



Electromagnetic Signatures of r-process Nucleosynthesis in Neutron Star Binary Mergers,
July 24 - August 18, 2017 (INT-17-2b)

Neutron star equation of state: the known and the unknown

Jérôme Margueron, IPN Lyon & INT Seattle

- 1- There exist many predictions for the nuclear EOS.
 - How could we quantify their differences?
 - How close there are from nuclear experimental knowledge?
- 2- Cold neutron stars: How nuclear experimental uncertainties propagate in extrapolation at high densities and isospin asymmetries?
- 3- Hot proto-neutron stars and ν propagation: What is the melting T of the pasta?

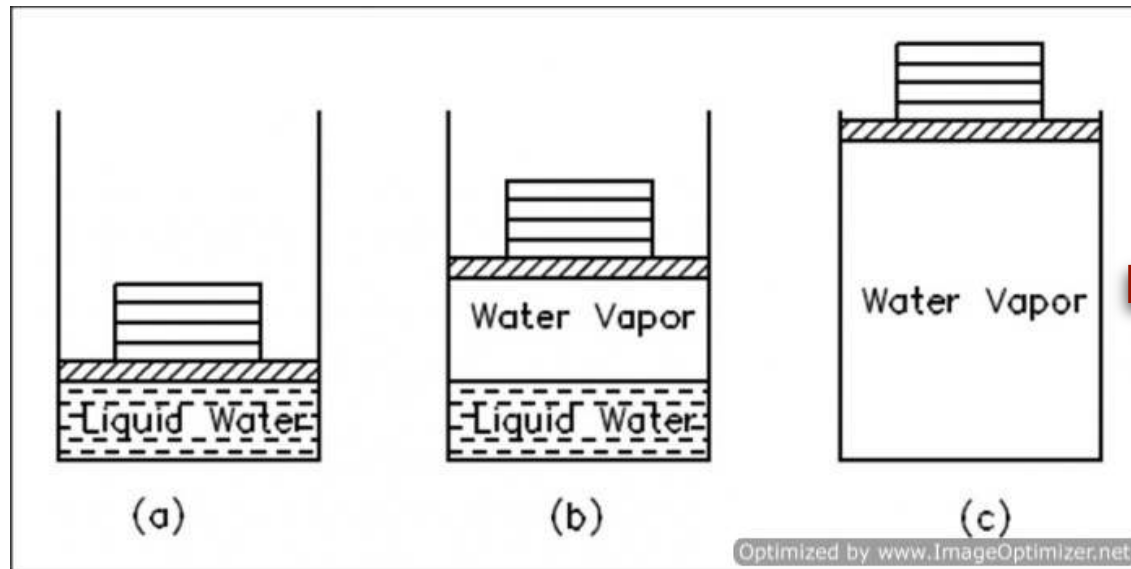
Collaboration with R. Casali (CTA Brazil), F. Gulminelli (LPC Caen), S. Reddy (INT), A. Roggero (INT).

Motivations

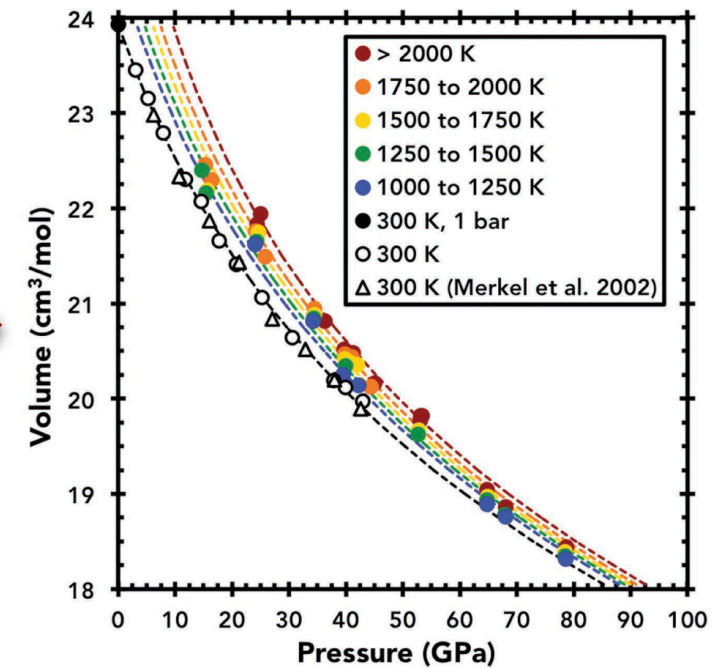
EOS: a nuclear physic challenge !

Equation of state in condensed matter physics :

Instrumental apparatus

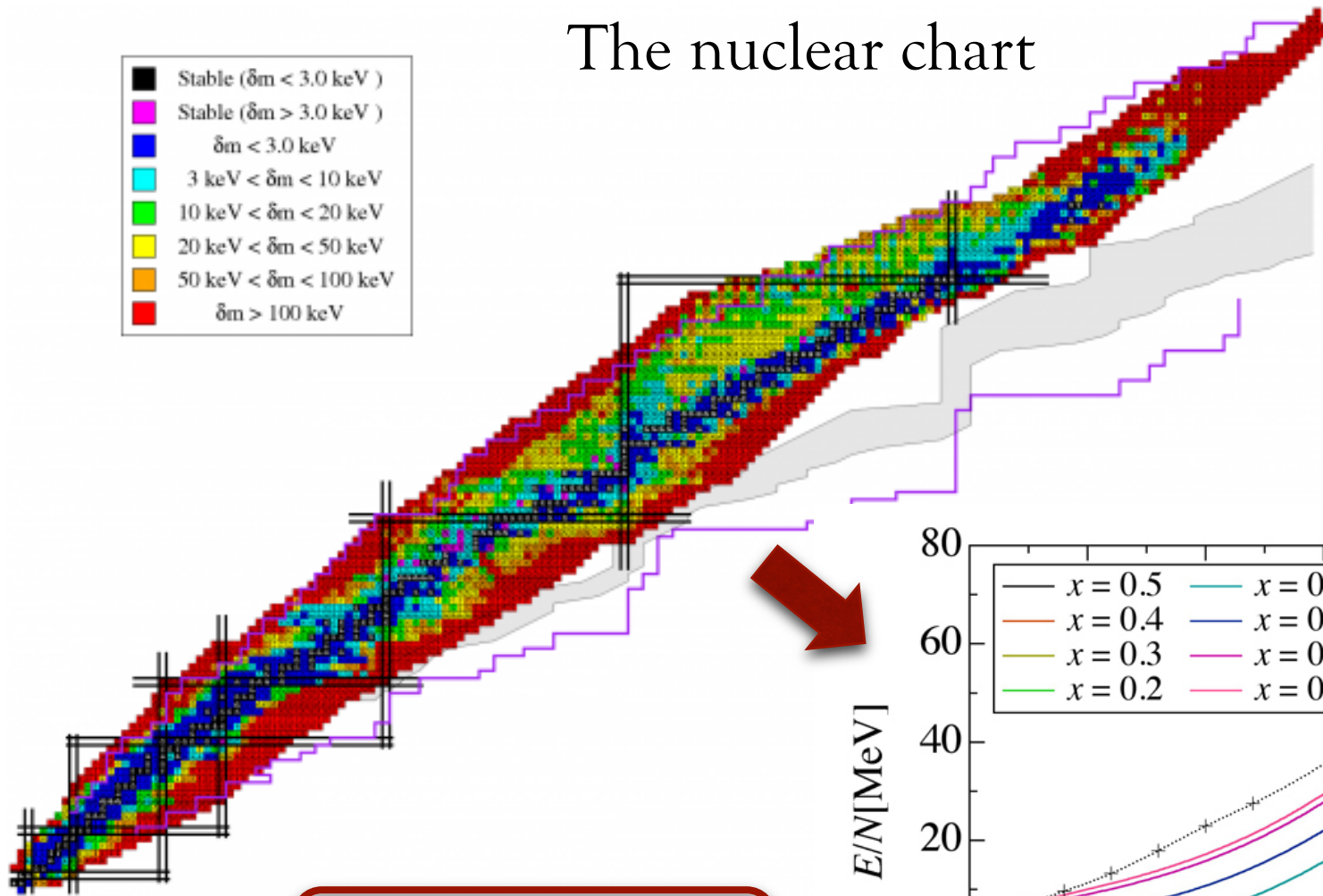
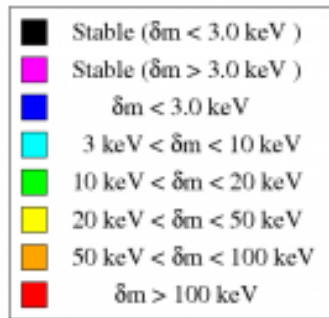


