Unpolarized TMD extractions

Alessandro Bacchetta

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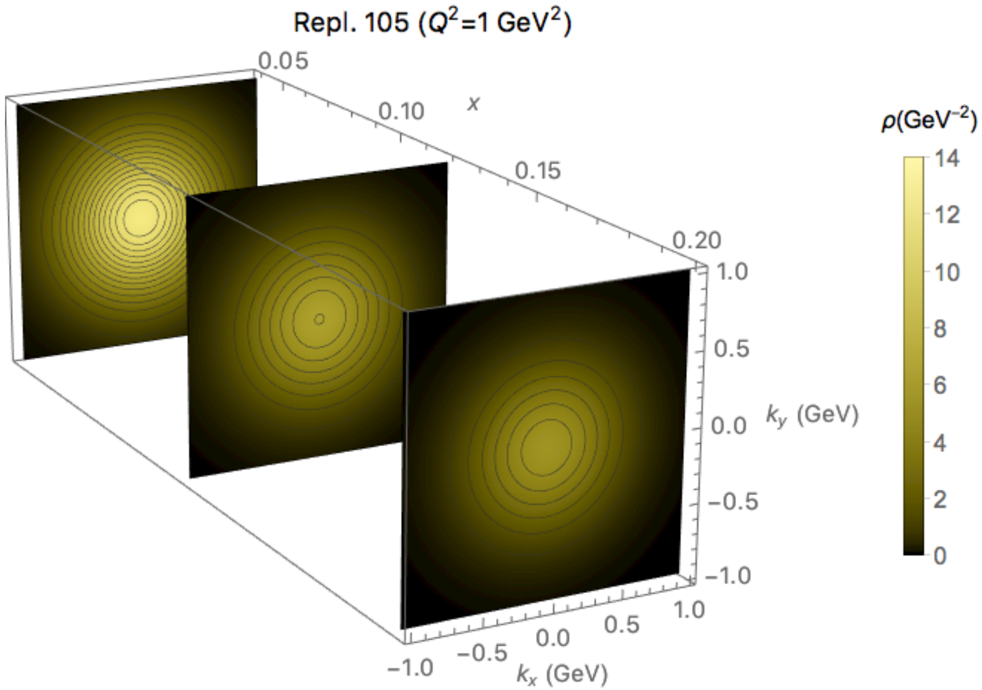
Questions



- Can we really trust the formalism? In which regions?
- How “wide” is the distribution? What’s the shape?
- Is there an x dependence of the width?
- Are valence quarks, sea quarks, gluons different?

Status of TMD phenomenology

Data, theory, fits: we start being in a position to validate the formalism

Only first attempts

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

Twist-2 TMDs

Lu, Ma, Schmidt, arXiv:0912.2031
Lefky, Prokudin arXiv:1411.0580
Barone, Boglione, Gonzalez, Melis, arXiv:1502.04214

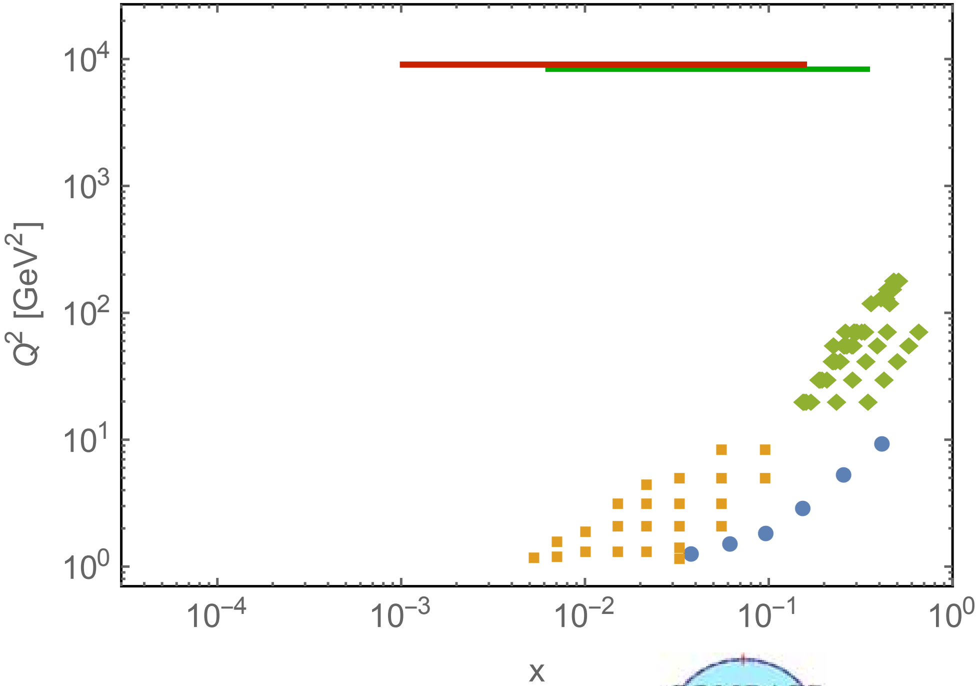
Limited data, theory, fits

see, e.g, Bacchetta, Radici, arXiv:1107.5755
Anselmino, Boglione, Melis, arXiv:1204.1239
Echevarria, Idilbi, Kang, Vitev, arXiv:1401.5078
Anselmino, Boglione, D'Alesio, Murgia, Prokudin, arXiv:1612.06413
Anselmino et al., arXiv:1303.3822
Kang et al. arXiv:1505.05589

