Extraction of structure functions and TMDs from azimuthal asymmetries in SIDIS

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Spatial and Momentum Tomography of Hadrons and Nuclei
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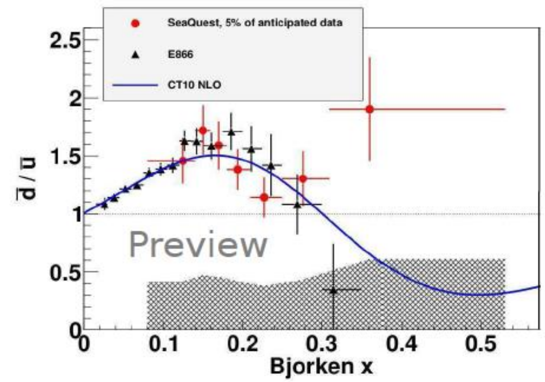
Outline

- Introduction
- Experimental factors affecting extraction of SFs
    - The experiment
    - Efficiency and acceptance
    - Radiative Corrections
- Data output for 3D PDF (TMD, GPD) studies
- Testing procedure using MC
- Extraction and Validation Framework (EVA) for 3D PDFs
- Summary

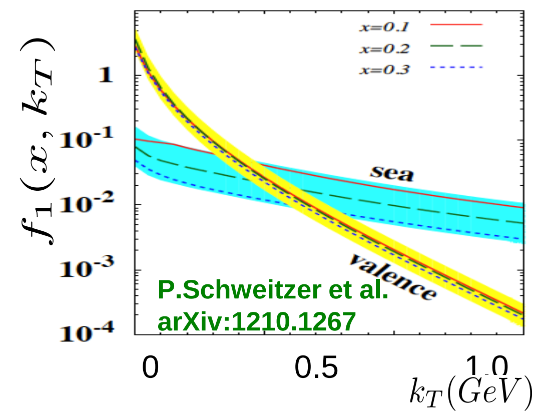
Features of partonic 3D non-perturbative distributions

Non-perturbative sea in nucleon is a key to understand the nucleon structure

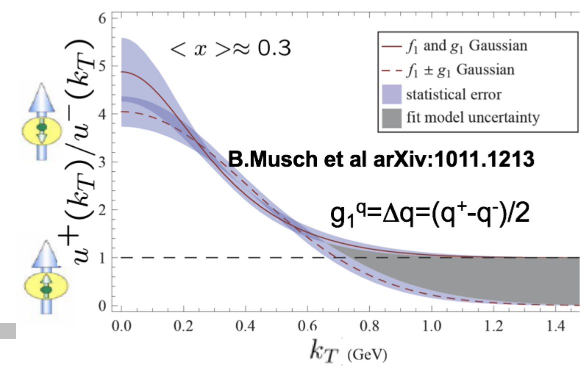
-- Large flavor asymmetry as evidence $\bar{d} > \bar{u}$



- Predictions from dynamical model of chiral symmetry breaking [Schweitzer, Strikman, Weiss JHEP 1301 (2013) 163]
  -- k_T (sea) >> k_T (valence)
- d-quarks may be wider (lattice)
- anti-alligned with proton spin quarks may be wider



- spin and momentum of struck quarks may be correlated with remnant
- correlations of spins of q-q-bar with valence quark spin and transverse momentum should lead to observable effects



SIDIS x-section

SIDIS $\ell(l) + N(P,S) \to \ell(l) + h(P_h) + X$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+\,S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

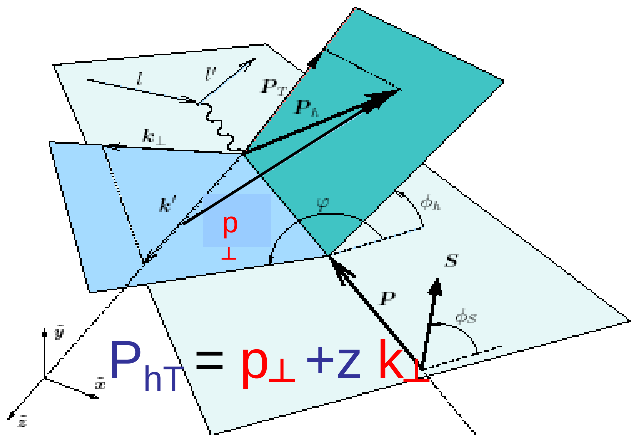
$$+\,|S_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\,\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$

$$+\,|S_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\,\sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},$$



$$F_{UU,T} = x\sum_q e_q^2 \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^{(2)}(zk_\perp + p_\perp - P_{hT})\,f^q(x,k_\perp)\,D^{q\to h}(z,p_\perp)$$

or

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2;\mu^2)\int d\boldsymbol{k}_\perp\,d\boldsymbol{P}_\perp\,f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\,D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

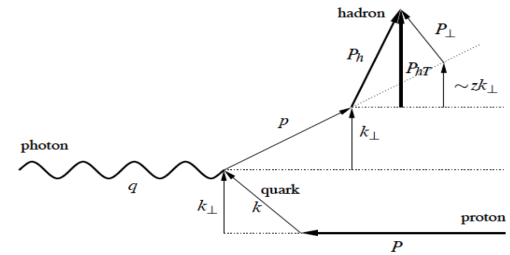
$$+\,Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M/Q)\,.$$

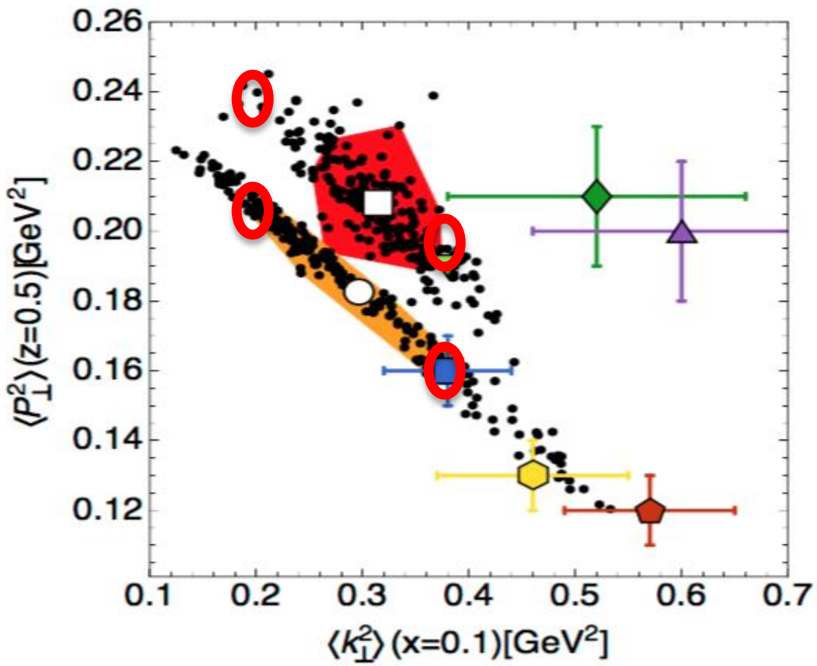
or any representation of structure functions!!!

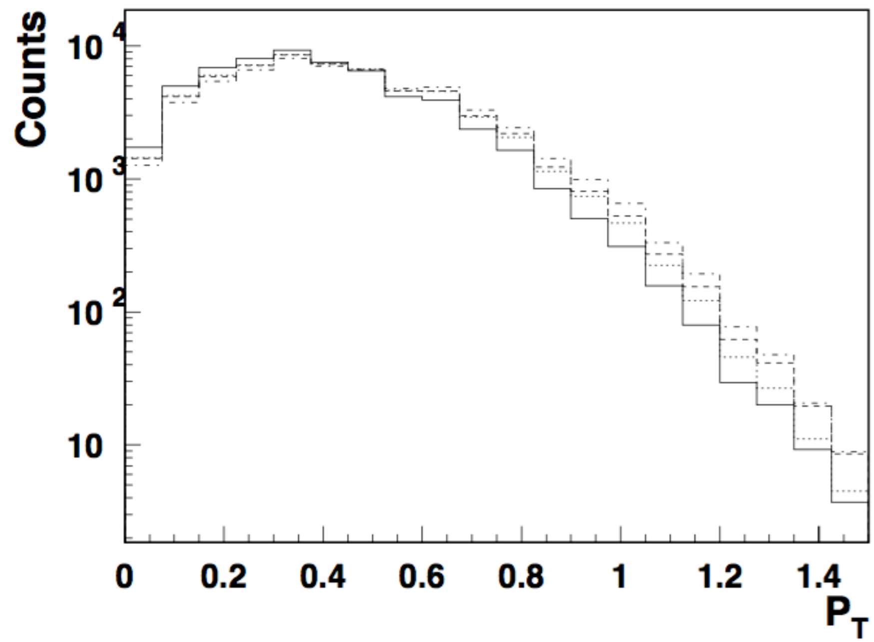
Extracting the average transverse momenta

Andrea Signori,[1,*] Alessandro Bacchetta,[2,3,†] Marco Radici,[3,‡] and Gunar Schnell[4,5,§]

$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2)\int dk_\perp\, dP_\perp\, f_1^a(x,k_\perp^2;\mu^2)\, D_1^{a\to h}(z,P_\perp^2;\mu^2)\,\delta(zk_\perp - P_{hT} + P_\perp) + Y_{UU,T}(Q^2,P_{hT}^2) + \mathcal{O}(M/Q).$$







$$m_N^h(x,z,P_{hT}^2) = \frac{\pi}{\sum_a e_a^2 f_1^a(x)} \times \sum_a e_a^2\, f_1^a(x)\, D_1^{a\to h}(z)\, \frac{e^{-\boldsymbol{P}_{hT}^2/\left(z^2\langle \boldsymbol{k}_{\perp,a}^2\rangle + \langle \boldsymbol{P}_{\perp,a\to h}^2\rangle\right)}}{\pi\left(z^2\langle \boldsymbol{k}_{\perp,a}^2\rangle + \langle \boldsymbol{P}_{\perp,a\to h}^2\rangle\right)}$$

- Extraction very sensitive to input (replicas)
- Most sensitive to parameters is the large P_T region
- Multiplicity alone may not be enough to separate $\langle k_T\rangle$ from average $\langle p_T\rangle$

Experiment-Theory interaction

Theorist predicts effects and observables sensitive to them

Experimentalists submit a proposal and make measurement (~3-4 years)

Observables in form of a table bin# <average kin> | observable | <err>stat | <err>syst

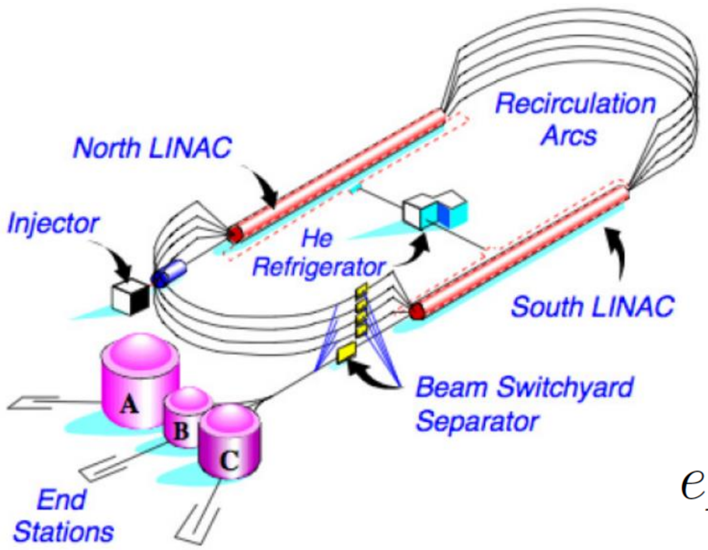
Experiment measures "unexpected" effects

Theorist come up with possible interpretation

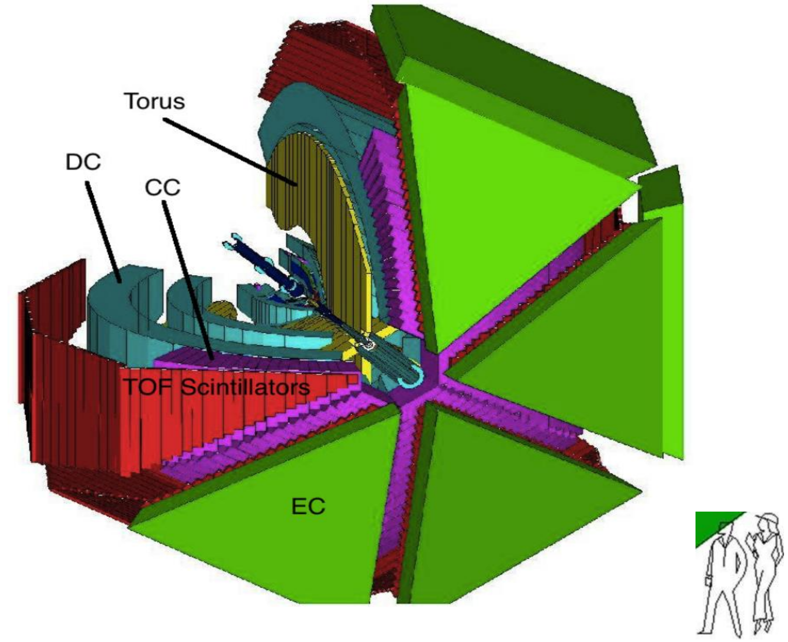
What will be the most efficient format for the data (and metadata)?

- Data required for certain analysis may require event by even info
- How to store and preserve the data (for unbined analysis)
- Alternative to store full events (all tracks
  - Should provide easy access for theory)

CLAS: e1f data set



$$ep \to e\pi^{\pm}X$$



- Two 0.4 GeV linear accelerators.
- Nine recirculation arcs for five loops around the track.
- Continuous, polarized electron beam up to 6 GeV delivered simultaneously to 3 experimental halls.
- High luminosity of 0.5 x 10^{34} $(\text{cm}^2\ \text{s})^{-1}$

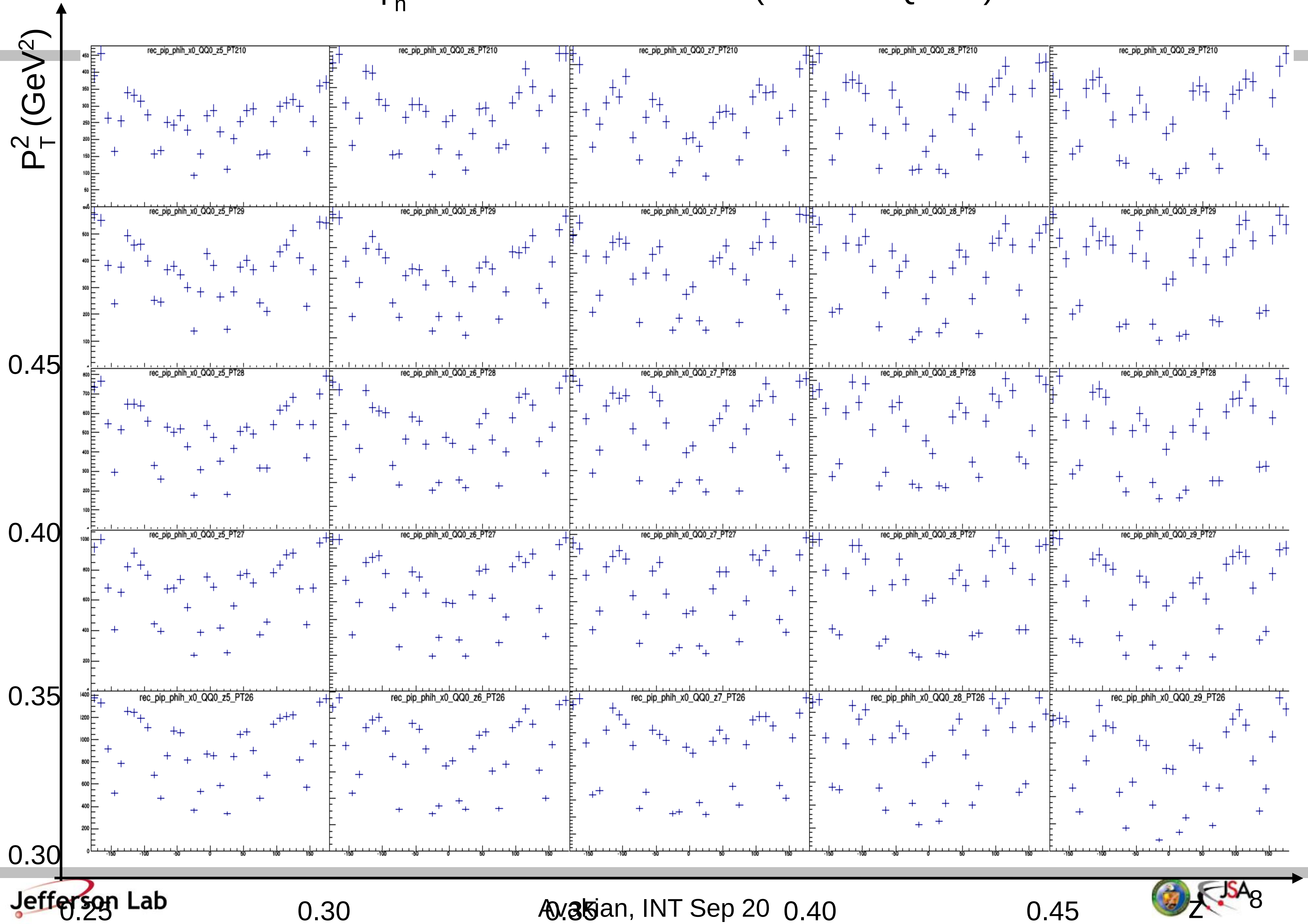
- E1-f run: 5.498 GeV electron beam with ~75% polarization (averaged over for this analysis); unpolarized liquid hydrogen target; about 2 billion events; broad and comparable kinematic range for two channels:

- Electromagnetic Calorimeter (EC) and Čerenkov Counter (CC) used in electron identification.

- Drift Chamber (DC) (3 regions) and time of flight Scintillators (SC) record position and timing information for each charged track.

- Torus magnet creates toroidal magnetic field which causes charged tracks to curve while preserving the φ_{lab} angle.

φ_h distributions - raw data (lowest x-Q^2 bin)



Analysis of azimuthal moments in SIDIS/HEP

Data (contains N events with 4 vectors of reconstructed particles, N~1B)

MC +RC (contains M events with 4 vectors of generated and reconstructed particles, M~10-100N)

Define x-sections/normalized counts

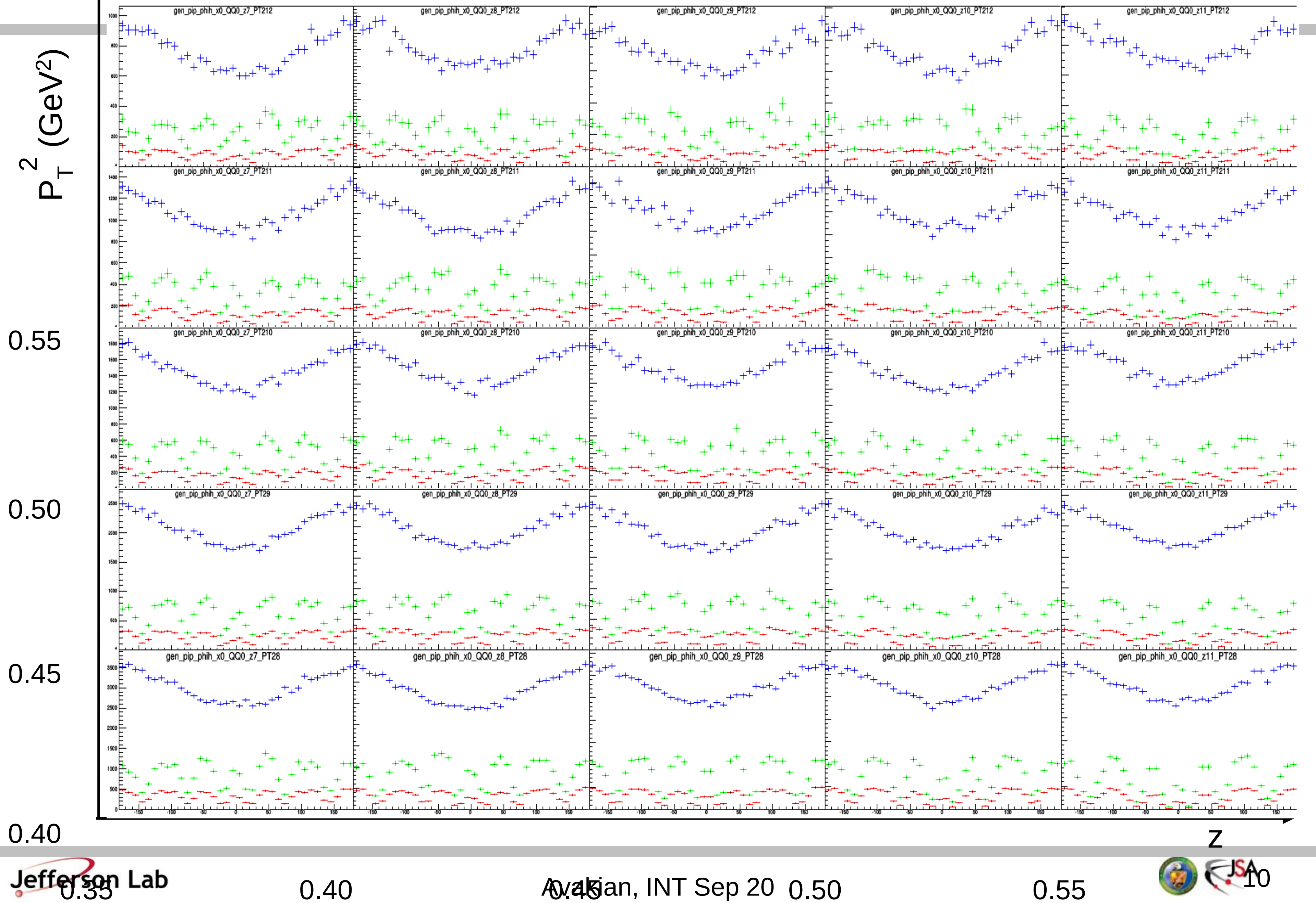
Compare generated with reconstructed

Acceptance in "small" bins (counts in λ,Λ,x,y,[z,P_T][t], ϕ) defining reconstruction efficiency and material on path of leptons

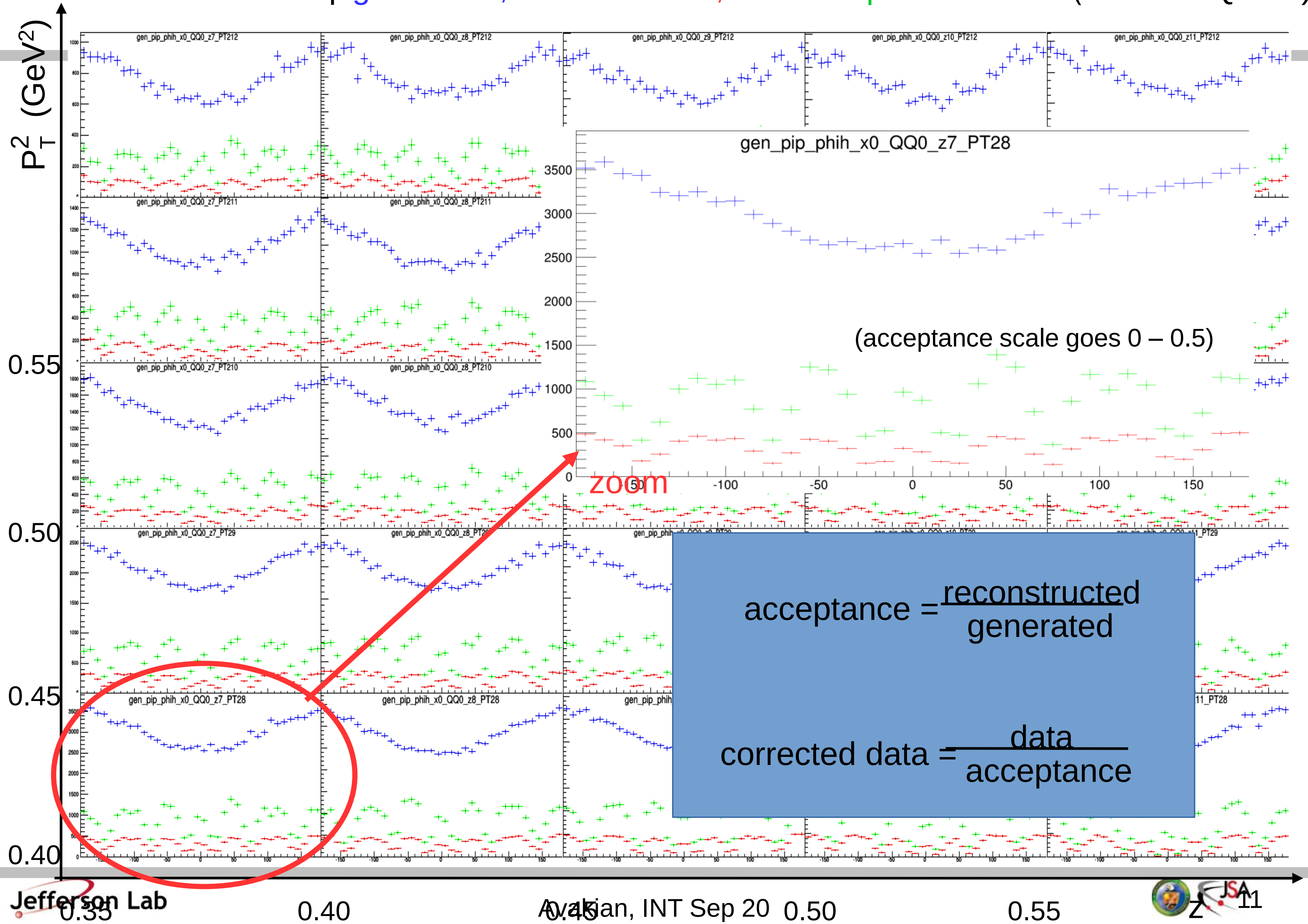
Counts in "small" bins in λ,Λ,x,y, [z,P_T][t],ϕ,RC corrected for detector acceptance and efficiency

- Counts in a given bin corrected by rec.efficiency and radiative effects
- Size of the bins dictated by the statistics allowing fits for extraction of azimuthal moments

Monte Carlo φ generated, reconstructed, and acceptance for π+ (lowest x-Q^2 bin)



Monte Carlo φ generated, reconstructed, and acceptance for π+ (lowest x-Q^2 bin)

