Equation of State and Uncertainty Quantification in Hot and Dense Matter

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March 1, 2017

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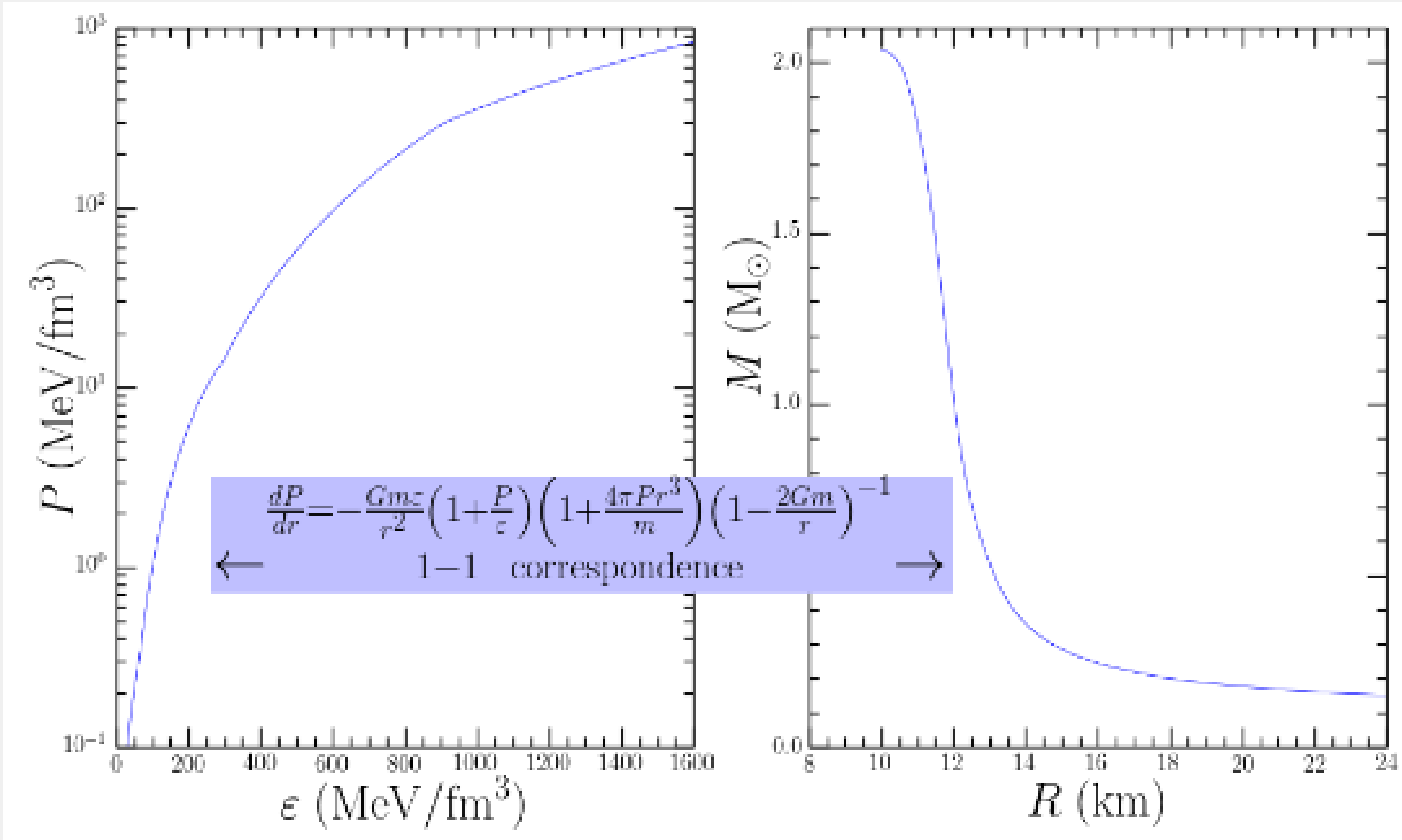
Outline

- Neutron star structure
- Masses and radii and the equation of state
- Bayesian Inference
- Inverse Problems
- Correlations
- Uncertainty
- New EOSs
- Unphysical Correlations
- Accreting NSs

Neutron Star Masses and Radii and the EOS

- Neutron stars (to better than 10%) all lie on one universal mass-radius curve

  (Largest correction is rotation)



- Two 2 $M_\odot$ neutron stars

  Demorest et al. (2010), Antoniadis et al. (2013)

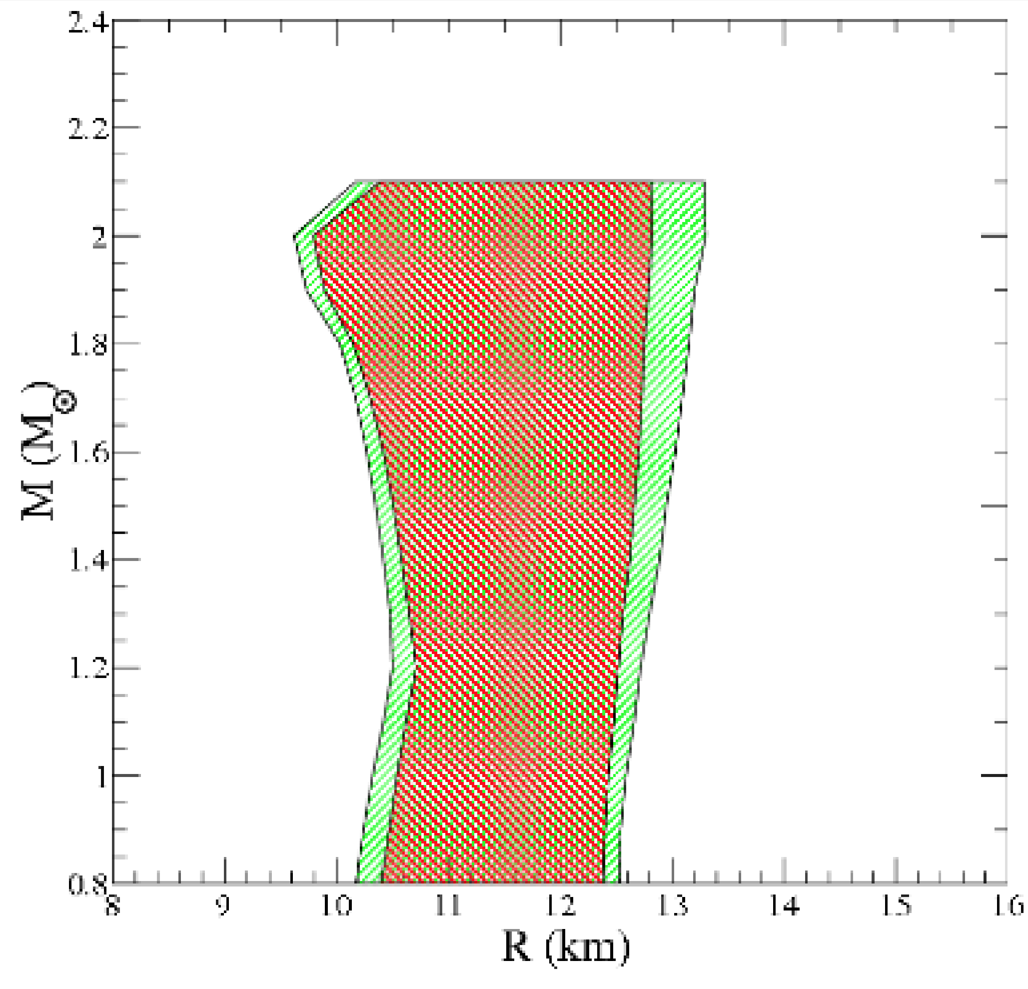
- As of 2007, neutron star radii ranged from 8-16 km

  Lattimer and Prakash (2007)

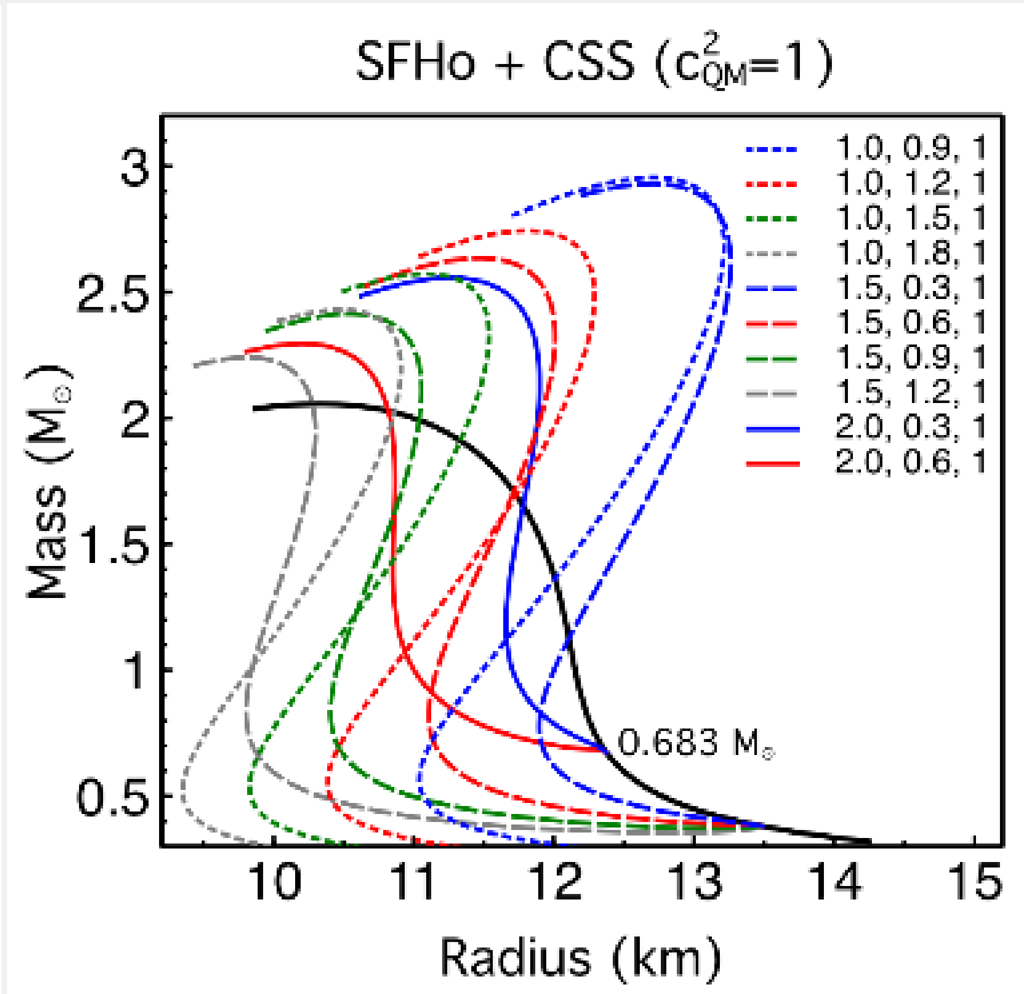
- Now 10-13 km is more likely

  Steiner, Lattimer, and Brown (2013)

M-R and EOS results



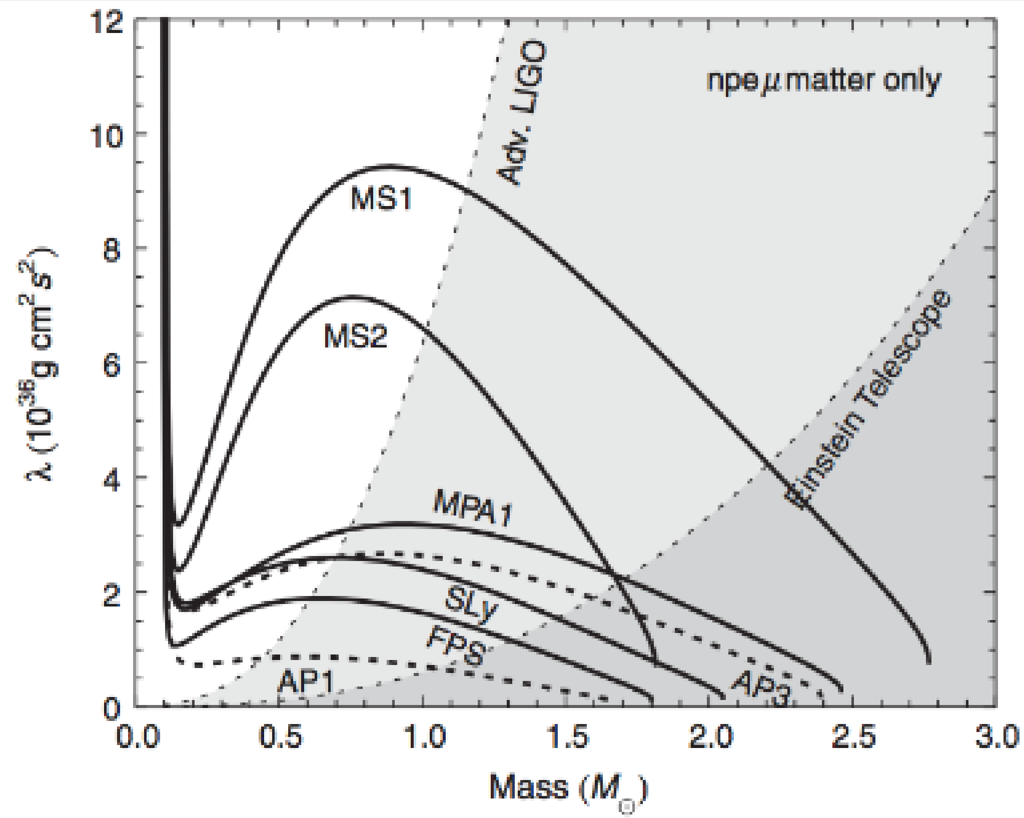
Steiner, Lattimer, and Brown (2013)