Quark unpol. TMD: measurements

Z production@ LHC

Z production@ Tevatron



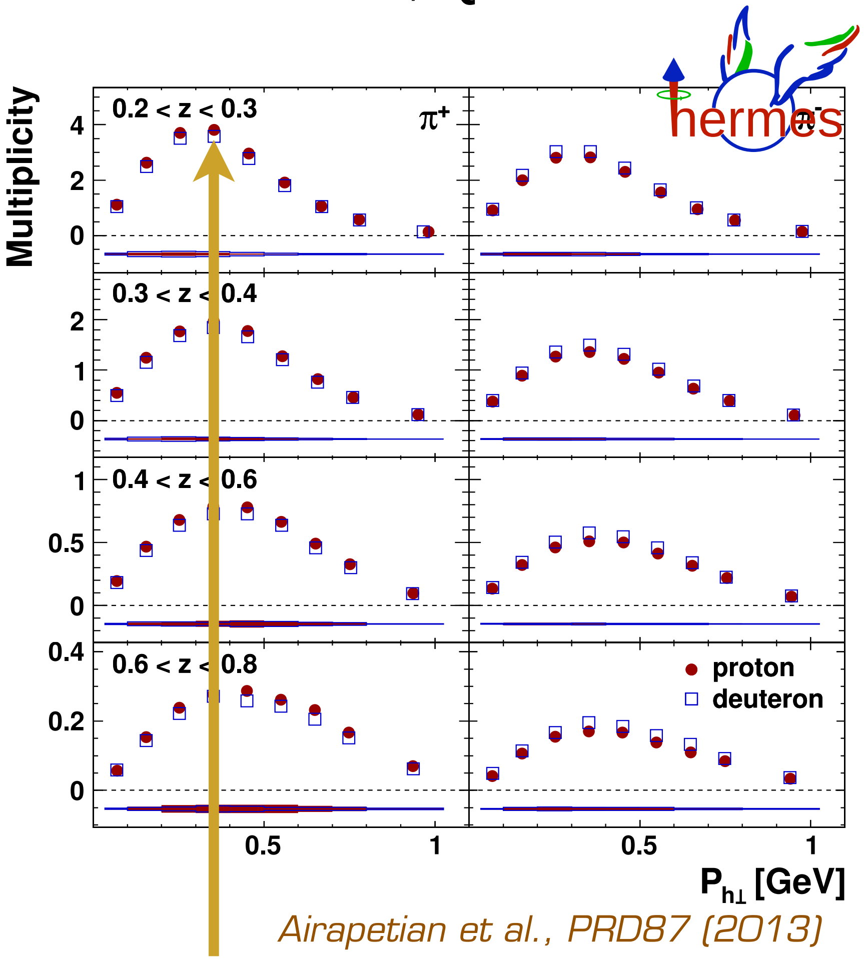
Drell-Yan@ Fermilab

SIDIS@ hermes

SIDIS@ COMPASS

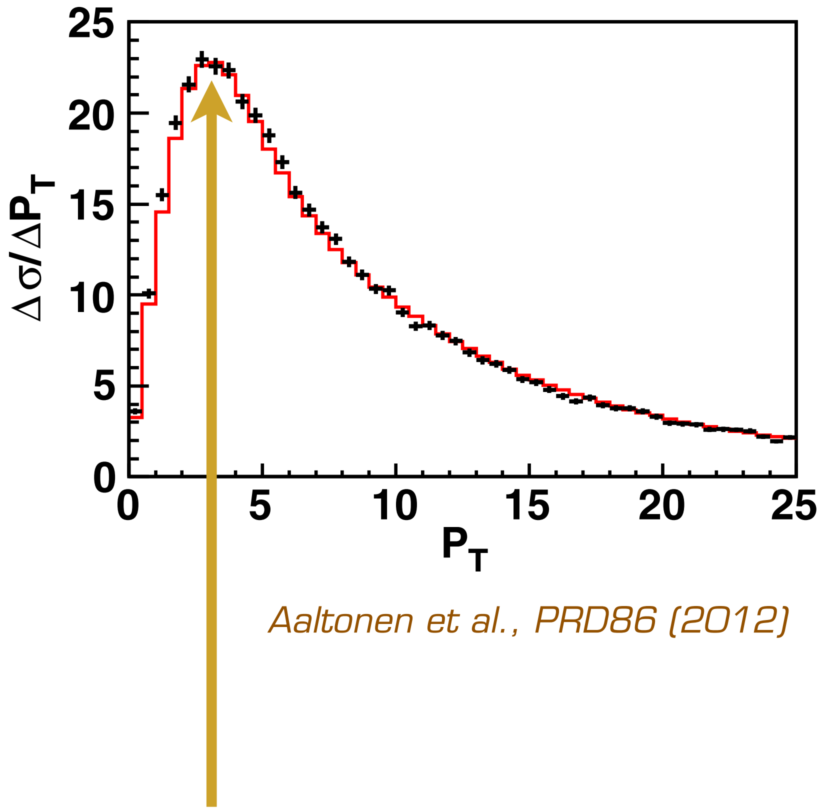
TMD evolution

HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

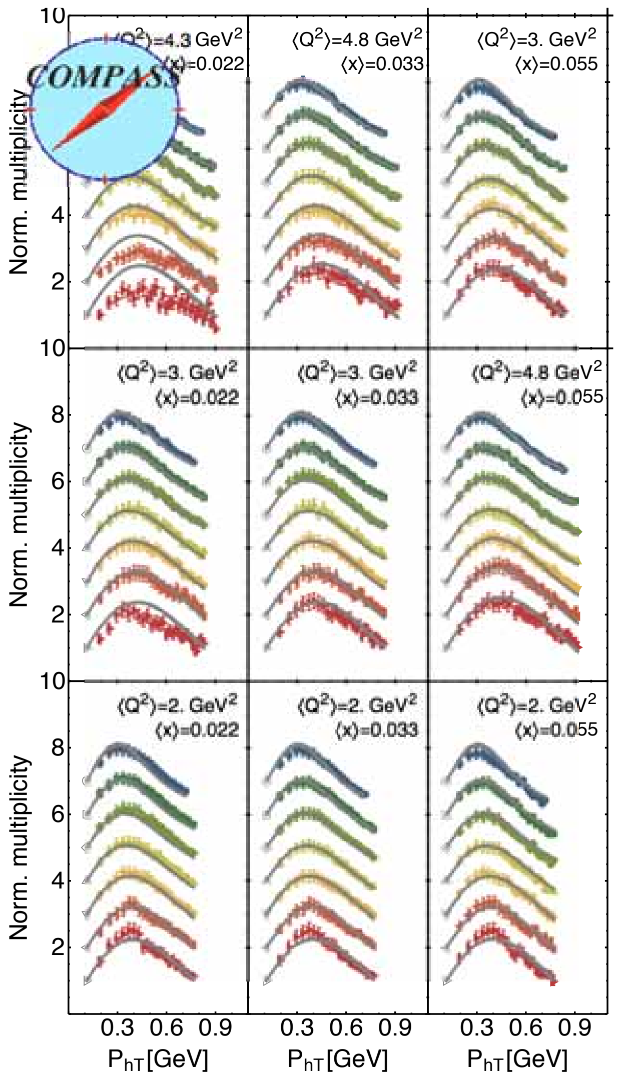
Width of TMDs changes of one order of magnitude: can we explain this in detail? (TMD evolution)

Quark unpol. TMD: extractions

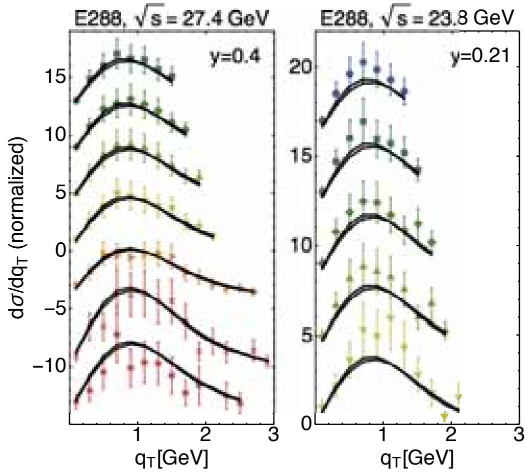
	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	NLL / NLO	✘	✘	✔	✔	98
Pavia 2013 arXiv:1309.3507	No evo	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	No evo	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL / NLO	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	NLL / LO	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2016 arXiv:1703.10157	NLL / LO	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLL / NNLO	✘	✘	✔	✔	309

First global fit of TMDs

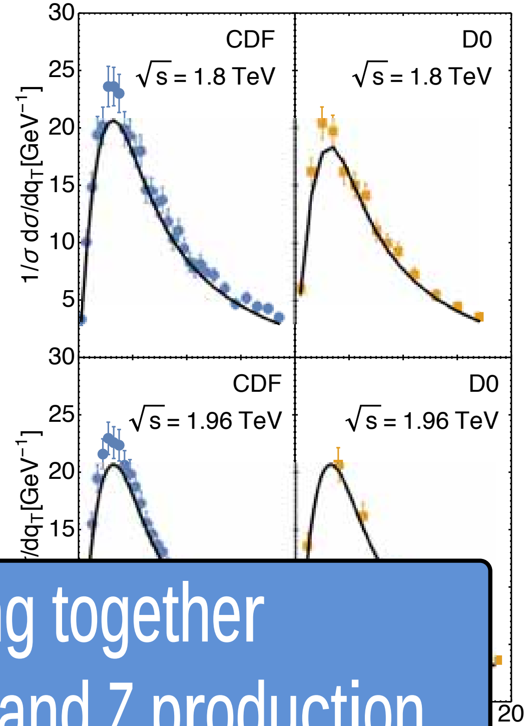
SIDIS



Drell-Yan



Z production



Number of data points: 8059

Global χ^2/dof = 1.55

Pavia2016: first fit putting together semi-inclusive DIS, Drell-Yan and Z production

Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

Pavia 2016 results in a nutshell

Total number of data points: 8059

Total number of free parameters: 11
(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)

Total χ^2 / dof = 1.55±0.05

Comparison with SV 2017 results

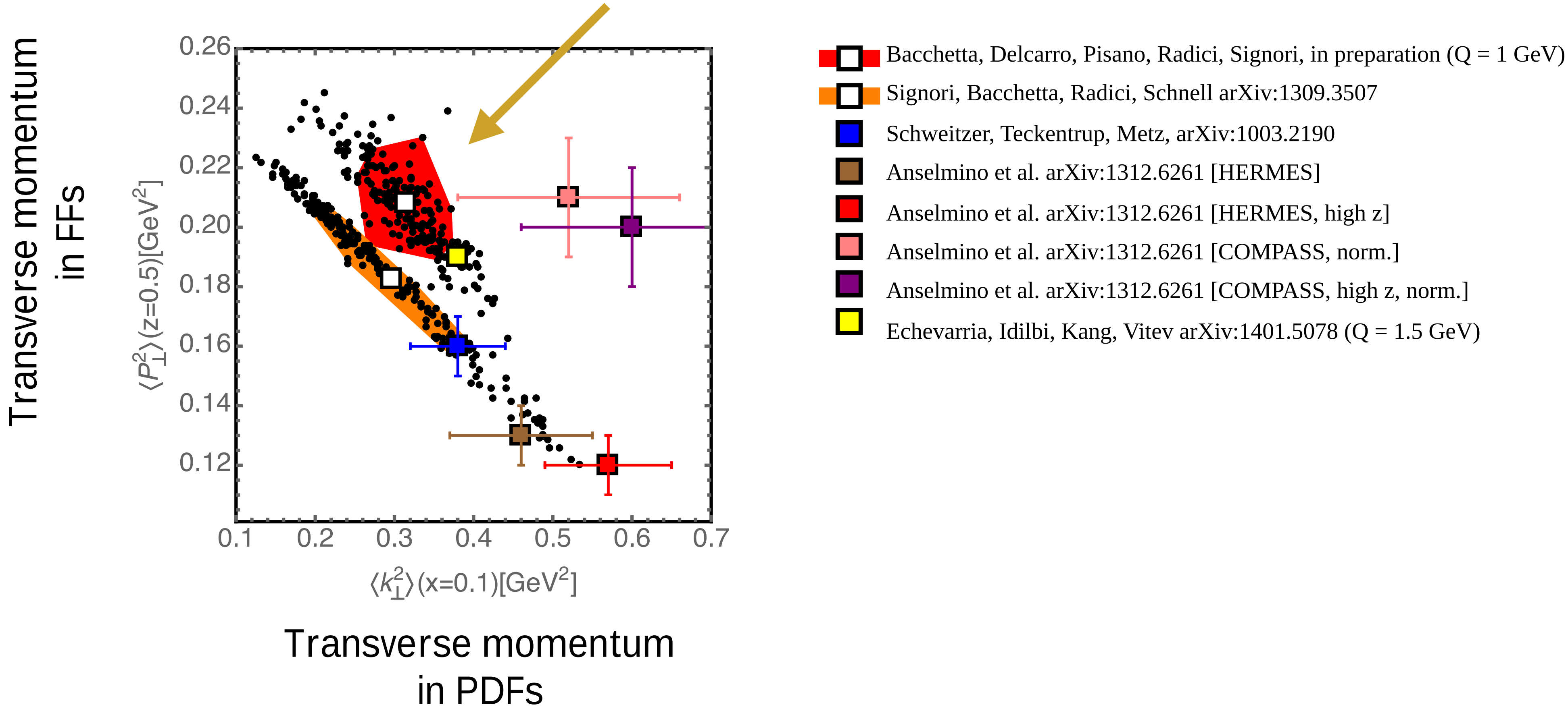
Total number of data points: 309

Total number of free parameters: 2 or 3
(2 for TMD PDFs, 1 for TMD evolution)

Total χ^2 / dof = 1.79 - 1.84

Mean transverse momentum squared

Pavia2016 results, Q^2=1 GeV^2



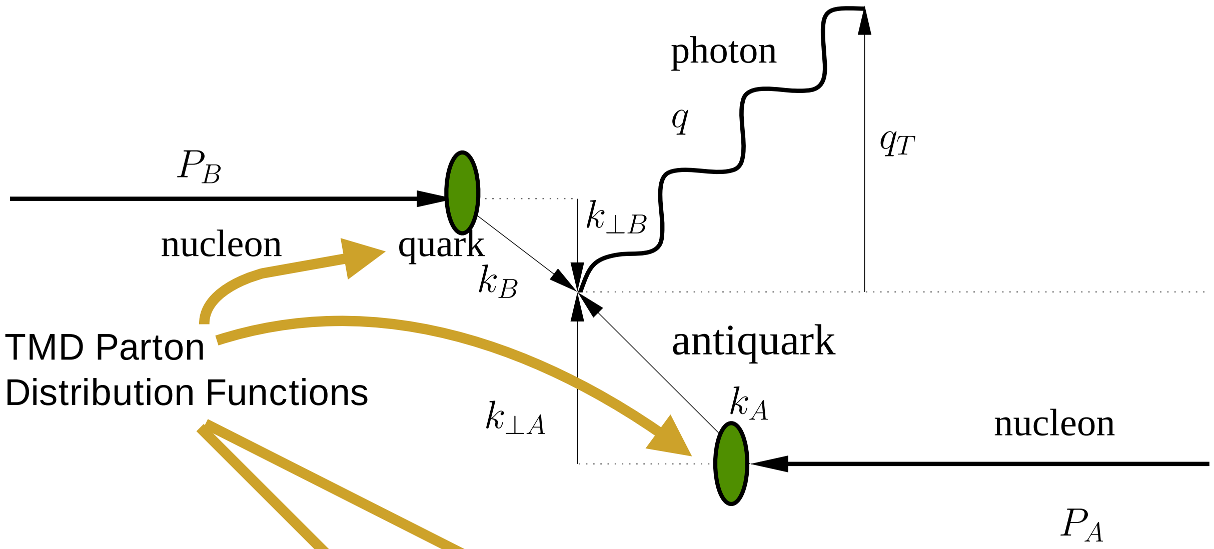
CAVEAT: intrinsic transverse momentum depends on TMD evolution "scheme" and its parameters. Not the best quantity to consider.