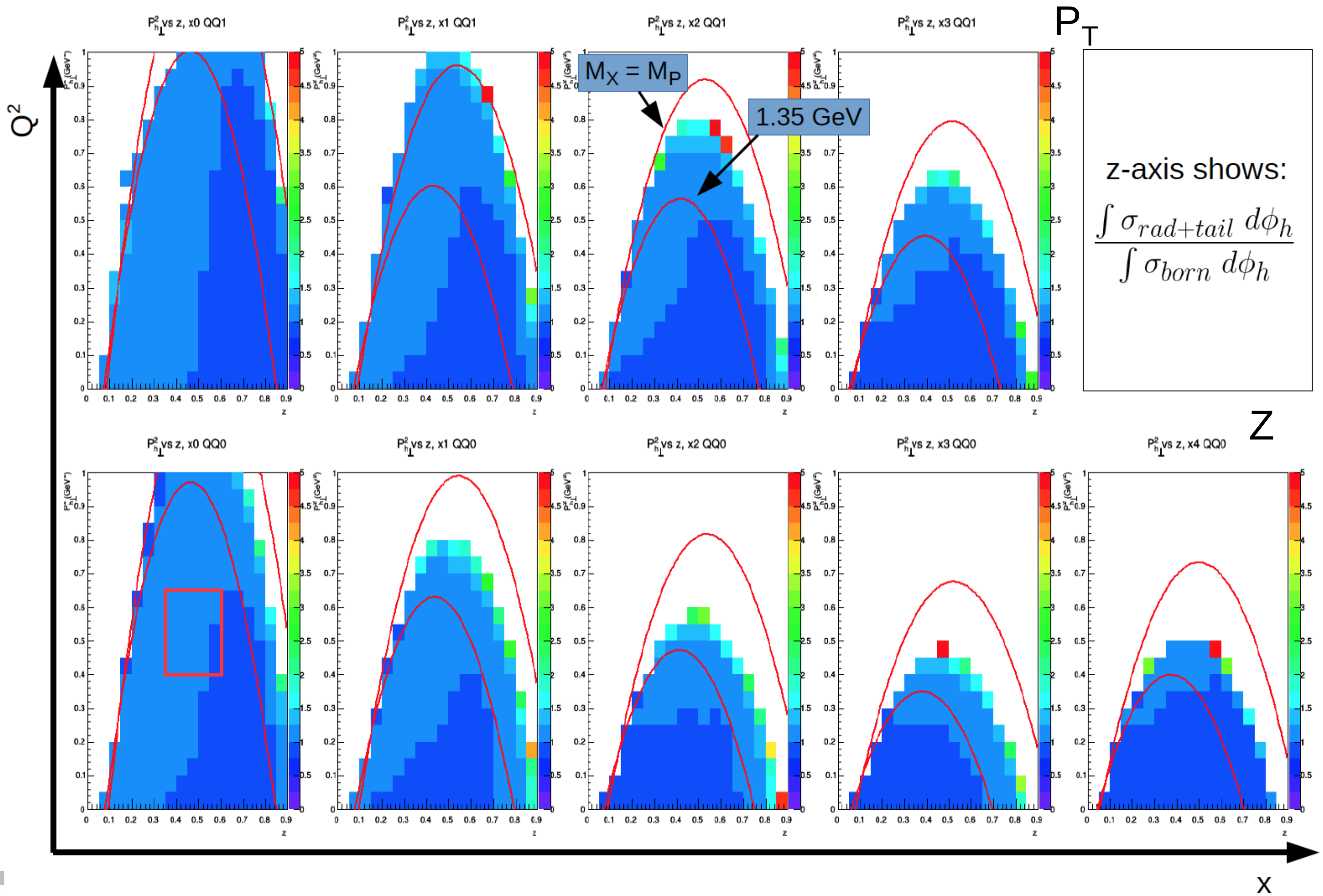


$$\text{acceptance} = \frac{\text{reconstructed}}{\text{generated}}$$

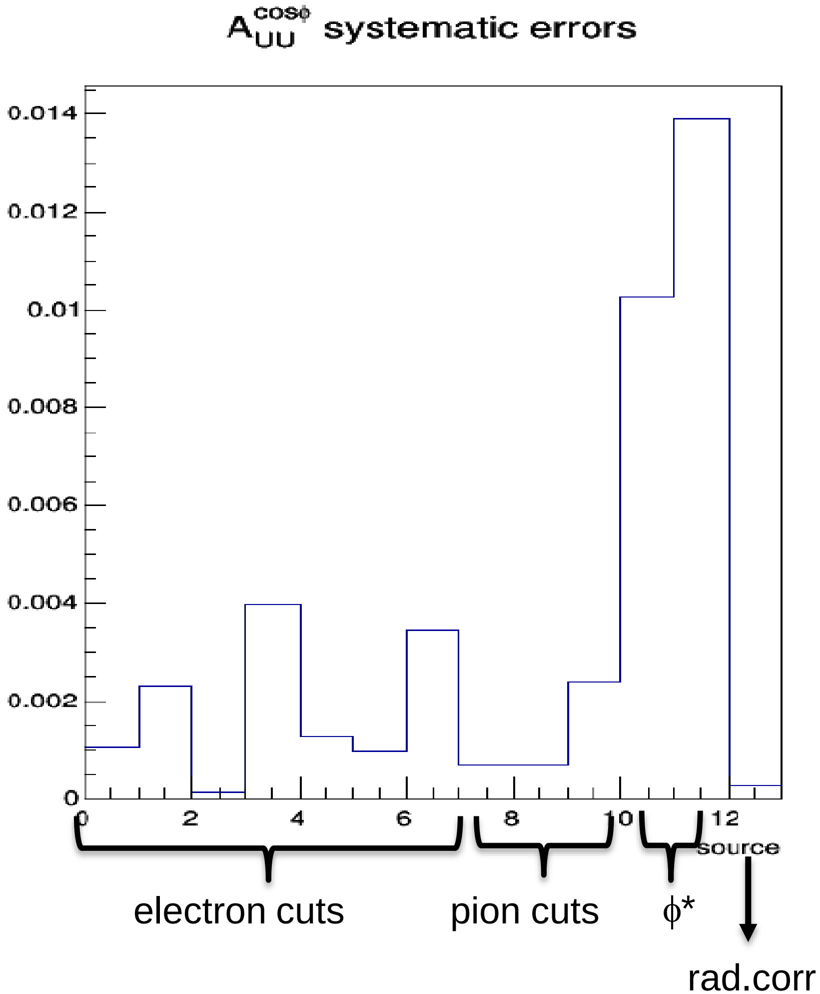
$$\text{corrected data} = \frac{\text{data}}{\text{acceptance}}$$

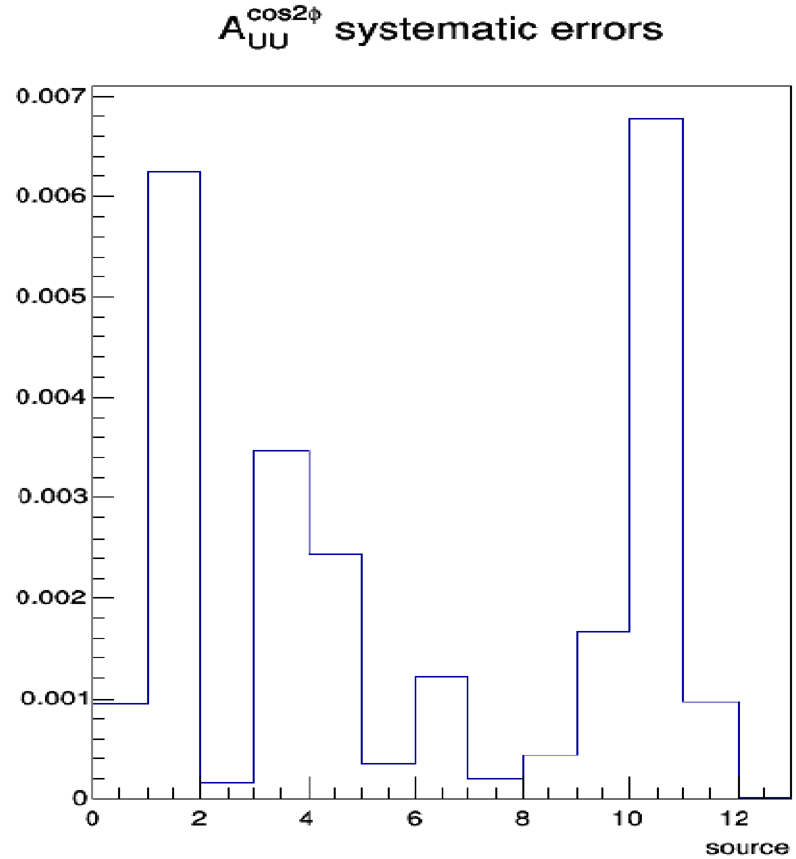
Additional complications: Experiment has limited energy

In $F_{XY}^{h}(x,y,z,P_T,\phi)$ variables independent, while in real life even for 100% acceptance they are limited



Systematics





Systematics from different factors considered uncorrelated

Radiative Corrections

- Radiative effects, such as the emission of a photon by the incoming or outgoing electron, can change all five SIDIS kinematic variables.

- Furthermore, exclusive events can enter into the SIDIS sample because of radiative effects ("exclusive tail").

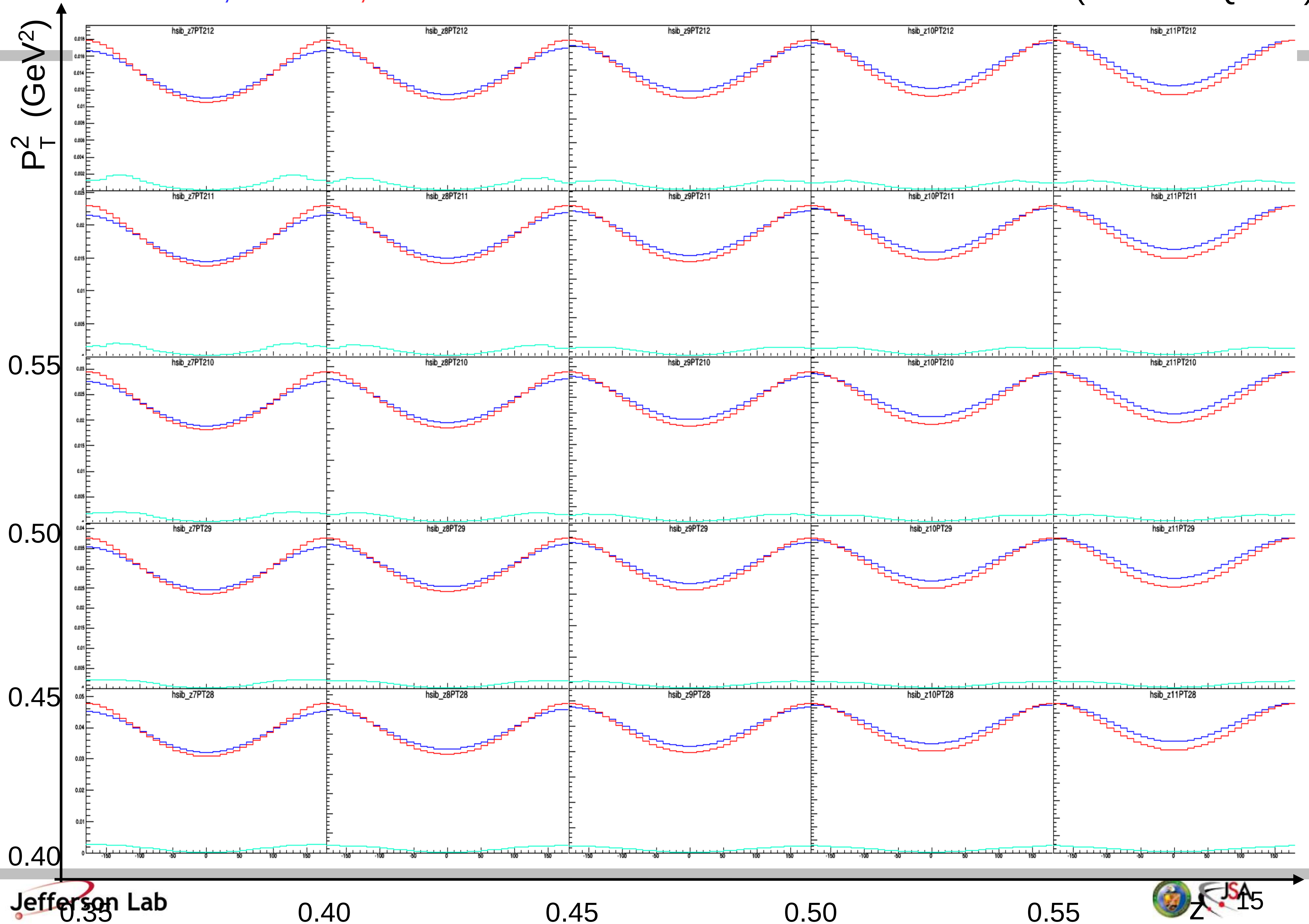
- HAPRAD 2.0 is used to do radiative corrections.

- For a given $\sigma_{Born}(x, Q^2, z, P_{h\perp}^2, \phi_h)$ (obtained from a model), HAPRAD calculates $\sigma_{rad+tail}\left(x, Q^2, z, P_{h\perp}^2, \phi_h\right)$. The correction factor is then:

$$RC\ factor = \frac{\sigma_{rad+tail}\left(x, Q^2, z, P_{h\perp}^2, \phi_h\right)}{\sigma_{Born}\left(x, Q^2, z, P_{h\perp}^2, \phi_h\right)}$$

- 3 different models were used to study model dependence.

Born, radiated, and exclusive tail cross-sections from HAPRAD (lowest x-Q^2 bin)



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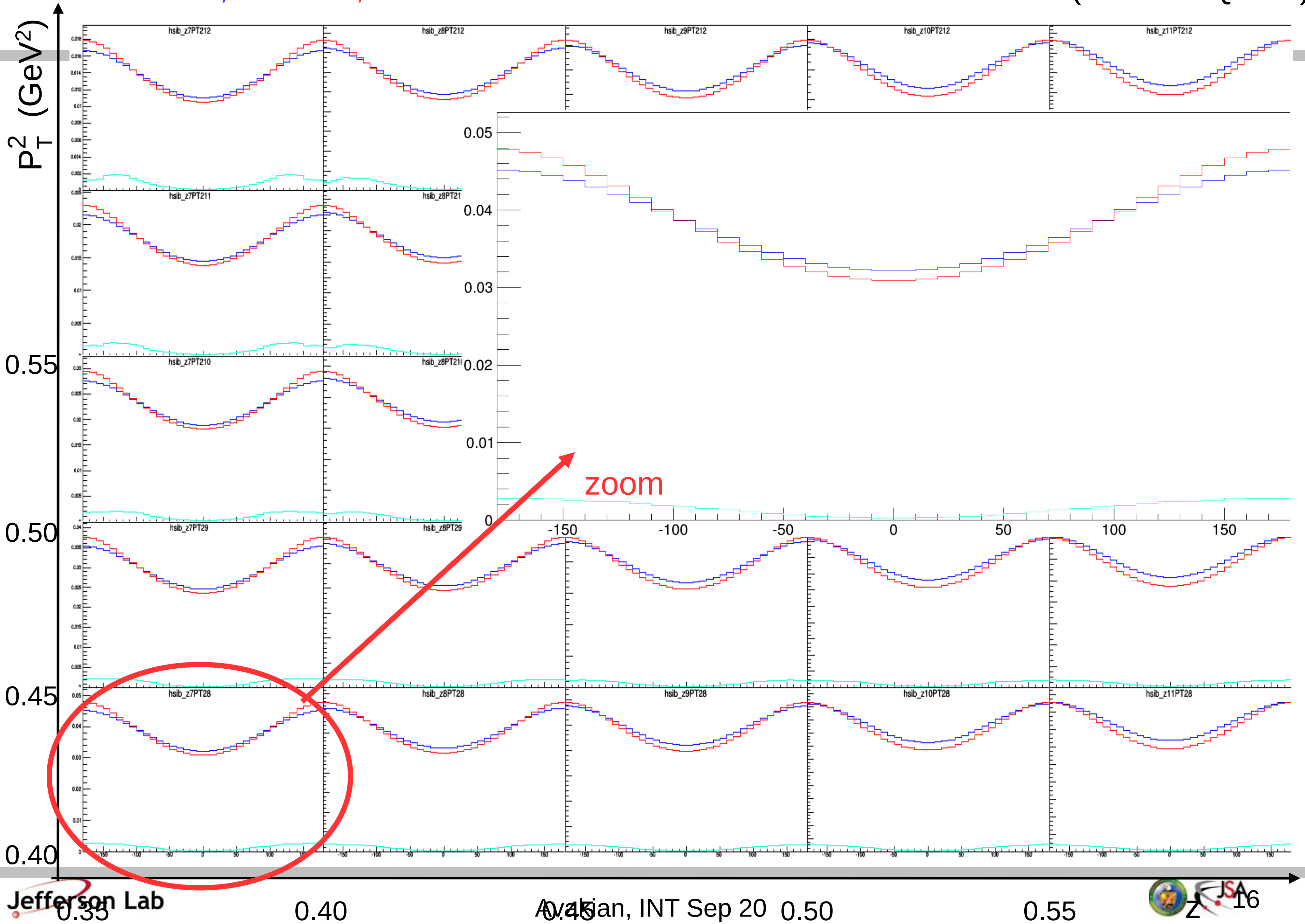
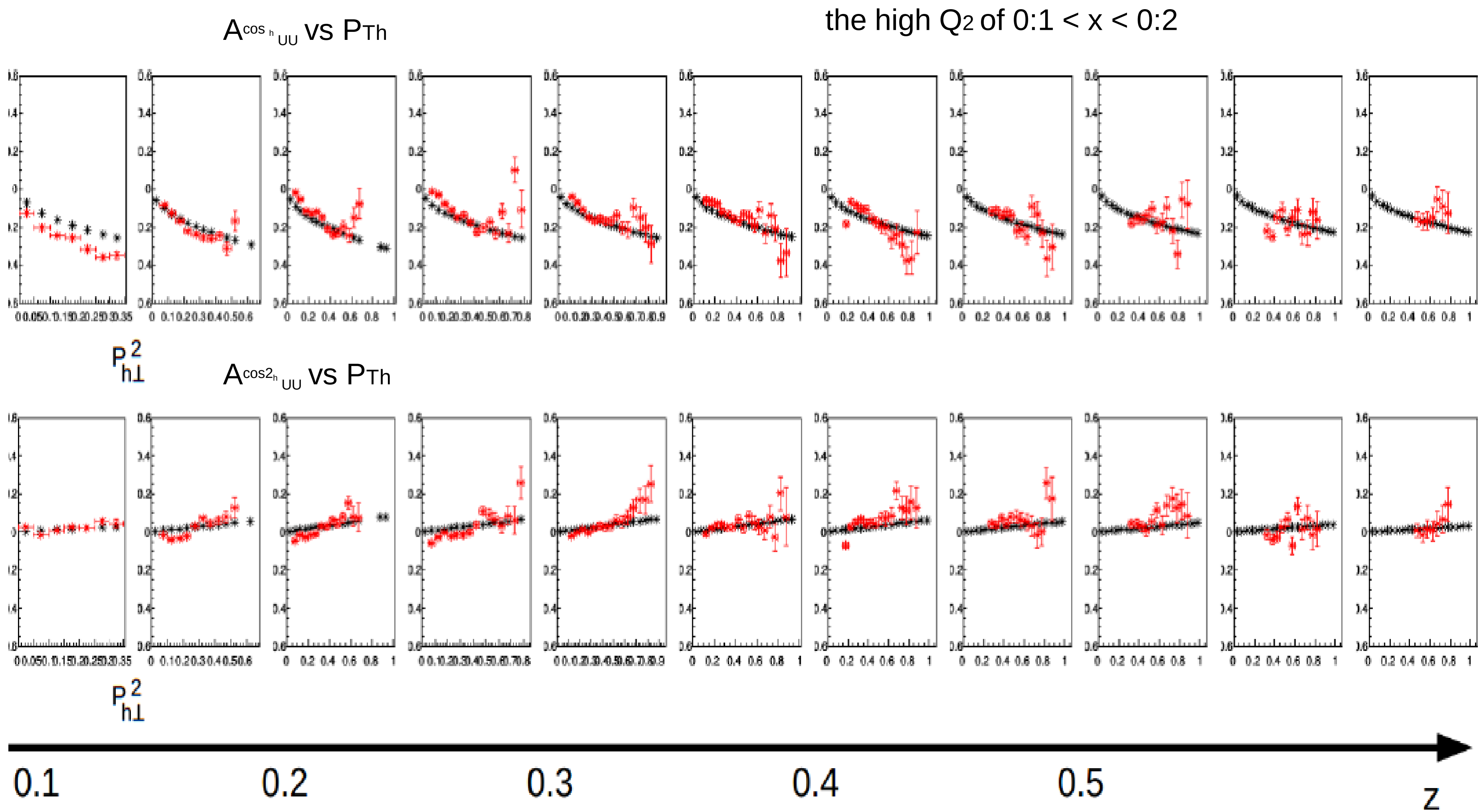


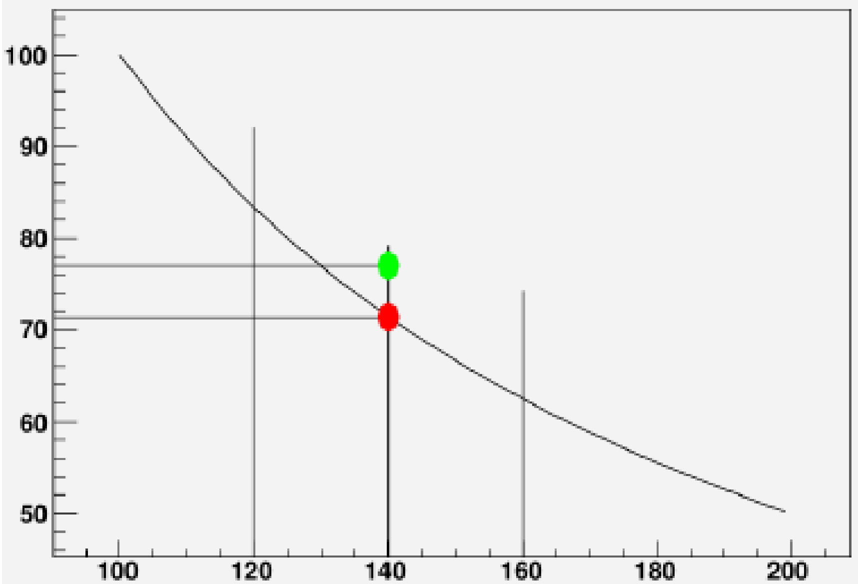
Radiative Corrections



Model for azimuthal moments after few iterations, roughly consistent with the input.

Bin Centering Corrections

$$BCC\ factor = \frac{v}{V}\frac{\sigma_{averaged}}{\sigma_{center}}$$



v - 5-dimensional “volume" of the micro bin at the center of the normal bin

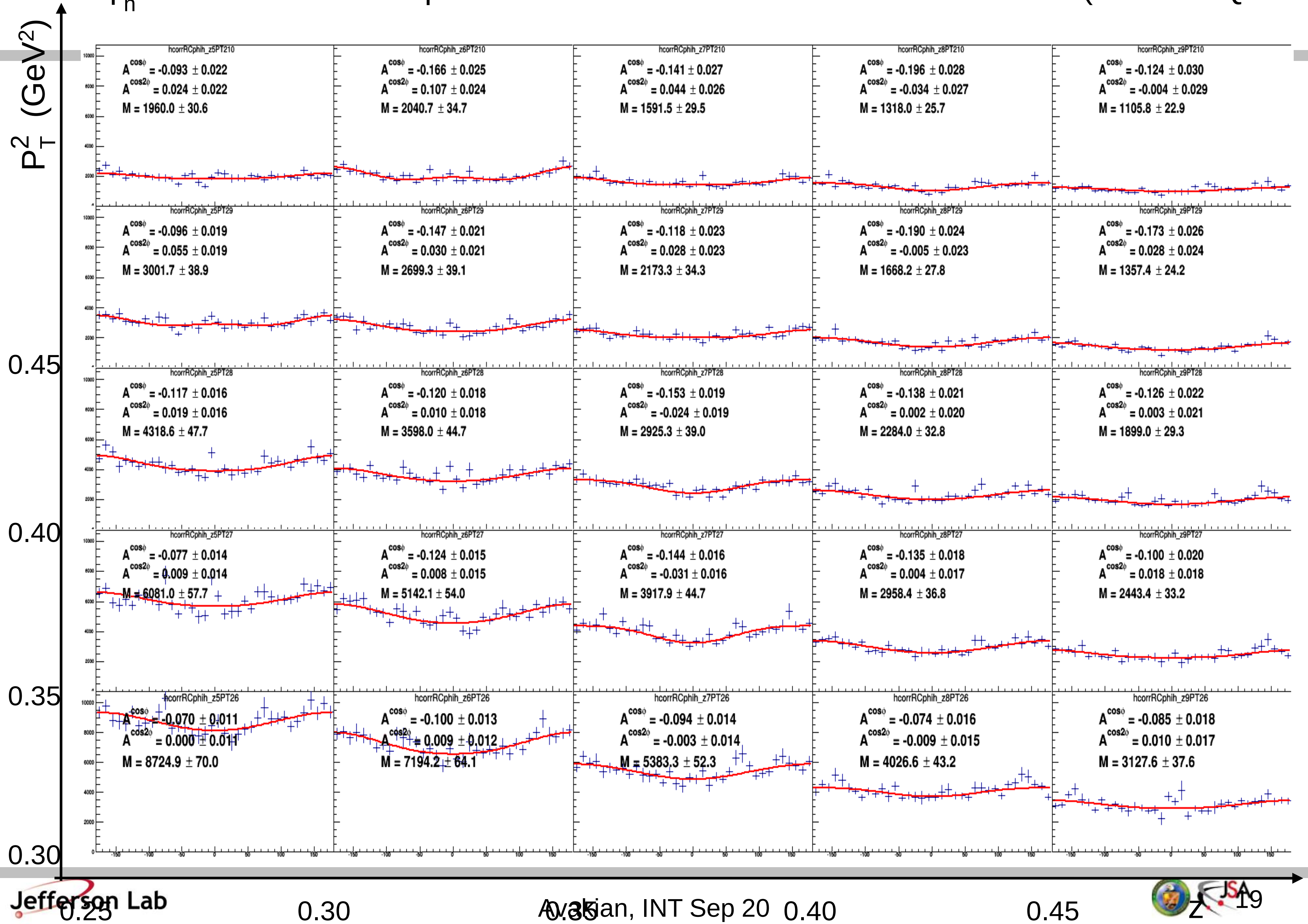
V - the “volume” of the normal bin,

$\sigma_{averaged}$ - cross-section averaged over the “normal bin”

σ_{center} - cross-section at the micro-bin at the center of the normal bin

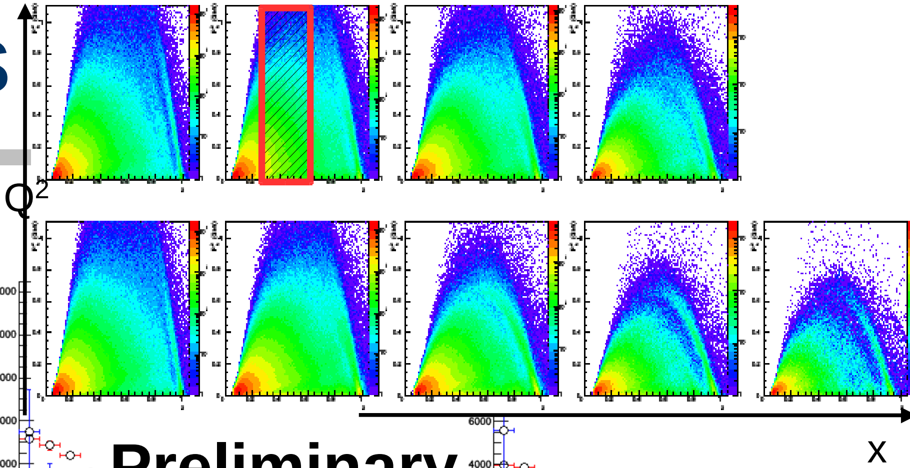
Bin centering corrections are approximated using a model based on the results of the measurement. Using the model, the cross-section is calculated In “micro-bins" (bins much smaller than the “normal bins” used for the final analysis.

