$:	10, 12, 14	12, 14, 16	10, 12, 14, 16, 18

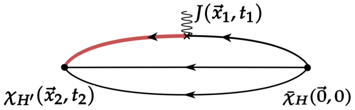
C

MainStream HS

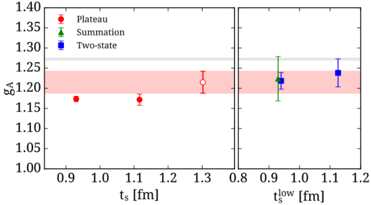
Proton Spin

Axial Charge (connected)

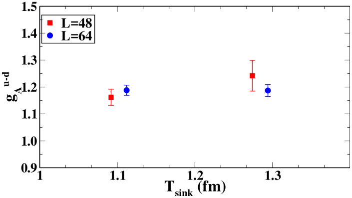




Excited States

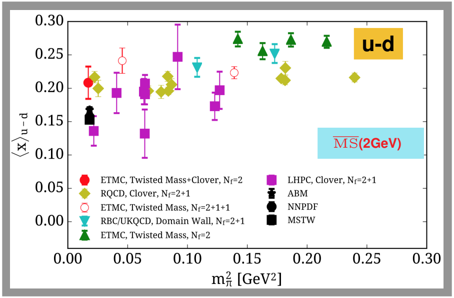


Volume effects

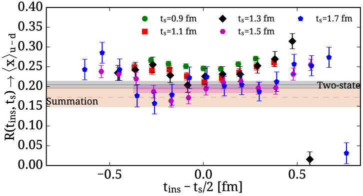


Mild excited states and volume effects

Quark Momentum Fraction (connected)



Excited States



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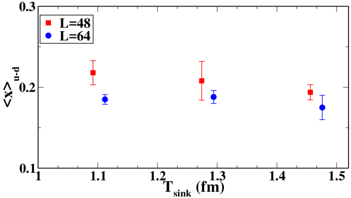
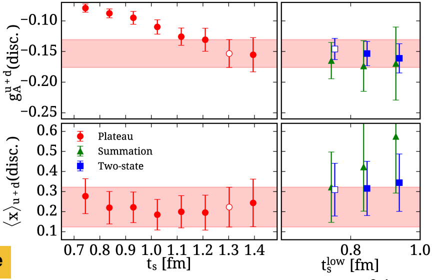
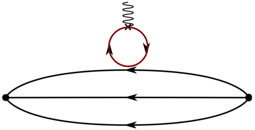


Excited states and volume effects non-negligible

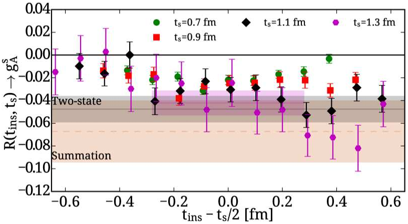
Disconnected contributions

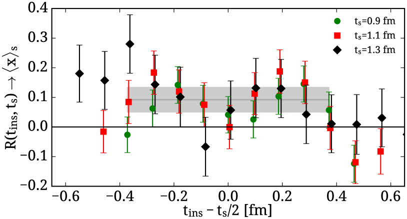
u+d





strange



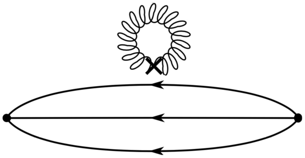


★ **Taking into account the disconnected contributions is crucial**

Gluon Momentum Fraction

Direct Calculation

$$\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$$



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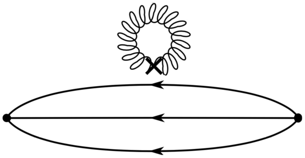

Challenges

★ Disconnected diagram:

- **Small signal-to-noise ratio**
- **Requires special techniques**

★ Renormalization

- **Mixing with operator for** $\langle x \rangle_{u+d}$ — **Unavoidable**
- **Mixing with other Operators** — **Vanish in physical matrix elements**