Something about the technical details

Factorization

- Factorization has to do with perturbative QCD more than with the nonperturbative side.
- In TMDs, there are several intriguing technical details related to regularisation of divergences of all kinds (infrared, ultraviolet, rapidity), much more sophisticated than collinear factorization.
- We are one of the “communities” that is most deeply involved into the study of perturbative QCD. Any “expert” in QCD acknowledges the relevance of these studies.

DY structure functions and TMDs



$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2) = \sum_a \mathcal{H}_{UU}^{1a}(Q^2)$$

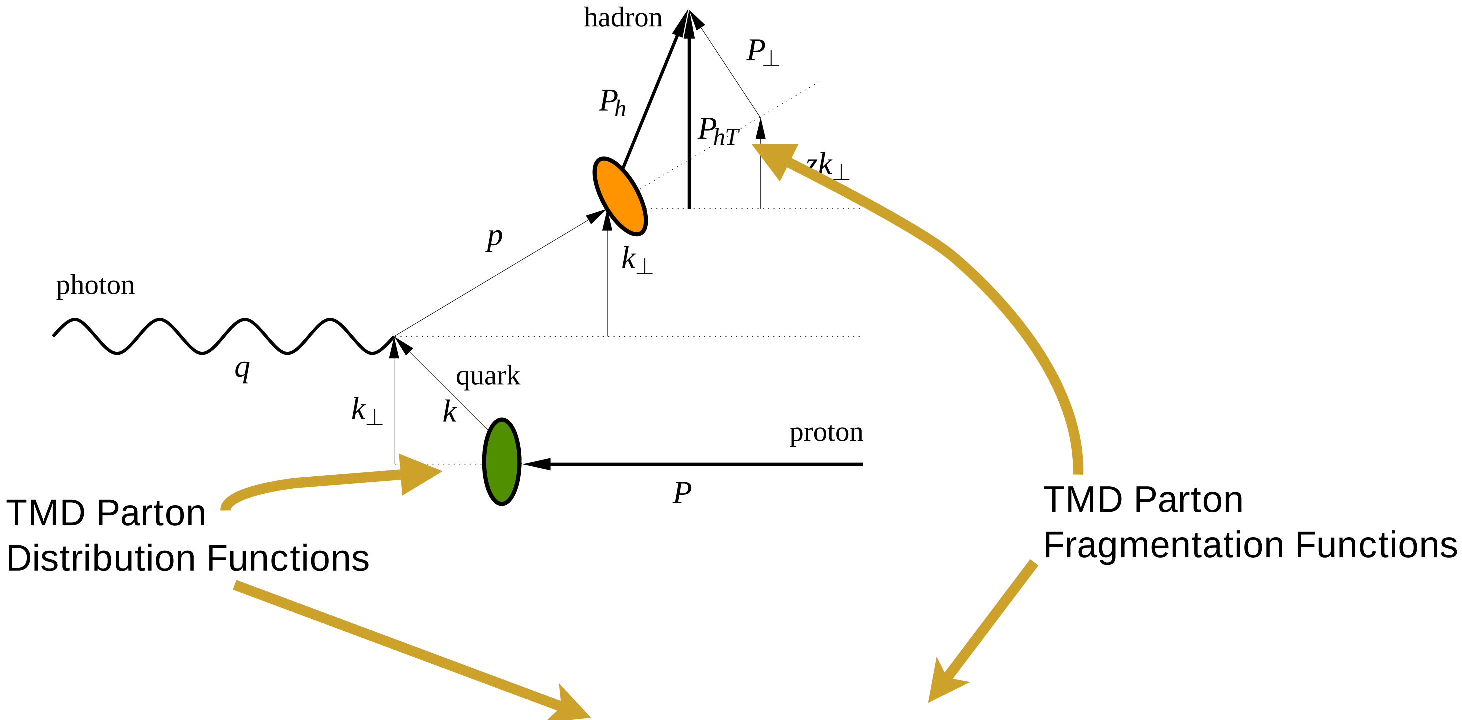
$$\times \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}_{\perp A}^2; Q^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}_{\perp B}^2; Q^2)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

$$+ Y_{UU}^{1}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}\big(M^2/Q^2\big)\,.$$

not implemented in present fits

$$\mathcal{H}_{UU,\gamma}^{1a}(Q^2) \approx \frac{e_a^2}{N_c}, \qquad \mathcal{H}_{UU,Z}^{1a}(Q^2) \approx \frac{V_a^2 + A_a^2}{N_c}\,,$$

SIDIS structure functions and TMDs



$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2) \int d\boldsymbol{k}_\perp\, d\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$+\, Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

not implemented in present fits

TMD evolution: Fourier transform

$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_\perp e^{-i b_\perp \cdot k_\perp} \widetilde{f}_1^a(x, b_\perp; \mu^2)$$

for simplicity, here I am using one scale, but in reality there are two independent ones

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

see, e.g., Rogers, Aybat, PRD 83 (11)
Collins, "Foundations of Perturbative QCD" (11)
Collins, Soper, Sterman, NPB250 (85)

Perturbative ingredients

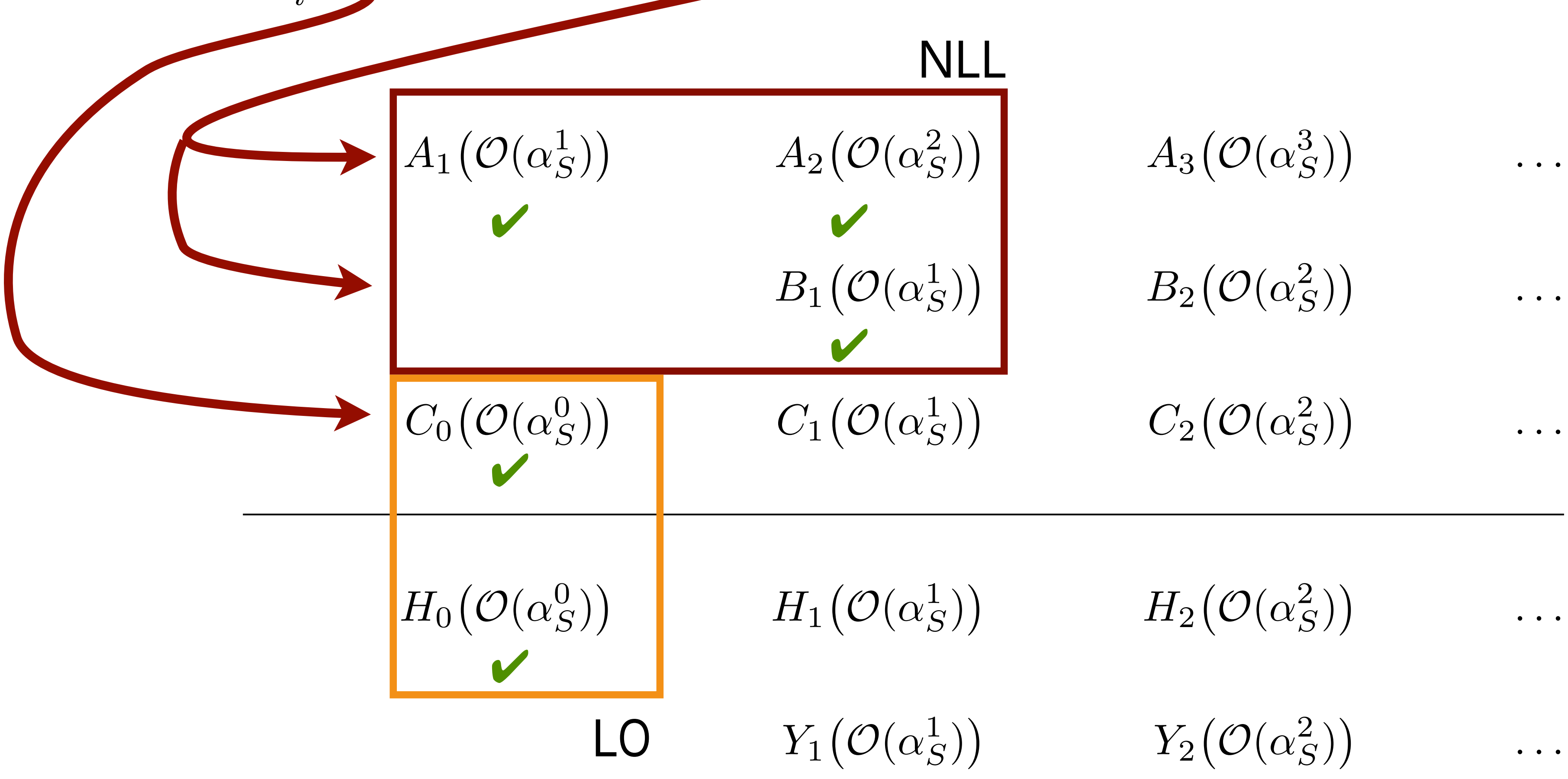
$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\mathrm{NP}}(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ $\quad A_2(\mathcal{O}(\alpha_S^2))$ $\quad A_3(\mathcal{O}(\alpha_S^3))$ $\quad \ldots$

$B_1(\mathcal{O}(\alpha_S^1))$ $\quad B_2(\mathcal{O}(\alpha_S^2))$ $\quad \ldots$

Pavia 2016 perturbative ingredients

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$



SV 2017 perturbative ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{NP}^a(x, b_T)$$

		NLL	NNLL	
$A_1(\mathcal{O}(\alpha_S^1))$ ✔		$A_2(\mathcal{O}(\alpha_S^2))$ ✔	$A_3(\mathcal{O}(\alpha_S^3))$	$\ldots$
		$B_1(\mathcal{O}(\alpha_S^1))$ ✔	$B_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$C_0(\mathcal{O}(\alpha_S^0))$ ✔		$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$H_0(\mathcal{O}(\alpha_S^0))$ ✔		$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\ldots$ NNLO
	LO	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\ldots$

μ and b* prescriptions

Choice **Choice**

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{NP}^a(x, b_T)$$

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$ — *Collins, Soper, Sterman, NPB250 (85)*