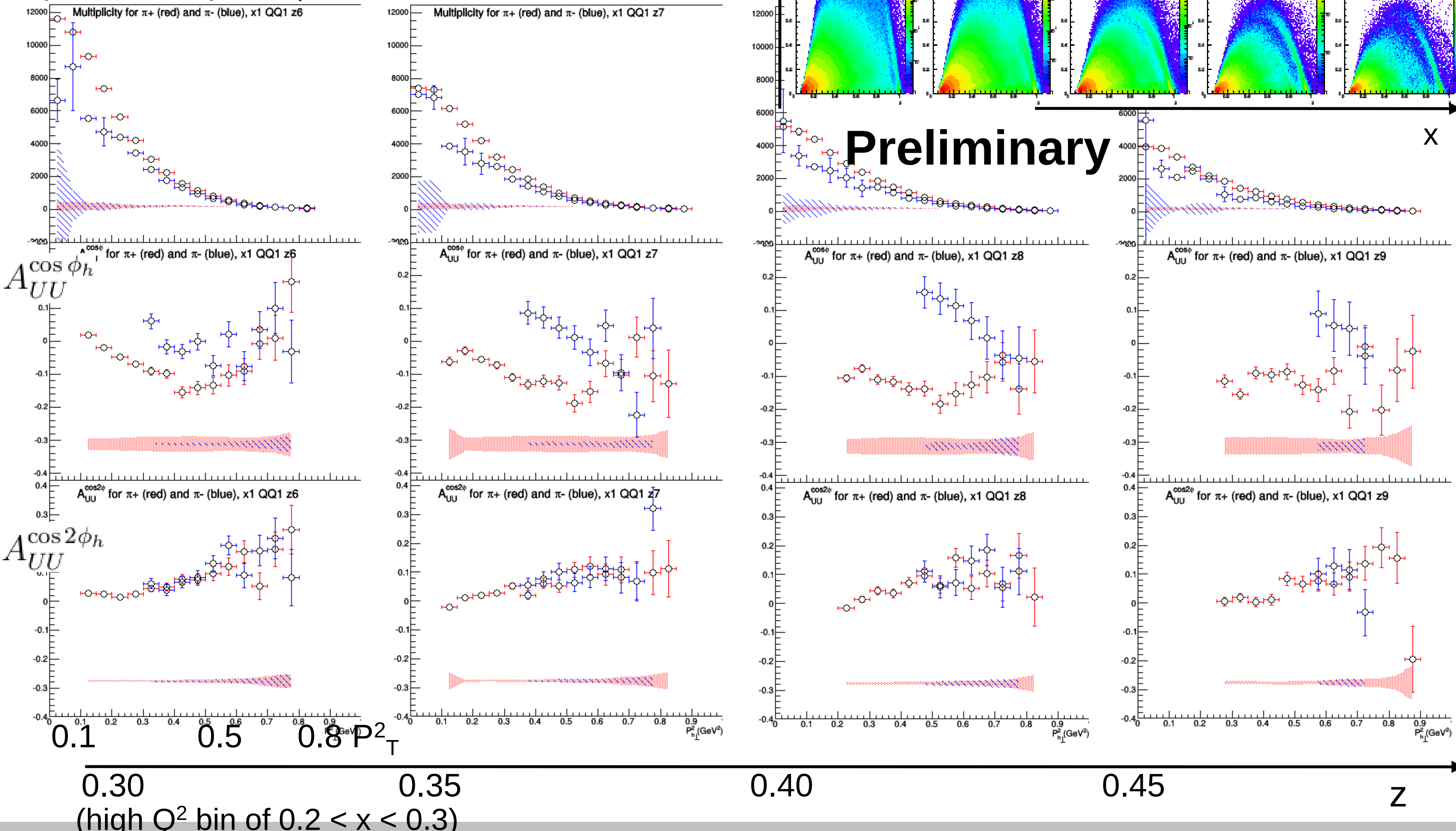
φ_h distributions – acceptance and radiative corrected with fit results (lowest x-Q^2 bin



Representative Results

A_0 (top row), $A_{UU}^{\cos\phi_h}$ (middle row), and $A_{UU}^{\cos 2\phi_h}$ (bottom row) vs P_T^2 for π+ and π-





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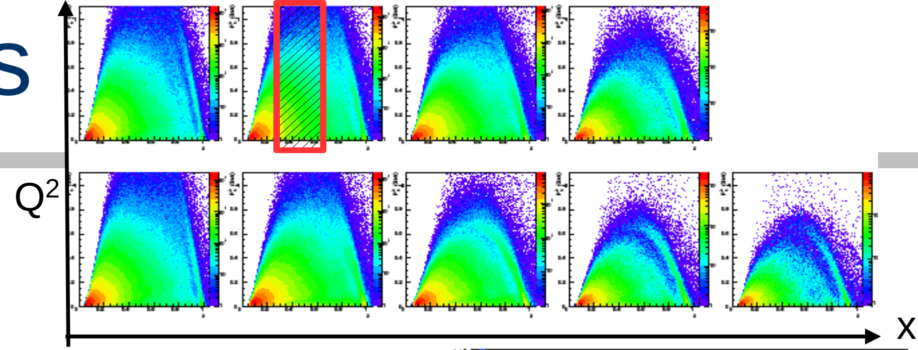
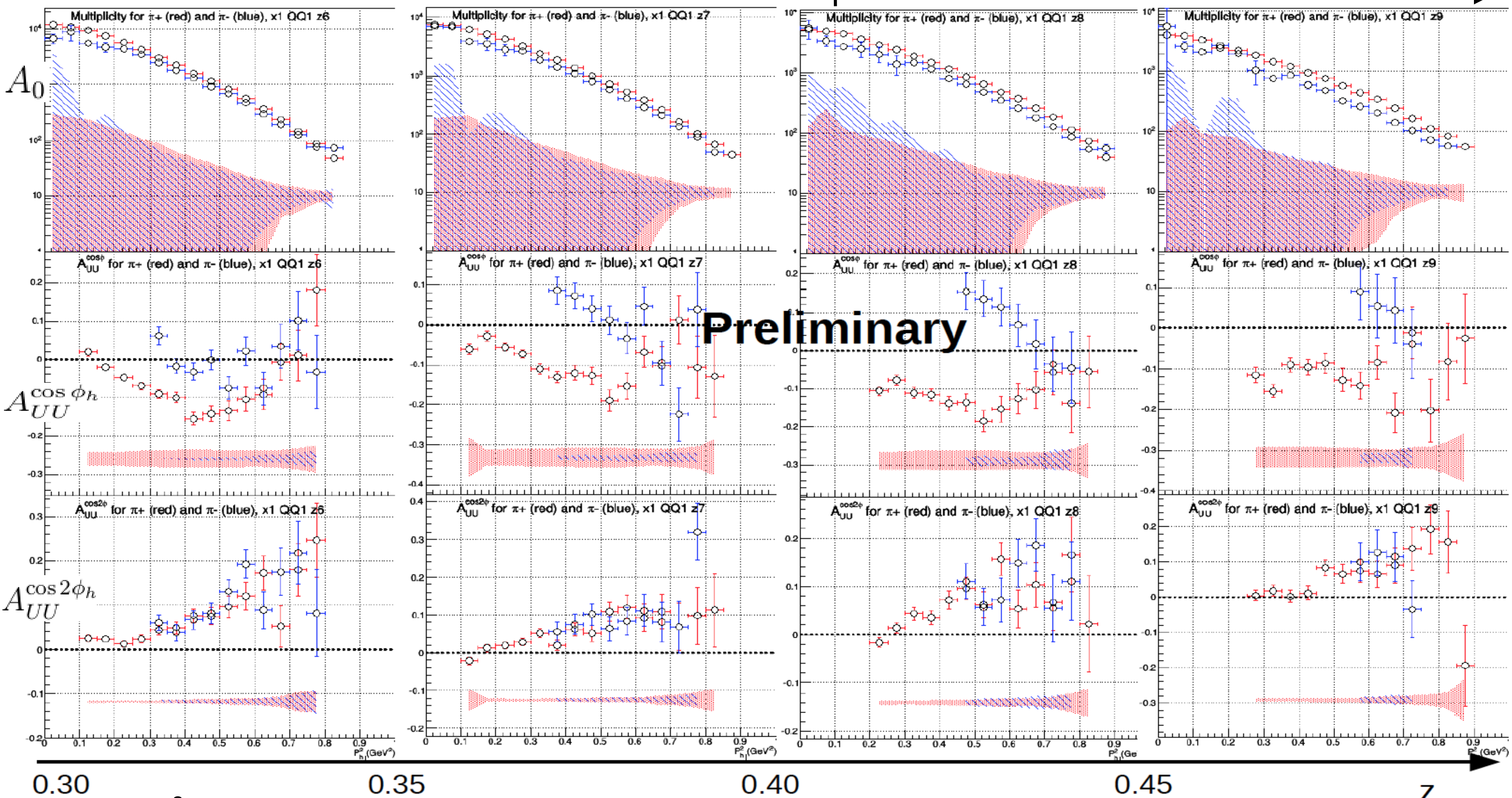




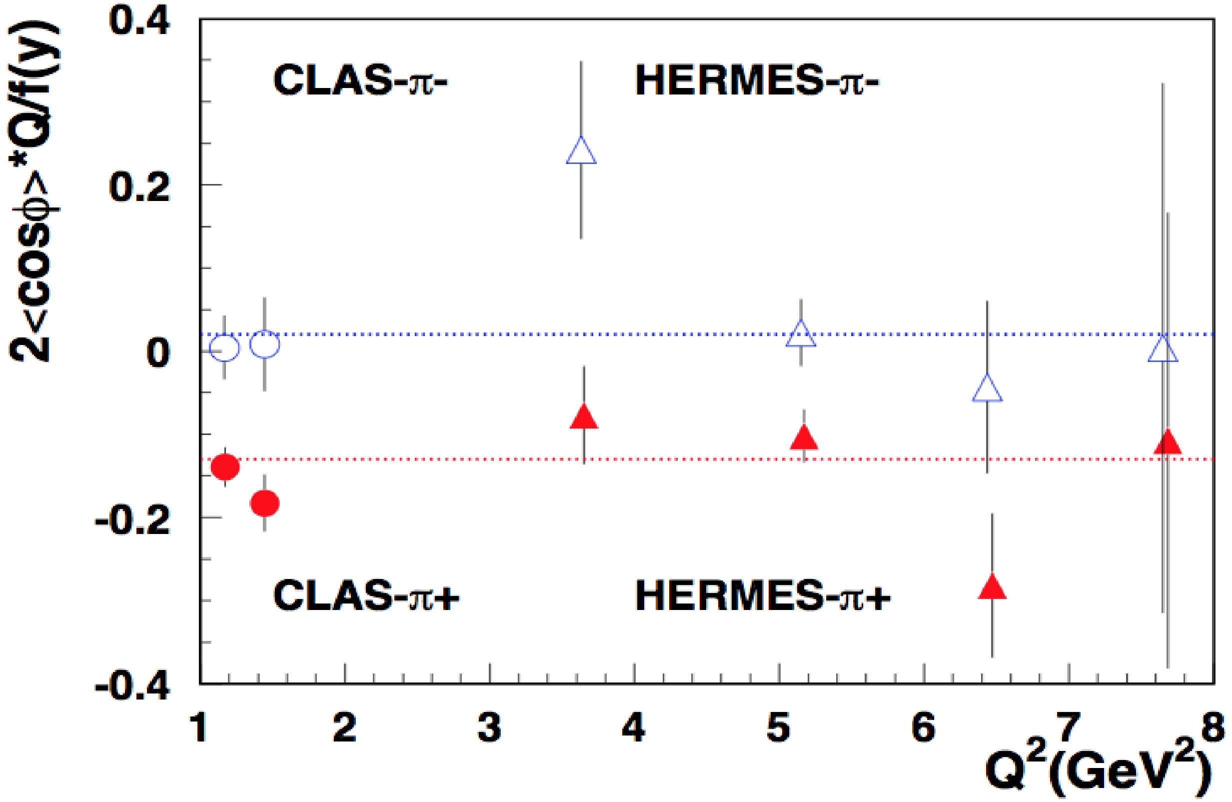
(high Q^2 bin of $0.2 < x < 0.3$)

Comparing with HERMES

$$F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\ \cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

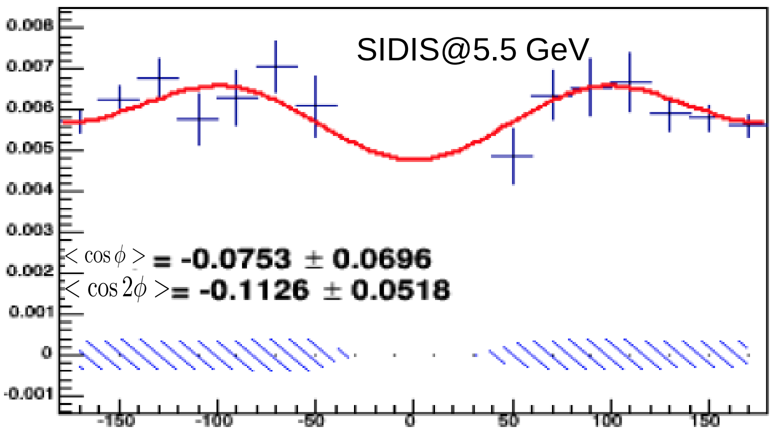
$x=0.19, z=0.45, P_T=0.42$ GeV

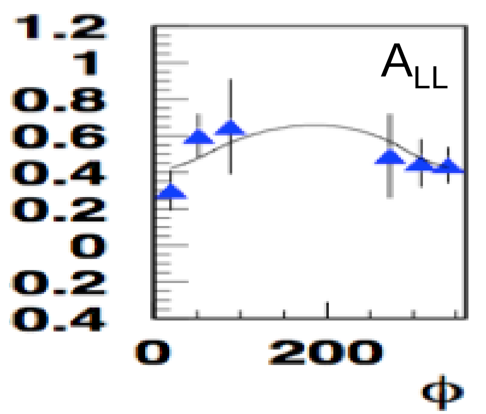


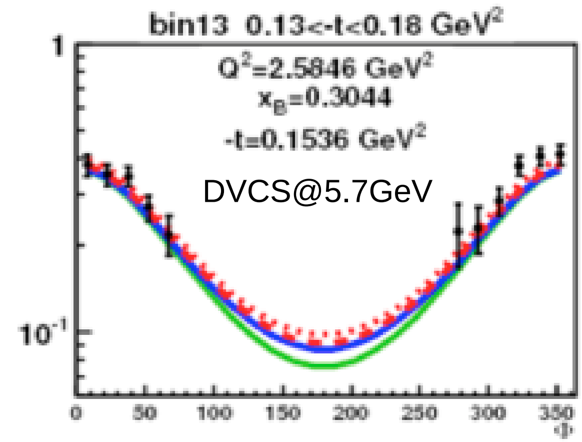
CLAS data consistent with HERMES (27.5 GeV)

Additional complications: Experiment has limited acceptance

Limited kinematical coverage (acceptance) in particular at acceptance edges, large Q^2 and P_T





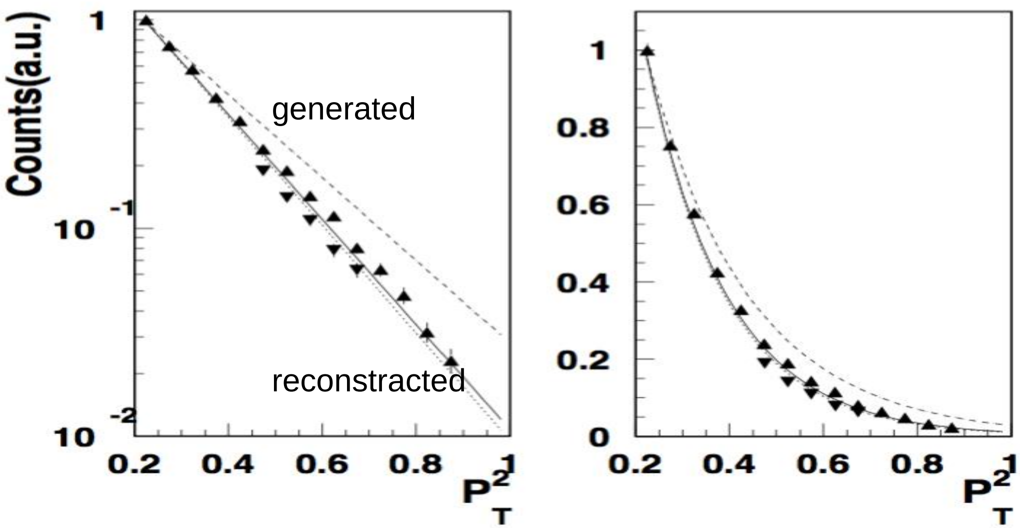


Ignoring other variables (ϕ-in particular) doesn't mean integrating over them

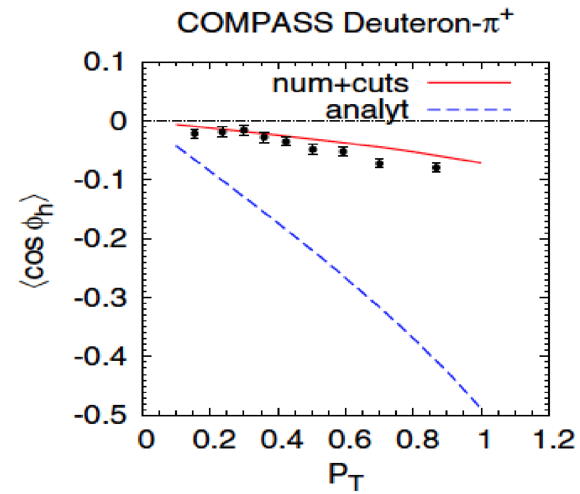
Experiment measures ϕ – counts involving also HT contributions !!!

Additional complications: limited phase space

M. Boglione, S. Melis & A. Prokudin
Phys. Rev. D 84, 034033 2011

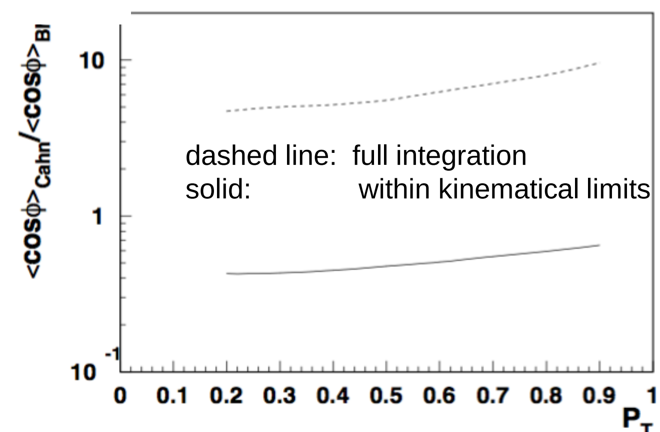


EVA tests: Cahn vs BM



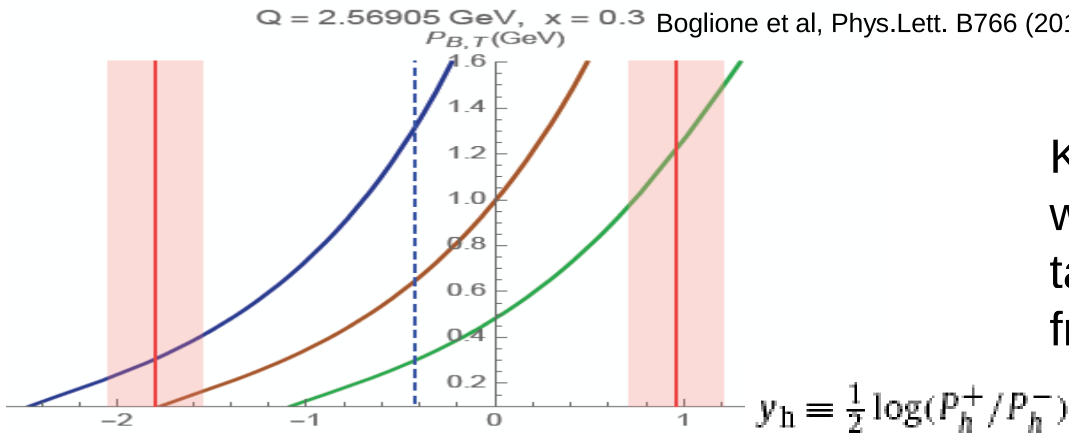
$$C[w, fD] = x \sum_a e_a^2 \int_0^{k_{\perp max}} k_\perp dk_\perp \int_0^{2\pi} d\phi \, w(\boldsymbol{k}_\perp, \boldsymbol{p}_\perp(\boldsymbol{k}_\perp)) f^a(x, \boldsymbol{k}_\perp^2) \, D^a(z, (\boldsymbol{P}_{h\perp} - z\boldsymbol{k}_\perp)^2)$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q} C\left[\frac{\hat{h}\cdot \boldsymbol{p}_\perp}{zM_h}\frac{k_\perp^2}{M^2} h_1^\perp H_1^\perp - \frac{\hat{h}\cdot \boldsymbol{k}_\perp}{M} z f_1 D_1\right]$$



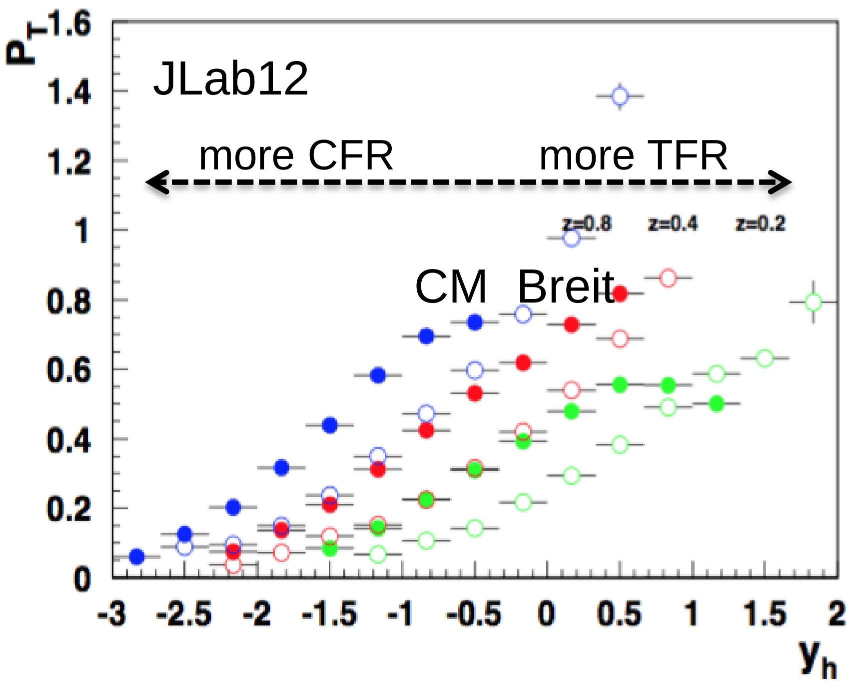
BM contribution seem to be less sensitive to phase space limitations
Need cross check.

Additional complications: Experiment covers ranges described by different SFs



Boglione et al, Phys.Lett. B766 (2017) 245-253

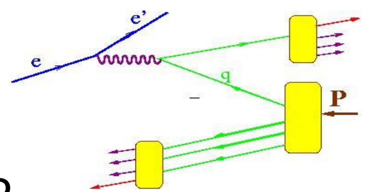
Kinematics covers regions with different fractions from target and current fragmentation



Understanding of the scale of ignored contributions (M/Q^2,P_T/Q^2, Target/Current correlations,…) will define the limits on precision for other involved contributions (ex. evolution).

Multidimensional bins (<u>x,y,z,P_T,ϕ</u>) are crucial for separation of different contributions

Target fragmentation in SIDIS

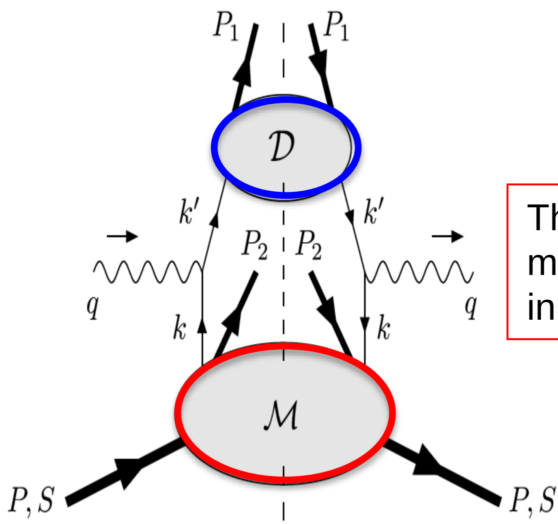


P_1

P_2

M. Anselmino, V. Barone and A. Kotzinian, Physics Letters B 713 (2012)

$$\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)} = \frac{|\vec{P}_{1\perp}\vec{P}_{2\perp}|}{m_N m_2}\mathcal{C}[w_5 M_L^{\perp,h} D_1]$$



$$A_{LU} = -\frac{y\left(1-\frac{y}{2}\right)}{\left(1-y+\frac{y^2}{2}\right)}\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}\sin\Delta\phi$$

Leading Twist

	U	L	T
U	M	$M_L^{\perp,h}$	$M_T^h, M_T^\perp$
L	$\Delta M^{\perp,h}$	ΔM_L	$\Delta M_T^h, \Delta M_T^\perp$
T	$\Delta_T M_T^h, \Delta_T M_T^\perp$	$\Delta_T M_L^h$ $\Delta_T M_L^\perp$	$\Delta_T M_T, \Delta_T M_T^{hh}$ $\Delta_T M_T^{\perp\perp}, \Delta_T M_T^{\perp h}$

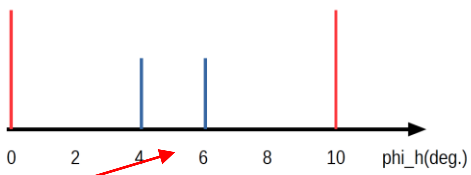
The beam–spin asymmetry appears, at leading twist and low transverse momenta, in the deep inelastic inclusive lepto-production of two hadrons, one in the target fragmentation region and one in the current fragmentation region.

