High-Precision QCD Predictions for DVCS

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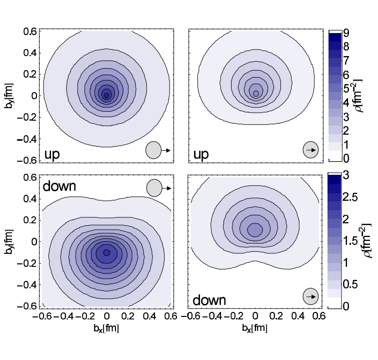
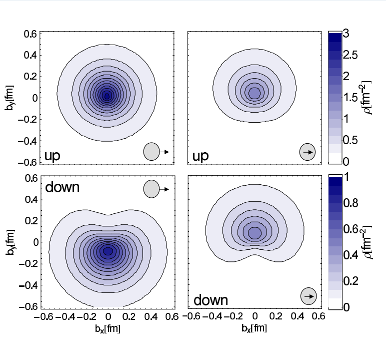
INT Seattle, 08/29/2017

Nucleon Tomography

access to three-dimensional picture of the nucleon (M. Burkardt)





$\hookrightarrow$ first two moments of transverse spin parton density

computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

Precision Frontier

Medium-Term Projection

- DIS: NNNLO unpolarized PDF fits (four-loop); NNLO polarized
- NNLO TMD factorization
- Two-loop multijet amplitudes (Higgs physics)
- Three-loop MOM/MS and/or X-scheme matching for lattice calculations

Tomography: Have to go over from GPD “models” to “parametrizations”

- DVCS: NLO $\rightarrow$ NNLO (Three-loop evolution, two-loop coefficient functions)
- DVCS: Beyond leading twist

In this talk:

- **NNLO evolution equations for Generalized Parton Distributions**

  V. Braun, A. Manashov, S. Moch, M. Strohmaier, JHEP 1603 (2016) 142;
  JHEP 1706 (2017) 037

- **Kinematic power corrections to DVCS**
  - Ambiguity of the leading-twist approximations
  - Finite t and target mass corrections, t/Q^2 and m^2/Q^2

    V. Braun, A. Manashov, D. Müller, B. Pirnay, Phys.Rev. D89 (2014) 074022
  - Scalar targets

    V. Braun, A. Manashov, B. Pirnay, Phys.Rev. D86 (2012) 014003

Multiplicatively renormalizable leading-twist operators

- **One loop:**
  anomalous dimensions + conformal symmetry → full anomalous dimension matrix

$$\mathcal{O}_N \sim (\partial_{z_1} + \partial_{z_2})^N C_N^{3/2}\left(\frac{\partial_{z_1} - \partial_{z_2}}{\partial_{z_1} + \partial_{z_2}}\right) \bar{q}(z_1 n)\gamma_+ q(z_2 n)\Big|_{z_{1,2}\to 0}$$

Makeenko, 1980

- **D. Müller, '94-'00**

  — off-diagonal elements of the n-loop mixing matrix are determined by the $(n-1)$-loop conformal anomaly

- **Two loops:**

  Belitsky, Mueller, 2000

- **Three loops:**

  Braun, Manashov, Moch, Strohmaier, work in progress

Different approach:

Instead of studying consequences of conformal symmetry breaking in QCD we make use of *exact* conformal symmetry of a modified theory:

Large N_f QCD in $4-\epsilon$ dimension at critical coupling

QCD in $d = 4 - 2\epsilon$

Consider renormalized QCD in $d = 4 - 2\epsilon$ dimensions. In MS-like schemes

$$\beta^{QCD}(a_s) = 2a_s\left[-\epsilon - \beta_0 a_s + \ldots\right] \qquad Z = 1 + \sum_{j=1}^{\infty} \epsilon^{-j} \sum_{k=j}^{\infty} a_s^k \mathbb{Z}_{jk}$$

- scale and conformal invariance at the critical point — Banks, Zaks, '82

$$a_s^* = -4\pi\epsilon/\beta_0 + \ldots \qquad \beta^{QCD}(a_s^*) = 0$$

- Z_{jk} do not depend on ϵ by construction, thus

$$\boxed{\mathbb{H}(a_s^*) = a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu \; + \; \mathbb{H}(a_s^*)\Big)[\mathcal{O}](z_1, z_2) = 0$$

$\Longleftrightarrow$

$$\boxed{\mathbb{H}(a_s) = a_s\,\mathbb{H}^{(1)} + a_s^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbb{H}(a_s)\Big)[\mathcal{O}(z_1, z_2)] = 0$$

“Hidden” conformal invariance of QCD RG equations in MS-like schemes

$$\Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbb{H}(a_s)\Big)[\mathcal{O}(z_1, z_2)] = 0$$