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv b_{\max}\left(1 - e^{-\frac{b_T^4}{b_{\max}^4}}\right)^{1/4}$ — *Bacchetta, Echevarria, Mulders, Radici, Signori* *arXiv:1508.00402*

$\mu_b = Q_0 + q_T$ $\quad b_* = b_T$ — *DEMS 2014*

Complex-b prescription — *Laenen, Sterman, Vogelsang, PRL 84 (00)*

ζ prescription — *SV 2017, see talk by A. Vladimirov*

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$

$e^{-\frac{b_T^2}{\langle b_T^2 \rangle}}$	*many*
$e^{-\frac{b_T^2}{\left\langle b_T^2(x) \right\rangle_a}}$	*Pavia 2013, KN 2006*
$e^{-\lambda_1 b_T} \left(1 + \lambda_2 b_T^2\right)$	*DEMS 2014, SV 2017*
$\exp\left(\frac{-\lambda_q z \boldsymbol{b}^2}{\sqrt{1 + z^2 \boldsymbol{b}^2 \frac{\lambda_q^2}{\lambda_1^2}}}\right)$	*SV 2017*

Nonperturbative ingredients 2

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$-g_2 \frac{b_T^2}{2}$$

Collins, Soper, Sterman, NPB250 (85)

$$-2\, g_2 \ln\left(1 + \frac{b_T^2}{4}\right)$$

Aidala, Field, Gamberg, Rogers
arXiv:1401.2654

$$-g_0(b_{\rm max})\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{\rm max}) b_{\rm max}^2}\right]\right)$$

Collins, Rogers
arXiv:1412.3820

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

- The justification is to recover the integrated result (“unitarity constraint”)
- Modification at low b_T is allowed because resummed calculation is anyway unreliable there

Pavia 2016 “choices”

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$g_K = -g_2 \frac{b_T^2}{2} \qquad \mu_0 = 1\,\mathrm{GeV}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\max}\left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \qquad b_{\max} = 2e^{-\gamma_E}$$

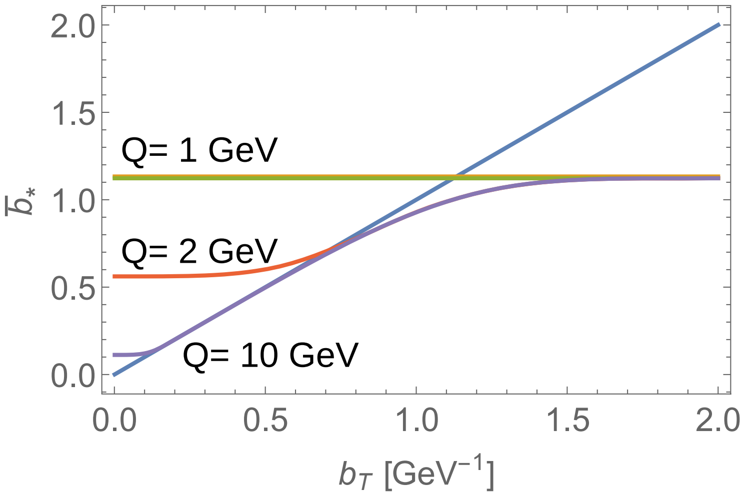
$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

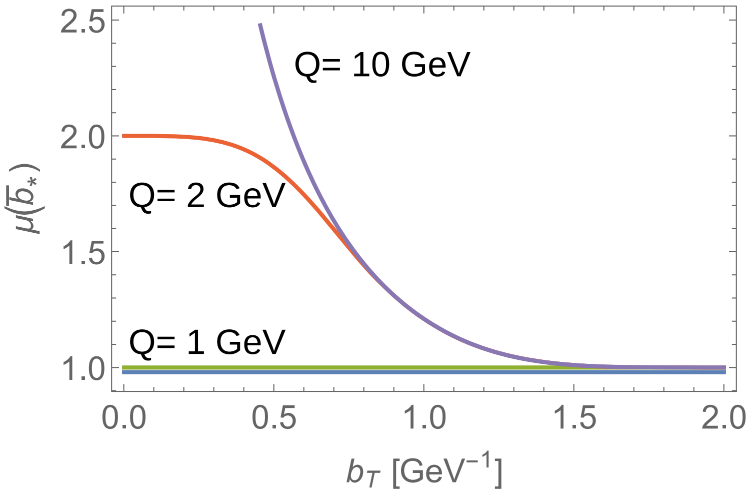
These are all choices that should be at some point checked / challenged

Effects of $\bar{b}_*$ prescription

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \qquad b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

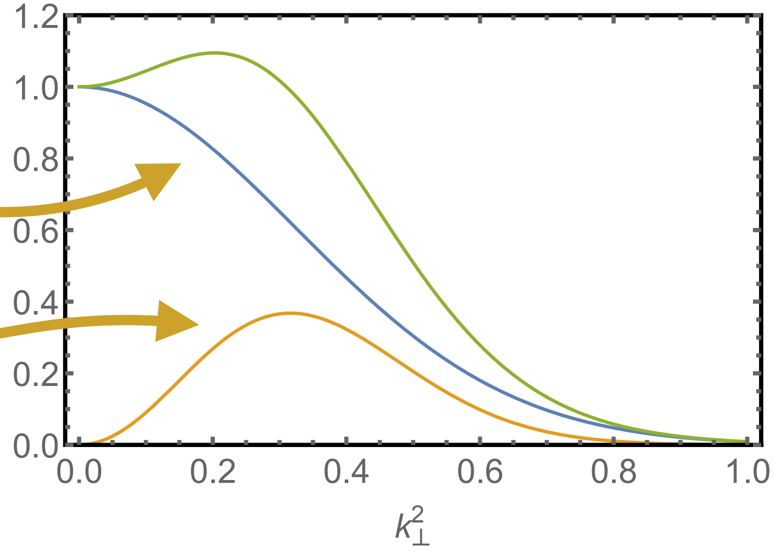




No significant effect at high Q, but large effect at low Q
(inhibits perturbative contribution)

Functional form of TMDs at 1 GeV

$$\hat{f}^a_{\rm NP} = \text{F.T. of} \left(e^{-\frac{k_\perp^2}{\langle k_{\perp,a}^2 \rangle}} + \lambda k_\perp^2 e^{-\frac{k_\perp^2}{\langle k_{\perp,a}^2 \rangle'}} \right)$$



$$\langle \boldsymbol{k}_{\perp,a}^2 \rangle(x) = \langle \hat{\boldsymbol{k}}_{\perp,a}^2 \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma},$$

where $\langle \hat{\boldsymbol{k}}_{\perp,a}^2 \rangle \equiv \langle \boldsymbol{k}_{\perp,a}^2 \rangle(\hat{x})$, and $\hat{x} = 0.1$.

Fragmentation function is similar
Including TMD PDFs and FFs, in total: 11 free parameters
(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)

Data selection

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5 \text{ GeV}$$

Total number of data points: 8059
Total χ^2 / dof = 1.55

We checked also

$$P_{hT} < \text{Min}[0.2\,Q, 0.5\,Qz] + 0.3\,\text{GeV}$$

Total number of data points: 3380
Total χ^2 / dof = 0.96

$$P_{hT} < 0.2\,Qz$$

Total number of data points: 477
Total χ^2 / dof = 1.02

Data selection SV 2017

$q_T < \delta_T\, Q$

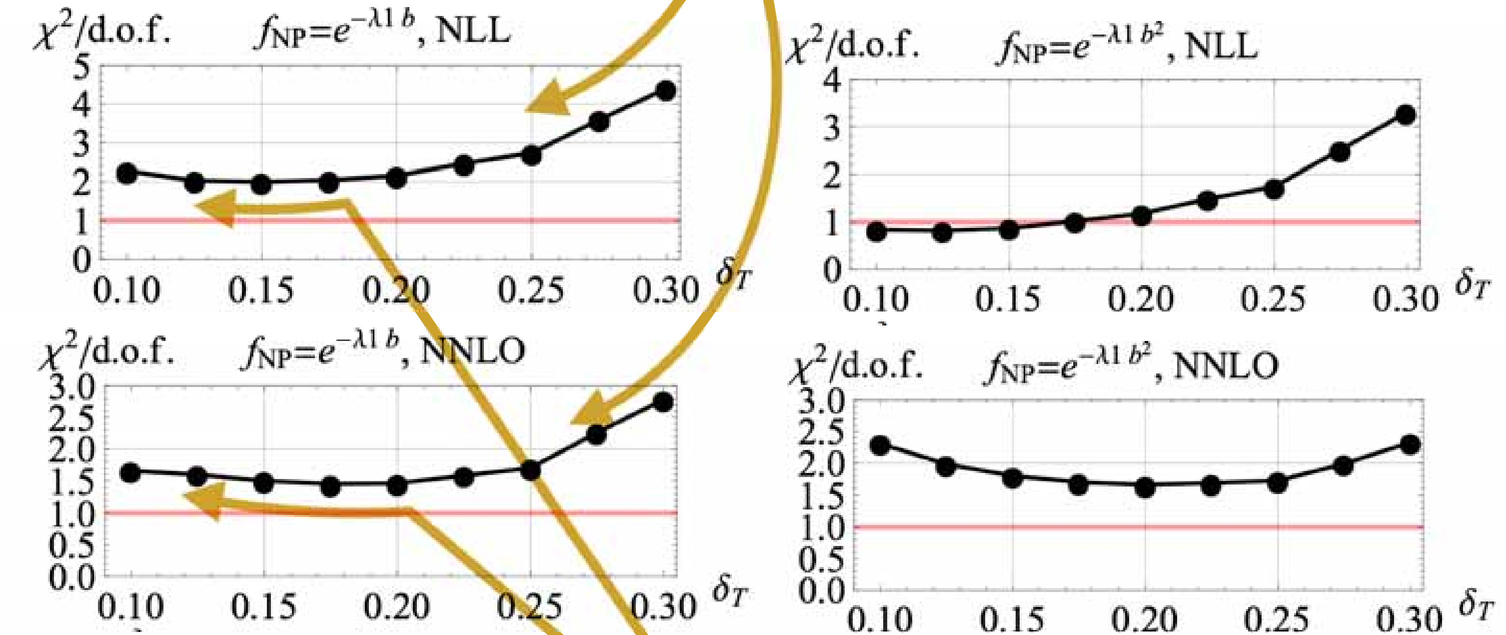
Choosing a narrower region, the χ^2 / dof improves