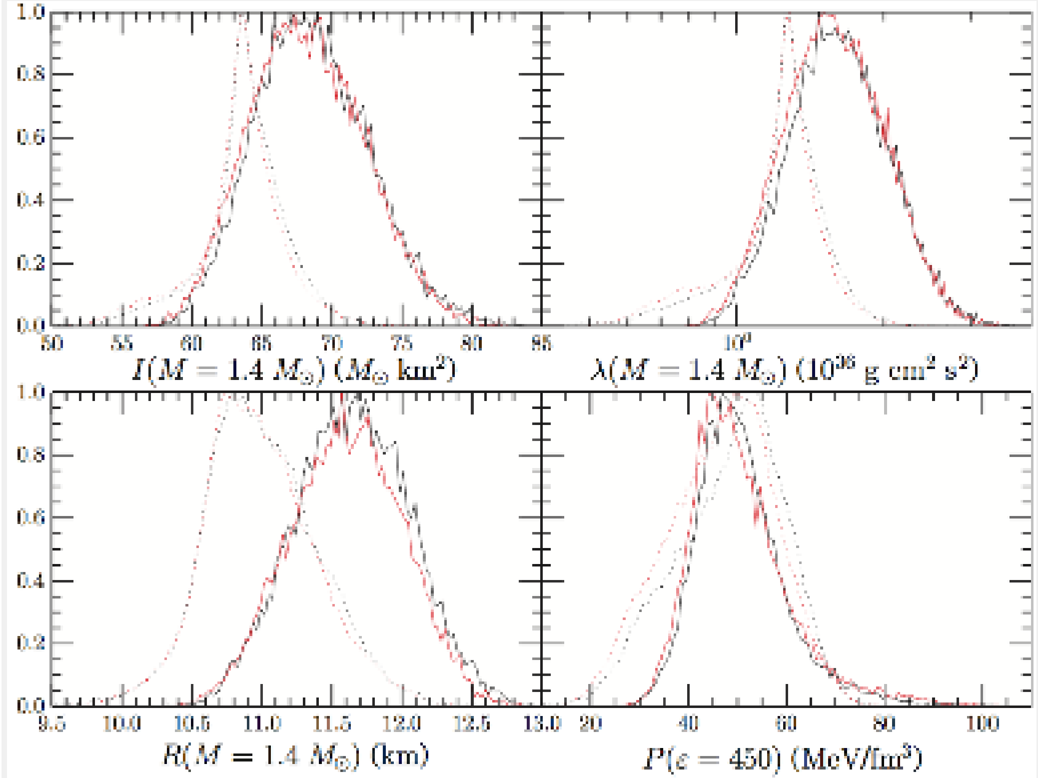
Han, Du, and Steiner

- 10 – 13 km radii
- From 2013, but this result still stands
- Affects the amount of mass ejected in a merger

Tidal deformabilities



Hinderer (2010)



Steiner et al. (2015)

- Large tidal deformabilities also ruled out
- Information on the EOS and moments of inertia from neutron star radius measurements

Bayesian Inference vs. χ^2 fitting

$$\chi^2 = \sum_i \left[\frac{(\text{data})_i - (\text{model})_i}{(\text{err})_i}\right]^2 \quad ; \quad \mathcal{L} = \exp(-\chi^2/2)$$

- Bayes theorem

$$P[\mathcal{M}_i|D] \propto P[D|\mathcal{M}_i]P[\mathcal{M}_i] = \mathcal{L} \times \text{prior}$$

- Determine parameters through marginalization, i.e.

$$P(\mathcal{M}_i^0) = \int \delta(\mathcal{M}_i - \mathcal{M}_i^0)P[D|\mathcal{M}_i]P[\mathcal{M}_i]\, d\mathcal{M}$$

- Iterative process of improving probability distributions

$$P[\mathcal{M}_i|D_1 D_2 \ldots D_N] = P[D_N|\mathcal{M}_i]P[D_{N-1}|\mathcal{M}_i] \ldots P[D_1|\mathcal{M}_i]P[\mathcal{M}_i]$$

- Coherence: probability has a sensible meaning
- "Inverse problem": determine unobserved probability distributions

Propagating Uncertainties in the R-process

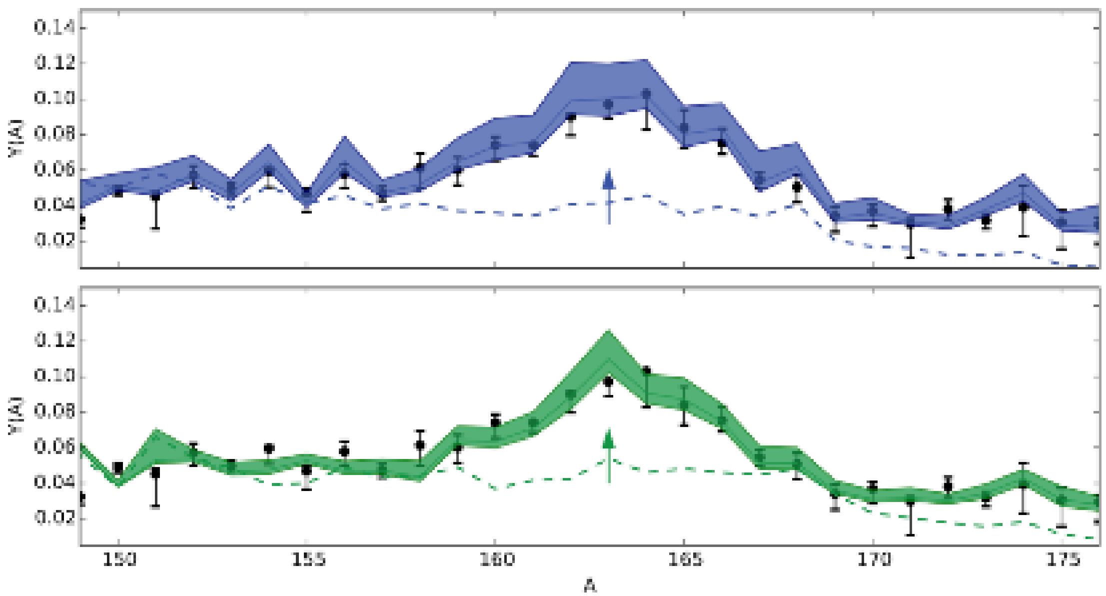


Figure 9. Final abundance predictions in the rare earth region for individual trajectories: hot - traj. 1 (top panel), cold - traj. 4 (middle panel) and very neutron-rich cold - traj. 8 (bottom panel) Metropolis runs. The application of the reverse engineering framework starts with the baseline (dashed curves) and produces the shaded region in each case.

Mumpower, McLaughlin, Surman, and Steiner (2016)

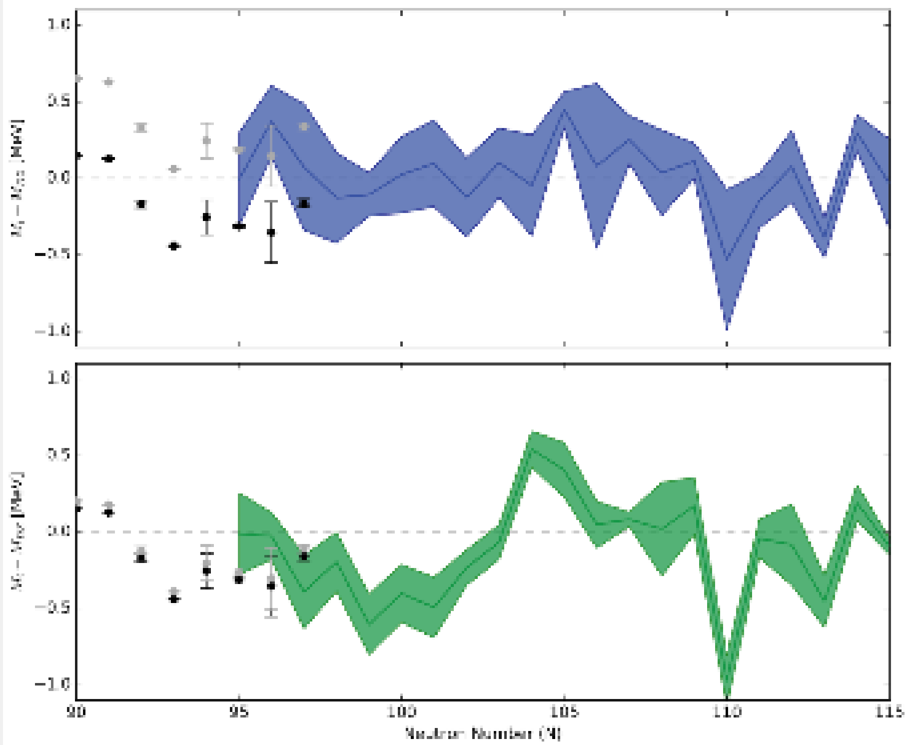


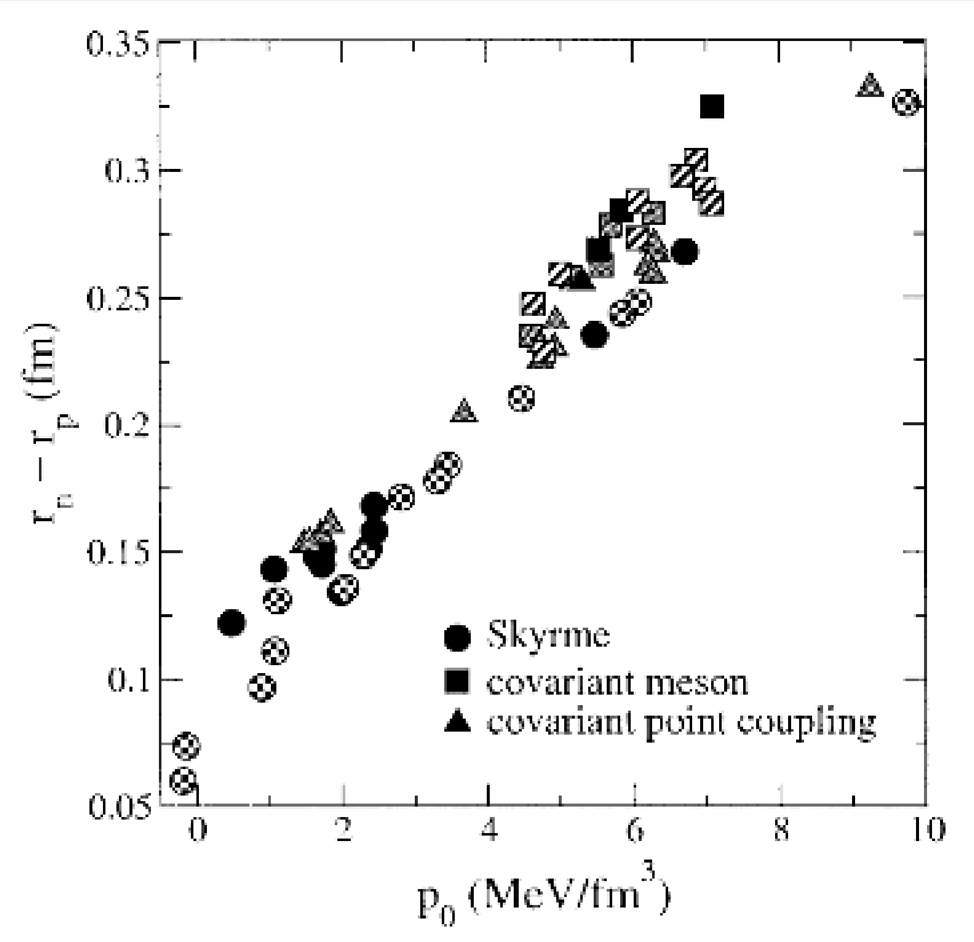
Figure 11. Combined mass surfaces for hot (top panel), cold (middle panel), and very neutron-rich cold (bottom panel) r-process conditions assuming a persistent feature ($Z = 40$) in proton number. The error band in these calculations represents a spreading of the parameter space which produces the rare earth peak due to the differences in evolution of similar astrophysical conditions.

