Measurements of TMD effects and related issues

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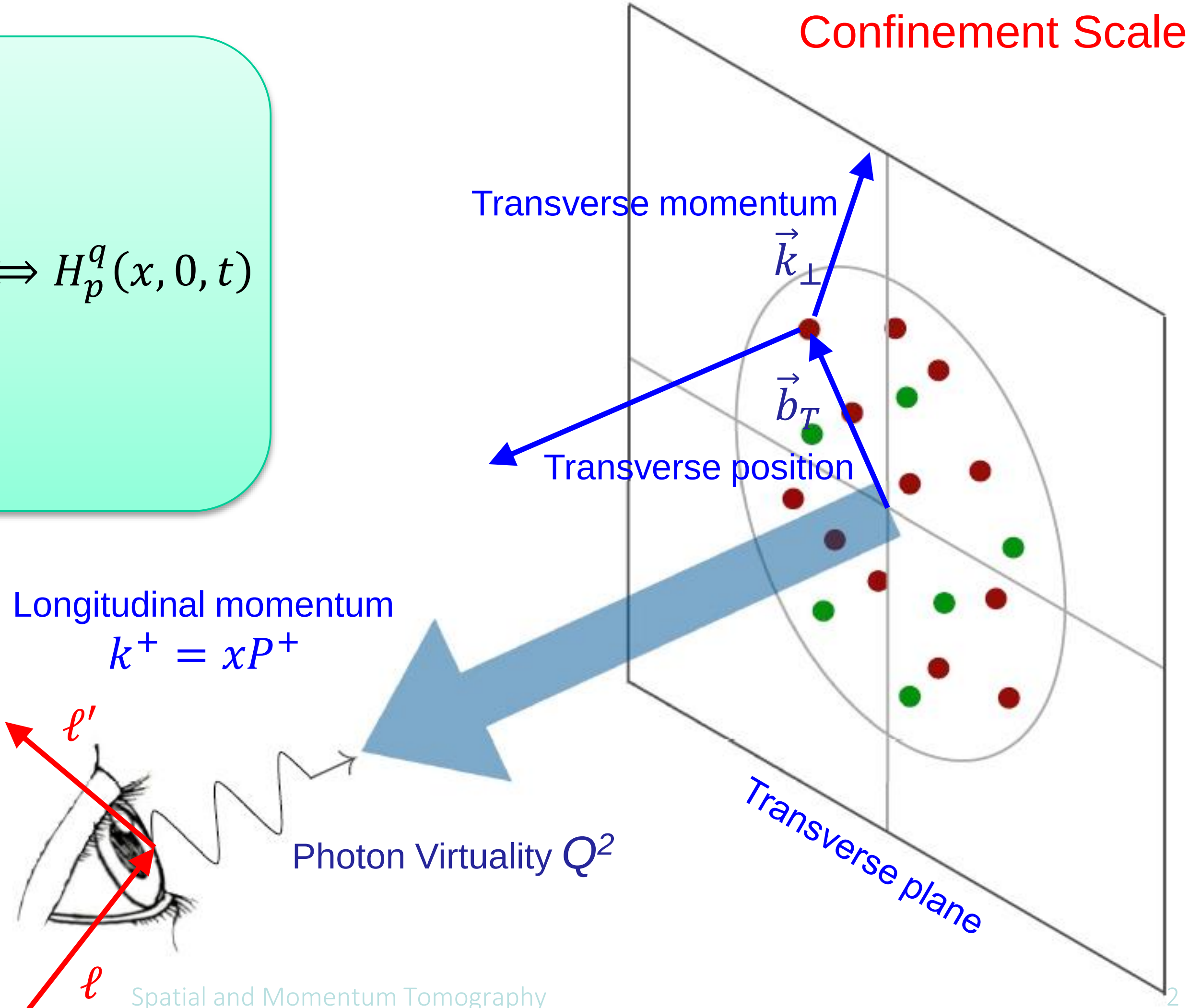
SPATIAL AND MOMENTUM TOMOGRAPHY OF HADRONS AND NUCLEI (INT-17-3), 25-29/09/2017 -- SEATTLE (WA)

Transverse structure of the Nucleon

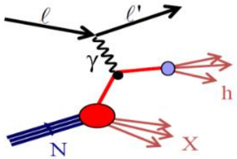
$$\begin{tikzcd}
& W_p^q(x,\vec{k}_\perp,\vec{b}_T) \arrow[dl, "d^2b_T"'] \arrow[dr, "d^2k_\perp"] & \\
f_p^q(x,\vec{k}_\perp) \arrow[dr, "d^2k_\perp"'] & & f_p^q(x,\vec{b}_T) \Leftrightarrow H_p^q(x,0,t) \arrow[dl, "d^2b_T"] \\
& f_p^q(x) &
\end{tikzcd}$$

High Energy Probe
Hard Scale



Accessing TMD PDFs and FFs

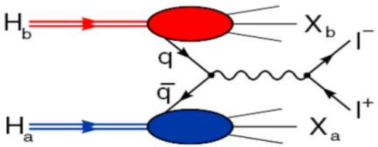
- SIDIS off polarized p, d, n targets



HERMES
COMPASS
JLab
future: ***EIC***

$$\sigma^{\ell p\to\ell' hX}\sim q(x)\otimes\hat{\sigma}^{\gamma q\to q}\otimes D_q^h(z)$$

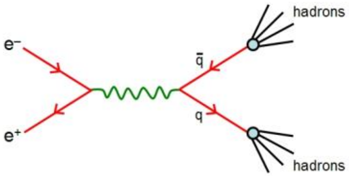
- polarised Drell-Yan



COMPASS
RHIC
FNAL
future: ***FAIR, JPark, NICA***

$$\sigma^{hp\to\mu\mu}\sim\bar{q}_h(x_1)\otimes q_p(x_2)\otimes\hat{\sigma}^{\bar{q}q\to\mu\mu}(\hat{s})$$

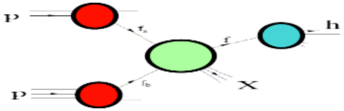
- $e^+e^-\to h_1h_2$



BaBar
Belle
Bes III

$$\sigma^{e^+e^-\to h_1h_2}\sim\hat{\sigma}^{\ell\ell\to\bar{q}q}(\hat{s})\otimes D_q^{h_1}(z_1)\otimes D_q^{h_2}(z_2)$$

- pp hadron reactions (challenging for theory)



RHIC

$$\sigma^{pp\to hX}\sim\otimes q(x_1)\otimes q(x_2)\hat{\sigma}^{qq\to qq}(\hat{s})\otimes D_q^h$$

Future: ***FAIR, JPark, NICA***

SIDIS access to TMDs

$$\sigma^{\ell p\to\ell' hX} \sim f_{q,p}(x,k_\perp^2) \otimes \sigma^{\ell q\to q} \otimes D_{1q}^h(z,p_\perp^2)$$

TMDs $(x,\vec{k}_\perp)$

FFs $(z,\vec{p}_\perp)$

Nucleon polarization

Parton polarization	U	T	L
U	f_1	$f_{1T}^\perp$	
T	$h_1^\perp$	$h_1, h_{1T}^\perp$	$h_{1L}^\perp$
L		g_{1T}	g_{1L}

Hadron polarization

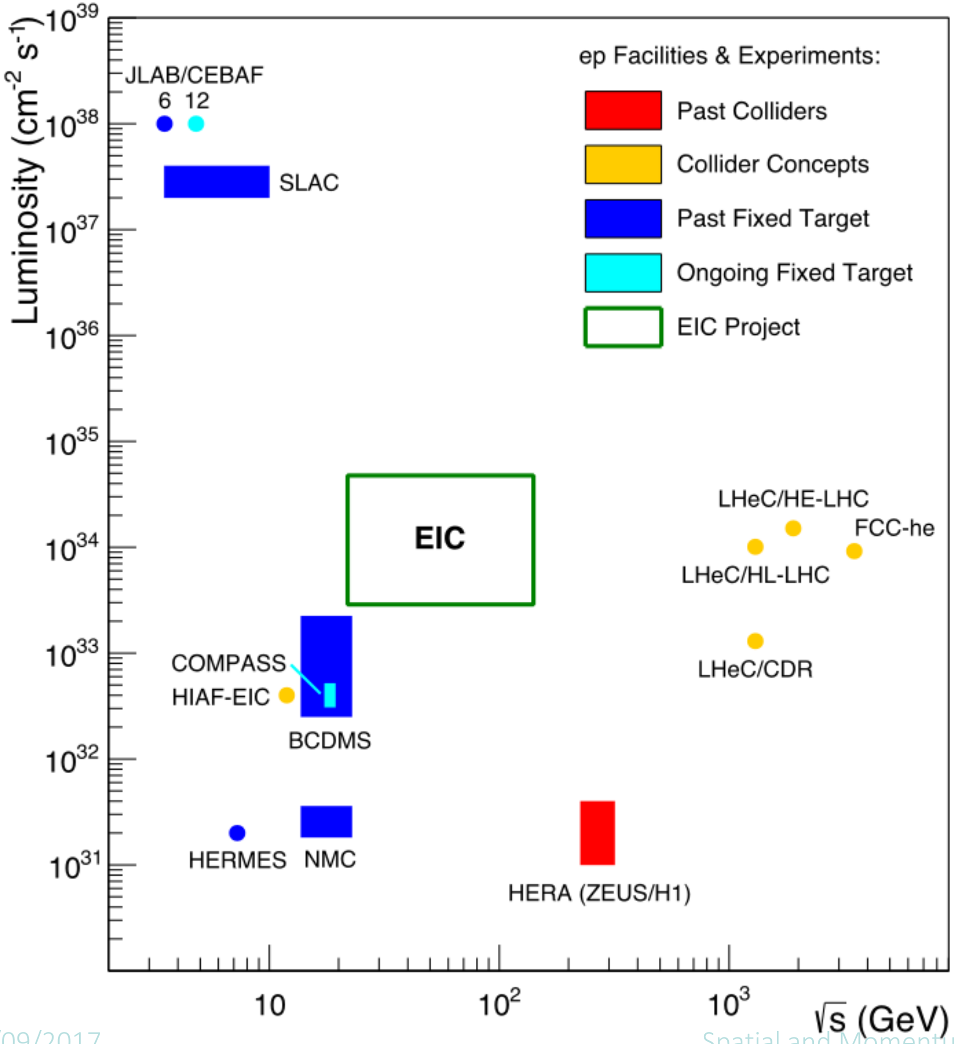
Parton polarization	U	T	L
U	D_1	$D_{1T}^\perp$	
T	$H_1^\perp$	H_1 , $H_{1T}^\perp$	$H_{1T}^\perp$
L		G_{1T}	G_{1L}

T odd

chiral odd

Factorisation (Collins & Soper, Ji, Ma, Yuan, Qiu & Vogelsang, Collins & Metz...)

The CM Energy vs Luminosity Landscape



HIAF-EIC = Chinese version of Electron-Ion Collider

("A dilution-free mini-COMPASS")

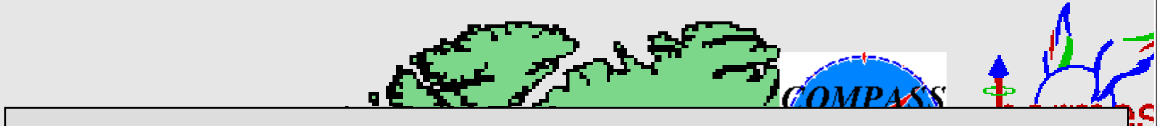
JLEIC = EIC@Jlab

eRHIC = EIC@BNL

LHeC = ep/eA collider @ CERN

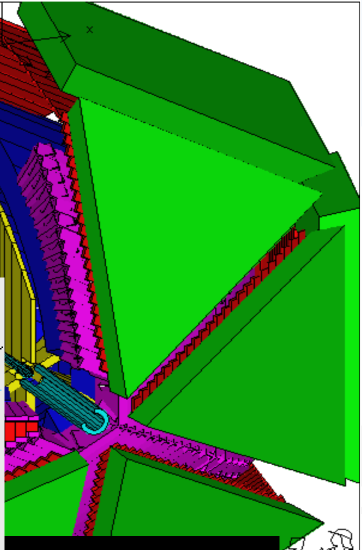
HERA

Players on SIDIS playground

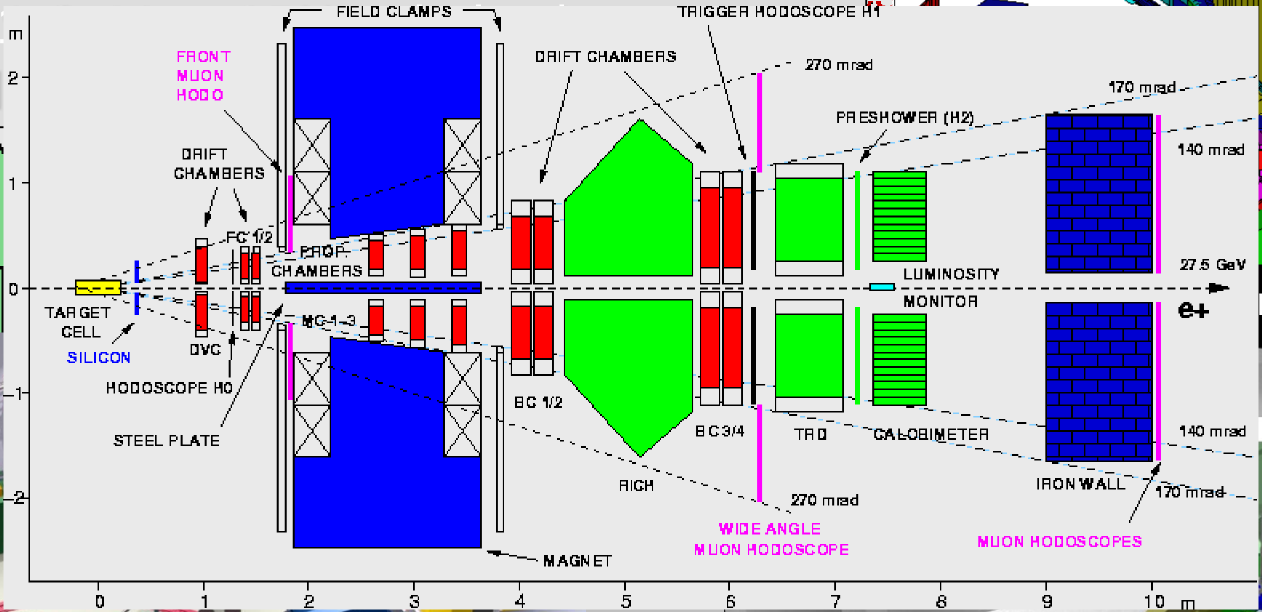


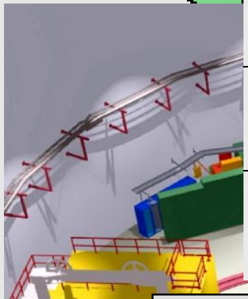
Beam: 6-1

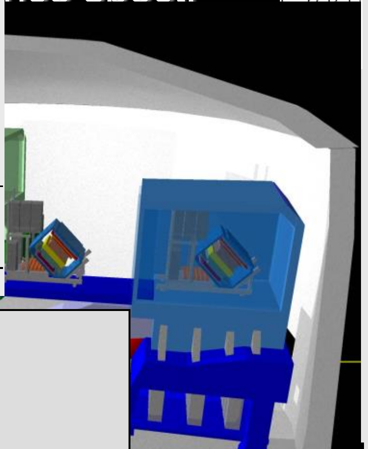
Target: pol



nce spect.







