# Twist-3 GPDs

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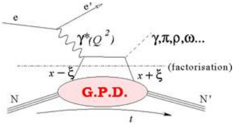
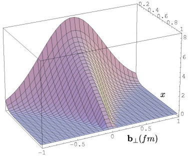


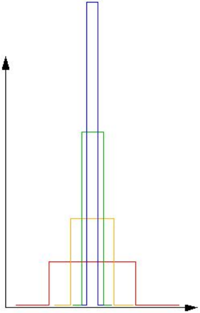



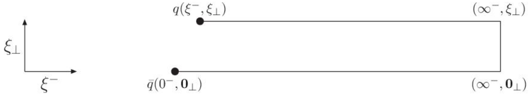

- Motivation: why twist-3 GPDs
  - twist-3 GPD $G_2^q \longrightarrow L^q$
  - twist 3 PDF $g_2(x) \longrightarrow \perp$ force
  - twist 2 GPDs $\longrightarrow \perp$ imaging (of quark densities)

  $\hookrightarrow$ twist 3 GPDs $\overset{?}{\rightsquigarrow} \perp$ imaging of $\perp$ forces
- $\delta(x)$ contributions to twist-3 PDFs

$\hookrightarrow$ twist-3 GPDs $\longrightarrow$ discontinuities at $x \pm \xi$

- making the world safer for twist-3 factorization
- Summary
- Outlook





# OAM from twist 3 GPDs 

## twist-3 GPDs

Polyakov & Kitpily

$$\int dz^- e^{ixz^-\bar{p}^+} \langle p'|\bar{q}(z^-/2)\gamma^x q(-z^-/2)|p\rangle$$

$$= \frac{1}{2\bar{p}^+}\bar{u}(p')\left[\frac{\Delta^x}{2M}G_1 + \gamma^x(H+E+G_2) + \frac{\Delta^x\gamma^+}{\bar{p}^+}G_3 + \frac{i\Delta^y\gamma^+\gamma_5}{\bar{p}^+}G_4\right]u(p)$$

## Lorentz invariance relations

- $\int dx G_1^q(x,\xi,t) = 0$
- $\int dx G_2^q(x,\xi,t) = 0$
- $\int dx G_3^q(x,\xi,t) = 0$
- $\int dx G_4^q(x,\xi,t) = 0$

## QCD Eqs. of motion

Polyakov & Kitpily

- $\int dx\, x G_2^q(x,0,0) = -L^q$
- same relation also derived in scalar Yukawa

## Tests

- test above relations in scalar diquark model & QED
- $\mathcal{L}(x) \stackrel{?}{=} -\int_x^1 dy G_2(y)$

## issues

- $\delta(x)$ in $G_2^q$?
- $G_2^q$ from DVCS?

- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$ (nonrelativistic)
  reference point is center of mass
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction $x$

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction $x$

- careful: cannot measure longitudinal momentum ($x$) and longitudinal position simultaneously (Heisenberg)

$\hookrightarrow$ consider purely transverse momentum transfer

## Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

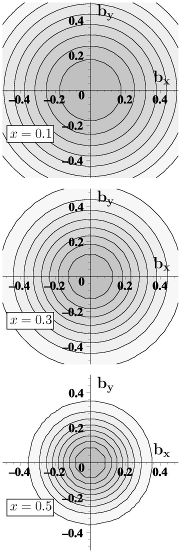
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

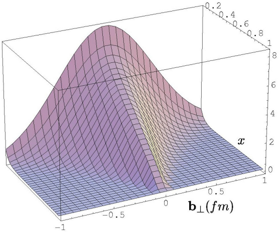
- No relativistic corrections (Galilean subgroup!)

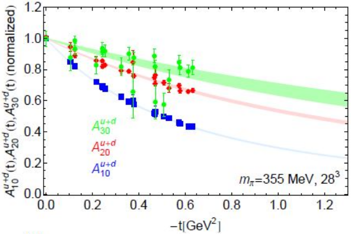
$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free of relativistic corrections ($\rightarrow$G.Miller)

- probabilistic interpretation

$q(x, \mathbf{b}_\perp)$ for unpol. p




## unpolarized proton

MB,PRD 62, 071503 (2000)

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $F_1(-\boldsymbol{\Delta}_\perp^2) = \int dx H(x, 0, -\boldsymbol{\Delta}_\perp^2)$
- $x$ = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small $x$: large 'meson cloud'
- larger $x$: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$