- Conformal symmetry implies existence of three generators that satisfy usual SL(2) relations and commute with the renormalization kernel

$$[S_k, \mathbb{H}] = 0$$

$$[S_+, S_-] = 2S_0 \qquad [S_0, S_+] = S_+ \qquad [S_0, S_-] = -S_-$$

- True to all orders in perturbation theory (in MS-like schemes)
- Complete RG kernel in $d = 4$, a digression to $d = 4 - \epsilon$ is an intermediate step

In a free theory, in coordinate representation

generators $j = 1$ for quarks

$$
\begin{aligned}
S_+ &= z^2\partial_z + 2jz \\
S_0 &= z\partial_z + j \\
S_- &= -\partial_z
\end{aligned}
$$

$SL(2)$ algebra

$$
\begin{aligned}
[S_+, S_-] &= 2S_0 \\
[S_0, S_+] &= S_+ \\
[S_0, S_-] &= -S_-
\end{aligned}
$$

- In the interacting theory the generators are modified by quantum corrections

$$
\begin{aligned}
S_+ &= S_+^{(0)} + a_s^* S_+^{(1)} + (a_s^*)^2 S_+^{(2)} + \dots \\
S_0 &= S_0^{(0)} + a_s^* S_0^{(1)} + (a_s^*)^2 S_0^{(2)} + \dots \\
S_- &= S_-^{(0)}
\end{aligned}
$$

- General structure

$$
\begin{aligned}
S_- &= S_-^{(0)}\,, \\
S_0 &= S_0^{(0)} - \epsilon + \frac{1}{2}\mathbb{H}(a_s^*)\,, \\
S_+ &= S_+^{(0)} + (z_1+z_2)\Big(-\epsilon + \frac{1}{2}\mathbb{H}\Big) + (z_1-z_2)\Delta_+\,,
\end{aligned}
$$

$$a_s = \frac{\alpha_s^*}{4\pi}$$

where

$$
\begin{aligned}
\mathbb{H}(a_s^*) &= a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + (a_s^*)^3\,\mathbb{H}^{(3)} + \ldots \\
\Delta_+(a_s^*) &= a_s^*\,\Delta_+^{(1)} + (a_s^*)^2\,\Delta_2^{(2)} + (a_s^*)^3\,\Delta_3^{(3)} + \ldots
\end{aligned}
$$

- Modification of S_0 can be written in terms of the evolution kernel

…but $\Delta_+(a_s^*)$ requires explicit calculation

- Light-ray operator representation *Balitsky, Braun '89*

$$[\mathcal{O}](z_1,z_2) \equiv [\bar q(z_1 n)\not{n} q(z_2 n)] \equiv \sum_{m,k}\frac{z_1^m z_2^k}{m!k!}[(D_n^m \bar q)(0)\not{n}(D_n^k q)(0)]$$

$$\mathbb{H}[\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha,\beta)\,[\mathcal{O}](z_{12}^\alpha, z_{21}^\beta)$$

$$z_{12}^\alpha \equiv z_1\bar\alpha + z_2\alpha \qquad \bar\alpha = 1-\alpha$$

one-loop result Braun, Manashov, PLB **734**, 137 (2014)

$$\Delta_+^{(1)}[\mathcal{O}](z_1,z_2) = -2C_F\int_0^1 d\alpha\left(\frac{\bar\alpha}{\alpha}+\ln\alpha\right)\Big[[\mathcal{O}](z_{12}^\alpha,z_2)-[\mathcal{O}](z_1,z_{21}^\alpha)\Big]$$

Two-loop conformal anomaly

- two-loop result — Braun, Manashov, Moch, Strohmaier: JHEP **1603** (2016) 142

$$[\Delta_+^{(2)}\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \Big[\omega(\alpha,\beta) + \omega^{\mathbb{P}}(\alpha,\beta)\mathbb{P}_{12}\Big]\Big[\mathcal{O}(z_{12}^\alpha, z_{21}^\beta) - \mathcal{O}(z_{12}^\beta, z_{21}^\alpha)\Big]$$
$$+ \int_0^1 du \int_0^1 dt\, \varkappa(t) \Big[\mathcal{O}(z_{12}^{ut}, z_2) - \mathcal{O}(z_1, z_{21}^{ut})\Big].$$

$$\omega^{\mathbb{P}} = 2\Big[C_F^2 - \frac{1}{2}C_F C_A\Big]\Big[\Big(\bar\alpha - \frac{1}{\bar\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\alpha}{\bar\beta}\Big) - \mathrm{Li}_2(\alpha) - \ln\bar\alpha \ln\bar\beta\Big] + \alpha\bar\tau\ln\bar\tau + \frac{\beta^2}{\bar\beta}\ln\bar\alpha\Big]$$