Mumpower, McLaughlin, Surman, and Steiner (2016)

- R-process abundances in the rare earth peak are best reproduced by particular masses in neutron-rich nuclei
- ...depending on the astrophysical site of the r-process
- Demonstration of an inverse problem

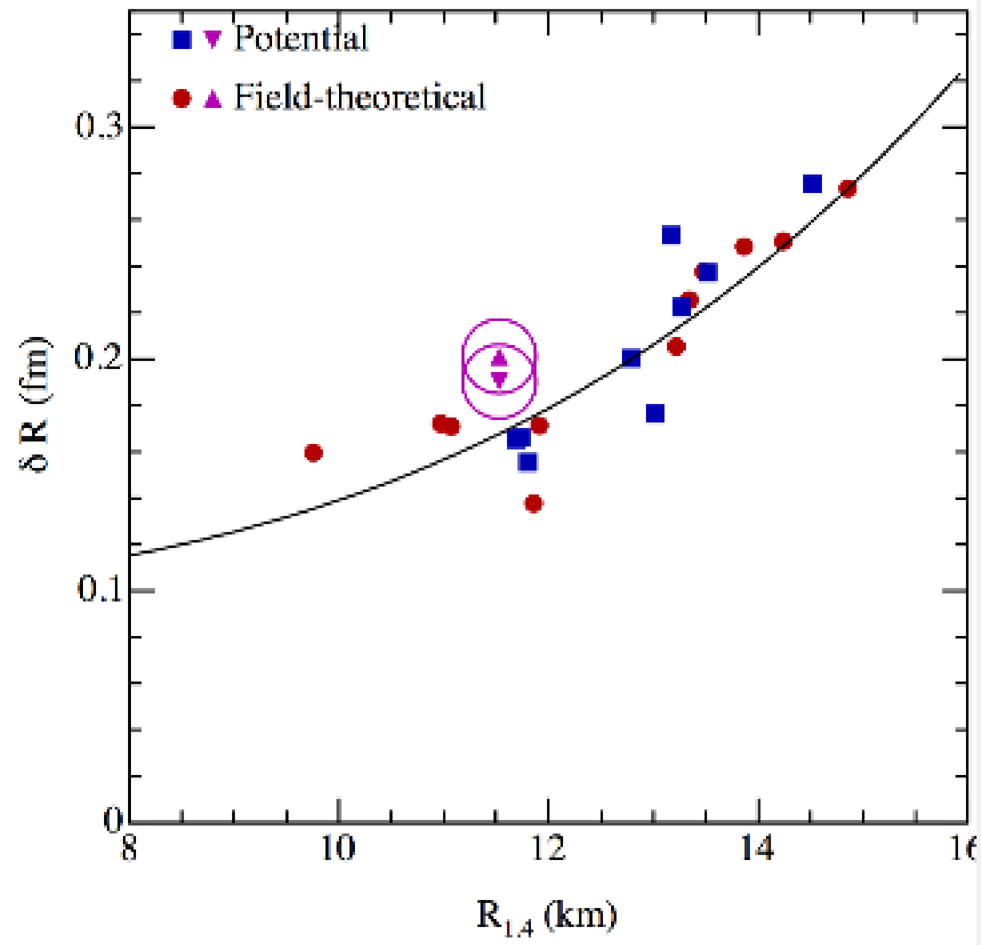
Correlations and/or "Universal Relations"

- Take advantage of relationships between observables
- Quantification of our current theoretical ignorance



Furnstahl (2002)

(p_0 is related to L)

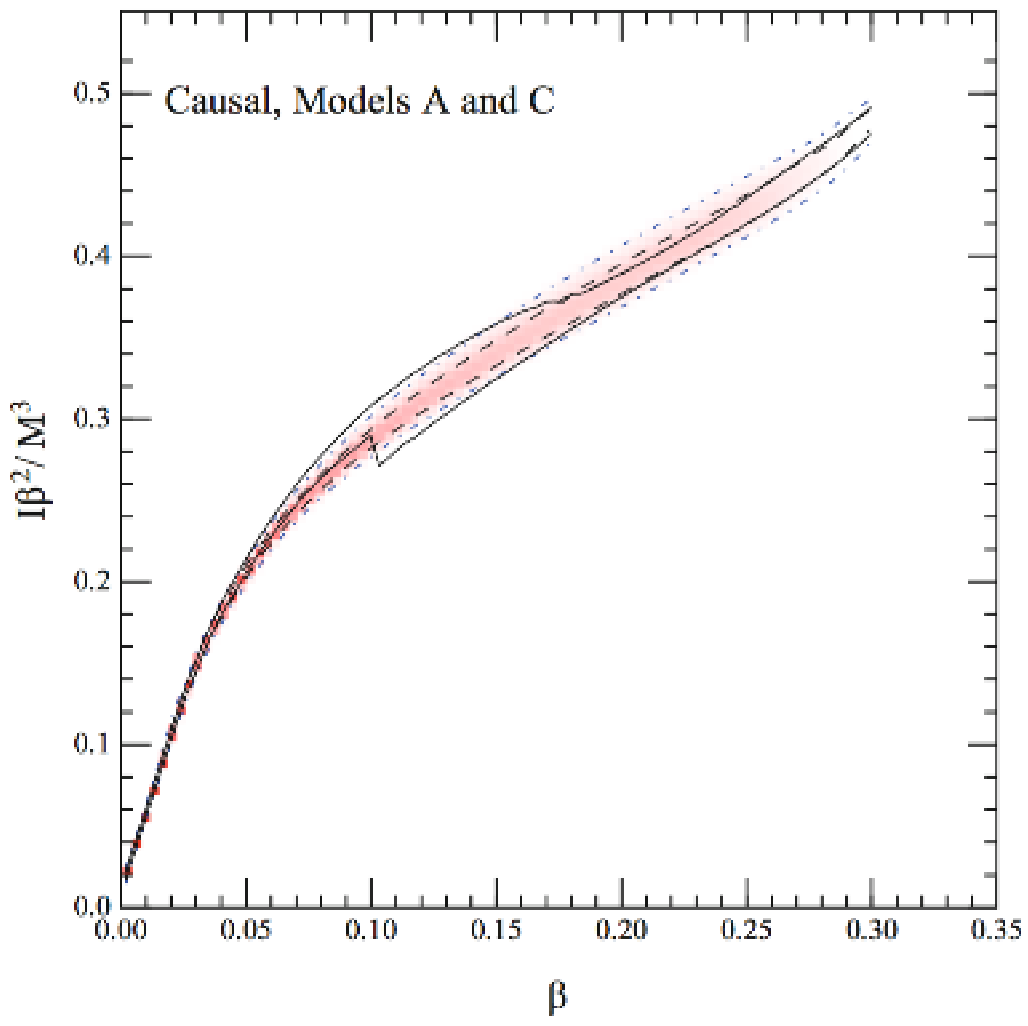


Steiner et al. (2005)

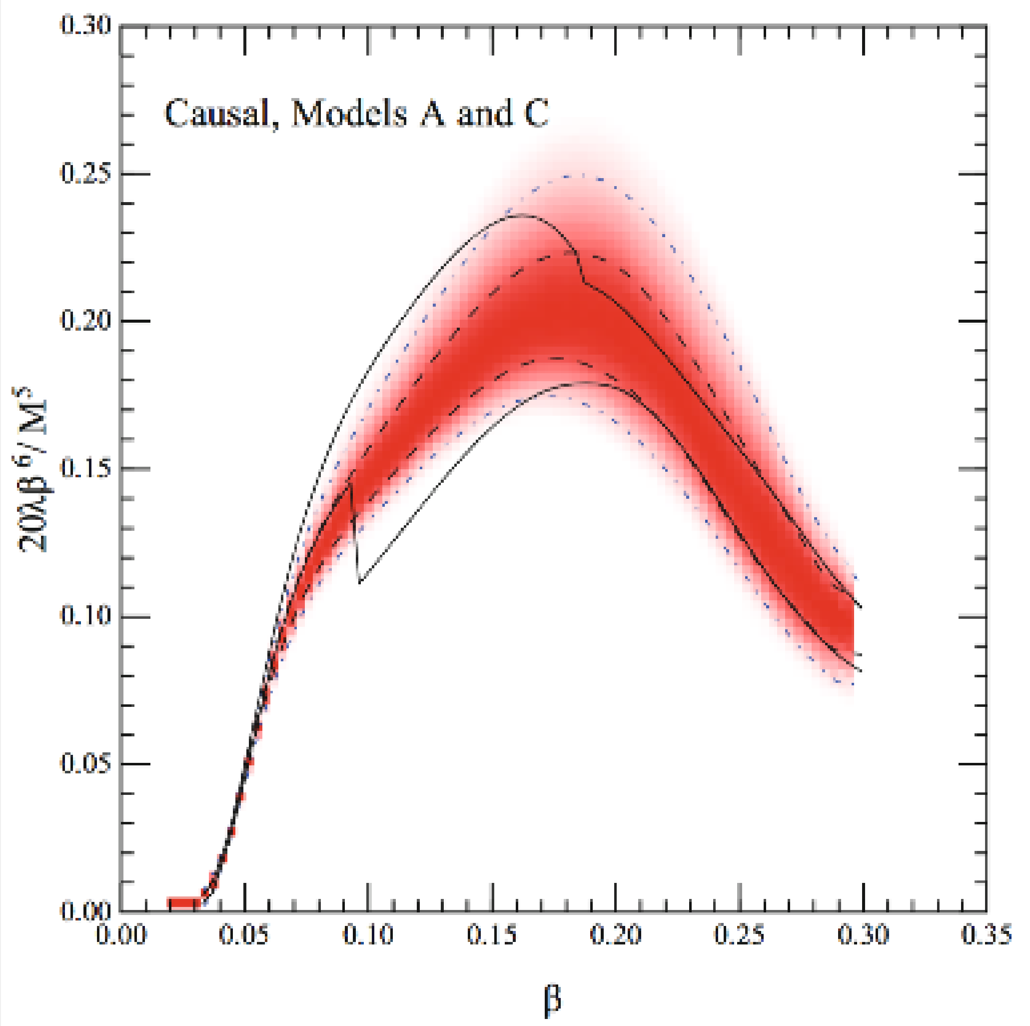
based on Horowitz and Piekarewicz (2001)

- 2-3 models and a dozen parameterizations (not optimally chosen) for each model
- Plots like this can be dangerous!

Correlations without Radius Information



Steiner, Lattimer, and Brown (2016)



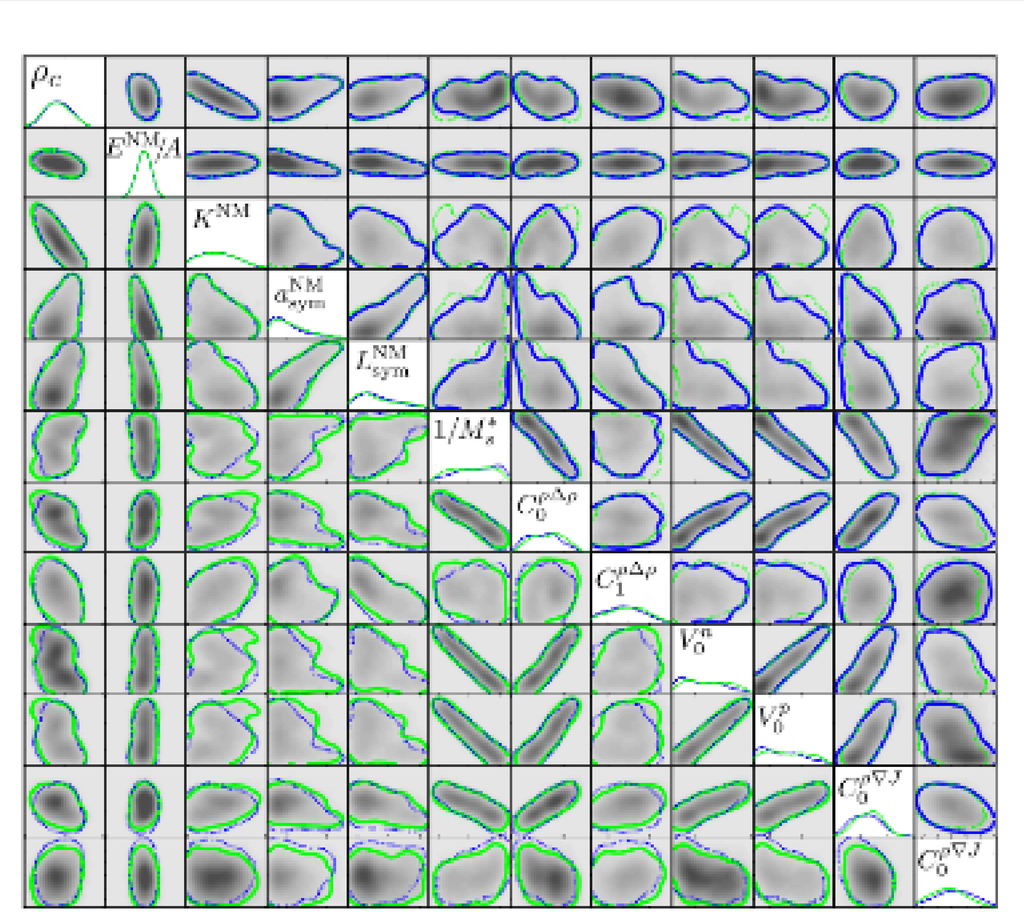
Steiner, Lattimer, and Brown (2016)

