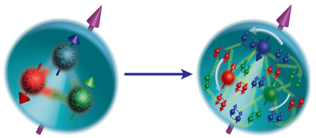
INT Program INT-17-3

Spatial and Momentum Tomography of Hadrons and Nuclei

August 28 – September 29, 2017



Phenomenology of TMDs and TMD Evolution: perturbative and non-perturbative aspects.

Mariaelena Boglione

INFN Torino

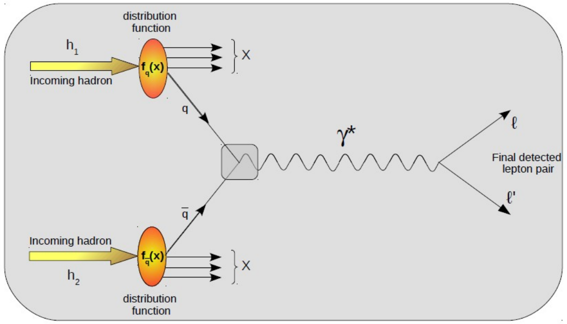
UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

Scuola di Scienze della Natura

In collaboration with J.O. Gonzalez Hernandez, S. Melis and A. Prokudin
and with J. Collins, L. Gamberg, T. Rogers, N. Sato, R. Taghavi

Where do we learn about TMDs ?

Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x, k_\perp) \otimes f_{\bar{q}}(x, k_\perp) \otimes \hat{\sigma}^{q\bar{q} \to \ell\ell'}$$

Allows extraction of distribution functions

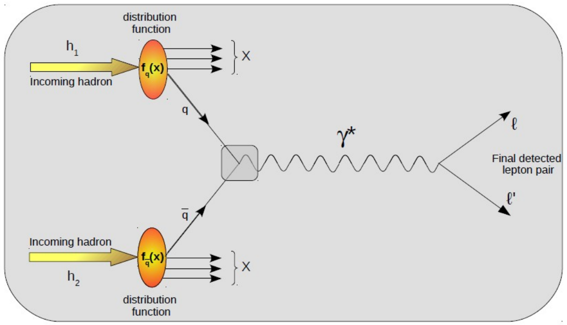
Fermilab

BROOKHAVEN NATIONAL LABORATORY

COMPASS

Where do we learn about TMDs ?

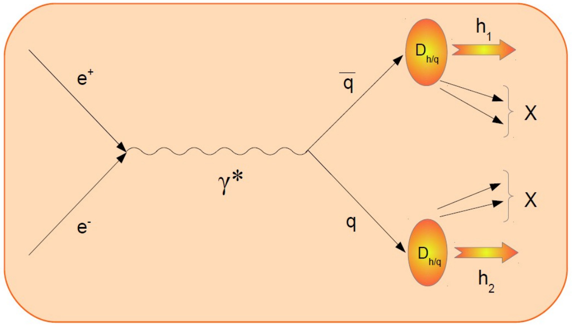
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

$e^+ e^- \to h_1\, h_2\, X$

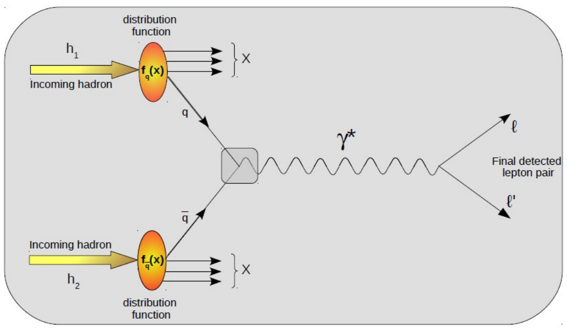


$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

Where do we learn about TMDs ?

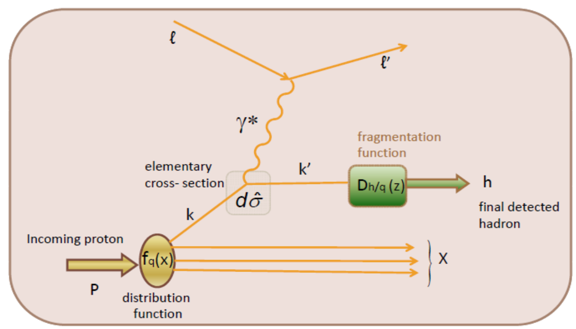
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

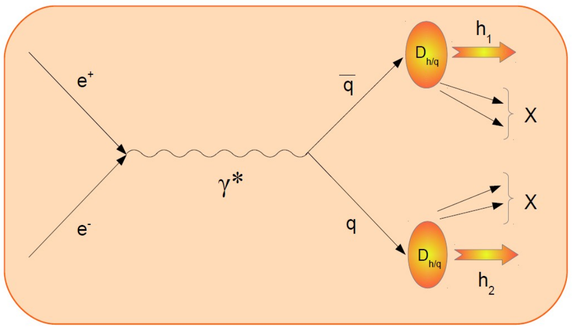
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

$e^+ e^- \to h_1 h_2 X$



$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

Drell-Yan Processes

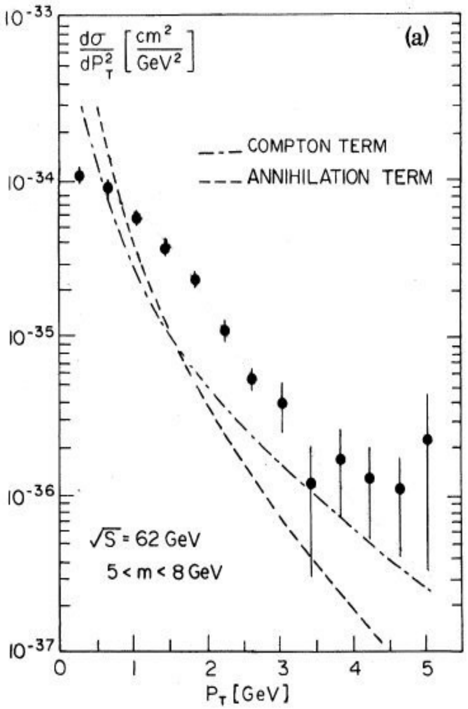
Naive TMD approach

- Calculating a cross section which describes a hadronic process over the whole q_T range is a highly non-trivial task

Let's consider Drell Yan processes (for historical reasons)

- Fixed order calculations cannot describe DY data at **small q_T**:

  At Born Level the cross section is vanishing
  At order α_s the cross section is divergent...



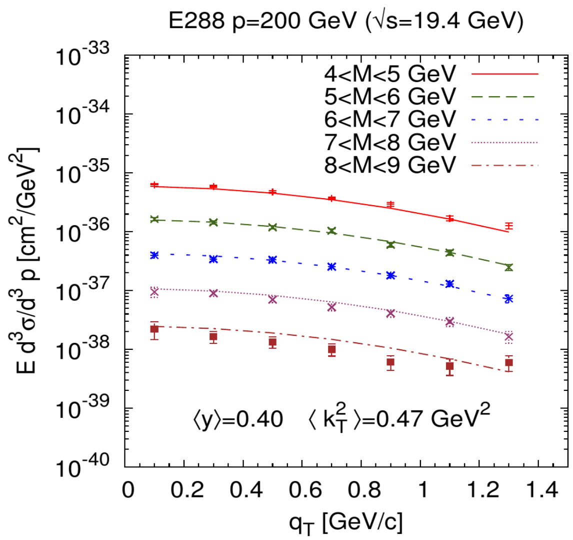
$$q_T \to 0$$

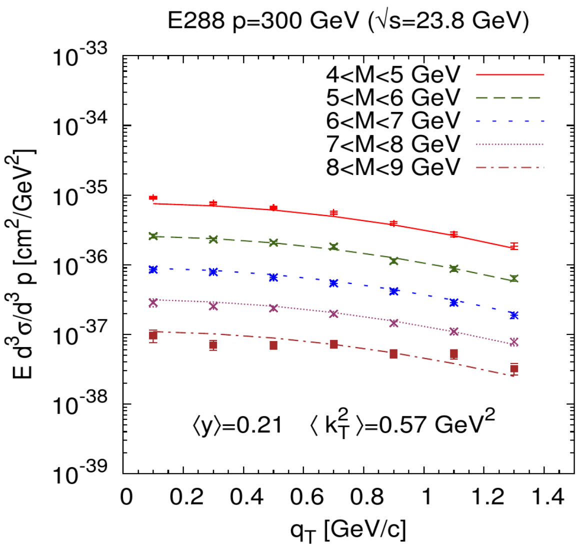
$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

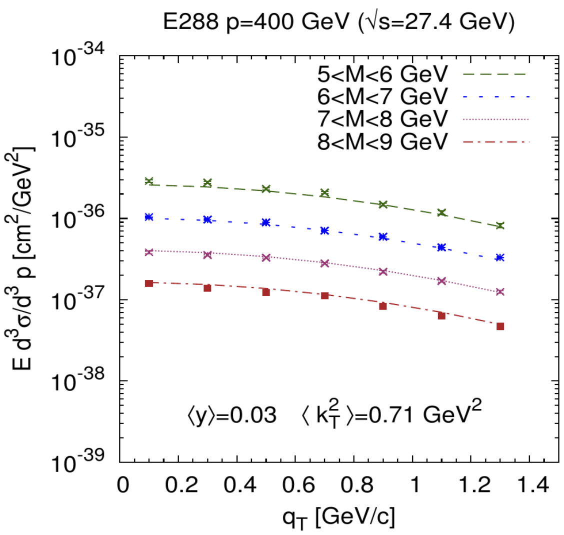
Naive TMD approach

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$

- Considering the same DY process at different energies:





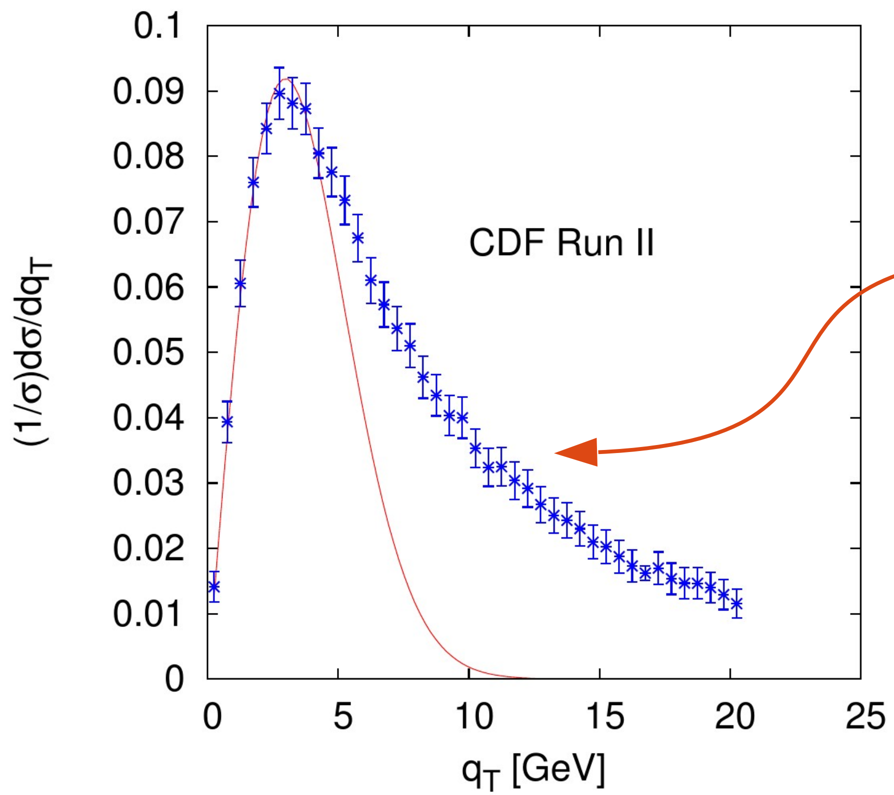


Each data set is Gaussian but with a different width

Drell-Yan phenomenology

- Does the q_T distribution behave like a Gaussian ?

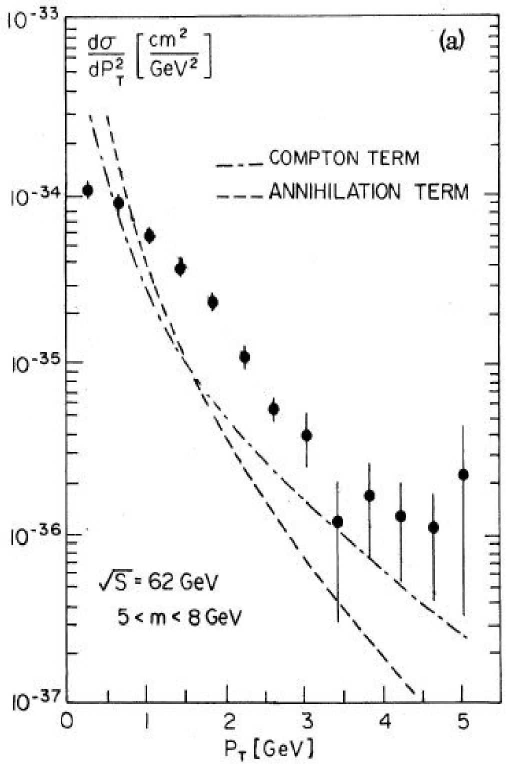
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$



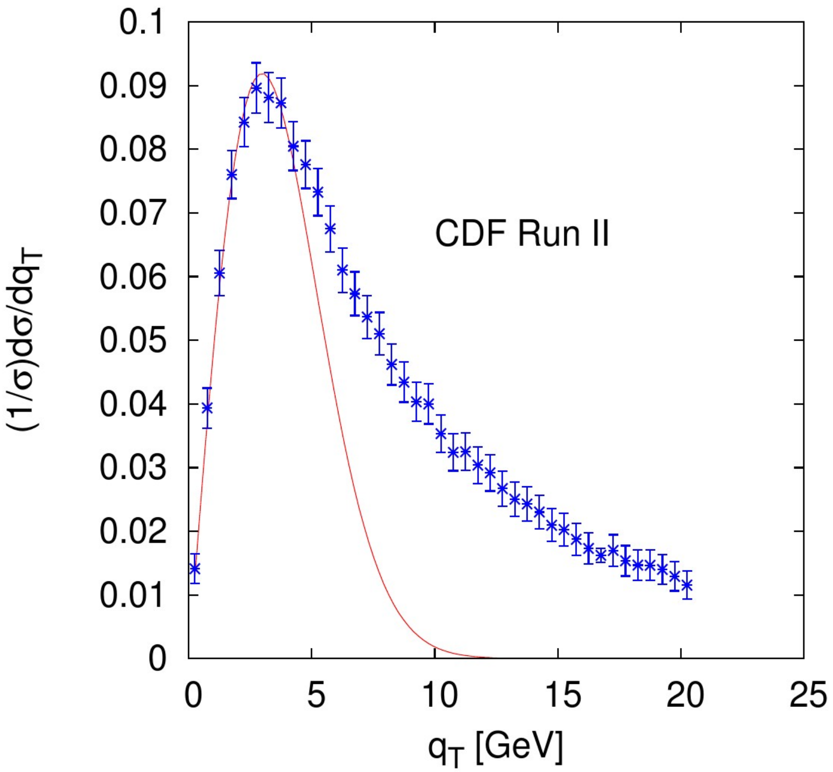
Clearly this is not
a Gaussian tail !