## $d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between $g_2$ and $\frac{1}{Q^2}$ corrections to $g_1$

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

## color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral
$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

## sign of $d_2$

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of $d_2^q$: opposite Sivers

## magnitude of $d_2$

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

- take $x^2$ moment of twist-3 GPDs ($\xi = 0$)
- subtract twist-2 parts
- take 2D Fourier transform

$$\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \int dx\, x^2 \tilde{G}_2^{tw\,3}(x, 0, -\Delta_\perp^2)$$

$\hookrightarrow$ $\langle R_\perp = 0, S_\perp | \bar{q}(b_\perp)\gamma^+ gF^{+y}(b_\perp) q(b_\perp) | R_\perp = 0, S_\perp \rangle$

$\hookrightarrow$ distribution of $\perp$ force in $\perp$ plane for transversely polarized target & unpol. quarks

- $x^2$ moments of other twist-3 GPDs provide info about $\perp$ force tomography for other spin correlations

$\hookrightarrow$ twist-3 GPDs $\Rightarrow$ 2D $\perp$ force maps

- could be done immediately in lattice QCD
- need to address some issues regarding experimental access...

## example: scalar diquark

$$q_\Gamma(x,k_\perp)=\int dk^-\,\bar{u}(P,S)\frac{\not{k}+m}{k^2-m^2+i\varepsilon}\Gamma\frac{\not{k}+m}{k^2-m^2+i\varepsilon}u(P,S)\frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- similar for quark target (QCD)
- $k^+ = xp^+$

## denominator integral

$$I_{den} \equiv \int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2}\frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- $k^2 = 2k^+k^- - k_\perp^2$, $(P-k)^2 = 2(P^+-k^+)(P^- - k^-) - k_\perp^2$
- $I_{den} = 0$ for $k^+ < 0$: all $k^-$ poles in UHP
- $I_{den} = 0$ for $k^+ > P^+$: all $k^-$ poles in LHP
- $I_{den} = \frac{-\pi i}{P^+(1-x)x^2}\frac{1}{\left[2P^+P^- - \frac{k_\perp^2+m^2}{x} - \frac{k_\perp^2+\lambda^2}{1-x}\right]^2}$
- twist-2: $\Gamma$ contains $\gamma^+$; $\not{k} = k^-\gamma^+$

$\hookrightarrow$ numerator only function of $x$, $k_\perp$ as $\gamma^+\gamma^+ = 0$ $\quad\Rightarrow$ straightforward!

## example: scalar diquark

$$q_\Gamma(x,k_\perp)=\int dk^- \bar{u}(P,S)\frac{\not{k}+m}{k^2-m^2+i\varepsilon}\Gamma\frac{\not{k}+m}{k^2-m^2+i\varepsilon}u(P,S)\frac{1}{(P-k)^2-\lambda^2+i\varepsilon}$$

- similar for quark target (QCD)
- similar for 1-loop corrections

## twist-3: example $\Gamma = \mathbb{1}$

- numerator $(\not{k}+m)^2 = k^2+m^2+2m\not{k}$
- $\bar{u}(P,S)\not{k}u(P,S) = 2P^+k^- + ...$
- $2k^- = \frac{(P-k)^2-\lambda^2}{P^+-k^+} - \left[P^- - \frac{k_\perp^2+\lambda^2}{P^+-k^+}\right]$
- $2^{nd}$ term canonical (from LF Hamiltonian pert. theory → SJB)
- $1^{st}$ term cancels spectator propagator

↪ $I_\delta = \int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = \int dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2+i\varepsilon)^2} = ?$

- $I_\delta = 0$ for $k^+=0$ as pole can be avoided
- $\int d^2k_L \frac{1}{(k^2-m^2+i\varepsilon)^2} \equiv \int dk^+dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = \frac{\pi i}{k_\perp^2+\lambda^2} \Rightarrow I_\delta = \frac{\pi i}{k_\perp^2+\lambda^2}\delta(k^+)$

# Relevance for QCD



## sum rules for twist-3 PDFs

MB, PRD **52**, 3841 (1995)

- $\int_{-1}^{1} dx g_T(x) = \int_{-1}^{1} dx g_1(x)$
- $\int_{-1}^{1} dx h_L(x) = \int_{-1}^{1} dx h_1(x)$
- $\int_{-1}^{1} dx e(x) = \frac{1}{2M}\langle P|\bar{q}q|P\rangle$ ($\sigma$-term sum rule)
- first two are Lorentz invariance (LI) relations

If sum rule is tested by evaluating e.g. $\lim_{\varepsilon\to 0}\int_{\varepsilon}^{1} dx\,[h_L(x)+h_L(-x)]$ then presence of $\delta(x)$ in $h_L$ would result in violation of LI relation!

