(Towards a new formalism for the study of) Azimuthal asymmetries in unpolarised SIDIS

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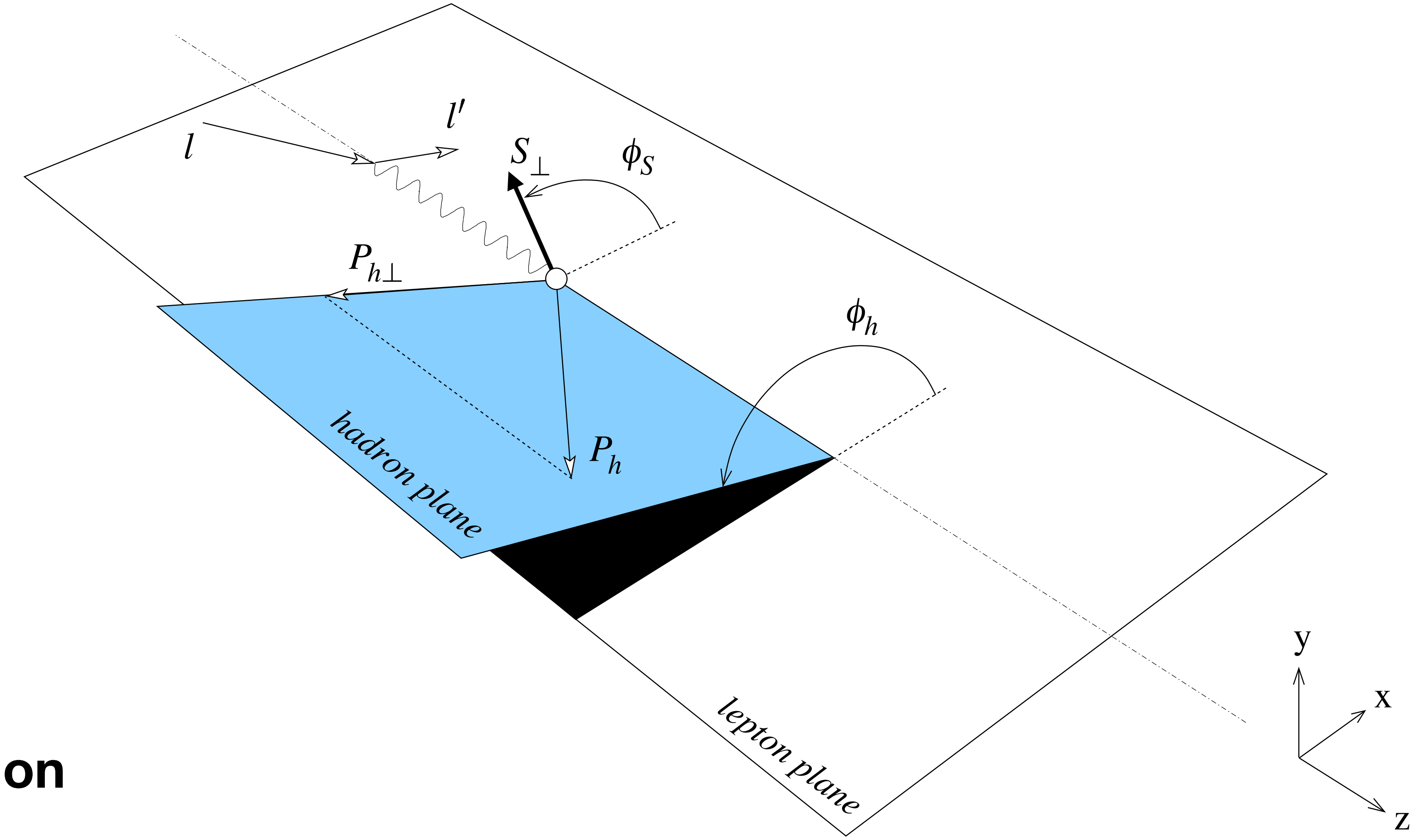
Unpolarised SIDIS