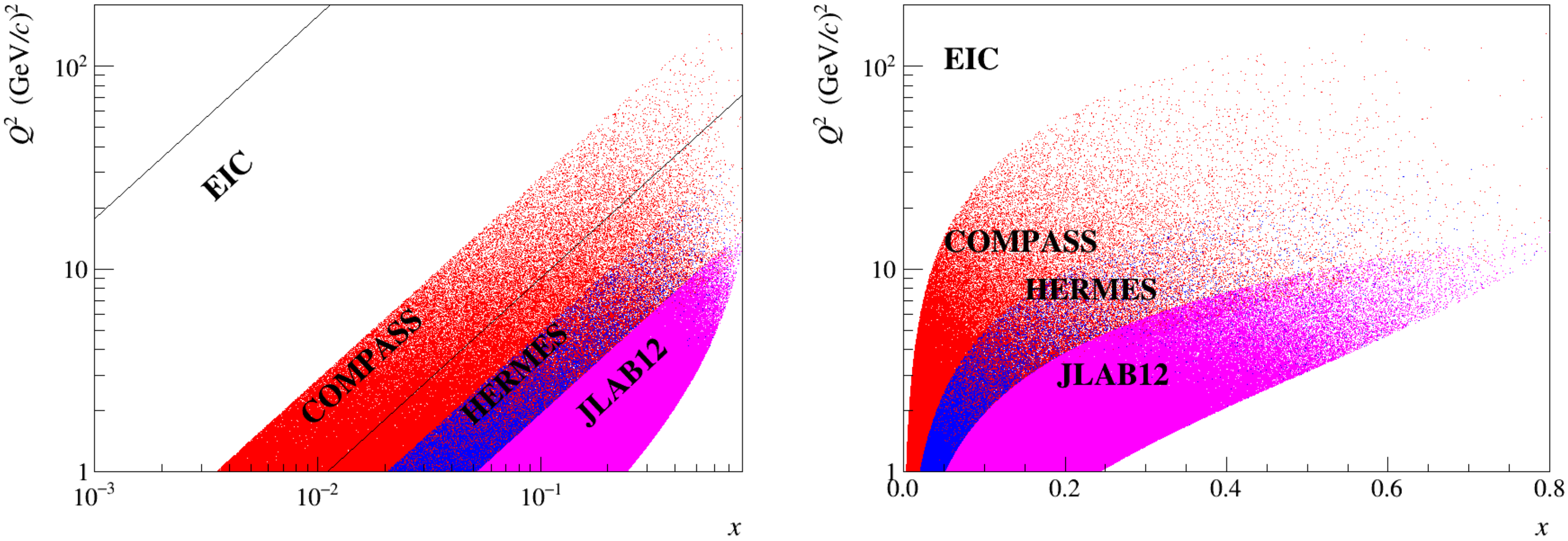
Beam: 27.5 GeV $e^{\pm}$; ⟨50⟩% polarization

Target: polarized p gas targets; ⟨85%⟩ polarization

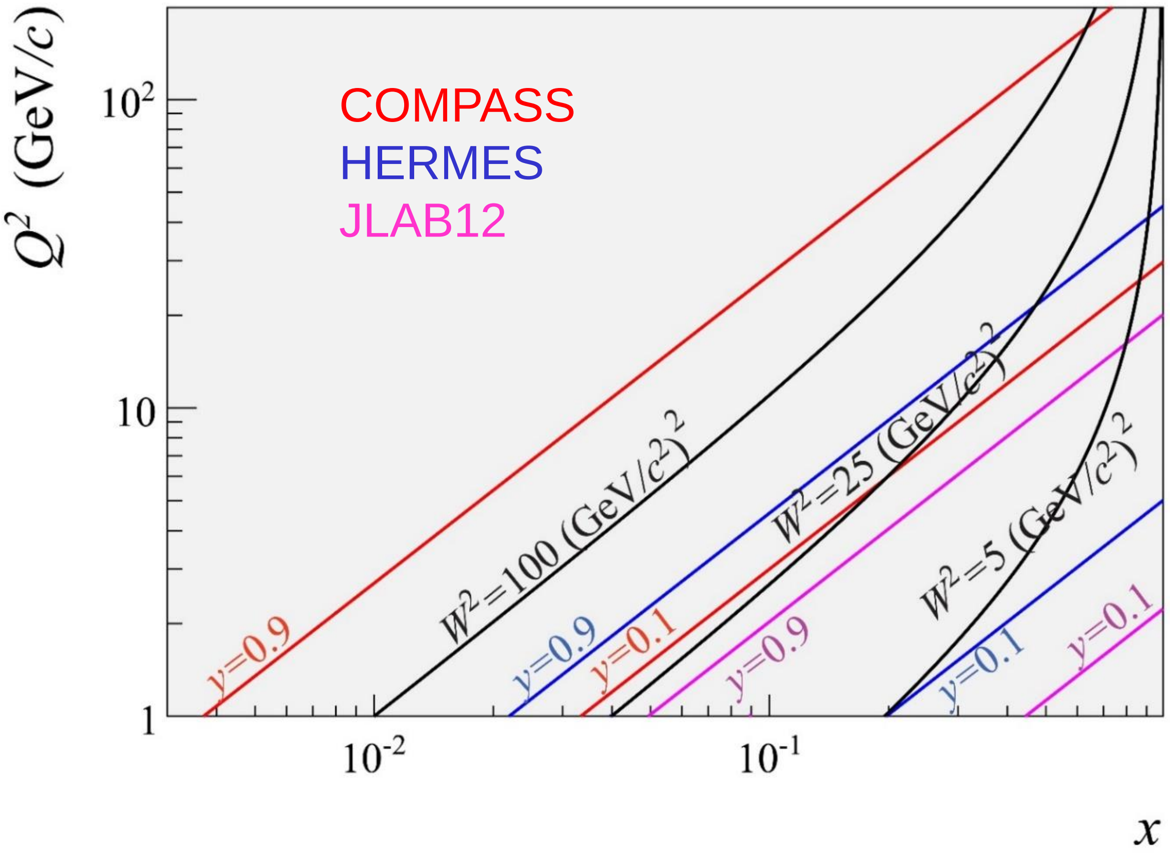
7 GeV spectrometer,
1.8 GeV spectrometer,
large installation experiments

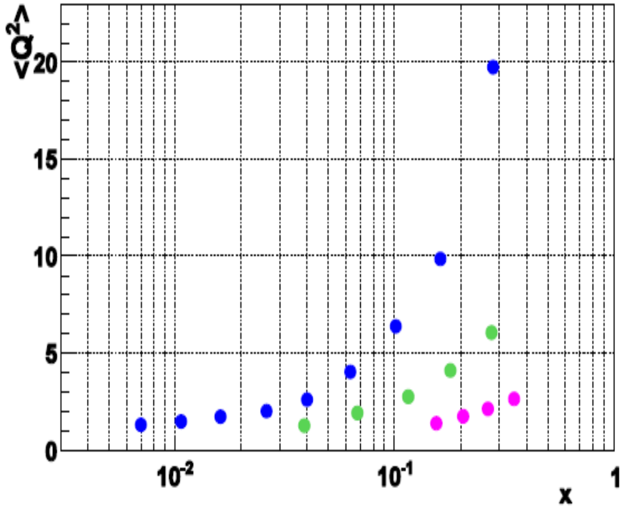
Two high-resolution
4 GeV spectrometers

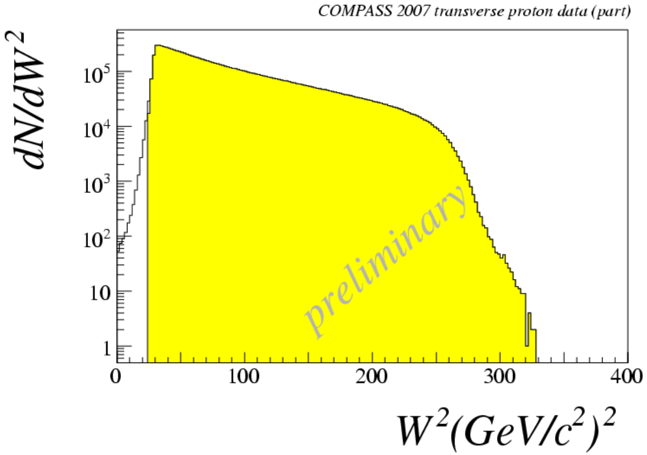
Kinematic coverage



Kinematic coverage







SIDIS Experiments

SIDIS Experiment must/should:

- Have large/flat acceptances on all the relevant variables x, Q^2, z, P_{hT}, ϕ
- Use different targets (p, d, n) and identify hadrons to allow flavor separation
- Be at different energies to cover PDFs from the valence region down to small-x
- Large luminosity to allow multidimensional results needed by the complexity of TMDs
- **The polarized lepton-nucleon collider will be a mandatory tool to reach the level of ordinary PDF**

Multiplicity distributions

- Unpolarized hadron multiplicity distributions are the basic material for studying the mechanisms of P_{hT} generation and the applicability of TMD factorization.

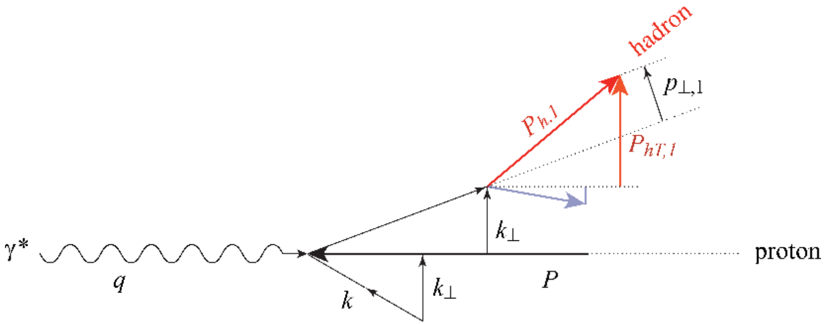
- It is important to have differential distributions in kinematic variables x, Q^2, z besides P_{hT}

- Not only low P_{hT}. Tails at $P_{hT} > 1$ GeV carries important perturbative & non-perturbative information

Importance of unpolarized SIDIS

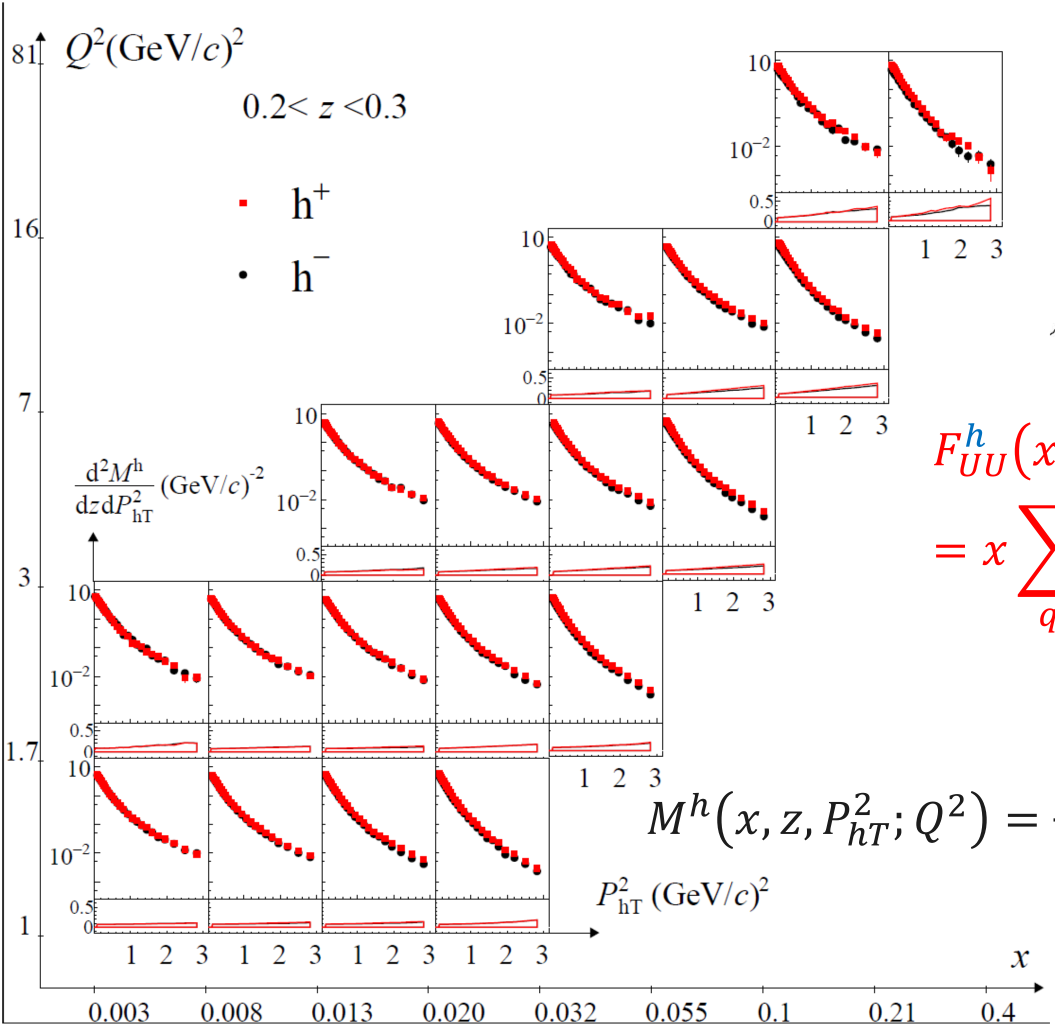
- The cross section dependence from P_{hT} results from:
  - intrinsic $k_\perp$ of the quarks
  - $p_\perp$ generated in the quark fragmentation
  - A Gaussian ansatz for $k_\perp$ and $p_\perp$ leads to

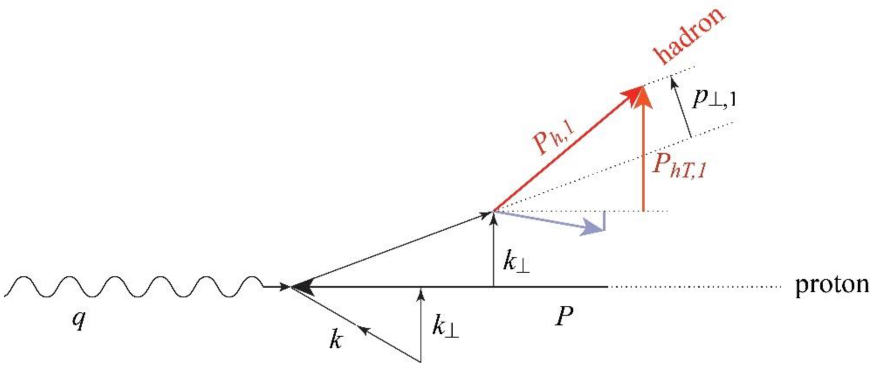
    $\langle P_{hT}^2\rangle = z^2\langle k_\perp^2\rangle + \langle p_\perp^2\rangle$



- The azimuthal modulations in the unpolarised cross sections comes from:
  - Intrinsic $k_\perp$ of the quarks
  - The Boer-Mulders PDF
- Difficult measurements were one has to correct for the apparatus acceptance
- COMPASS and HERMES have
  - results on ${}^6LiD\ (\sim d)$ and d and on p (Hermes only)
  - No COMPASS measurements on p since on $NH_3\ (\sim p)$ nuclear effects may be important
- ⇒COMPASS-II, measurements on LH$_2$ in parallel with DVCS

Importance of unpolarized SIDIS



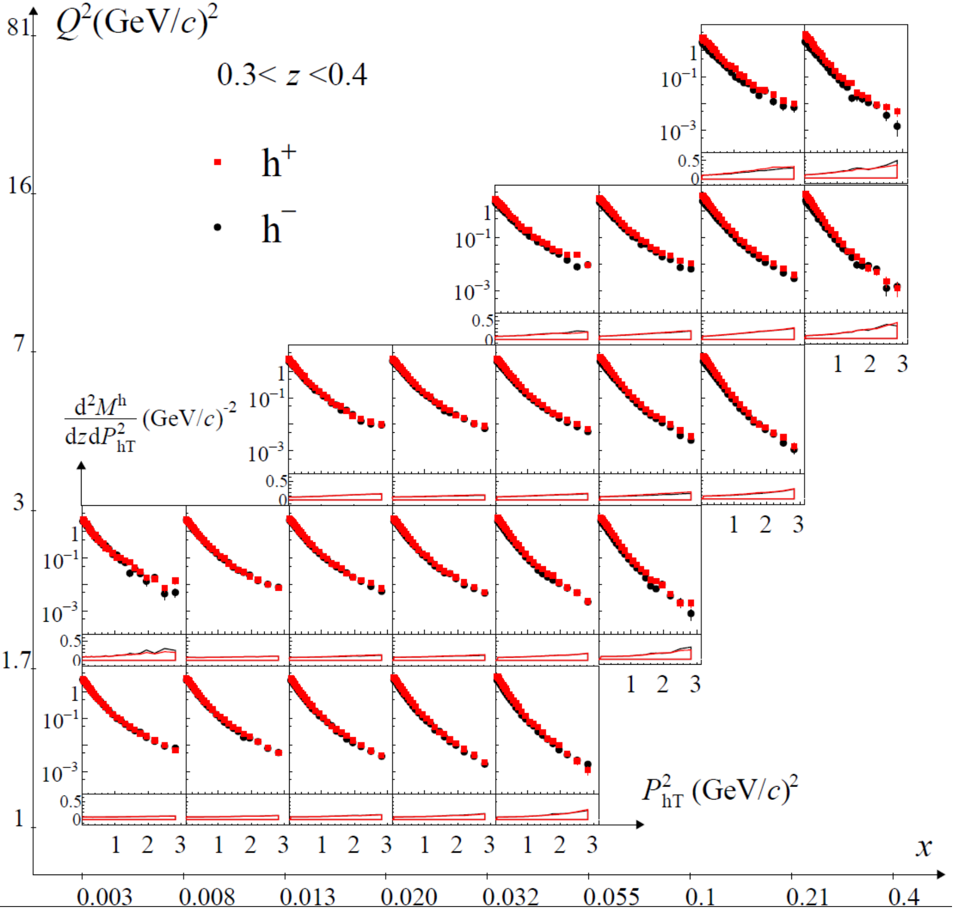


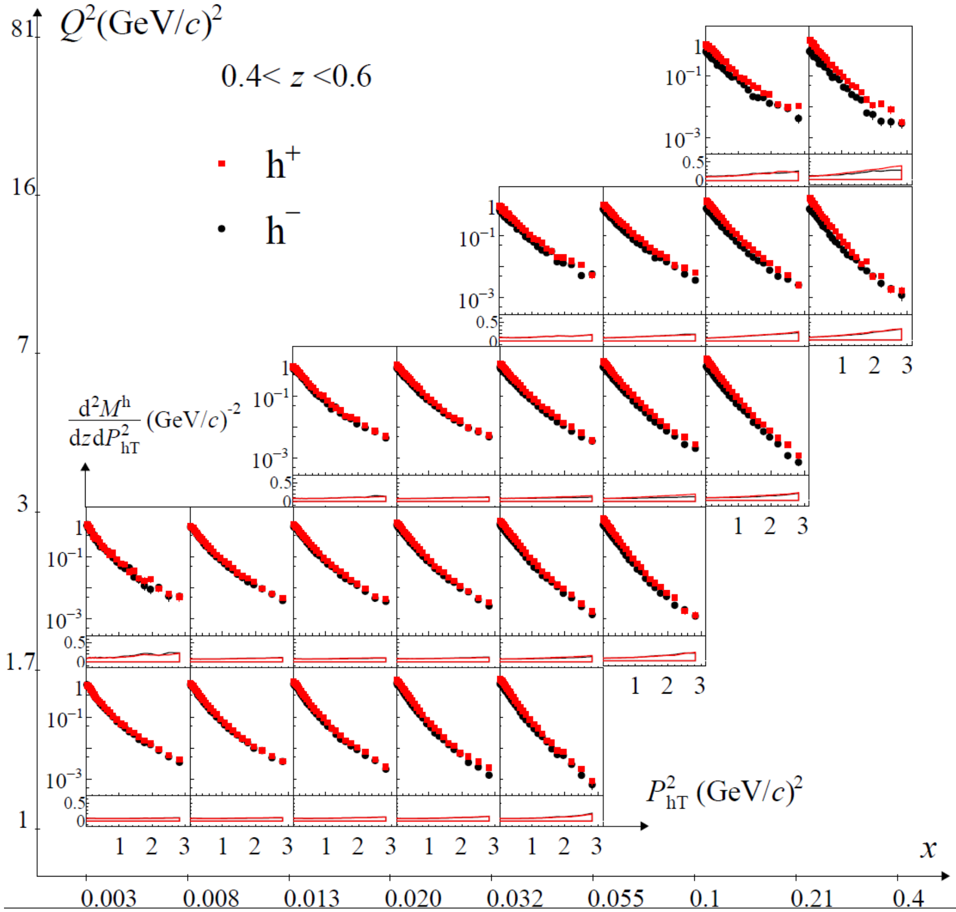
$$F^h_{UU}(x,z,P^2_{hT};\ Q^2)$$

$$= x\sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta(\vec{p}_\perp - z\vec{k}_\perp$$