Two-loop conformal anomaly (II)

- two-loop result — Braun, Manashov, Moch, Strohmaier: JHEP **1603** (2016) 142

$$\omega(\alpha,\beta) = C_F^2\,\omega_{FF}(\alpha,\beta) + C_F C_A\,\omega_{FA}(\alpha,\beta)$$

$$\begin{aligned}\omega_{FF} = 4\Big\{&\Big(\alpha-\frac{1}{\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-\mathrm{Li}_2(\alpha)-\frac14\ln^2\bar\alpha\Big]-\alpha\Big[\mathrm{Li}_2(\alpha)-\mathrm{Li}_2(1)\Big]\\ &-\frac{\alpha+\beta}{2}\ln\alpha\ln\bar\alpha+\frac14\Big[\beta\ln^2\bar\alpha-\alpha\ln^2\alpha\Big]-\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]\\ &+\frac14\Big[\beta-2\bar\alpha+\frac{2\beta}{\alpha}\Big]\ln\bar\alpha+\frac12\Big[\bar\alpha-\frac{\alpha}{\bar\alpha}-3\beta\Big]\ln\alpha-\frac{15}{4}\alpha\Big\},\end{aligned}$$

$$\begin{aligned}\omega_{FA} = 2\Big\{&\Big(\frac{1}{\alpha}-\alpha\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-2\,\mathrm{Li}_2(\alpha)-\ln\alpha\ln\bar\alpha\Big]\\ &+\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]-\bar\beta\ln\alpha-\frac{\bar\alpha}{\alpha}\ln\bar\alpha\Big\},\end{aligned}$$

$$\boxed{\tau=\frac{\alpha\beta}{\bar\alpha\bar\beta}}$$

- result for $\varkappa(t)$ of similar complexity

Evolution equations from operator algebra

- **Expanding the commutation relations in powers of a_s^***

$$[S_+^{(0)}, \mathbb{H}^{(1)}] = 0\,,$$
$$[S_+^{(0)}, \mathbb{H}^{(2)}] = [\mathbb{H}^{(1)}, S_+^{(1)}]\,,$$
$$[S_+^{(0)}, \mathbb{H}^{(3)}] = [\mathbb{H}^{(1)}, S_+^{(2)}] + [\mathbb{H}^{(2)}, S_+^{(1)}]\,, \qquad \text{etc.}$$

- A nested set of inhomogenious first order differential equations for $\mathbb{H}^{(k)}$
  Their solution determines $\mathbb{H}^{(k)}$ up to an $SL(2)$-invariant term
- The r.h.s. involves $\mathbb{H}^{(k)}$ and $S_+^{(m)}$ at one oder less compared to the l.h.s. D.Müller

Solution

Braun, Manashov, Moch, Strohmaier, JHEP 1706 (2017) 037

$$\mathbb{H}^{(1)} = \mathbf{H}^{(1)}_{\rm inv}\,,$$

$$\mathbb{H}^{(2)} = \mathbf{H}^{(2)}_{\rm inv} + \mathbb{T}^{(1)}\Big(\beta_0 + \frac{1}{2}\mathbf{H}^{(1)}_{\rm inv}\Big) + [\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(1)}]\,,$$

$$\begin{aligned}\mathbb{H}^{(3)} = {}& \mathbf{H}^{(3)}_{\rm inv} + \mathbb{T}^{(1)}\Big(\beta_1 + \frac{1}{2}\mathbf{H}^{(2)}_{\rm inv}\Big) + \mathbb{T}^{(2)}_1\Big(\beta_0 + \frac{1}{2}\mathbf{H}^{(1)}_{\rm inv}\Big)^2 + \Big(\mathbb{T}^{(2)} + \frac{1}{2}\big(\mathbb{T}^{(1)}\big)^2\Big)\Big(\beta_0 + \frac{1}{2}\mathbf{H}^{(1)}_{\rm inv}\Big)\\ &+ [\mathbf{H}^{(2)}_{\rm inv}, \mathbb{X}^{(1)}] + \frac{1}{2}\Big[\mathbb{T}^{(1)}\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(1)}\Big] + \frac{1}{2}\Big[\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(2,1)}\Big]\mathbf{H}^{(1)}_{\rm inv} + [\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(2)}_{\rm I}]\\ &+ \beta_0\Big(\Big[\mathbb{T}^{(1)}, \mathbb{X}^{(1)}\Big] + \Big[\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(2,1)}\Big]\Big) + \frac{1}{2}\Big[\Big[\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(1)}\Big], \mathbb{X}^{(1)}\Big] - \frac{1}{2}\Big[\mathbf{H}^{(1)}_{\rm inv}, \mathbb{X}^{(2,2)}\Big],\end{aligned}$$