EOS of pyrite to 80 GPa and 2400 K,
Thompson et al.,
American Mineralogist 101 (2016) 1046



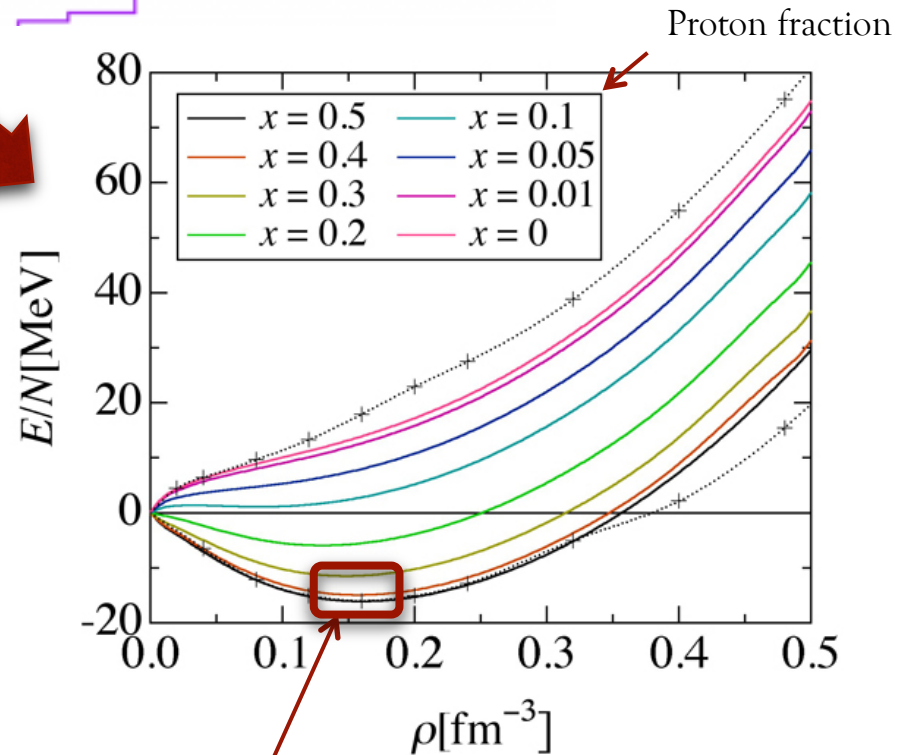
In nuclear physics, we only have 1 density (almost) at pressure $P=0$!

The nuclear chart



$0.4 < x_p = Z/A < 0.6$
 $0.12 \text{ fm}^3 < n_{\text{sat}} < 0.18 \text{ fm}^3$

→ A very small portion of the phase diagram



Saturation density n_{sat}

J. MARGUERON

Could NS observation data provide the nuclear EoS ?

Recent attempts to deduce the nuclear EoS from:

- thermal X-rays emission,
- X-ray pulsations (NICER),
- Photospheric expansion,
- ...

Hebeler 2013
Ozel 2010, 2012, 2014
Steiner 2010, 2013

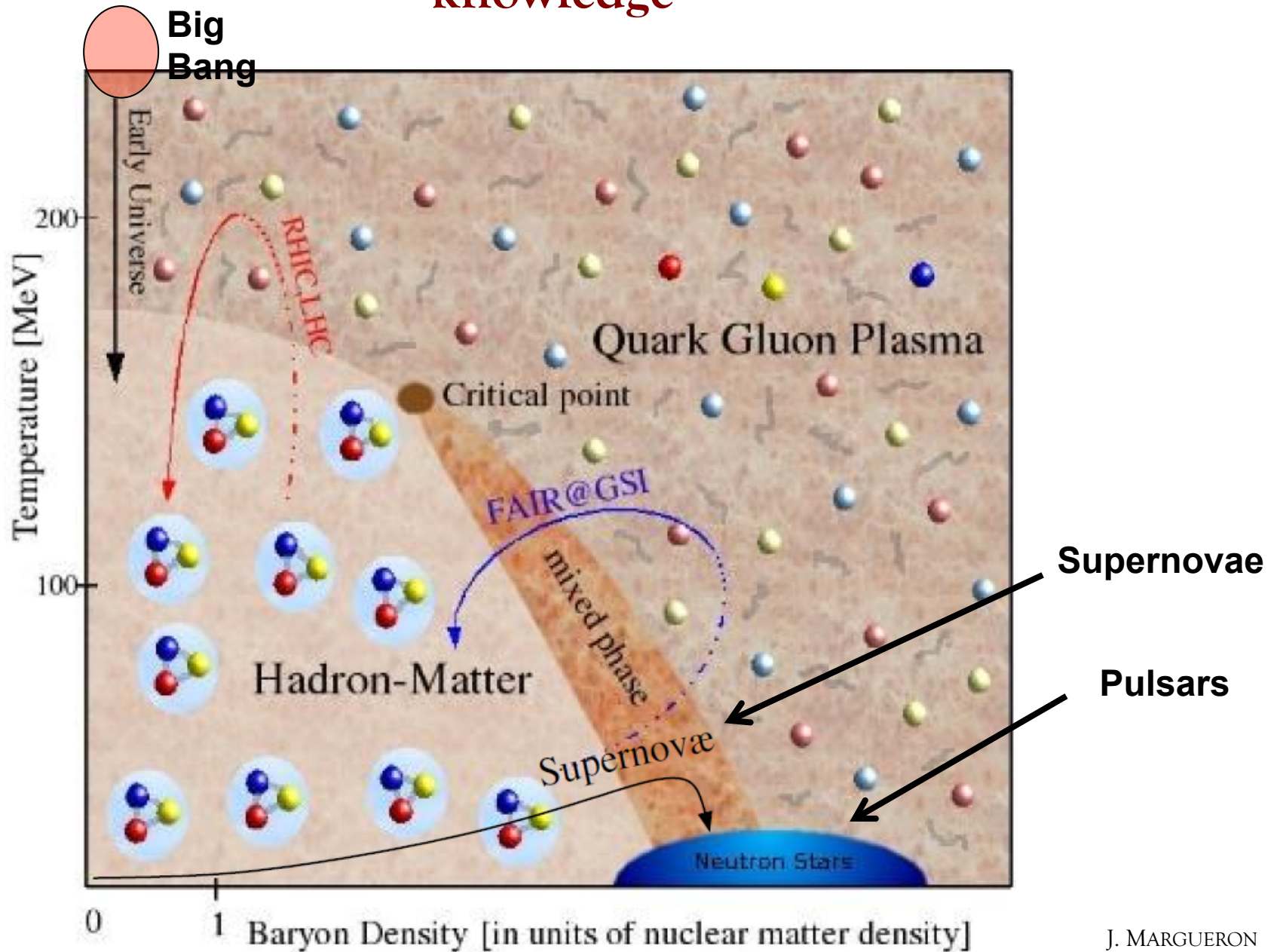
...