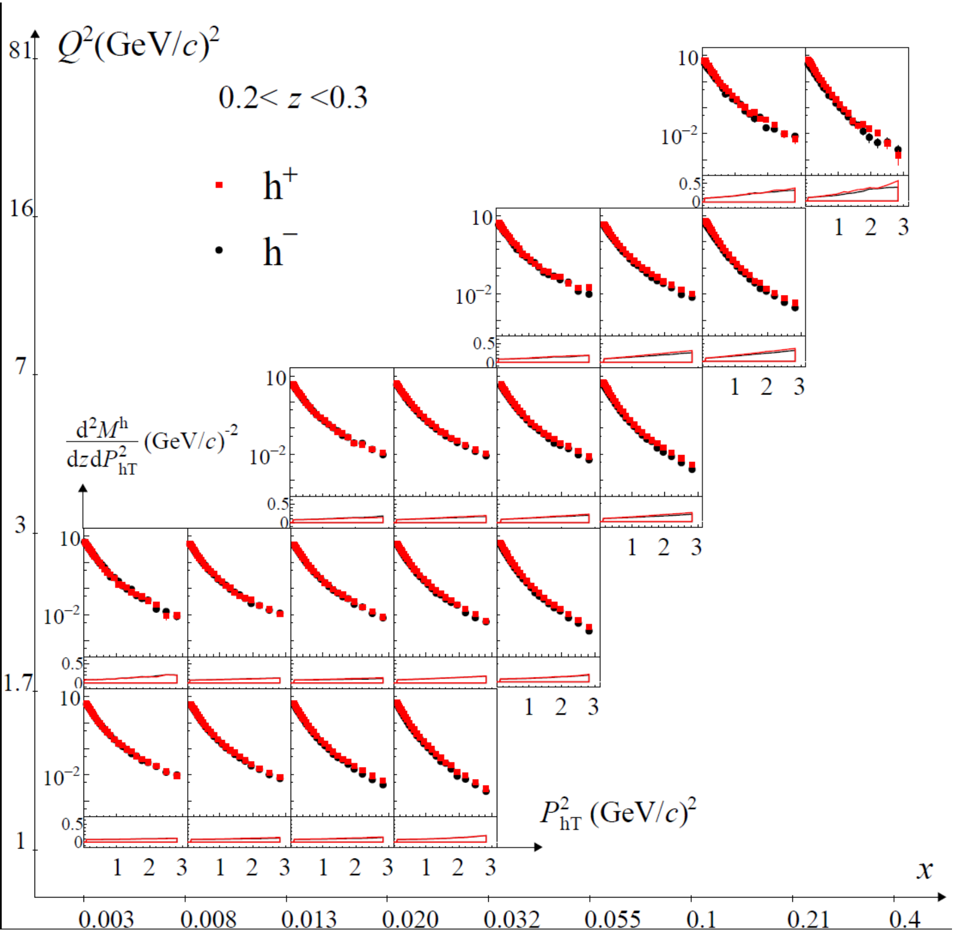
$$M^h(x,z,P^2_{hT};Q^2) = \frac{d^5\sigma^h/dxdQ^2dzd^2\vec{P}_{hT}}{d^2\sigma^{DIS}/dxdQ^2} \sim \frac{F^h_{UU}(x,z,P^2_{hT};\ Q^2)}{F_{UU,T}+\varepsilon F_{UU,L}}$$