Drell-Yan phenomenology

Fixed order calculations cannot describe correctly DY cross sections at small q_T



DY cross sections do not show a Gaussian behaviour at large q_T

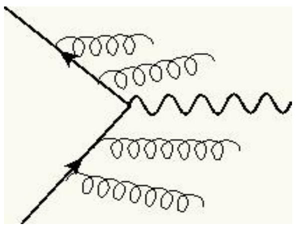


Resummation / TMD evolution

- Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

- These divergencies are taken care of by TMD evolution/resummation



The cross section is written in **b_T space**:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 \overbrace{W_j(x_1,x_2,b_T,Q)}^{\text{Resummed term}} + \overbrace{Y(x_1,x_2,q_T,Q)}^{\text{Finite term}}$$

Resummation / TMD evolution

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 \overbrace{W_j(x_1,x_2,b_T,Q)}^{\text{Resummed term}} + \overbrace{Y(x_1,x_2,q_T,Q)}^{\text{Finite term}}$$

$$\boxed{Y = \sigma^{FO} - \sigma^{ASY}}$$

- The W term is designed to work well at low and moderate q_T, when $q_T << Q$. (Notice that W is devised to work down to $q_T \sim 0$, however collinear-factorization works up to $q_T > M$; therefore, TMD-factorization and collinear-factorization can be simultaneously applied only when $q_T >> M$).
- The W term becomes unphysical at larger q_T, when $q_T \geq Q$, where it becomes negative (and large).
- The Y term corrects for the misbehavior of W as q_T gets larger, providing a consistent (and positive) q_T differential cross section.
- The Y term should provide an effective smooth transition to large q_T, where fixed order perturbative calculations are expected to work.

Resummation / TMD evolution

- Example: the CSS resummation scheme:

at small b_T OPE works
→ collinear PDFs

$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right] \sum_{i,k} C_{ji} \otimes f_i(x_1, C_1^2/b_T^2)\; C_{\bar{j}k} \otimes f_k(x_2, C_1^2/b_T^2)$$

$$S_j(b_T, Q) = -\int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa)) \ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

At large b_T the scale μ becomes too small!

$$\mu = \frac{C_1}{b_T}$$

Non-trivially connected to the physical region: $Q^2 \gg q_T^2 \simeq \Lambda_{QCD}^2$

- **All TMD evolution schemes require a model to deal with the non-perturbative region**

- **Working in b_T space makes phenomenological analyses more difficult, as we lose intuition and direct connection with “real world experience”. (Experimental data are in q_T space).**

Non perturbative region

- This is a perturbative scheme. All the scales must be frozen when reaching the non perturbative region:

$$b_T \longrightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu = \frac{C_1}{b_T} \longrightarrow \mu_b = C_1/b_*$$

- Then we define a non perturbative function for large b_T:

$$\frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q)$$

$$W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp\left[S_j(b_*, Q)\right] \left[C_{ji} \otimes f_i\,(x_1, \mu_b)\right] \left[C_{\bar{j}k} \otimes f_k\,(x_2, \mu_b)\right] F_{NP}(x_1, x_2, b_T, Q)$$

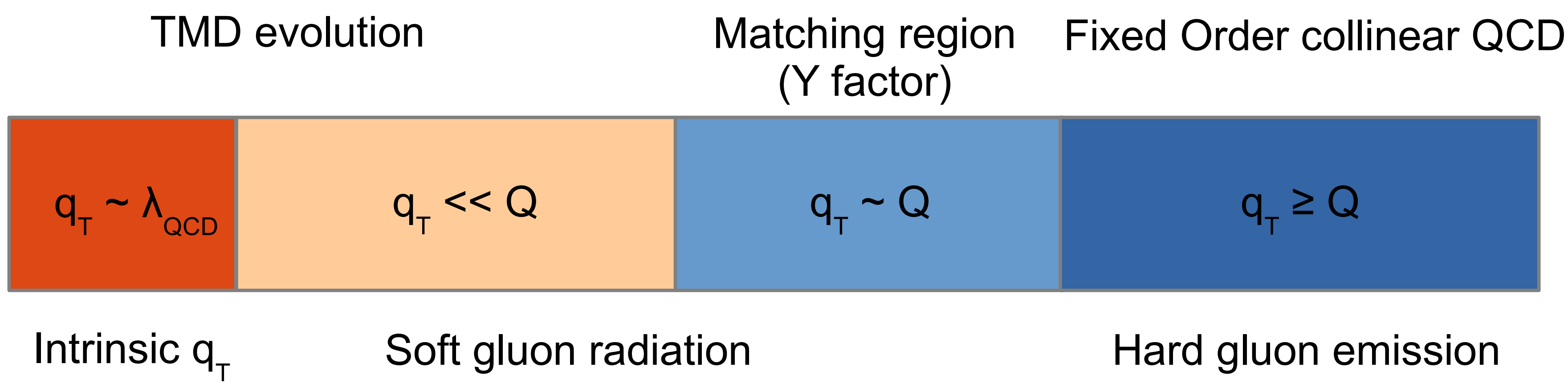
$b_*,\ \mu_b$ $\qquad\qquad$ b_T

$$C_1 = 2\exp(-\gamma_E)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

TMD regions

- For this scheme to work, 4 distinct kinematic regions have to be identified
- They should be large enough and well separated



CSS for DY processes

To perform phenomenological studies we need a non perturbative function.

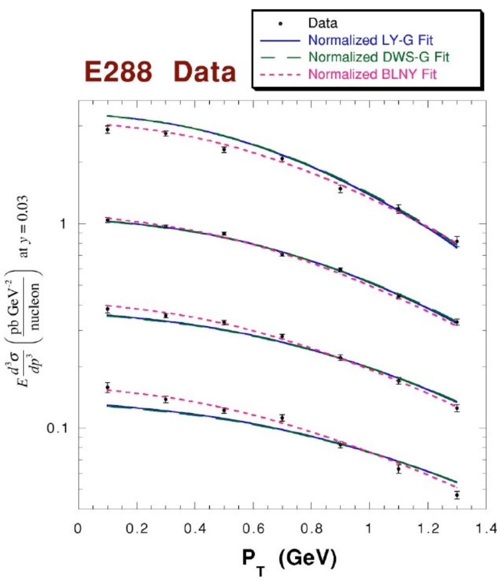
$$F_{NP}(x_1, x_2, b_T, Q)$$

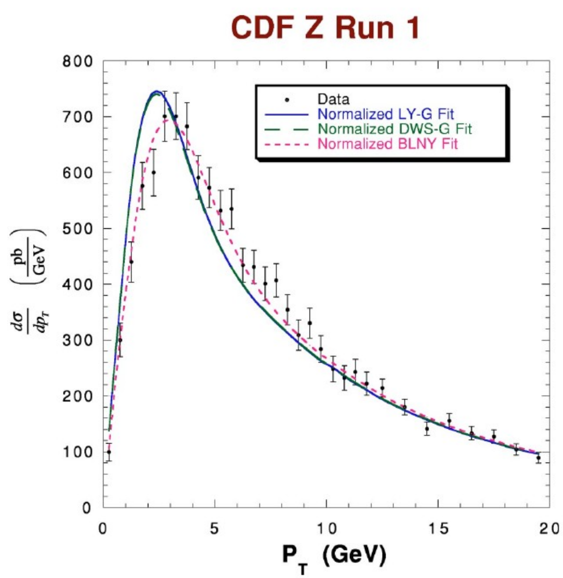
Davies-Webber-Stirling (DWS)	$\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2;$
Ladinsky-Yuan (LY)	$\exp\left\{\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 - [g_1 g_3 \ln(100 x_1 x_2)] b\right\};$
Brock-Landry-Nadolsky-Yuan (BLNY)	$\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$

Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS for DY processes

Nadolsky et al.* analyzed successfully
low energy DY data and Z_0 production data
using different parametrizations





Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF Z Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ Z Run-1 N_{fit}	1.01	1.01	1.00
CDF Z Run-1 N_{fit}	0.89	0.90	0.89
χ^2	416	407	176
χ^2/DOF	3.47	3.42	1.48

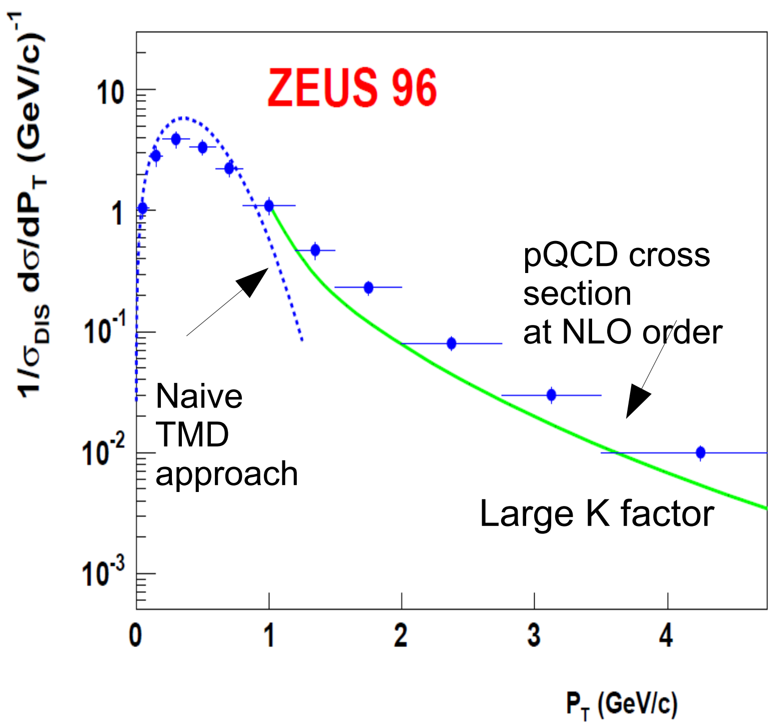
$b_{max} = 0.5\ \text{GeV}^{-1}$

**Nadolsky et al., Phys.Rev. D67,073016 (2003)*

SIDIS processes

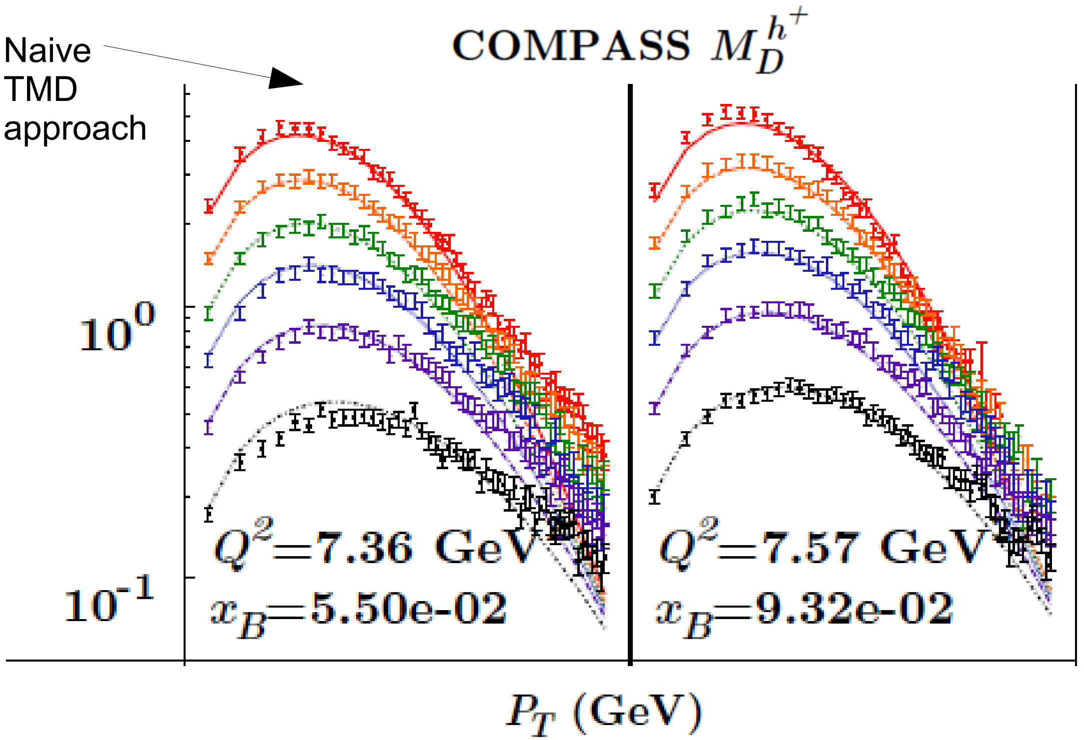
Resummation in SIDIS

- As mentioned above
  - fixed order pQCD calculation fail to describe the SIDIS cross sections at small q_T,
  - the cross section tail at large q_T is clearly non-Gaussian.



Anselmino, Boglione, Prokudin, Turk, Eur.Phys.J. A31 (2007) 373-381

ZEUS Collaboration (M. Derrick), Z. Phys. C 70, 1 (1996)



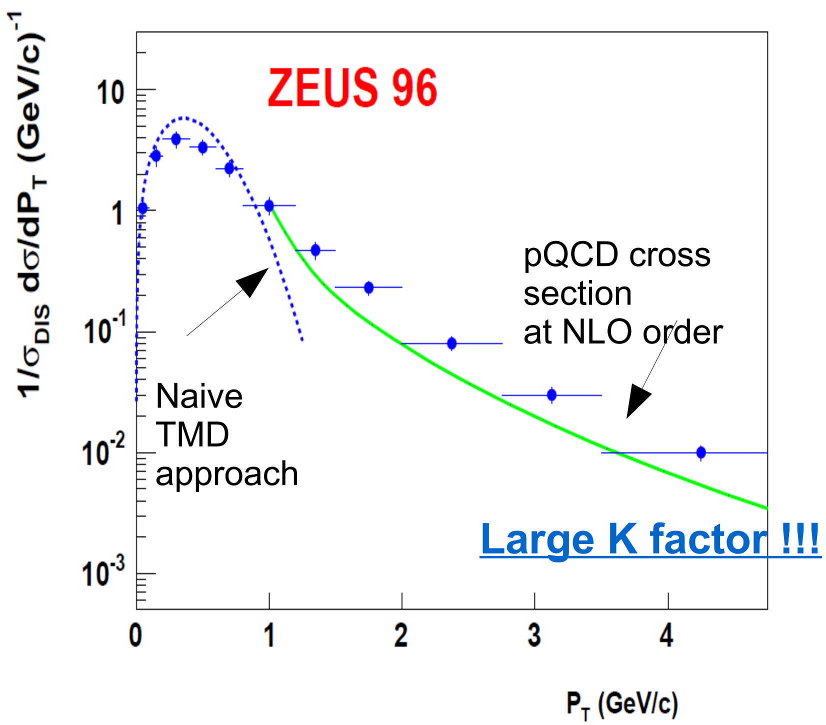
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005

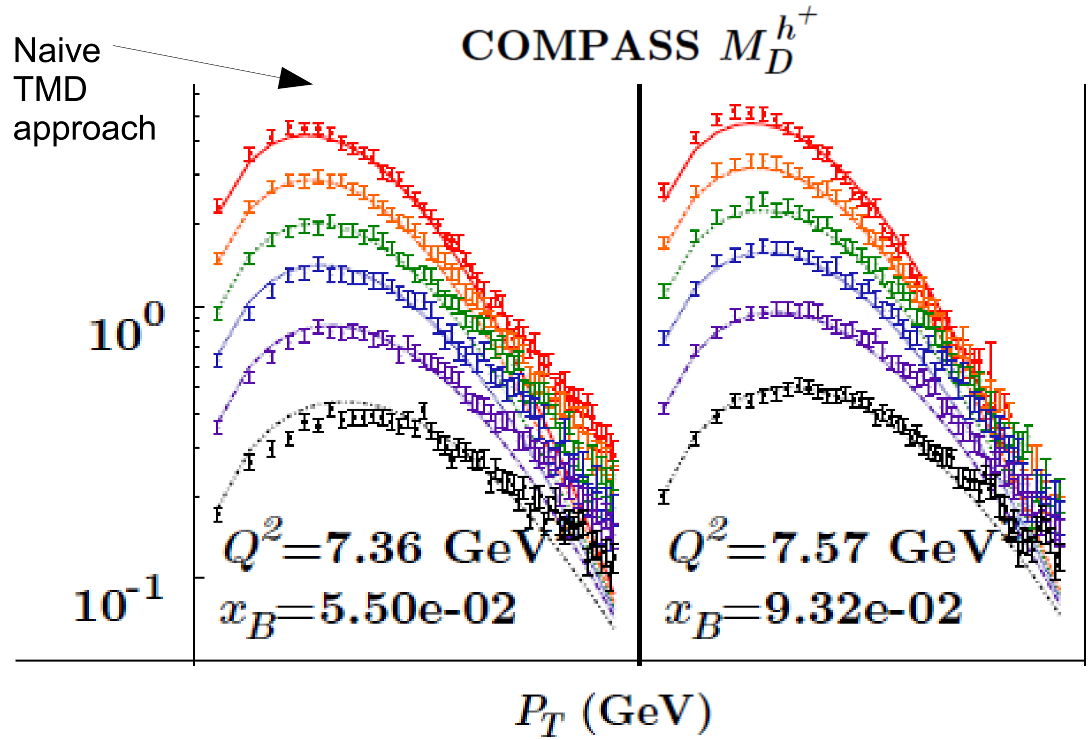
COMPASS, Adolph et al., Eur. Phys. J. C 73 (2013) 2531

Need resummation of large logs and matching perturbative to non-perturbative contributions

Resummation in SIDIS

- As mentioned above
  - fixed order pQCD calculation fail to describe the SIDIS cross sections at small q_T,
  - the cross section tail at large q_T is clearly non-Gaussian.