- All $\mathbb{T}$ and $\mathbb{X}$ kernels known analytically
- Invariant kernels are solutions of homogenious equations $[S_+^{(0)}, \mathbf{H}_{\rm inv}]^{(k)} = 0$ (functions of conformal ratio)

Examples

$$\mathbb{X}^{(1)} f(z_1, z_2) = 2C_F \int_0^1 \frac{d\alpha}{\alpha} \ln\alpha \Big[2f(z_1, z_2) - f(z_1, z_{21}^\alpha) - f(z_{12}^\alpha, z_2) \Big]$$

$$\mathbb{T}^{(1)} f(z_1, z_2) = 4C_F \bigg\{ \int_0^1 \frac{d\alpha}{\alpha} \bar\alpha \ln\bar\alpha \Big[2f(z_1, z_2) - f(z_1, z_{21}^\alpha) - f(z_{12}^\alpha, z_2) \Big] - \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \ln(1-\alpha-\beta) f(z_{12}^\alpha, z_{21}^\beta) \bigg\}$$

Invariant kernels (I)

$$\mathbf{H}_{\rm inv}[\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha,\beta)\,[\mathcal{O}](z_{12}^\alpha, z_{21}^\beta)$$

$$z_{12}^\alpha \equiv z_1\bar\alpha + z_2\alpha \qquad \bar\alpha = 1-\alpha$$

$$\boxed{[S_+^{(0)}, \mathbf{H}_{\rm inv}] = 0} \qquad \Longrightarrow \qquad h(\alpha,\beta) = h\left(\frac{\alpha\beta}{\bar\alpha\bar\beta}\right) = h(\tau)$$

i.e. a function of two variables reduces to a function of one variable
$\longrightarrow$ can be restored from anomalous dimensions

$$\gamma_N = \int_0^1 d\alpha \int_0^1 d\beta\,(1-\alpha-\beta)^{N-1} h(\alpha,\beta)$$

- One-loop — Balitsky, Braun, 1989

$$\mathbf{H}^{(1)}_{\rm inv} f(z_1,z_2) = 4C_F\Big\{\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[2f(z_1,z_2) - f(z_{12}^\alpha,z_2) - f(z_1,z_{21}^\alpha)\Big] - \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, f(z_{12}^\alpha,z_{21}^\beta) + \frac12 f(z_1,z_2)\Big\}$$

- Combined DGLAP, ERBL and GPD evolution equations in a compact form

Invariant kernels (II)

Braun, Manashov, PLB **734** (2014) 137

- Two loops

$$\mathbf{H}^{(2)}_{\rm inv} = 4C_F\Big\{\beta_0\left(\frac{13}{12}+\frac{5}{3}\mathcal{H}_{\langle+\rangle}-\frac{11}{3}\mathcal{H}_{\langle 1\rangle}\right)+2C_A\left(\frac{19}{48}-\frac{1}{3}\mathcal{H}_{\langle+\rangle}-\frac{2}{3}\mathcal{H}_{\langle 1\rangle}-\frac{1}{4}\mathcal{H}^2_{\langle 1\rangle}\right)$$
$$+\frac{2}{N_c}\Big[\left(3\zeta(3)-\frac{\pi^2}{3}+\frac{11}{16}\right)-\frac{\pi^2-6}{6}\left(\mathcal{H}_{\langle+\rangle}-\mathcal{H}_{\langle 1\rangle}\right)+\mathcal{H}_{\langle\frac{1}{\tau}\ln\bar\tau\rangle}-\frac{3}{4}\mathcal{H}^2_{\langle 1\rangle}-\mathcal{H}^3_{\langle 1\rangle}$$
$$-\frac{1}{2}\mathbb{P}_{12}\left(\mathcal{H}_{\langle\ln^2\bar\tau\rangle}-2\mathcal{H}_{\langle\tau\ln\bar\tau\rangle}\right)\Big]\Big\}.$$

in this notation

$$\mathbf{H}^{(1)}_{\rm inv} = 4C_F\Big\{\mathcal{H}_{\langle+\rangle}-\mathcal{H}_{\langle 1\rangle}+\frac{1}{2}\Big\}$$

Invariant kernels (III)