Final choice

$q_T < 0.2\, Q$

exponential

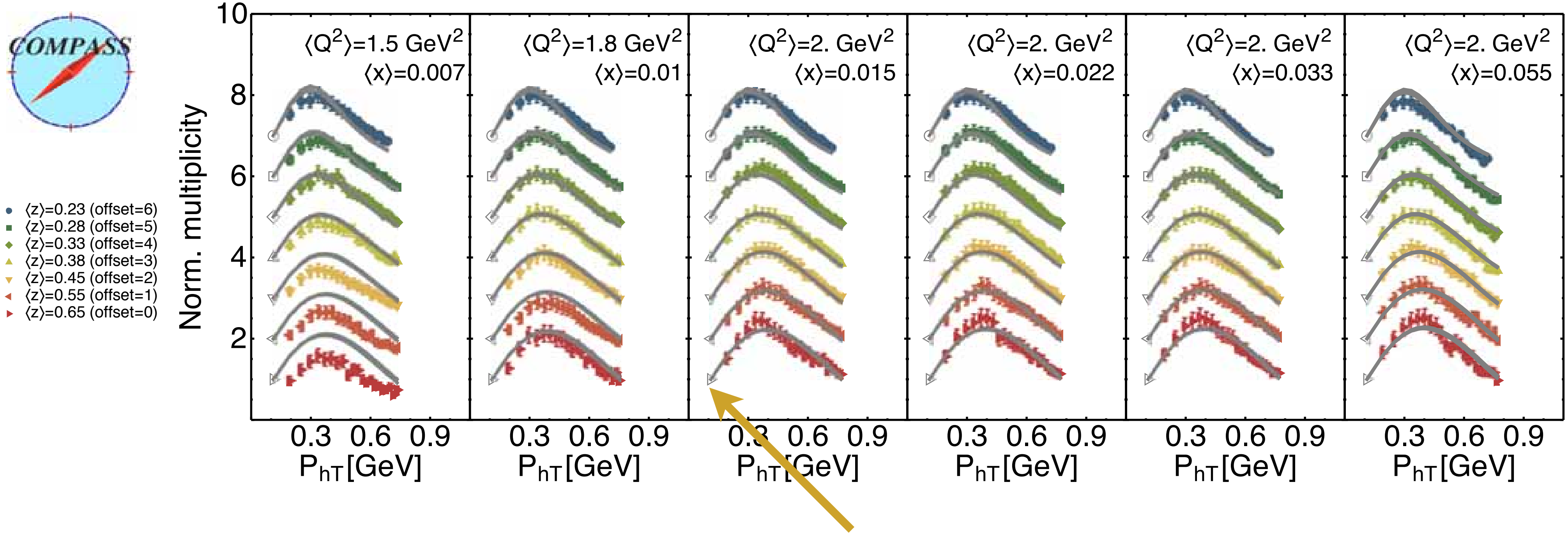
Gaussian



At a certain point, the χ^2 / dof either reaches a plateau, or even deteriorates

Something about the results

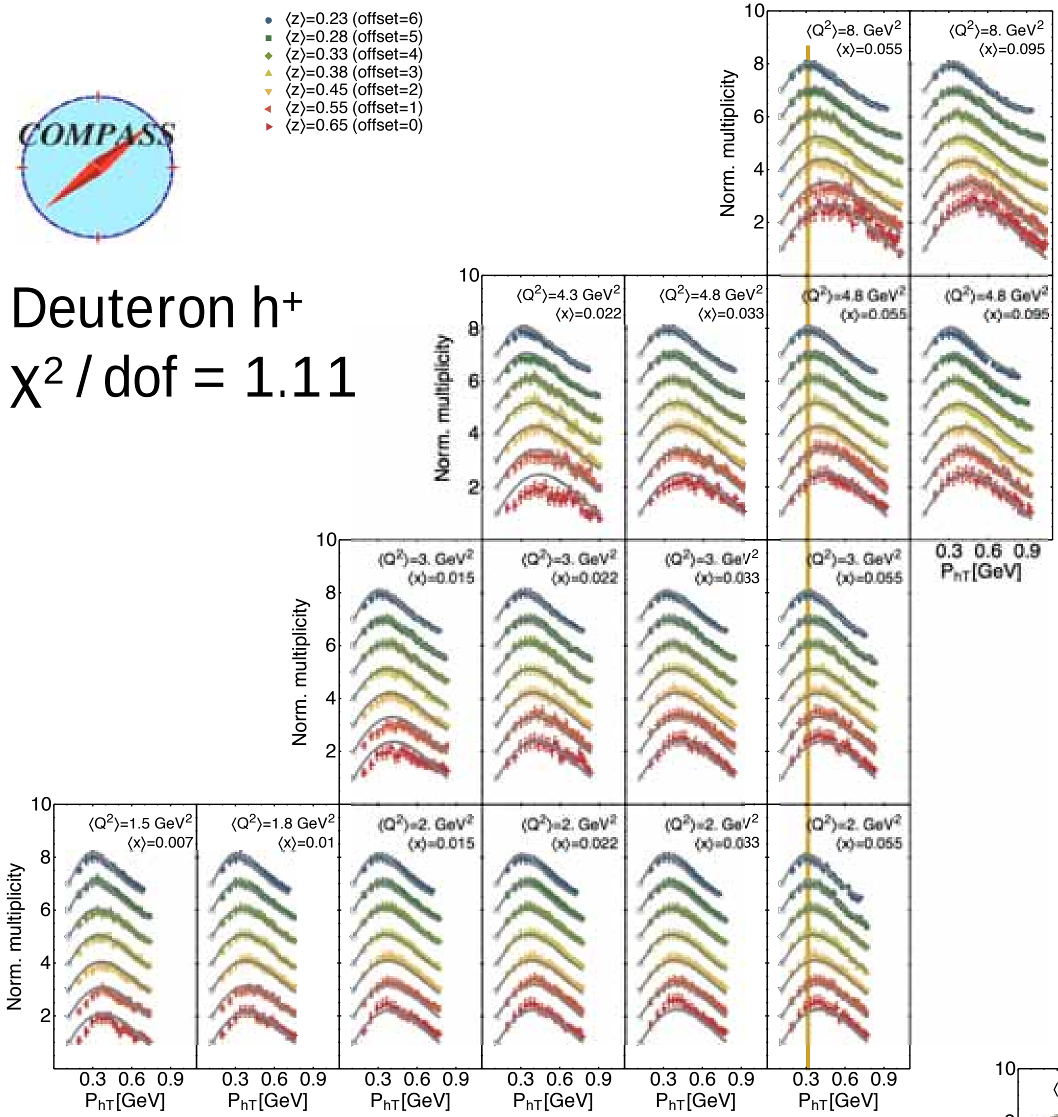
COMPASS selected bins



Deuteron h^- $\chi^2 / dof = 1.61$

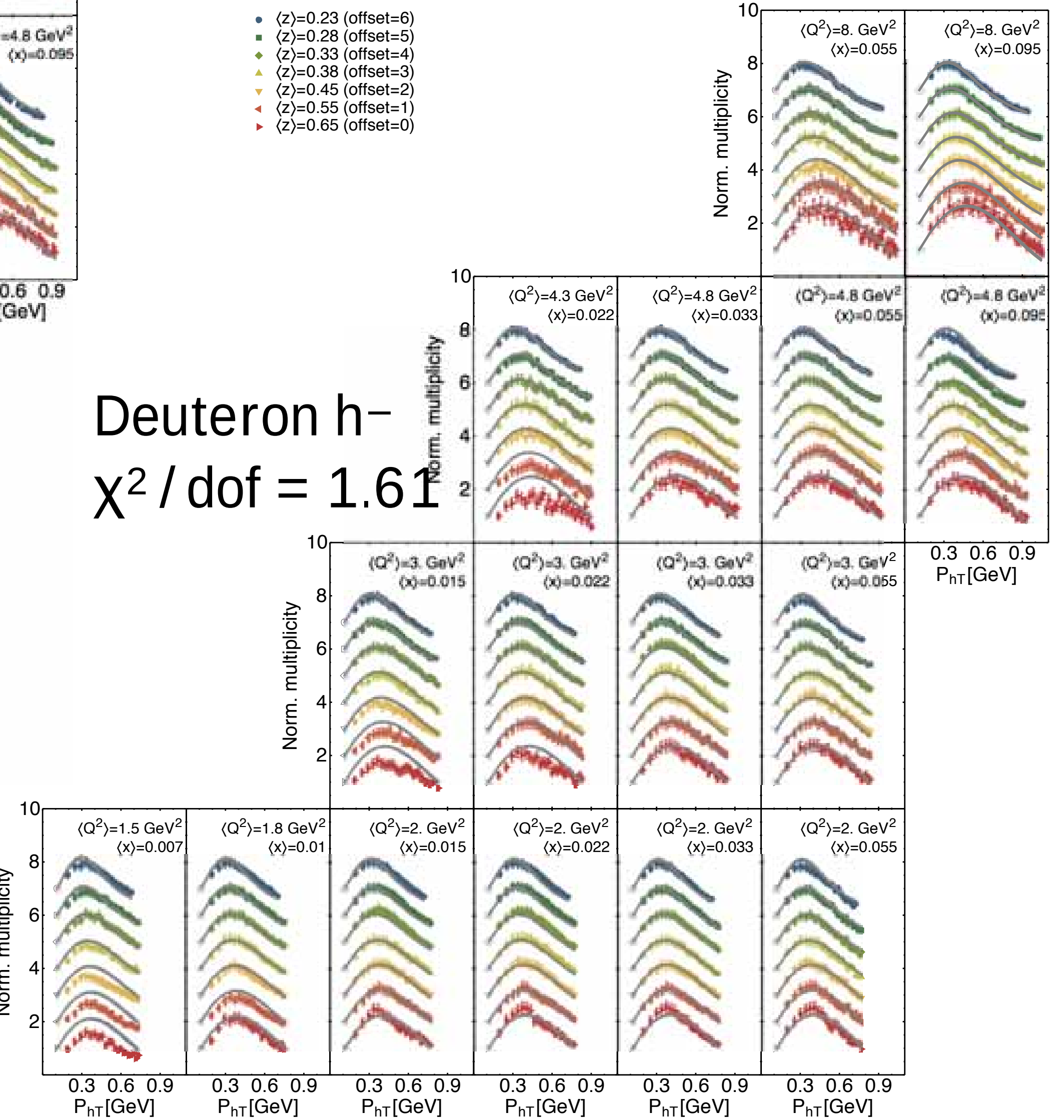
First points are not fitted, but used as normalization