... NS merging (expected data from VIRGO/LIGO)



Maybe.. maybe not.. but nuclear physics experimental knowledge could help !

Prediction over the dense phase diagram challenges our knowledge



The empirical parameters:
a way to characterise the nuclear EOS
and
to link them to nuclear experimental data

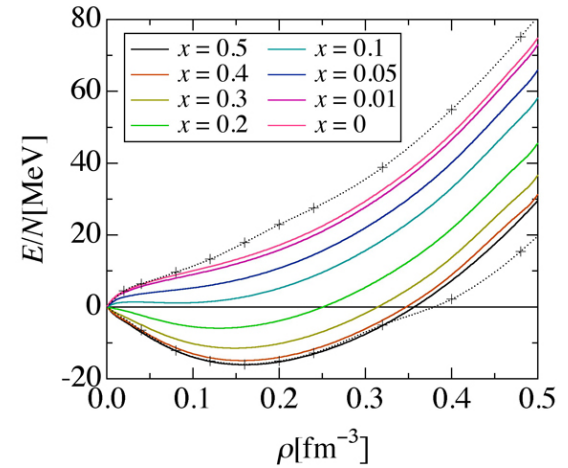
Empirical parameters

The empirical parameters code the **bulk** properties of the nuclear EoS.

Taylor expansion of the EoS around ρ_0 (theory):

$$\frac{E}{A} = E_{sat} + E_{sym}\delta^2 + (L_{sym}\delta^2)x + \frac{1}{2}(K_{sat} + K_{sym}\delta^2)x^2 + \dots$$

$$\text{where } x = \frac{n - n_{sat}}{3n_{sat}} \quad \text{and} \quad \delta = \frac{n_n - n_p}{n}$$



Phenomenological mass formula (experiment):

$$\bullet \text{ LD formula: } B(A, Z) = \overbrace{E_{sat} + E_{sym}I^2}^{\text{bulk}} + \overbrace{E_{surf}A^{-1/3}}^{\text{surface}} + \dots$$

• Droplet formula: including skin contribution

$$\bullet \text{ Compressible LD formula: } B(A, Z) = \frac{E}{A}(n_A, I) + E_{surf}A^{-1/3}$$

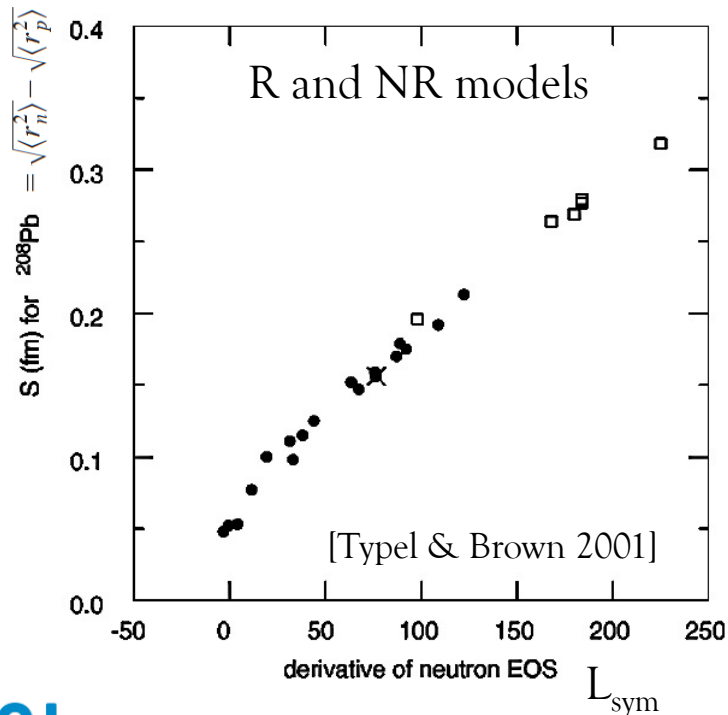
$$n_A = n_{sat} \left(1 - \frac{3L_{sym}}{K_{sat}} I^2 \right)$$

$$A = N + Z$$

$$I = \frac{N - Z}{A}$$

Experimental determination of the empirical parameters

- $E_{\text{sat}}, E_{\text{sym}}$: From fit of LDM through the nuclear chart, or from DFT adjustment.
- K_{sat} : from ISGMR [Blaizot, 1980]
 - better correlated to M_c [Khan, J.M. 2012]
- $L_{\text{sym}}, K_{\text{sym}}$: more difficult
 - Neutron skin in Pb,
 - ISGMR in neutron rich nuclei (K_{sym}, K_{τ}) [Garg+2010]



Measurement of neutron skin:

With strong probes:

- p-N elastic diffusion
- π, α, d scattering
- π photoproduction
- Heavy-ion collisions
- Electric dipole polarizability

With weak probes:

- PREX / C-REX

How well do we know the empirical parameters ?

Definition:

$$\text{Around } n_{\text{sat}}: \quad \frac{E}{A}(n, \delta) \approx e_{\text{sat}}(n) + e_{\text{sym}}(n)\delta^2 + e_{\text{sym},4}(n)\delta^4 + \dots$$

$$\text{with } e_{\text{sat}}(n) = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{6}Q_{\text{sat}}x^3 + \frac{1}{24}Z_{\text{sat}}x^4 + \dots$$

$$e_{\text{sym}}(n) = E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 + \frac{1}{6}Q_{\text{sym}}x^3 + \frac{1}{24}Z_{\text{sym}}x^4 + \dots$$

Determination:

- Systematic comparison of model predictions:
 - Non relativistic interactions: Skyrme, Gogny, ...
 - Relativistic: RMF, RHF
- For models being adjusted on nuclear properties (binding energy, charge radius, etc...)