Braun, Manashov, Moch, Strohmaier, JHEP 1706 (2017) 037

- Three loops

$$\mathbf{H}^{(3)}_{\rm inv} f(z_1,z_2) = \Gamma^{(3)}_{\rm cusp}\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big(2f(z_1,z_2) - f(z_{12}^\alpha,z_2) - f(z_1,z_{21}^\alpha)\Big) + \chi_0^{(3)} f(z_1,z_2)$$
$$+\int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\Big(\chi^{(3)}_{\rm inv}(\tau) + \chi^{\mathbb{P}(3)}_{\rm inv}(\tau)\mathbb{P}_{12}\Big) f(z_{12}^\alpha, z_{21}^\beta)\,.$$

restored from the known spectrum

$$\mathbf{H}^{(3)}_{\rm inv}(z_1-z_2)^N = \gamma^{(3)}_{\rm inv}(N)\,(z_1-z_2)^N; \qquad \gamma^{(3)}_{\rm inv}(N) = \gamma^{(3+)}_{\rm inv}(N) + (-1)^N\gamma^{(3-)}_{\rm inv}(N)$$

$$\chi^{(3)}_{\rm inv}(\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\,(2N+3)\,\Delta\gamma^{(3+)}_{\rm inv}(N)\,P_{N+1}\left(\frac{1+\tau}{1-\tau}\right)$$
$$\chi^{\mathbb{P}(3)}_{\rm inv}(\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\,(2N+3)\,\gamma^{(3-)}_{\rm inv}(N)\,P_{N+1}\left(\frac{1+\tau}{1-\tau}\right)$$

$P_N(x)$ are Legendre functions

Invariant kernels (IV)

Braun, Manashov, Moch, Strohmaier, JHEP 1706 (2017) 037

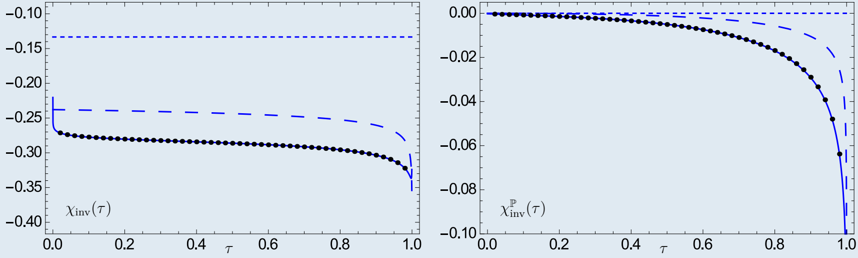


Figure: Invariant functions $\chi_{\rm inv}(\tau)$ (left panel) and $\chi^{\mathbb{P}}_{\rm inv}(\tau)$ (right panel) for $\alpha_s/\pi = 0.1$. The LO result (short dashes) is shown together with the NLO (long dashes) and NNLO (solid curves). The NNLO results using exact $\mathcal{O}(a^3)$ functions obtained by the numerical integration are shown by black dots for comparison.

- Flavor-singlet operators: in planning

Byproduct: pion distribution amplitude

- mixing matrix for the Gegenbauer polynomials expansion: 1703.0953

$$a = \frac{\alpha_s}{4\pi}, \qquad n_f = 4$$

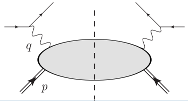
$$a^2 \begin{pmatrix} * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 11.1+179a & 0 & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 22.6+290a & 0 & * & 0 & 0 & 0 & 0 \\ -3.47-13.2a & 0 & 24.5+297a & 0 & * & 0 & 0 & 0 \\ 0 & 6.12+93.2a & 0 & 24.3+291a & 0 & * & 0 & 0 \\ -5.94-49.3a & 0 & 9.74+120a & 0 & 23.5+282a & 0 & * & 0 \\ 0 & 0.0764+35.6a & 0 & 11.4+131a & 0 & 22.6+272a & 0 & * \end{pmatrix}.$$

HIGHER TWISTS

Planar vs. non-planar kinematics

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

DIS

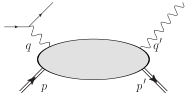


Define (p, q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

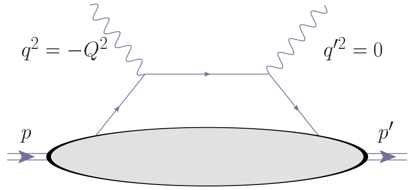
etc.

$\Rightarrow$ parton fraction $2\xi = x_B[1+\mathcal{O}\left(\frac{t}{Q^2}\right)]$, redefinition of helicity amplitudes

- **Ambiguity is resolved by adding "kinematic" power corrections t/Q^2, m^2/Q^2**

"Photon" reference frame

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2+t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \;\sim\; t_{\rm min} - t$$

where

$$P = \frac{1}{2}(p+p')\,, \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} \;=\; \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu}P^\nu$$

Relating CFFs in the laboratory and photon reference frame

$$\mathcal{F}_{++}^{\text{lab}} = \mathcal{F}_{++}^{\text{phot}} + \frac{\varkappa}{2}\left[\mathcal{F}_{++}^{\text{phot}} + \mathcal{F}_{-+}^{\text{phot}}\right] - \varkappa_0\, \mathcal{F}_{0+}^{\text{phot}},$$
$$\mathcal{F}_{0+}^{\text{lab}} = -(1+\varkappa)\,\mathcal{F}_{0+}^{\text{phot}} + \varkappa_0\left[\mathcal{F}_{++}^{\text{phot}} + \mathcal{F}_{-+}^{\text{phot}}\right]$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Defining the Leading Twist approximation