- Moment of inertia and tidal deformability correlated with compactness
- Millions of parameterizations

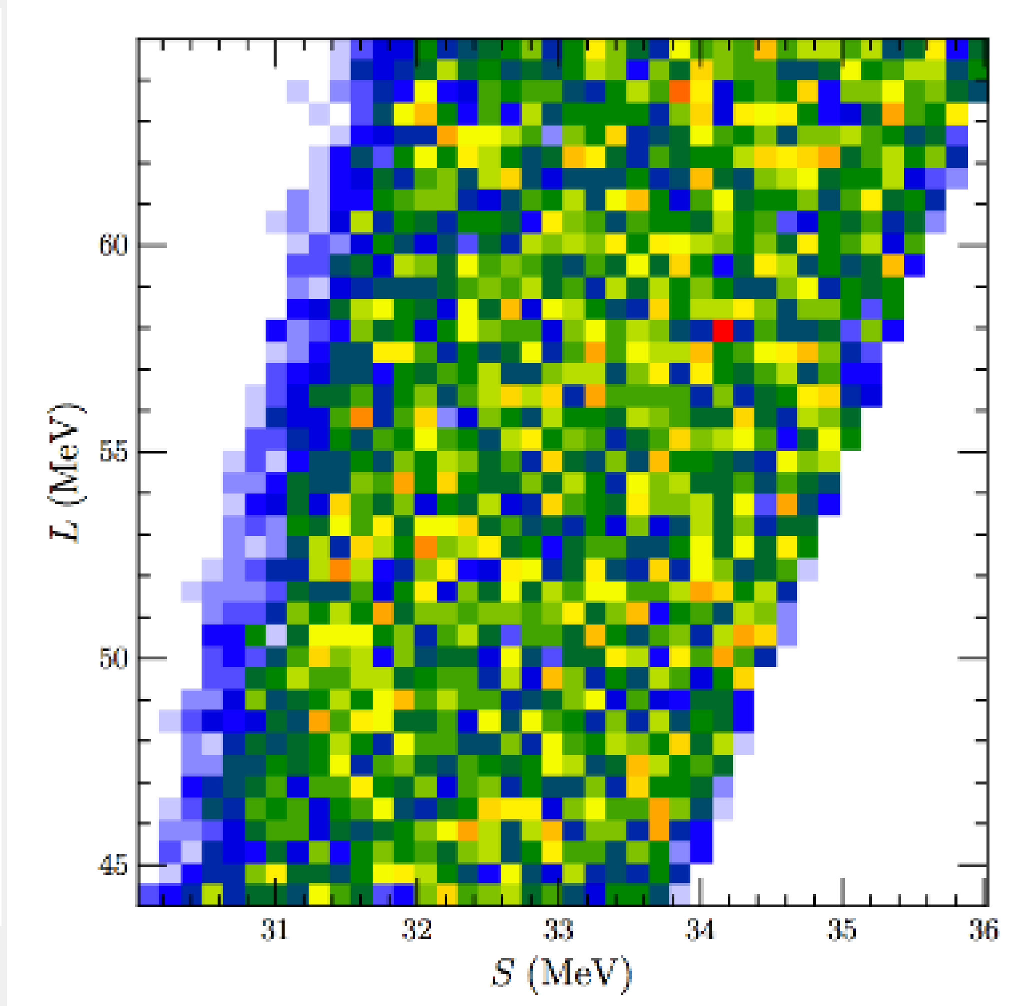
Correlations to Uncertainty

- Correlations describe the shape of our probability distributions
- The peak of the probability distribution is the "best fit"
- Correlations describe the shape of our uncertainties
- 1. Discovering new correlations and testing old correlations requires we characterize our uncertainties
- 2. Long-term goal is a large inverse problem: determine unobserved probabilities from merger data

"Thousands" of New Equations of State



McDonnell et al. (2015)



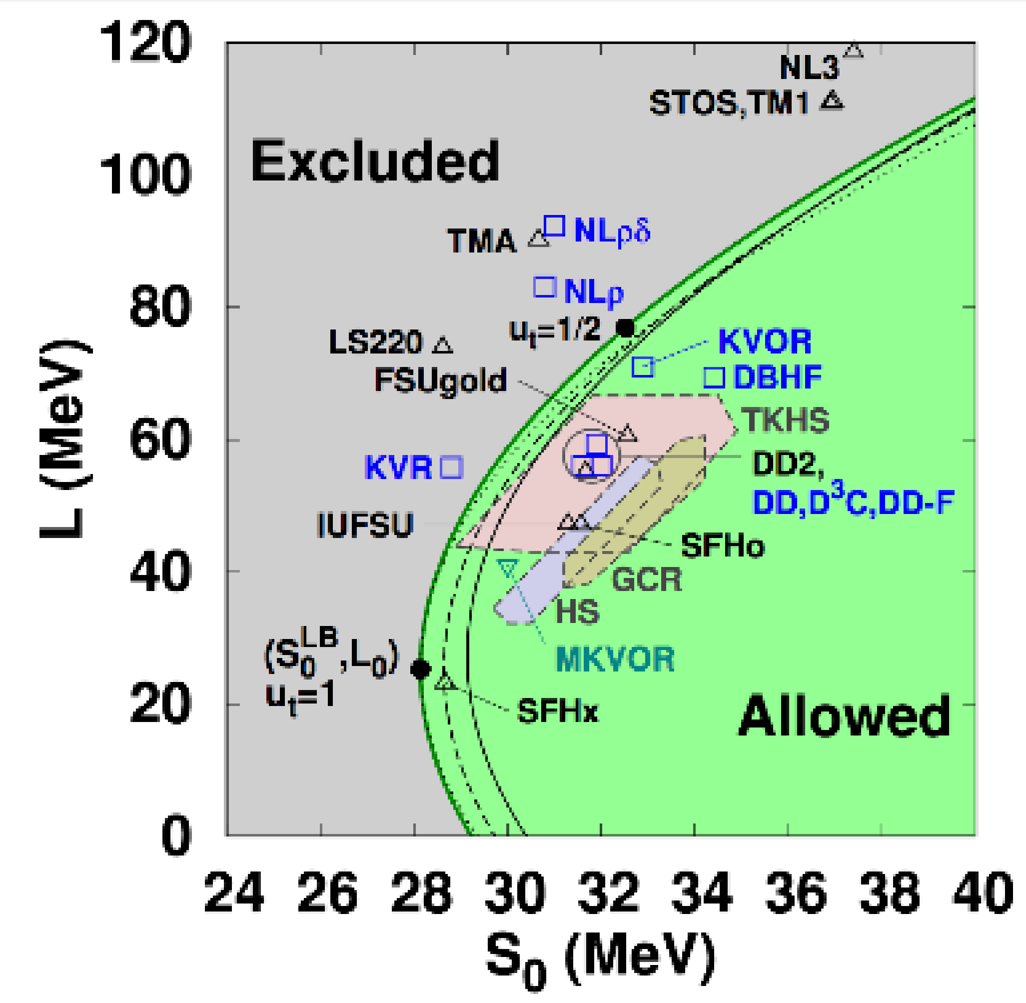
Du and Steiner

- Take advantage of previous steps in an iterative Bayesian process
- Bayesian inference based on nuclear masses and charge radii

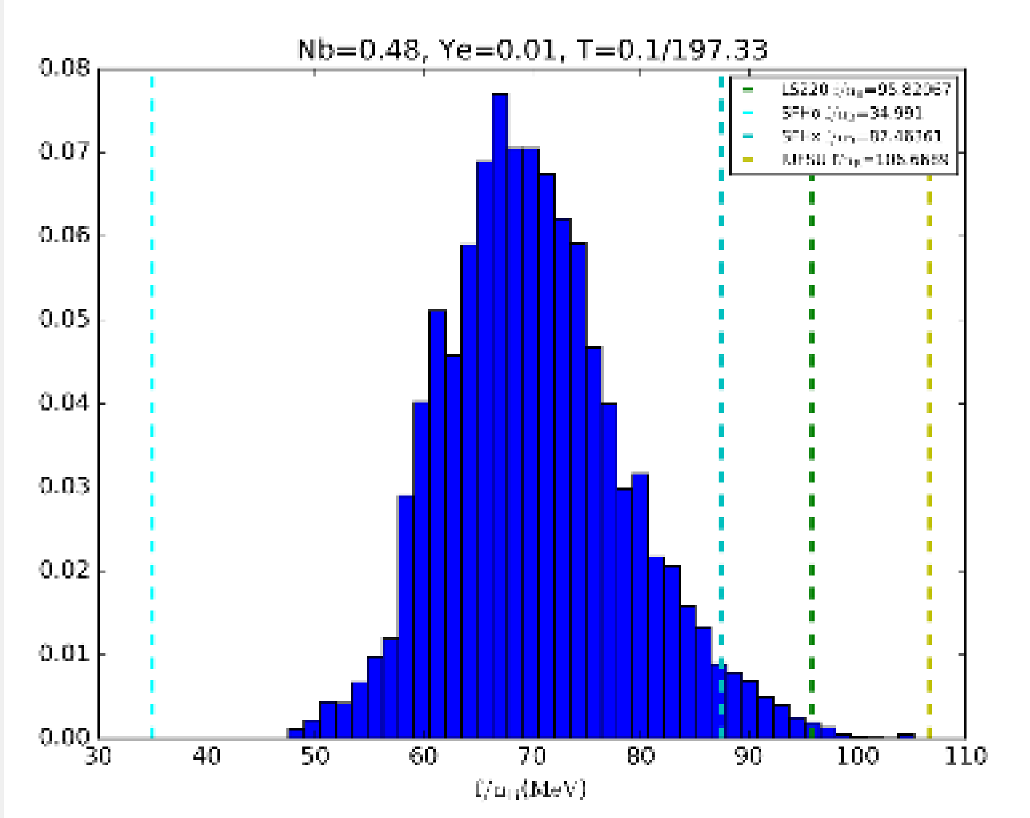
- Nuclei do not accurately specify S, L, or M^*
- Results from quantum Monte Carlo

"Thousands" of New Equations of State II

- Consistency with virial EOS, neutron star observations, and more



Kolomeitsev et al. (2017)



Very preliminary results from Du and Steiner

- Will automatically satisfy these constraints (as before)

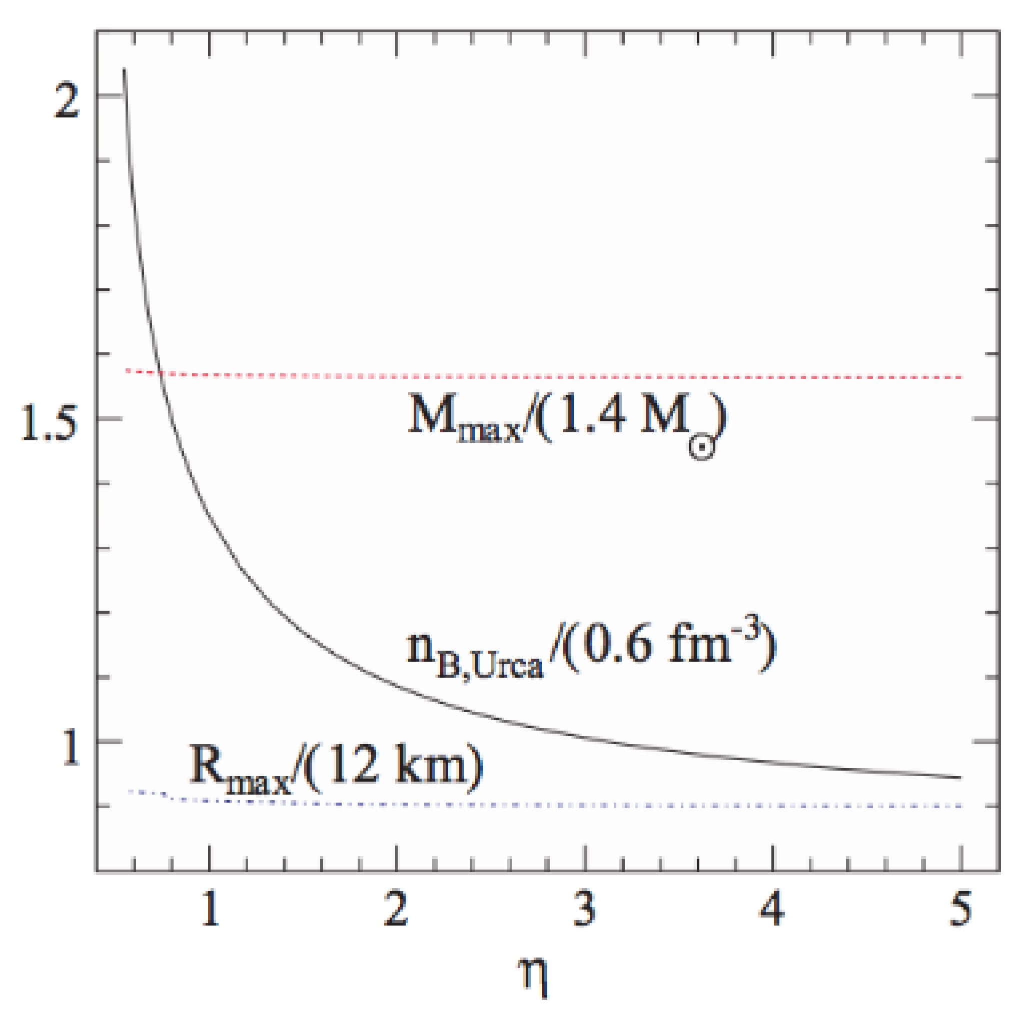
- One plot like this for every point in (n_B, Y_e, T) space