Empirical parameters from various effective approaches

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
RHF	Average	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
	σ	0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

$$e_{sat}(n) = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{6}Q_{sat}x^3 + \frac{1}{24}Z_{sat}x^4 + \dots$$

$$e_{sym}(n) = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{6}Q_{sym}x^3 + \frac{1}{24}Z_{sym}x^4 + \dots$$

Empirical parameters from various effective approaches

1 % accuracy

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm ⁻³	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
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Empirical parameters from various effective approaches

10-20 % accuracy

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
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Empirical parameters from various effective approaches

50 % accuracy

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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Empirical parameters from various effective approaches

Very large inaccuracy

Model	ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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Empirical parameters from various effective approaches

fixed

Explore inside small interval

Consider large interval

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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	σ	0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

- In the following, we neglect any correlation among the empirical parameters.
- Another approach is possible: apply the eEOS directly in nuclei and fix the uncertainties directly from the experimental data ++ bring physical correlations (work in progress).

Phenomenological + ab-initio approaches

Model (N_α)	der. order	E_{sat} MeV	E_{sym} MeV	n_{sat} fm^{-3}	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV	m_{sat}^*/m	$\Delta m_{sat}^*/m$	κ_ν	K_τ MeV
		0	0	1	1	2	2	3	3	4	4	-	-	-	-
Phenomenological approaches															
Skyrme (16)	Average σ	-15.88 0.15	30.25 1.70	0.1595 0.0011	47.8 16.8	234 10	-130 66	-357 22	378 110	1500 169	-2219 617	0.73 0.10	0.08 0.24	0.46 0.27	-344 25
Skyrme (35)	Average σ	-15.87 0.18	30.82 1.54	0.1596 0.0039	49.6 21.6	237 27	-132 89	-349 89	370 188	1448 510	-2175 1069	0.77 0.14	0.127 0.310	0.44 0.37	-354 45
RMF (11)	Average σ	-16.24 0.06	35.11 2.63	0.1494 0.0025	90.2 29.6	268 34	-5 88	-2 393	271 357	5058 2294	-3672 1582	0.67 0.02	-0.09 0.03	0.40 0.06	-549 153
RHF (4)	Average σ	-15.97 0.08	33.97 1.37	0.1540 0.0035	90.0 11.1	248 12	128 51	389 350	523 237	5269 838	-9956 4156	0.74 0.03	-0.03 0.01	0.34 0.07	-572 169
Total (50)	Average σ_{tot} Min Max	-16.03 0.20 -16.35 -15.31	33.30 2.65 26.83 38.71	0.1543 0.0054 0.1450 0.1746	76.6 29.2 9.9 122.7	251 29 201 355	-3 132 -394 213	13 431 -748 950	388 289 -86 846	3925 2270 -903 9997	-5268 4282 -16916 -5	0.72 0.09 0.38 1.11	0.01 0.20 -0.47 1.02	0.39 0.22 0.00 2.02	-492 166 -835 -254
Ab-initio approaches															
APR (1)	Average σ	-16.0 - [†]	33.12 0.30	0.16 - [†]	50.0 1.3	270 2	-199 13	-665 30	923 67	337 94	-2053 125	1.0 - [†]	0.0 - [†]	0.0 - [†]	-376 30
χ -EFT Drischler 2016 (7)	Average σ_{tot} Min Max	-15.16 1.24 -16.92 -13.23	32.01 2.09 28.53 34.57	0.171 0.016 0.140 0.190	48.1 3.6 43.9 53.5	214 22 182 242	-172 40 -224 -108	-139 104 -310 24	-164 234 -640 96	1306 214 901 1537	-2317 379 -2961 -1750	- - - -	- - - -	- - - -	-428 63 -534 -334

[†] This parameter is fixed.

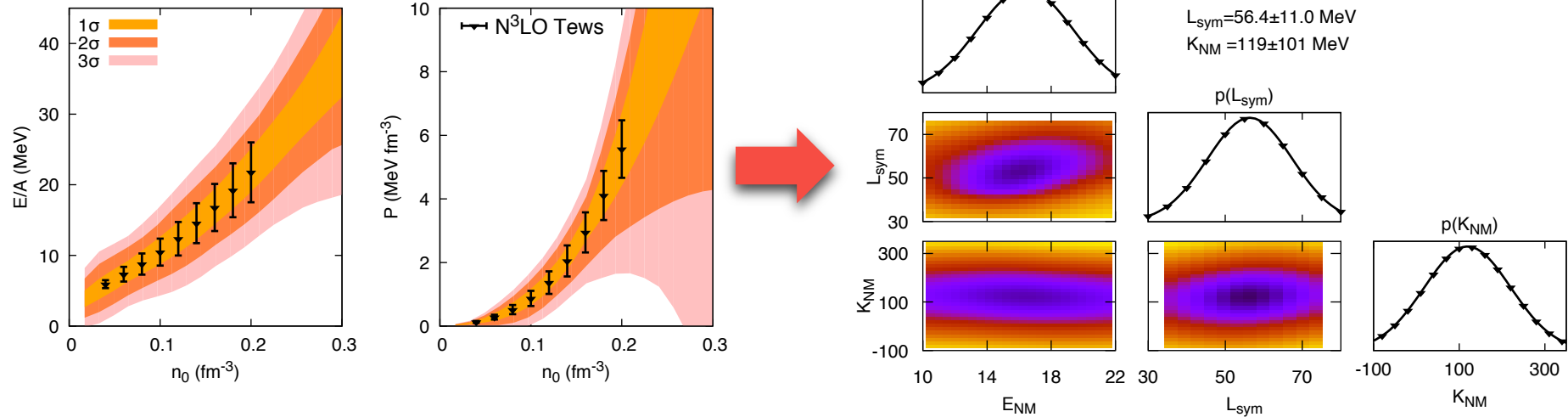
In neutron matter

Model (N_α)		E_{NM} MeV	L_{sym} MeV	K_{NM} MeV	Q_{NM} MeV	Z_{NM} MeV
Phenomenological approaches						
Skryme (35)	Average σ	14.95 1.72	49.6 21.6	106 116	21 276	-727 1580
RMF (11)	Average σ	18.86 2.69	90.2 29.6	263 121	269 750	1386 3876
RHF (4)	Average σ	17.99 1.46	90.0 11.1	376 63	912 587	-4686 4994
Ab-initio approaches						
APR (1)	Average σ	17.27 0.30	50.0 1.3	71 15	258 97	-1716 219
GCR 2012 (7)	Average σ	16.76 1.39	45.8 9.7	77 43	80 29	-131 15
χ -EFT Tews 2013	Average σ	16.39 2.97	56.4 11.0	119 101	- -	- -
χ -EFT Drischler 2016 (7)	Average σ	16.93 0.92	48.3 3.5	41 33	-314 226	-991 349



Bayesian analysis of chiral EFT predictions

I. Tews et al., PRL 110, (2013)



$$\text{Estimation of the error: } \chi^2 = \frac{1}{2M-3} \sum_{i=1}^M \left(\frac{e_i - e_{ELFc}(n_0^i)}{\varepsilon_i^e} \right)^2 + \left(\frac{p_i - p_{ELFc}(n_0^i)}{\varepsilon_i^p} \right)^2$$

$$\text{Likelihood probability: } p(E_{NM}, L_{sym}, K_{NM}) = \exp(-\chi^2/2)$$

$$\text{1-parameter probability: } p(A) = \sum_{B,C} p(A, B, C)$$

The « model independent » EOS can include constraints from ab-initio calculations.