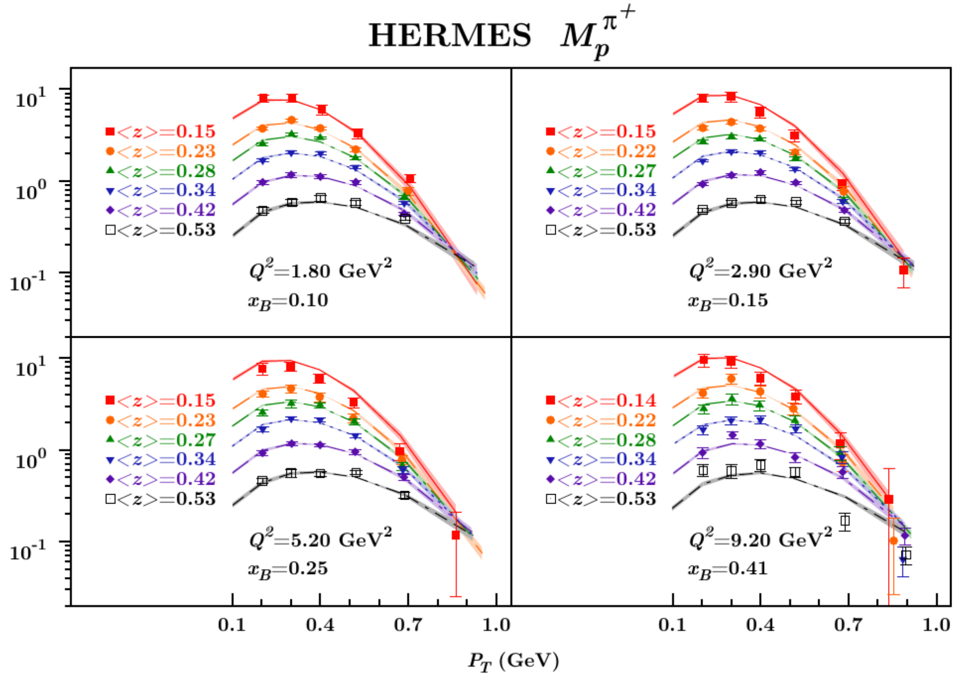
The NLO collinear SIDIS cross section is not correctly normalized !

Naive TMD approach

- Simple <u>phenomenological</u> ansatz can reproduce low q_T data

$$f_{q/p}(x,k_\perp) = f(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle} \qquad \langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



Airapetian et al, Phys. Rev. D 87 (2013) 074029

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2\rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

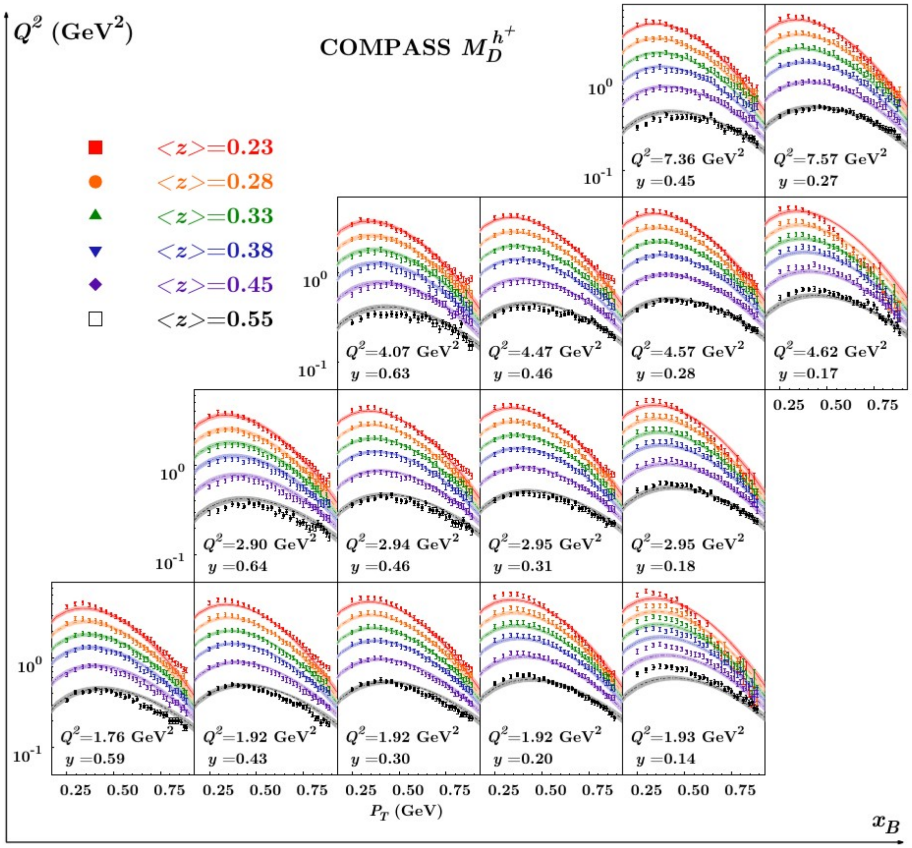
$$\chi^2_{\rm dof} = 1.69$$

Anselmino et al. JHEP 1404 (2014) 005

Naive TMD approach

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B)\, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi \langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2 \langle k_\perp^2\rangle$$



Anselmino et al. JHEP 1404 (2014) 005

$$\langle k_\perp^2\rangle = 0.60 \pm 0.14 \text{ GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20 \pm 0.02 \text{ GeV}^2$$
$$\chi^2_{\rm dof} = 3.42$$

Fit over 6000 data points with 2 free parameters !

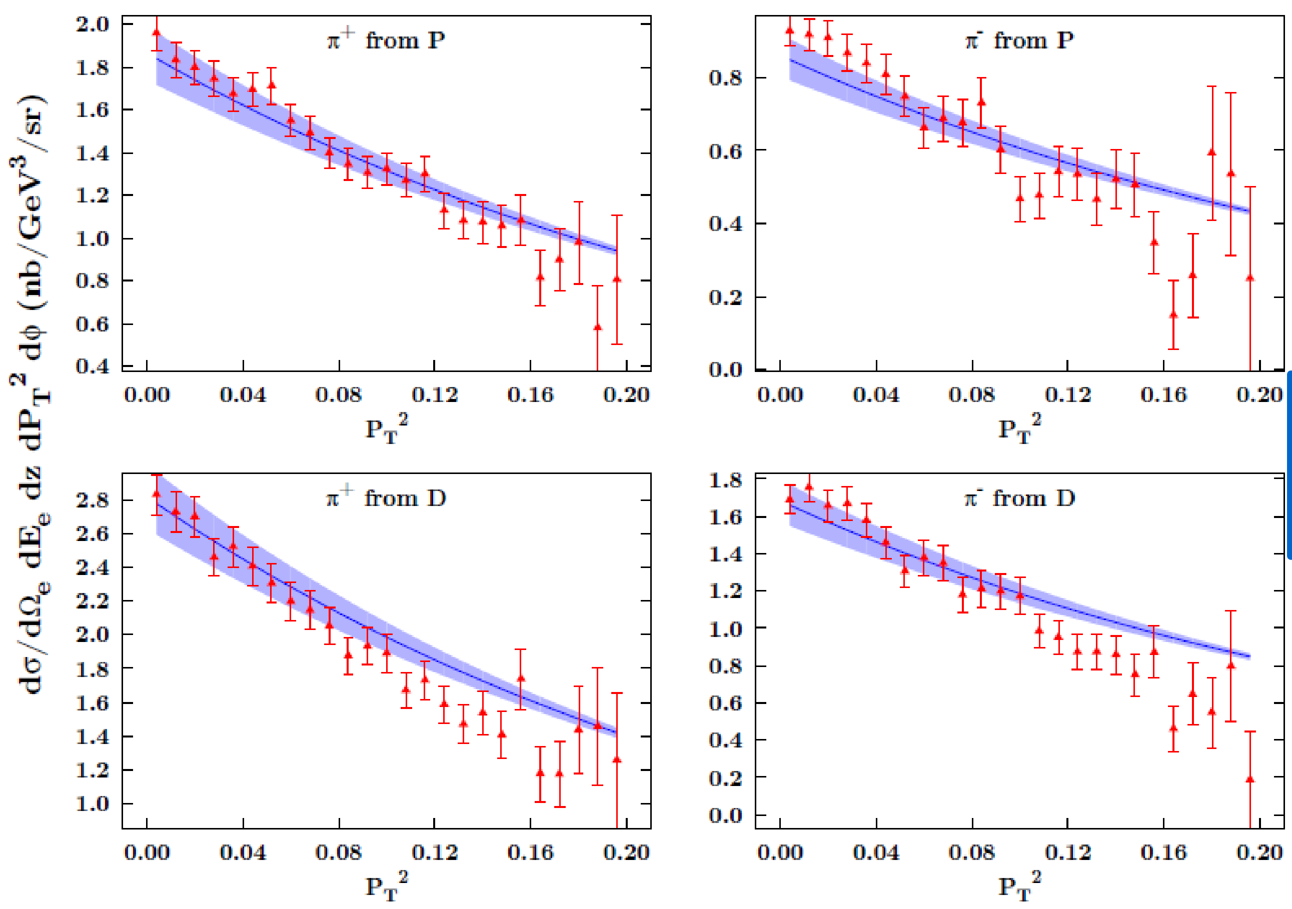
$$N_y = A + B\,y$$

"The point-to-point systematic uncertainty in the measured multiplicities as a function of p_T^2 is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the p_T^2-integrated multiplicities depends on z and y and can be as large as 40%".