Deuteron h+
$\chi^2 / dof = 1.11$

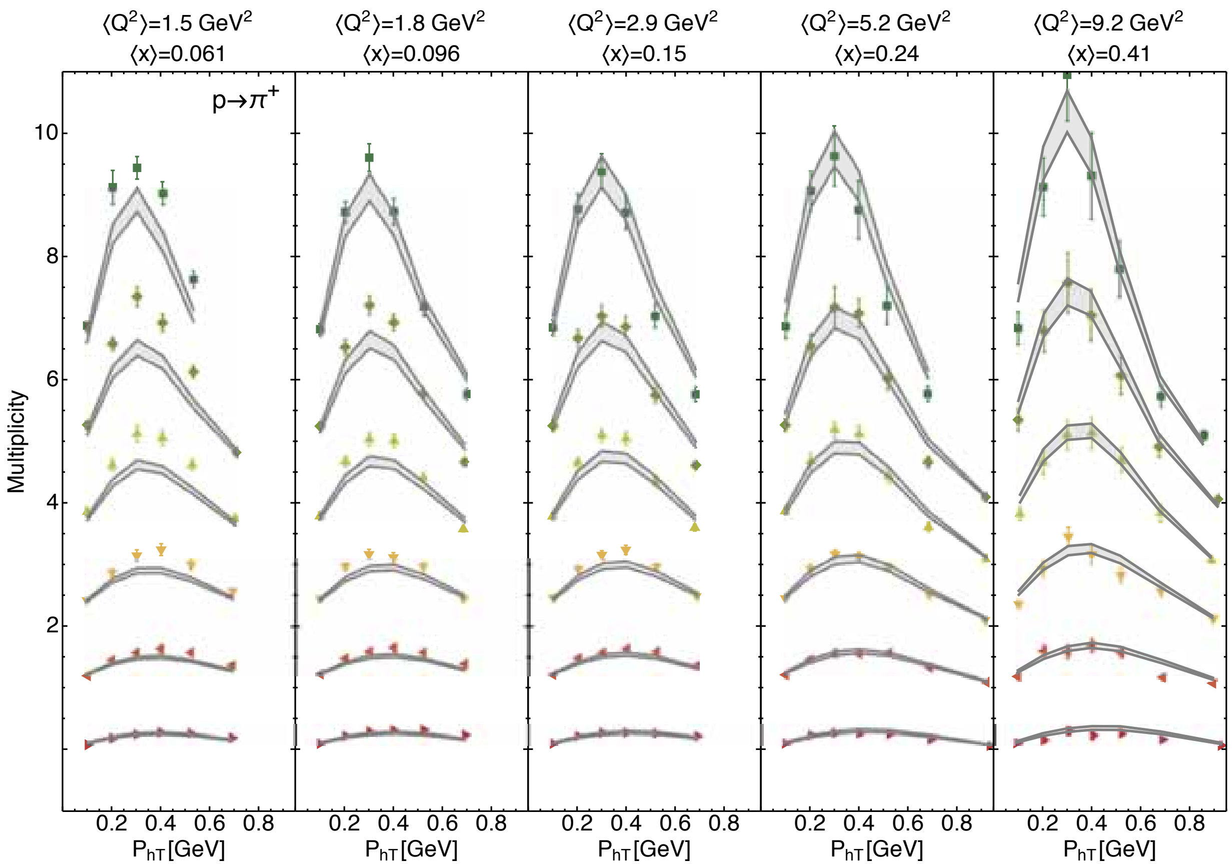


Aidala, Field, Gamberg, Rogers
arXiv:1401.2654

Deuteron h−
$\chi^2 / dof = 1.61$



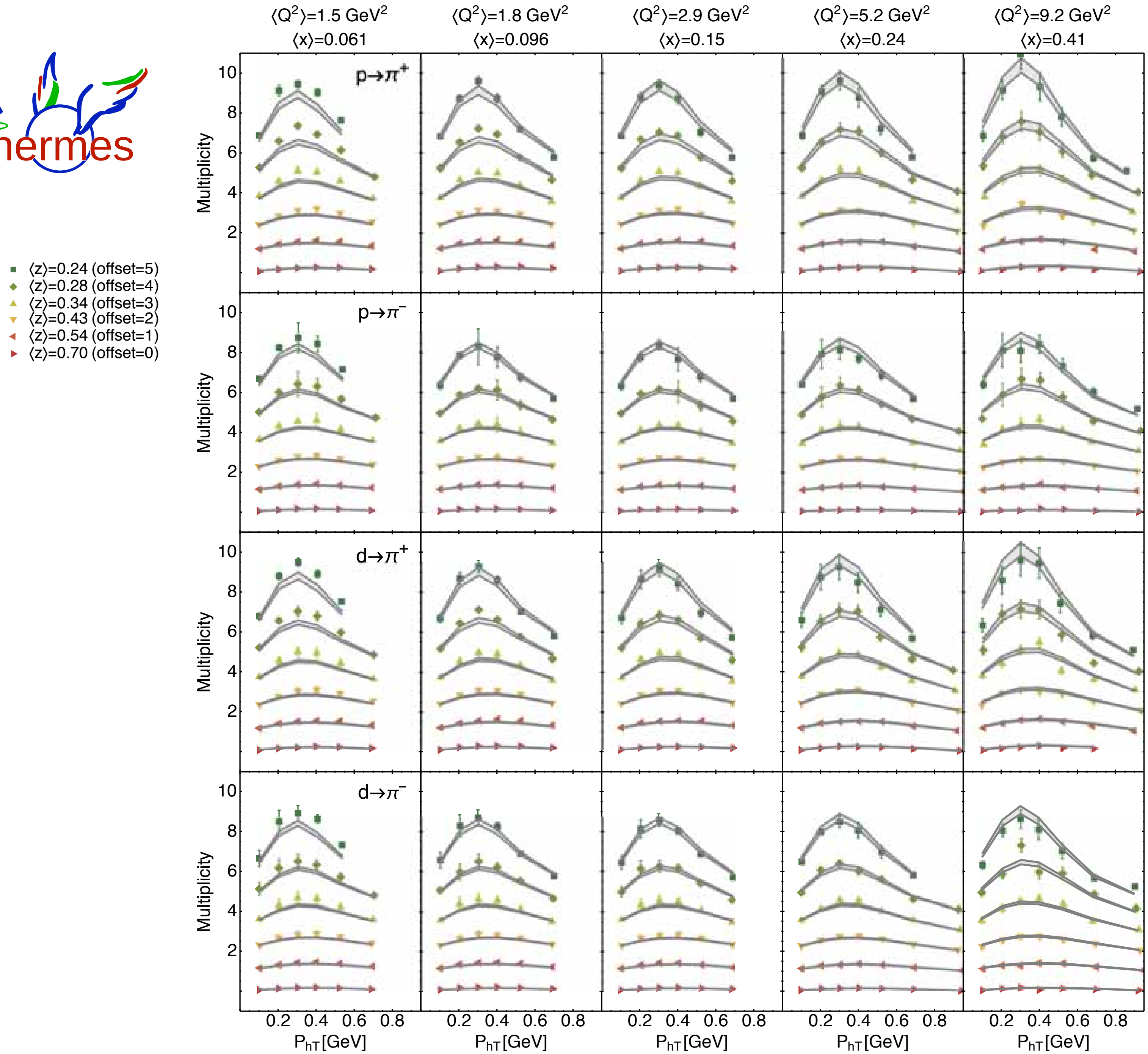
HERMES, selected bins



χ^2 / dof = 4.83

The worst of all channels...

However normalizing the theory curves to the first bin, without changing the parameters of the fit, χ^2 / dof becomes good