In Summary

In the following, we consider the following central values and uncertainties (1σ):

P_α	E_{sat} MeV	E_{sym} MeV	n_{sat} fm^{-3}	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV	m_{sat}^*/m	$\Delta m_{sat}^*/m$
$\langle P_\alpha \rangle$	-15.8	32	0.155	60	230	-100	300	0	-500	-500	0.75	0.1
σ_{P_α}	± 0.3	± 2	± 0.005	± 15	± 20	± 100	± 400	± 400	± 1000	± 1000	± 0.1	± 0.1

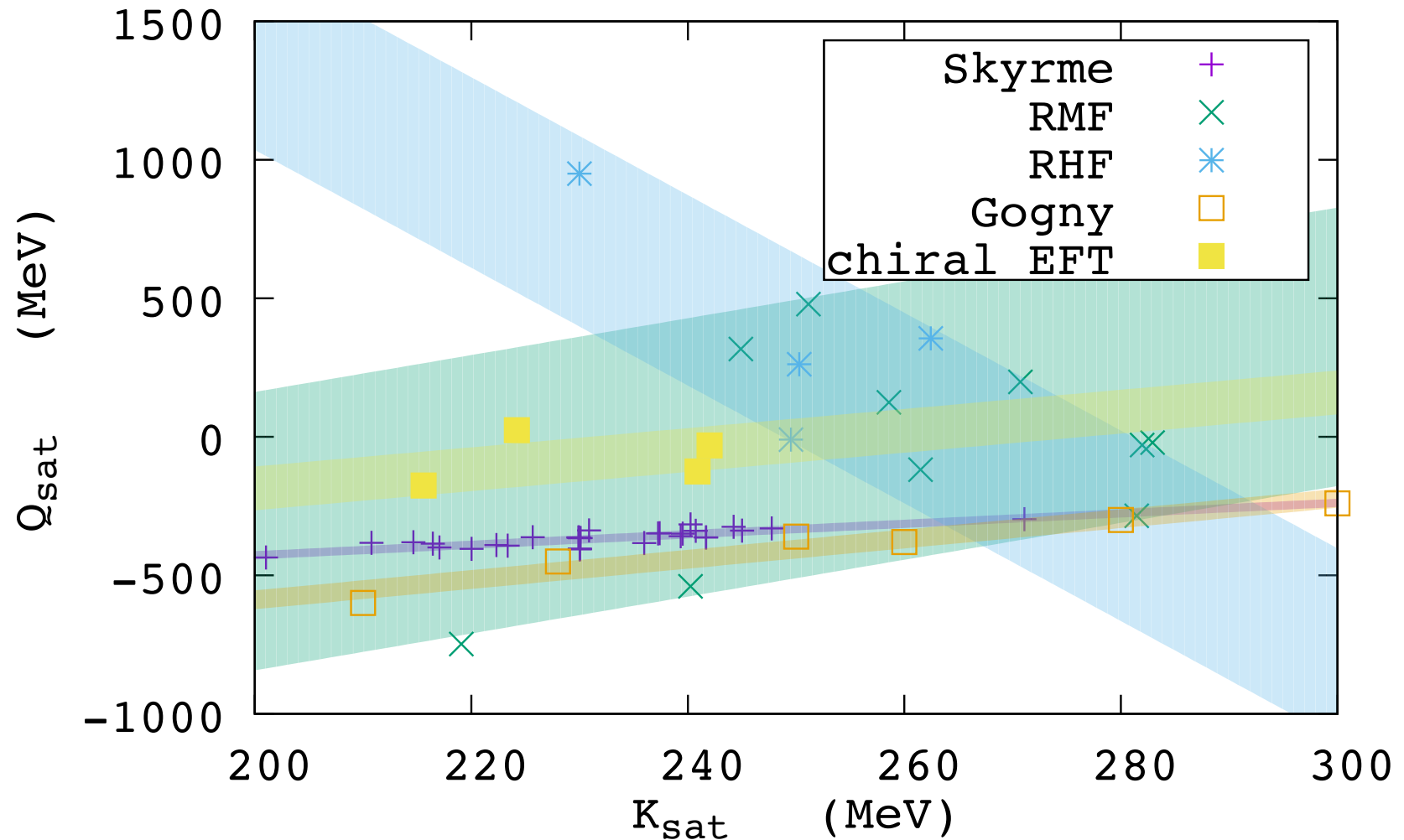


Large uncertainties

→ What impact for the nuclear EOS?

A “model independent” nuclear EOS

The question of the spurious correlations



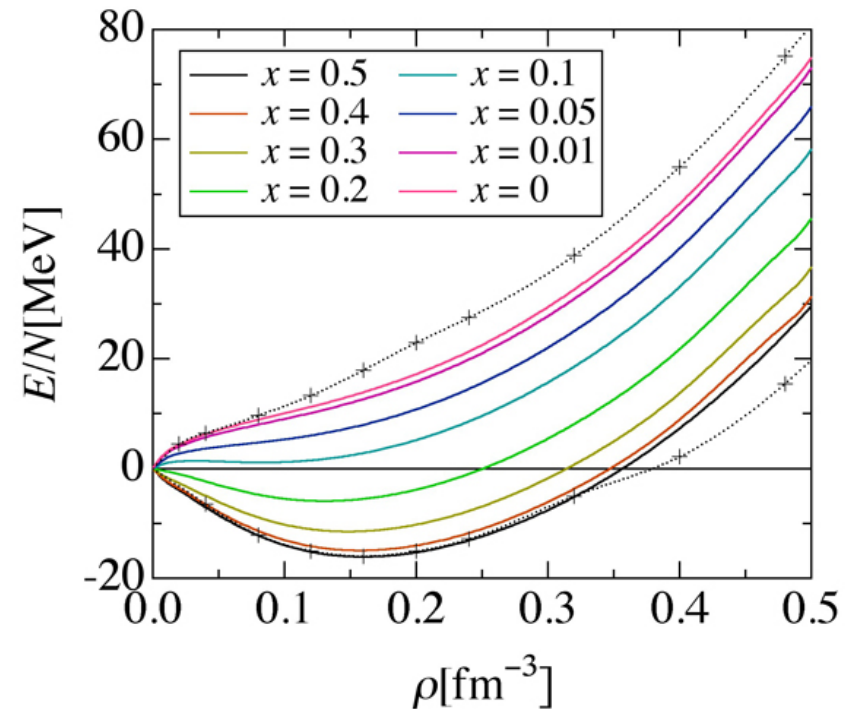
- To few parameters in symmetric matter.
- General issues: results can be model-dependent.

Needs for a “model independent” nuclear EoS

- For the analysis of NS observation (thermal X-rays, X-ray pulsations, etc...)
 - Extraction of NS radius
- For implementation in hydro-dynamical codes (CCSN, NS mergers, etc...)
- For Heavy Ion Collisions (Hadron physics) and analysis of the results

The nuclear EoS is a function of the density, isospin asymmetry (n/p ratio) and temperature.