Erratum Eur.Phys.J. C75 (2015) 2, 94

Comparison with Jlab 6 data HALL C

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

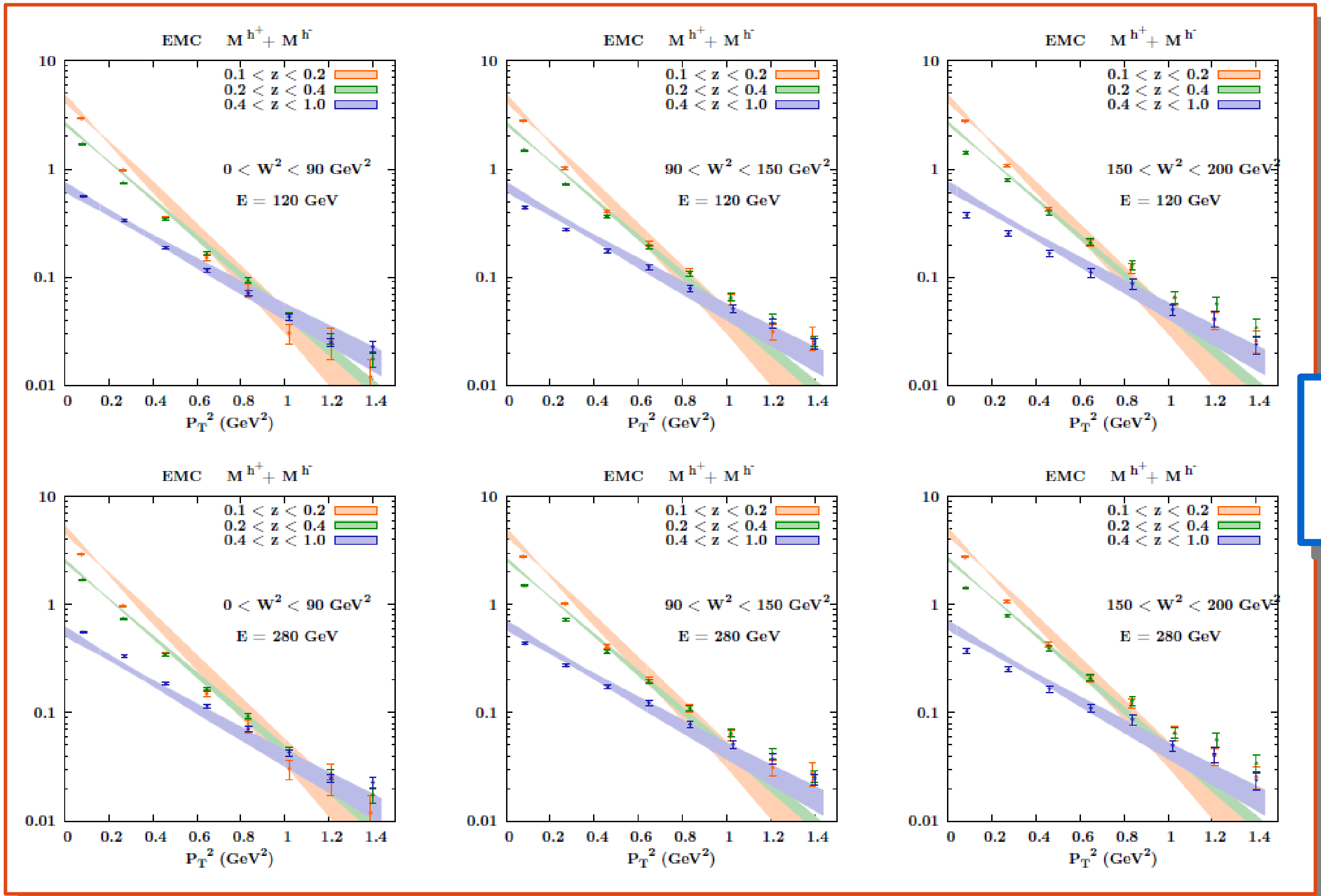


Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Comparison with EMC data

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261



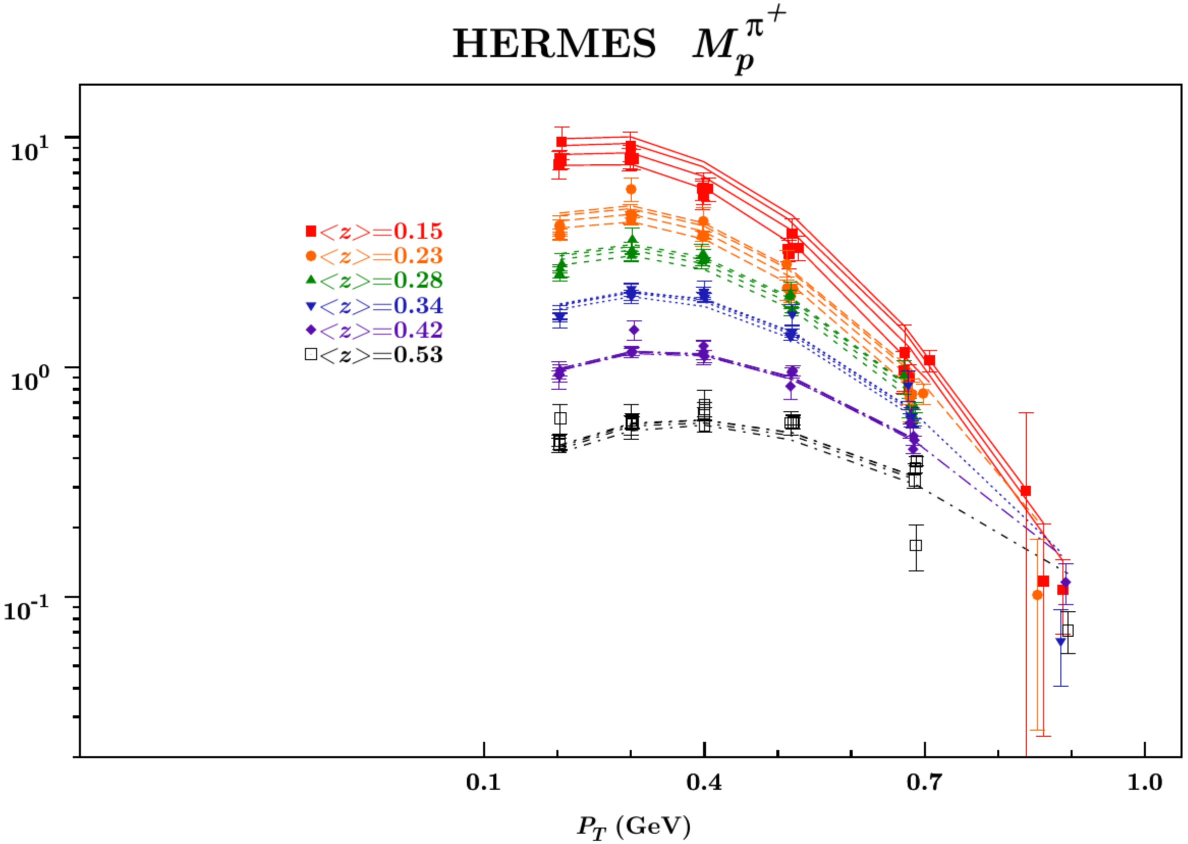
Predictions obtained by using the parameter values extracted from COMPASS multiplicities

J. Ashman et al. (European Muon Collaboration) Z. Phys. C52,361 (1991)

Q^2 dependence of HERMES data...

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\, \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



Anselmino et al. JHEP 1404 (2014) 005

$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2\rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi^2_{\rm dof} = 1.69$$

All four bins have been overlapped in the same panel

Hard to decouple the Q^2 dependence from HERMES data alone

Resummation of large logarithms

- To ensure momentum conservation, write the cross section in the Fourier conjugate space

$$\delta^2(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots)}$$

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \left[\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} X_{div}(b_T)\right] + Y_{reg}(q_T)}$$

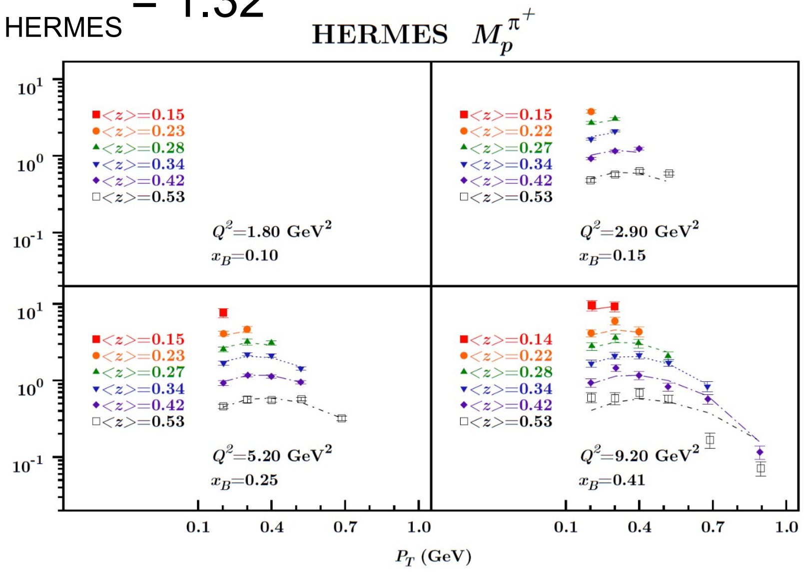
$$X_{div}(b_T) \longrightarrow W(b_T) = \exp\left[S(b_T)\right] \times (\text{PDFs and Hard coefficients})$$

$$\boxed{\frac{d\sigma^{total}}{dx\, dy\, dz\, dq_T^2} = \pi\sigma_0^{DIS}\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} \underbrace{W^{SIDIS}(x,z,b_T,Q)}_{\text{Resummed part}} + \underbrace{Y^{SIDIS}(x,z,q_T,Q)}_{\text{Regular part}}}$$

Fit of HERMES and COMPASS data
Attempting "Resummation" in SIDIS …

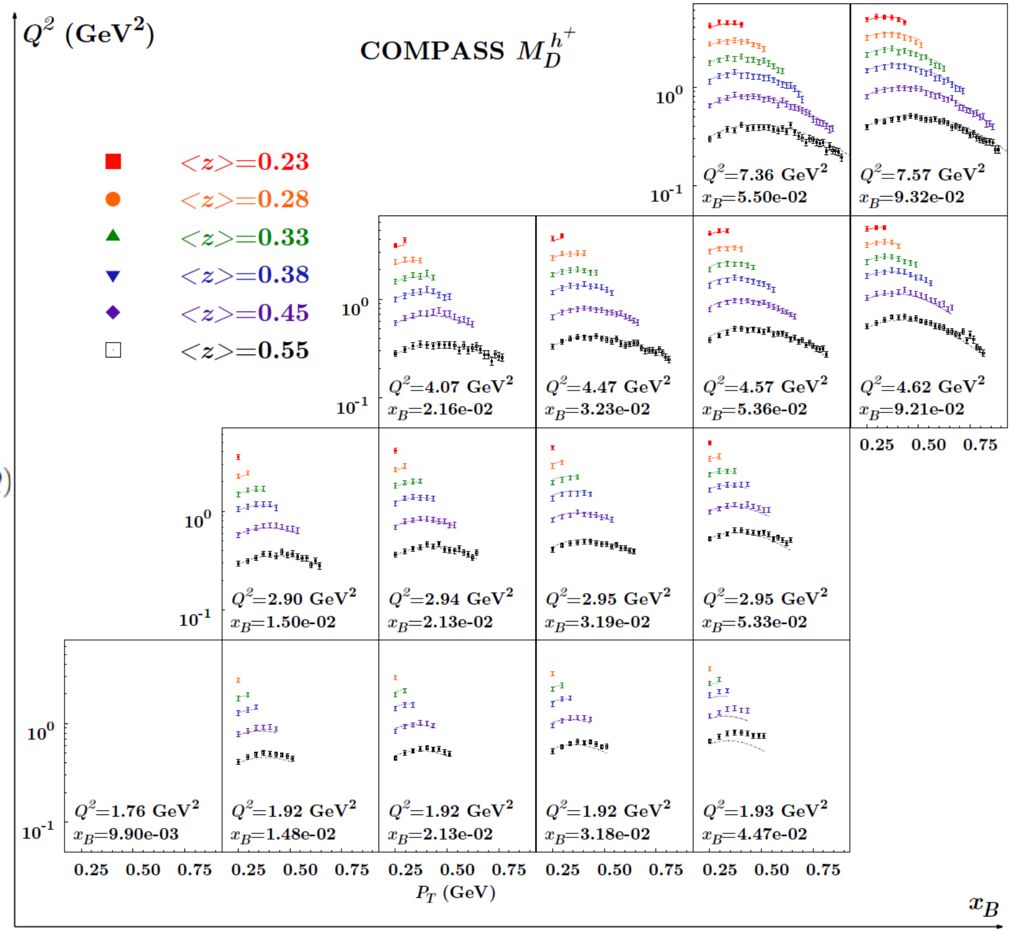
J. Osvaldo Gonzalez Hernandez, work in progress

$\chi^2_{\mathrm{HERMES}} = 1.32$



$\chi^2_{\mathrm{tot}} = 1.17$

$\chi^2_{\mathrm{COMPASS}} = 1.12$



$$\frac{d\sigma}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\left\{\int\frac{d^2\mathbf{b}_T e^{i\mathbf{q}_T\cdot\mathbf{b}_T}}{(2\pi)^2}W^{SIDIS}(x,z,b_*,Q,C_1,C_2,C_3)F_{NP}^{SIDIS}(x,z,b_T,Q)\right.$$
$$\cancel{+Y^{SIDIS}(x,z,q_T,Q,C_4)},$$

$$F_{NP}^{SIDIS}(x,z,Q) = \exp\left\{\left[-\frac{g_1+g_{1f}/z^2}{2} - g_2\ln(Q/(2Q_0)) - g_1 g_3\ln(10x)\right]b_T^2\right\}$$

N ~ 2 (One overall normalization parameter is required)
g1 ~ 0.5 (too large compared to the value extracted from DY data)
g2 ~ 0.5
g3 ~ - 0.03

TMD regions

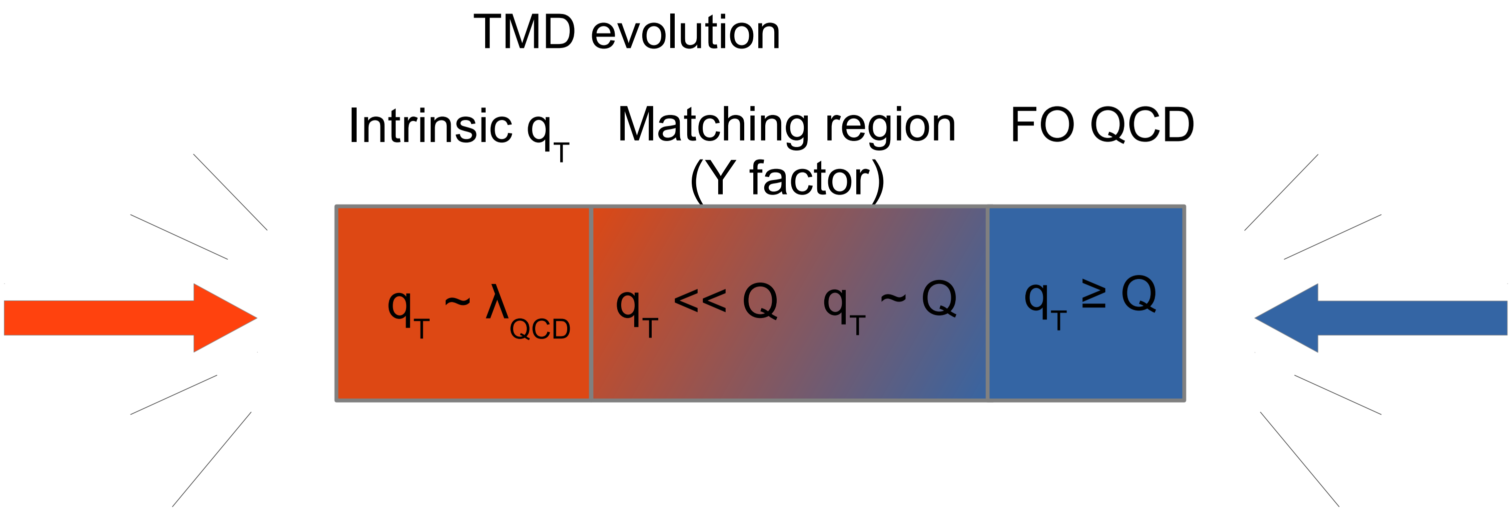
- For this scheme to work, 4 distinct kinematic regions have to be identified
- They should be large enough and well separated

TMD evolution		Matching region (Y factor)	Fixed Order collinear QCD
$q_T \sim \lambda_{QCD}$	$q_T << Q$	$q_T \sim Q$	$q_T \geq Q$
Intrinsic q_T	Soft gluon radiation		Hard gluon emission

TMD regions

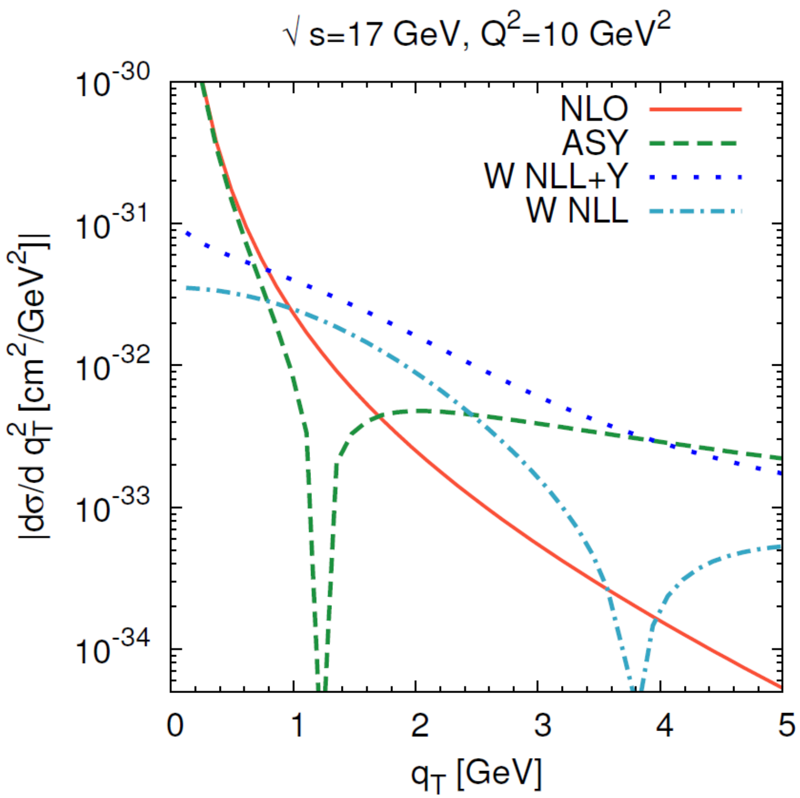
- For this scheme to work, 4 distinct ki… have to be identified
- They should be large… separated

Does not work in SIDIS !



