Covariant extension of Generalized Parton Distributions

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Outline

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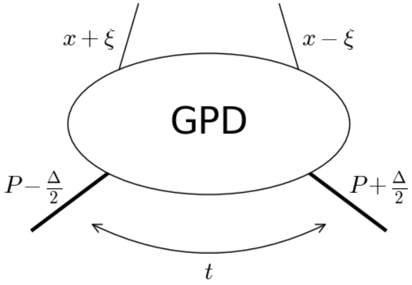
1. Introduction to Generalized Parton Distributions
   - Definition and properties
   - Experimental access
2. Representations of Generalized Parton Distributions
   - Overlap of Light-cone wave functions
   - Double Distributions
3. Covariant extention of Generalized Parton Distributions
   - Motivation
   - Inversion of Incomplete Radon Transform
   - Results
4. Conclusion

Definition of GPDs

- Quark GPD (twist-2, spin-0 hadron): **(Müller et al., 1994; Radyushkin, 1996; Ji, 1997)**

$$H^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,x\,P^+z^-} \left\langle P+\frac{\Delta}{2}\left|\bar{q}(-z)\,\gamma^+ q(z)\right| P-\frac{\Delta}{2}\right\rangle\Big|_{z^+=0,\,z_\perp=0} . \quad (1)$$



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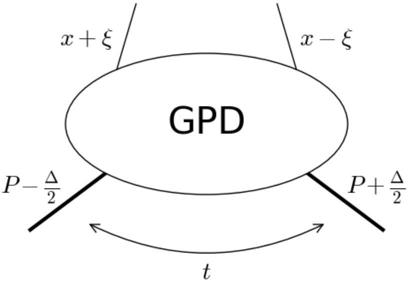

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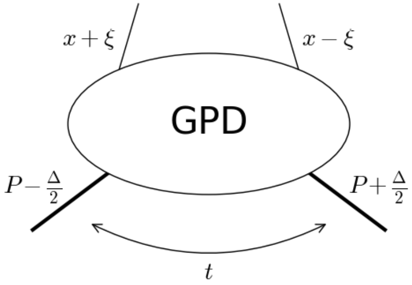

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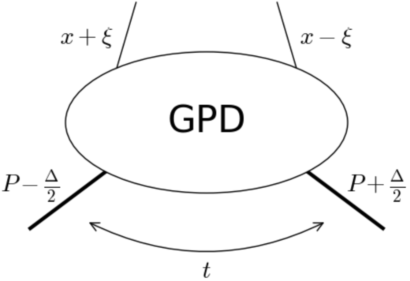

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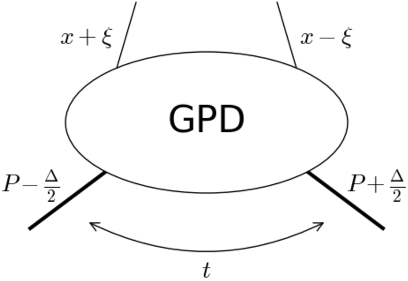

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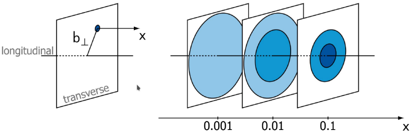

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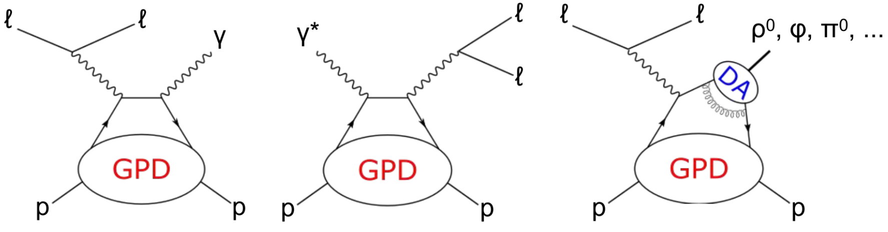

DVCS TCS DVMP

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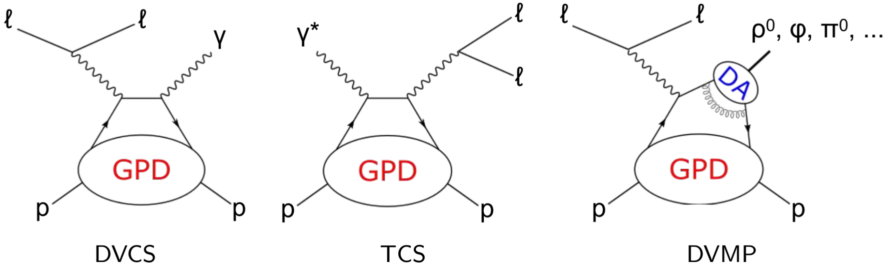

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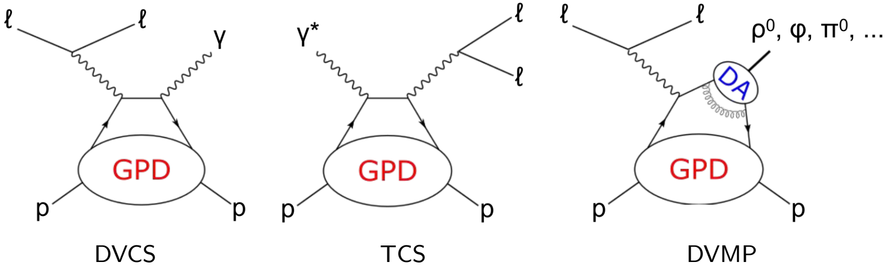