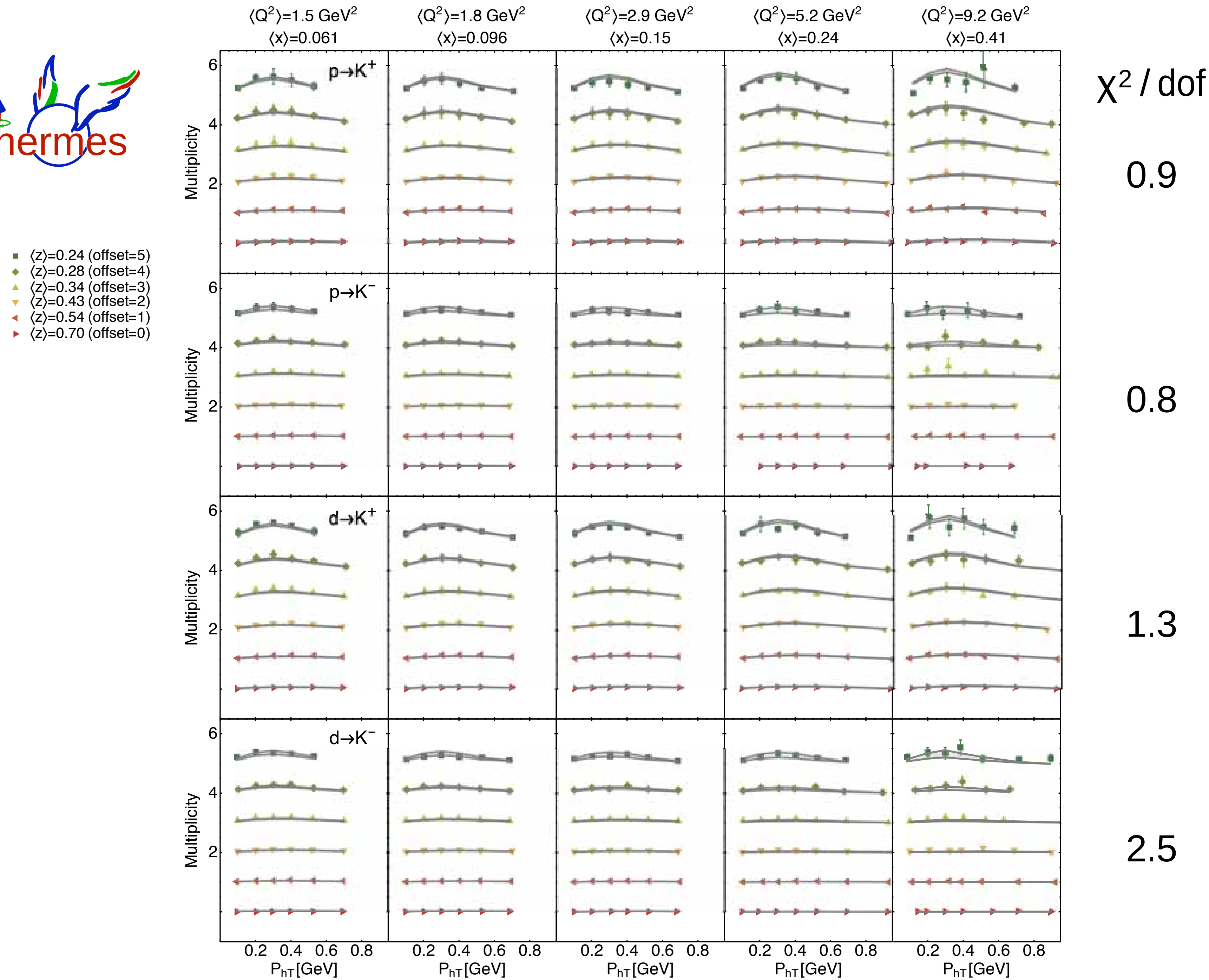
χ^2 / dof

4.8

2.5

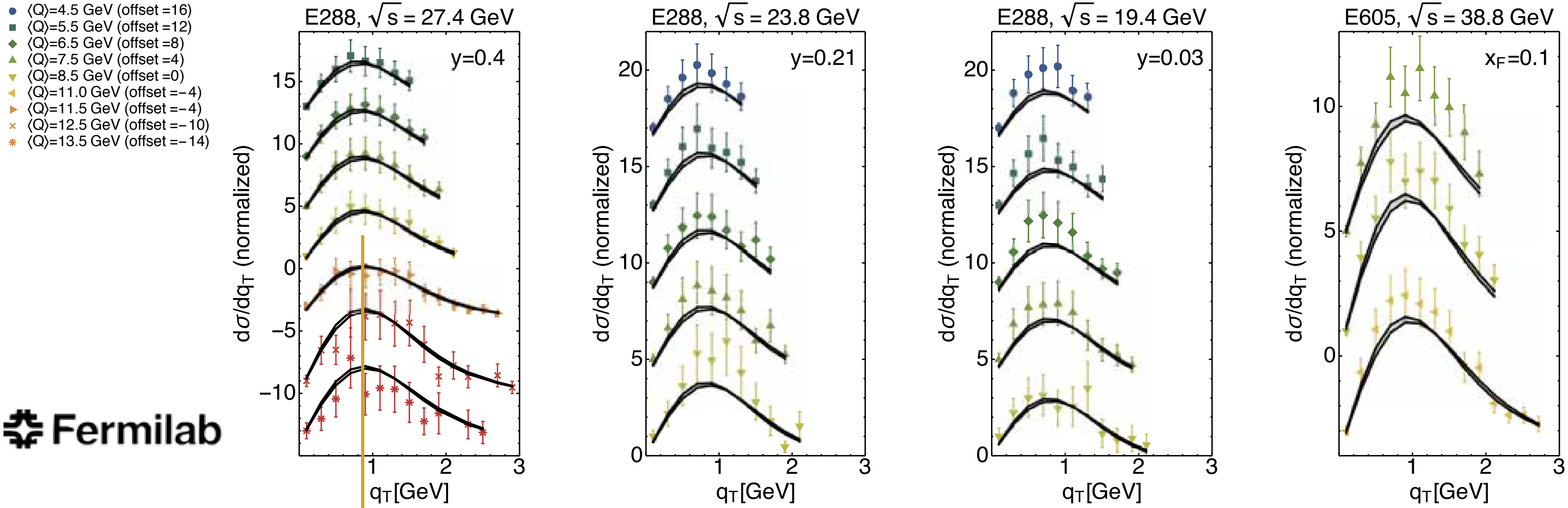
3.5

2.0



⟨Q²⟩=1.5 GeV² ⟨x⟩=0.061
⟨Q²⟩=1.8 GeV² ⟨x⟩=0.096
⟨Q²⟩=2.9 GeV² ⟨x⟩=0.15
⟨Q²⟩=5.2 GeV² ⟨x⟩=0.24
⟨Q²⟩=9.2 GeV² ⟨x⟩=0.41
p→K+
p→K−
d→K+
d→K−
Multiplicity
P_hT [GeV]
⟨z⟩=0.24 (offset=5)
⟨z⟩=0.28 (offset=4)
⟨z⟩=0.34 (offset=3)
⟨z⟩=0.43 (offset=2)
⟨z⟩=0.54 (offset=1)
⟨z⟩=0.70 (offset=0)
χ² / dof
0.9
0.8
1.3
2.5

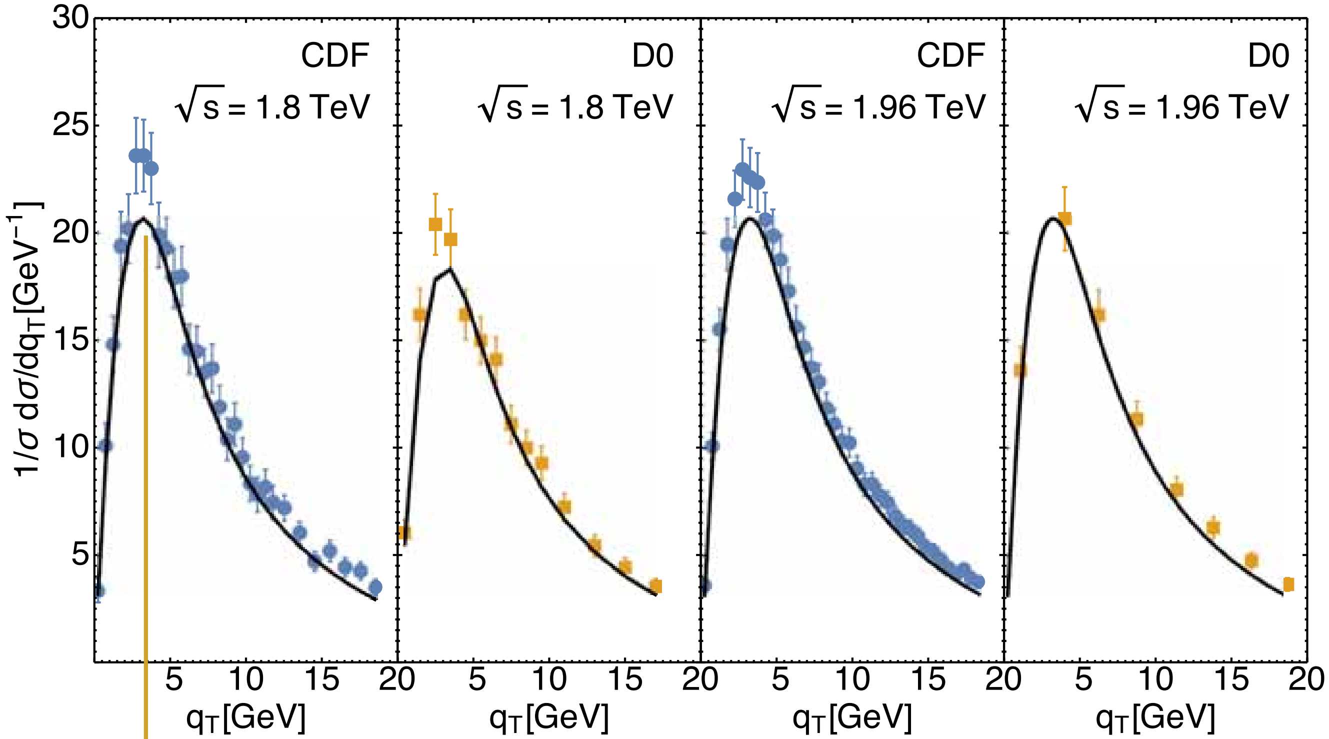
Drell-Yan data



χ^2 / dof 0.32 0.84 0.99 1.12

the peak was at 0.4 GeV and now is at 1 GeV

Z-boson data



χ^2 / dof 1.4 1.1 2.0 1.7

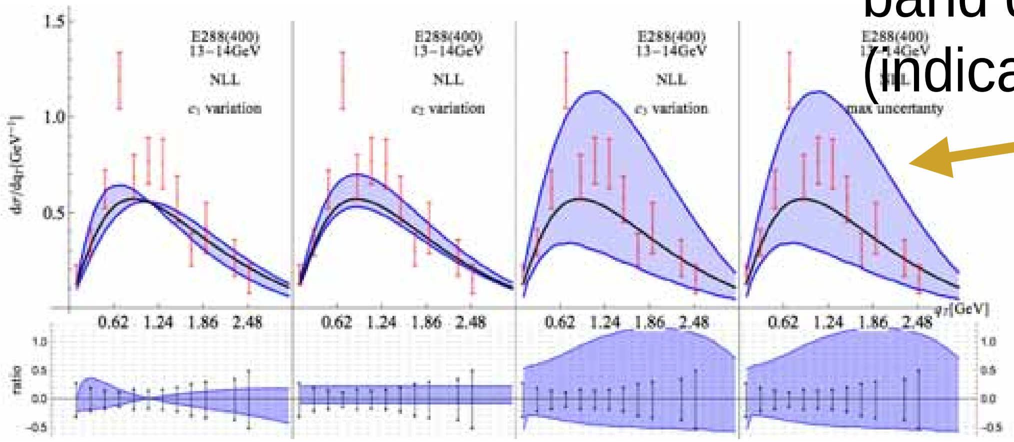
the peak now is at 4 GeV

Most of the χ^2 due to normalization, not to shape

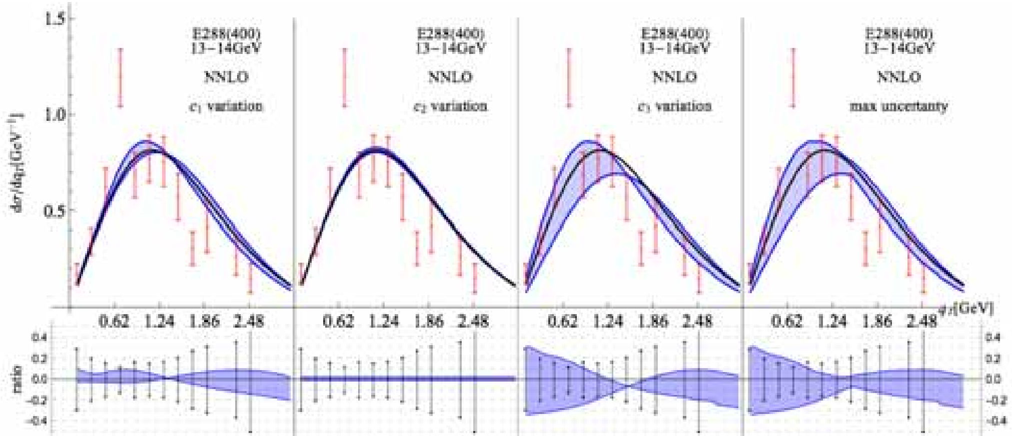
SV2017 and perturbative accuracy

band due to scale variation
(indication of theory accuracy)