What's wrong ???

SIDIS - Y factor



$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y$$

- The Y factor is very large (even at low q_T)
- However, it could be affected by large theoretical uncertainties

Boglione, Gonzalez, Melis, Prokudin, JHEP 02 (2015) 095

The Y factor cannot be neglected !!!

- New prescription for Y factor, b* and W

Collins, Gamberg, Prokudin, Rogers, Sato, Wang, Phys. Rev. D 94 (2016) 034014

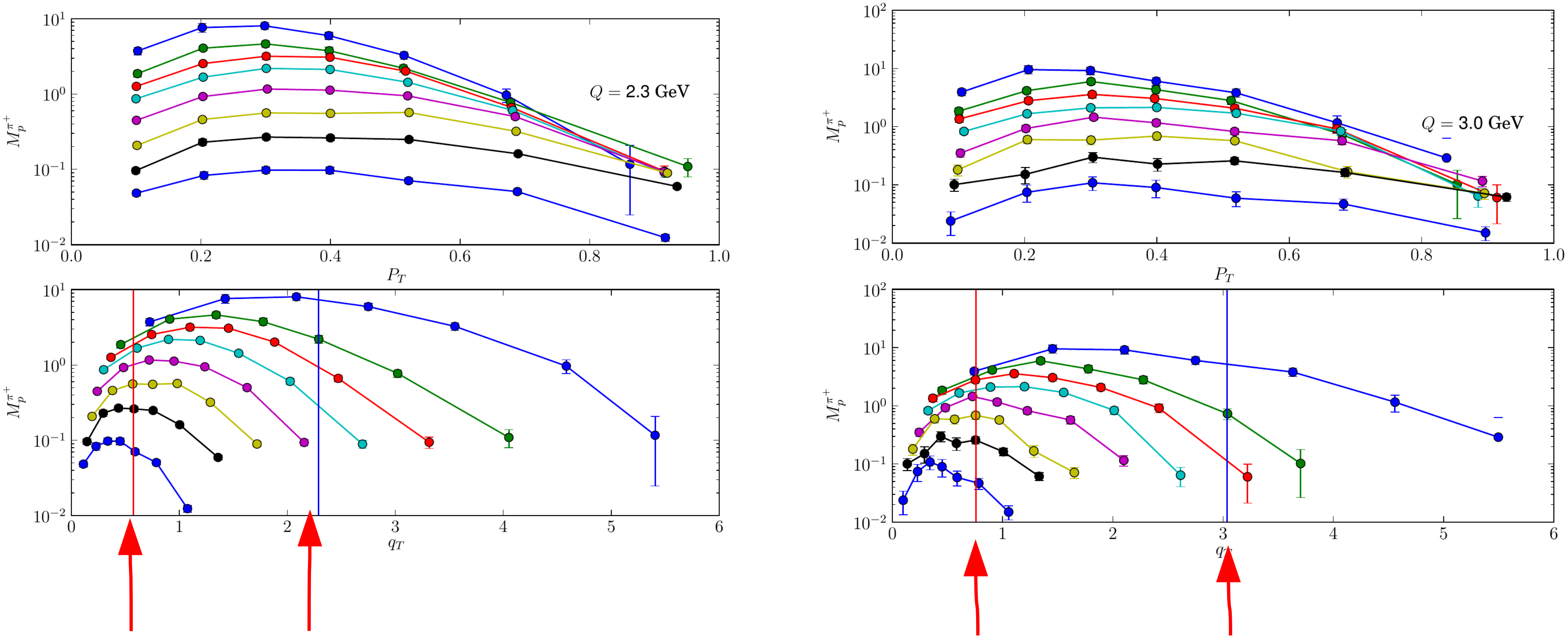
$\sigma^{ASY} = Q^2/q_T^2$ [A $\ln(Q^2/q_T^2)$ + B + C]

Other issues related to TMD regions ...

- TMD regions are defined in terms of q_T and not in terms of P_T

P_T

$q_T=P_T / z$



$q_T=Q/4$ $q_T=Q$

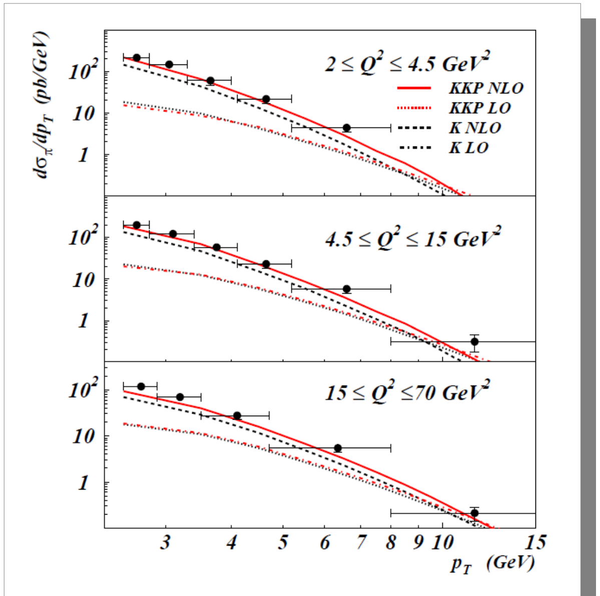
$q_T=Q/4$ $q_T=Q$

Possible issues ...

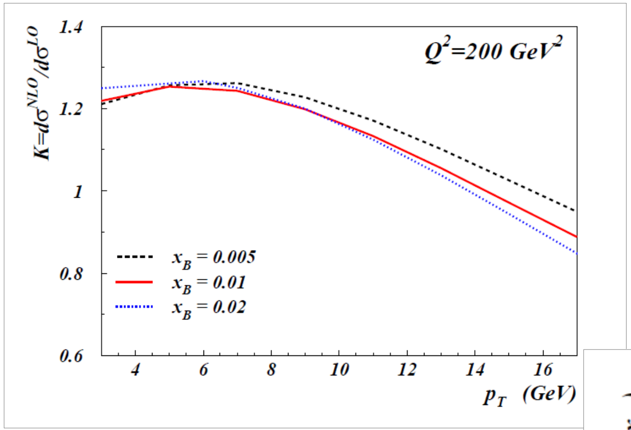
- This fit gives a very high quality description of a wide amount of data points
- However, there are a few issues that are worth mentioning:
  - The NLL SIDIS cross section is not correctly normalized $\rightarrow$ N ~ 2
  - The Y factor has been neglected
  - More work required to include Drell-Yan data into the fit

See global fit by
Bacchetta, Delcarro, Pisano, Radici Signori
JHEP 1706 (2017) 081,
which includes SIDIS and DY data.

Normalization and K factor

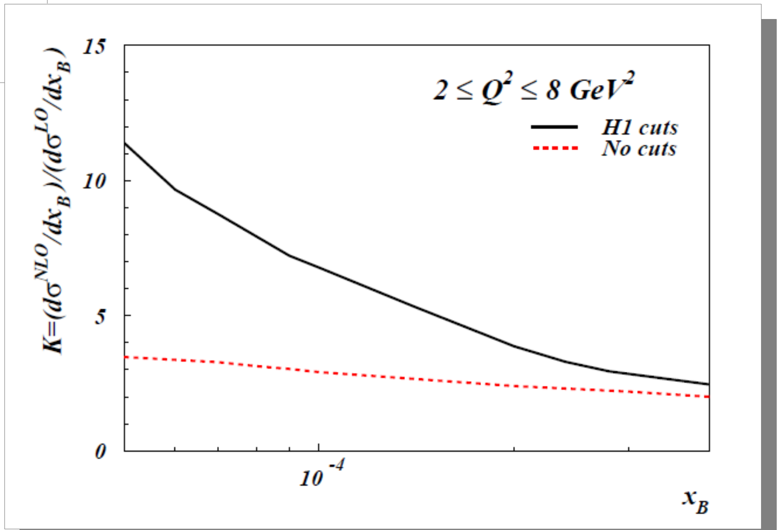


How can we address the normalization problem ???



- K factor depends on p_T
- Kinematics cuts can affect the size of K factors … up to a factor 10 !

Stringent cuts on the pion production angle in H1 data suppresses LO and NLO contributions in a different way



Daleo, De Florian, Sassot, Phys.Rev. D71 (2005) 034013

Daleo, De Florian, Sassot, Braz.J.Phys. 37 (2007) 585-590

Aktas et al., H1 Collaboration, Eur. Phys. J. C36 (2004) 441

"The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant ('leading-order') channel, and not to the 'genuine' increase in the partonic cross section [...]. The dominance of the new channel is due to the size of the gluon distribution at small x_B and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \to g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the gg component represents less than 25% of the total NLO contribution at small x_B."

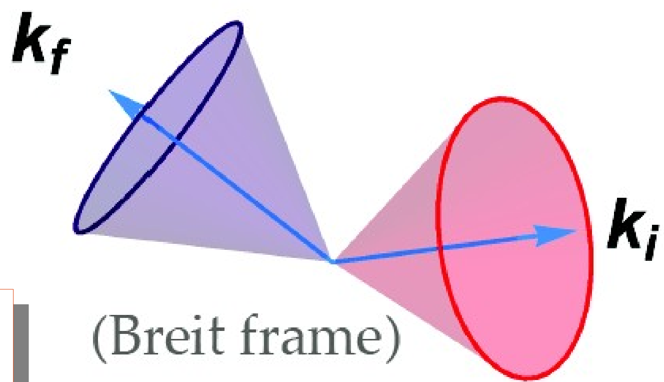
Daleo, De Florian, Sassot, Phys.Rev. D71 (2005) 034013

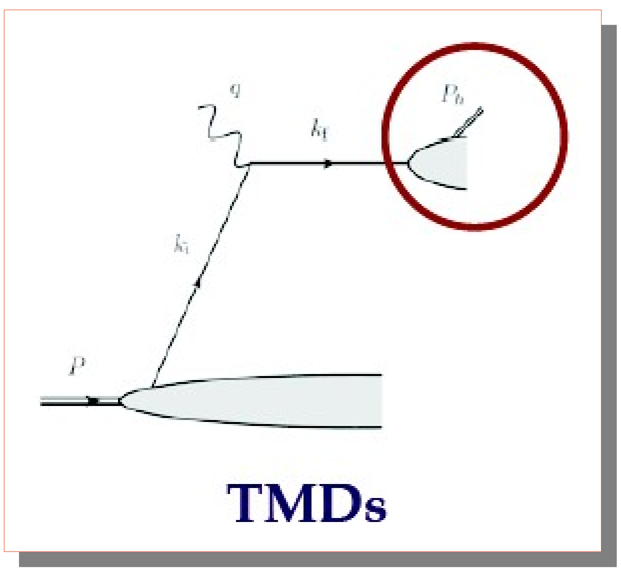
Daleo, De Florian, Sassot, Braz.J.Phys. 37 (2007) 585-590

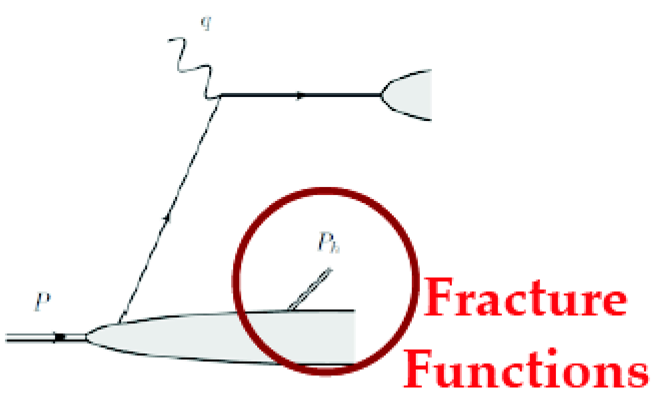
Kinematics of current region

Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato
Phys. Lett. B766 (2017) 245

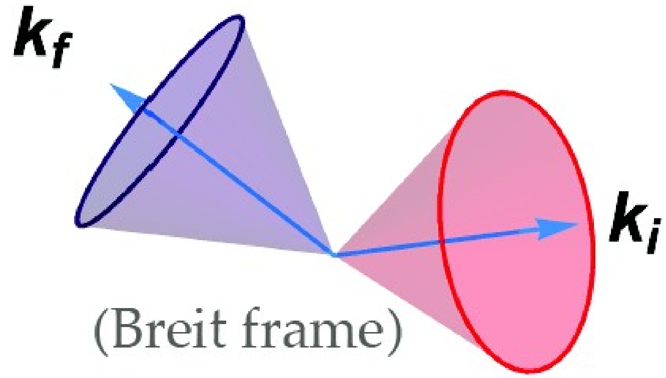
Need a quantitative way to identify the region of validity of TMD factorization (**current region**)