- (Location of vertical lines may be incorrect)

Correlations From Uncertainty Underestimates

$$(E/A)(n,\alpha) = (E/A)_{\rm nuc}(n,\alpha) + \alpha^2 S(n) + \alpha^4 Q(n)$$

- Below saturation, quartic terms are likely "small", above saturation densities, they may be important
- Only weak connection between radius and proton fraction (at any density)
- More recent work (Holt et al. 2016) shows this expansion is not analytic



Steiner (2006); $T = 0$ threshold for direct Urca is related to proton fraction

Heating Curves

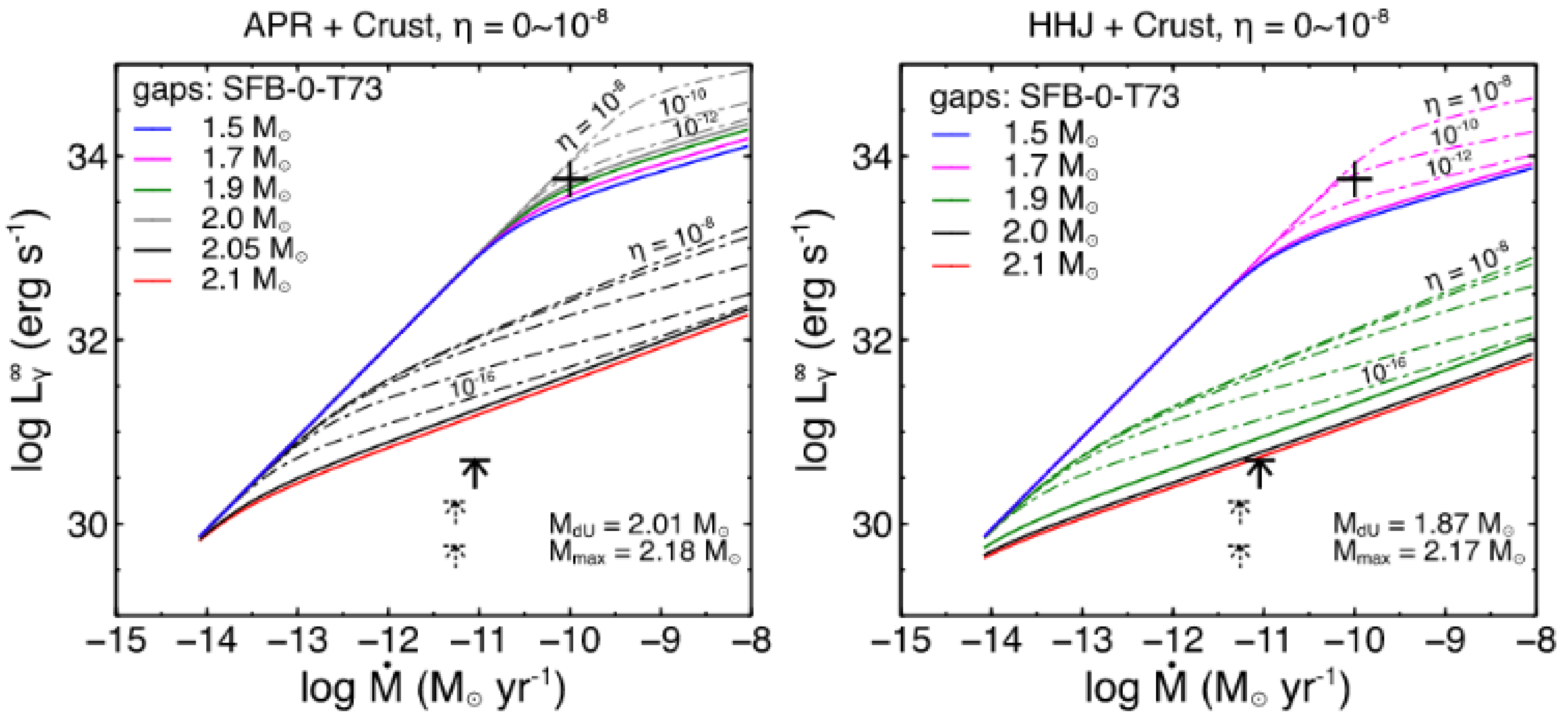


FIG. 4: (Color online) Heating curves for APR/HHJ EoS, with superfluidity models "SFB-0-T73" (vanishing n^3P_2 gap). Effects from light-element layer in the envelope (see Eq. (7)) are shown for two different masses (APR: $2.0\,M_\odot, 2.05\,M_\odot$; HHJ: $1.7\,M_\odot, 1.9\,M_\odot$) below and above the direct Urca onset.

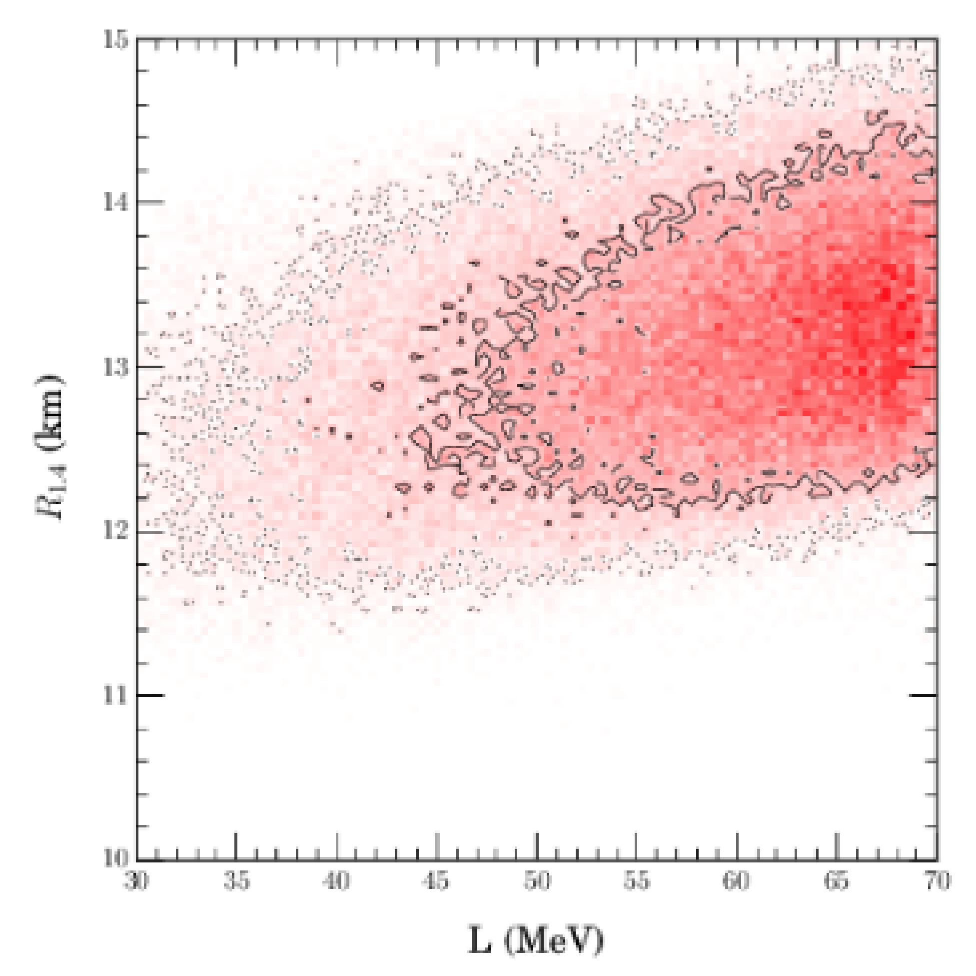
Han and Steiner (2017)

- Variations in the EOS, mass, and envelope composition required to reproduce the data (14 parameters)
- Choose two most extreme data points

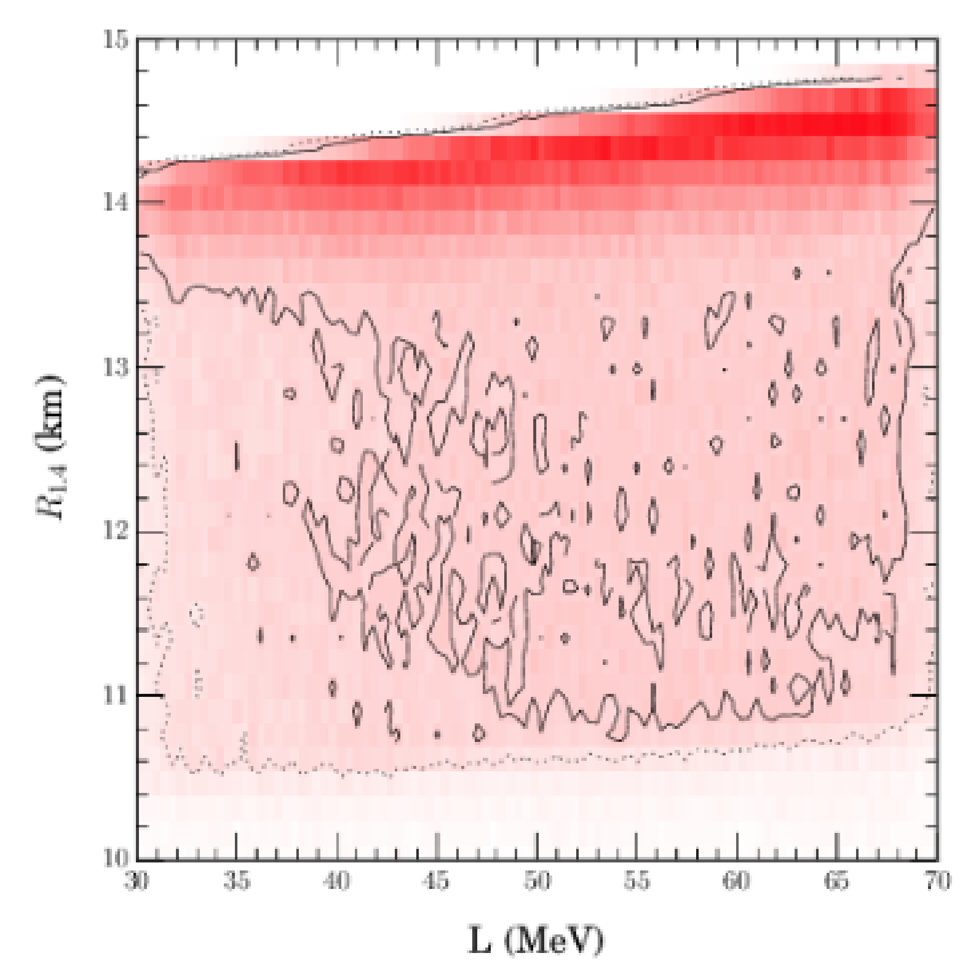
Summary

- Some EOSs are ruled out
- Grand plan: uncertainty quantification from terrestrial experiments all the way to astronomical observations
- Continue to incorporate more data
- New EOSs being developed

Quantifying Correlations



line-segments in $\log P - \log \varepsilon$



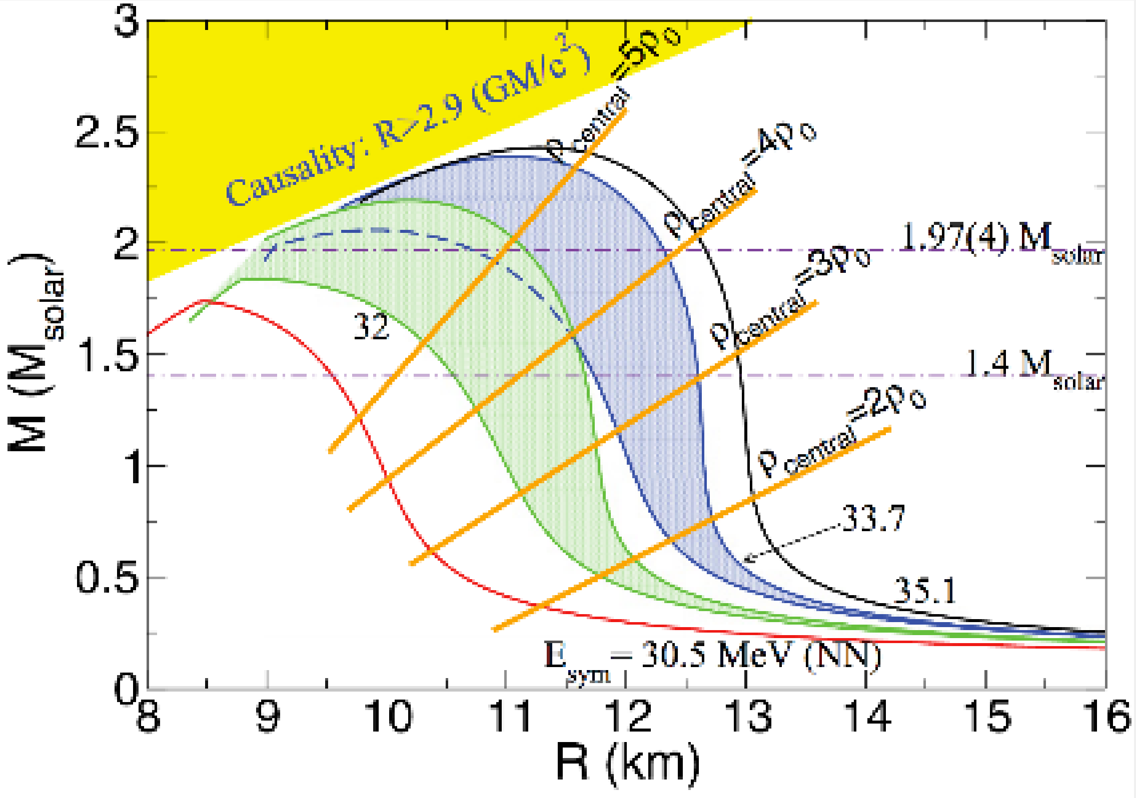
line-segments in $P - \varepsilon$

Based on Steiner, Lattimer, and Brown (2016)