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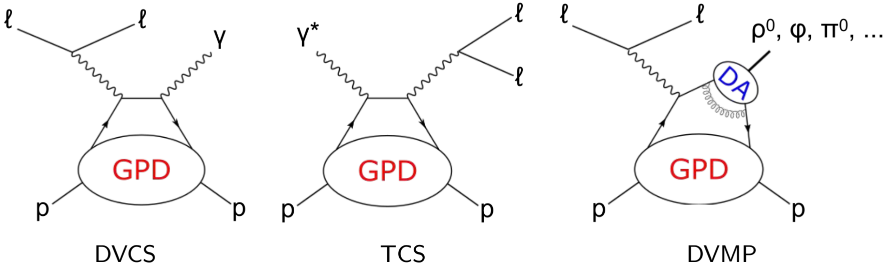

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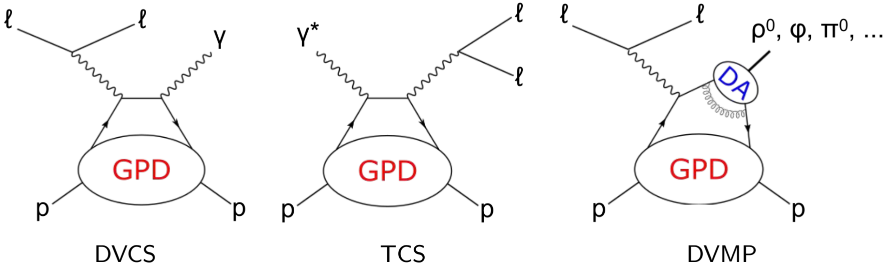

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  - a hard-scattering kernel, calculated with perturbative QCD (short distance interactions).

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$$H(x,\xi,t) \propto \int_\Omega \mathrm{d}\beta\, \mathrm{d}\alpha\, h(\beta,\alpha,t)\, \delta(x-\beta-\alpha\xi)\ . \tag{13}$$

- DD h is defined on the support $\Omega = \{|\beta| + |\alpha| \leq 1\}$.
- **Polynomial** in ξ:

$$\begin{aligned} \int_{-1}^{1} \mathrm{d}x\, x^m\, H(x,\xi,t) &\propto \int \mathrm{d}x\, x^m \int_\Omega \mathrm{d}\beta\, \mathrm{d}\alpha\, h(\beta,\alpha,t)\, \delta(x-\beta-\alpha\xi) \\ &\propto \int_\Omega \mathrm{d}\beta\, \mathrm{d}\alpha\, (\beta+\xi\alpha)^m\, h(\beta,\alpha,t)\ . \end{aligned} \tag{14}$$

- Positivity not manifest...

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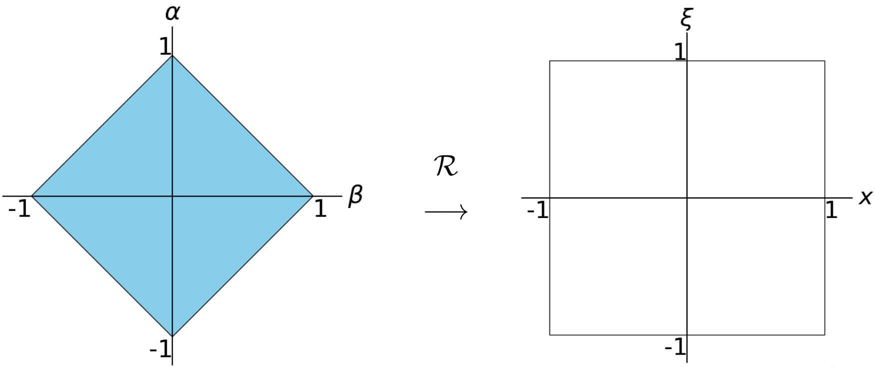

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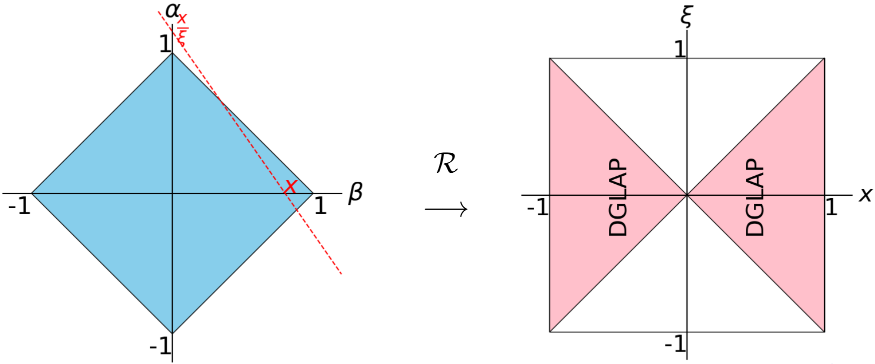

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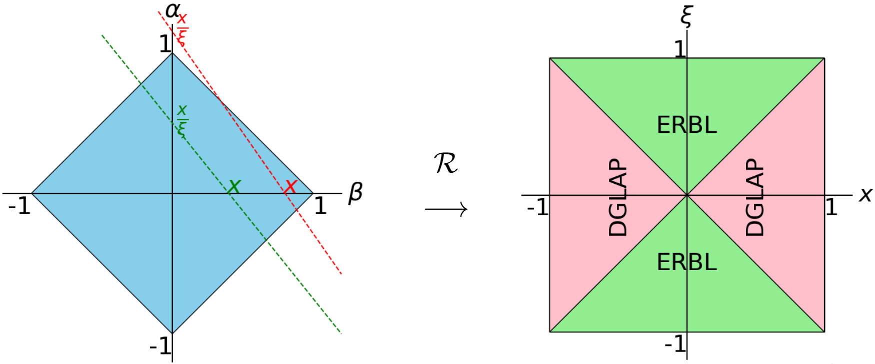
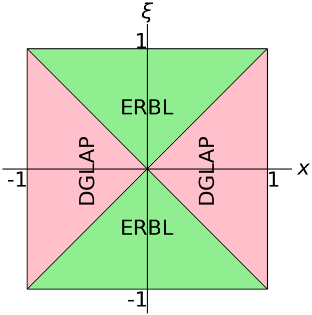