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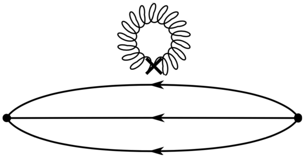

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- **Mixing with operator for $\langle x\rangle_{u+d}$** — **Unavoidable**
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$$\langle x\rangle^R_g = Z_{gg}\langle x\rangle^B_g + Z_{gq}\sum_q\langle x\rangle^B_q$$

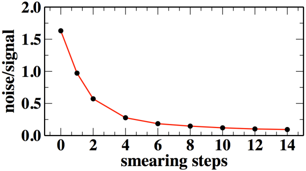
$$\sum_q\langle x\rangle^R_q = Z_{qq}\sum_q\langle x\rangle^B_q + Z_{qg}\langle x\rangle^B_g$$

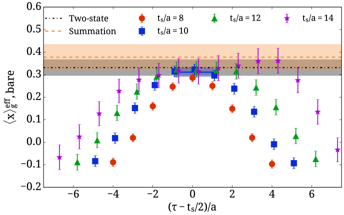
Pert. Theory: computation of mixing coefficients

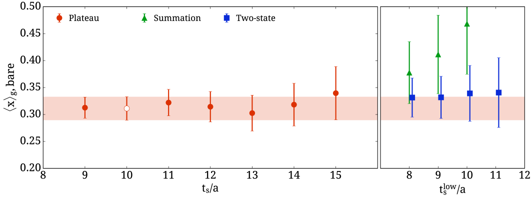
Lattice Results

$N_f = 2$ TM fermions, $m_\pi = 130$MeV

Smearing improves signal







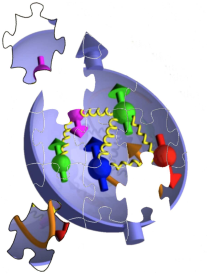
Upon disentangling the gluon momentum fraction from the quark:

$$\langle x \rangle_g^R = 0.267(22)(19)(24)$$

[C. Alexandrou et al. (ETMC), 1611.06901]

Proton Spin: Can we put the puzzle together?

Spin Structure from First Principles



Spin Sum Rule:

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q\right) + J^G$$

L_q: **Quark orbital angular momentum**

$\Delta\Sigma_q$: **intrinsic spin**

J^G: **Gluon part**

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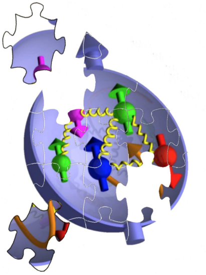

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Extraction from LQCD:

$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right)\,, \quad L^q = J^q - \Sigma^q\,, \quad \Sigma^q = g_A^q$$

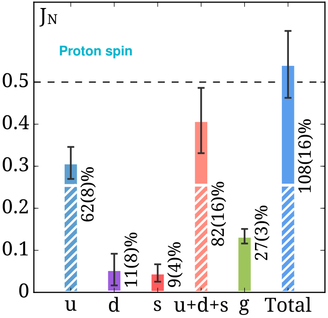
★ **Individual quark contributions: disconnected insertion contributes**

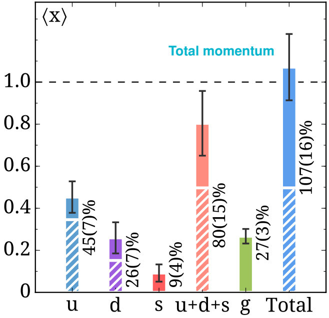
Collected Results

★ **Satisfaction of spin and momentum sum rule is not forced**

⇓

★ **important check of results and the systematic uncertainties**





D

Novel directions in HS

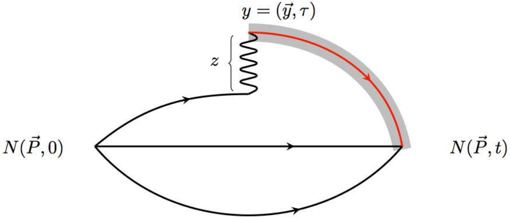
quasi-PDFs

Access of PDFs on a Euclidean Lattice

★ **quasi-PDF purely spatial for nucleons with finite momentum**

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}$$

- $\mathcal{A}(z,0)$: **Wilson line from** $0 \to z$
- z: **distance in any spatial direction** (**momentum boost in** z **direction**)