Understanding of Target Fragmentation Region (TFR) is important for interpretation of the Current FR

- Need a consistent theoretical description for TFR
- Measure/model fracture functions

From data to phenomenology: EBC

bin#	x	Q^2	y	W	M_X	ϕ	z	P_T	λ	Λ	N(counts)	RC
1 ... N												



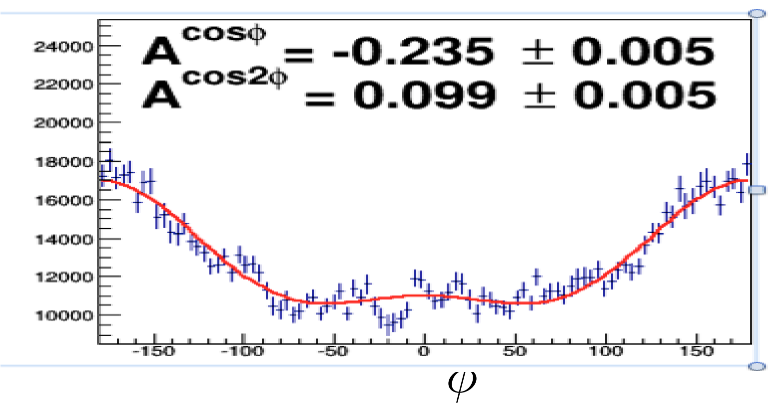
For precision studies of TMDs we need x-sections/muliplicities in smallest possible bins in x,y,z,P_T,ϕ for all hadrons and all relevant polarization states

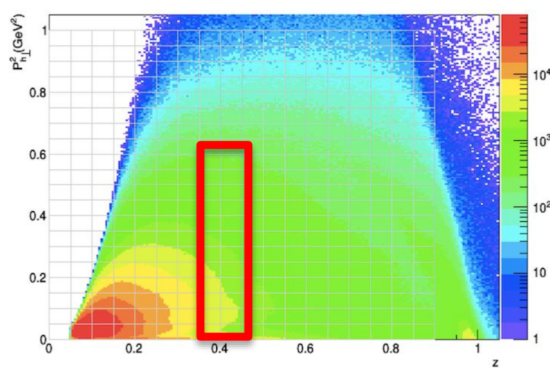
Elementary Bins vs macroscopic bins

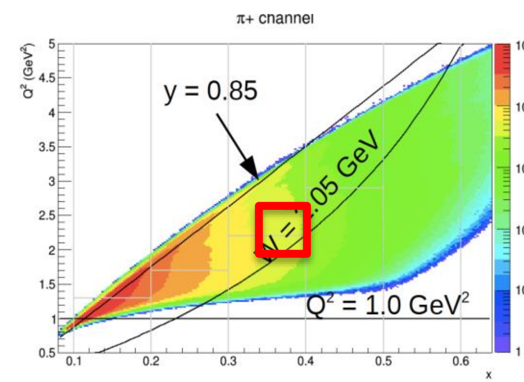
Pros:
1)can go to wider bins,
2)smaller bin centering corrections
3) smaller acceptance/radiative correcions.
4) can perform also Bessel weighting
5)Can re-calculate for any other kinematical variables (η,P_T/z,…)
……………………..

Cons:
1)Requires huge MC sample

EBC: bin sizes limited by resolutions







Examples of data from SIDIS experiments

HERMES

```
*********************************************************

   These are published data of the HERMES Collaboration.
   You are free to use these data in any publication. However, you must
   make a reference to the following publication:

   A. Airapetian et al, Phys. Lett. B562 (2003) 182 - 192

*****************************************************************************
```

```
*************************************
DEUTERIUM TARGET     Pi^+
*************************************

 <x>   A_UL^sin\phi  stat     sys
------------------------------------
0.039     0.007      0.004    0.002
0.068     0.009      0.004    0.002
0.115     0.014      0.004    0.002
0.179     0.022      0.007    0.002
0.276     0.025      0.009    0.002

<pt>   A_UL^sin\phi  stat     sys
------------------------------------
0.17      0.003      0.004    0.001
0.32      0.015      0.004    0.002
0.47      0.012      0.005    0.002
0.65      0.014      0.005    0.002
0.95      0.018      0.009    0.002
```

COMPASS

```
*dataset:
*location: Table 2
*dscomment: ASYMUU(SIN(PHI(HADRON))) asymmetries
*reackey: MU+ LI6DEUT --> MU+ HADRONS X
*obskey: ASYM
*qual: . : POSITIVE HADRONS : NEGATIVE HADRONS
*qual: PT(HADRON) IN GEV : 0.1 TO 1.0
*qual: Q**2 IN GEV**2 : > 1
*qual: RE : MU+ LI6DEUT --> MU+ HADRONS X
*qual: THETALAB(GAMMA*) IN MILLIRAD : < 60
*qual: W IN GEV : > 5
*qual: Y : 0.2 TO 0.9
*qual: Z : 0.2 TO 0.85
*yheader: ASYMUU(SIN(PHI(HADRON)))
*xheader: XB
*data: x : y : y
 0.003 TO 0.008; 0.021 +- 0.009; 0.010 +- 0.009;
 0.008 TO 0.013; 0.026 +- 0.008; 0.017 +- 0.008;
 0.013 TO 0.02; -0.007 +- 0.009; -0.026 +- 0.01;
 0.02 TO 0.032; 0.036 +- 0.011; 0.009 +- 0.011;
 0.032 TO 0.05; 0.020 +- 0.013; -0.022 +- 0.015;
 0.05 TO 0.08; 0.020 +- 0.015; -0.013 +- 0.017;
 0.08 TO 0.13; 0.022 +- 0.019; -0.016 +- 0.022;
*dataend:
```

http://hepdata.cedar.ac.uk/view/ins1278730

Experiment measures ϕ-dependence and performes fits to extract different moments

Need wide bins in kinematical variables to provide moments!

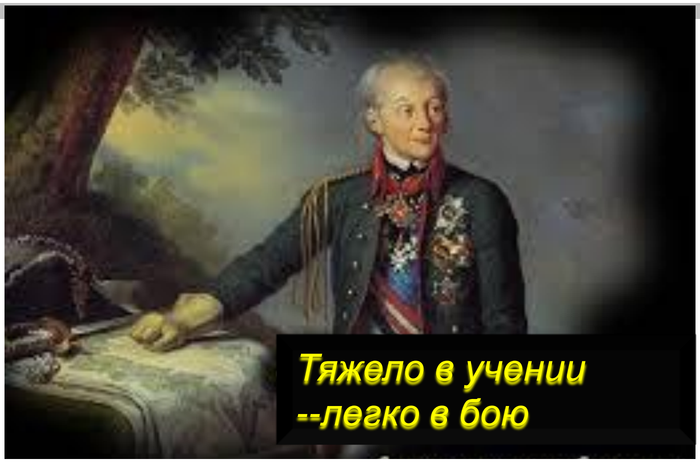
Standard output: CLAS e1f at 5.5 GeV

D. Riser

(JavaScript Object Notation used for serializing and transmitting structured data)


```
#!  {
#!     "data-set": ["E1-F"],
#!     "reference": "Exploring the Structure of the Proton via Semi-Inclusive Pion Production, Nathan Harrison",
#!     "web-source": "https://www.jlab.org/Hall-B/general/thesis/Harrison_thesis.pdf",
#!     "particle": "pi+",
#!   "lepton-polarization": "0",
#!   "nucleon-polarization": "0",
#!   "target": "hydrogen",
#!     "beam-energy": "5.498 GeV",
#!     "variables": ["counts-corrected","stat-err","rad-corr"],
#!     "axis": [
#!         { "name": "a", "bins":   5, "min":    0.10, "max":   0.60, "scale":"arb", "description":"Bjorken x"},
#!         { "name": "b", "bins":   1, "min":    1.00, "max":   4.70, "scale":"arb", "description":"Q^2"},
#!         { "name": "c", "bins":  18, "min":    0.00, "max":   0.90, "scale":"lin", "description":"hadron frac. energy"},
#!         { "name": "d", "bins":  20, "min":    0.00, "max":   1.00, "scale":"lin", "description":"transverse momentum"},
#!         { "name": "e", "bins":  36, "min": -180.00, "max": 180.00, "scale":"lin", "description":"azimuthal angle"},
#!      ]
#!  }
0 0 15 2 0 0.153135 1.16888 0.772973 0.125044 -175 0.74663 3173.48 205.893 1.00537
0 0 15 2 1 0.153135 1.16888 0.772973 0.125044 -165 0.74663 3464.36 226.181 1.00307
0 0 15 2 2 0.153135 1.16888 0.772973 0.125044 -155 0.74663 3473.09 241.549 0.999228
0 0 15 2 3 0.153135 1.16888 0.772973 0.125044 -145 0.74663 3015.84 253.718 0.994561
0 0 15 2 4 0.153135 1.16888 0.772973 0.125044 -135 0.74663 4327.02 463.082 0.988254
```


- Full 5-dimentional table (7 with helicities) allowing rebining, proper integrations over other variables, web browsing, graphical presentation,…
- While keeping "human readable" the data will be machine readable (will need API)
- Reducing the size of the bins (limited by resolution and MC statistics for acceptance extraction



- the more you sweat in times of peace the less you bleed in war

Monte Carlo simulation is crucial for understanding of systematics of all steps and assumptions used in extraction of complex 3D nucleon structure

Questions to address

SIDIS and Hard Exclusive processes requiring multidimensional analysis, are a major challenge for experiment, theory, software extraction framework, claiming control of systematic uncertainties

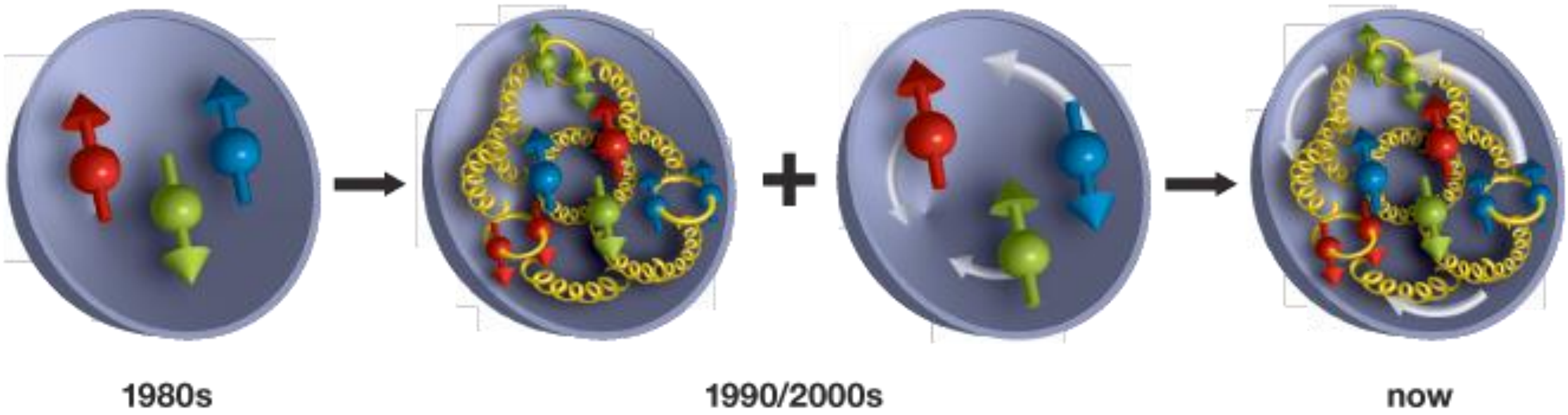
- At which step the experimental extraction should stop and theory extraction start?
- How a detailed MC could help to understand better different contributions in the x-section of single or double pion production?

- How the TMD/GPD libraries could be integrated into extraction process

- How we deal with "real" data with finite beam energies and limited phase space?
- Do we need "validation" of extracted TMDs and what that will include?

Studies of 3D Structure of Nucleon

http://www.int.washington.edu/PROGRAMS/14-55w/



The ultimate goal:
a precise mapping of the 3D nucleon structure and a detailed flavor decomposition of 3D parton distribution functions

J.Phys. G42 (2015) 034015

Organizers: Elke Aschenauer, Barbara Pasquini, Harut Avakian, Peter Schweitzer

Event generators for SIDIS studies

Main classes of event generators:

a)Full event generators where sets of outgoing particles are produced in the interactions between two incoming particles and a complete event is generated

Applications: attempt to reproduce the raw data
understand background conditions
estimating rates of certain types of events
planning and optimizing detector performances,…

b) Specific event generators (single hadron, di-hadron,…) , where only the final state particles of interest are generated

Applications: providing fast tests of analysis procedures with relatively simple integration of different input models.
developing analysis frameworks.

+unfolding measured data for acceptance and detector resolution effects

Nucleon structure & TMDs at leading twist

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+\,S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\,S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+\,|\boldsymbol{S}_\perp|\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\right.$$

$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2}$$

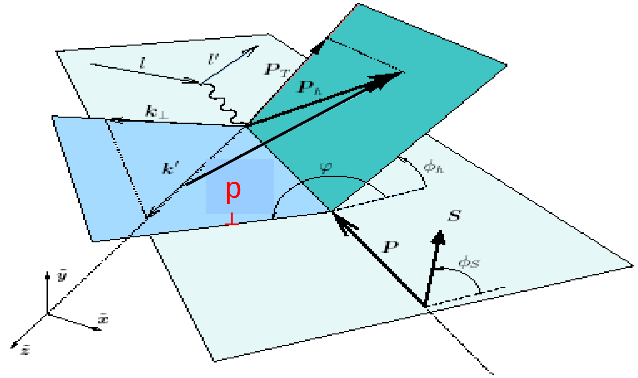
$$+\,|\boldsymbol{S}_\perp|\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F\right.$$

$$\left.\left.+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\right\}$$

Extraction of leading twist TMDs limited to formalism accounting for only leading twists will require some mechanisms for controlling the systematics (measure and simulate background effects).

What we miss in the leading twist picture?

Reproduce SIDIS output with MC



SIDIS MC in 7D (10D)

$$\frac{d\sigma_{\lambda\Lambda}^{eN\to e'hX}}{dx\,dQ^2\,dz\,dP_{hT}^2\,d\phi_h\,d\phi_l\,d\phi_s} = \sum_{l=1}^{L} SF_l$$

step-1 $\quad x_i, Q_i^2 z_i P_{hT}^{i2} \phi_h^i \phi_l^i \phi_s^i$

step-2 (for a given E_{beam},λ,Λ) $\quad P_i^{el}, P_i^h$

step-3 (detected for a given Detector configuration)

$$x_j, Q_j^2 z_j P_{hT,j}^2 \phi_h^j \phi_l^j \phi_s^j$$

Theory

Provide a set of SF_l

For a given model/theory based on underlying non-perturbative input calculate SF_l

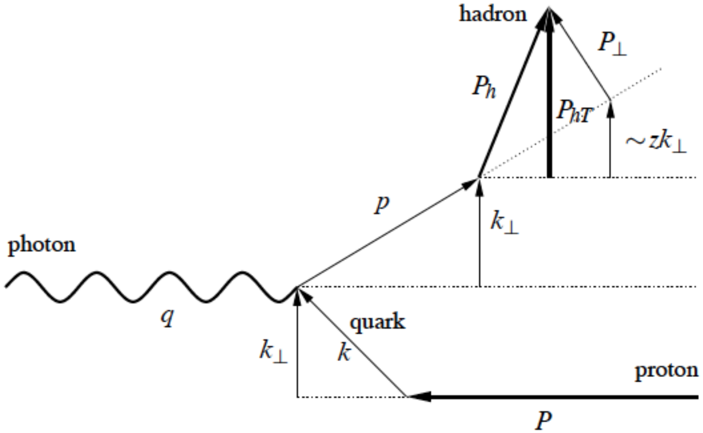
Output counts for a given energy and detector setup

A set of Structure Functions needed for x-section

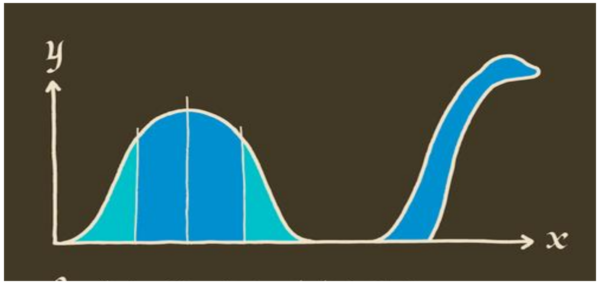
$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2;\mu^2) \int dk_\perp \, dP_\perp \, f_1^a(x,k_\perp^2;\mu^2) \, D_1^{a\to h}(z,P_\perp^2;\mu^2) \, \delta(zk_\perp - P_{hT} + P_\perp)$$
$$+ Y_{UU,T}(Q^2,P_{hT}^2) + \mathcal{O}(M/Q).$$

$$F_{UU,T} = x \sum_a e_a^2 f_1^a(x) \, D_1^{a\to h}(z) \, \frac{1}{\pi \langle P_{h\perp}^2 \rangle} \, e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle}$$

$$\langle P_{h\perp}^2 \rangle^2 = z^2 \langle k_{q,\perp}^2 \rangle + \langle p_{q\to h\perp}^2 \rangle .$$

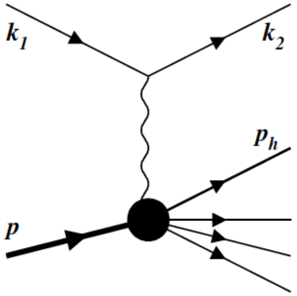


Framework should handle any SF input



Radiative SIDIS

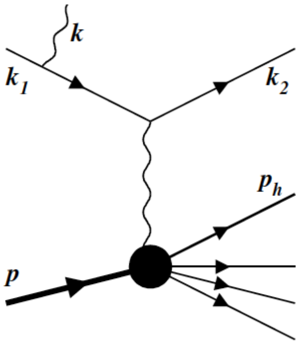
Akushevich&Ilyichev in progress



$$e(k_1,\xi)+n(p,\eta)\longrightarrow e(k_2)+h(p_h)+x(p_x)$$

$$\frac{d\sigma^B}{dx\,dy\,dz\,dp_t^2\,d\phi_h\,d\phi}$$

$$e(k_1,\xi)+n(p,\eta)\rightarrow e(k_2)+h(p_h)+x(\tilde{p}_x)+\gamma(k)$$



+.....

additional photon can be described by three additional variables:

$$R=2kp,\ \ \tau=\frac{kq}{kp},\ \ \phi_k$$

$$S_x=2p(k_1-k_2)$$

$$\lambda_Y=S_x^2-4M^2Q^2$$

The phase space of the real photon: $$\frac{d^3k}{k_0}=\frac{RdRd\tau d\phi_k}{2\sqrt{\lambda_Y}}.$$

ϕ_k is an angle between $(\mathbf{k}_1,\mathbf{k}_2)$ and $(\mathbf{k},\mathbf{q})$ planes.

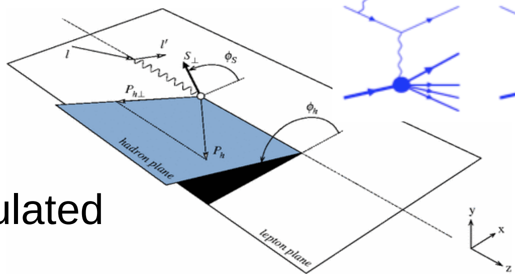
$$e(k_1,\xi)+n(p,\eta)\rightarrow e(k_2)+h(p_h)+u(p_u)+\gamma(k),$$

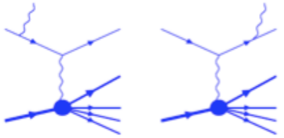
$$\delta^4(k_1+p-k_2-p_h-p_u-k)$$

Additional complications: Experiment can't measure just 1 SF

I. Akushevich et al

$$\sigma = \sigma_{UU} + \sigma_{UU}^{\cos\phi} cos\phi + S_T \sigma_{UT}^{\sin\phi_S} \sin\phi_S + ...$$





Due to radiative corrections, ϕ-dependence of x-section will get multiplicative R_M and additive R_A corrections, which could be calculated from the full Born (σ_0) cross section for the process of interest

$$\sigma_{Rad}^{ehX}(x,y,z,P_T,\phi,\phi_S) \to \sigma_0^{ehX}(x,y,z,P_T,\phi,\phi_S) \times R_M(x,y,z,P_T,\phi) + R_A(x,y,z,P_T,\phi,\phi_S)$$

Due to radiative corrections, ϕ-dependence of x-section will get more contributions

- Some moments will modify
- New moments may appear, which were suppressed before in the x-section

Simplest rad. correction

$$R(x,z,\phi_h) = R_0(1 + r\cos\phi_h)$$

Correction to normalization

$$\sigma_0(1+\alpha\cos\phi_h)R_0(1+r\cos\phi_h) \to \sigma_0 R_0(1+\alpha r/2)$$

Correction to SSA

$$\sigma_0(1+sS_T\sin\phi_S)R_0(1+r\cos\phi_h) \to \sigma_0 R_0(1+sr/2S_T\sin(\phi_h-\phi_S)+sr/2S_T\sin(\phi_h+\phi_S))$$

Correction to DSA

$$\sigma_0(1+g\lambda\Lambda+f\lambda\Lambda\cos\phi_h)R_0(1+r\cos\phi_h) \to \sigma_0 R_0(1+(g+fr/2)\lambda\Lambda)$$

Simultaneous extraction of all moments is important also because of correlations!

Suggested standard input for SFs

(JavaScript Object Notation for a single hadron production eN->e' hX)

```
=======================================================
# Header input information example to be  parsed as JSON
# for the lines that start with '#!'
    #=====================================================
    #!  {
    #!      "model": "VGD_Fuu_01",
    #!      "description": "Cahn contribution to cos",
    #!      "reference": "A.B. et al, PRL",
    #!      "web-source": "http://aaa.html",
    #!    "formula": "$sf1=-2*d/b*a*a*(1-a)^p0*c^p1*(1-c)^p2*c*p3/p4*exp(-d*d/p4)/p4$",
    #!    "moment": "$\\cos\\phi$",
    #!    "lepton-polarization": "0",
    #!    "nucleon-polarization": "0",
    #!    "particle": "pi+",
    #!    "target": "proton",
    #!    "variables": ["SF1","SF1Error"],
    #!       "axis": [
    #!          { "name": "a", "bins": 20, "min":  0.01, "max":   0.99, "scale":"log" ,"description":"Bjorken x"}
    #!          { "name": "b", "bins": 20, "min":  1.00, "max": 100.00, "scale":"log", "description":"Q^2"},
    #!          { "name": "c", "bins": 20, "min":  0.10, "max":   0.99, "scale":"lin", "description":"hadron frac. energy"},
    #!          { "name": "d", "bins": 25, "min":  0.00, "max":   1.50, "scale":"lin", "description":"transverse momentum"}
    #!       ],
    #!      "parameters": [
    #!            {"name":"p0", "value": 1.0},
    #!            {"name":"p1", "value": 0.2},
    #!            {"name":"p2", "value": 0.1},
    #!            {"name":"p3", "value": 0.1},
    #!            {"name":"p4", "value": 0.1}
    #!           ]
    #!  }
    #===================================================
```

Advantages:

- Table can be generated from any existing program for calculation of SFs for any given set of parameters, final state particles, target nucleon, polarization states.
- Corresponding API will allow rebining, summing of tables with different ranges, web browsing, graphical presentation, integrations and other operations (will need API)

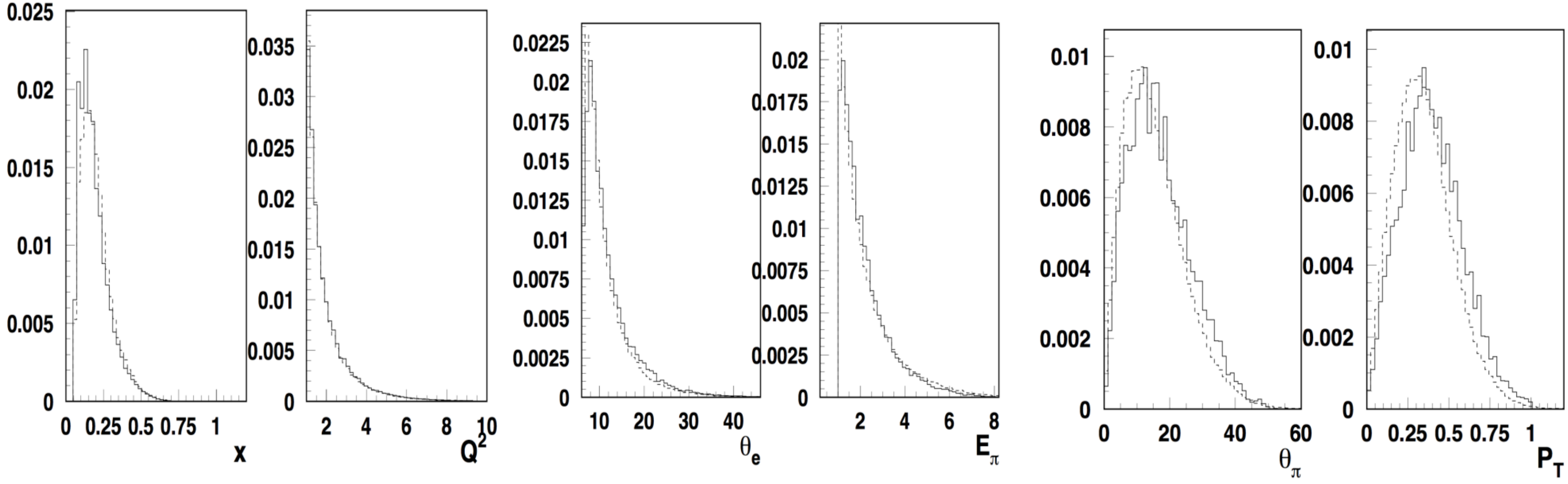
Suggested standard input for SFs:Example(model)


```
#!{
#!     "model": "VGD_Fuu_01",
#!     “description”: “Cahn contribution to cos”,
#!     "reference": “M. Boglione, S. Melis & A. Prokudin Phys. Rev. D 84, 034033  2011",
#!     "web-source": "http://aaa.html",
#!    "formula": "$sf1=-2*d/b*a*a*(1-a)^p0*c^p1*(1-c)^p2*c*p3/p4*exp(-d*d/(p4+c*c*p3)/p4$",
#!    "moment": “$A_{uu}\\cos\\phi$",
#!    “lepton-polarization”: “0”,
#!    “nucleon-polarization”: “0”,
#!    "particle": "pi+",
#!    "variables": ["AuuCos2","AuuCos2-Err"],
#!      "axis": [
#!         { "name": "a", "bins":  40, "min":  0.025, "max":   0.995, "scale":"arb" ,"description":"Bjorken x"}
#!         { "name": "b", "bins":  40, "min":  20.00, "max": 4.70, "scale":"arb”, ”description":"Q^2"},
#!         { "name": "c", "bins":  40, "min":  0.025, "max":   0.995, "scale":"lin", "description":"hadron frac. energy"},
#!         { "name": "d", "bins":  40, "min":  0.00, "max":   2.00, ”scale":"lin", "description":"transverse momentum"}
#!      ],
#!     "parameters": [
#!           {"name":"p0", "value": 1.0},
#!           {"name":"p1", "value": 0.2},
#!           {"name":"p2", "value": 0.1},
#!           {"name":"p3", "value": 0.33, "description":”average k_T2”},
#!           {"name":"p4", "value": 0.16, "description":”average pt_T2”}
#!                 ]
#!  }
```


```
0   0   0   0   -0.01285
0   0   0   1   -0.03736
0   0   0   2   -0.05850
0   0   0   3   -0.07459
0   0   0   4   -0.08467
.............
```