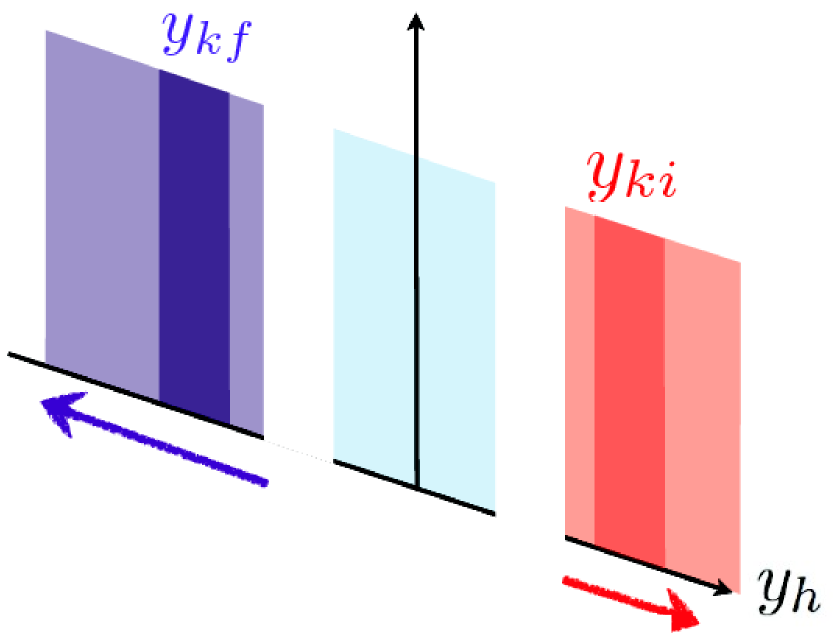


Kinematics of current region



Hadron rapidity

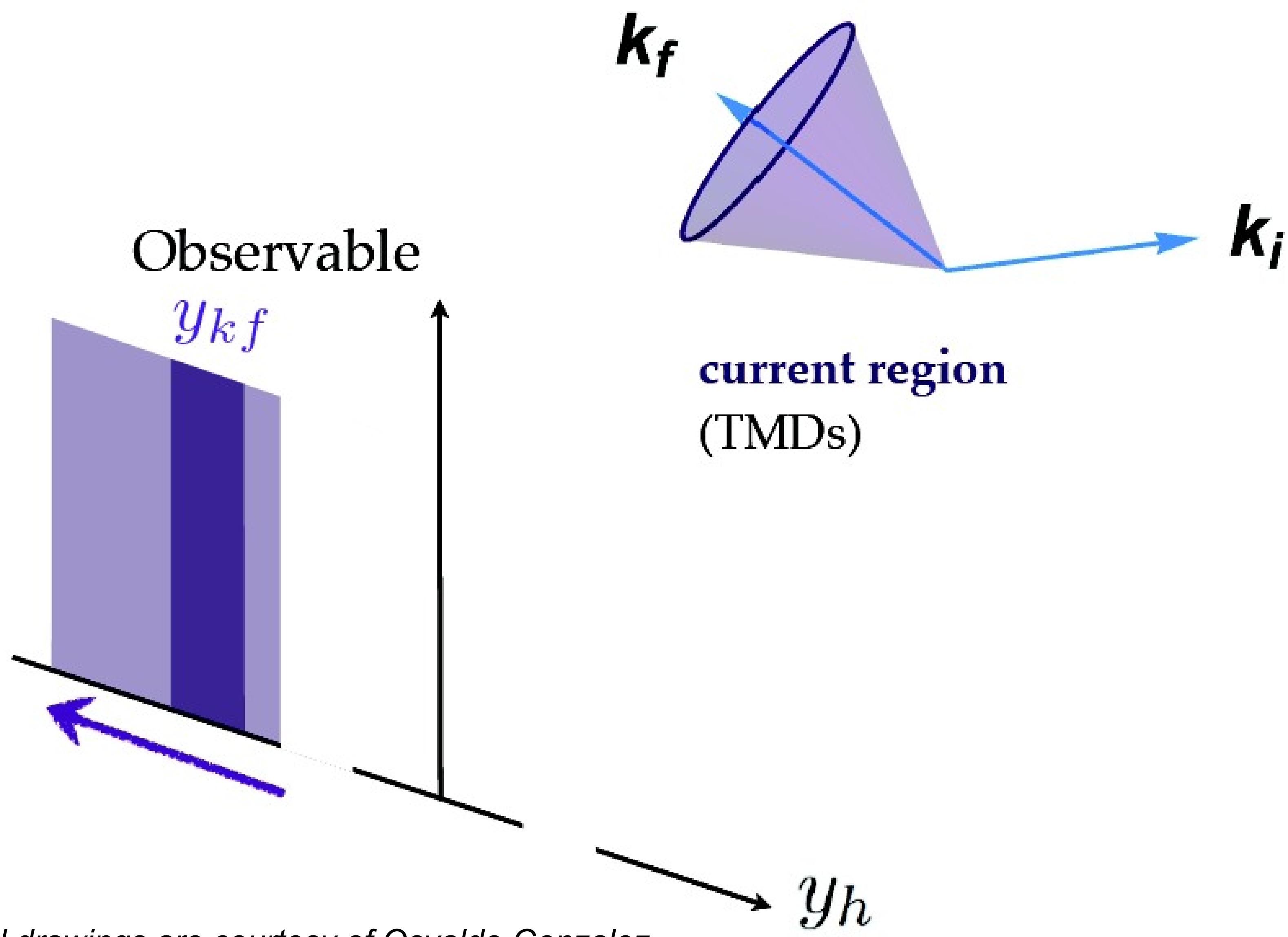
$$y_h \equiv \frac{1}{2}\log\frac{P_h^+}{P_h^-}$$



Current and fragmentation regions should be well separated in the observed hadron rapidity

These beautiful drawings are courtesy of Osvaldo Gonzalez

Kinematics of current region



These beautiful drawings are courtesy of Osvaldo Gonzalez

Kinematics of current region

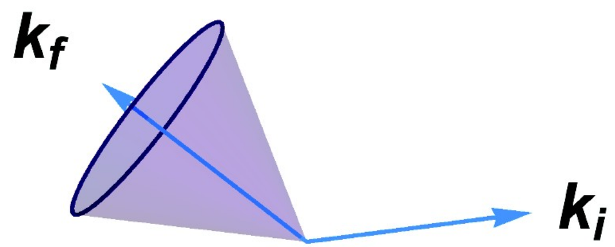
Factorization implies power counting for the momenta

Small mass

$$P_h \cdot k_f = O\left(m^2\right)$$

$$P_h \cdot k_i = O\left(Q^2\right)$$

Hard scale



Current region

$$\boxed{R(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}}$$

Collinearity must be small in the current region

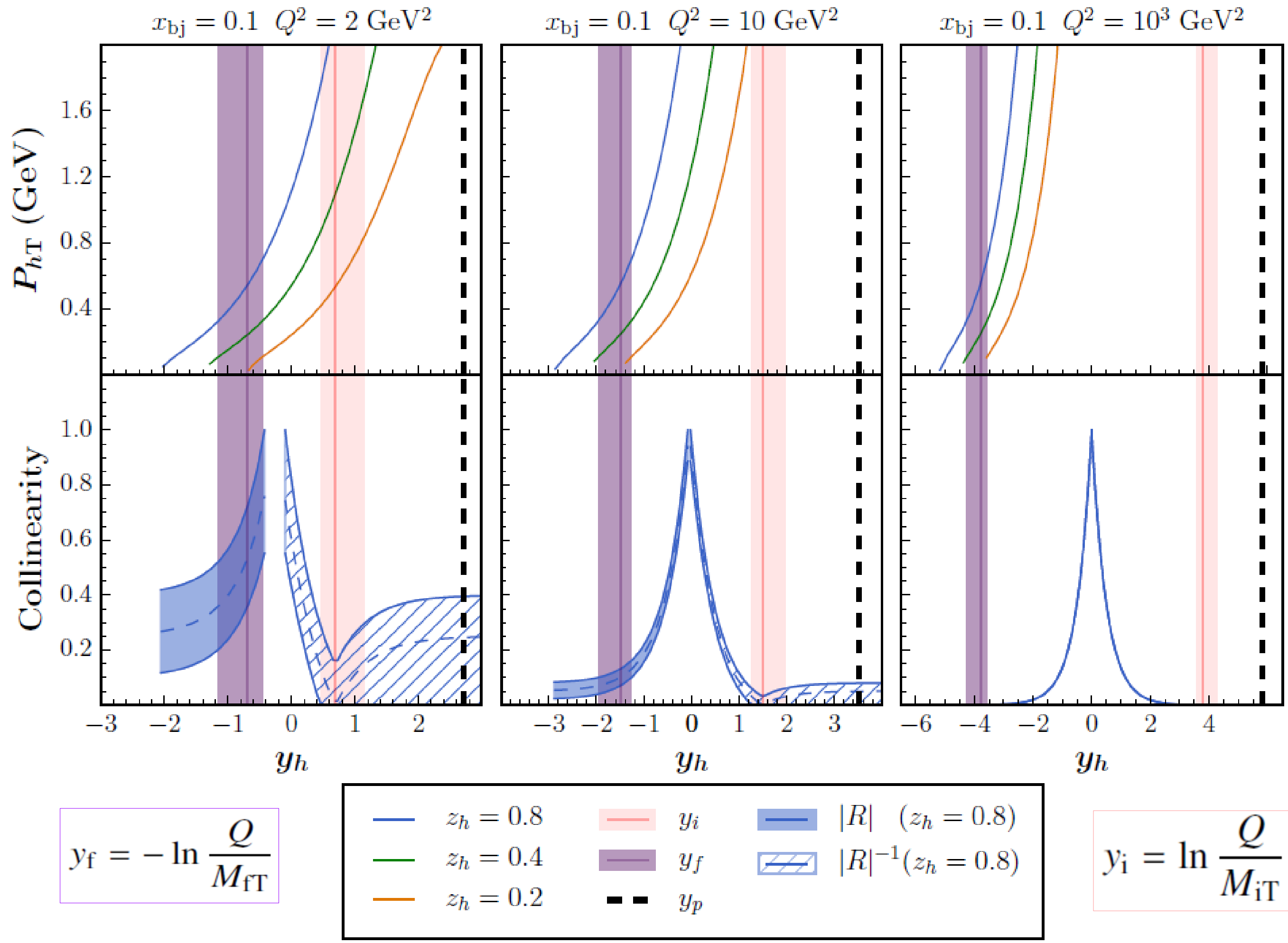
These beautiful drawings are courtesy of Osvaldo Gonzalez

Kinematics of current region

$$R(y_h, z_h, x_{bj}, Q) \ll 1: \text{ collinear to outgoing quark},$$
$$R(y_h, z_h, x_{bj}, Q)^{-1} \ll 1: \text{ collinear to incoming quark}.$$

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

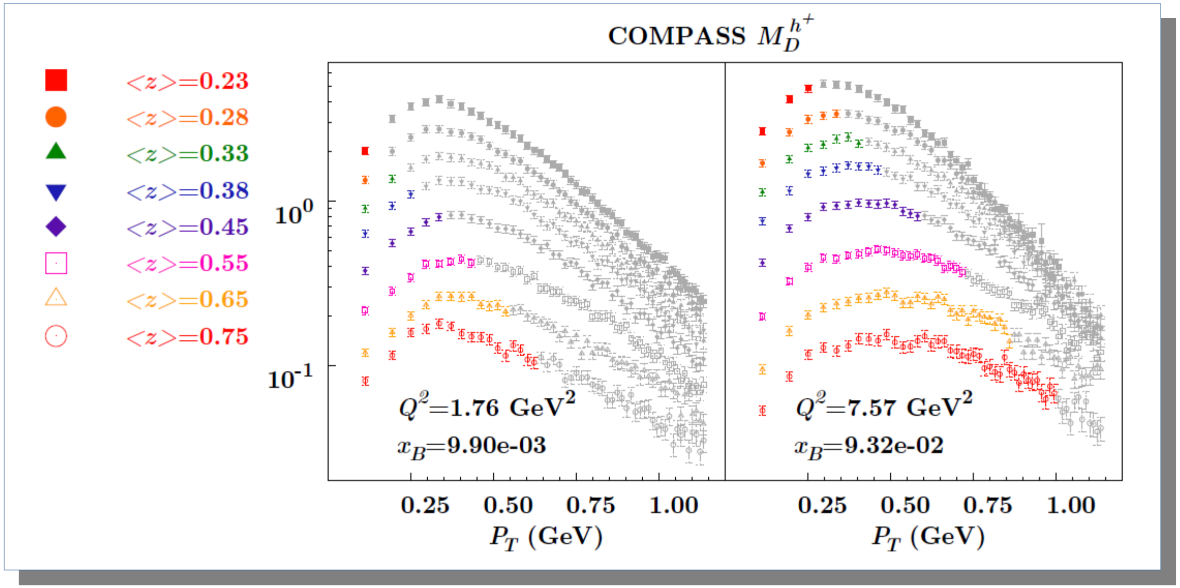
$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$



$$y_f = -\ln \frac{Q}{M_{fT}}$$

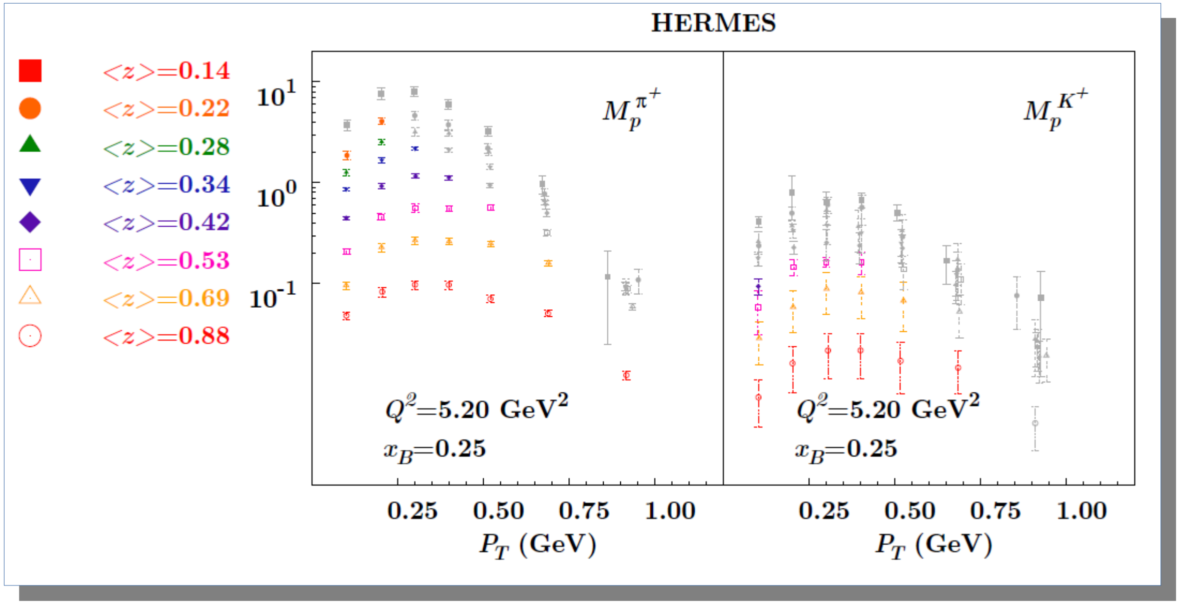
$$y_i = \ln \frac{Q}{M_{iT}}$$

Kinematics of current region



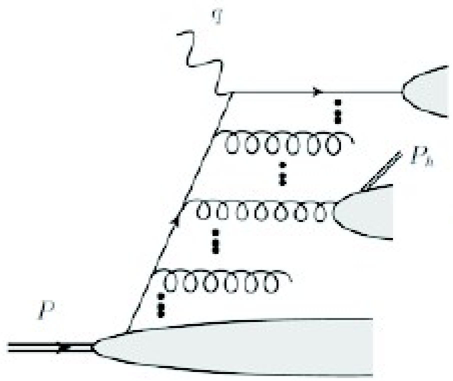
- Colored points belong to the current fragmentation region.
- Gray points are likely to be outside of current region

This is just an example!
The actual implementation of these cuts crucially depends on the choice of the non-perturbative parameters of the model

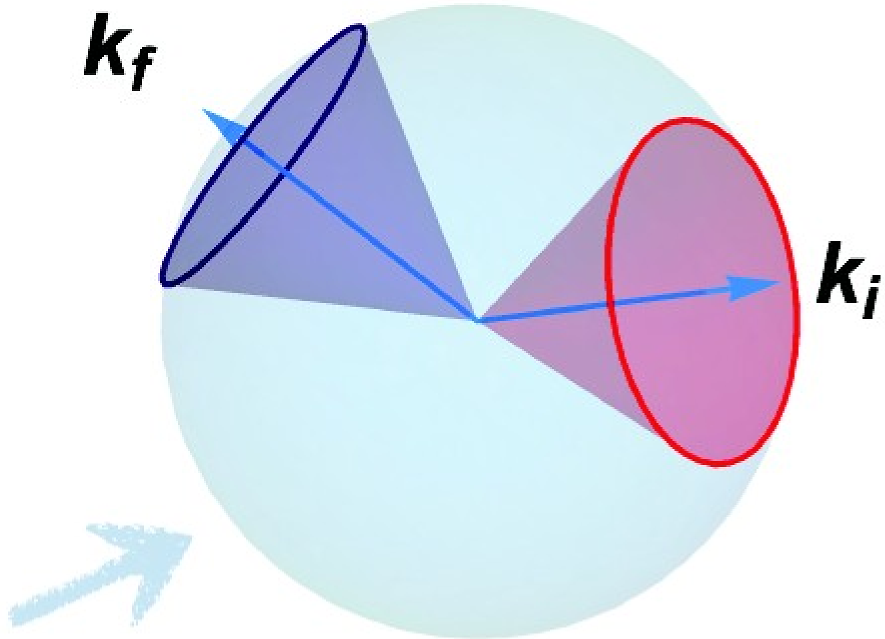


Kinematics of soft region

$$y_h \equiv \frac{1}{2}\log\frac{P_h^+}{P_h^-}$$



soft

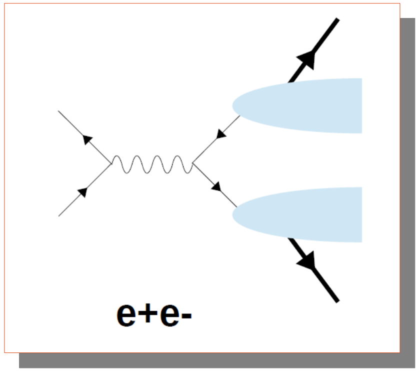


However, this neglects the soft fragmentation region

(No factorization theorem for this region)

e^+e^- scattering processes

e^+e^- scattering processes



- Recent data on Collins azimuthal asymmetries from BELLE, BaBar and BES III
- No modern data available (yet) on unpolarized cross sections