NLL / LO



NNLL / NNLO



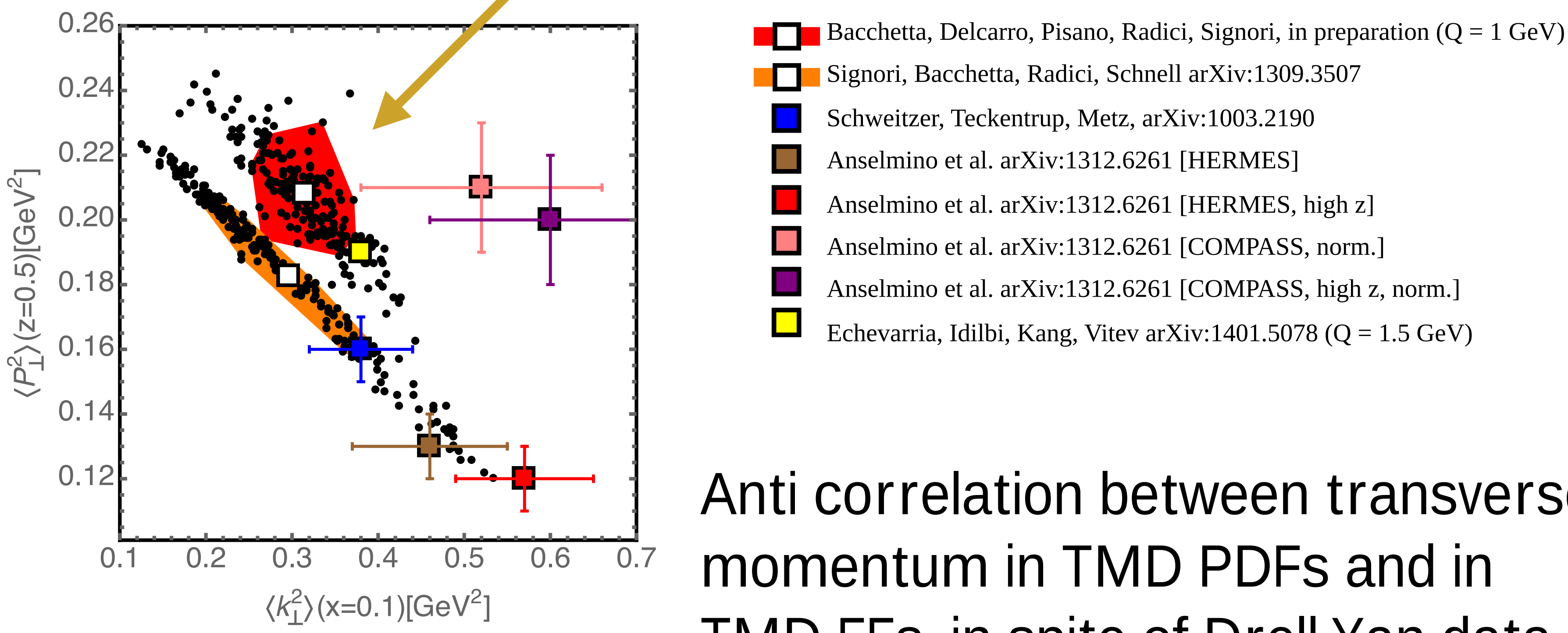
Huge improvement for low-energy experiments

SV 2017

Mean transverse momentum squared

Pavia2016 results, Q^2=1 GeV^2

Transverse momentum in FFs



Transverse momentum in PDFs

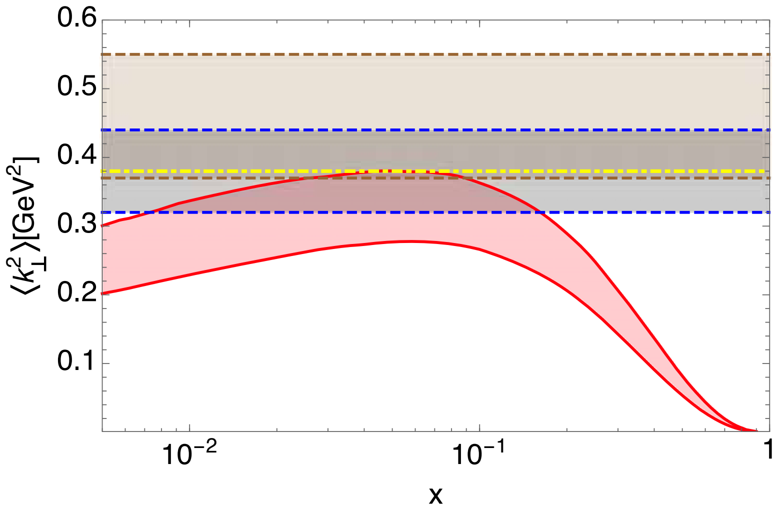
Anti correlation between transverse momentum in TMD PDFs and in TMD FFs, in spite of Drell-Yan data

CAVEAT: intrinsic transverse momentum depends on TMD evolution "scheme" and its parameters

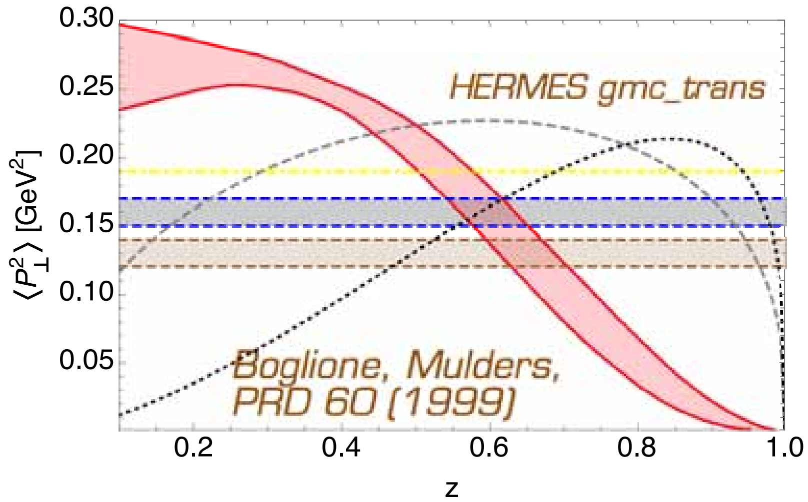
Mean transverse momentum squared

same color coding as previous slide

at Q =1 GeV

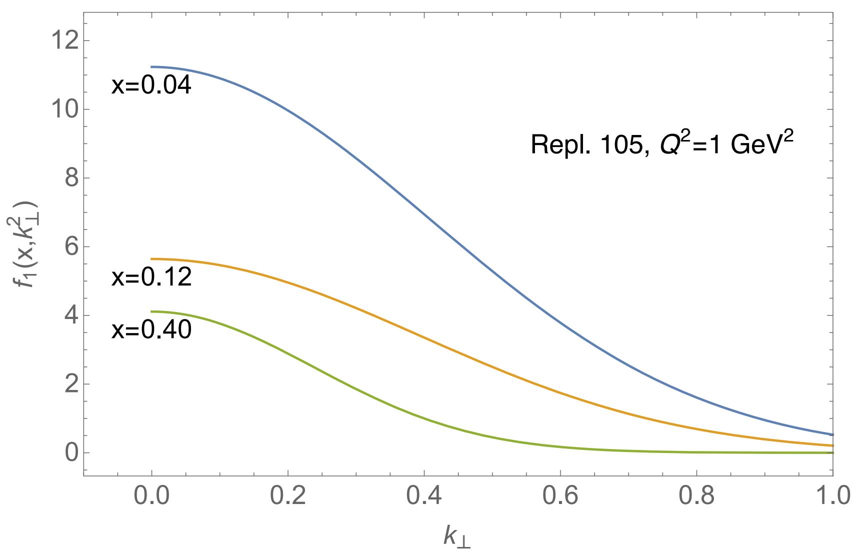


In TMD distribution functions

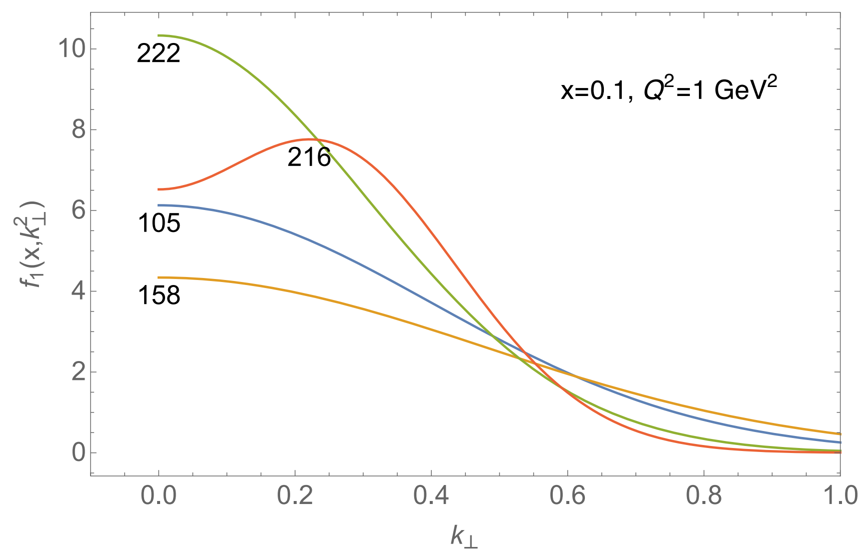


In TMD fragmentation functions

Do we know the shape?



x-dependence of a single replica.
Most of them are similar.



Shape of four selected replicas.
Still huge uncertainties.

Nonperturbative evolution parameters

TMD evolution is not uniquely determined by pQCD calculations. Nonperturbative input is needed to determine evolution precisely. Different schemes may behave differently.

	g_2 (GeV^2)	b_{max}(GeV^{-1})
BLNY 2003	0.68 ± 0.02	0.5
KN 2006	0.184 ± 0.018	1.5
EIKV 2014	0.18	1.5
Pavia 2016	0.12 ± 0.01	1.123
SV 2017	0.006 ± 0.006	1

Faster evolution: transverse momentum increases faster due to gluon radiation

Slower evolution: the effect of gluon radiation is weaker

Unpolarized TMDs open issues

- Different choices in implementation of TMD evolution: is there a better one?
- Limits of applicability of TMD factorization
- General problems with normalizations theory / experiment
- Flavor dependence and more flexible functional forms
- Matching with high-transverse momentum calculation with collinear PDFs
- More data needed to test formalism, particularly in the EIC region
- Improvements in the knowledge of fragmentation functions essential