Kumerički-Müller convention (KM)

Braun-Manashov-Pirnay convention (BMP)

$$\mathsf{LT}_{\rm KM}:\ \begin{cases}\mathcal{F}^{\sf lab}_{++} = T_0 \otimes F, & \mathcal{F}^{\sf lab}_{0+} = 0,\\ \mathcal{F}^{\sf lab}_{-+} = 0, & \xi_{\rm KM} = \xi^{\sf lab}\end{cases}$$

$$\mathsf{LT}_{\rm BMP}:\ \begin{cases}\mathcal{F}^{\sf phot}_{++} = T_0 \otimes F, & \mathcal{F}^{\sf phot}_{0+} = 0,\\ \mathcal{F}^{\sf phot}_{-+} = 0, & \xi_{\rm BMP} = \xi^{\sf phot}\end{cases}$$

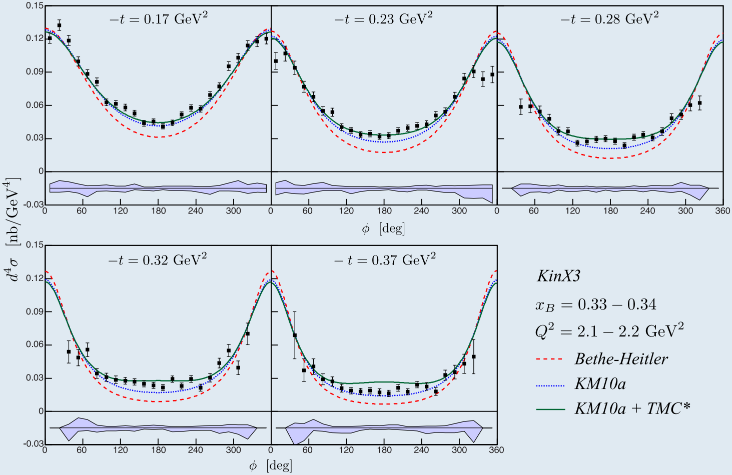
$$\Downarrow$$

$$\mathsf{LT}_{\rm BMP}:\ \begin{cases}\mathcal{F}^{\sf lab}_{++} = \left(1+\frac{\varkappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \varkappa_0\, T_0 \otimes F\\ \mathcal{F}^{\sf lab}_{-+} = \frac{\varkappa}{2}\, T_0 \otimes F, & \xi = \xi_{\rm BMP},\end{cases}$$

- **Changing frame of reference results in**
  - **Different skewedness parameter for a given x_B**
  - **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

Large effects for the total cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453

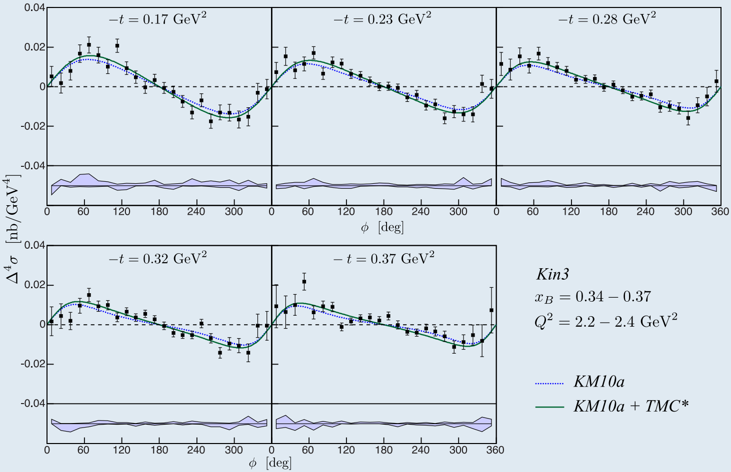


- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Small/moderate effects for asymmetries

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Lessons:

- **noncomplanarity makes separation of collinear directions ambiguous**
  - — hence "leading twist approximation" ambiguous
  - — related to violation of translation invariance and EM Ward identities
- **have to be repaired by adding power corrections of special type, "kinematic" PC**