Outline

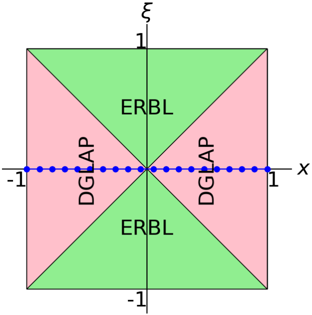
Covariant extension

- What do we want?



Covariant extension

- What do we want?
    - GPD at $\xi = 0$, for nucleon tomography.



Covariant extension

- What do we want?
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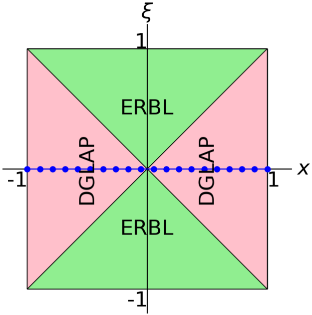

Covariant extension

- What do we want?
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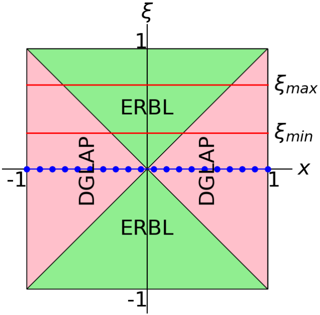
- What do we have?
  - Experimental access: integrals over x of GPD at $\xi > 0$.



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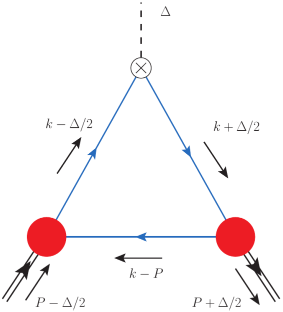
- Covariant approach?



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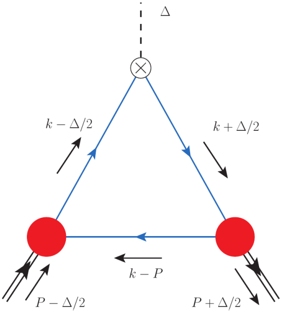
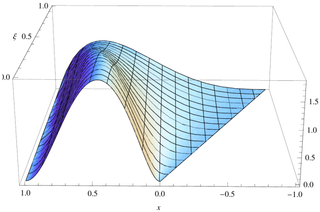
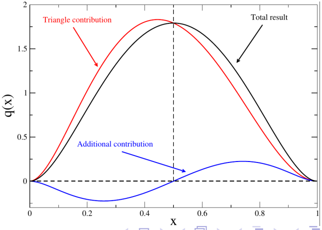
- Covariant approach?
  - Positivity?







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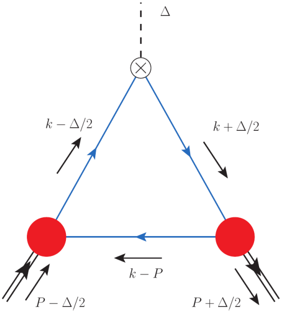

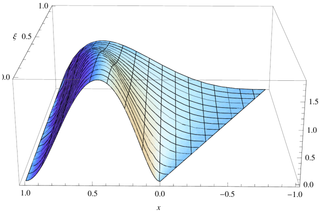

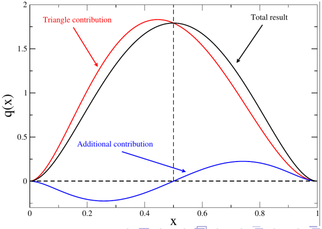

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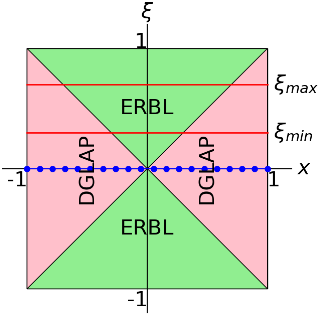

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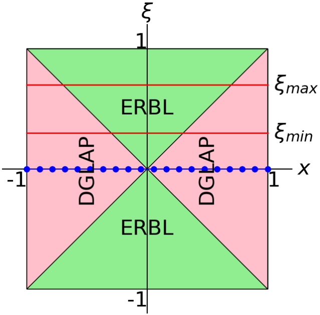

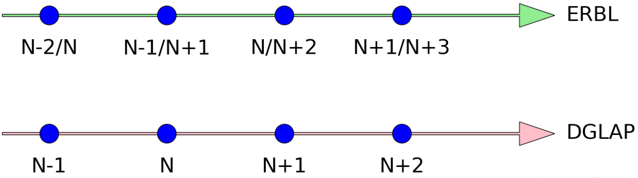

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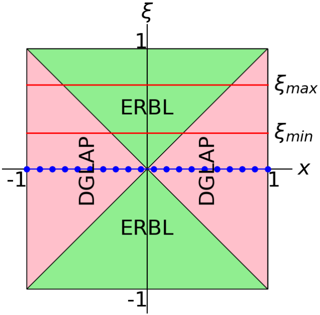