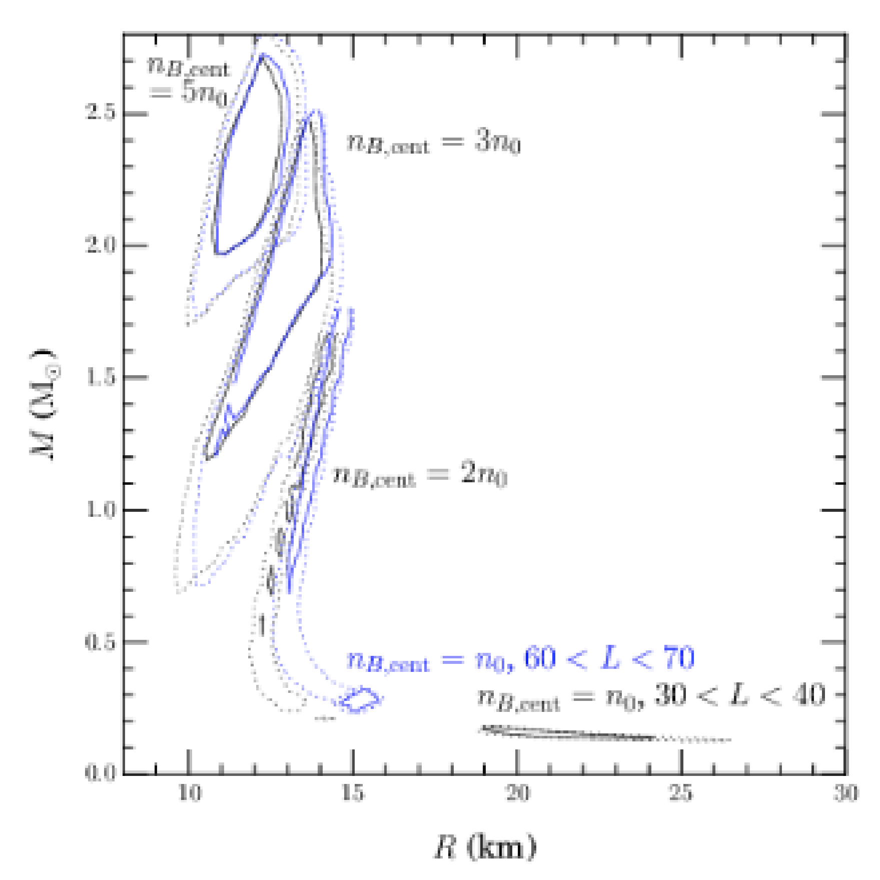
- Shown are probability distributions with no radius data
- Replace dozens of points with millions of points
- The correlation between L and $R_{1.4}$ is strongly dependent on model assumptions

  Phase transitions potentially break the correlation

Danger of Extrapolation



Gandolfi et al. (2012)



Model C from Steiner et al. (2016)

- Extrapolation gives an overly simplistic impression of the nature of dense matter

Extrapolation: The Tail Wags the Dog

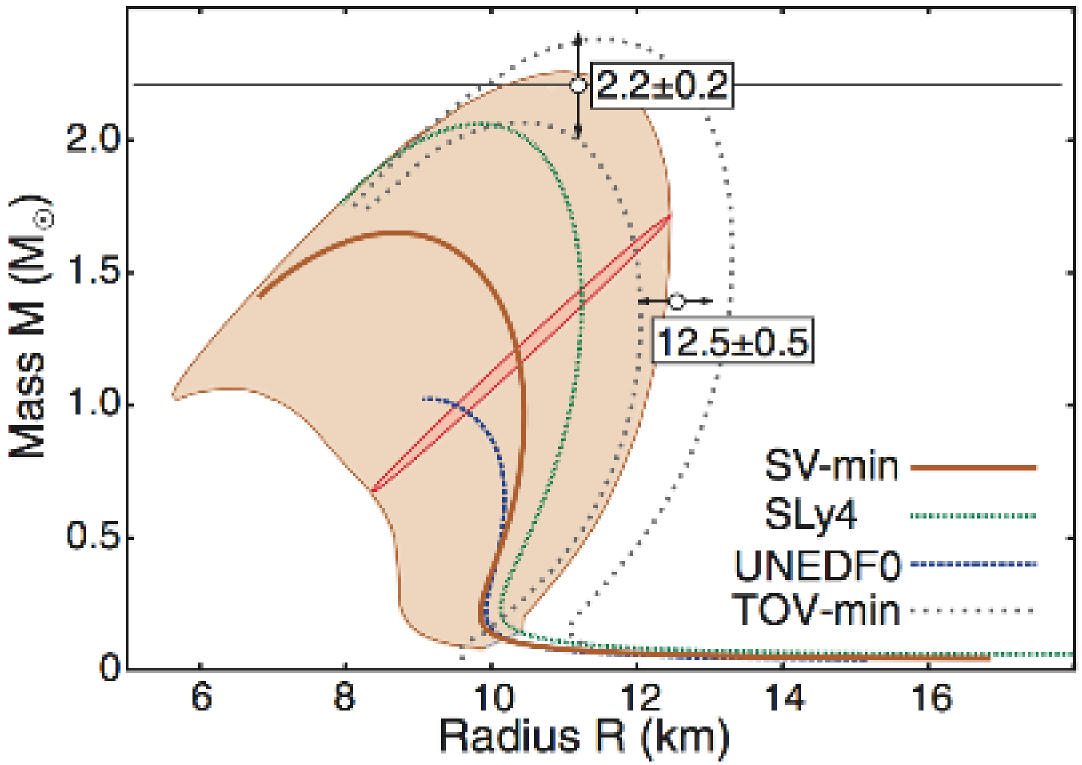


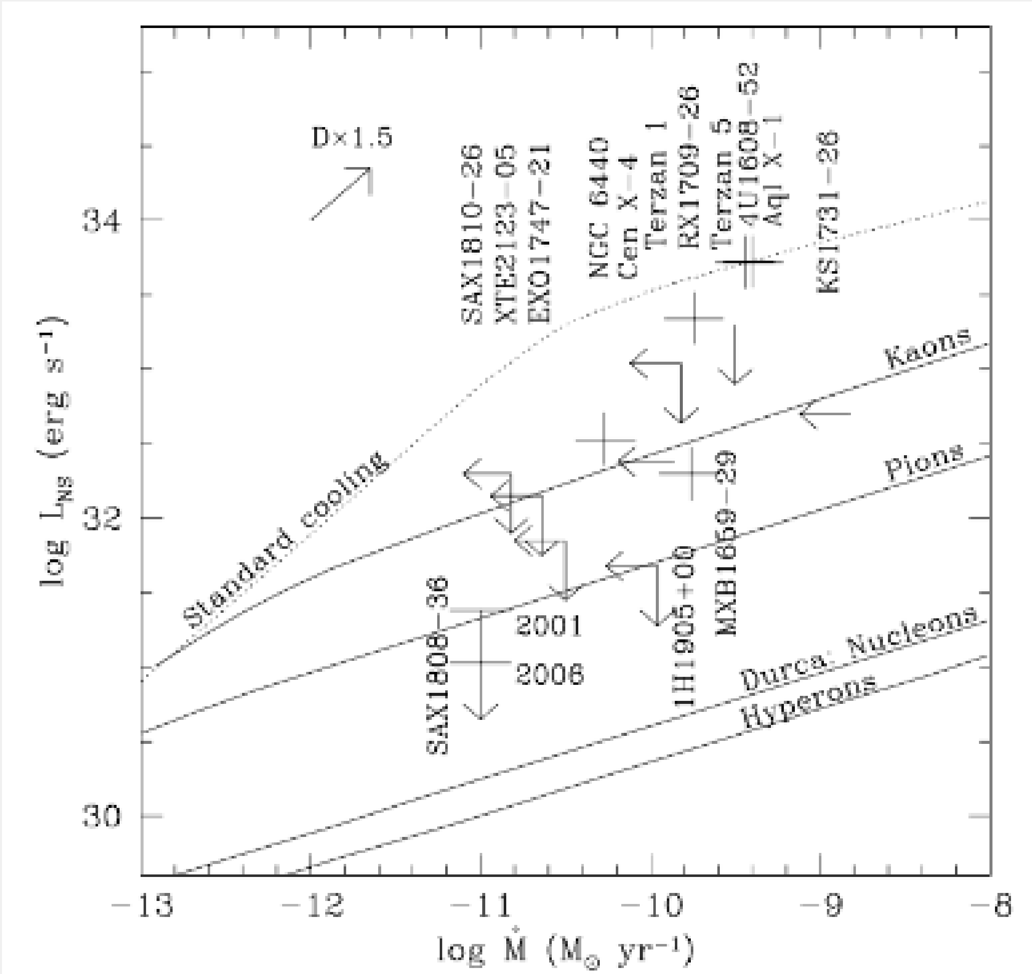
FIG. 2. (Color online) Mass-radius relation of SLy4 [1], UNEDF0 [22] and SV-min [24]. The uncertainty band for SV-min is shown. This band is estimated by calculating the covariance ellipsoid for the mass M and the radius R at each point of the SV-min curve as indicated by the ellipsoid. Also depicted (dotted lines) are uncertainty limits for TOV-min.

Erler et al. (2013)

- Many works extrapolate models of the nucleon-nucleon interaction up to very large densities
- Bayesian inference can alleviate this problem
- Arrange prior distribution to properly reweigh the dog and its tail
- Important issue when fitting disparate data sets to one model
- Related to "overfitting"

Accretion vs. Luminosity

- Presume steady state and use energy conservation: more accretion should lead to a larger photon luminosity in quiesence
- Reality does not follow this expectation
- Expected that the distinction is related to the neutron star mass
- Presume no exotic matter, then superfluidity also plays a role

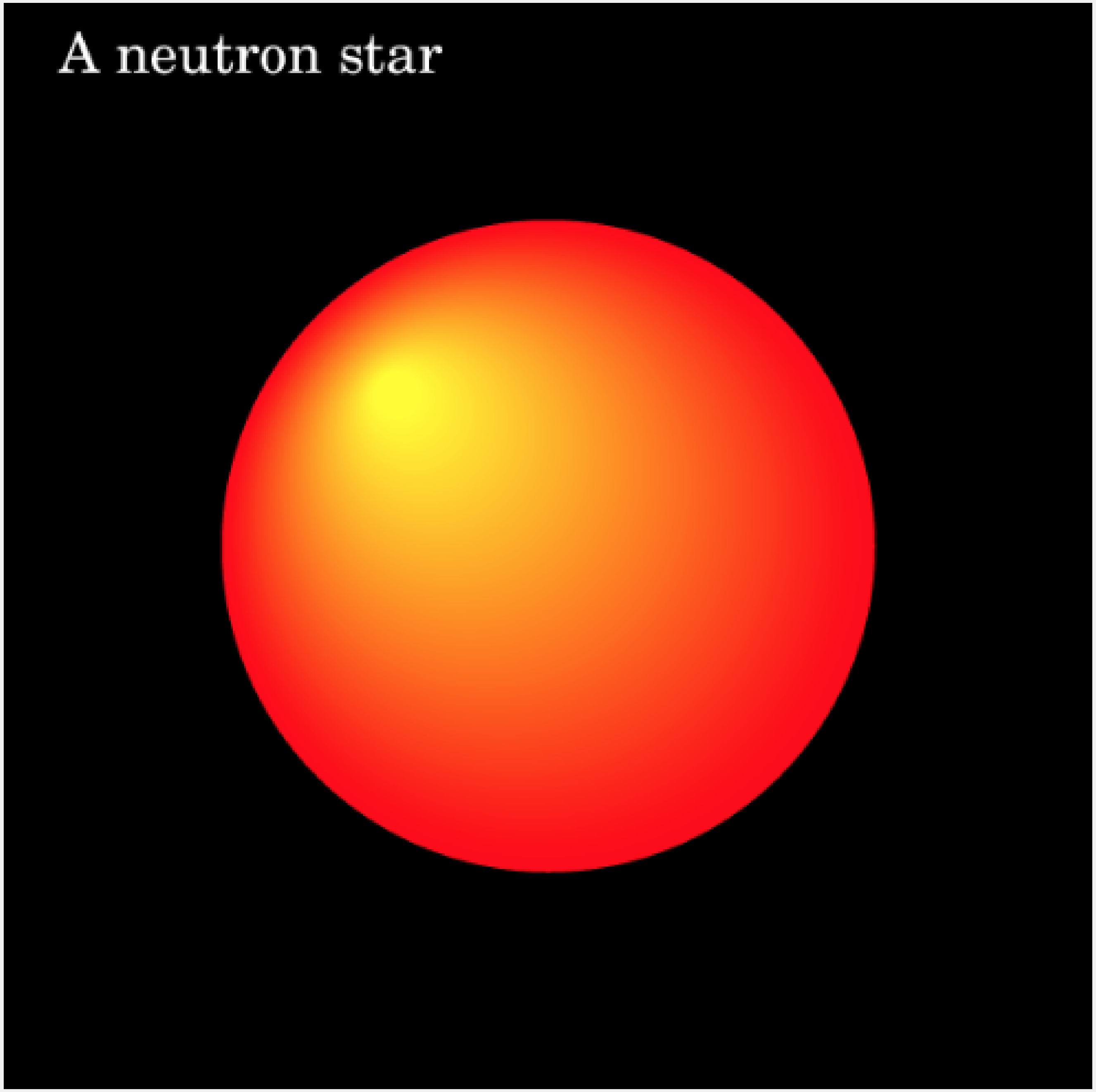


Heinke (2007)