★ **At finite but feasibly large momenta on the lattice:**

a large momentum EFT can relate Euclidean $\tilde{q}$ **to PDFs through a factorization theorem**

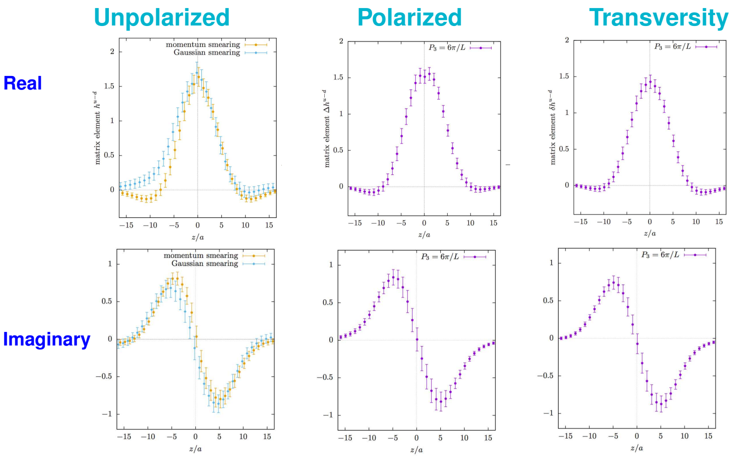
★ **use of Perturbation Theory for the matching**

Prior 2017

Bare Nucleon Matrix Elements

[C. Alexandrou et al. (ETMC), arXiv:1504.07455, arXiv:1610.03689]

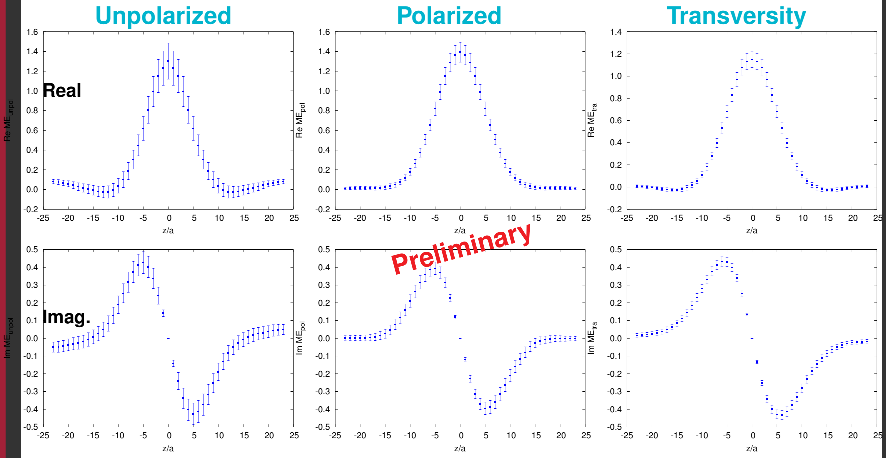
Twisted Mass Fermions, m_π=375MeV, P_3=6π/L



★ **Momentum smearing allows to reach higher momenta**

Bare Nucleon Matrix Elements

Twisted Mass Fermions & clover term, m_π=130MeV P_3=6π/L



★ **Currently increasing momentum to** P_3=8π/L, 12π/L

2017 and After

Renormalization... Finally!

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Based on:

M. Constantinou, H. Panagopoulos, Phys. Rev. D, in Press, [arXiv:1705.11193]

C. Alexandrou, et al., Nucl. Phys. B (Frontier Article), in Press, [arXiv:1706.00265]

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How can Perturbation Theory help?

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How can Perturbation Theory help?