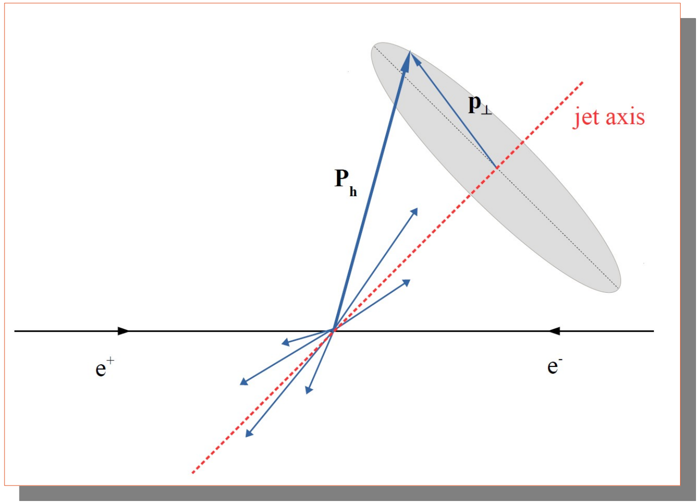
TASSO, MARKII data available for unpolarized $e^+e^- \to h\,X$

- pT distributions
- Different energies

TMD evolution

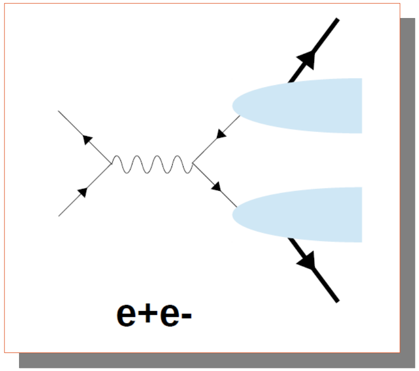
- Integrated over z

big limitation



- Direct observation of the hadron momentum component transverse to the fragmenting parent parton (jet axis)

e^+e^- scattering processes



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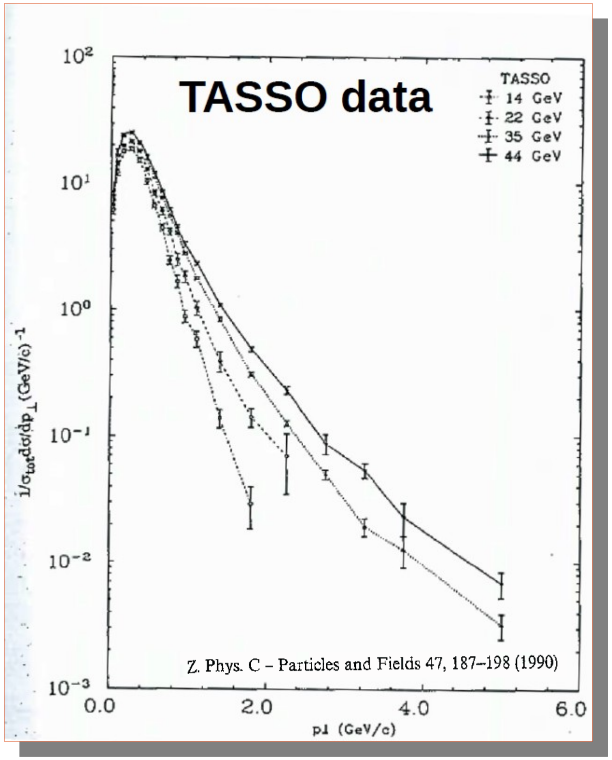


TASSO, MARKII data available for unpolarized $e^+e^- \to h\, X$

- pT distributions
- Different energies

TMD evolution

- Integrated over z

big limitation

- Direct observation of the hadron momentum component transverse to the fragmenting parent parton (jet axis)

Unpolarized cross section

$$\boxed{\frac{d\sigma^h}{dz\, d^2\boldsymbol{p}_\perp} = L_{\mu\nu}W^{\mu\nu} = \frac{4\pi\alpha^2}{3s}\, z\, F_1^h(z, p_\perp; Q^2)}$$

$$\left\{\begin{aligned} &W^{\mu\nu} = W^{\mu\nu}_{TMD} + W^{\mu\nu}_{coll} \\ &W^{\mu\nu}_{TMD} \propto \sum_f |\mathcal{H}_f(Q;\mu)|^{\mu\nu} D_{h/f}(z, \boldsymbol{p}_\perp; \mu, \zeta_D) \end{aligned}\right.$$

$$D_{h/f}(z, z\boldsymbol{k}_\perp; \mu, \zeta_D) \equiv \frac{1}{(2\pi)^2}\int d^2\boldsymbol{b}_\perp e^{-i\boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp} \tilde{D}_{h/f}(z, \boldsymbol{b}_\perp; \mu, \zeta_D)\,,$$

$$\tilde{D}_{h/f}(z, \boldsymbol{b}_\perp; \mu, \zeta_D) \equiv$$

$$\sum_j \left[\tilde{C}_{j/f} \otimes d_{h/j}(z; \mu_b)/z^2\right]$$

$$\times \exp\left\{\int_{\mu_b}^{\mu} \frac{d\tilde{\mu}}{\tilde{\mu}}\left[\gamma_D(\alpha_s(\tilde{\mu}); 1) - \gamma_K(\alpha_s(\tilde{\mu}))\log\left(\frac{\sqrt{\zeta_D}}{\tilde{\mu}}\right)\right]\right\}$$

$$\cancel{\times \exp\left\{\tilde{K}(b_*; \mu_b)\log\left(\frac{\sqrt{\zeta_D}}{\mu_b}\right)\right\}}$$

$$\times \exp\left\{g_{h/j}(z, b_\perp) + g_K(b_\perp)\log\left(\sqrt{\frac{\zeta_D}{\zeta_D^{(0)}}}\right)\right\}.$$

Calculable within perturbative QCD

Non-perturbative information

Modeling the cross section

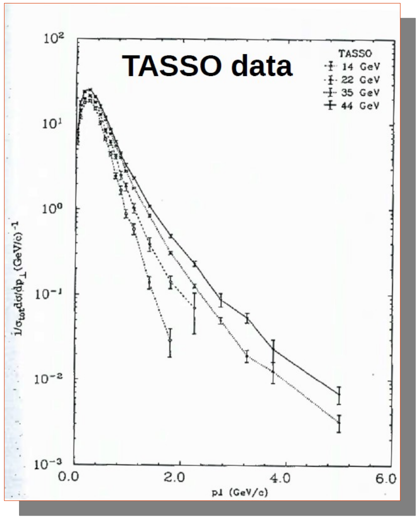
Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86

Assuming factorization ...

$$\left.\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right|_{model} = \frac{4\pi\alpha^2}{3s}\sum_q e_q^{\,2}\, D_q^h(z,p_\perp;Q^2)$$

$$= \frac{4\pi\alpha^2}{3s}\sum_q e_q^{\,2}\, D_q^h(z,Q^2)\, h(p_\perp)$$

To leading order



Use this ...

To get information about that ...

Issues to investigate:

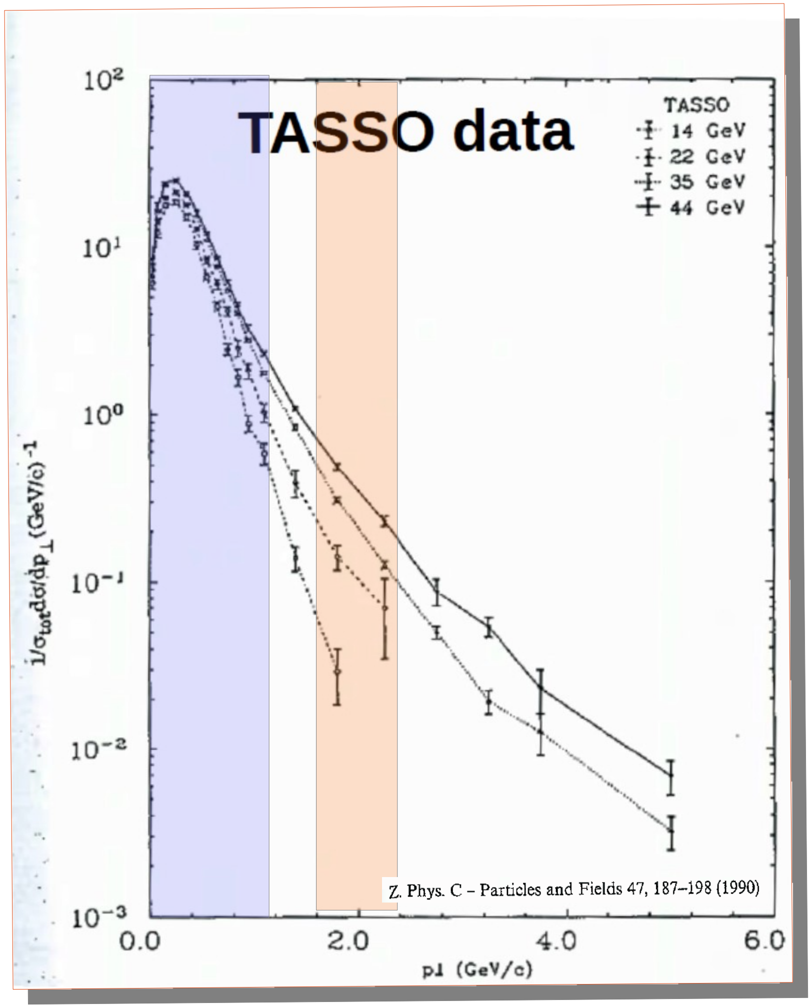
- Appropriate functional form for $g_{j/p}$
- Scale evolution induced by g_k

$$\tilde{D}_{h/q}(z,\boldsymbol{b}_\perp;Q) = \sum_i \left[\left(\tilde{C}_{j/q}\otimes\frac{d_{h/j}}{z^2}\right) e^{\Gamma_D(Q)}\right] \exp\left\{g_{j/P}(x,b_\perp) + g_K(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\}$$

Modeling the cross section

Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86

The lack of information about **z** hinders a full TMD extraction of the FF.



Identify the region where TMD effects are dominant

For fully differential cross sections, matching region is Expected to be at

$$p_\perp \sim zQ$$

Use experimental **<z>** to make an estimate

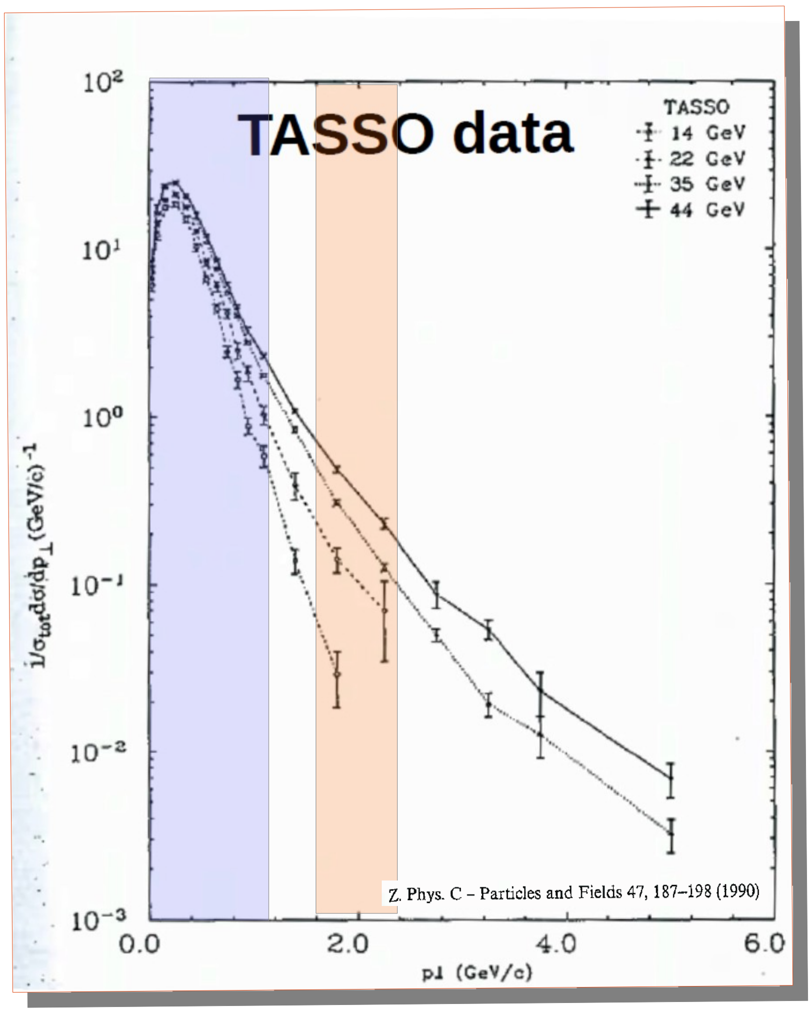
$$p_\perp \sim 2\,\mathrm{GeV}$$

We start by concentrating on a restricted range:

$$p_\perp < 1\ \mathrm{GeV}$$

Modeling the $p_\perp$ dependence

Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86



$$\boxed{D_q^h(z, p_\perp) = D_q^h(z)\, h(p_\perp)}$$

The lack of information about **z** hinders a full TMD extraction of the FF.