BMP helicity amplitudes

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q,q',p) &= i\int d^4x\, e^{-i(z_1 q - z_2 q')x}\langle p',s'|T\{J_\mu(z_1x)J_\nu(z_2x)\}|p,s\rangle \\ &= \varepsilon^+_\mu\varepsilon^-_\nu\mathcal{A}^{++} + \varepsilon^-_\mu\varepsilon^+_\nu\mathcal{A}^{--} + \varepsilon^0_\mu\varepsilon^-_\nu\mathcal{A}^{0+} \\ &+ \varepsilon^0_\mu\varepsilon^+_\nu\mathcal{A}^{0-} + \varepsilon^+_\mu\varepsilon^+_\nu\mathcal{A}^{+-} + \varepsilon^-_\mu\varepsilon^-_\nu\mathcal{A}^{-+} + q'_\nu\mathcal{A}^{(3)}_\mu\end{aligned}$$

for the calculation to the twist-4 accuracy one needs

- $\mathcal{A}^{++}, \mathcal{A}^{--}$: $1 + \frac{1}{Q^2}$
- $\mathcal{A}^{0+}, \mathcal{A}^{0-}$: $\frac{1}{Q}$ $\longleftarrow$ agree with existing results
- $\mathcal{A}^{-+}, \mathcal{A}^{+-}$: $\frac{1}{Q^2}$ $\longleftarrow$ straightforward

BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$\mathcal{A}_q^{a\pm} = \mathbb{H}^q_{a\pm} h + \mathbb{E}^q_{a\pm} e \mp \widetilde{\mathbb{H}}^q_{a\pm} \tilde{h} \mp \widetilde{\mathbb{E}}^q_{a\pm} \tilde{e}$$

with, e.g. Belitsky, Müller, Ji: NPB **878** (2014) 214

$$h = \frac{\bar{u}(p')\,(\not{q} + \not{q}')u(p)}{P\cdot(\not{q} + \not{q}')} \qquad \ldots$$

- The results read Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\mathbb{H}_{++} = T_0 \circledast H + \frac{t}{Q^2}\left[-\frac{1}{2}T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\right] \circledast H + \frac{2t}{Q^2}\xi^2 \partial_\xi \xi T_2 \circledast (H+E)$$

$$\mathbb{H}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left[\xi\partial_\xi T_1 \circledast H + \frac{t}{Q^2}\partial_\xi \xi\, T_1 \circledast (H+E)\right] - \frac{t}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \left[\xi\,(H+E) - \widetilde{H}\right]$$

$$\mathbb{H}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\left[\xi\partial_\xi^2 \xi\, T_1^{(+)} \circledast H + \frac{t}{Q^2}\partial_\xi^2 \xi^2\, T_1^{(+)} \circledast (H+E)\right]$$
$$+ \frac{2t}{Q^2}\xi\left[\xi\partial_\xi \xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\right]$$

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$$\mathbb{E}_{++} = T_0 \circledast E + \frac{t}{Q^2}\left[-\frac{1}{2}T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\right] \circledast E - \frac{8m^2}{Q^2}\xi^2 \partial_\xi \xi\, T_2 \circledast (H+E)$$

$$\mathbb{E}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left[\xi \partial_\xi T_1 \circledast E\right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \left[\xi\,(H+E) - \widetilde{H}\right]$$

$$\mathbb{E}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\left[\xi \partial_\xi^2 \xi\, T_1^{(+)} \circledast E\right] - \frac{8m^2}{Q^2}\xi\left[\xi \partial_\xi \xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\right]$$

etc.

where $F = H, E, \widetilde{H}, \widetilde{E}$ are C-even GPDs

$$T \circledast F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2\xi}\, T\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}\right) F(x,\xi,t)$$

the coefficient functions $T_k^{\pm}$ are given by the following expressions:

$$T_0(u) = \frac{1}{1-u}$$

$$T_1(u) \equiv T_1^{(-)}(u) = -\frac{\ln(1-u)}{u}$$

$$T_1^{(+)}(u) = \frac{(1-2u)\ln(1-u)}{u}$$

$$T_2(u) = \frac{\mathrm{Li}_2(1) - \mathrm{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u}$$

and

$$\mathbf{D}_\xi = \partial_\xi + 2\frac{|\xi P_\perp|^2}{t}\partial_\xi^2 \xi \;=\; \partial_\xi - \frac{t - t_{\min}}{2t}(1-\xi^2)\partial_\xi^2 \xi$$

Main features:

- **Complete results available to t/Q^2, m^2/Q^2 accuracy**
  - translation and gauge invariance restored
  - factorization valid
  - correct threshold behavior $t \to t_{\min}$, $\xi \to 1$
  - for many observables, complete results close to LT in "photon frame"
- **Two expansion parameters**

$$\frac{t}{Q^2}; \qquad \frac{t - t_{\min}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}$$

- **Most of mass corrections absorbed in $t_{\min} = -4m^2\xi^2/(1-\xi^2)$; always overcompensated by finite-t corrections in the physical region**
- **Some extra m^2/Q^2 corrections for nucleon due to spinor algebra; disappear in certain CFF combinations and for scalar targets**