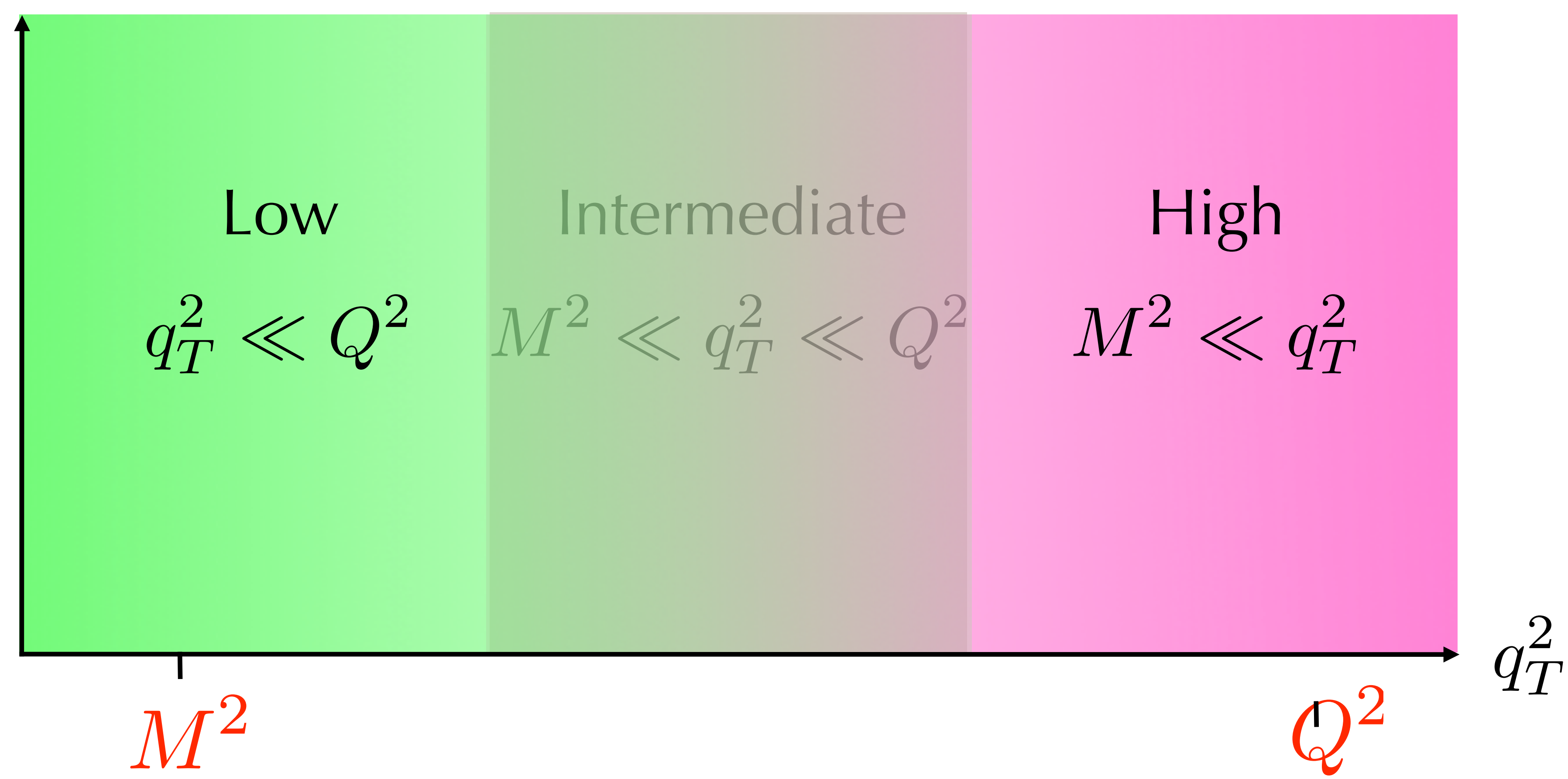
Differential cross section

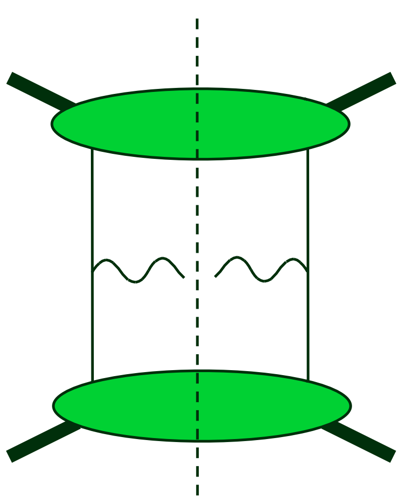
$$\frac{d\sigma}{d\phi_h dP_{h\perp}^2} \propto \frac{\alpha^2}{Q^2} \{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h}\}$$

Azimuthal Asymmetries

$$A_{UU}^{\cos\phi_h} = \frac{\sqrt{2\varepsilon(1+\varepsilon)} F_{UU}^{\cos\phi_h}}{F_{UU,T} + \varepsilon F_{UU,L}} . \qquad A_{UU}^{\cos 2\phi_h} = \frac{\varepsilon F_{UU}^{\cos 2\phi_h}}{F_{UU,T} + \varepsilon F_{UU,L}} .$$