## violation of twist-3 sum rules in QCD

MB & Y. Koike, NPB **632**, 311 (2002)

Using moment relations based on QCD eqs. of motion one finds

- $h_L^\delta(x) = -\frac{m_q}{2M}\left[g_1(0^+) - g_1(0^-)\right]$ (LI relation 'violated' at 1-loop)
- $g_T^\delta(x) = -\frac{m_q}{M}\left[h_1(0^+) - h_1(0^-)\right]$ (LI relation o.k. at 1-loop)
- $\sigma$-term sum rule 'violated' at 1-loop

## implications for twist-3 GPDs

what does presence of $\delta(x)$ in twist-3 PDFs imply for twist-3 GPDs?

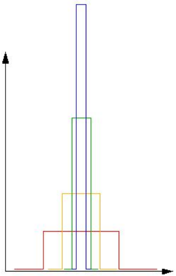
- relevant energy denominators:

$$\int dk^- \frac{1}{\left(k-\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{\left(k+\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{(P-k)^2 - \lambda^2 + i\varepsilon}$$

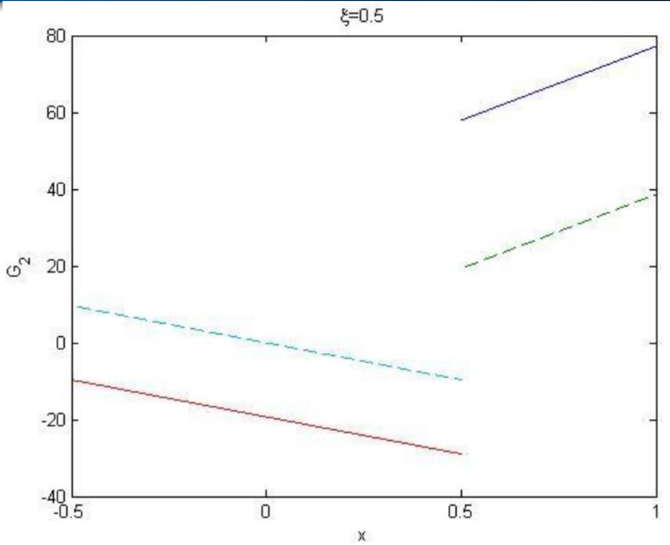
- twist-3: $k^-$ from Dirac numerator can cancel $(P-k)^2 - \lambda^2 + i\varepsilon$

$\hookrightarrow$ $\int dk^- \frac{1}{\left(k-\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{\left(k+\frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \sim \frac{\Theta\left(\frac{-\Delta^+}{2} < k^+ < \frac{\Delta^+}{2}\right)}{\Delta^+} \frac{1}{k_\perp^2 + m^2}$

- contribution to ERBL region only!
  - nonzero only for $-\xi < x < \xi$
  - discontinuous at $x \pm \xi$
  - $\propto \frac{1}{\xi}$ for $-\xi < x < \xi$
  
  $\hookrightarrow$ representation of $\delta$ function as $\xi \to 0$
- big issue: convergence of $\int \frac{dx}{x-\xi} GPD(x,\xi,t)$ when $GPD(x,\xi,t)$ discontinuous at $x \pm \xi$
- presence of such terms 'normal' for twist-3 GPDs

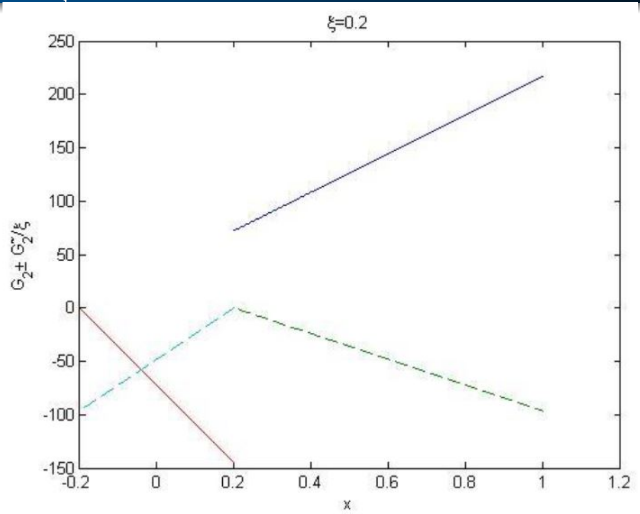


# $G_2, \tilde{G}_2$ in QCD (1 loop) 




- $G_2(\Gamma = \gamma_\perp)$, $\tilde{G}_2(\Gamma = \gamma_\perp\gamma_5)$ discontinuous at $x = -\xi$
- $\int \frac{dx}{x \pm \xi} G_2(x, \xi, t)$ divergent — oops!