Here we do not consider phase transitions to hyperon matter or quark matter.



An “model independent” nuclear EoS

- Hypothesis:**
- 1) Matter is non-relativistic ($\rightarrow E=T+V$),
 - 2) Nuclear potential quadratic in δ ,
 - 3) The EoS is analytic in x (\rightarrow polynomial expansion possible),
 - 4) $\lim_{\rho \rightarrow 0} e(\rho, \delta) \rightarrow 0$ for $\rho \rightarrow 0$.

Kinetic energy:
$$t^{eff}(\rho, \delta) = \frac{1}{2} t_0^{FG} \left(\frac{\rho}{\rho_0} \right)^{2/3} \left[f^{FG}(\delta) + \frac{\rho}{\rho_0} f^{eff}(\delta) \right]$$

Binding energy:
$$e^N(\rho, \delta) = t^{eff}(\rho, \delta) + \sum_{\alpha \geq 0}^N \left[v_{\alpha}^{s, is} + v_{\alpha}^{s, iv} \delta^2 \right] \frac{x^{\alpha}}{\alpha!} u_{\alpha}(\rho)$$

\rightarrow One-to-one correspondence between model parameters and empirical quantities:

\rightarrow Flexible model with no hidden correlations among parameters.

\rightarrow Can map most of nucleon EOS (up to $4n_{\text{sat}}$): “model independent” EOS.

$$v_{\alpha=0}^{s, is} = E_0 - t_0^{FG} (1 + \bar{M})$$

$$v_{\alpha=1}^{s, is} = -t_0^{FG} (2 + 5\bar{M})$$

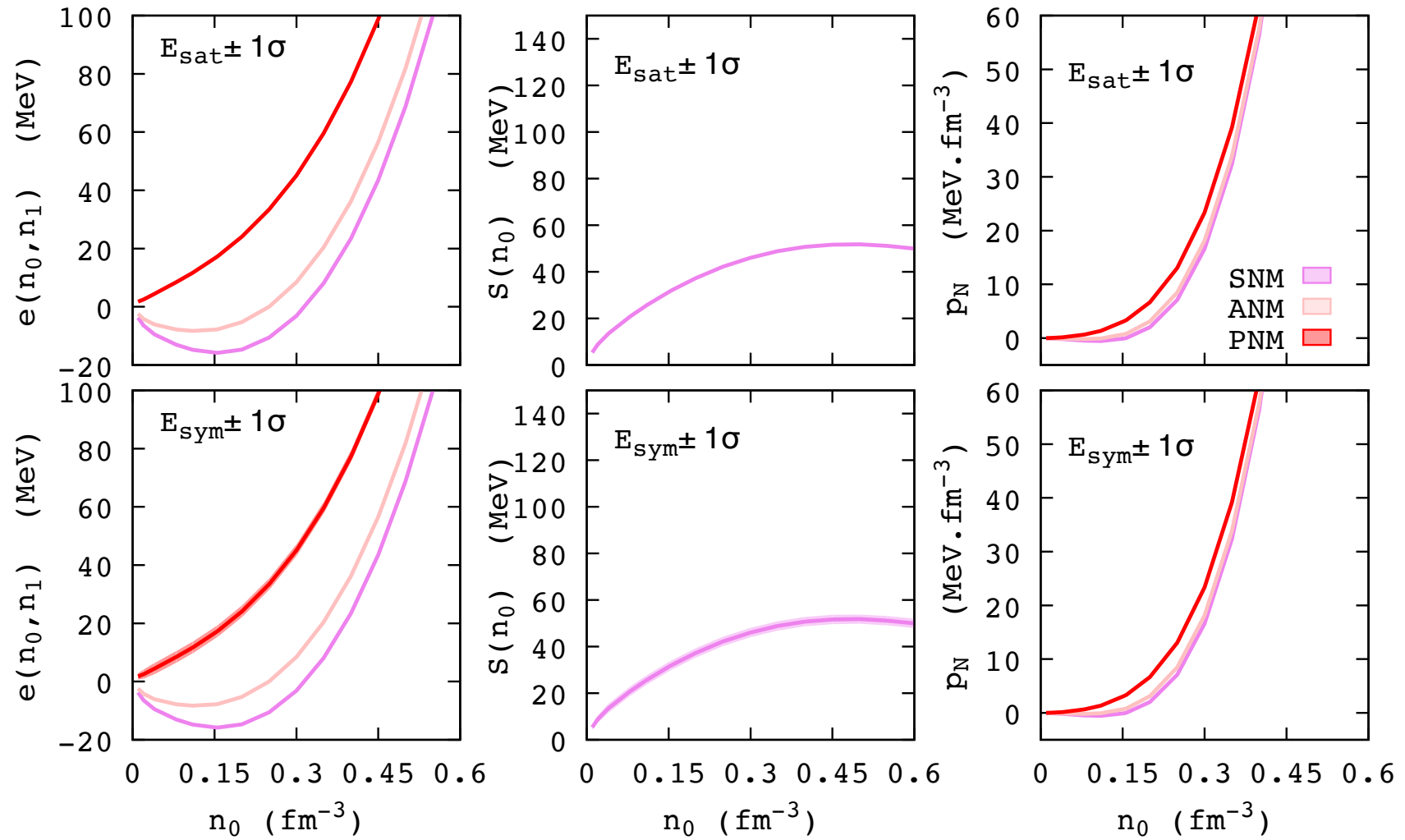
$$v_{\alpha=2}^{s, is} = K_0 - 2t_0^{FG} (-1 + 5\bar{M})$$