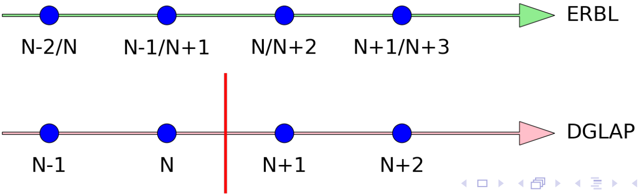

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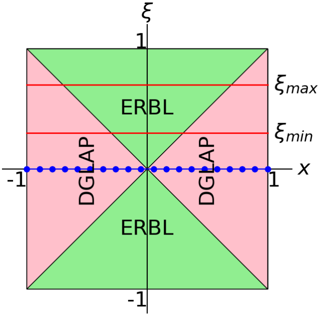

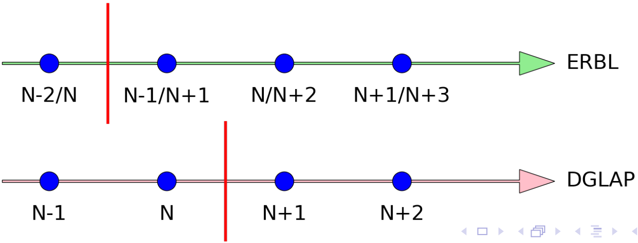

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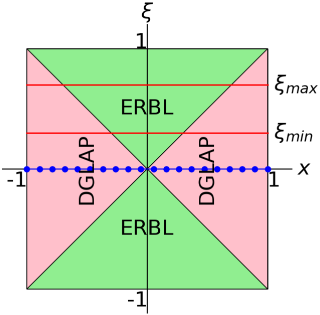

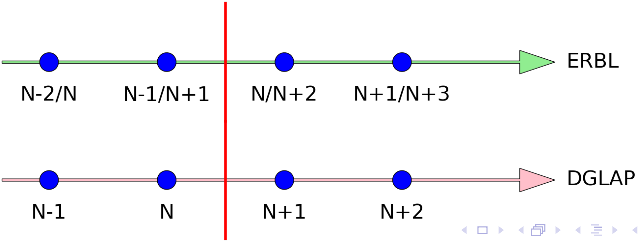

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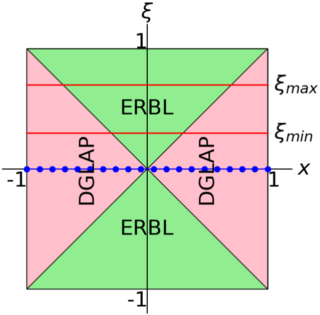

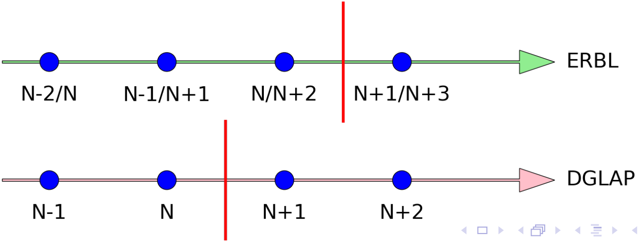

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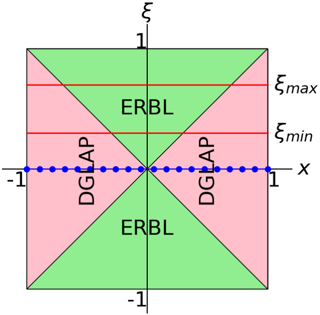

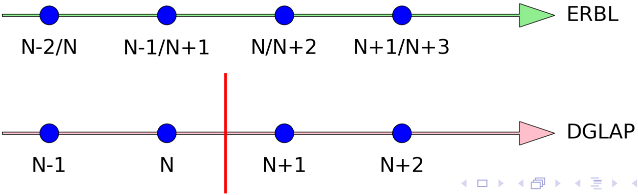

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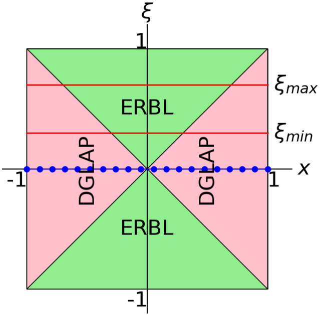