- ★ **Computation of conversion factor between various renormalization schemes**
- ★ **Explore renormalization pattern in Lattice Pert. Theory**

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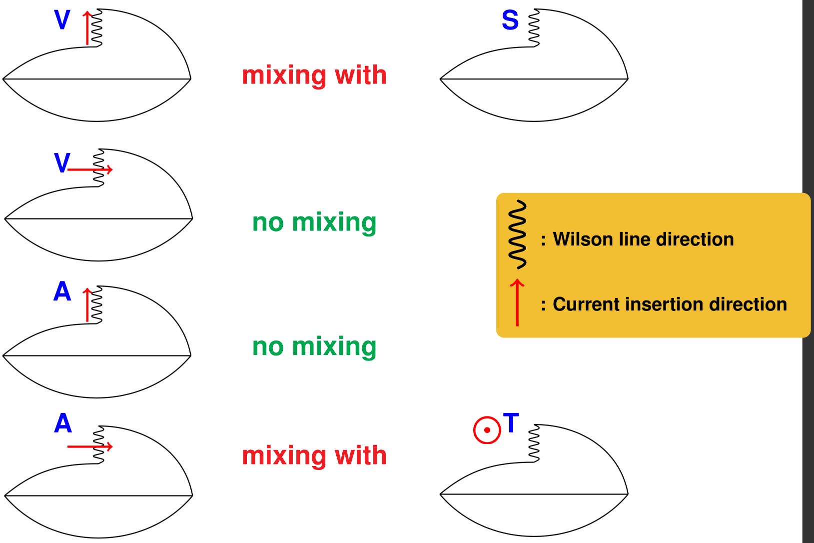
Mixing was revealed... unexpectedly !

Mixing pattern (Identified in PT)

Depends on the relation between the current & Wilson line direction



Non-perturbative Renormalization

- RI′ scheme
- Use **1-loop conversion factor** to convert to the $\overline{\text{MS}}$ at 2 GeV
- Vertex function has the same divergence as the nucleon matrix element

No mixing

Helicity & transversity

$$Z_{\mathcal{O}}(z) = \frac{Z_q}{\mathcal{V}_{\mathcal{O}}(z)}$$

$$\mathcal{V}_{\mathcal{O}} = \frac{\text{Tr}}{12}\left[\mathcal{V}(p)\left(\mathcal{V}^{\text{Born}}(p)\right)^{-1}\right]\Big|_{p=\bar{\mu}}$$

- Z_q: fermion field renormalization
- $Z_{\mathcal{O}}$ includes the linear divergence

Mixing

Unpolarized

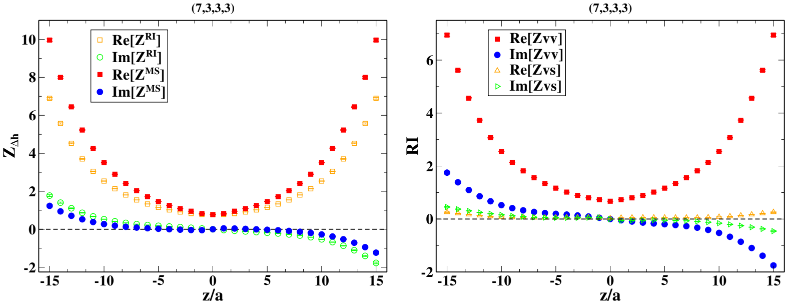
$$\begin{pmatrix}\mathcal{O}^R_V(P_3,z)\\ \mathcal{O}^R_S(P_3,z)\end{pmatrix} = \hat{Z}(z)\cdot\begin{pmatrix}\mathcal{O}_V(P_3,z)\\ \mathcal{O}_S(P_3,z)\end{pmatrix}$$

$$Z_q^{-1}\,\hat{Z}(z)\,\hat{\mathcal{V}}(p,z)\Big|_{p=\bar{\mu}} = \hat{1}$$

$$\begin{aligned} h^R_V(P_3,z) &= Z_{VV}(z)\;h_V(P_3,z)\\ &+ Z_{VS}(z)\;h_S(P_3,z)\end{aligned}$$