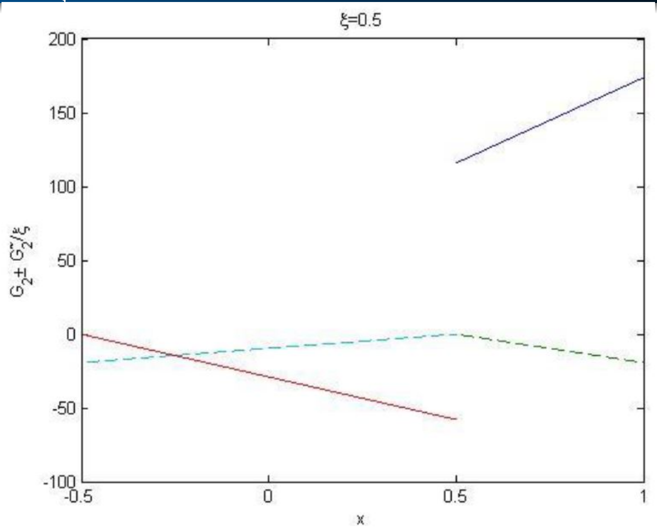
$\hookrightarrow$ factorization?




- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$

$\hookrightarrow$ makes world a lot safer for twist-3 factorization!

# $G_2 \pm \frac{1}{\xi}\tilde{G}_2$ in QCD (1 loop)




- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$

$\hookrightarrow$ makes world a lot safer for twist-3 factorization!

## quasi PDFs/TMDs

- Let $\rho_P^\Gamma(k_z, k_\perp)$ be momentum distribution of quarks
- $P$ momentum of nucleon (in $\hat{z}$-direction)
- $\Gamma$: Dirac structure of quark bilinear ($\Gamma = \gamma^z$ for twist 2, unpol.)

$\hookrightarrow$ $q_\Gamma(x, k_\perp) \equiv \lim_{P\to\infty} P\rho_P^\Gamma(xP, k_\perp)$ 'quasi-PDF' X.Ji++

## twist-3 quasi PDFs $\Gamma = \mathbb{1}$ (quark target model)

$\rho_P^{\mathbb{1}}(k_z, k_\perp) \sim \int dk_0 \frac{k^2+m^2+2p\cdot k}{[k^2-m^2]^2[(p-k)^2-\lambda^2]}$

- $2p\cdot k = p^2 + k^2 - (p-k)^2 = p^2 + k^2 - \lambda^2 - \left[(p-k)^2 - \lambda^2\right]$

$\hookrightarrow$ contribution to $\rho_P^\Gamma(k_z, k_\perp)$ that is independent of $P$

$$\rho_P^{\mathbb{1},\delta}(k_z, k_\perp) \sim \int dk^0 \frac{1}{\left[k^2 - m^2\right]^2}$$

$\hookrightarrow$ corresponding quasi PDF is representation of $\delta$ function!!!!!

- some quarks 'left behind' when hadron gets boosted

- GPDs $\stackrel{FT}{\longrightarrow} q(x, \mathbf{b}_\perp)$ '3d imaging'
- $x^2$ moment of twist-3 GPDs

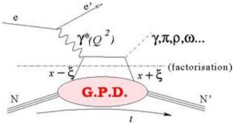
$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp} q$ distribution

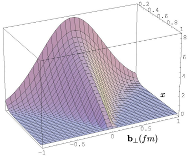
$\hookrightarrow$ $\perp$ force tomography

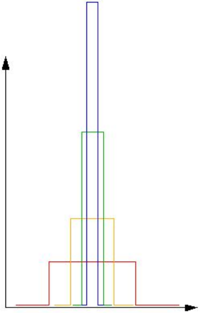
- $\delta(x)$ in twist-3 PDF

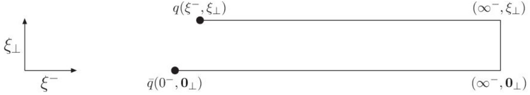
$\hookrightarrow$ discontinuities in twist-3 GPDs

- rep. of $\delta(x)$ as $\xi \to 0$
- cancel in DVCS amplitude $\sim G_2 \pm \tilde{G}_2$
- individual extraction of $G_2$ & $\tilde{G}_2$ questionable
- some quarks 'left behind' in IMF





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