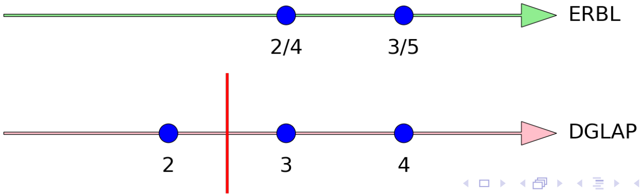

Inversion

Problem

Find $h(\beta, \alpha)$ on square $\{|\alpha| + |\beta| \leq 1\}$ such that

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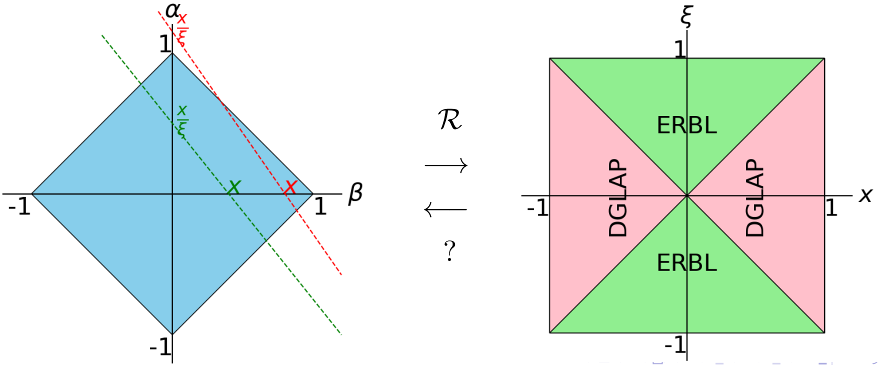
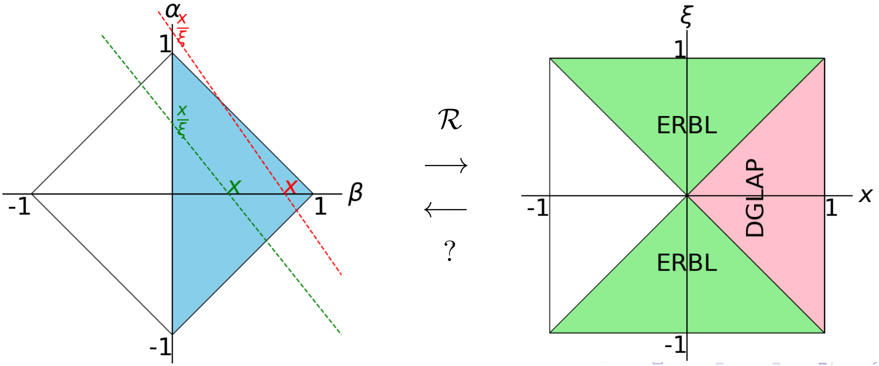


Inversion

- Quark GPD: $H(x,\xi) = 0$ for $-1 < x < -|\xi| \Longrightarrow h(\beta,\alpha) = 0$ for $\beta < 0$.



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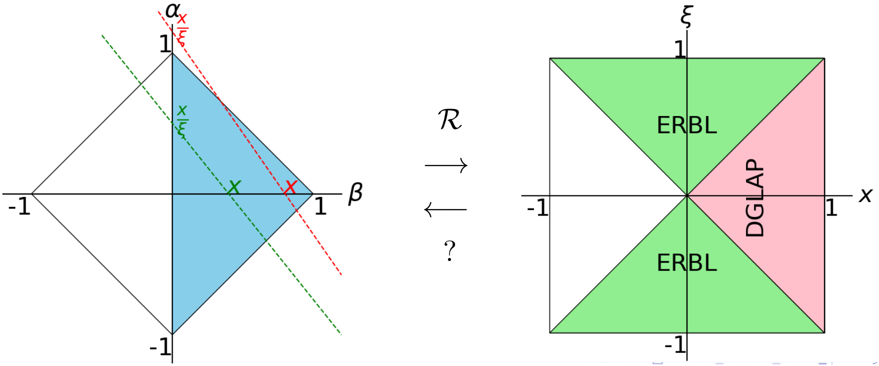

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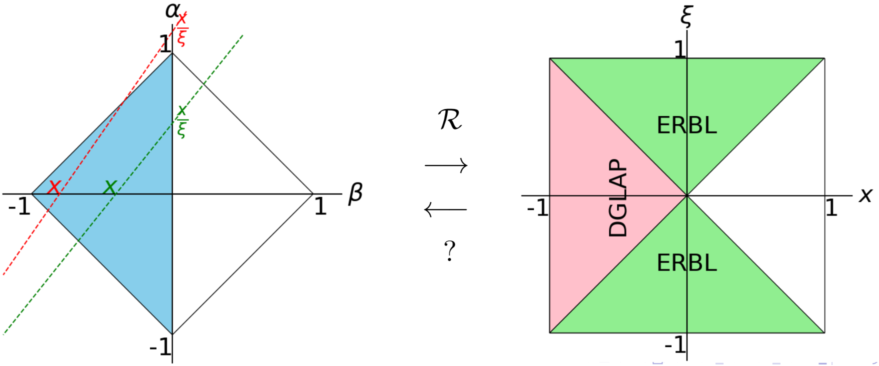

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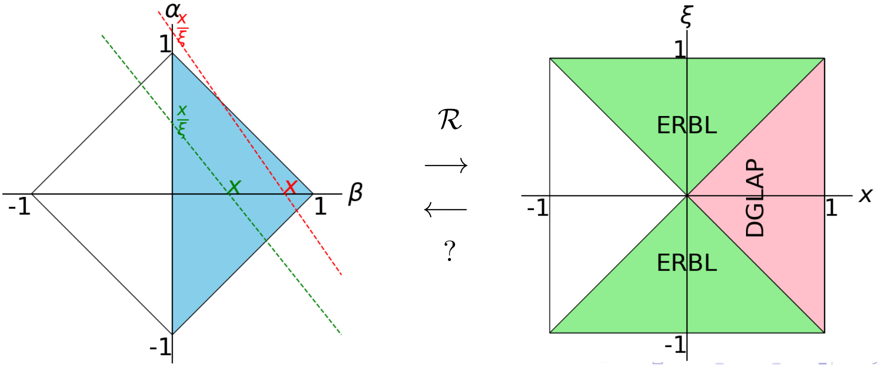