Multiple files for all relevant combinations of involved parameters

Kinematic distributions



eπX evnts compared with eπX events from PYTHIA tuned to data (dashed)

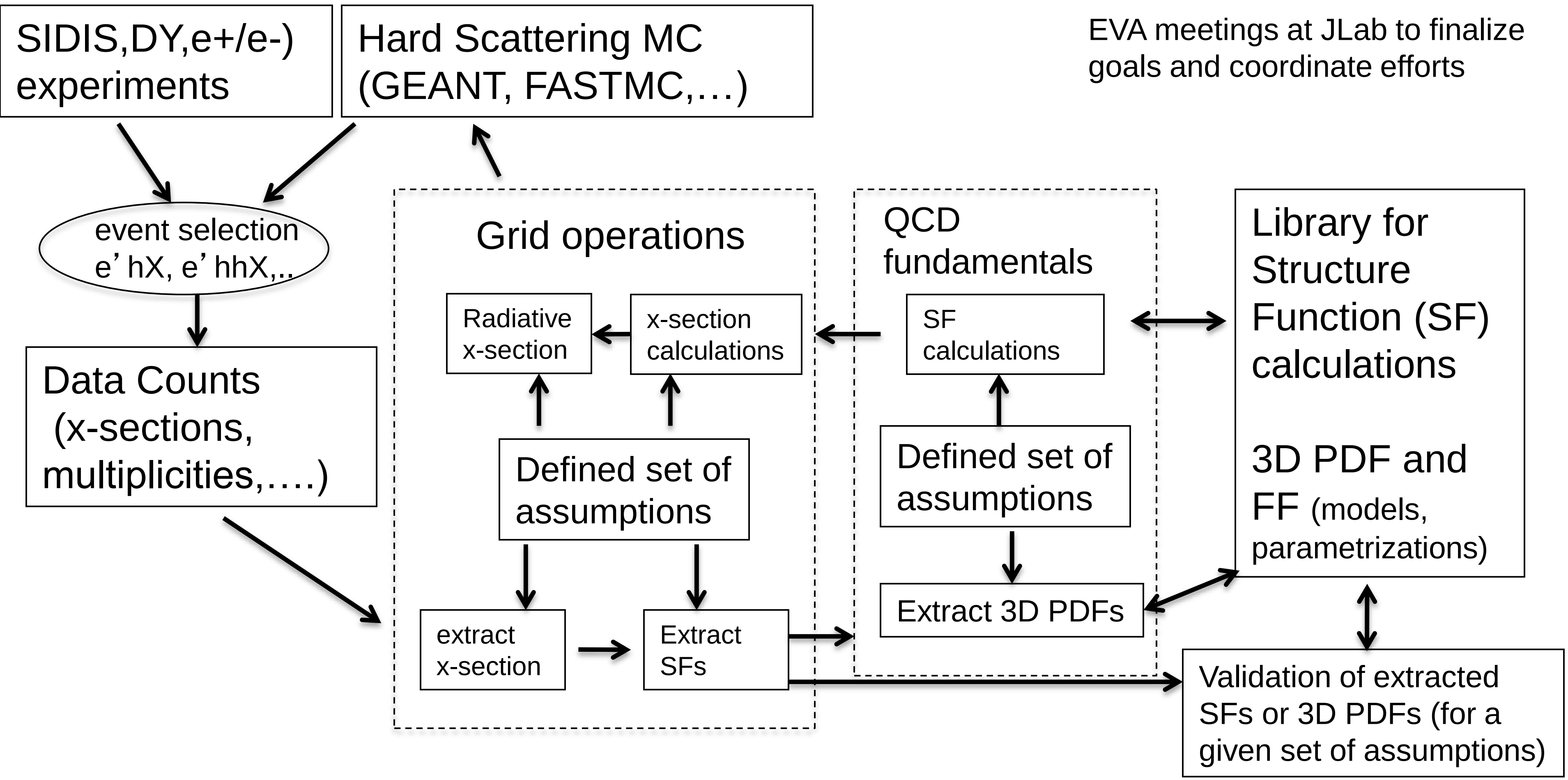
Simple event generator should be “reasonable”

Suggested standard input for SFs:Example (data)

(JavaScript Object Notation for a single hadron production eN->e' hX)


```
#!  {
#!     "model": "Data",
#!     “description”: “”,
#!     "reference": "Exploring the Structure of the Proton via Semi-Inclusive Pion Production, Nathan Harrison",
#!     "web-source": "https://www.jlab.org/Hall-B/general/thesis/Harrison_thesis.pdf",
#!    "moment": “$A_{uu}\\cos\ 2\phi$",
#!    “lepton-polarization”: “0”,
#!    “nucleon-polarization”: “0”,
#!    "particle": "pi+",
#!    "variables": ["AuuCos2","AuuCos2-Err"],
#!      "axis": [
#!         { "name": "a", "bins":  5, "min":  0.01, "max":   0.60, "scale":"arb" ,"description":"Bjorken x"}
#!         { "name": "b", "bins":  2, "min":  1.00, "max": 4.70, "scale":"arb”, ”description":"Q^2"},
#!         { "name": "c", "bins": 18, "min":  0.00, "max":   0.90, "scale":"lin", "description":"hadron frac. energy"},
#!         { "name": "d", "bins": 20, "min":  0.00, "max":   1.00, ”scale":"lin", "description":"transverse momentum"}
#!      ]
#!  }
0 0 1 0 -0.0162215 0.00242759
0 0 2 0 0.0264976 0.00306648
0 0 2 1 -0.000968785 0.00326021
0 0 2 2 -0.0183257 0.00427527
0 0 2 3 -0.00224623 0.00469542
0 0 3 0 0.04539 0.00433408
0 0 3 1 -0.00307352 0.00409825
0 0 3 2 -0.0403614 0.00503846
0 0 3 3 -0.034225 0.0061943
0 0 3 4 0.00820626 0.00610658
0 0 3 5 0.0013598 0.00762099
```

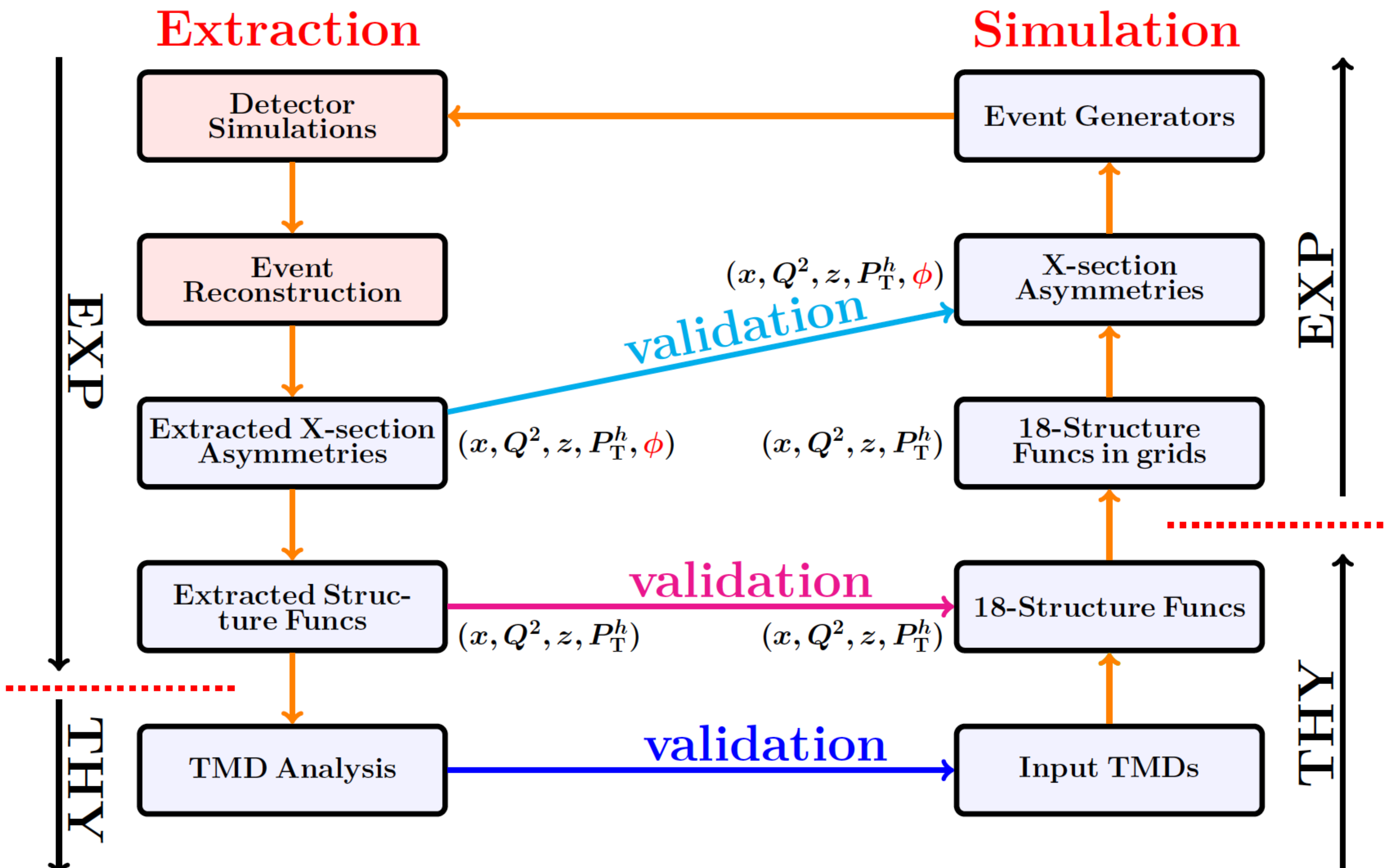
3D PDF Extraction and VAlidation (EVA) framework



Development of a reliable techniques for the extraction of 3D PDFs and fragmentation functions from the multidimensional experimental observables with controlled systematics requires close collaboration of experiment, theory and computing

Eva workflow

N.Sato & UConn



N.Sato & UConn

EVA: Extraction and Validation framework

- Dedicated numerical framework to study TMDs in SIDIS
- To be used as event generator for detector simulations
- To be used as TMD fitter
- Written in python to take advantage of extensive open-source libraries for data analysis
- Includes dedicated libraries for parallel computing needed to analyze big data sets such as multidimensional SIDIS measurements
- Current implementation is based on standard gaussian ansatz. It will be extend to include CSS formalism
- At present, the framework is being tuned to describe existing data from COMPASS, HERMES and JLab 6, using state-of-the-art Monte Carlo fitting techniques

SUMMARY

Need a collaboration of theorists, experimentalists and software experts to define the path to a flexible TMD/GPD extraction system with validation capabilities.

Suggestions:

- Define the data input (x-sections/multiplicities "Elementary Bin Counts" in ϕ-bins)
- Use MC to test extraction procedures
- Test the sensitivity to different assumptions in procedures for extraction of SFs and underlying 3D PDFs ("global fits")

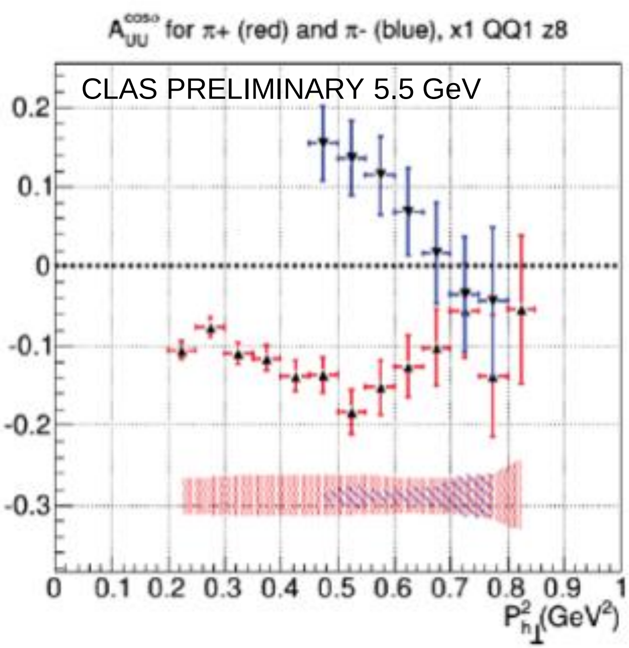
Plans

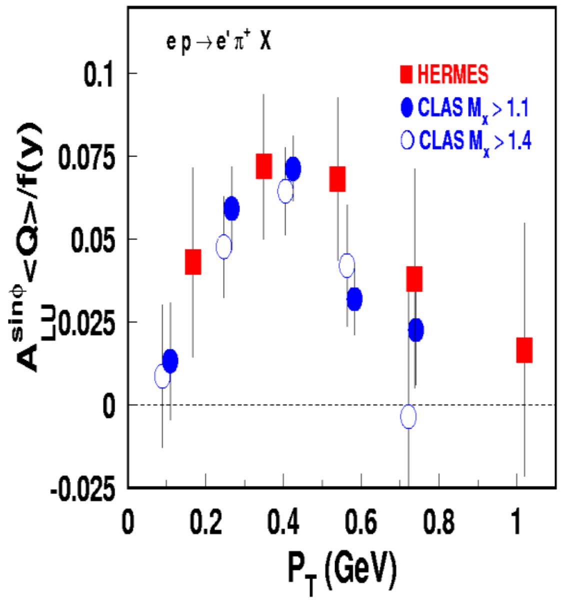
- Use CLAS/CLAS12 (any other) data/MC (FASTMC) for tests
- Apply different extraction procedures to define sensitivity to statistical and systematic uncertainties

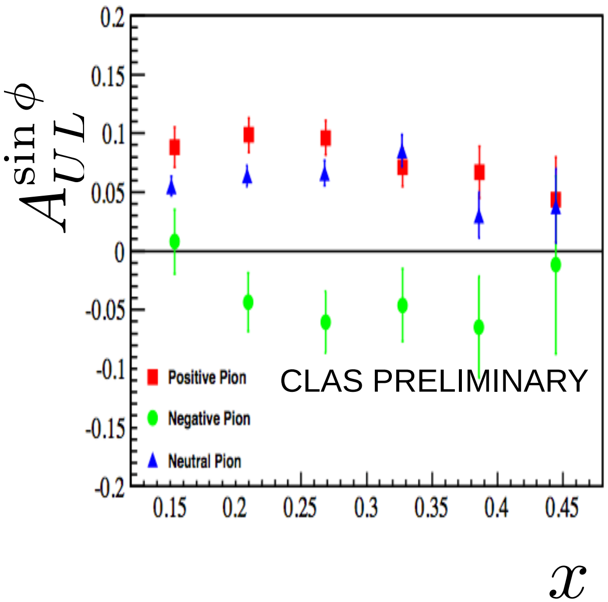
Support slides…

Additional complications(IV): Large higher twist structure functions

target mass corrections and HT SFs with strong dependence on flavor







presence of large corrections due to limited Q^2 make the estimate of systematics due to ignoring them important

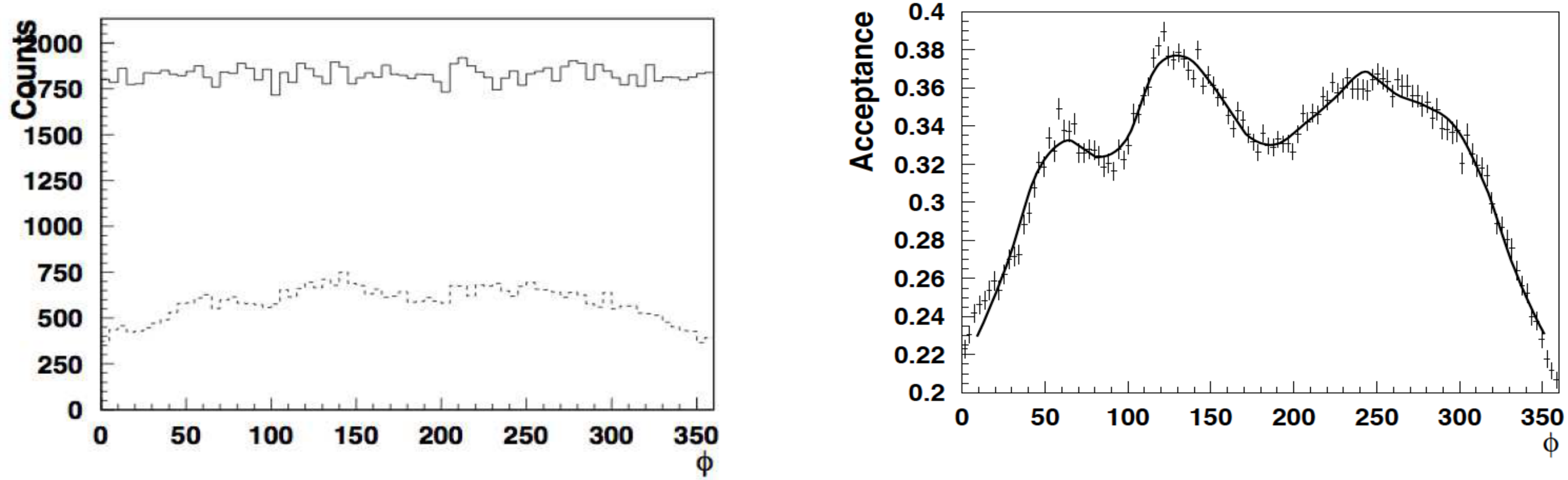
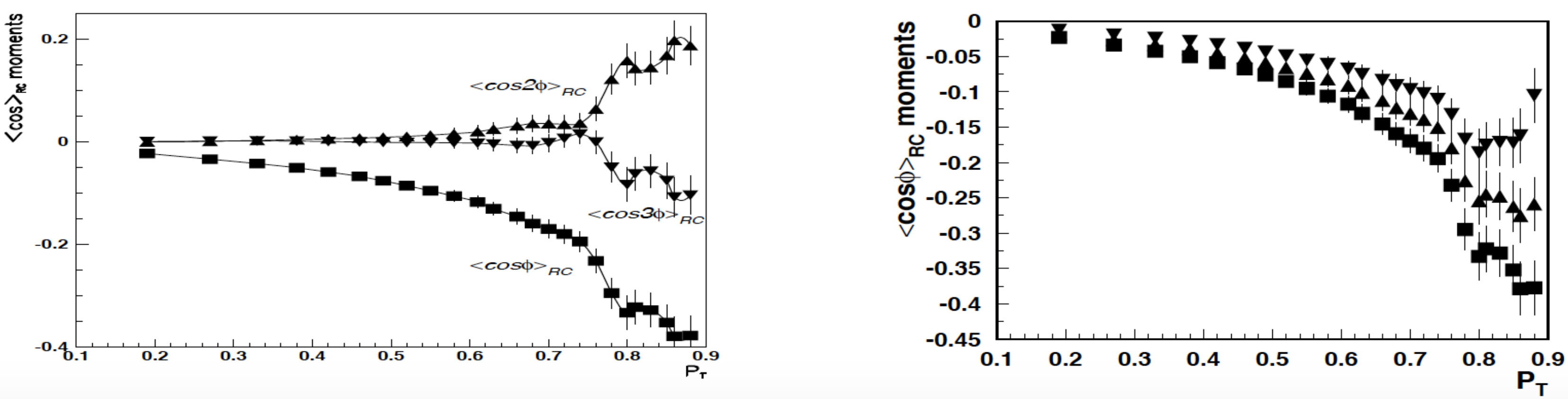
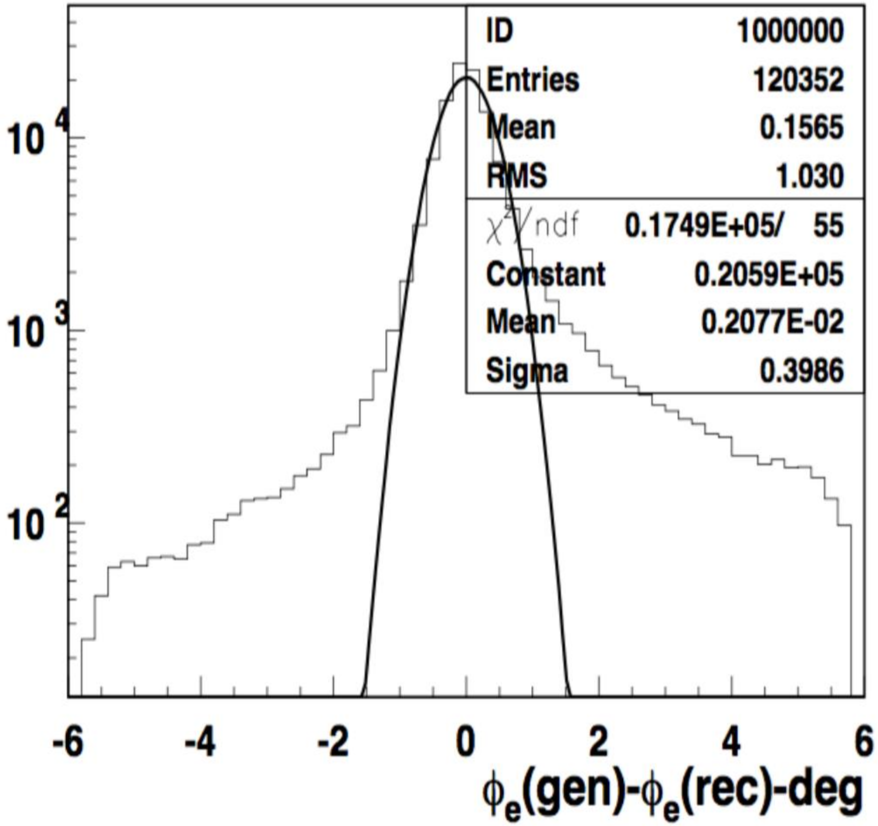
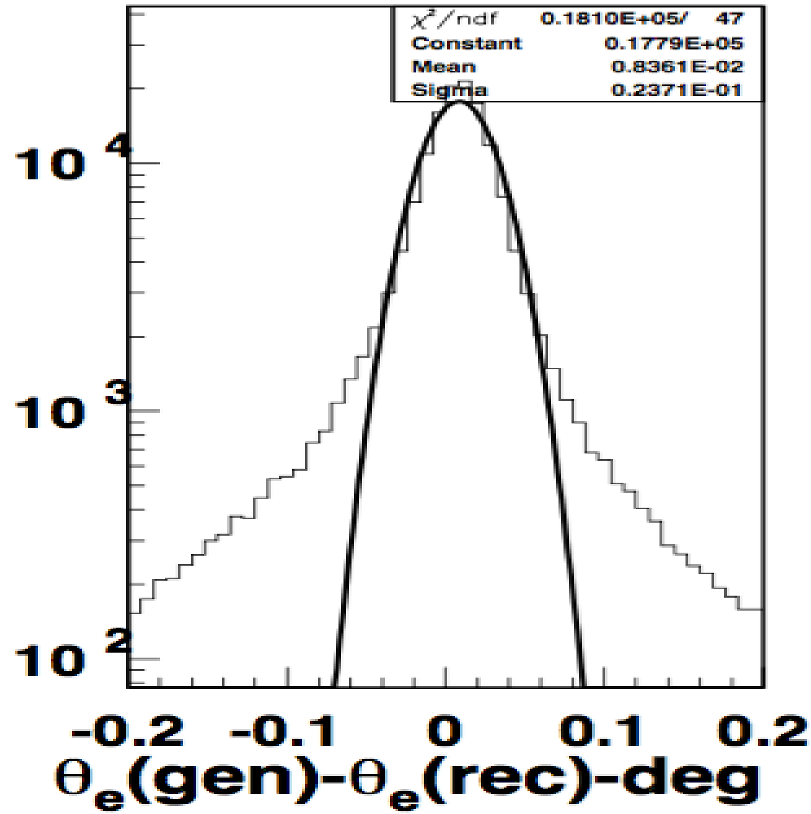


Figure 5. Left panel: generated (flat) and reconstructed ϕ distributions in $ep \to e'\pi^+X$ events for CLAS12 detector and 11 GeV electron beam energy. Right panel: acceptance (ratio of reconstructed to generated events) fitted with the Fourier series in Eq. (6), including the first ten cosine moments and the first four sine moments.