3 physical scales - 2 theoretical tools

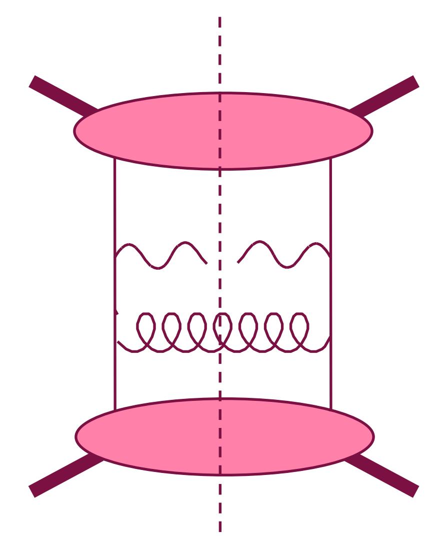




TMD

Do they describe the **same dynamics** or
two competing mechanisms
in the intermediate region?

(i.e., **interpolation** or **sum**?)



collinear PDF

Matches and mismatches

Bacchetta, Boer, Diehl, Mulders JHEP08(2008)

Low qT

$$F(q_T,Q) \overset{q_T\ll Q}{=} \frac{1}{M^2}\sum_n \left(\frac{q_T}{Q}\right)^{n-2} l_n\left(\frac{M}{q_T}\right) \overset{M\ll q_T\ll Q}{=} \frac{1}{M^2}\sum_{n,k} l_{n,k} \left(\frac{q_T}{Q}\right)^{n-2}\left(\frac{M}{q_T}\right)^{k}$$

$$l_{n,k} = h_{k,n}$$

same twist only if *k*=*n*

High qT

$$F(q_T,Q) \overset{M\ll q_T}{=} \frac{1}{M^2}\sum_n \left(\frac{M}{q_T}\right)^{n} h_n\left(\frac{q_T}{Q}\right) \overset{M\ll q_T\ll Q}{=} \frac{1}{M^2}\sum_{n,k} h_{n,k} \left(\frac{M}{q_T}\right)^{n}\left(\frac{q_T}{Q}\right)^{k-2}$$

Example of a MATCH: $M^2F(q_T,Q) \sim M^2\left(\frac{1}{q_T^2}+\frac{1}{Q^2}\right) \sim l_{2,2}+l_{4,2}$

$$F_{UU,T}$$

$l_{2,2} = h_{2,2}$ **leading** in both cases

$l_{4,2} = h_{2,4}$ **subleading** in both cases

Example of a MISMATCH: $M^2F(q_T,Q) \sim M^2\left(\frac{M^2}{q_T^4}+\frac{1}{Q^2}\right) \sim l_{2,4}+l_{4,2}$

$$F_{UU}^{\cos 2\phi_h}$$

$l_{2,4} = h_{4,2}$ **leading** for low qT, **subleading** for high qT

$l_{4,2} = h_{2,4}$ **leading** for high qT, **subleading** for low qT

The situation for unpolarised SIDIS

observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	leading powers match
$F_{UU,T}$	2	α_s **$l_{2,2}$**	$1/q_T^2$	2	α_s **$h_{2,2}$**	$1/q_T^2$	yes
$F_{UU,L}$	4	**$l_{4,2}$**		2	α_s **$h_{2,4}$**	$1/Q^2$	
$F_{UU}^{\cos\phi_h}$	3	α_s **$l_{3,2}$**	$1/(Q\,q_T)$	2	α_s **$h_{2,3}$**	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s **$l_{2,4}$**	$1/q_T^4$	2	α_s **$h_{2,4}$**	$1/Q^2$	no

Mismatch of non-logarithmic terms

No matching in the intermediate region

From high to intermediate qT

- convolution of PDFs and FFs with hard scattering coefficients

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$
$$\times\left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

- expansion of delta function for small qT/Q

$$\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right) = \delta(1-\hat{x})\,\delta(1-\hat{z})\,\ln\frac{Q^2}{q_T^2} + \frac{\hat{x}}{(1-\hat{x})_+}\,\delta(1-\hat{z})$$
$$+ \frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x}) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\ln\frac{Q^2}{q_T^2}\right),$$

- extraction of leading behaviour

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\big)(z)\right.$$
$$\left. + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\,D_1^a(z)\right],$$

(

From low to intermediate qT

- transverse-momentum convolution of TMD PDFs and FFs with kinematical functions

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\Big[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\Big]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,d^2\boldsymbol{l}_T\,\delta^{(2)}\big(\boldsymbol{p}_T-\boldsymbol{k}_T+\boldsymbol{l}_T+\boldsymbol{q}_T\big)$$
$$\times\, w(\boldsymbol{p}_T,\boldsymbol{k}_T)\,f^a(x,p_T^2)\,D^a(z,k_T^2)\,U(l_T^2)\,.$$