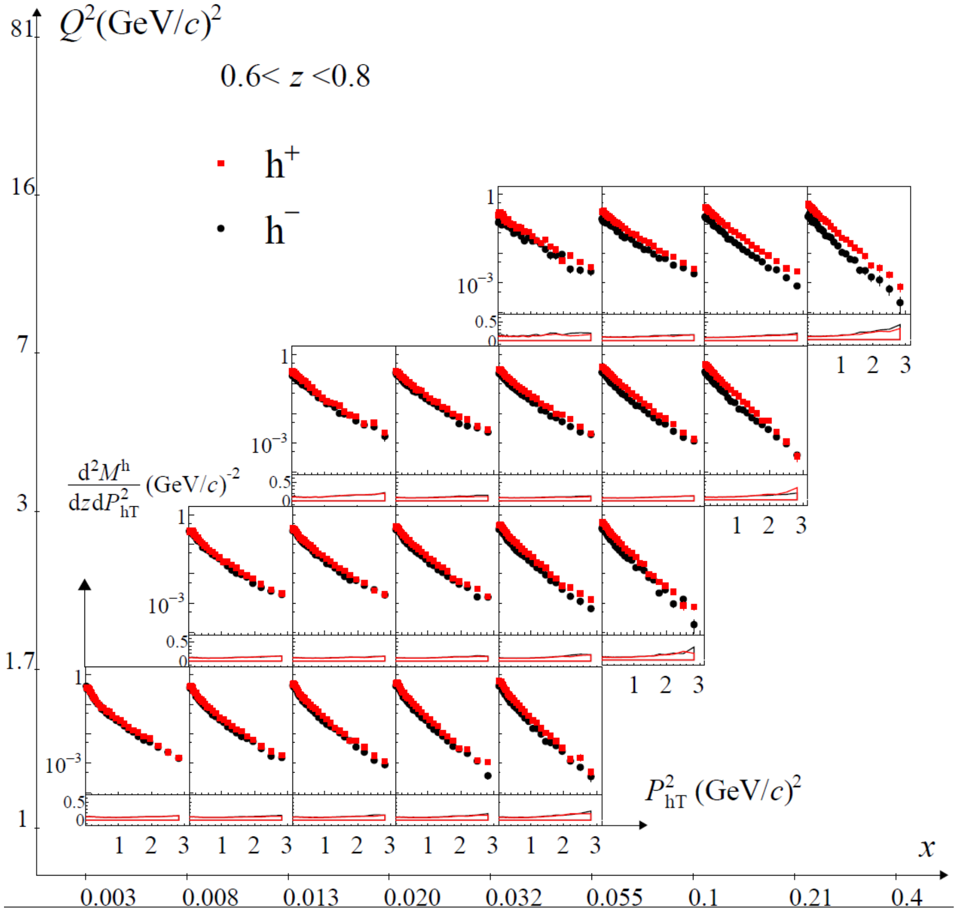
Importance of unpolarized SIDIS



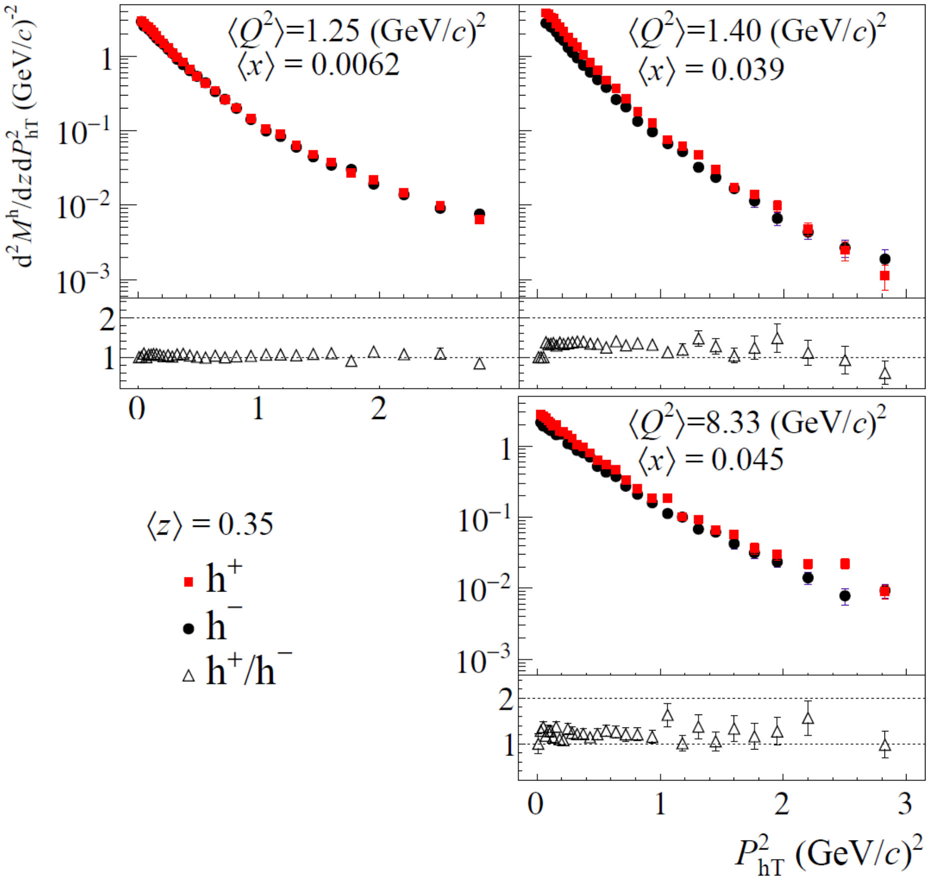


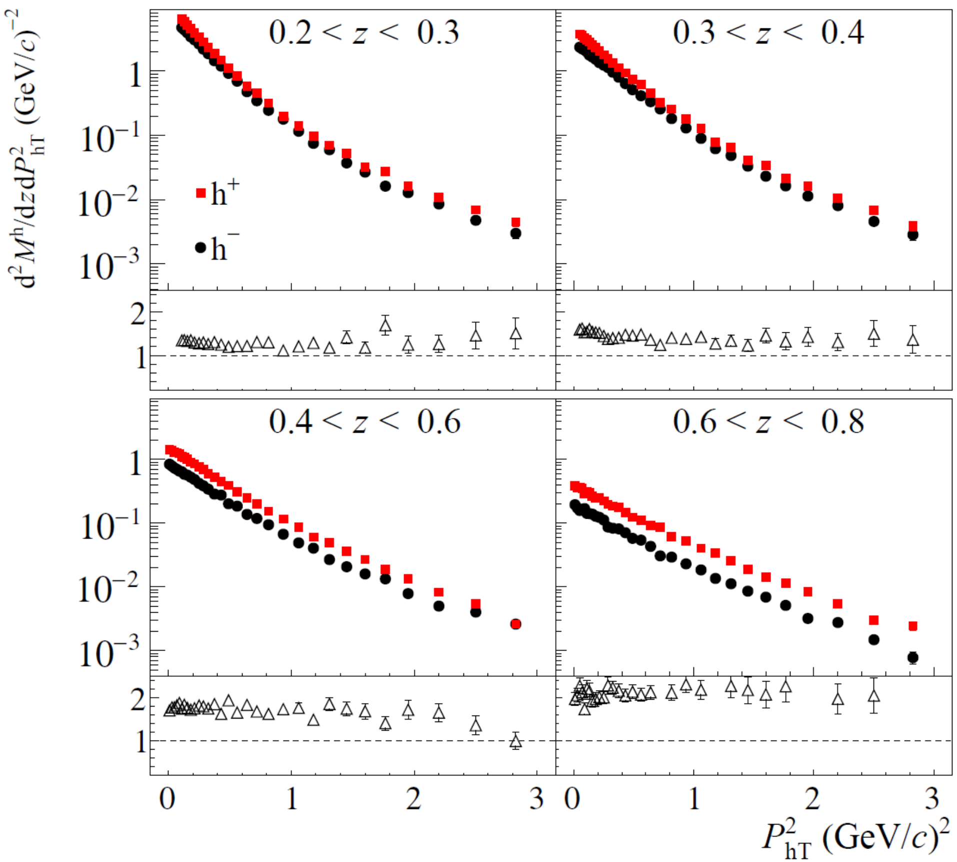
Importance of unpolarized SIDIS





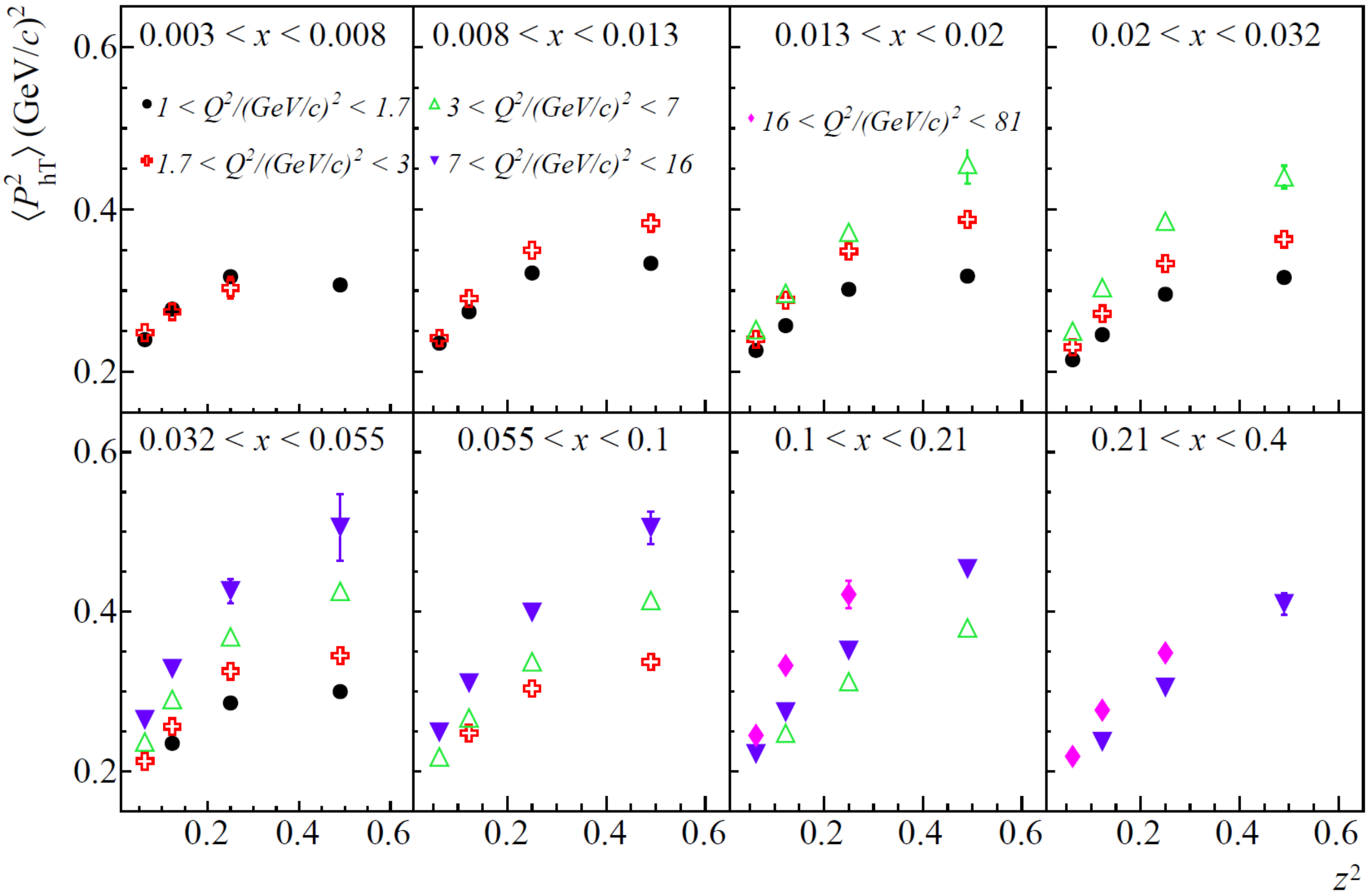
Importance of unpolarised SIDIS



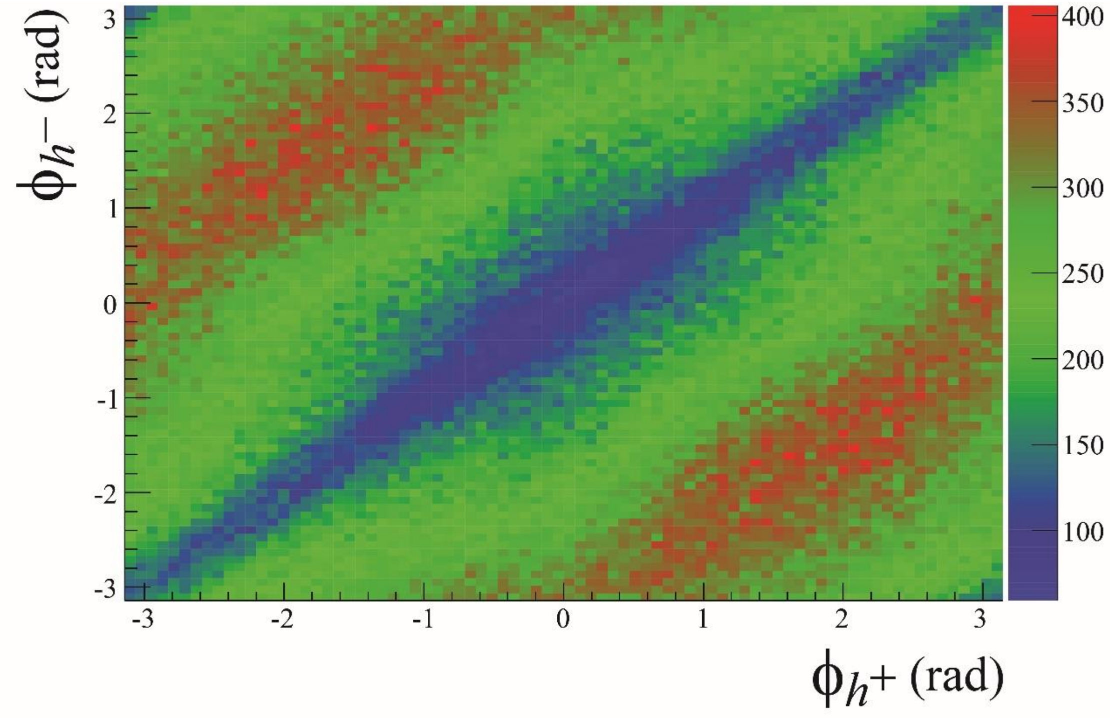


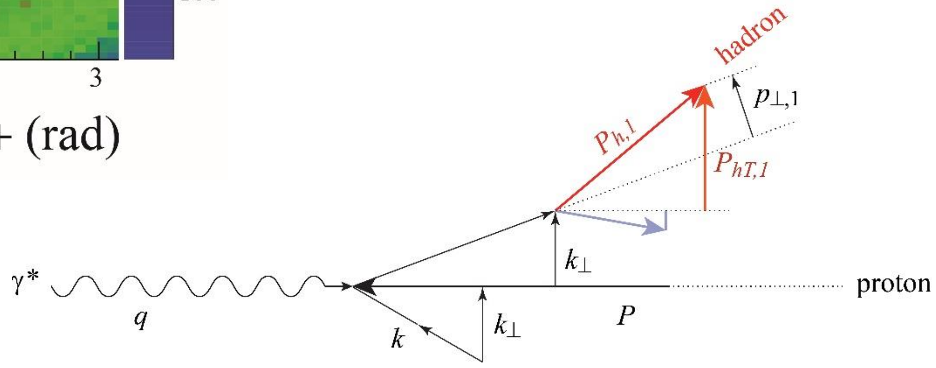
$\langle Q^2 \rangle = 9.78\ (\mathrm{GeV}/c)\quad \langle x \rangle = 0.149$

Importance of unpolarized SIDIS



Is correlation having an impact?





The asymmetries

- The asymmetries are:

$$A_{U(L),T}^{w(\phi_h,\phi_S)}(x,z,p_T;Q^2)=\frac{F_{U(L),T}^{w(\phi_h,\phi_S)}}{F_{UU,T}+\varepsilon F_{UU,L}}$$

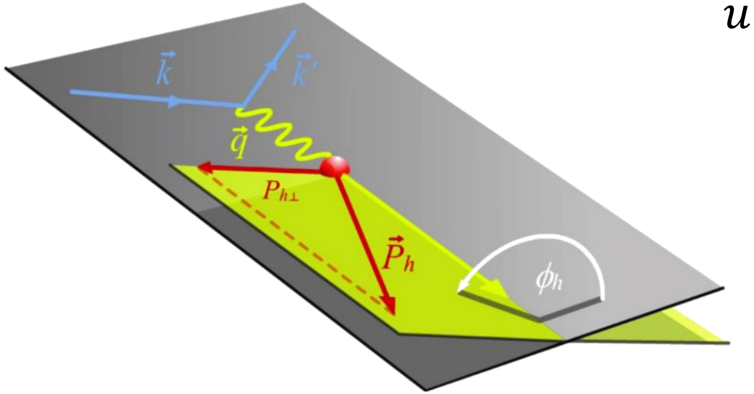
- When we perform 1D measurements

$$A_{U(L),T}^{w(\phi_h,\phi_S)}(x)$$

$$=\frac{\int_{Q_{min}^2}^{Q_{max}^2}dQ^2\int_{z_{min}}^{z_{max}}dz\int_{P_{hT,min}}^{P_{hT,max}}d^2\vec{P}_{hT}F_{U(L),T}^{w(\phi_h,\phi_S)}}{\int_{Q_{min}^2}^{Q_{max}^2}dQ^2\int_{z_{min}}^{z_{max}}dz\int_{P_{hT,min}}^{P_{hT,max}}d^2\vec{P}_{hT}\left(F_{UU,T}+\varepsilon F_{UU,L}\right)}$$

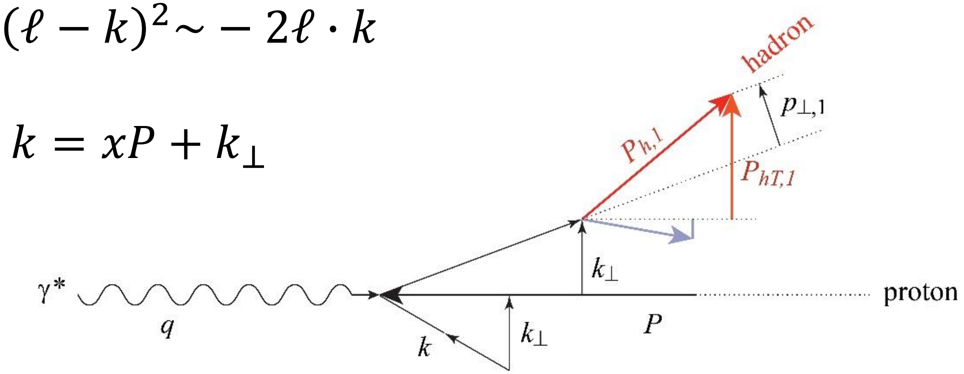
Unpolarised Azimuthal Modulation

The semi inclusive cross-section for $\ell p \to \ell' h X$ is given by $d\sigma^{\ell p \to \ell' h X} = \sum_q f_q(x, Q^2) \otimes d\sigma^{\ell q \to \ell' q} \otimes D_q^h(z, Q^2)$. The cross section for the partonic process is simply given by $d\sigma^{\ell q \to \ell' q} = \hat{s}^2 + \hat{u}^2$



$$s := (\ell + k)^2 \sim 2\ell \cdot k$$
$$u := (\ell - k)^2 \sim -2\ell \cdot k$$

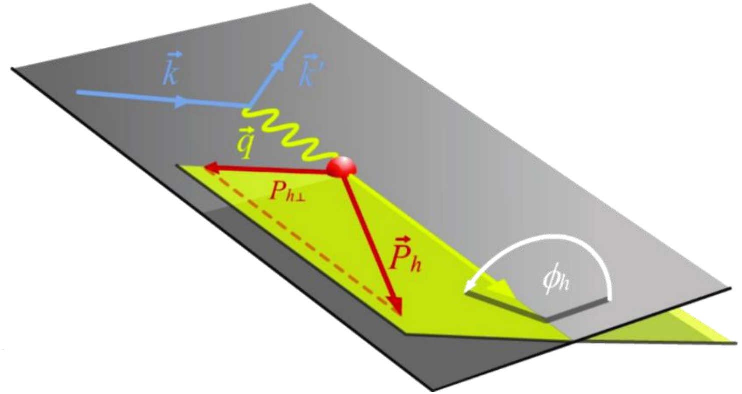
$$k = xP + k_\perp$$



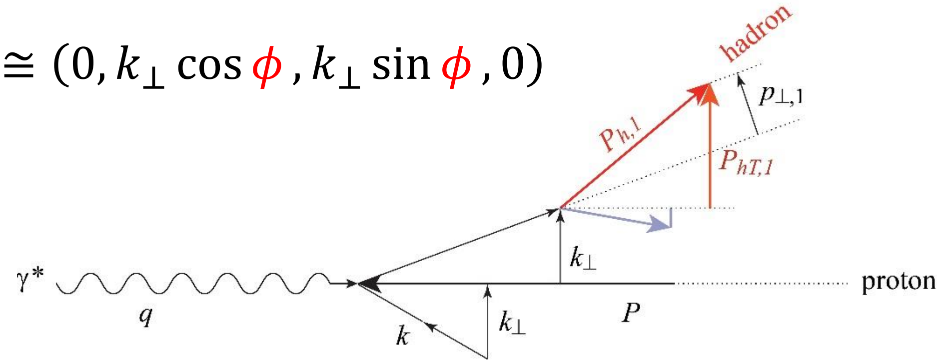
In collinear PM $d\sigma^{\ell q \to \ell' q} = \hat{s}^2 + \hat{u}^2 = x[1 + (1 - y)^2]$, i.e. no ϕ_h dependence.

Unpolarised Azimuthal Modulation

$k_\perp$ has only components outside the lepton scattering plane:



$$k_\perp \cong (0, k_\perp \cos\phi\,, k_\perp \sin\phi\,, 0)$$



Taking into account the parton transverse momentum in the kinematics leads to:

$$\hat{s} = sx\left[1 - \frac{2k_\perp}{Q}\sqrt{1-y}\cos\phi_h\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right) \quad \hat{u} = sx(1-y)\left[1 - \frac{2k_\perp}{Q\sqrt{1-y}}\cos\phi_h\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right)$$

Resulting in the $\cos\phi_h$ and $\cos 2\phi_h$ modulations observed in the azimuthal distributions

Unpolarised Azimuthal Modulation

The full cross section for the unpolarised case is written as:

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi}$$

$$= \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\left\{1+\cos\phi_h\sqrt{2\varepsilon(1+\varepsilon)}A_{UU}^{\cos\phi_h}\right.$$

Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel JHEP 0702:093 (2007)

$$A_{UU}^{x}\left(x,z,dP_{hT}^2,Q^2\right)=\frac{F_{UU}^{x}}{F_{UU,T}+\varepsilon F_{UU,L}}$$

$$\varepsilon=\frac{1-y-\frac{1}{4}y^2\gamma^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}\quad and\quad \gamma=\frac{2xM}{Q}$$

$$F_{UU}=C[f_1D_1]=x\sum_q e_q^2\int d\vec{p}_\perp d\vec{k}_\perp\,\delta^2\left(z\vec{k}_\perp+\vec{p}_\perp-\vec{P}_{hT}\right)f_1^q(x,k_\perp,Q^2)D_{1,q}^h(z,p_\perp,Q^2)$$

Unpolarised Azimuthal Modulation

When looking at the content of the structure functions/modulations in terms of TMD PDFs for the $\cos\phi_h$ and $\cos 2\phi_h$ we can write:

$$F_{UU}^{\cos\phi_h} = -\frac{2M}{Q} C\left[\frac{\hat{h}\cdot\vec{k}_\perp}{M} f_1 D_1 - \frac{p_\perp k_\perp}{M}\frac{\vec{P}_{hT} - z\left(\hat{h}\cdot\vec{k}_\perp\right)}{zM_h M} h_1^\perp H_1^\perp\right] + \text{twists} > 3$$

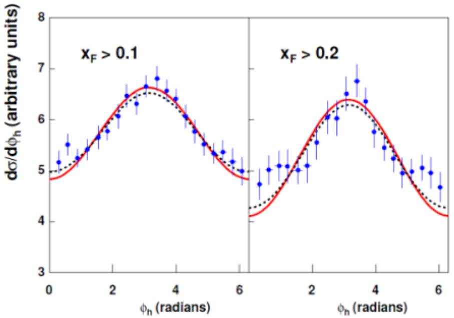
$$F_{UU}^{\cos 2\phi_h} = C\left[\frac{\left(\hat{h}\cdot\vec{k}_\perp\right)\left(\hat{h}\cdot\vec{p}_\perp\right) - \vec{p}_\perp\cdot\vec{k}_\perp}{MM_h} h_1^\perp H_1^\perp\right] + \text{twists} > 3$$

In the $\cos 2\phi_h$ Cahn effects enters only at twist4

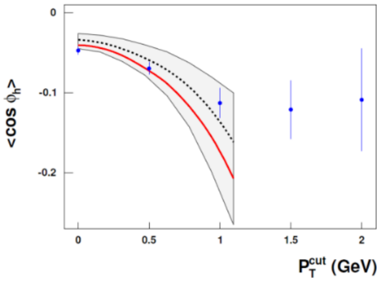
$$F_{\text{Cahn}}^{\cos 2\phi_h} \approx \frac{2}{Q^2} C\left[\left\{2\left(\hat{h}\cdot\vec{k}_\perp\right)^2 - k_\perp^2\right\} f_1 D_1\right]$$

Experimental status

- Azimuthal modulations in $\ell p \to \ell' h X$ measured by
  - EMC
  - E665



EMC

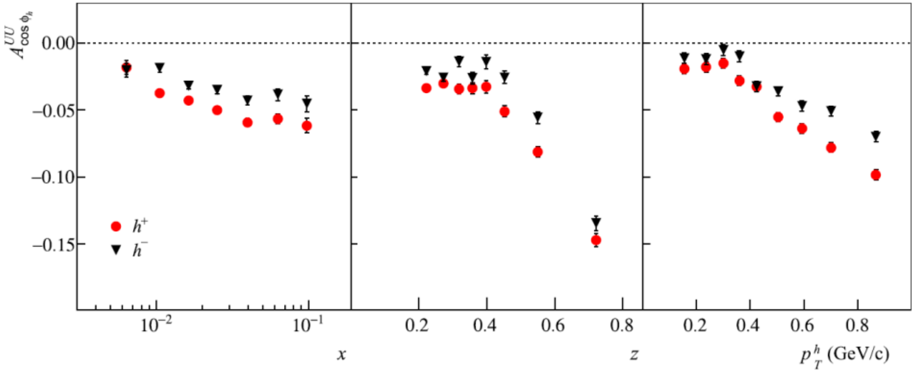


E665

Fits from M. Anselmino, V. Barone, E. Boglione,U. D'Alesio, F. Murgia, A. Prokudin, A. Kotzinian, and C. Turk

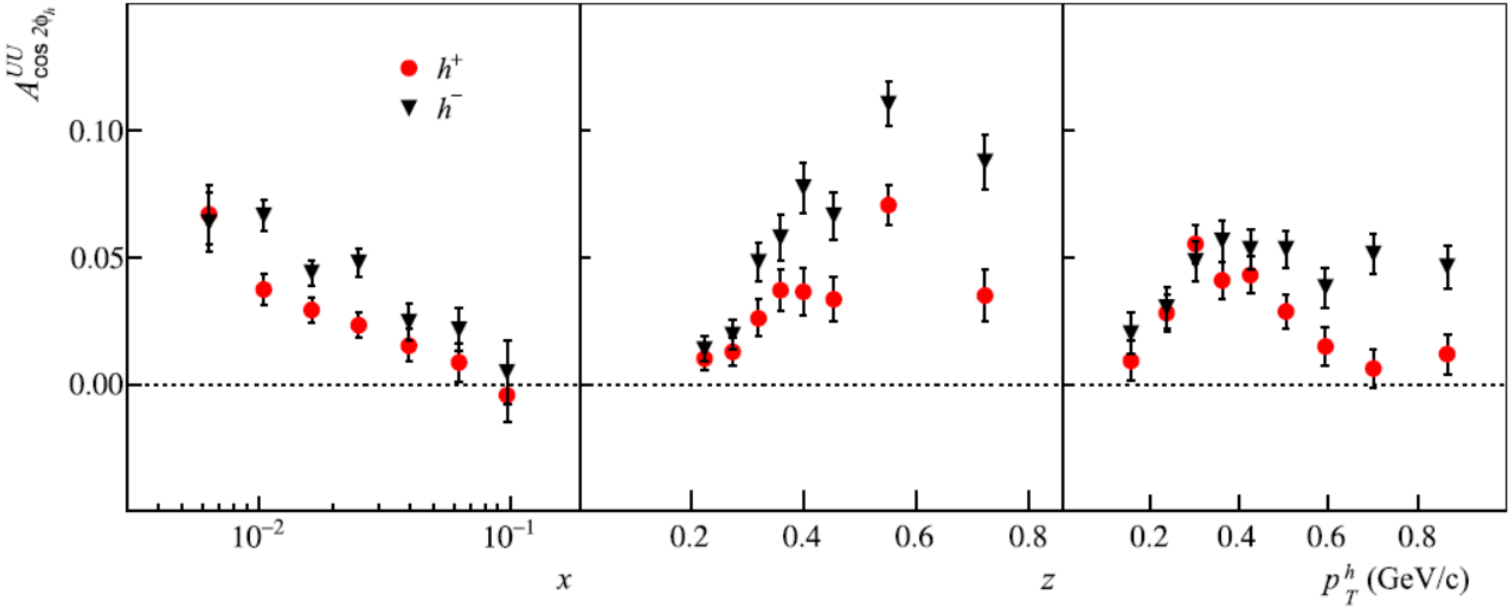
- Large modulations up to 40% for $\cos\phi$, while $\cos 2\phi$ is smaller, about 5%