The Parameters

- The amount of deep crustal heating (1)
- The radial extent and strength of superfluidity in the core (6)
  - From proton $^1S_0$ superconductivity and neutron $^3P_2$ superfluidity
- The high-density part of the EOS (2)
- The envelope composition of Aq X-1 (1)
  - Represents our uncertainty in the composition of the crust
- The masses of SAX J1808 and Aq X-1 (2)
- The threshold density and a broadening factor for the direct Urca process
  - Represents our uncertainty in the proton-to-neutron ratio
- Total of 14 parameters for 2 data points

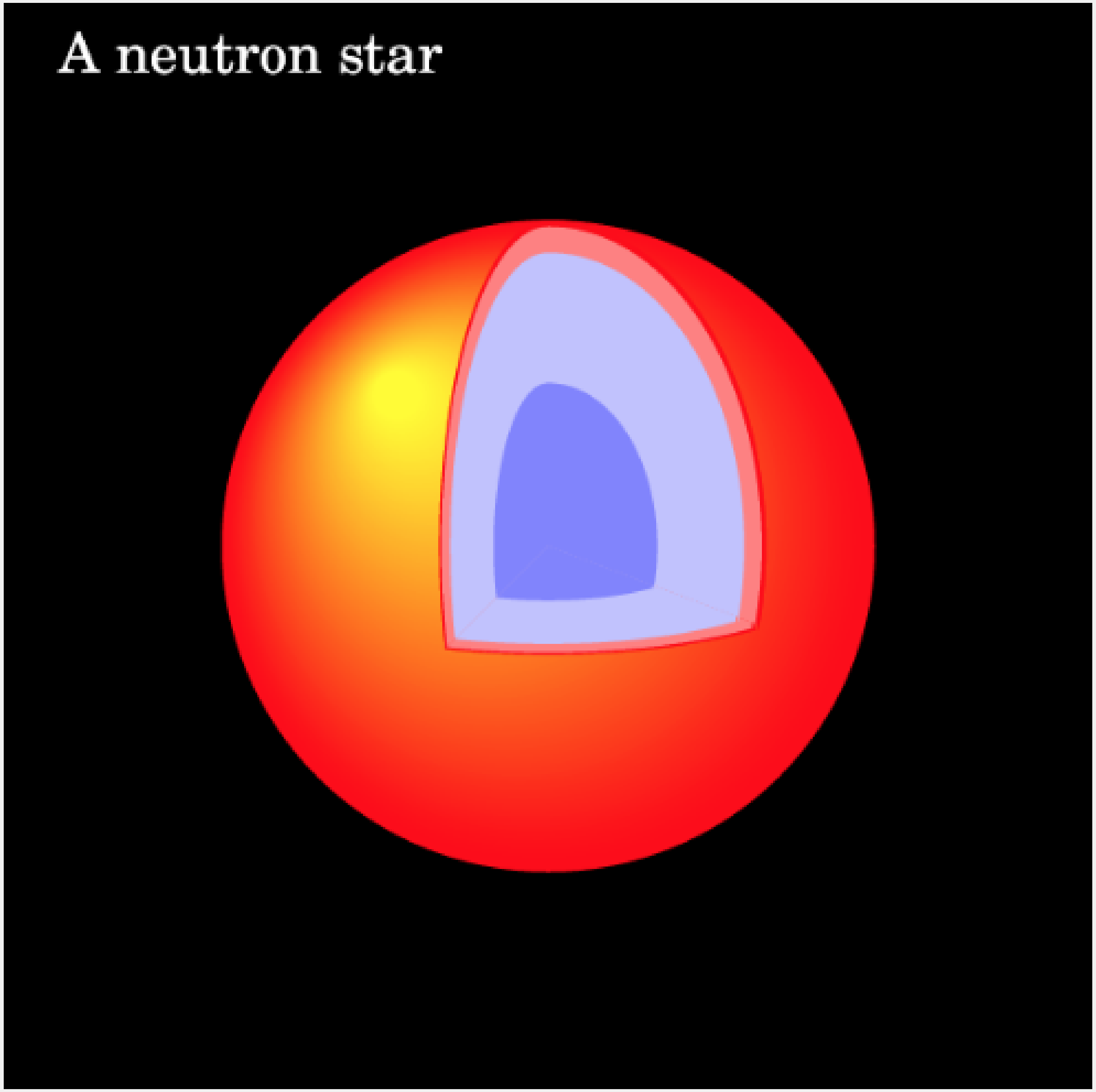
Neutron Star Composition



- Spherical nucleus with 10^{57} nucleons

Inspired by D. Page; open source (python)

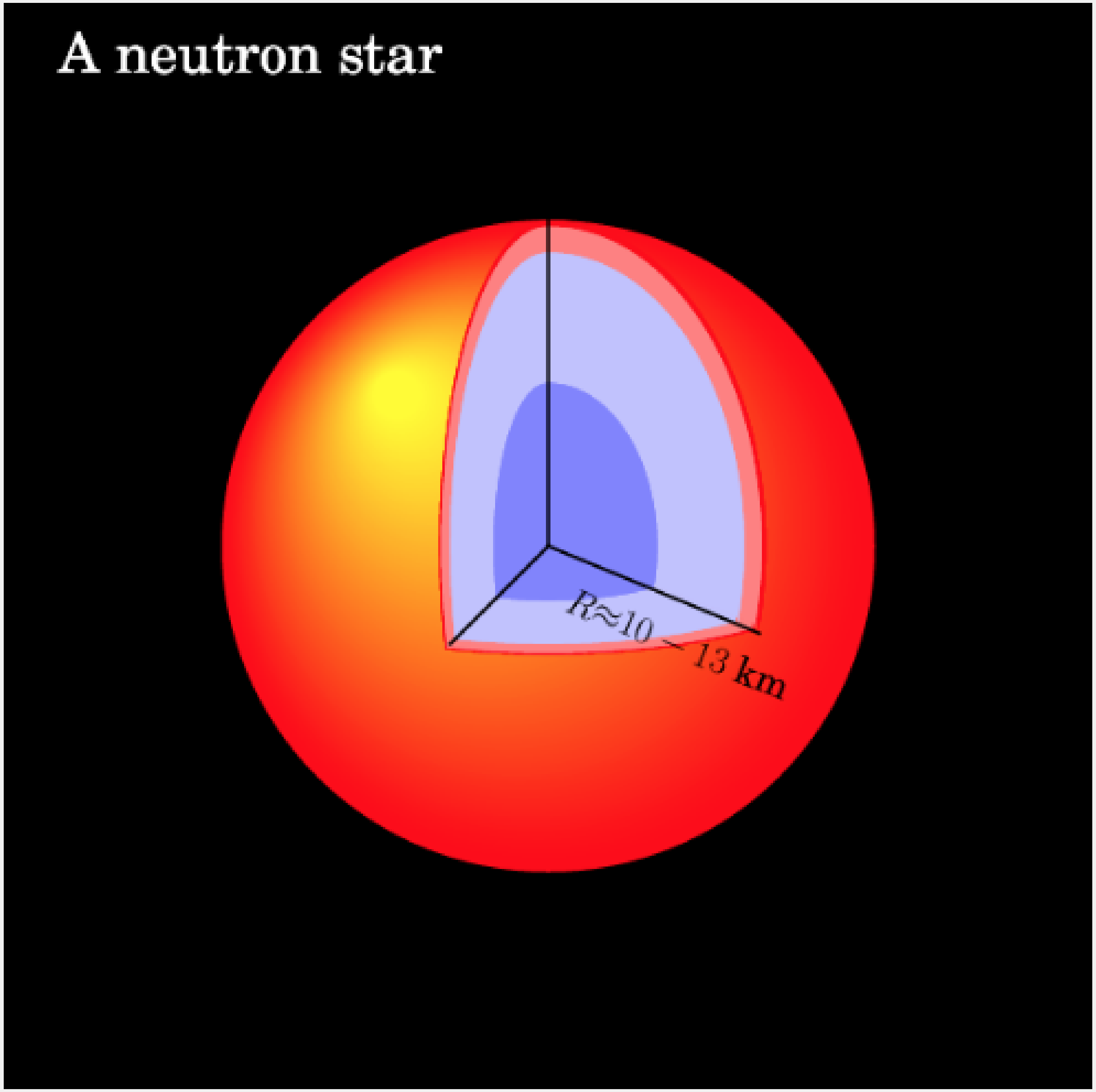
Neutron Star Composition



- Spherical stars
- Much like the earth: solid crust and fluid core

Inspired by D. Page; open source (python)

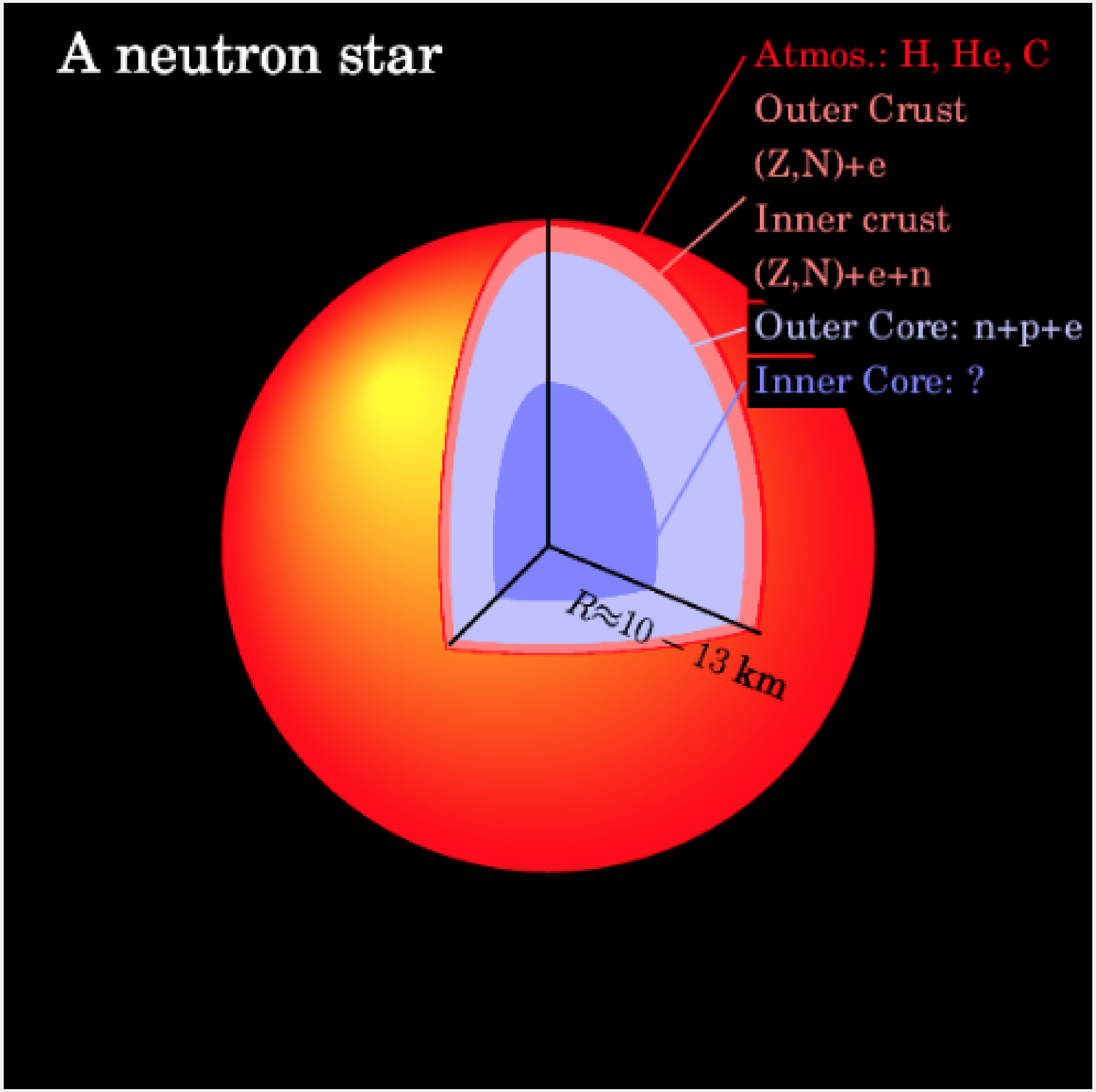
Neutron Star Composition



- Spherical stars
- Much like the earth: solid crust and fluid core
- Size of Chicago, mass of the sun