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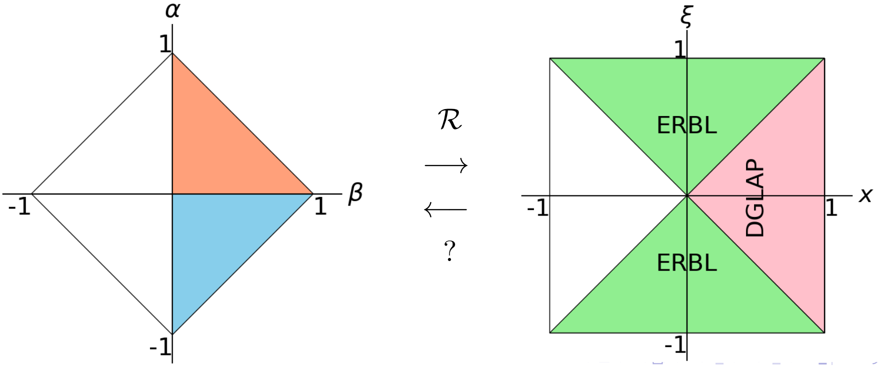

Discretization

- Expansion of the DD into basis functions $\{v_j\}$:

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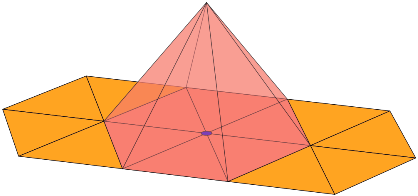

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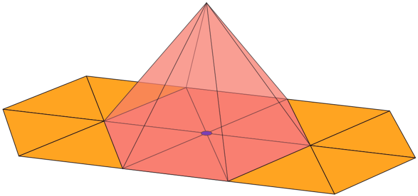
  - n columns of the matrix.



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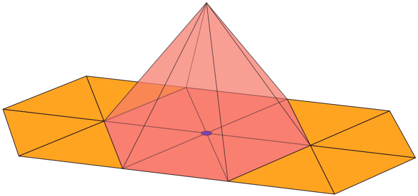

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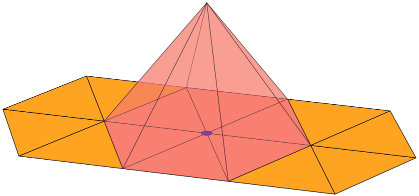

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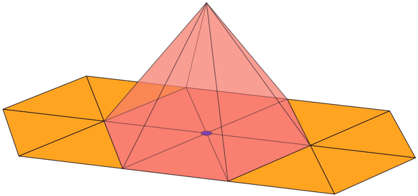

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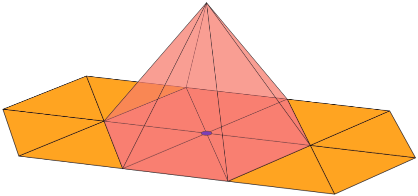

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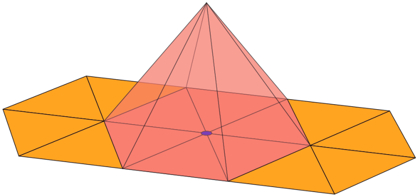
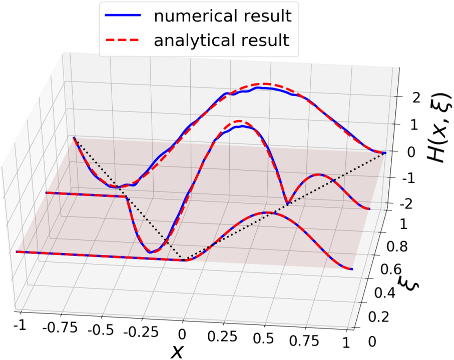
  - Trade-off between noise and convergence.



Some examples (Dyson-Schwinger model)



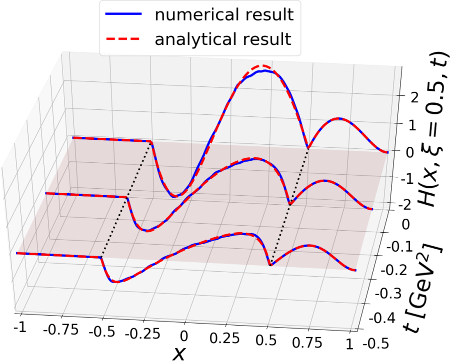


Figure: Extension of GPDs for the pion DSE model of Refs. **(Mezrag, 2015; Mezrag et al., 2016)**. Comparison to the analytical result. Left: Plot for fixed ξ values 0, 0.5 and 1, at $t = 0$ GeV^2. Right: Plot for fixed t values 0, -0.25 and -0.5 GeV^2, at $\xi = 0.5$.

Some examples (Spectator model)

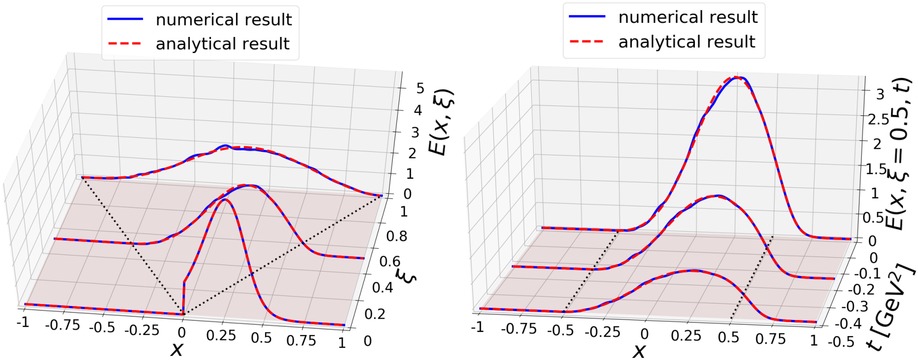


Figure: Extension of GPD E for the nucleon model of Ref. **(Hwang and Mueller, 2008)**. Comparison to the analytical result of the authors. Left: Plot for fixed ξ values 0, 0.5 and 1, at $t = 0$ GeV^2. Right: Plot for fixed t values 0, -0.25 and -0.5 GeV^2, at $\xi = 0.5$.