COMPASS



Boer-Mulders in $\cos 2\phi$

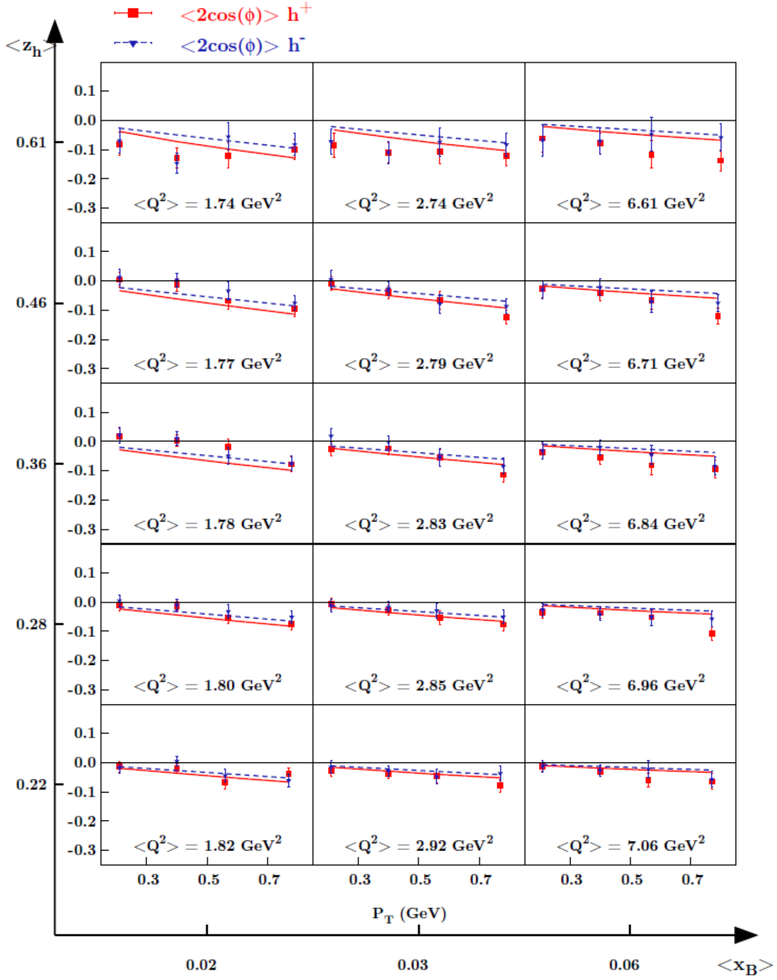
1064 C. Adolph et al. / Nuclear Physics B 886 (2014) 1046–1077

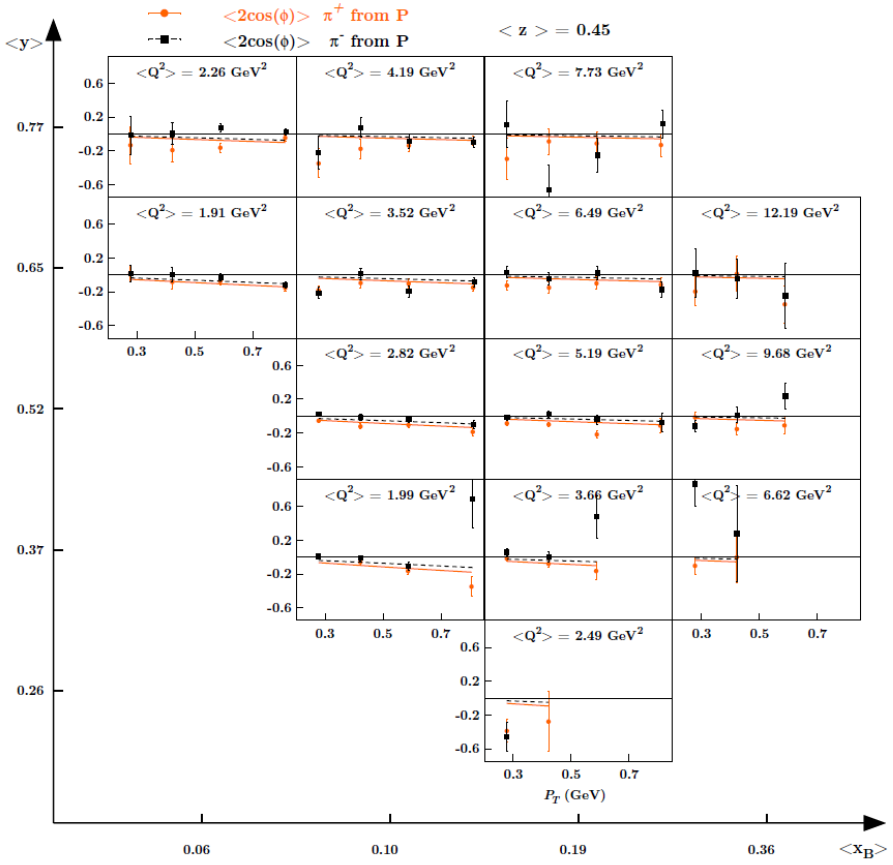


$$F_{UU}^{\cos 2\phi}(x, z, P_{hT}^2;\ Q^2)$$

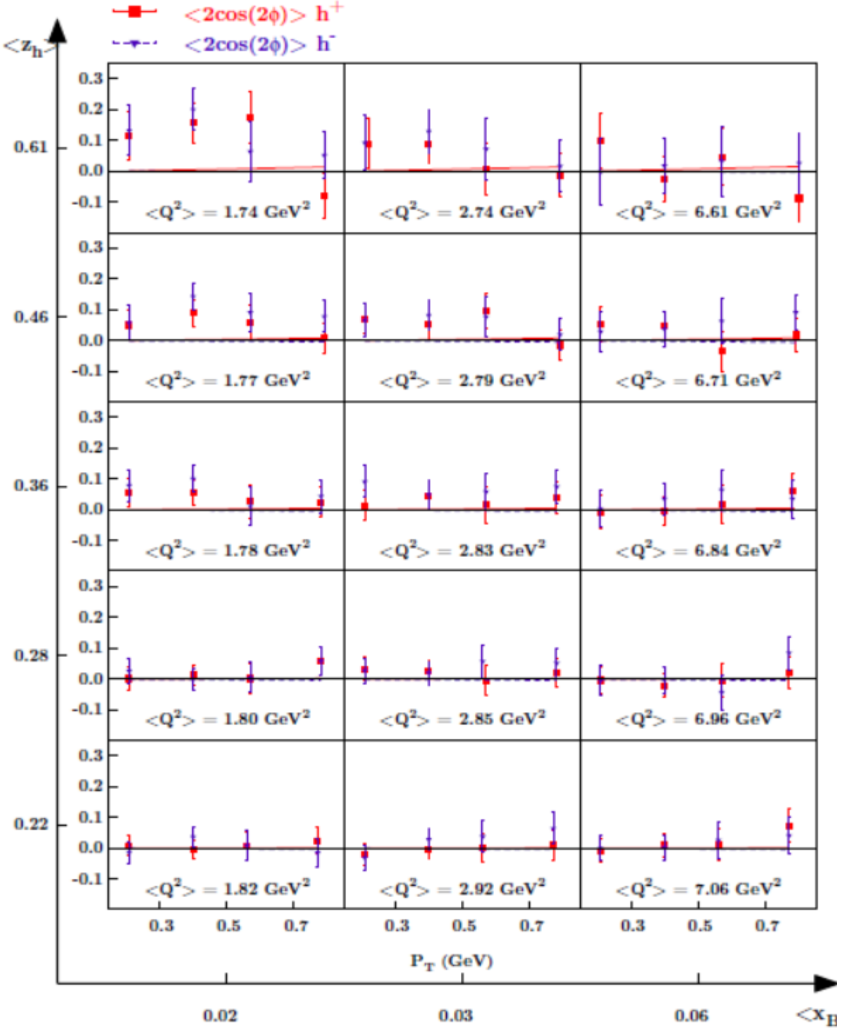
$$= -x \sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \frac{2(\hat{h}\cdot\vec{k}_\perp)(\hat{h}\cdot\vec{k}_\perp) - \vec{k}_\perp\cdot\vec{p}_\perp}{M m_h}\ h_1^{\perp,q}(x, k_\perp^2; Q^2) H_1^{\perp,q\to h}(z, p_\perp^2; Q^2)$$