Inspired by D. Page; open source (python)

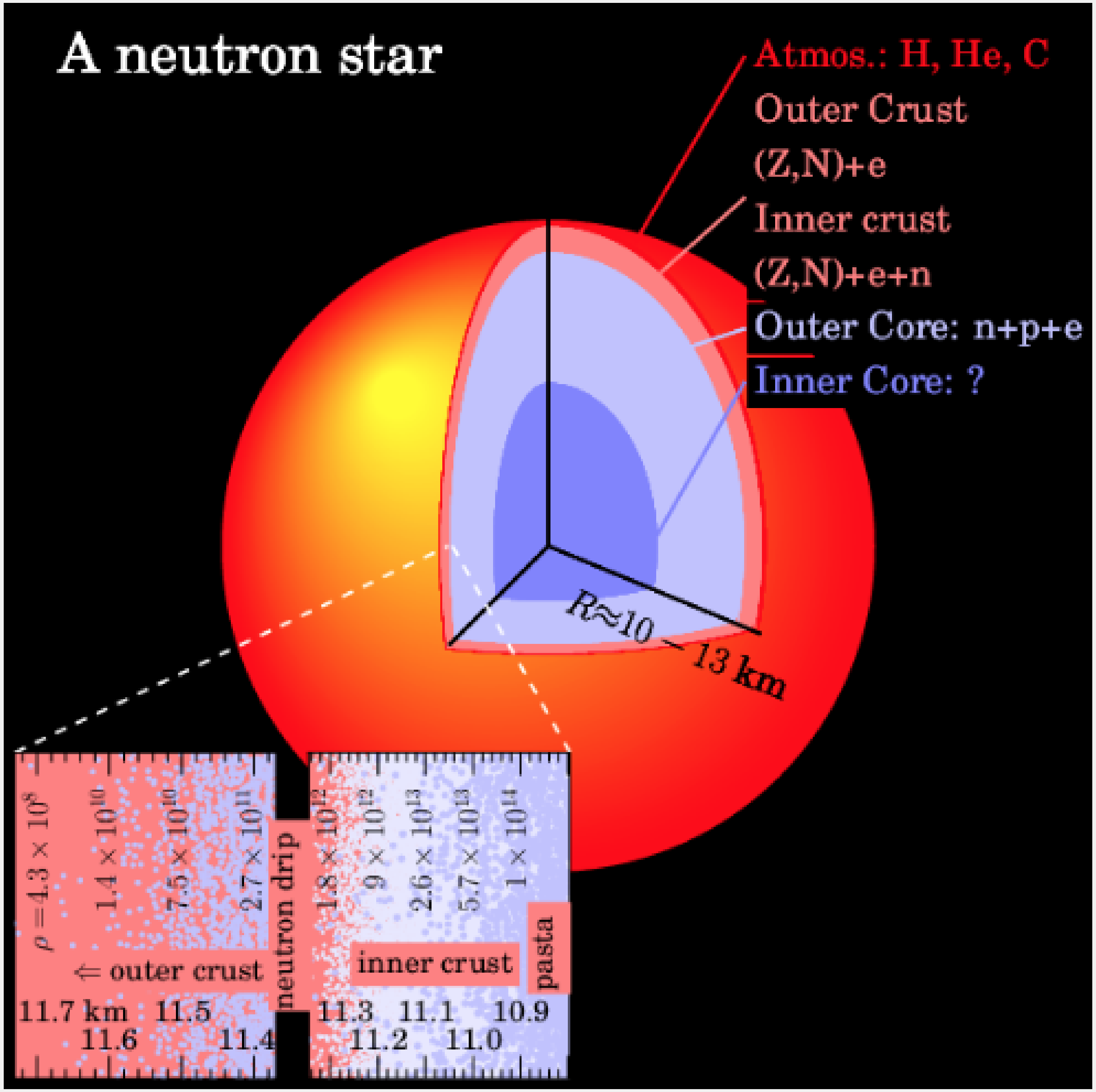
Neutron Star Composition



- Outer crust: of neutron-rich nuclei
- Inner crust: neutron-rich nuclei embedded in a sea of quasi-free superfluid neutrons
- Outer core: fluid of neutrons, protons, and electrons
- Inner core: hyperons? Bose condensates? deconfined quark matter?

Inspired by D. Page; open source (python)

Neutron Star Composition



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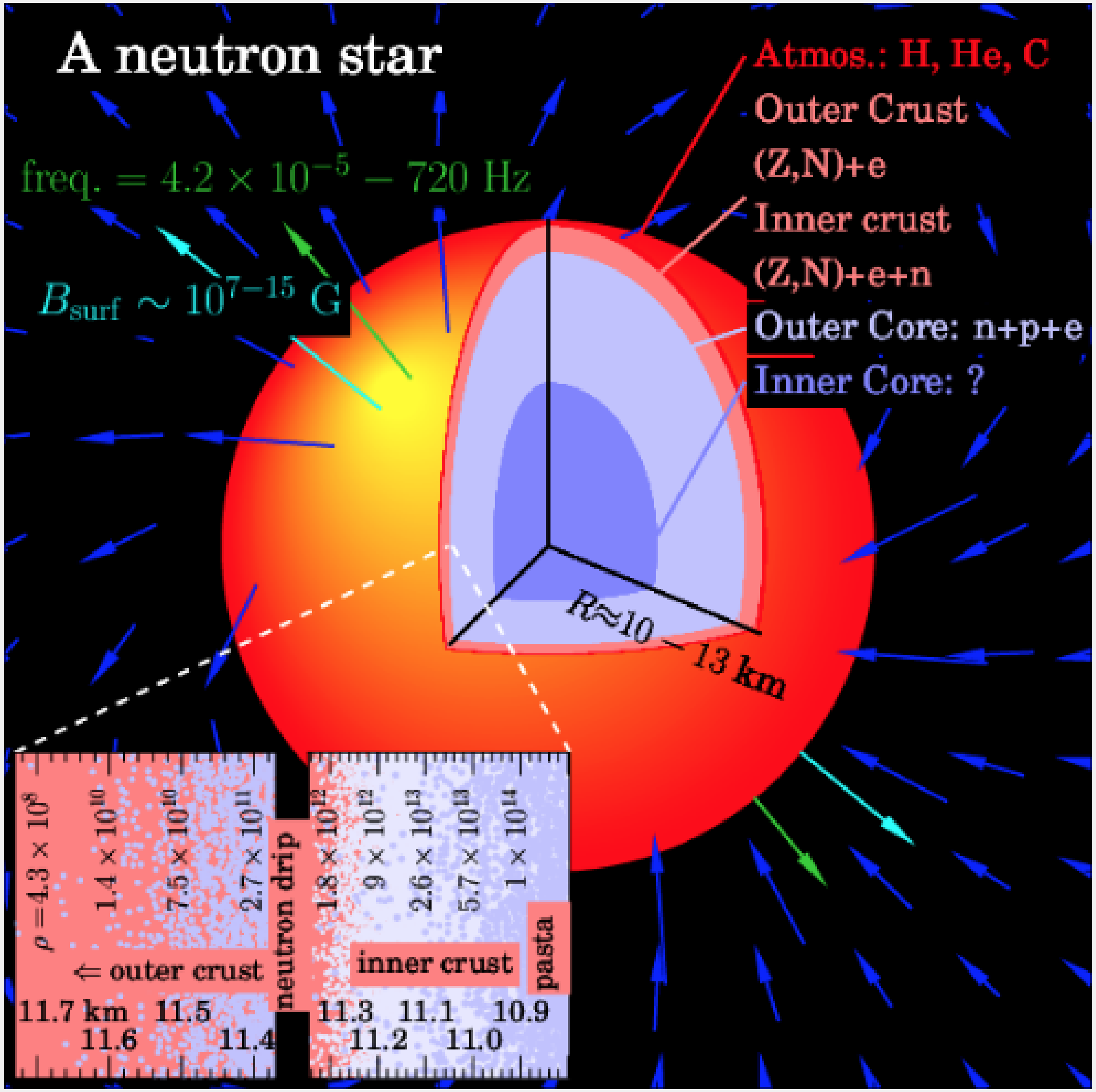

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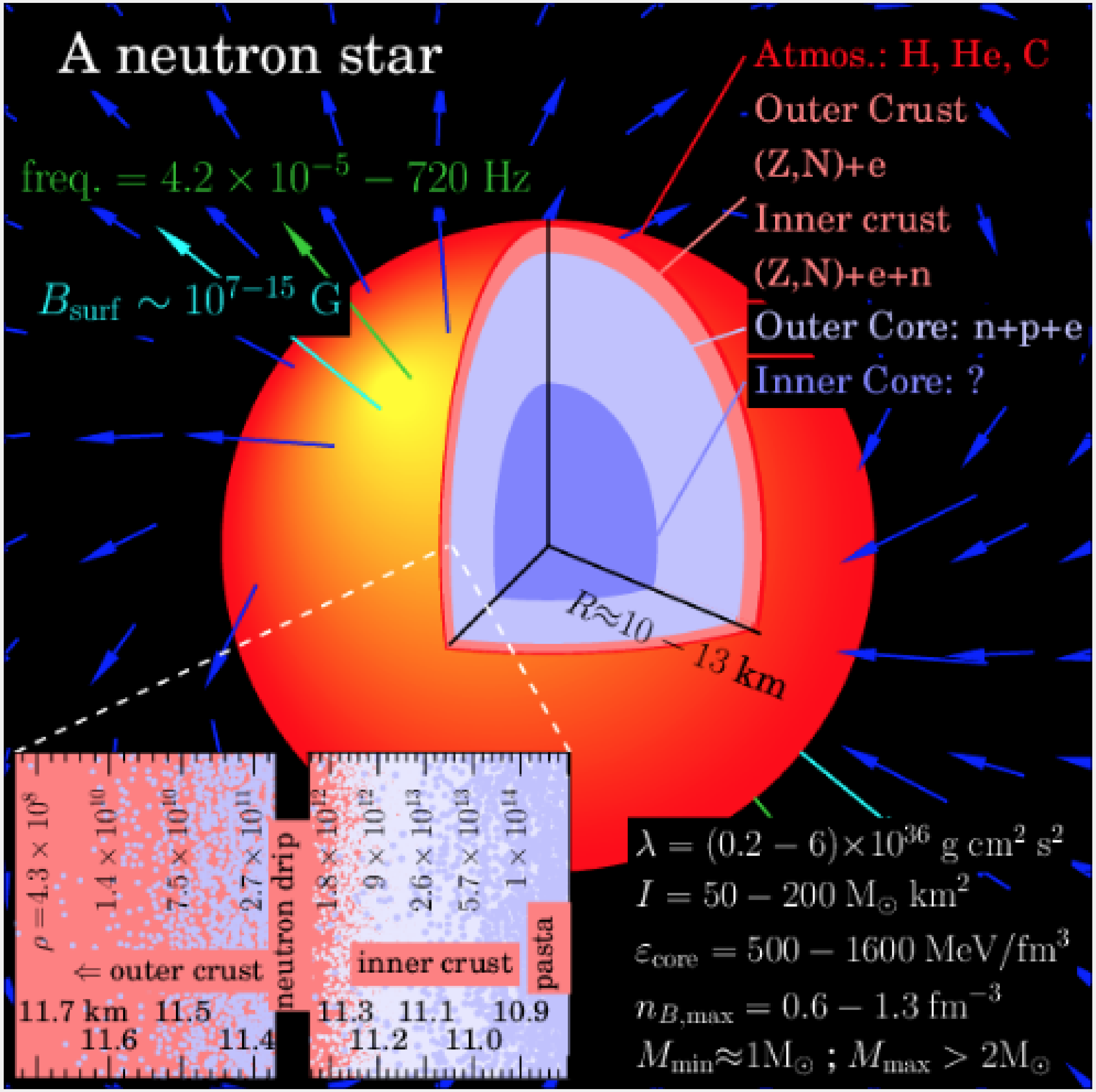
- Magnetic fields and rotation

Inspired by D. Page; open source (python)

Neutron Star Composition



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What are the correct degrees of freedom for the effective field theory which describes dense matter?

Inspired by D. Page; open source (python)