- Gaussian $p_\perp$ dependence

$$h(p_\perp) = \frac{e^{-p_\perp{}^2/\langle p_\perp{}^2\rangle}}{\pi \langle p_\perp{}^2\rangle}$$

- Power law $p_\perp$ dependence

$$h(p_\perp) = 2(\alpha - 1) M^{2\,(\alpha-1)} \frac{1}{(p_\perp^2 + M^2)^\alpha}$$

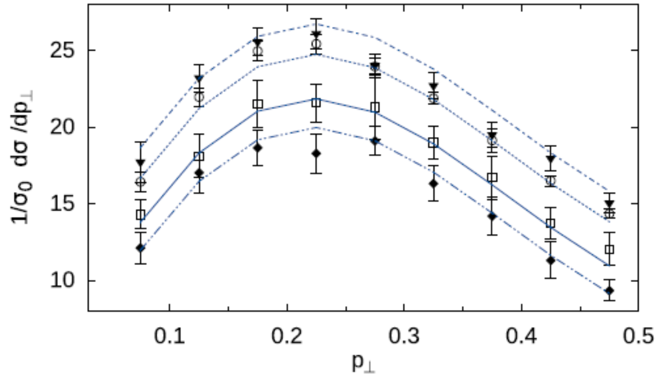
$$M^2 = (2\alpha - 1)\, p_{0\perp}{}^2$$
$$p_{0\perp} = 0.212\,\mathrm{GeV}\,.$$

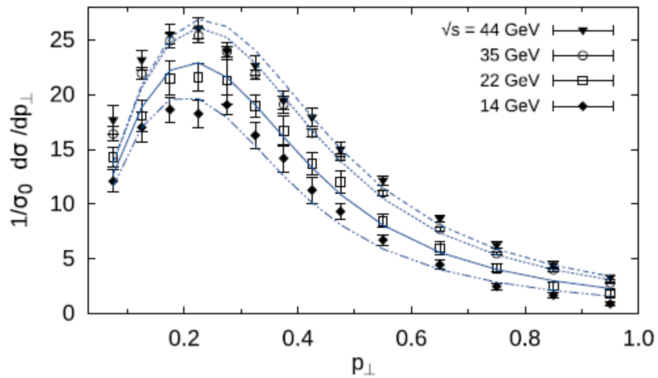
Modeling the $p_\perp$ dependence

Fit of TASSO data,
using **gaussian** $p_\perp$ dependence

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dp_\perp}\bigg|_{model} \longrightarrow \pi\, p_\perp N \left[\int dz\, \frac{\sum_q e_q^2 D_q^h(z;Q^2)}{\sum_q e_q^2}\right] h(p_\perp) + \delta\, Q$$

Normalization problematic !





$$\langle p_\perp^2\rangle = 2\,g_1 + 2\,g_2\, z^2 \log\left(\frac{Q}{2\,Q_0}\right)$$

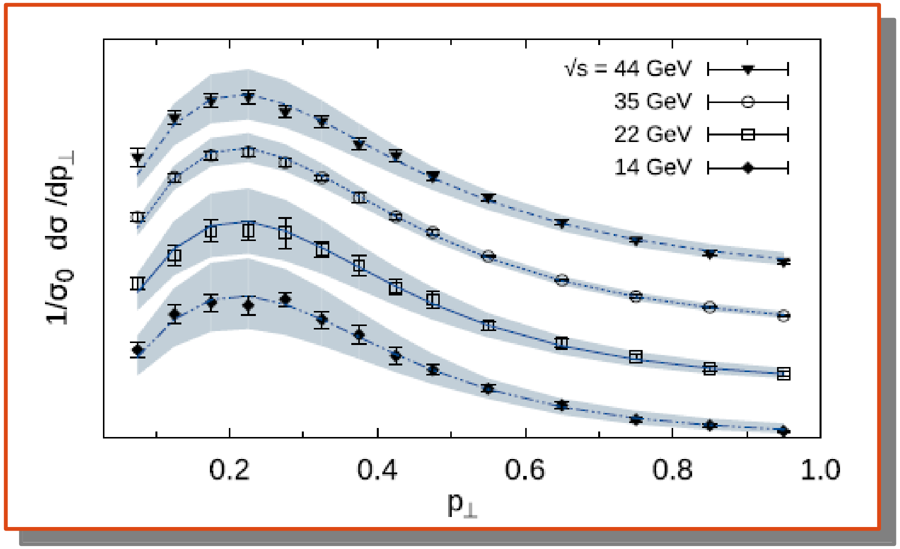
Tension between the shape of the peak and that of the tale

The lack of information about **z** hinders a full TMD extraction of the FF.

Fit of TASSO data,
using **power law** $p_\perp$ dependence

$$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2+\mathrm{M}^2\right)^\alpha}$$

$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$



Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86

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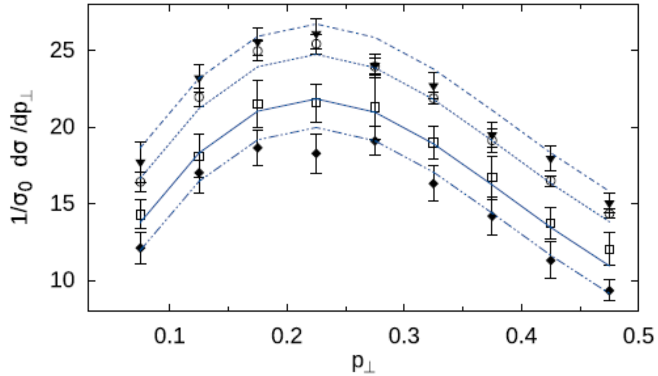

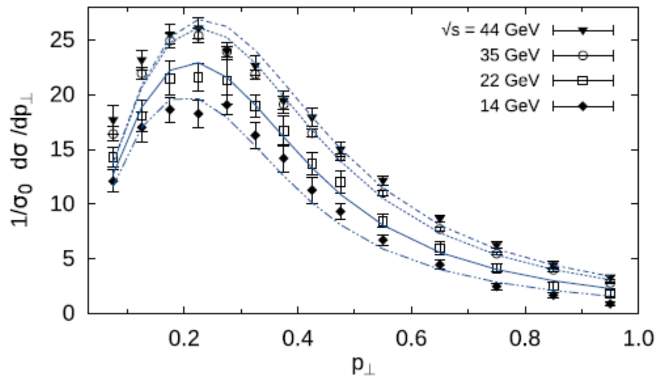

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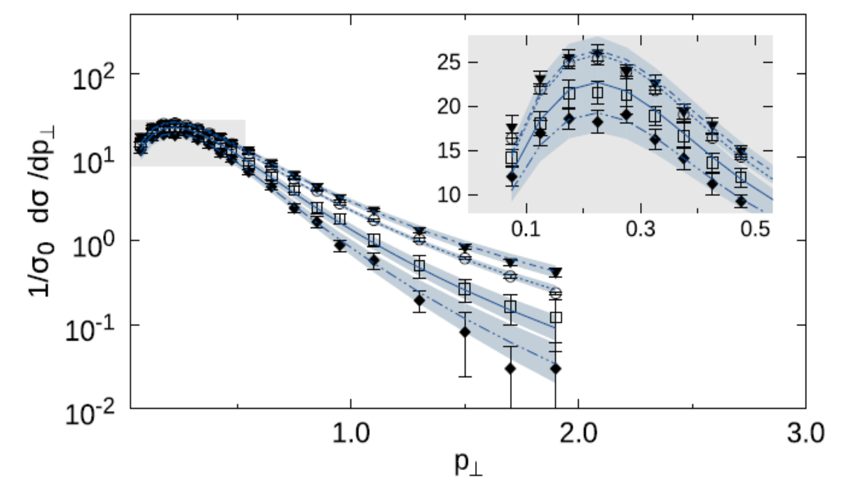

Boglione, Gonzalez-Hernandez, Taghavi,
Phys. Lett. B772 (2017) 78-86

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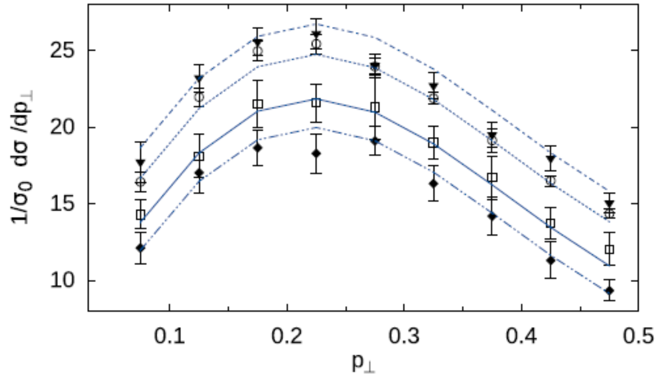

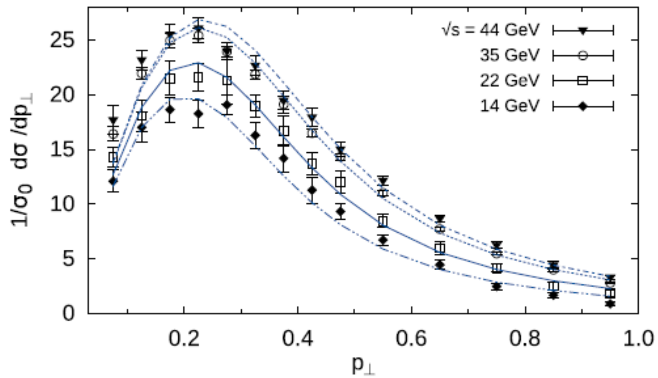

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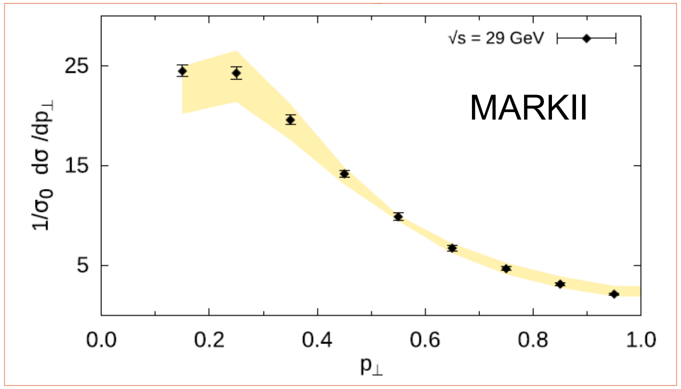
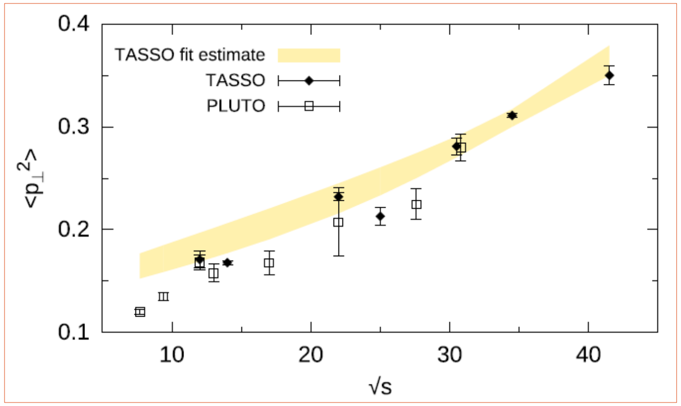
$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$





Interpreting our results ...

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\left(\lambda_\Gamma(b_*) + g_K(b_\perp)\right)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

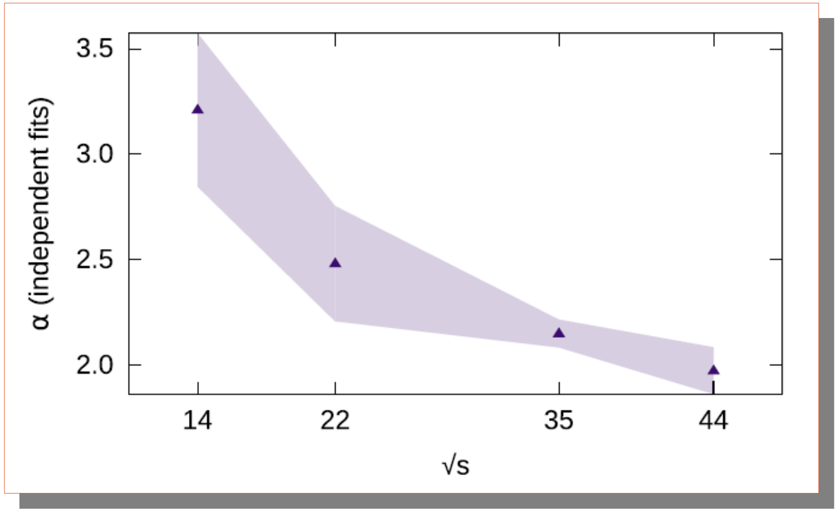
MODEL $$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{\left(p_\perp^2+\mathrm{M}^2\right)^\alpha}\right\} \xrightarrow{\text{large } b_\perp} \frac{1}{2^\alpha\,\pi\,\Gamma(\alpha)}\left(\frac{b_\perp}{\mathrm{M}}\right)^{\alpha-1}\sqrt{\frac{\pi}{2}}\,\frac{e^{-b_\perp \mathrm{M}}}{\sqrt{b_\perp \mathrm{M}}}\left[1 + \mathcal{O}\left(\frac{1}{b_\perp \mathrm{M}}\right)\right]$$

Interpreting our results ...

MODEL

$$h(p_\perp) = 2(\alpha - 1)M^{2\,(\alpha-1)} \frac{1}{(p_\perp^2 + M^2)^\alpha}$$



TMD scheme

(under the assumption that integration over z does not alter the structural form of the non perturbative exponential)

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\tilde{g}(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\}$$

$$b_\perp^{\alpha_0}\exp\left\{\tilde{g}(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\} \propto b_\perp^{\alpha}$$

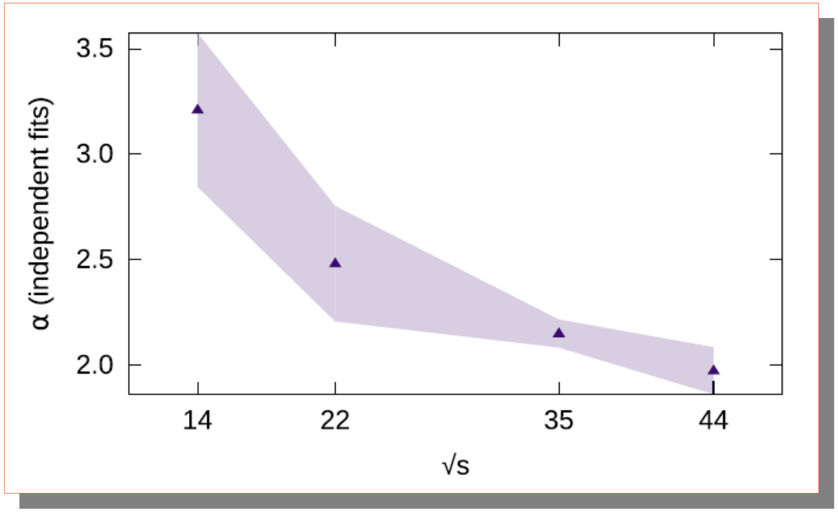
Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\boxed{\begin{gathered}\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right) \\ g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)\end{gathered}}$$

Interpreting our results ...

MODEL

$$h(p_\perp) = 2(\alpha - 1)\mathrm{M}^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$



TMD scheme

(under the assumption that integration over z does not alter the structural form of the non perturbative exponential)

- **There are caveats on this interpretation**: it is consistent with theoretical expectations but it is not unique.
- Lack of information on z-dependence of the TMD FF in the TASSO and MARK II measurements (and possible correlations between Q and z of different origin) hinders a more solid conclusion about TMD evolution effects in these data sets.

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha} \log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu\, b_\perp)$$

Conclusions

- Phenomenological studies of TMD factorization and evolution have come a long way.
  Many aspects of the interplay between perturbative and non-perturbative contributions are now better understood.

- Some issues remain open and need further investigation, especially as far as phenomenology is concerned:
  - Difficult to work in b_T space where we loose phenomenological intuition
  - F.T. involves integration of an oscillating function over b_T up to infinity:
    upon integration one loses track of what was small b_T and what was large b_T.
  - ...

- P_T distributions of SIDIS cross sections over the full P_T range will have to be further investigated.

- Simultaneous fits of SIDIS, Drell-Yan and e^+e^- annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.

- Data selection is crucial in global fitting:
  - not too many
    (only data within the ranges where the TMD evolution schemes work should be considered)
  - not too few
    (too strict a selection can bias the fit results and neglect important information from experimental data) → see our new criteria to select current fragmentation region events !