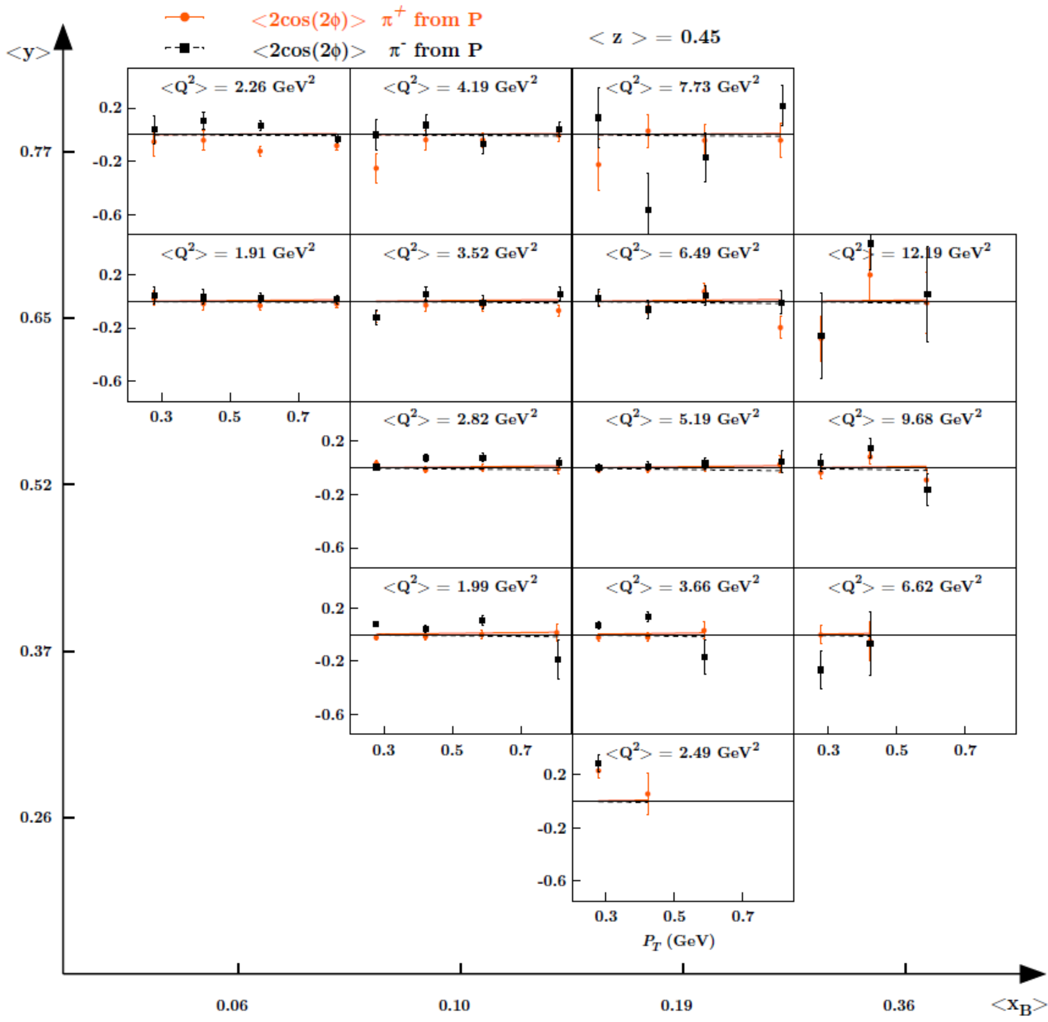
cos ϕ modulation





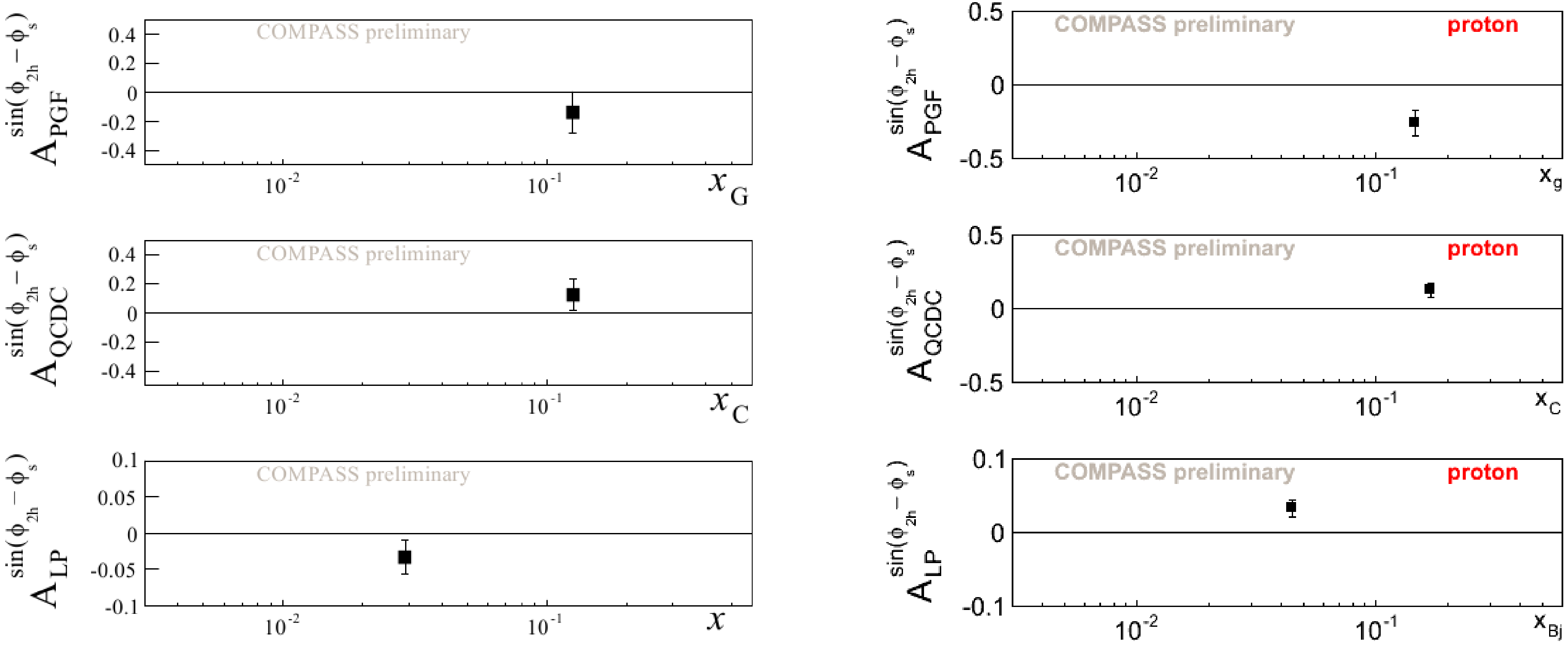
Boer-Mulders in cos 2ϕ and in cos ϕ





Sivers asymmetry on deuteron and proton for Gluons

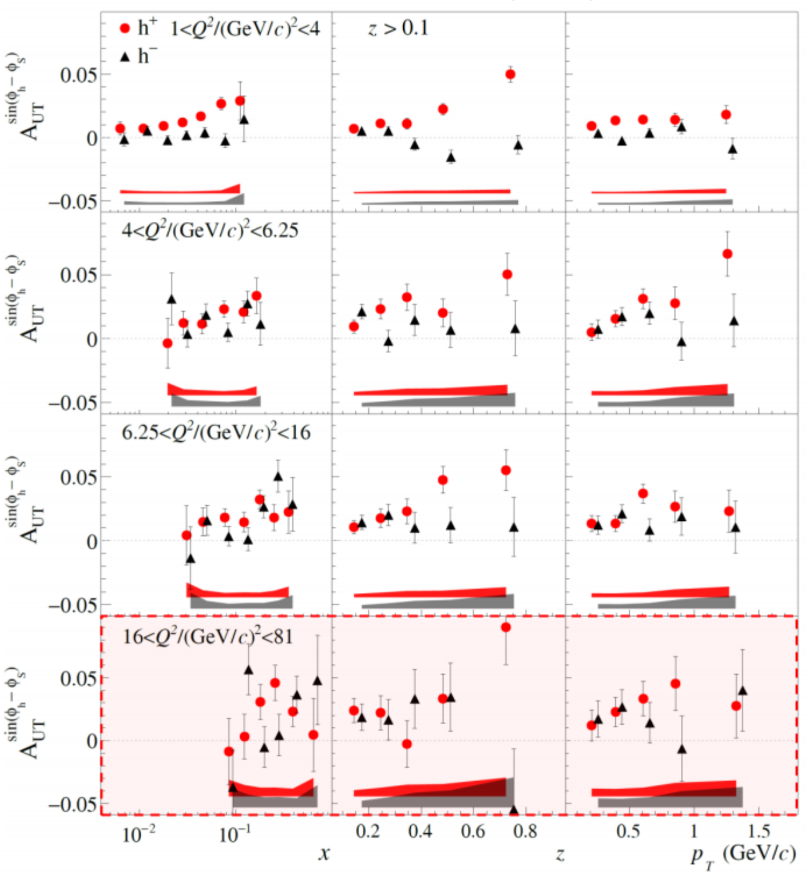
PLB 772 (2017) 854

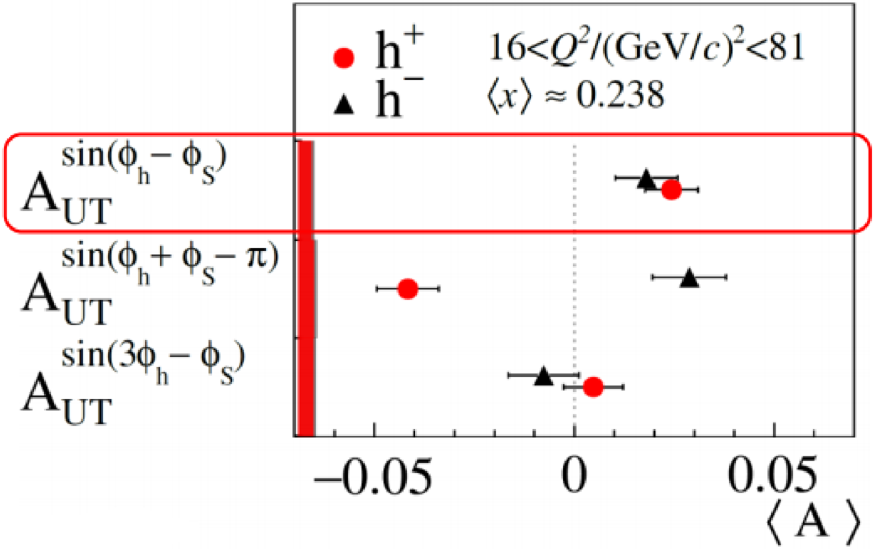


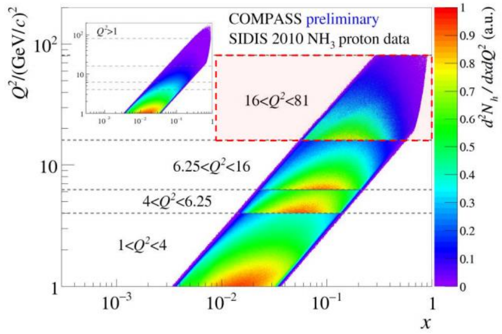
Sivers asymmetry on proton. DY range

Sivers asymmetries in the DY range

COMPASS PLB 770 (2017) 138



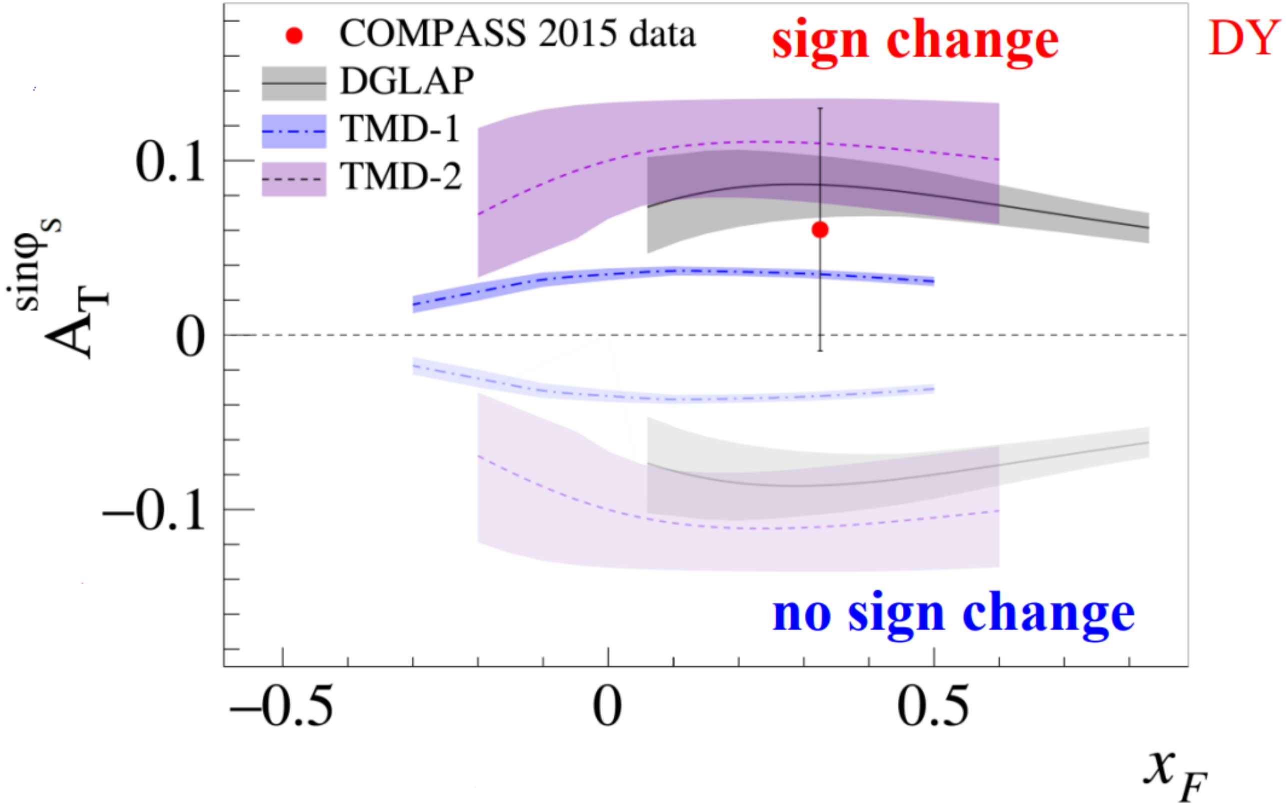


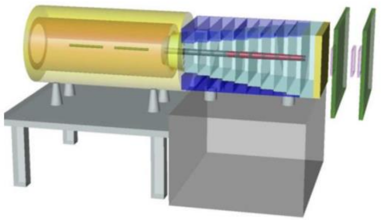


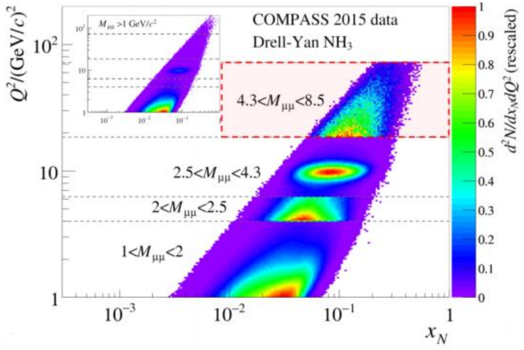
Sivers asymmetry on proton. Drell-Yan measurement

CERN-EP/2017-059

hep-ex/1704.00488

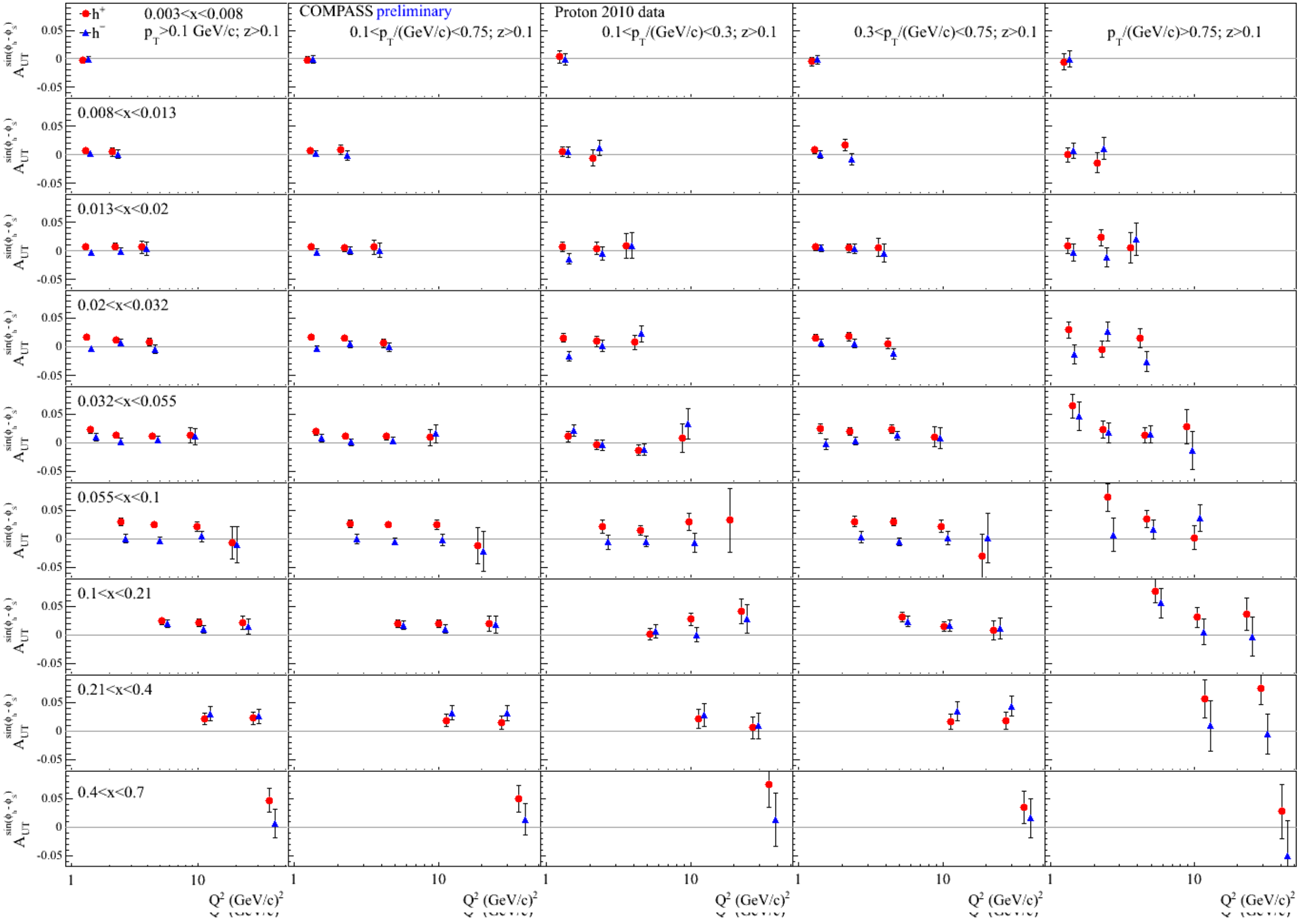


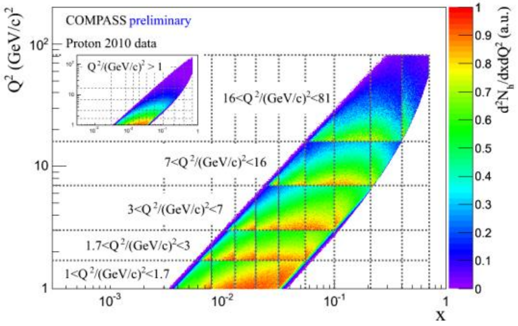


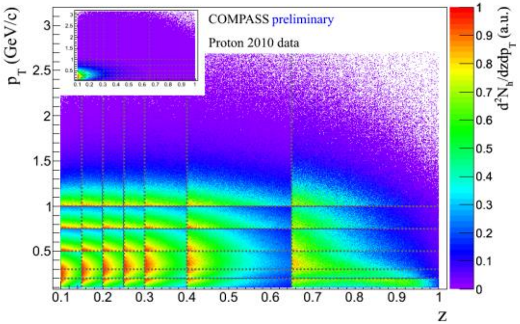


Sivers asymmetry on proton. Multidimensional

First ever extraction of TSAs within such a Multi-D $(x\colon Q^2\colon z\colon p_T)$ approach



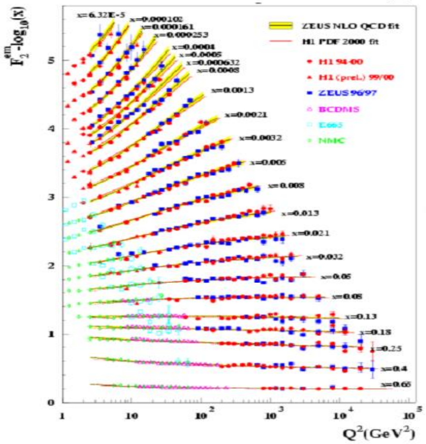




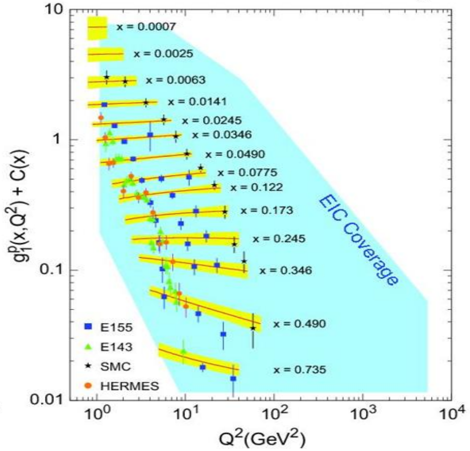
Conclusions

- The study of TMDs has entered the phase of multidimensional analysis
- An important step in this direction is the large sample of precise unpolarised data, both as multiplicities and as azimuthal modulations
- In the next years more of such data will be available both from COMPASS and from JLab12
- Waiting for the EIC to extend the accessible phase space, the description of such data is a mandatory task for the theory of TMDs