Clas12 resolutions

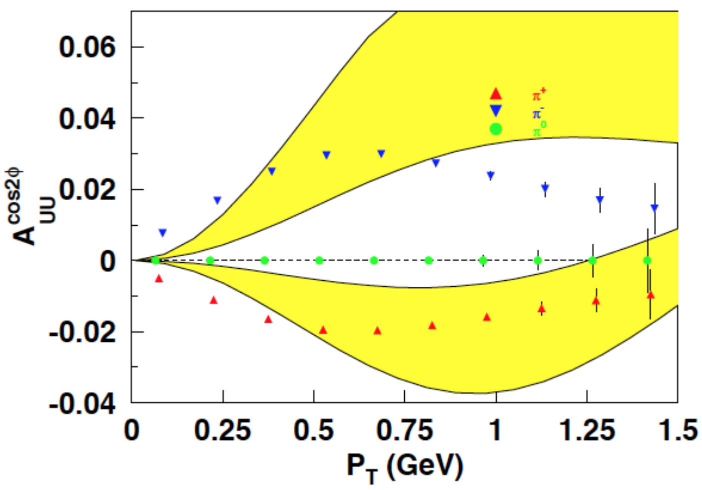




Angular and momentum resolutions define the EBC size

systematics

clas12 proposals



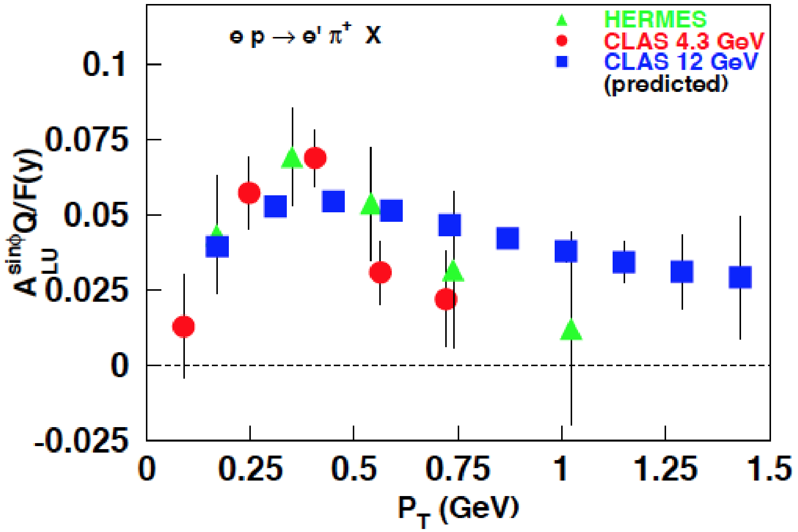
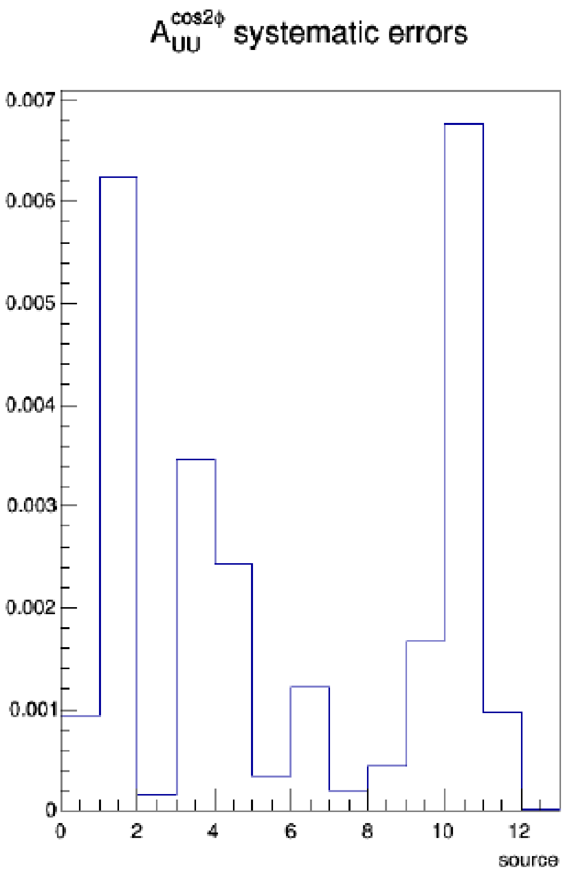


Table 2: Expected systematic uncertainties for azimuthal moments

Source	$\Delta A^{\cos\phi}$	$\Delta A^{\cos 2\phi}$	$\Delta A^{\sin\phi}$
Beam polarization			2%
ϕ acceptance	3%	1%	1%
other moments	1%	2%	1%
Radiative corrections	2%	1%	1%
Total	< 4%	< 3%	< 3%

Systematics

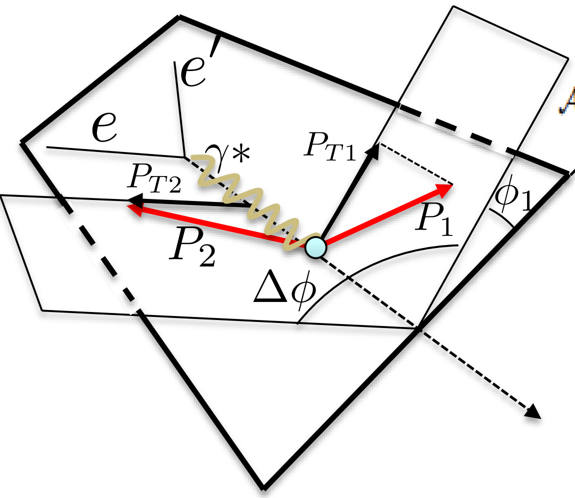


Label	Source	# of variations	Description
0	e- z-vertex cut	2	cut value is loosened or tightened by 0.2 cm on each side
1	e- EC sampling cut	2	see figure 10.1
2	e- EC outer vs inner cut	2	cut value is loosened or tightened by 0.005 GeV
3	e- EC geometric cut	2	see figure 10.1
4	e- CC θ matching cut	2	see figure 10.1
5	e- region 1 fiducial cut	2	see figure 10.1
6	e- region 3 fiducial cut	2	see figure 10.1
7	e- CC fiducial cut	2	see figure 10.1
8	pion β cut	2	cut is loosened or tightened by 0.25σ on both the low and high side
9	pion region 1 fiducial cut	2	see figure 10.2
10	ϕ_h fiducial cut	2	a bin (10°) on each side is added or removed
11	acceptance model dependence	1	the second to last iteration is used
12	radiative correction model dependence	1	the second to last iteration is used

B2B hadron production in SIDIS: First measurements

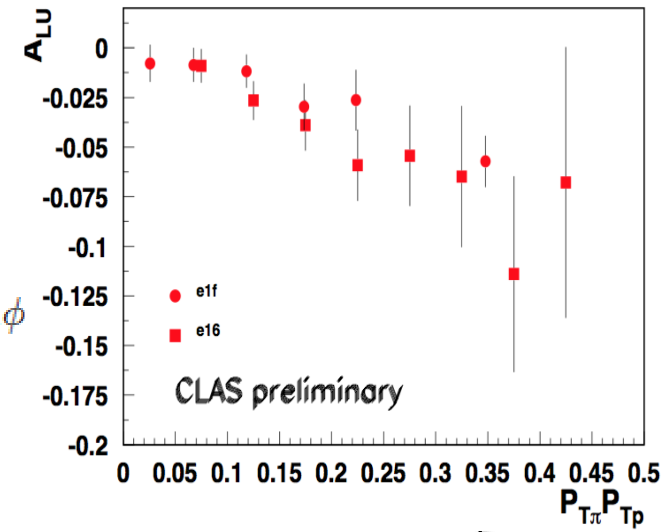
M. Anselmino, V. Barone and A. Kotzinian,
Physics Letters B 713 (2012)

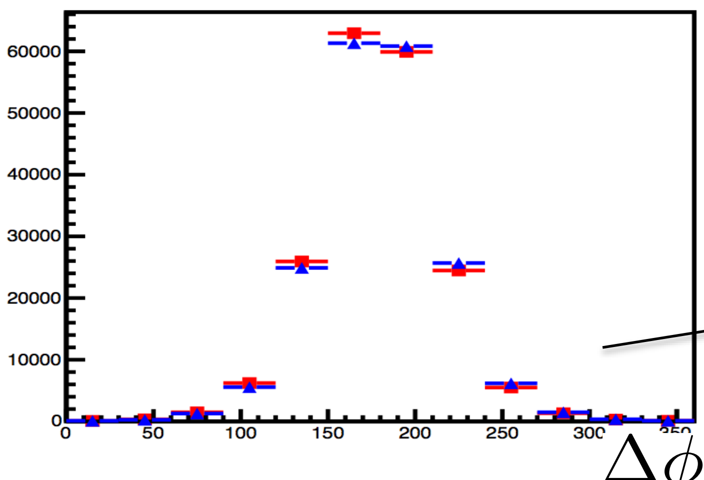
$$ep \rightarrow e' p \pi^+ X$$

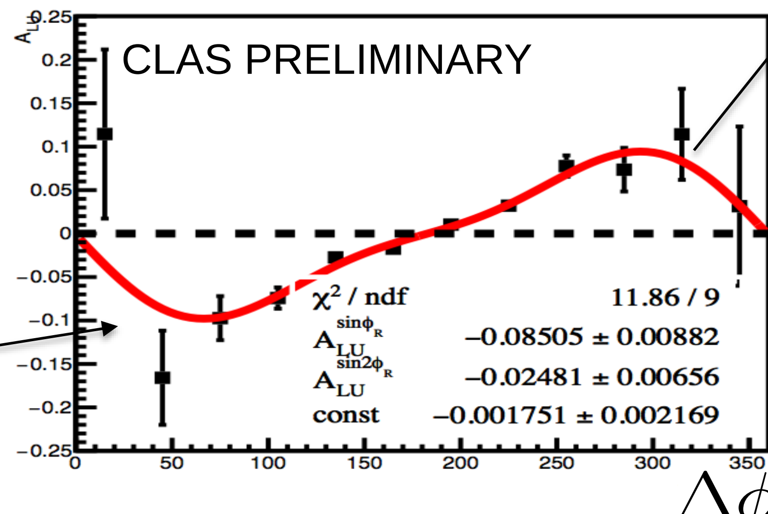


$$A_{LU} = -\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})} \frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}} \sin\Delta\phi$$

$$= -\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N m_2} \frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})} \frac{\mathcal{C}[w_5 M_L^{\perp,h} D_1]}{\mathcal{C}[M D_1]} \sin\Delta\phi$$

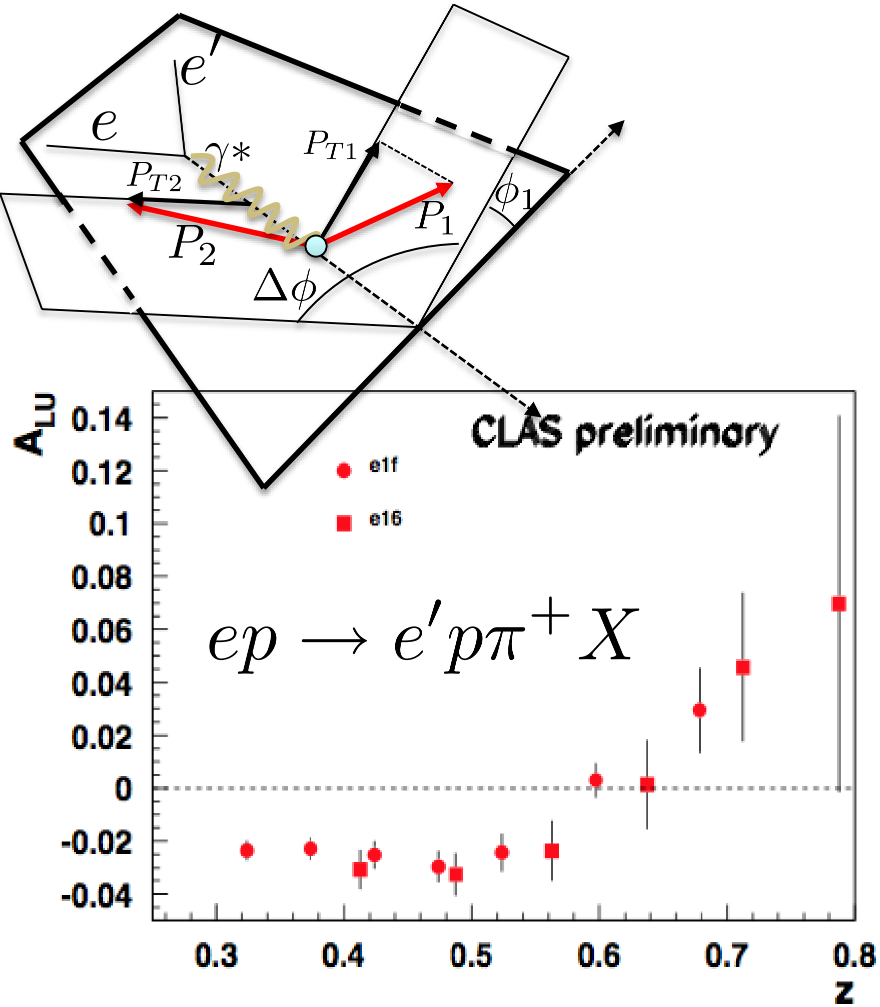


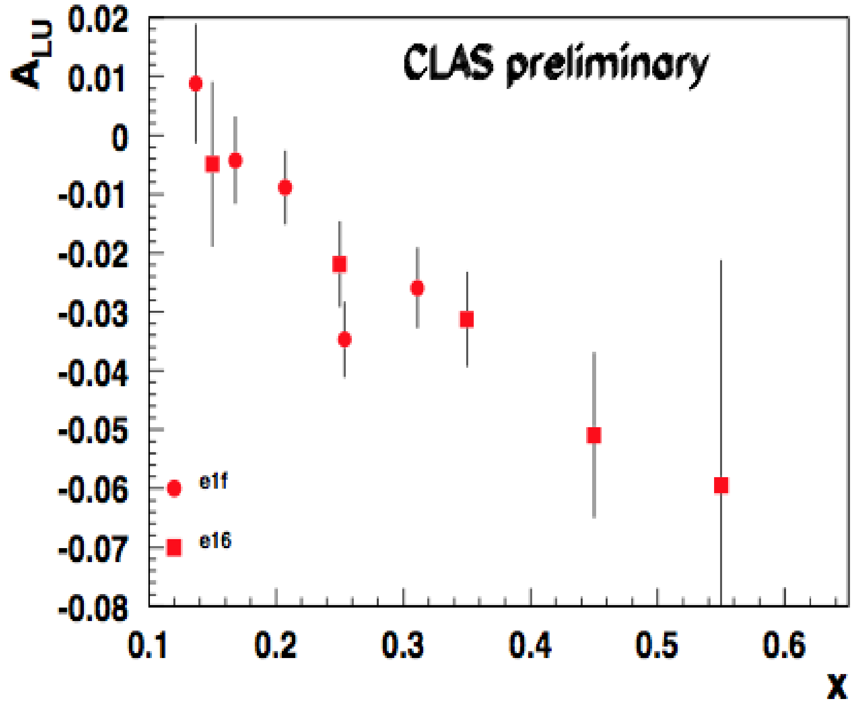




Significant asymmetries observed by CLAS at 6 GeV

A_{LU} comparing CLAS data sets e16 and e1f





e1f: weaker field, lower Q^2 and x

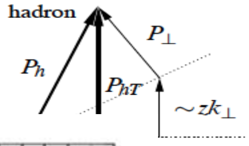
e16: higher field, higher average Q^2

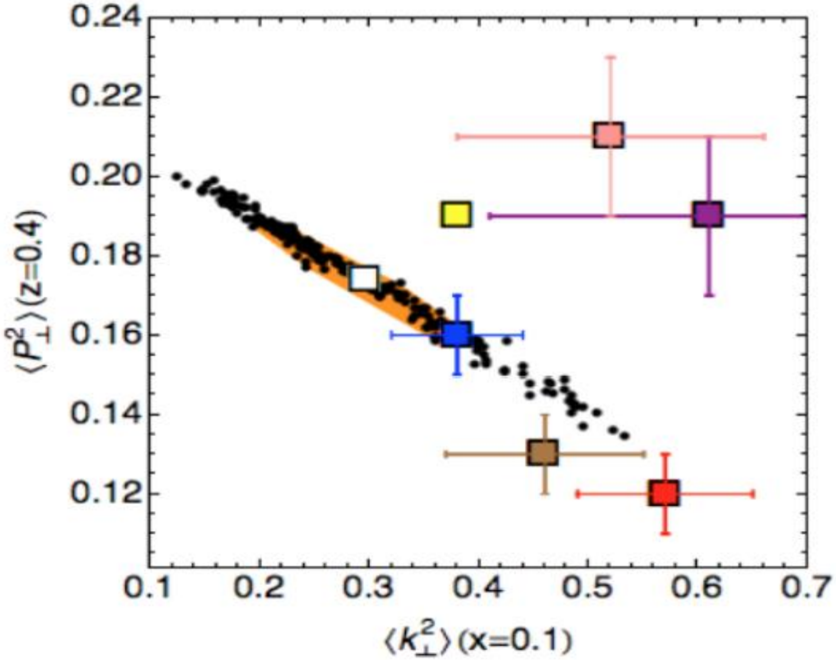
- Asymmetries may change the sign in the exclusive limit
- Asymmetries are large in the large x-region

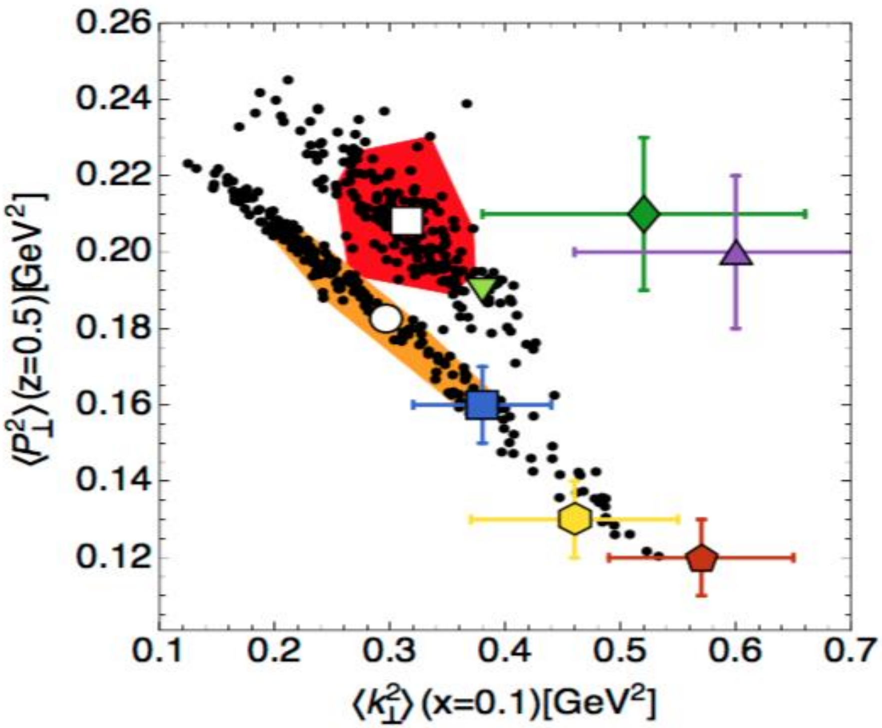
Extracting the average transverse momenta

Andrea Signori,[1,*] Alessandro Bacchetta,[2,3,†] Marco Radici,[3,‡] and Gunar Schnell[4,5,§]

$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2;\mu^2) \int dk_\perp\, dP_\perp\, f_1^a(x,k_\perp^2;\mu^2)\, D_1^{a\to h}(z,P_\perp^2;\mu^2)\, \delta(zk_\perp - P_{hT} + P_\perp)$$
$$+ Y_{UU,T}(Q^2,P_{hT}^2) + \mathcal{O}(M/Q).$$







$$m_N^h(x,z,P_{hT}^2) = \frac{\pi}{\sum_a e_a^2\, f_1^a(x)}$$
$$\times \sum_a e_a^2\, f_1^a(x)\, D_1^{a\to h}(z)\, \frac{e^{-P_{hT}^2/\left(z^2\langle k_{\perp,a}^2\rangle + \langle P_{\perp,a\to h}^2\rangle\right)}}{\pi\left(z^2\langle k_{\perp,a}^2\rangle + \langle P_{\perp,a\to h}^2\rangle\right)}$$

difference?

Suggested standard input for SFs:Example

(JavaScript Object Notation for a single hadron production eN->e' hX)

```
===================================================================
# Header input information example to be  parsed as JSON
# for the lines that start with '#!'
    #==========================================================
    #!  {
    #!       "model": "VGD_Fuu_01",
    #!       "description": "Cahn contribution to cos",
    #!       "reference": "A.B. et al, PRL",
    #!       "web-source": "http://aaa.html",
    #!     "formula": "$sf1=-2*d/b*a*a*(1-a)^p0*c^p1*(1-c)^p2*c*p3/p4*exp(-d*d/p4)/p4$",
    #!     "moment": "$\\cos\\phi$",
    #!     "lepton-polarization": "0",
    #!     "nucleon-polarization": "0",
    #!     "particle": "pi+",
    #!     "target": "proton",
    #!     "variables": ["SF1","SF1Error"],
    #!        "axis": [
    #!           { "name": "a", "bins": 20, "min":  0.01, "max":   0.99, "scale":"log" ,"description":"Bjorken x"}
    #!           { "name": "b", "bins": 20, "min":  1.00, "max": 100.00, "scale":"log", "description":"Q^2"},
    #!           { "name": "c", "bins": 20, "min":  0.10, "max":   0.99, "scale":"lin", "description":"hadron frac. energy"},
    #!           { "name": "d", "bins": 25, "min":  0.00, "max":   1.50, "scale":"lin", "description":"transverse momentum"}
    #!        ],
    #!       "parameters": [
    #!             {"name":"p0", "value": 1.0},
    #!             {"name":"p1", "value": 0.2},
    #!             {"name":"p2", "value": 0.1},
    #!             {"name":"p3", "value": 0.1},
    #!             {"name":"p4", "value": 0.1}
    #!            ]
    #!  }
    #=====================================================
```

"reference: "M. Boglione, S. Melis & A. Prokudin **Phys. Rev. D 84, 034033 2011"**

"formula"
begin{align}
F_{UU} &= \sum_{q} \, e_q^2 \,\xbj \, f_1^{q}(\xbj)\,D_{h/q}(z_h)
\frac{e^{-\pth^2/\wpth}}{\pi\wpth}}, \\
\end{align} "

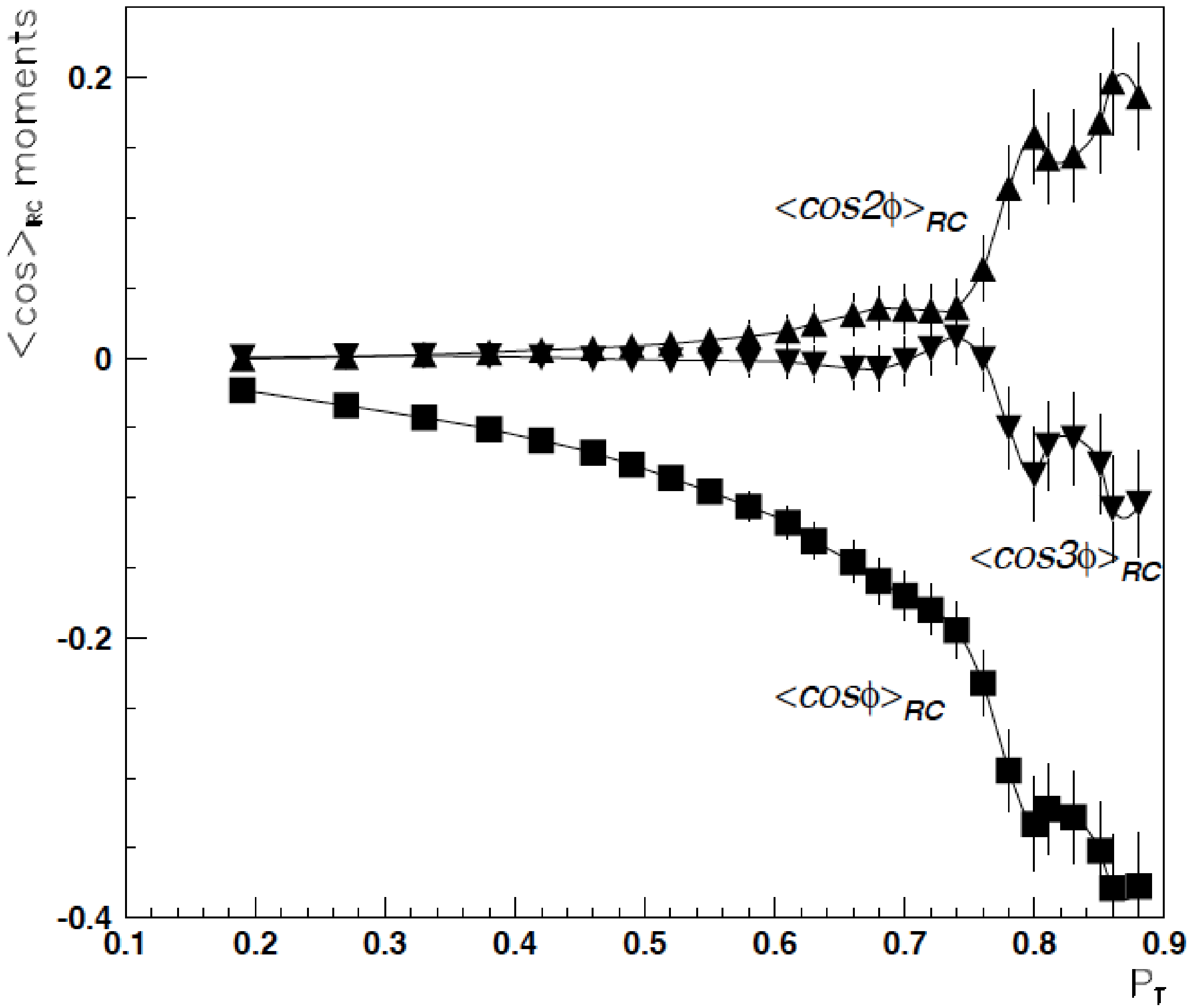
$$F_{UU} = \sum_q e_q^2 \, x \, f_1^q(x) \, D_{h/q}(z_h) \frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle}}{\pi\langle P_{h\perp}^2\rangle}$$

$$\langle P_{h\perp}^2\rangle^2 = z^2\langle k_{q,\perp}^2\rangle + \langle p_{q\to h\perp}^2\rangle$$

p1 p2

The p_T-dependence of the cross section

I.Akushevich

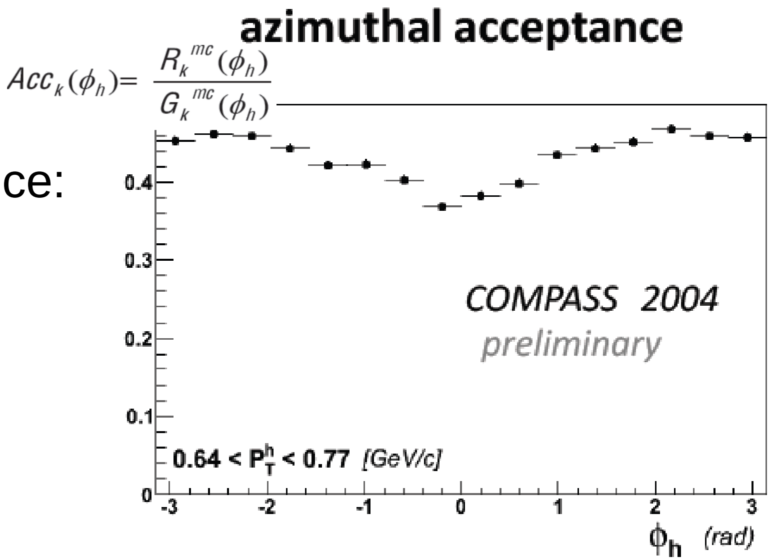


This plot illustrates that the RC may be very significant at large PT. Also the plot illustrates occurrence of the effects not observed at the level of the Born cross section (i.e., $< cos(3\phi) >$.