$$v_{\alpha=3}^{s, is} = Q_0 - 2t_0^{FG} (4 - 5\bar{M})$$

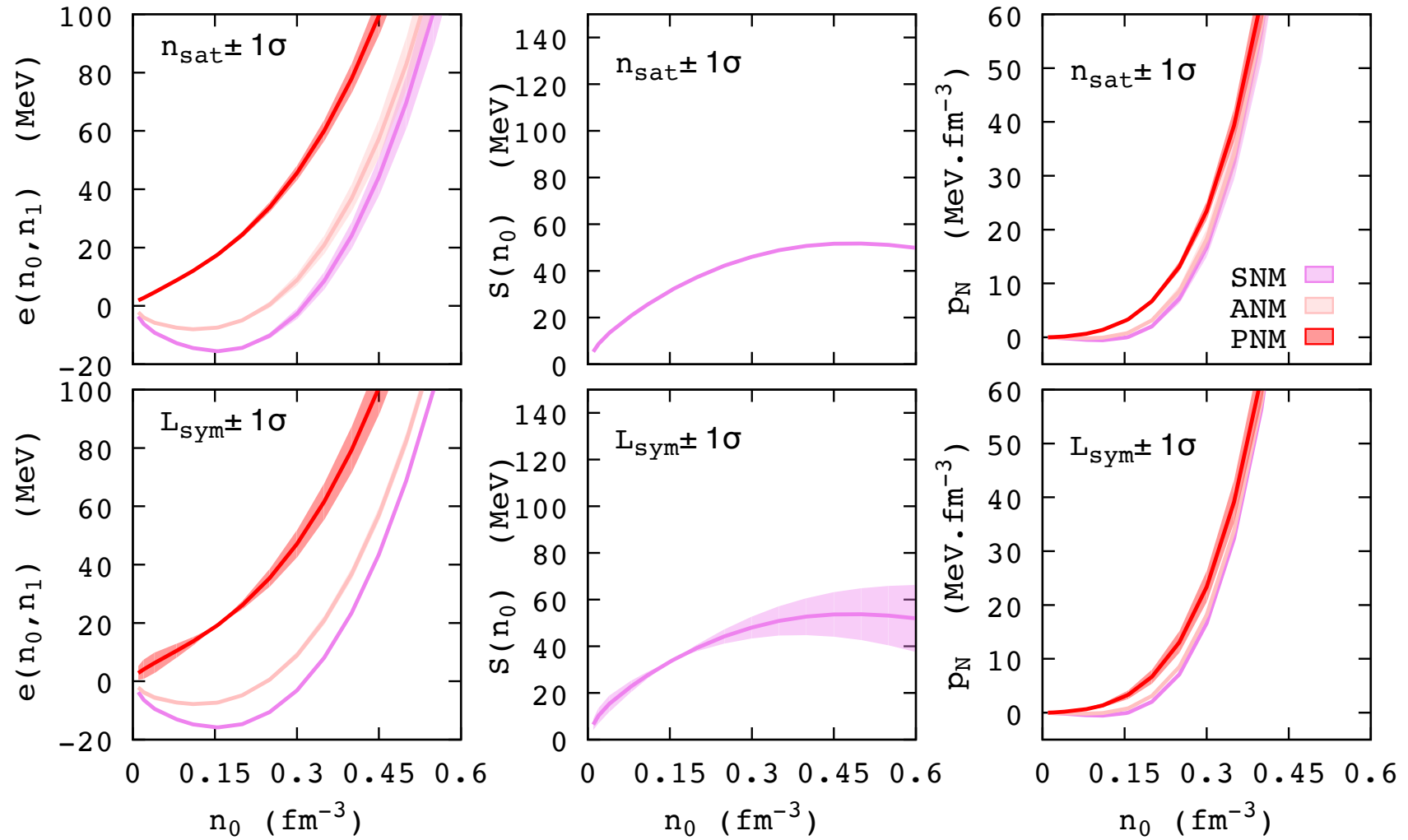
$$v_{\alpha=4}^{s, is} = Z_0 - 8t_0^{FG} (-7 + 5\bar{M})$$