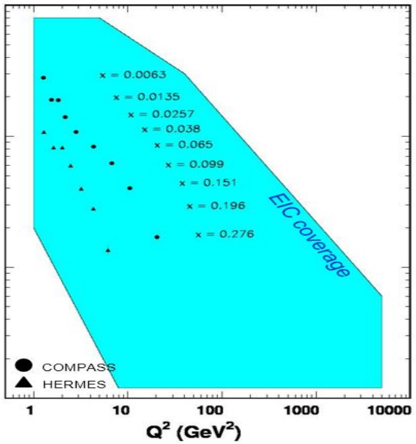
World Data on F_2^p



World Data on g_1^p

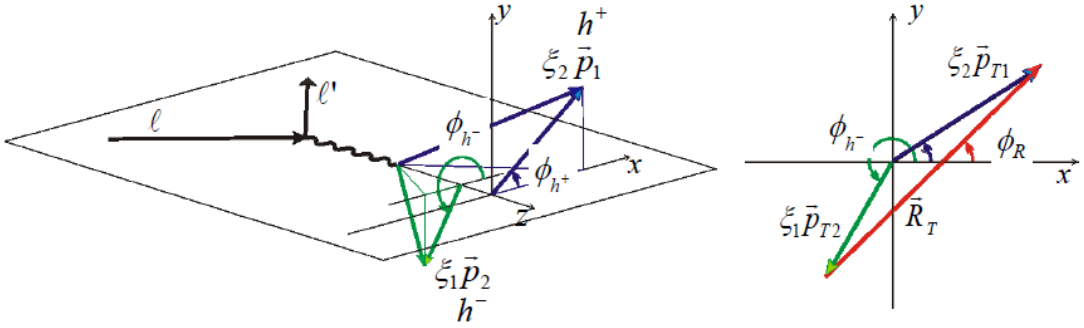


World Data on f_1^p

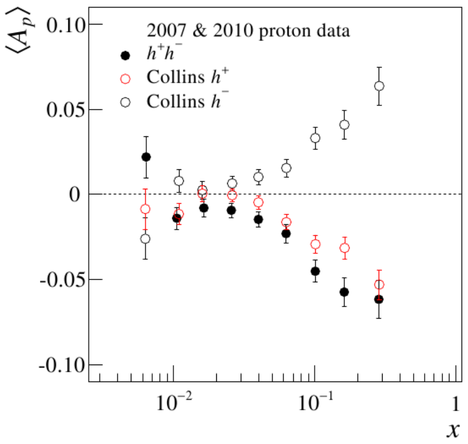


Thank you

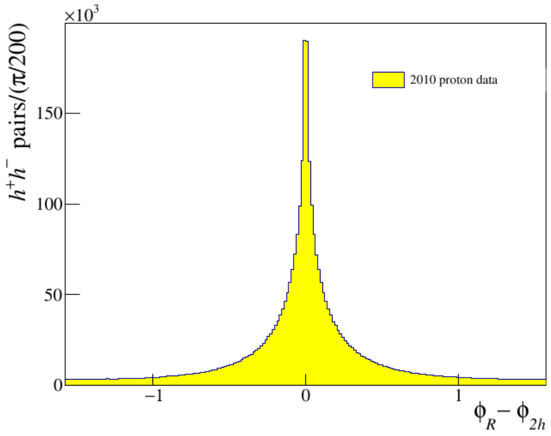
Hadron correlations

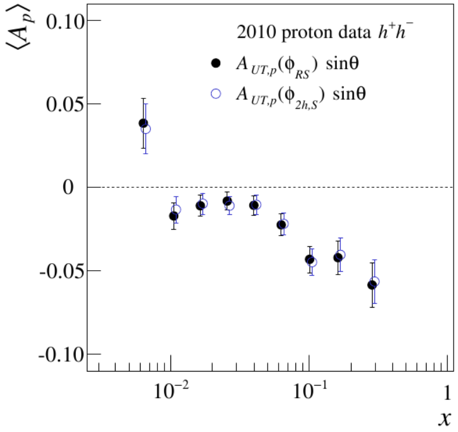


Interplay between Collins and IFF asymmetries

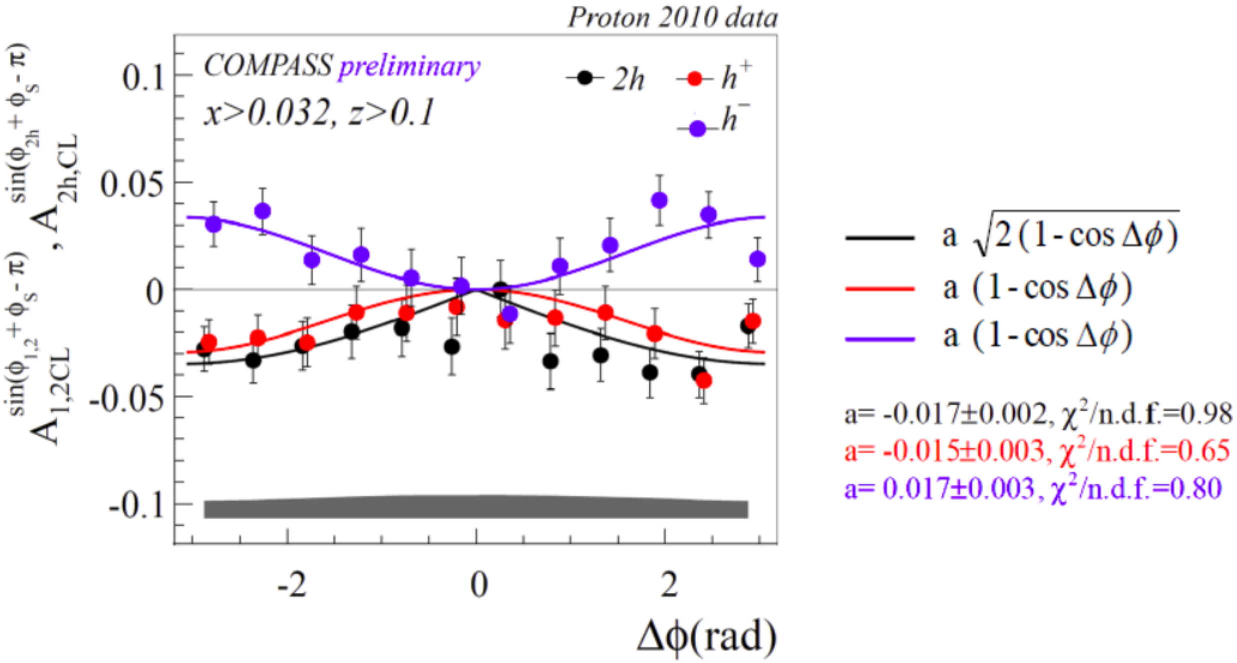


common hadron sample for Collins and 2h analysis





Asymmetries for $x > 0.032$ vs $\Delta\phi = \phi_{h^+} - \phi_{h^-}$



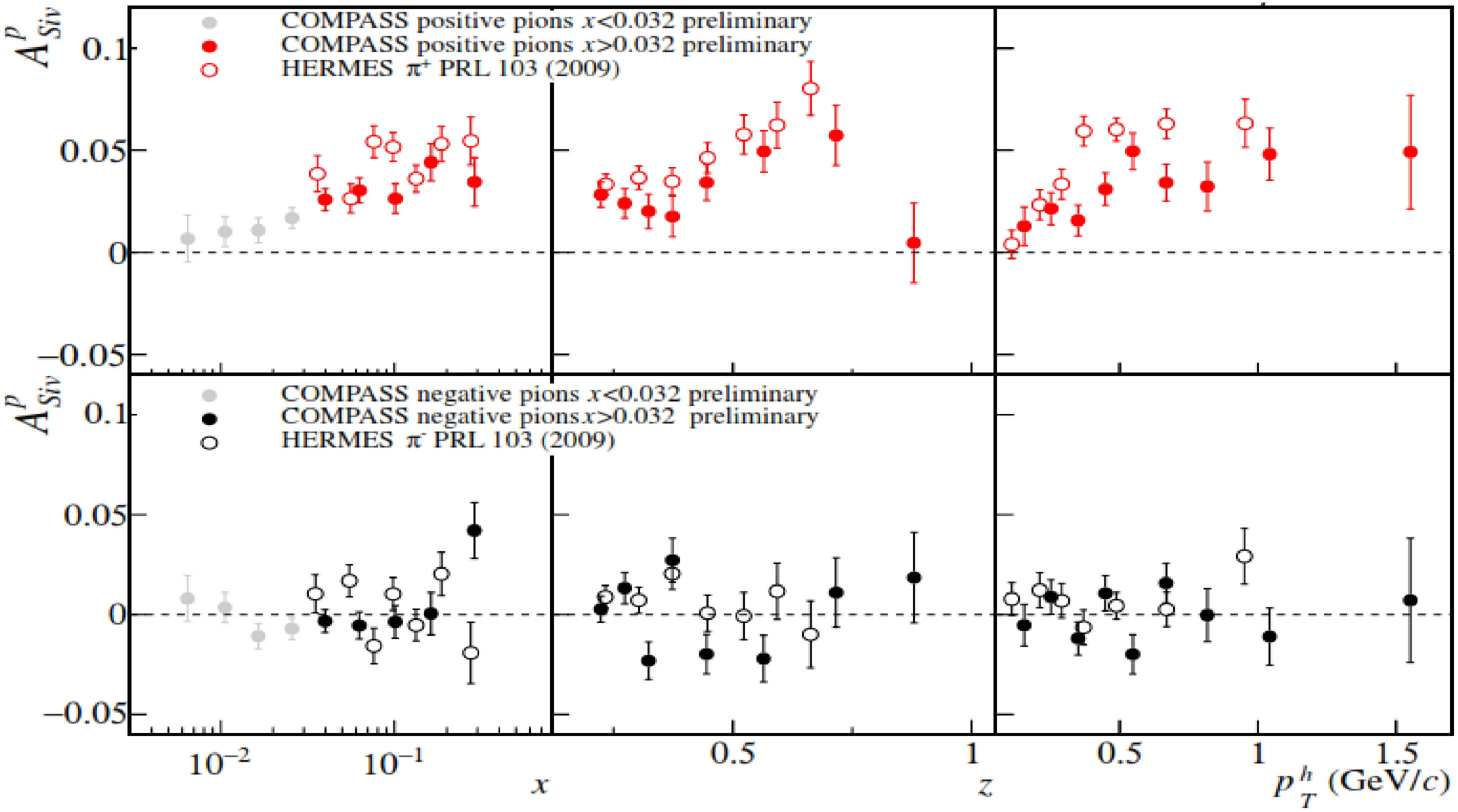
$$a = \frac{\sigma_{1C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

$$= -\frac{\sigma_{2C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

ratio of the integrals compatible with $4/\pi$

Sivers asymmetry on p

charged pions (and kaons), HERMES and COMPASS



π+

●

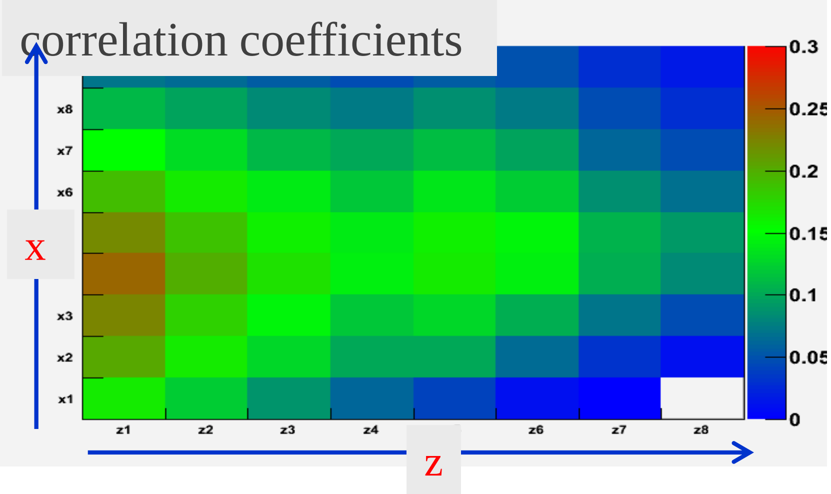
○

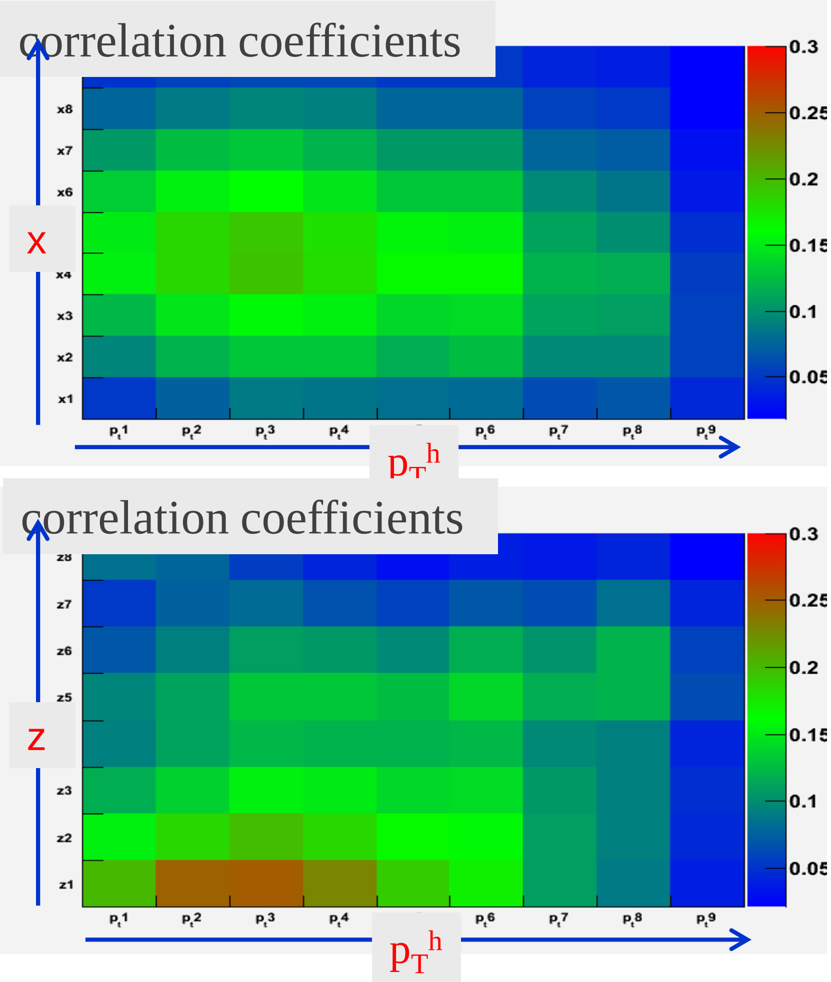
π-

●

○

Statistical correlations





charged pions

also available for

charged hadrons

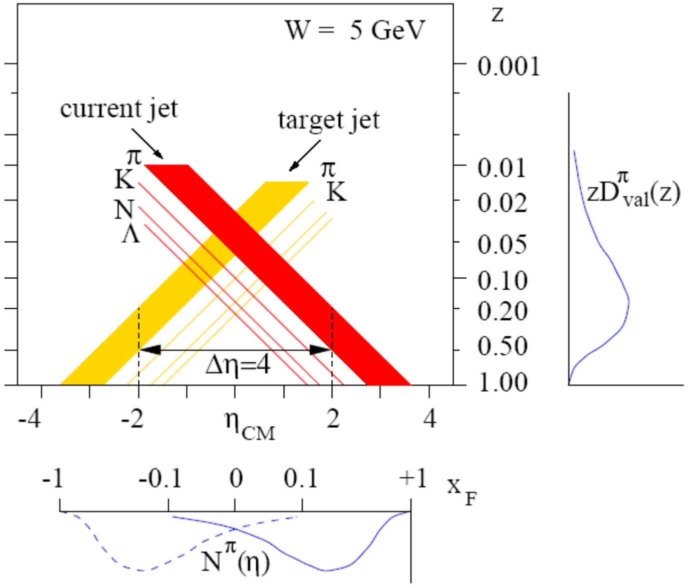
charged kaons

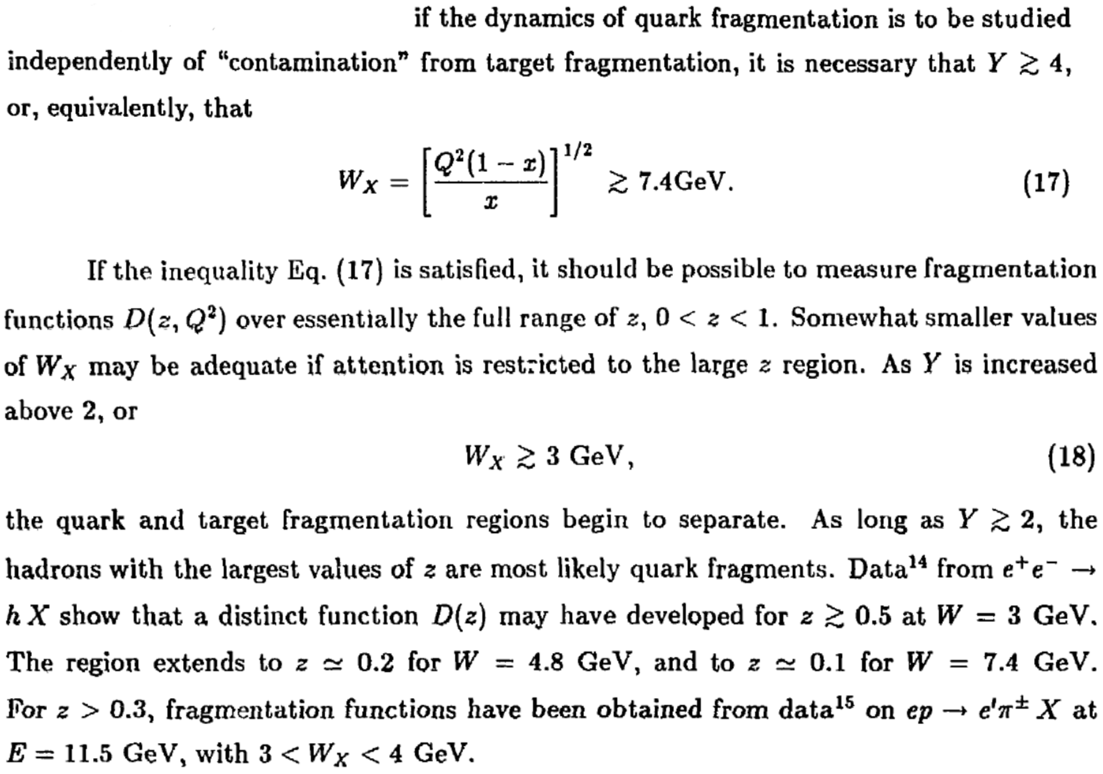
have to be taken into account

Berger criterion (separation of CFR &TFR)

The typical hadronic correlation length in rapidity is

$$\Delta y_h \simeq 2$$





if the dynamics of quark fragmentation is to be studied independently of "contamination" from target fragmentation, it is necessary that $Y \gtrsim 4$, or, equivalently, that

$$W_X = \left[\frac{Q^2(1-x)}{x}\right]^{1/2} \gtrsim 7.4\mathrm{GeV}. \qquad (17)$$

If the inequality Eq. (17) is satisfied, it should be possible to measure fragmentation functions $D(z, Q^2)$ over essentially the full range of z, $0 < z < 1$. Somewhat smaller values of W_X may be adequate if attention is restricted to the large z region. As Y is increased above 2, or

$$W_X \gtrsim 3\ \mathrm{GeV}, \qquad (18)$$

the quark and target fragmentation regions begin to separate. As long as $Y \gtrsim 2$, the hadrons with the largest values of z are most likely quark fragments. Data[14] from $e^+e^- \to h\,X$ show that a distinct function $D(z)$ may have developed for $z \gtrsim 0.5$ at $W = 3$ GeV. The region extends to $z \simeq 0.2$ for $W = 4.8$ GeV, and to $z \simeq 0.1$ for $W = 7.4$ GeV. For $z > 0.3$, fragmentation functions have been obtained from data[15] on $ep \to e'\pi^{\pm} X$ at $E = 11.5$ GeV, with $3 < W_X < 4$ GeV.

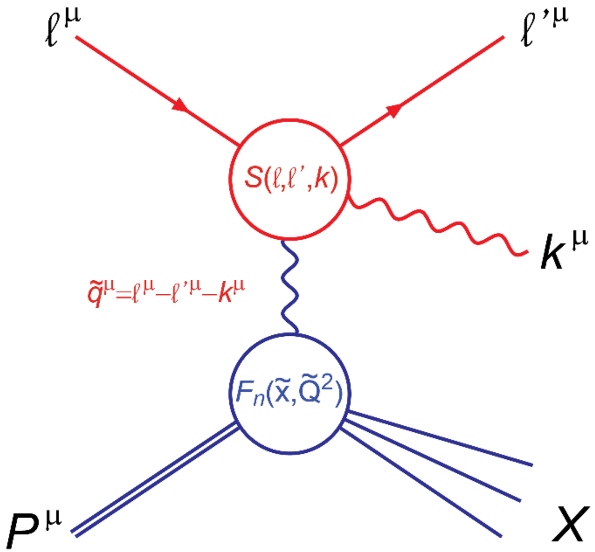
RADIATIVE CORRECTIONS

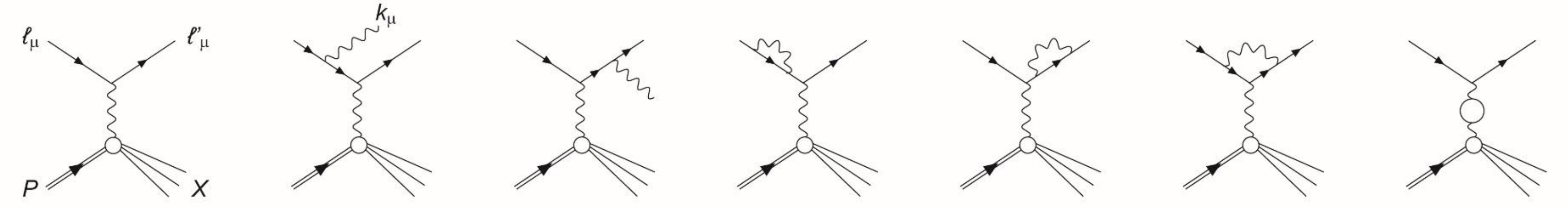
- Three basic channels contribute to lepton-nucleus ℓA scattering at different Q^2 and ν
- These are the
  - Elastic scattering $(\nu = Q^2/2M_A)$
  - Quasi elastic scattering $(\nu \sim Q^2/2M_N)$
  - Inelastic scattering$(\nu > Q^2/2M_N + m_\pi)$
- At Born level, Q^2 and ν are fixed by measuring energy and scattering angle of the lepton and the we can distinguish between the three processes.
- In case of the presence of a radiated photon (i.e. at the level of radiative corrections) the fixing of Q^2 and from θ and E' is removed and the photon has to be included in the kinematic calculation.
- This not done brings to wrong values of Q^2 and ν and mixing between processes.