Some examples (gaussian model)

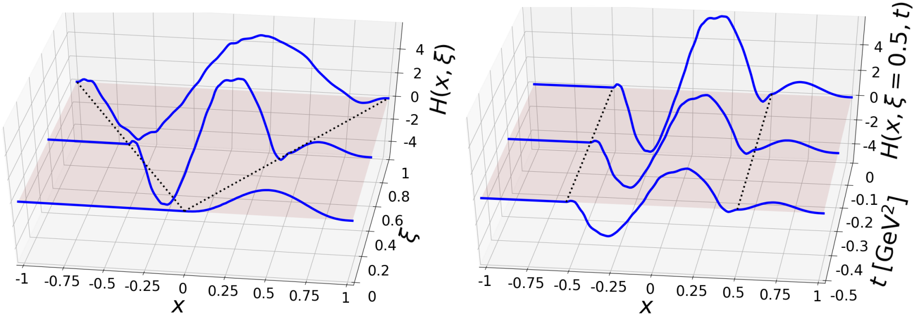


Figure: Extension of GPD for a gaussian pion model (in the vein of AdS/QCD). Left: Plot for fixed ξ values 0, 0.5 and 1, at $t = 0$ GeV^2. Right: Plot for fixed t values 0, -0.25 and -0.5 GeV^2, at $\xi = 0.5$.

Some examples (Regge behavior)

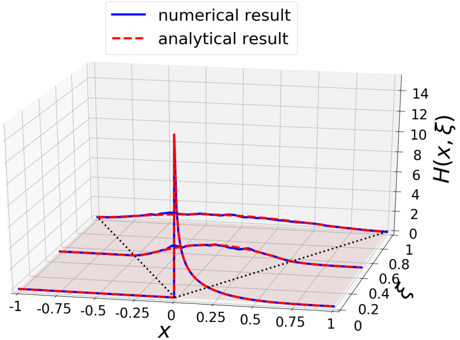


Figure: Extension of GPD for a nucleon toy model with Regge behavior. Plot for fixed ξ values 0, 0.5 and 1.

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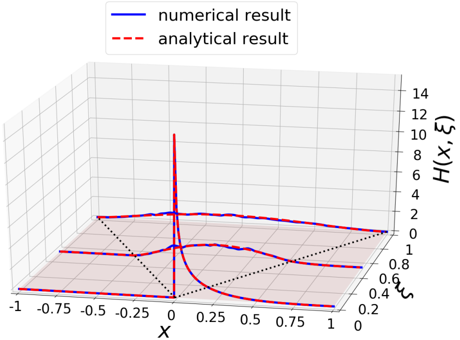

Figure: Extension of GPD for a nucleon toy model with Regge behavior. Plot for fixed ξ values 0, 0.5 and 1.

- Integrable singularity for the GPD at $x \sim 0$: $H(x, \xi) \propto \frac{1}{\sqrt{x}}$.

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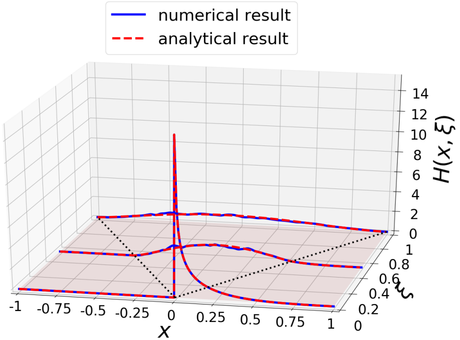

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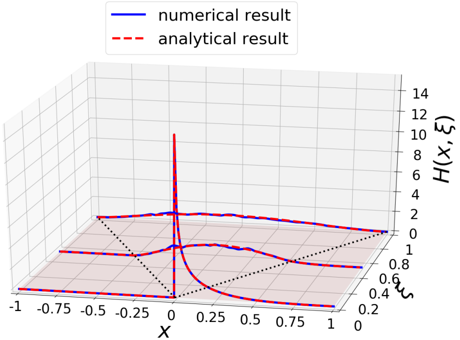

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  - We solve for $\sqrt{\beta}\, h(\beta,\alpha)$ instead of $h(\beta,\alpha)$!

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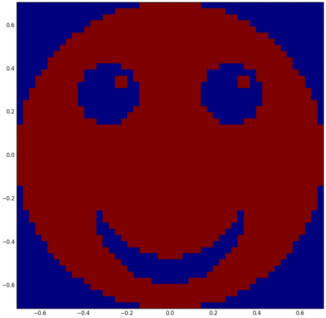
- Thank you!



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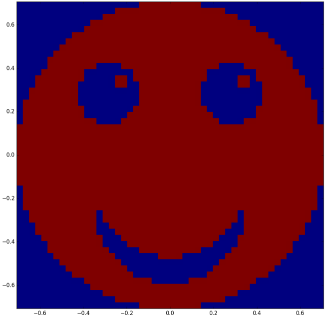
- Thank you!
  - Any questions?