- Taylor-expand the functions of pT in the integrand and use the delta function to perform the integral

- extraction of leading behaviour

$$F_{UU}^{\cos 2\phi_h} \sim \frac{M^2}{q_T^4}\,\alpha_s\,\mathcal{F}\big[h_1^{\perp(1)}\,H_1^{\perp(1)},\ldots\big]$$

$$\mathcal{F}\big[fD\big] = \frac{1}{z^2}\sum_{a,i} e_a^2\,\Big[\big(K_i\otimes f^i\big)(x)\,D^a(z) + f^a(x)\,\big(D^i\otimes L_i\big)(z)\Big]$$

Our work

A new Fourier-transformed expression for the structure functions

$$F_{UU}^{\cos\phi_h} = -\frac{2MM_h}{Q}\,\mathcal{B}_1\left[\left(x\widehat{h}\,\widehat{H}_1^{\perp(1)} + \frac{M_h}{M}\,\widehat{f}_1\frac{\widehat{\tilde{D}}^{\perp(1)}}{z}\right) + \frac{M}{M_h}\left(x\widehat{f}^{\perp(1)}\widehat{D}_1 + \frac{M_h}{M}\,\widehat{h}_1^{\perp(1)}\frac{\widehat{\tilde{H}}}{z}\right)\right],$$

$$F_{UU}^{\cos 2\phi_h} = -MM_h\,\mathcal{B}_2\left[\widehat{h}_1^{\perp(1)}\widehat{H}_1^{\perp(1)}\right] + \frac{M^4}{Q^2}\,\mathcal{B}_2\left[x\widehat{f}_3^{\perp(2)}\widehat{D}_1\right],$$

$$\mathcal{B}_n\left[\widehat{f}\widehat{D}\right] = 2\pi\sum_a e_a^2 x\int_0^\infty d\xi_T\xi_T^{n+1}J_n\left(\xi_T|\boldsymbol{P}_{hT}|/z\right)\widehat{f}^a(x,\xi_T^2)\widehat{D}^a(z,\xi_T^2)$$

A new expression for twist-3 TMDs

$$x\widehat{f}^{\perp a}(x,\xi_T^2;Q^2) = \frac{1}{2}\sum_{i=q,\bar{q},g}\left(C'_{a/i}\otimes f_1^i\right)(x,\xi_T,\mu_b^2)\;e^{S(\mu_b^2,Q^2)}\left(\frac{Q^2}{Q_0^2}\right)^{g_K(\xi_T)}\widehat{f}_{\rm NP}^{a\perp}(x,\xi_T^2)$$

$$x\widehat{f}_3^{\perp a}(x,\xi_T^2;Q^2) = \sum_{i=q,\bar{q},g}\left(C''_{a/i}\otimes f_1^i\right)(x,\xi_T,\mu_b^2)\;e^{S(\mu_b^2,Q^2)}\left(\frac{Q^2}{Q_0^2}\right)^{g_K(\xi_T)}\widehat{f}_{\rm NP3}^{a\perp}(x,\xi_T^2)$$

$$\frac{1}{z}\widehat{\tilde{D}}^{\perp a\to h}(z,\xi_T^2;Q^2) = \frac{1}{2}\sum_{i=q,\bar{q},g}\left(\hat{C}'_{a/i}\otimes D_1^{i\to h}\right)(z,\xi_T,\mu_b^2)\;e^{S(\mu_b^2,Q^2)}\left(\frac{Q^2}{Q_0^2}\right)^{g_K(\xi_T)}\widehat{\tilde{D}}_{\rm NP}^{\perp a\to h}(z,\xi_T^2)$$

Preliminary results

$F_{UU}^{\cos\phi_h}$ **we recover the *whole* leading term of the high qT expansion**

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Q q_T}\,\frac{\alpha_S}{2\pi^2 z^2}\sum_a e_a^2 x\, f_1^a(x)\, D_1^{a\to h}(z)\, L\left(\frac{Q^2}{q_T^2}\right)$$

$$L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$$

$F_{UU}^{\cos 2\phi_h}$ **we recover the *matching* of the 1/Q² at low and high qT**

$$F_{UU}^{\cos 2\phi_h} = \frac{1}{2}\,\frac{1}{Q^2}\,\frac{\alpha_S}{2\pi^2 z^2}\sum_a e_a^2 x f_1^a(x)\, D_1^{a\to h}(z)\, L\left(\frac{Q^2}{q_T^2}\right)$$

Outlook

- Examine all possible twist-4 contributions
- Recover the whole cos2phi result
- Extend the new formalism to Drell-Yan
- Compare with experiment (azimuthal modulations)

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