Extracting the moments

Moments mix in experimental azimuthal distributions

Acceptance:



Simplest correction $1 + A\cos\phi$

Correction to normalization

$$(1+\alpha\cos\phi)(1+A\cos\phi) \rightarrow 1 + A\alpha/2$$

$$(1+\beta\lambda\Lambda+\gamma\lambda\Lambda\cos\phi)(1+A\cos\phi)$$
$$\rightarrow 1 + (\beta+\gamma A/2)\lambda\Lambda$$

Correction to DSA

$$(1+S_T\delta\sin\phi_S)(1+A\cos\phi)$$
$$\rightarrow 1 + S_T/2\delta A(\sin\phi-\phi_S) + \ldots$$

Correction to SSA

Moments/asymmetries:

$$\frac{1+\beta\lambda\Lambda}{1+a\cos\phi} \rightarrow 1 - a\beta\lambda\Lambda\cos\phi$$

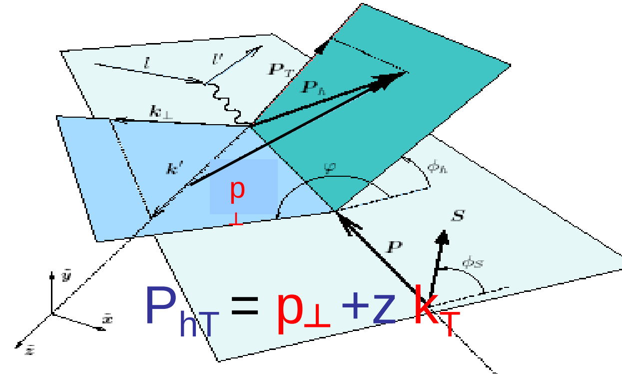
Fake DSA cos

Virtual photon angle:

$$\sin\theta_\gamma = \sqrt{\frac{4M^2x^2}{Q^2+4M^2x^2}\left(1-y-\frac{M^2x^2y^2}{Q^2}\right)}$$

Simultaneous extraction of all moments is important also because of correlations!

MC (level-I) for CLAS12



SIDIS MC in 7D $(x,y,z,\phi,\phi_S,p_T,\lambda,\pi)$

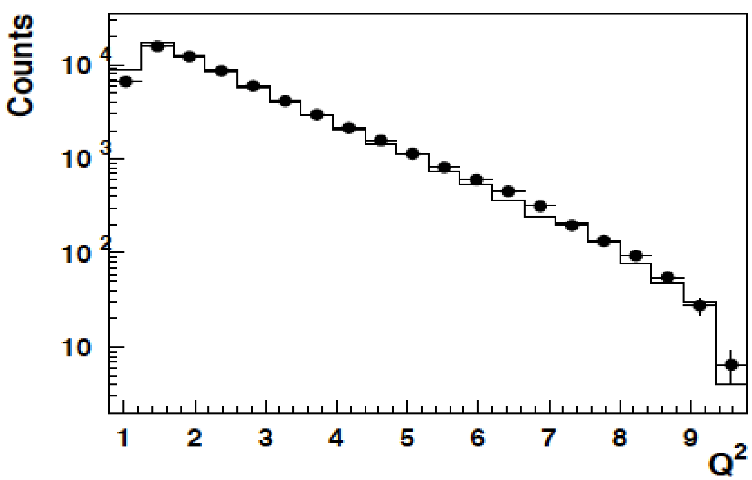
$$f_q(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

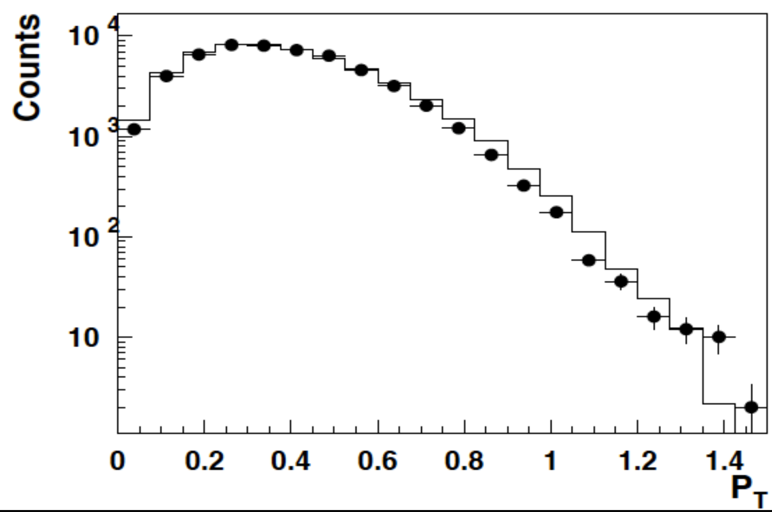
↓

CLAS12 acceptance & resolutions

↓

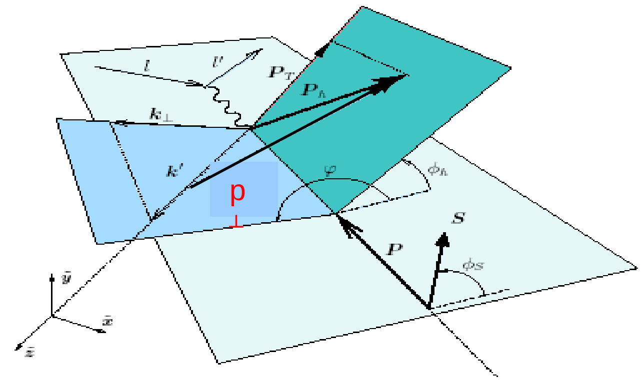
Events in CLAS12





Can achieve a reasonable agreement of kinematic distributions with realistic LUND simulation

MC(level-II) for CLAS12



SIDIS MC in 7D->9D

$P_{hT} = p_\perp + z\, k_\perp$

$$F_{UU,T} = x \sum_q e_q^2 \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^{(2)}(zk_\perp + p_\perp - P_{hT}) f^q(x,k_\perp) D^{q\to h}(z,p_\perp)$$

$$f_q(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

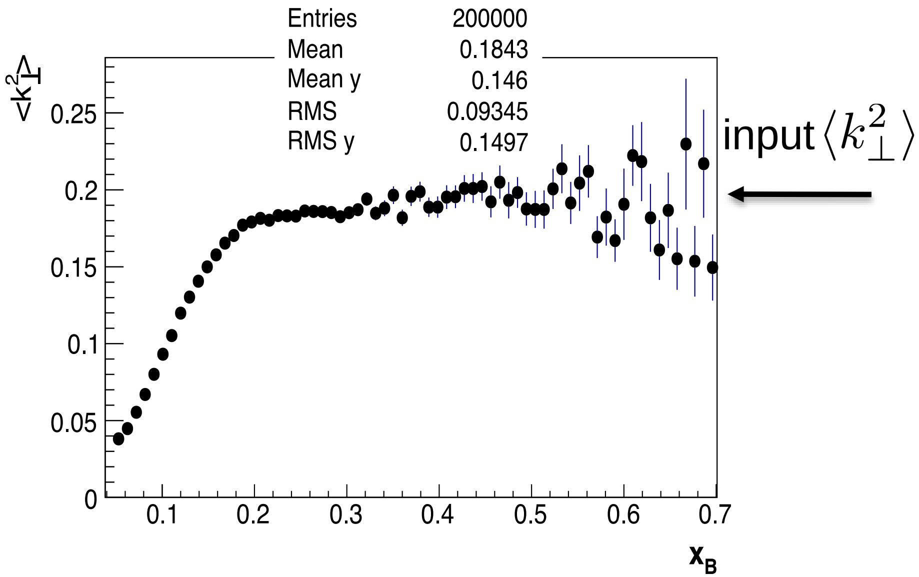
$$D^{q\to h}(z,p_\perp) = D^{q\to h}(z)\frac{1}{\pi\langle p_\perp^2\rangle}\exp^{-\frac{p_\perp^2}{\langle p_\perp^2\rangle}}$$

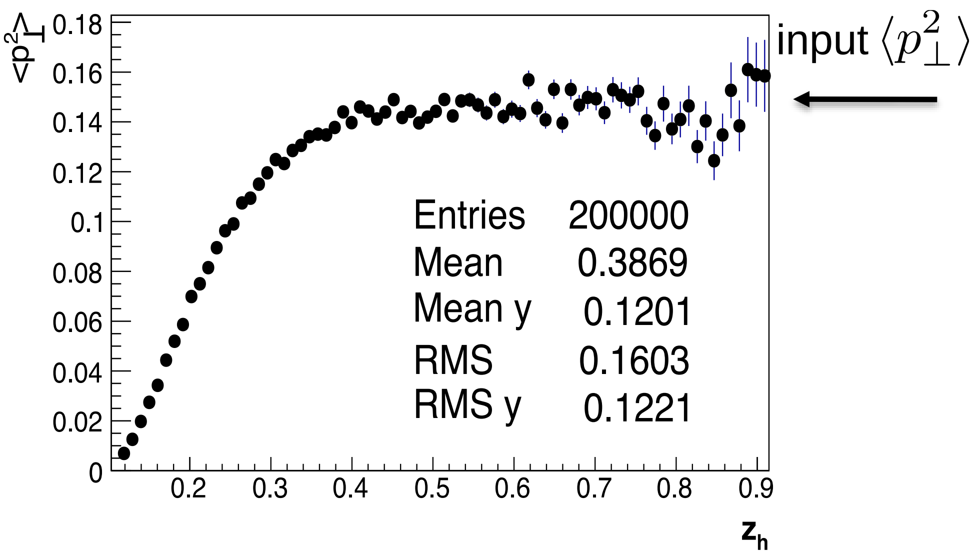
Not trivial to realize in a self consistent way,

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_l\,d\phi_h} \longrightarrow \frac{d\sigma}{dx\,dy\,dz\,dp_\perp^2\,dk_\perp^2\,d\phi_l\,d\phi_h\,d\phi_k}$$

what we learn starting MC at quark level?

Partonic Transverse Motion at 11 GeV





Kinematical limits on transverse momentum size provided by the parton model transfer directly to the experimental observables

Average values of the transverse momentums are not constant!

Analysis of azimuthal moments in SIDIS/HEP

Data (contains N events with 4 vectors of reconstructed particles, N~1B)

MC +RC (contains M events with 4 vectors of generated and reconstructed particles, M~10-100N)

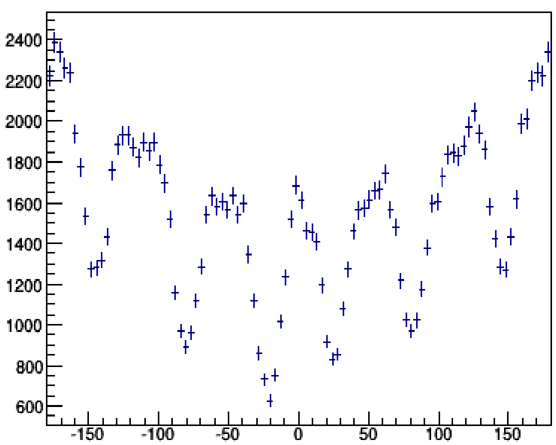
Define x-sections/normalized counts

Compare generated with reconstructed

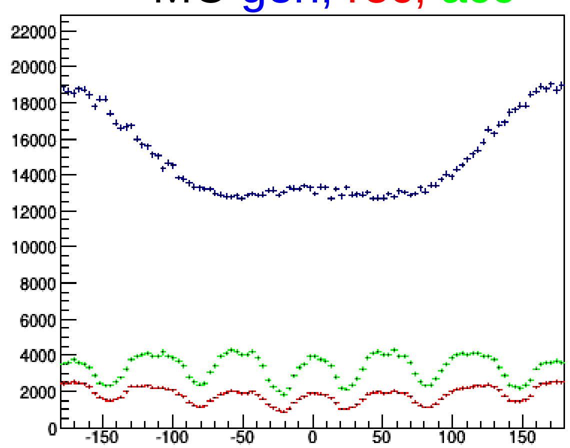
Counts in "small" bins in λ,Λ,x,y, [z,P_T][t],ϕ,RC corrected for detector acceptance and efficiency

Acceptance in "small" bins (counts in λ,Λ,x,y,[z,P_T][t], ϕ) defining reconstruction efficiency and material on path of leptons

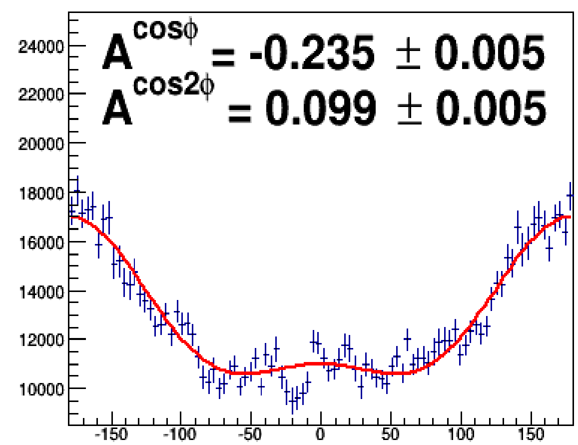
data



MC gen, rec, acc



corrected data

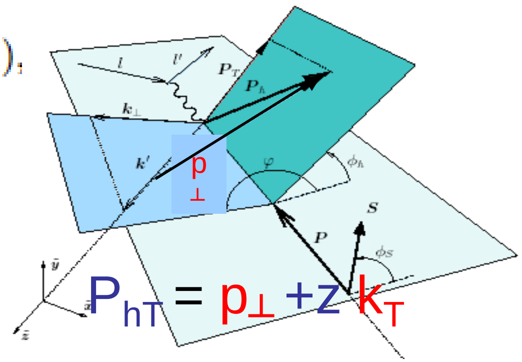


Experimental input to phenomenology: x-sections, moments

SIDIS with Bessel weighting

$$F_{UU,T} = x \sum_a e_a^2 \int d^2p_T\, d^2k_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

$$\delta^{(2)}(z\boldsymbol{p}_T + \boldsymbol{K}_T - \boldsymbol{P}_{h\perp}) = \int \frac{d^2\boldsymbol{b}_\mathbf{T}}{(2\pi)^2}\, e^{i\boldsymbol{b}_T(z\boldsymbol{p}_T + \boldsymbol{K}_T - \boldsymbol{P}_{h\perp})}$$

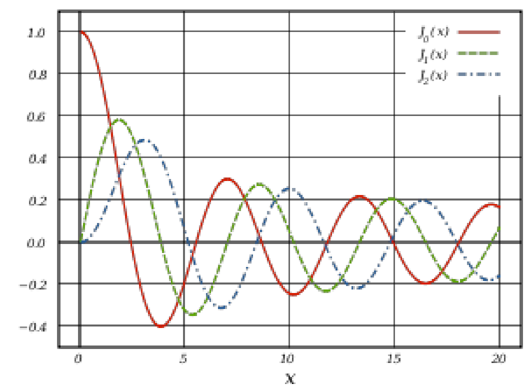


$$F_{UU,T} = x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, J_0(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\; \tilde{f}_1(x, z^2 b_T^2)\; \tilde{D}_1(z, b_T^2)$$

$$\int_0^\infty d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\, J_n(|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{b}_T|)\, J_n(|\boldsymbol{P}_{h\perp}|\,\mathcal{B}_T) = \frac{1}{\mathcal{B}_T}\delta(|\boldsymbol{b}_T| - \mathcal{B}_T)$$

$$\tilde{f}_1^q(x, z^2 b_T^2)\, \tilde{D}_1^{q\to\pi}(z, b_T^2)$$

$$\tilde{f}(x, b_T^2) \equiv \int d^2p_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\, f(x, \boldsymbol{p}_T^2) = 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f(x, \boldsymbol{p}_T^2)$$



$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, J_0(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\; \tilde{g}_{1L}(x, z^2 b_T^2)\; \tilde{D}_1(z, b_T^2)$$

- the formalism in **b_T-space** avoids convolutions
- provides a model independent way to study kinematical dependences of TMD

BGMP: extraction of k_T-dependent PDFs

Need: project x-section onto Fourier mods in b_T-space to avoid convolution

Boer, Gamberg, Musch &Prokudin arXiv:1107.5294

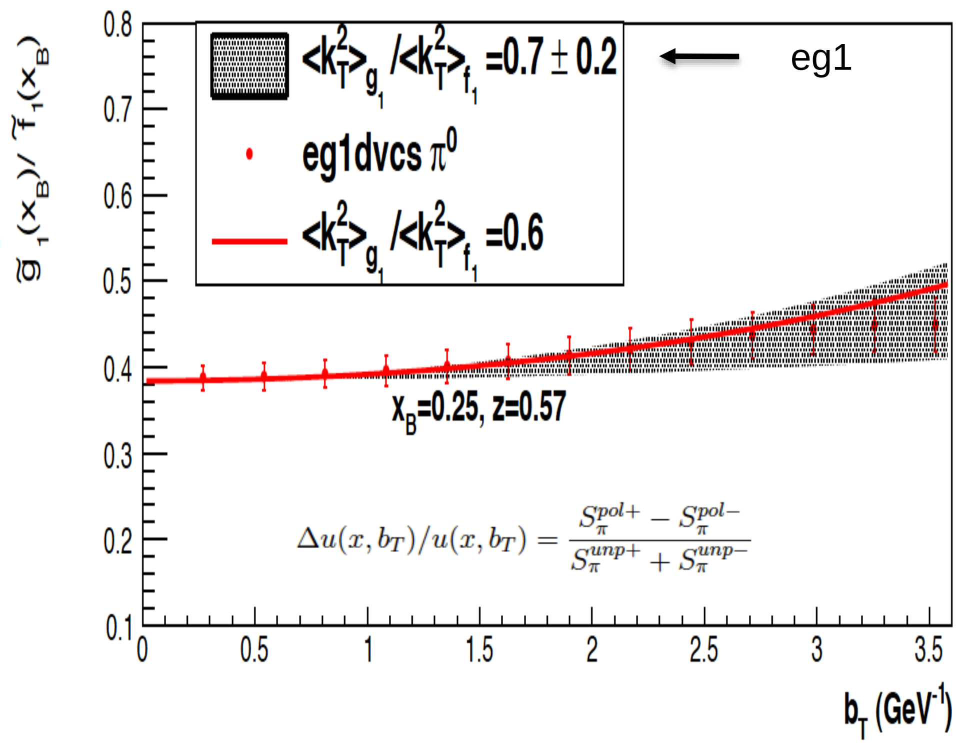
$$\int_0^\infty d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,J_0(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)\left[\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|}\right]$$

$$S_\pi^{unp\pm}(x_i,z_i,b_{Tj}) = \sum_{i=1}^{N_\pi^+/N_\pi^-} J_0(b_{Tj}P_{Ti})/\eta_i/A(x_i,y_i)$$

acceptance

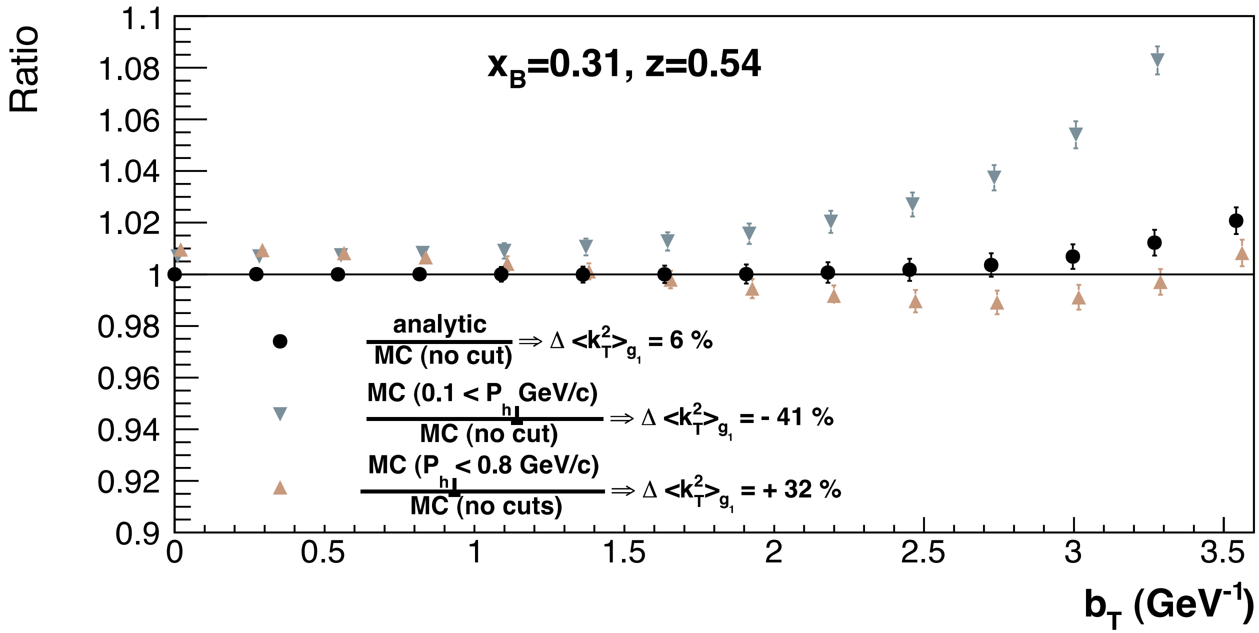
$$A(x,y) = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$

$$\tilde{f}_1^q(x,z^2b_T^2)\,\tilde{D}_1^{q\to\pi}(z,b_T^2)$$



- the formalism in **b_T-space** avoids convolutions
  →easier to perform a model independent analysis of TMDs
- Widths extracted from eg1dvcs π^0s consistent with eg1

Bessel method: sensitivity to cuts



- P_T cuts affects the value of extraction and the shape of b_T dependence!
- The correlation is direct consequence of the energy and momentum conservation when we account for intrinsic motion of the quarks
- The correlation is not sensitive to the details of the models used for the extraction.

Accounting for nuclear effects

[hep-ph/0801.0434].

Under the "maximal two gluon approximation", the TMD quark distribution in a nucleus for leading twist

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_q^N(x,\ell_\perp).$$

for higher twist

$$f_q^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \left(1+\frac{\Delta_{2F}}{2\vec{k}_\perp^2}\vec{k}_\perp\cdot\vec{\partial}_{k_\perp}\right) \int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_q^{\perp N}(x,\ell_\perp)$$

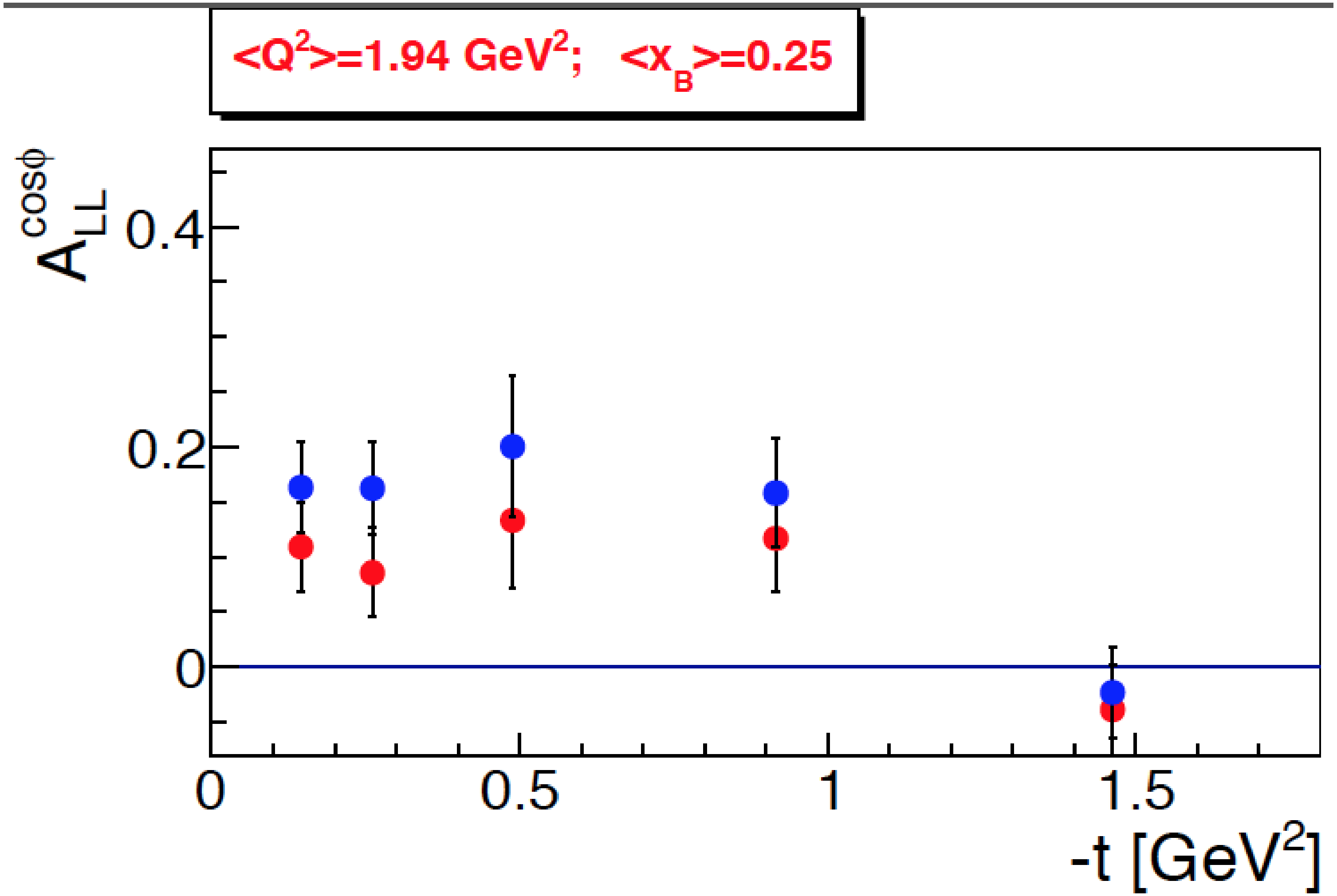
for simple Gaussian

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi(\langle k_\perp^2\rangle_{f_1}+\Delta_{2F})} f_q^N(x) e^{-k_\perp^2/(\langle k_\perp^2\rangle_{f_1}+\Delta_{2F})},$$

$$f_q^{\perp A}(x,k_\perp) \approx \frac{A\langle k_\perp^2\rangle_{f^\perp}}{\pi(\langle k_\perp^2\rangle_{f^\perp}+\Delta_{2F})^2} f_q^{\perp N}(x) e^{-k_\perp^2/(\langle k_\perp^2\rangle_{f^\perp}+\Delta_{2F})}.$$