DVCS on a scalar target

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

- Helicity amplitudes

$$\mathcal{A}_{\mu\nu} = -\, g^{\perp}_{\mu\nu}\, \mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}} \left(q_\mu - q'_\mu \frac{q^2}{(qq')} \right) g^{\perp}_{\nu\rho} P^\rho\, \mathcal{A}^{(1)}$$
$$+ \frac{1}{2} \left(g^{\perp}_{\mu\rho} g^{\perp}_{\nu\sigma} - \epsilon^{\perp}_{\mu\rho}\epsilon^{\perp}_{\nu\sigma} \right) P^\rho P^\sigma\, \mathcal{A}^{(2)} + \cancel{q'_\nu \mathcal{A}^{(3)}_\mu}$$

where

$$\mathcal{A}^{(0)} = -\,2 \Bigg\{ \left(1 - \frac{t}{2Q^2}\right) \int dx \frac{H(x,\xi,t)}{x+\xi-i\epsilon} + \frac{t}{Q^2} \int dx \frac{H(x,\xi,t)}{x-\xi} \ln\left(\frac{x+\xi}{2\xi} - i\epsilon\right)$$
$$- \frac{2}{Q^2}\left(\frac{t}{\xi} + 2|\xi P_\perp|^2 \partial_\xi\right) \xi^2 \partial_\xi \int dx \frac{H(x,\xi,t)}{x-\xi} \left[\frac{1}{2} \ln\left(\frac{x+\xi}{2\xi} - i\epsilon\right) + \mathrm{Li}_2\left(\frac{x+\xi}{2\xi} + i\epsilon\right) - \mathrm{Li}_2(1) \right] \Bigg\}$$

$$\mathcal{A}^{(1)} = \frac{8}{Q}\, \xi^2 \partial_\xi \int dx \frac{H(x,\xi,t)}{x-\xi} \ln\left(\frac{x+\xi}{2\xi} - i\epsilon\right),$$

$$\mathcal{A}^{(2)} = \frac{8}{Q^2}\, \xi^3 \partial^2_\xi \int dx \frac{x\, H(x,\xi,t)}{x-\xi} \ln\left(\frac{x+\xi}{2\xi} - i\epsilon\right)$$

DVCS on a scalar target — *continued*

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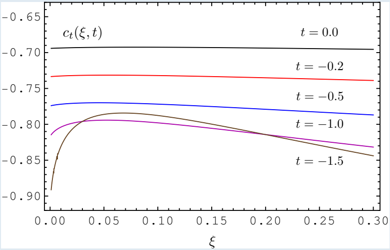
define

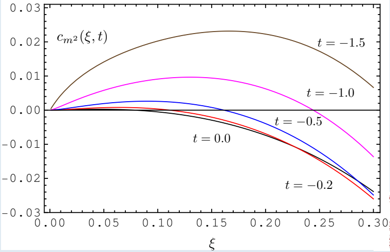
$$\frac{\mathrm{Im}\mathcal{A}^{(0)} - \mathrm{Im}\mathcal{A}^{(0)}_{LO}}{\mathrm{Im}\mathcal{A}^{(0)}_{LO}} = \frac{t}{Q^2}\, c_t(\xi, t) + \frac{m^2}{Q^2}\, c_{m^2}(\xi, t)$$

$$\mathrm{Im}\mathcal{A}^{(0)}_{LO} = 2\, H(\xi, \xi, t)$$



$$\left.\frac{\mathrm{Im}\mathcal{A}^{(0)} - \mathrm{Im}\mathcal{A}^{(0)}_{LO}}{\mathrm{Im}\mathcal{A}^{(0)}_{LO}}\right|_{t=t_{\min}} \simeq -(0.62 - 0.65)\,\frac{t_{\min}}{Q^2}$$

$$\sim \frac{\xi^2 m^2}{Q^2}$$



What can/should be done?

- Leading Twist
  - NNLO flavor-singlet
  - NNLO factorization schemes: conformal, BLM, etc.
  - NNLO solutions: Mellin space vs. momentum space, computer code
- Higher Twist

  short/medium term

  - Bulk of the twist-four corrections captured in “photon” frame for generic H, E?
  - Direct calculation of DVCS observables starting from “photon” frame
  - “Standard” code combining twist-4 + NLO
  - time-like DVCS

  long(er) term

  - resummation of $(t/Q^2)^k$ and $(m^2/Q^2)^k$ corrections to all powers
  - NLO corrections to $(t/Q^2)^k$ and $(m^2/Q^2)^k$, gluon constributions
  - Matching $(t/Q^2)^k$ and $(m^2/Q^2)^k$ with BFKL resummation in the small-x limit
  - Generalization to DVMP