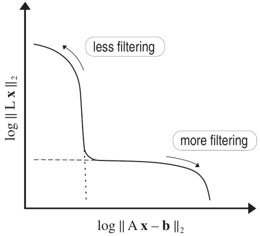


Ill-posed problems and Regularization

- Ill-posed problems?
  - For example the inversion of a Fredholm equation of the first kind:

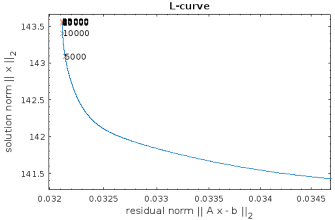
$$\int K(x,y)\, f(y)\, \mathrm{d}y = g(x)\,. \tag{16}$$

  - The inverse is not continuous: an arbitrarily small variation Δg of the rhs can lead to an arbitrarily large variation Δf of the solution.
- The corresponding discrete problem needs to be regularized.
  - E.g Tikhonov regularization: $\min\left\{\|AX - B\|^2 + \epsilon\,\|X\|^2\right\}$.



Theoretical "L-curve": curve parameterized by the regularization factor.

(fig. taken from Ref. (Hansen, 2007))



L-curve with the iteration number as regularization factor.

D-term considerations

- Polynomiality property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H(x, \xi, t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\xi^k \,. \tag{17}$$

- Recast polynomiality property for $H - D$:

$$\int_{-1}^{1} \mathrm{d}x\, x^m \left(H(x, \xi, t) - D\left(\frac{x}{\xi}, t\right)\right) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m} c_k^{(m)}(t)\xi^k \,, \tag{18}$$

where $D\left(\frac{x}{\xi}, t\right)$ is the so-called D-term with support on $-\xi < x < \xi$.

- $H - D$ is a Radon Transform:

$$H(x, \xi, t) - D\left(\frac{x}{\xi}, t\right) = \int_{\Omega} \mathrm{d}\beta\, \mathrm{d}\alpha\, h_{\mathrm{PW}}(\beta, \alpha)\, \delta(x - \beta - \alpha\xi) \,. \tag{19}$$

  - **The DGLAP region gives no information on the D-term.**

- With other DD representations, we can generate intrinsic D-terms, e.g. Pobylitsa representation:

$$H(x, \xi, t) = (1 - x) \int_{\Omega} \mathrm{d}\beta\, \mathrm{d}\alpha\, h_{\mathrm{P}}(\beta, \alpha)\, \delta(x - \beta - \alpha\xi) \,. \tag{20}$$

  - **Still freedom of extra D-term.**

Bibliography I

D. Müller, D. Robaschik, B. Geyer, F. M. Dittes, and J. Hořejši, "Wave functions, evolution equations and evolution kernels from light ray operators of QCD", *Fortsch. Phys.* **42** (1994) 101–141, arXiv:hep-ph/9812448 [hep-ph].

A. V. Radyushkin, "Scaling limit of deeply virtual Compton scattering", *Phys. Lett.* **B380** (1996) 417–425, arXiv:hep-ph/9604317 [hep-ph].

X.-D. Ji, "Deeply virtual Compton scattering", *Phys. Rev.* **D55** (1997) 7114–7125, arXiv:hep-ph/9609381 [hep-ph].

M. Burkardt, "Impact parameter dependent parton distributions and off forward parton distributions for zeta —> 0", *Phys. Rev.* **D62** (2000) 071503, arXiv:hep-ph/0005108 [hep-ph], [Erratum: Phys. Rev.D66,119903(2002)].

B. Pire, J. Soffer, and O. Teryaev, "Positivity constraints for off - forward parton distributions", *Eur. Phys. J.* **C8** (1999) 103–106, arXiv:hep-ph/9804284 [hep-ph].

A. V. Radyushkin, "Double distributions and evolution equations", *Phys. Rev.* **D59** (1999) 014030, arXiv:hep-ph/9805342 [hep-ph].

A. V. Belitsky, D. Mueller, and A. Kirchner, "Theory of deeply virtual Compton scattering on the nucleon", *Nucl. Phys.* **B629** (2002) 323–392, arXiv:hep-ph/0112108 [hep-ph].

S. J. Brodsky, T. Huang, and G. P. Lepage, "Hadronic wave functions and high momentum transfer interactions in quantum chromodynamics", *Conf. Proc.* **C810816** (1981) 143–199.

Bibliography II

M. Diehl, T. Feldmann, R. Jakob, and P. Kroll, “The overlap representation of skewed quark and gluon distributions”, *Nucl. Phys.* **B596** (2001) 33–65, arXiv:hep-ph/0009255 [hep-ph], [Erratum: Nucl. Phys.B605,647(2001)].

M. Diehl, “Generalized parton distributions”, *Phys. Rept.* **388** (2003) 41–277, arXiv:hep-ph/0307382 [hep-ph].

S. R. Deans, “The Radon Transform and Some of Its Applications”, Wiley-Interscience, 1983.

O. V. Teryaev, “Crossing and radon tomography for generalized parton distributions”, *Phys. Lett.* **B510** (2001) 125–132, arXiv:hep-ph/0102303 [hep-ph].

C. Mezrag, “Generalised Parton Distributions : from phenomenological approaches to Dyson-Schwinger equations”, PhD thesis, IRFU, SPhN, Saclay, 2015.

C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, “From Bethe–Salpeter Wave functions to Generalised Parton Distributions”, *Few Body Syst.* **57** (2016), no. 9, 729–772, arXiv:1602.07722 [nucl-th].

D. S. Hwang and D. Mueller, “Implication of the overlap representation for modelling generalized parton distributions”, *Phys. Lett.* **B660** (2008) 350–359, arXiv:0710.1567 [hep-ph].

P. C. Hansen, “Regularization Tools version 4.0 for Matlab 7.3”, *Numerical Algorithms* **46** (2007), no. 2, 189–194.