Numerical Results

- ★ **Twisted Mass fermions,** m_π=375MeV, $32^3 \times 64$, **HYP smearing**
- ★ **Conversion** & **Evolution to** $\overline{\text{MS}}$**(2GeV)** **(Perturbatively)**



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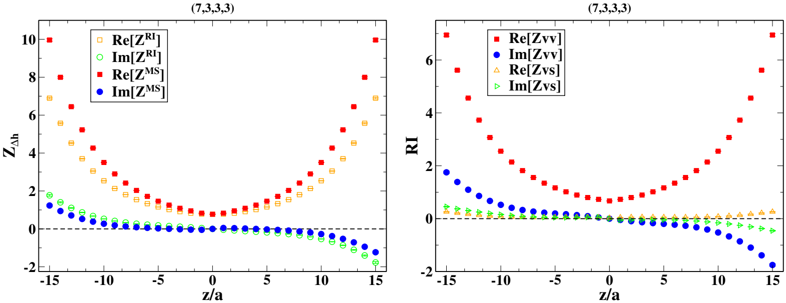
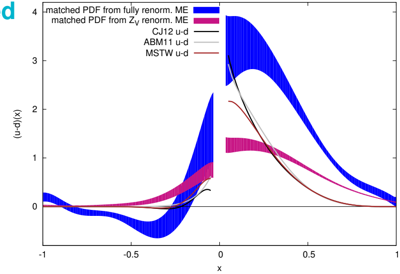


★ **Systematics need to be addresses (Upper bounds in arXiv:1706.00265) :**

Effect	$\text{Re}\,[Z^{\overline{\text{MS}}}_{\Delta h}]$	$\text{Im}\,[Z^{\overline{\text{MS}}}_{\Delta h}]$
Lattice artifacts	2-5%	$\lesssim 10$%
Conversion truncation	$\lesssim 2$%	$\lesssim 100$%

Renormalized PDFs @ $P_z = 6\pi/L$

Unpolarized

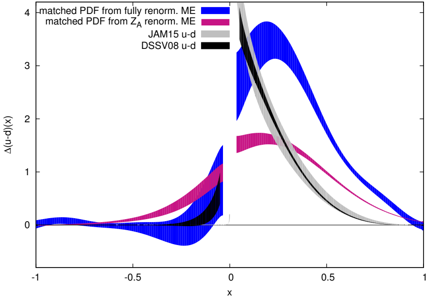


Mixing not included

Twisted Mass fermions:

Mixing with Pseudoscalar ($\mathcal{O}(a)$)

Polarized



- ★ **Results are promising**
- **Renormalization brings lattice data closer to the phenomenological estimates**
- **Need to reach higher momenta**

E

DISCUSSION

DISCUSSION

Lattice QCD:

★ **Addressing open questions, e.g. spin:**

- **quarks:** $\sim 80\%$ **of spin and** 75% **of momentum**
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THANK YOU

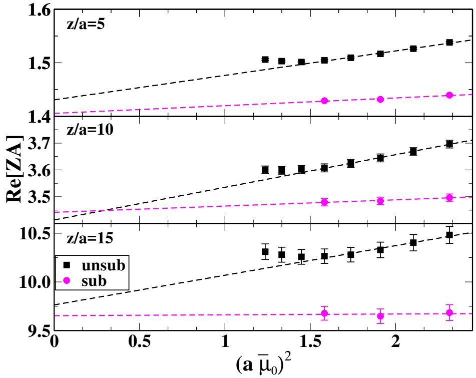
BACKUP SLIDES

Refining Renormalization

★ **Improvement Technique:**

- **Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2\, a^\infty)$**
- **Subtraction of lattice artifacts from non-perturbative estimated**

★ **Application to the quasi-PDFs:** **PRELIMINARY**



Quark Orbital Angular Momentum