RADIATIVE CORRECTIONS

- The radiative leptonic tensor $S(\ell, \ell', k)$ is
  - Gauge invariant
  - Infrared finite
  - Universal (for 1γ exchange)
  - The kinematic is shifted

    $\tilde{q}^{\mu} = q^{\mu} - k^{\mu}$

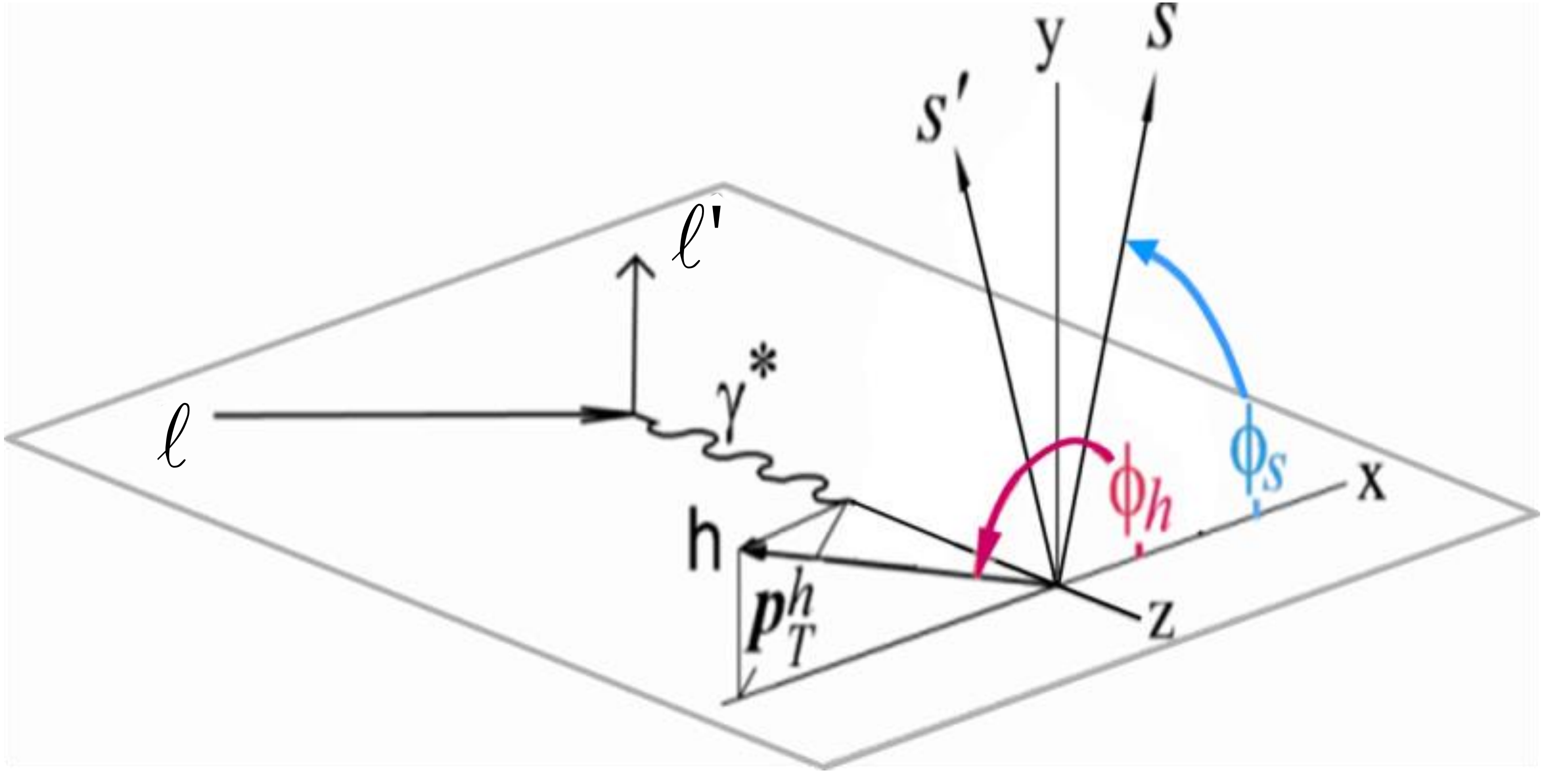




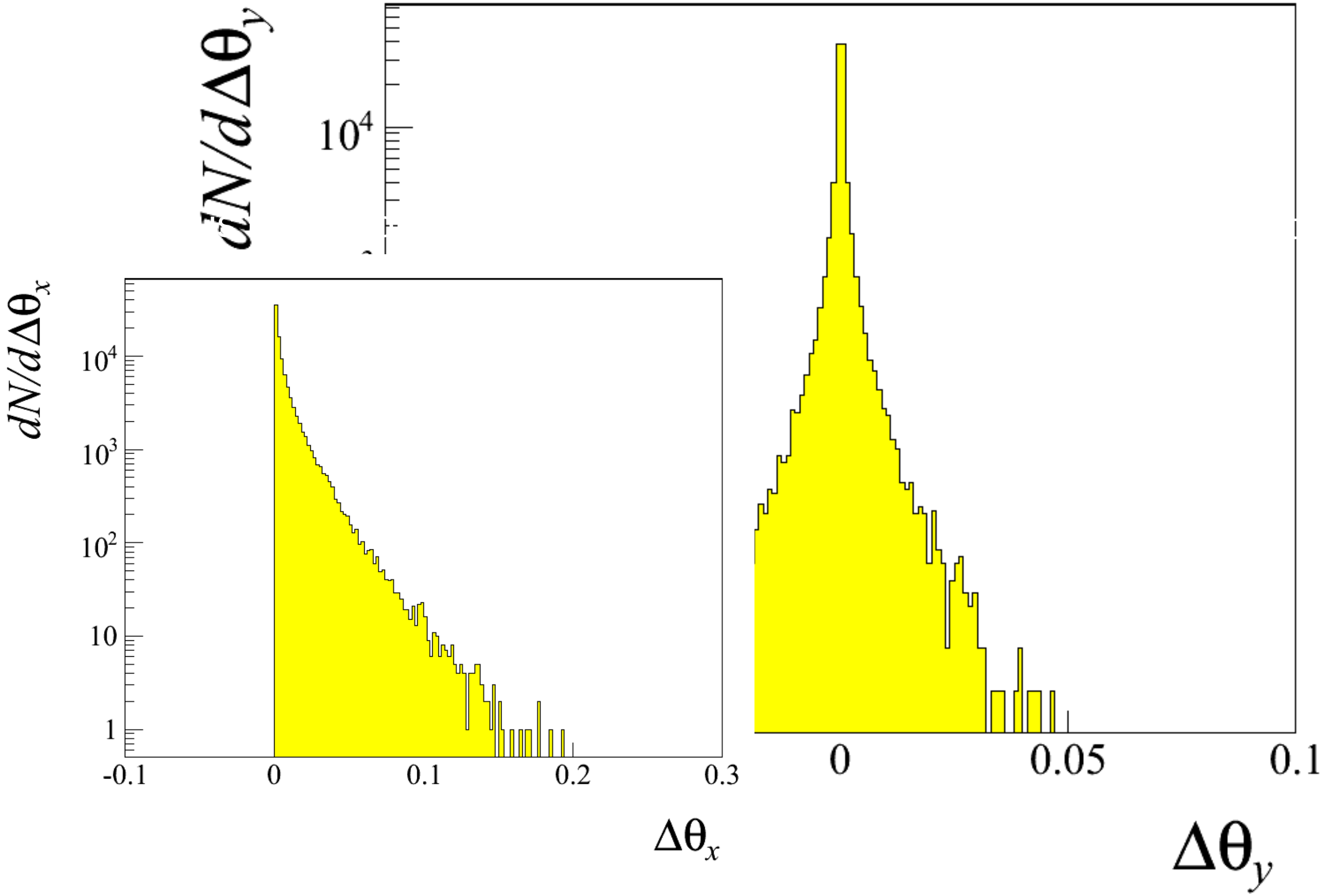
The problem for SIDIS

- Photon radiation from the muon lines changes the DIS kinematics on the event by event basis
- The direction of the virtual photon is changed with respect to the one reconstructed from the muons
  - This introduces false asymmetries in the azimuthal distribution of hadrons calculated with respect to the virtual photon direction
  - Smearing of the kinematic distributions (f.i. z and P_{hT})
- Due to the energy unbalance, in the lepton plane the true virtual photon direction is always at larger angles with respect to the reconstructed one
- In SIDIS, having an hadron in the final state, only the inelastic part of the radiative corrections plays a role

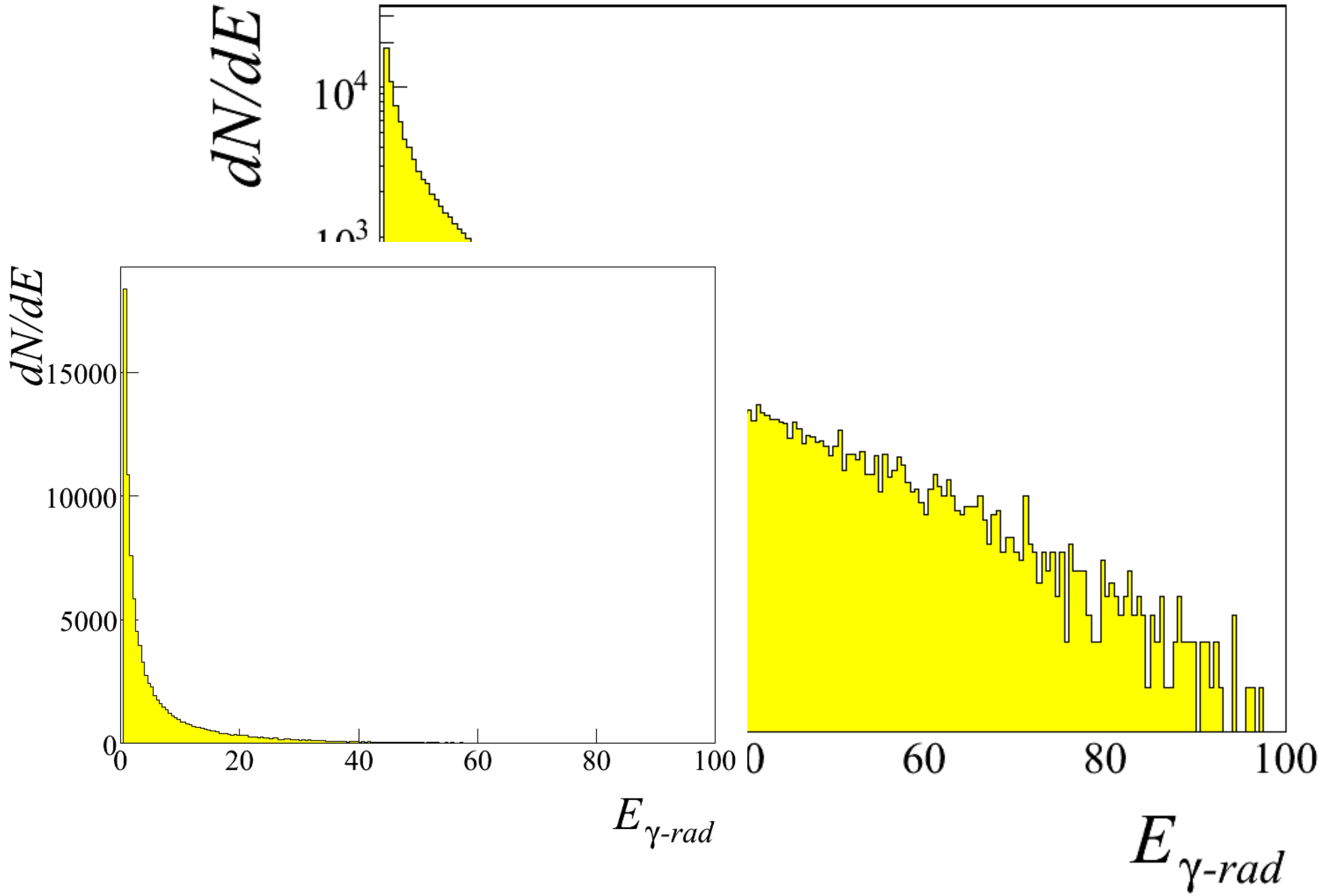
Azimuthal asymmetries in SIDIS



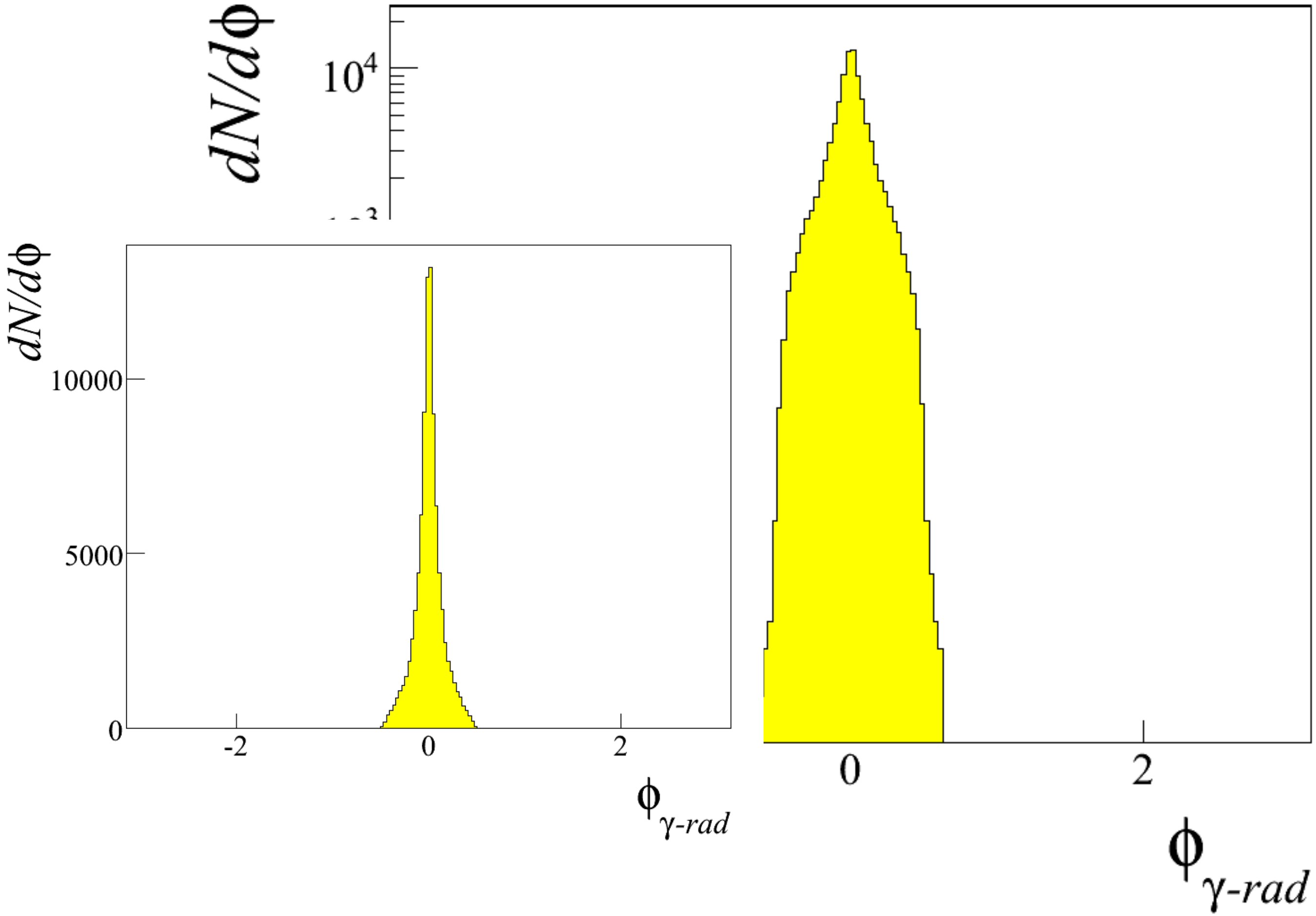
Virtual photon D’s (Qtrue-Qm)



Radiated Photon: Energy



Radiated Photon: Azimuthal angle in GNS



TMD PDFs

- When we consider the TMD we take into account also the transverse motion of the quark $k_\perp$ The field-theoretical expression is quite complicated due to the structure of the gauge link, which now connects two space-time points with a transverse separation

$$f_{q/N}(x,k_\perp) = \frac{1}{8\pi}\int dr^- \frac{dr_\perp^2}{(2\pi)^2} e^{-iMxr^-/2+ik_\perp\cdot r_\perp}$$

$$\langle N(P)|\bar{q}(r^-,r_\perp)\gamma^+ W[r^-,r_\perp;0]q(0)|N(P)\rangle|_{r^+\sim 1/\nu\to 0}$$

- The Wilson line W is no longer on the light-cone axis and may introduce a **process dependence**

Parity and Time reversal invariance ⇒

$$\left(f_{1Tq}^{\perp}\right)_{DY} = -\left(f_{1Tq}^{\perp}\right)_{SIDIS}$$

Most critical test to TMD approach to SSA