The broadening width Δ_{2F} or the total average squared transverse momentum broadening, is given by the quark transport parameter depending on the spatial nucleon number density inside the nucleus and the gluon distribution function in a nucleon

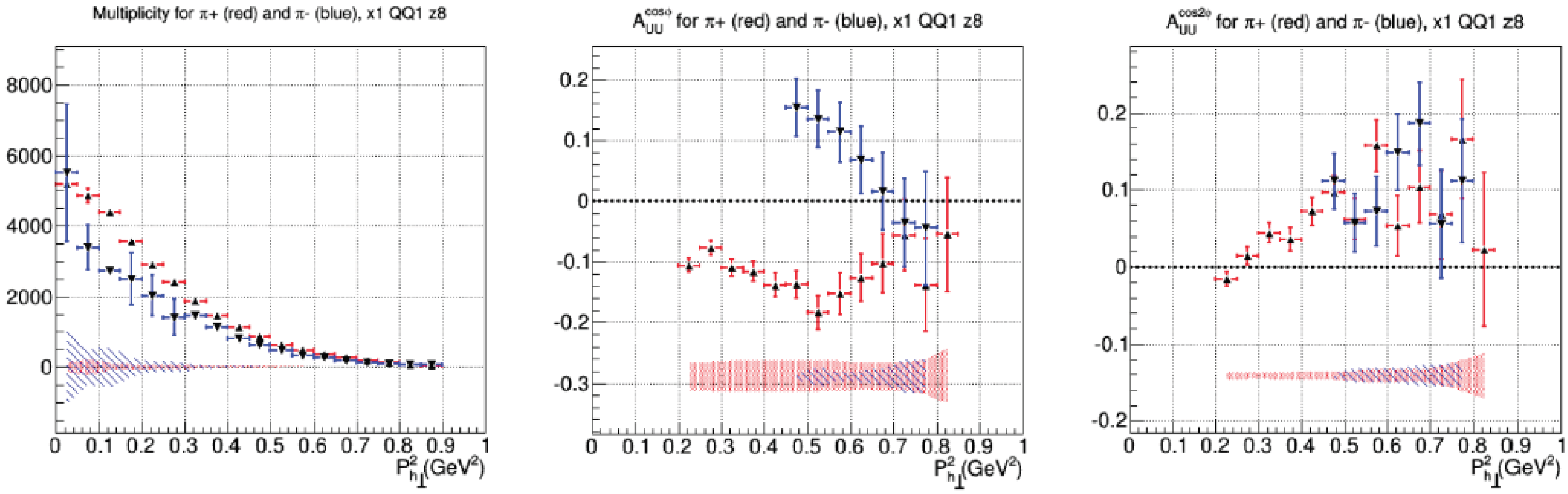
Correlations between moments



Unpolarized cosϕ (sets correspond to 0 and 0.1) , affects polarized sin2ϕ,cosϕ moments

Measuring SIDIS cross section

Fit with $a(1 + b\cos\phi_h + c\cos 2\phi_h)$

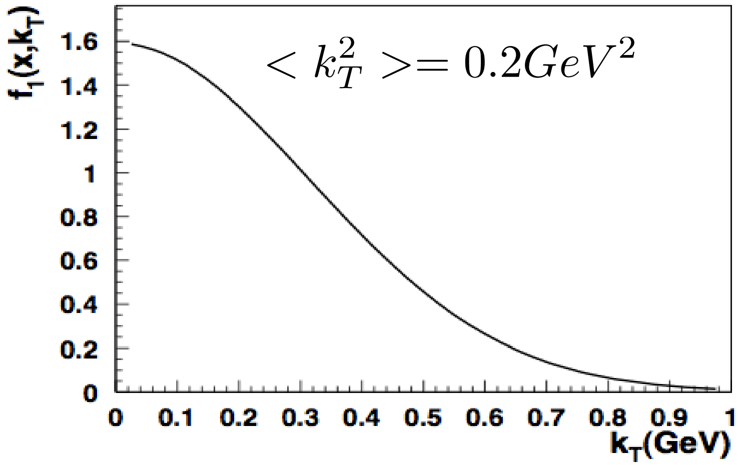


Simetric behaviour indicates large BM contribution

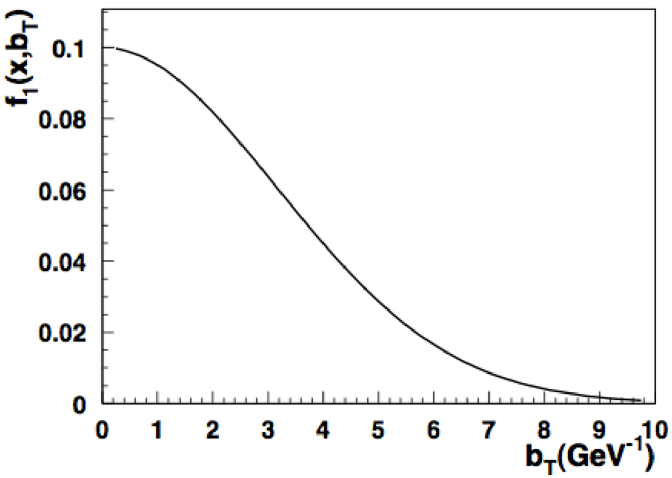
SIDIS with Bessel weighting

$$\tilde{f}(x,b_T^2) \equiv \int d^2 \boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \, f(x,\boldsymbol{p}_T^2) = 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x,\boldsymbol{p}_T^2)$$

$$f_1(x,k_T) = \frac{N}{\pi < k_T^2 >} e^{-\frac{k_T^2}{<k_T^2>}}$$



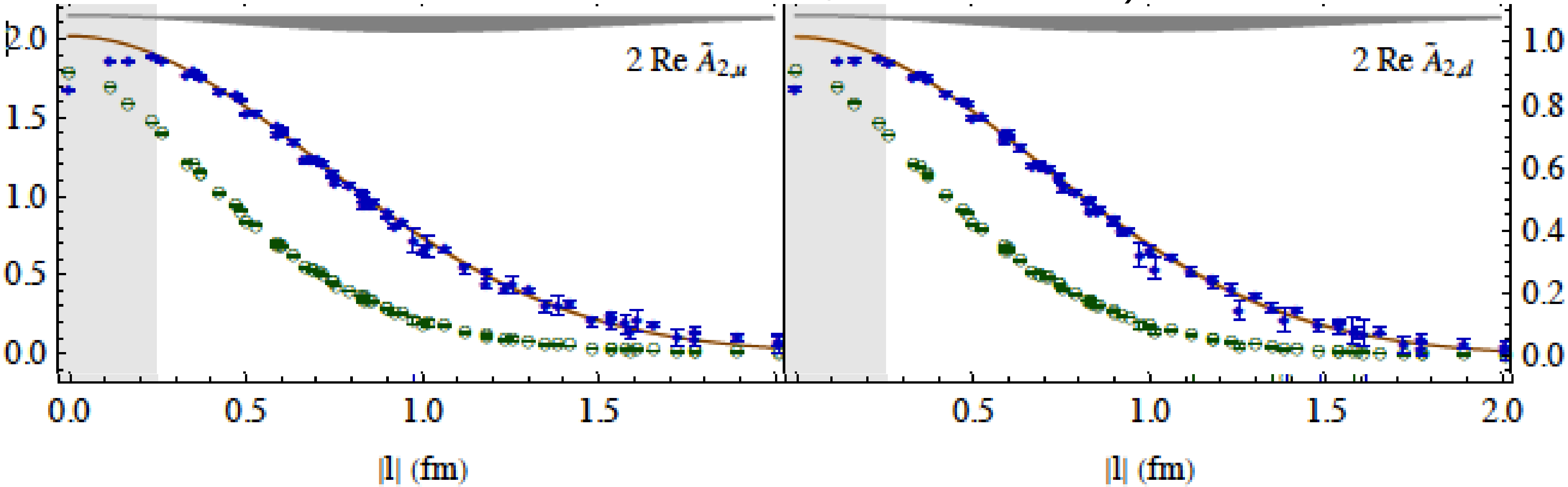
$$\tilde{f}_1(x,b_T^2) = \frac{1}{2} < k_T^2 > N e^{-\frac{<k_T^2>b_T^2}{4}}$$



•the data analysis can be performed in the **b_T-space**.

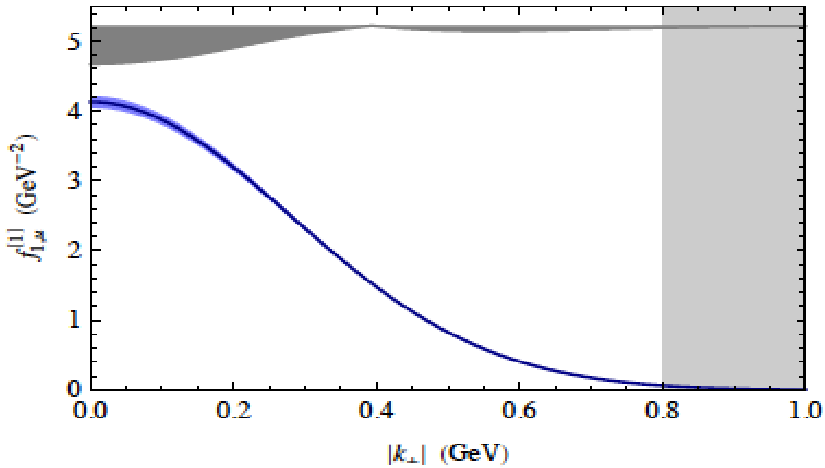
Lattice calculations and b_T-space

(PDFs in terms of Lorenz invariant amplitudes $\tilde{A}_i$
Musch et al, arXiv:1011.1213)



	c_2	σ_2
$\tilde{A}_{2,u}$	$2.0186 \pm 0.0063 \pm 0.0008$	$1.001 \pm 0.010 \pm 0.068$
$\tilde{A}_{2,d}$	$1.0171 \pm 0.0064 \pm 0.0005$	$0.975 \pm 0.012 \pm 0.063$

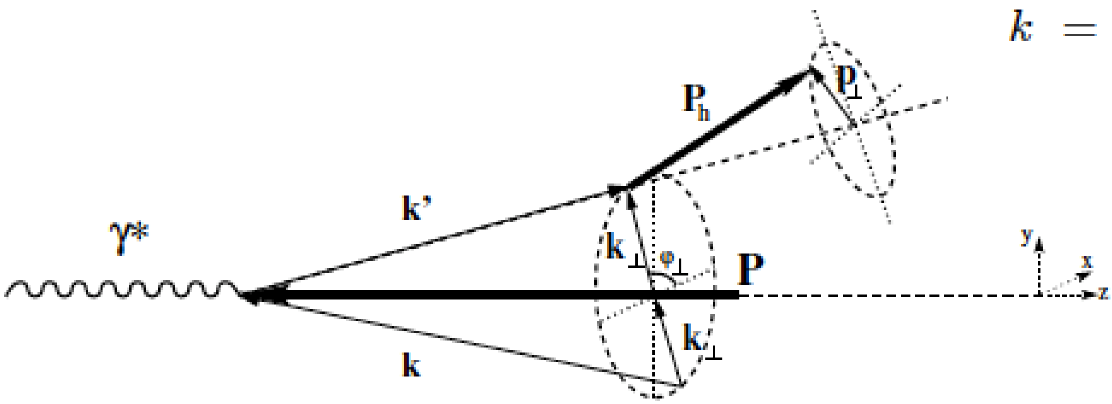
$$f_1^{[1]}(\boldsymbol{k}_\perp^2) = \frac{c_2\,\sigma_2^2}{4\pi} e^{-\frac{\boldsymbol{k}_\perp^2}{(2/\sigma_2)^2}}$$



Quarks Intrinsic Motion in MC

- New event generator based on M. Anselmino Phys. Rev. D, 71, 7, 2005 is developed (non zero hadrons mass approximation).
- As an input user can give his preferable distribution and fragmentation functions.

$$\frac{d\sigma}{dx\,dy\,dz\,dp_\perp^2\,dk_\perp^2\,d\phi_l\,d\phi_h\,d\phi_k} = K(x,y) \times [f_1(x,k_\perp)D_1(z,p_\perp) + ...]$$



$$k = \left(xP_0 + \frac{k_\perp^2}{4xP_0}, \boldsymbol{k}_\perp, -xP_0 + \frac{k_\perp^2}{4xP_0}\right)$$

Quark light-cone momentum fraction

$$x = \frac{1}{2}x_B\left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}}\right)$$

$$x \simeq x_B\,, \qquad z \simeq z_h \qquad \boldsymbol{p}_\perp \simeq \boldsymbol{P}_T - z_h\boldsymbol{k}_\perp\,. \qquad \mathcal{O}(k_\perp^2/Q^2).$$

Kinematic correlations at finite Q^2

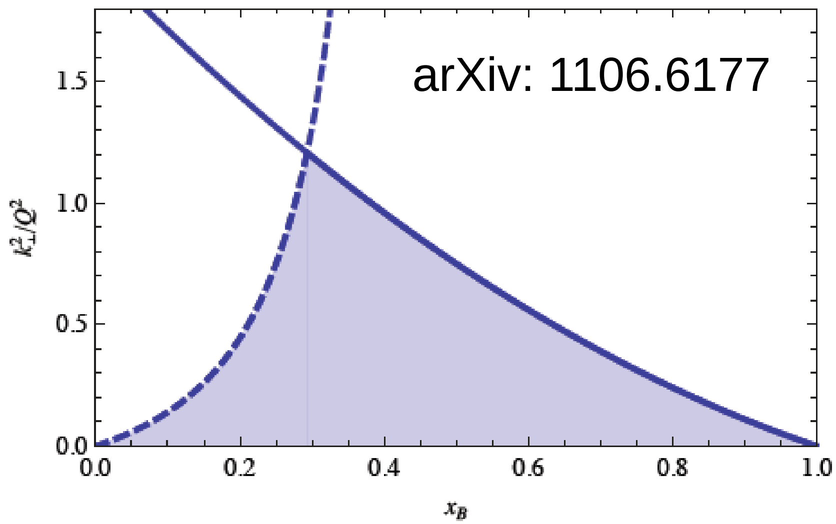
From energy/momentum conservation

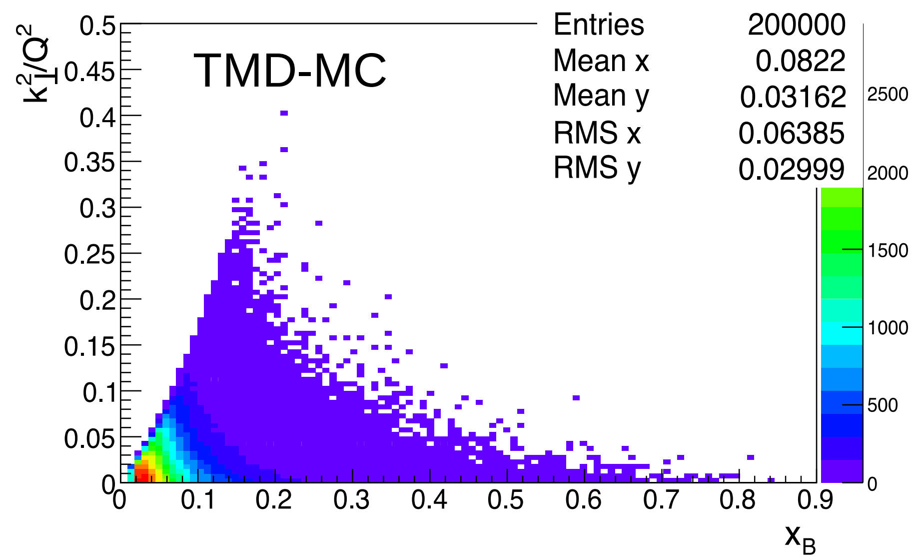
$$f_q(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$xP_0 + \frac{k_\perp^2}{4xP_0} \le P_0 \;\Rightarrow\; k_\perp^2 \le 4x(1-x)P_0^2$$

$$\Rightarrow\; k_\perp^2 \le \frac{x(1-x)}{x_B(1-x_B)}Q^2$$

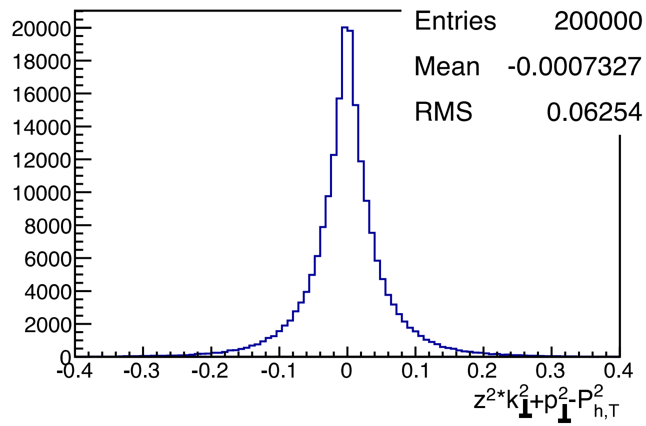
$$k_\perp^2 \le (2-x_B)(1-x_B)Q^2$$

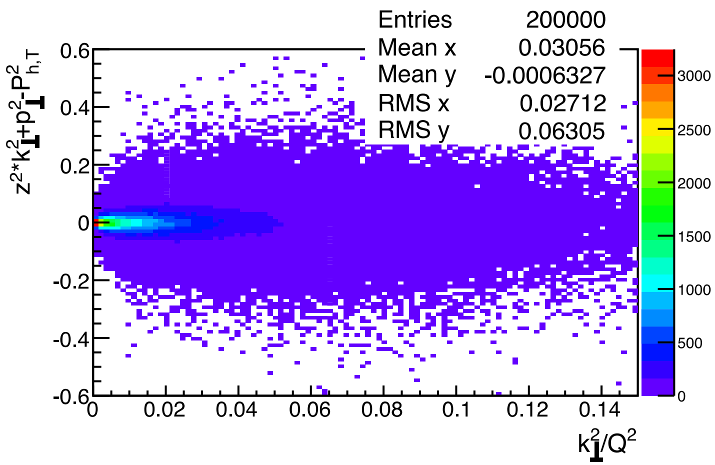




x and k_T are not independent at low Q^2 even in factorized Gaussian approach!

Output of MC in terms of physics





Well known $\delta(z^2k_\perp^2 + p_\perp^2 - P_{h,T}^2)$ function for each event and its dependence from $k_\perp^2/Q^2$ shows clear peak and smaller sigma at low $k_\perp^2/Q^2$, where TMD Factorization holds.

BGMP: extraction of k_T-dependent PDFs

Need: project x-section onto Fourier mods in b_T-space to avoid convolution

Boer, Gamberg, Musch &Prokudin arXiv:1107.5294

$$\int_0^\infty d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,J_0(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)\left[\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|}\right]$$

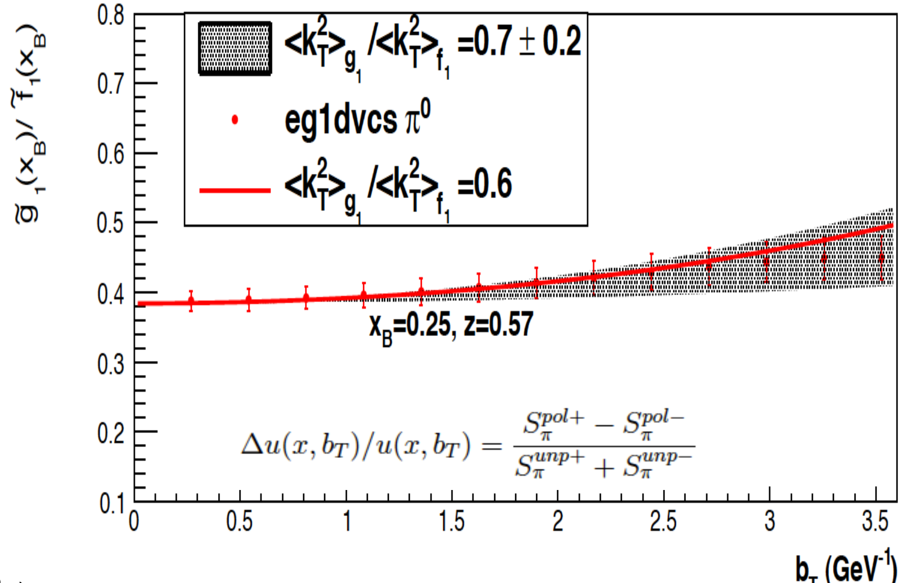
$$S_\pi^{unp\pm}(x_i, z_i, b_{Tj}) = \sum_{i=1}^{N_\pi^+/N_\pi^-} J_0(b_{Tj}P_{Ti})/\eta_i/A(x_i, y_i)$$

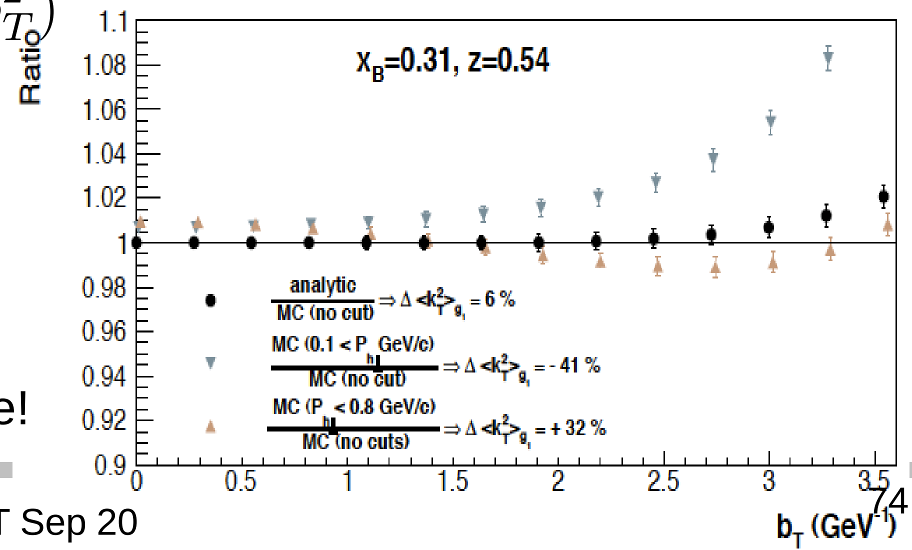
acceptance

$$A(x,y) = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$

$$\tilde{f}_1^q(x, z^2 b_T^2)\tilde{D}_1^{q\to\pi}(z, b_T^2)$$

- the formalism in **b_T-space** avoids convolutions $\rightarrow$ easier to perform a model independent analysis of TMDs





P_T cuts affect not only on the value of extraction also the shape of b_T dependence!

BGMP: extraction of k_T-dependent PDFs

Need: project x-section onto Fourier mods in b_T-space to avoid convolution

Boer, Gamberg, Musch &Prokudin arXiv:1107.5294

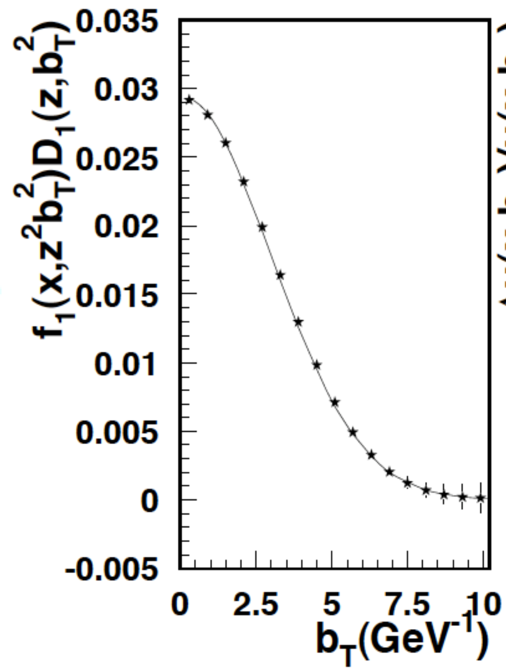
$$\int_0^\infty d|P_{h\perp}|\,|P_{h\perp}|\,J_0(|P_{h\perp}||b_T|)\left[\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|P_{h\perp}|d|P_{h\perp}|}\right]$$

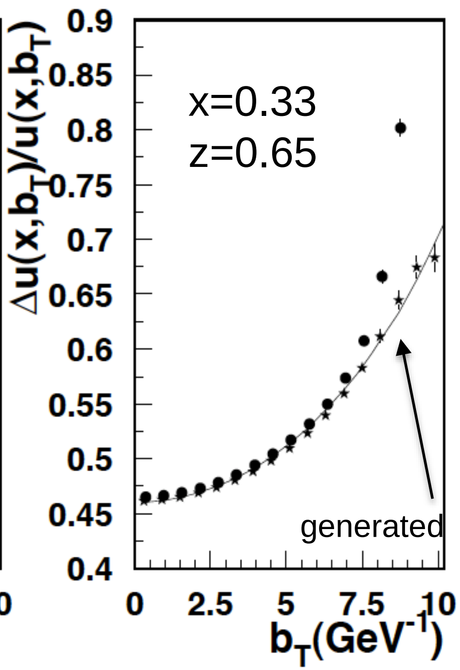
$$S_\pi^{unp\pm}(x_i,z_i,b_{Tj}) = \sum_{i=1}^{N_\pi^+/N_\pi^-} J_0(b_{Tj}P_{Ti})/\eta_i/A(x_i,y_i)$$

acceptance

$$A(x,y) = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$

$$\tilde{f}_1^q(x,z^2b_T^2)\,\tilde{D}_1^{q\to\pi}(z,b_T^2)$$





$$\Delta u(x,b_T)/u(x,b_T) = \frac{S_\pi^{pol+} - S_\pi^{pol-}}{S_\pi^{unp+} + S_\pi^{unp-}}$$

- BGMP provides a model independent way to extract k_T-dependences of helicity distributions
- requires wide range in hadron P_T

BGMP: extraction of k_T-dependent TMDs

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, Mz\, \tilde{f}_{1T}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)$$

$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|\, J_0(|b_T||P_{h\perp}|)\, \tilde{g}_{1L}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)\ ,$$

$$F_{LT}^{\cos(\phi_h-\phi_s)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, Mz\ \tilde{g}_{1T}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)\ ,$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, M_h z\, \tilde{h}_1(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)\ ,$$

$$F_{UU}^{\cos(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T||P_{h\perp}|) M M_h z^2\, \tilde{h}_1^{\perp(1)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)\ ,$$

$$F_{UL}^{\sin(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T||P_{h\perp}|) M M_h z^2\, \tilde{h}_{1L}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)\ ,$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^4 J_3(|b_T||P_{h\perp}|) \frac{M^2 M_h z^3}{4} \tilde{h}_{1T}^{\perp(2)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)\ .$$

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

- BGMP provides a model independent way to extract k_T-dependences of TMD
- requires wide range in hadron P_T