\swarrow
satisfy the limit $\rho \rightarrow 0$

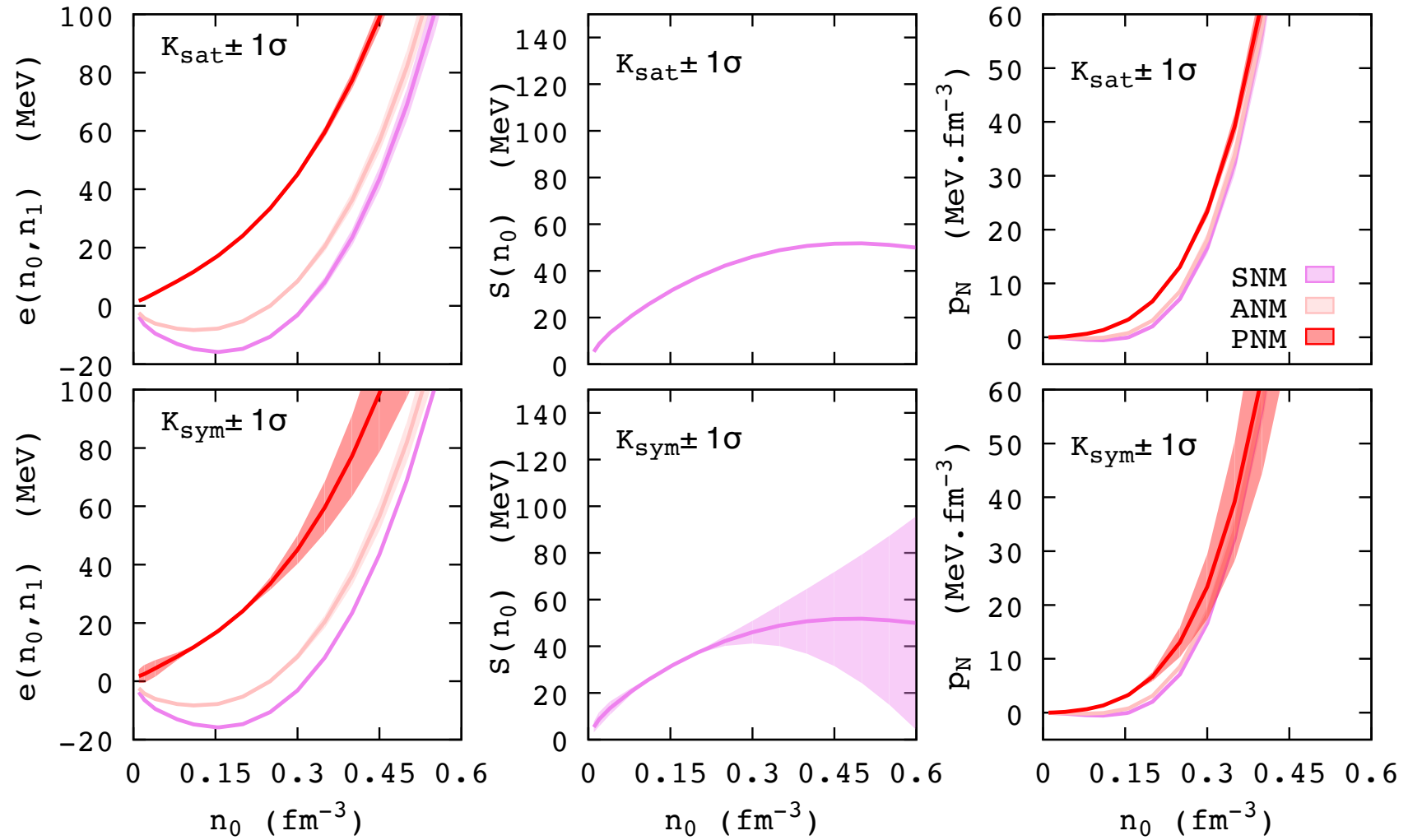
Impact of E_{sat} & E_{sym}



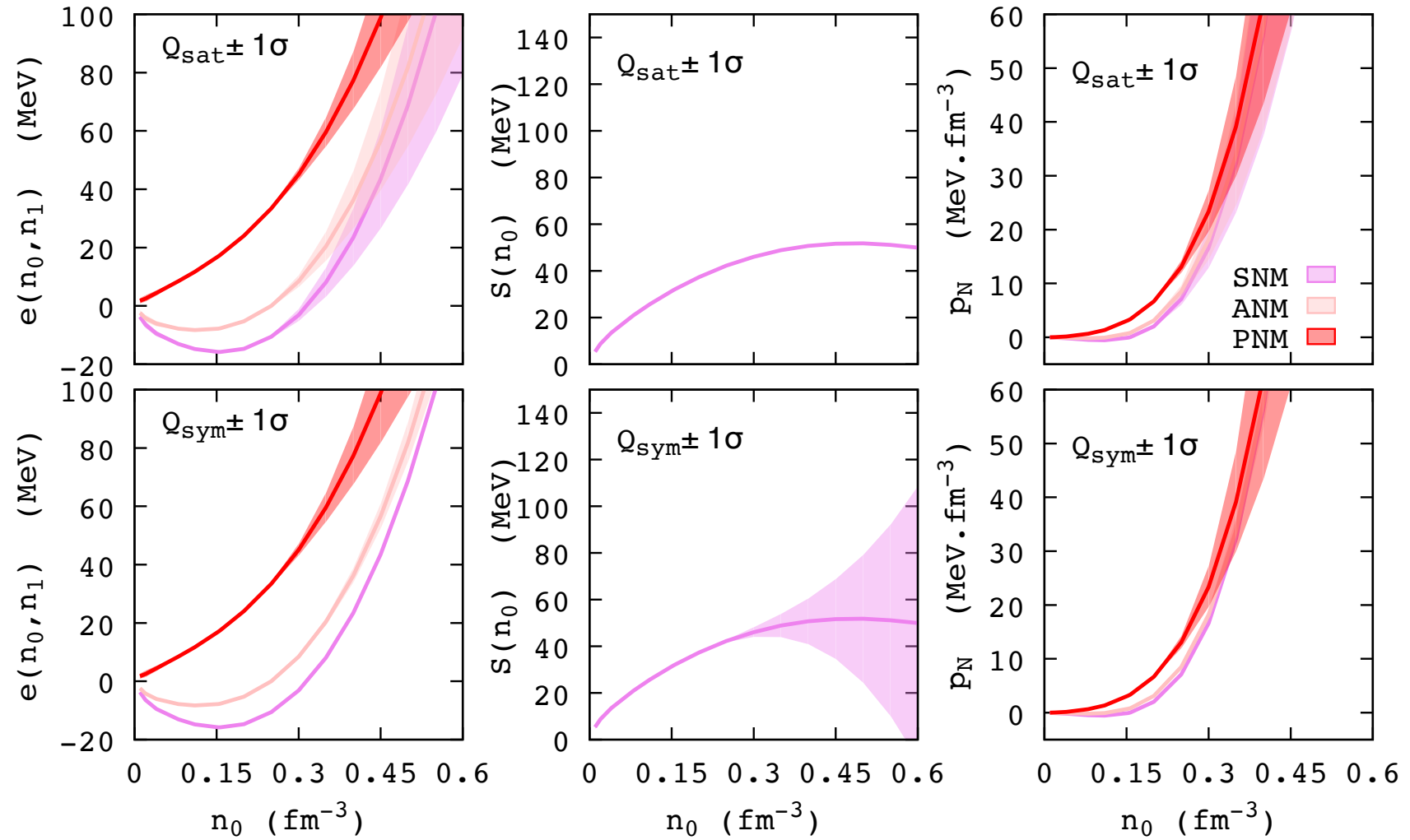
Impact of n_{sat} & L_{sym}



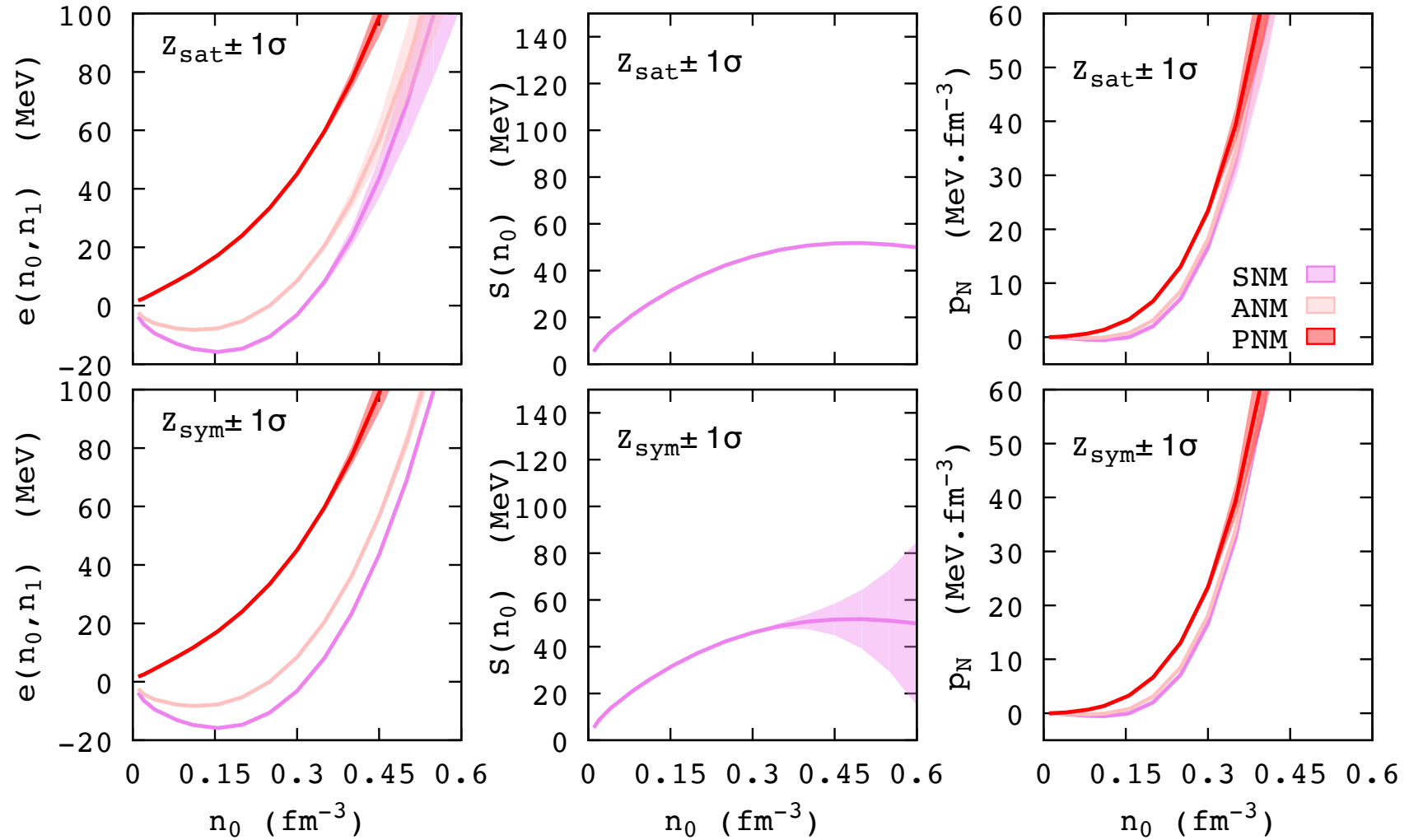
Impact of K_{sat} & K_{sym}



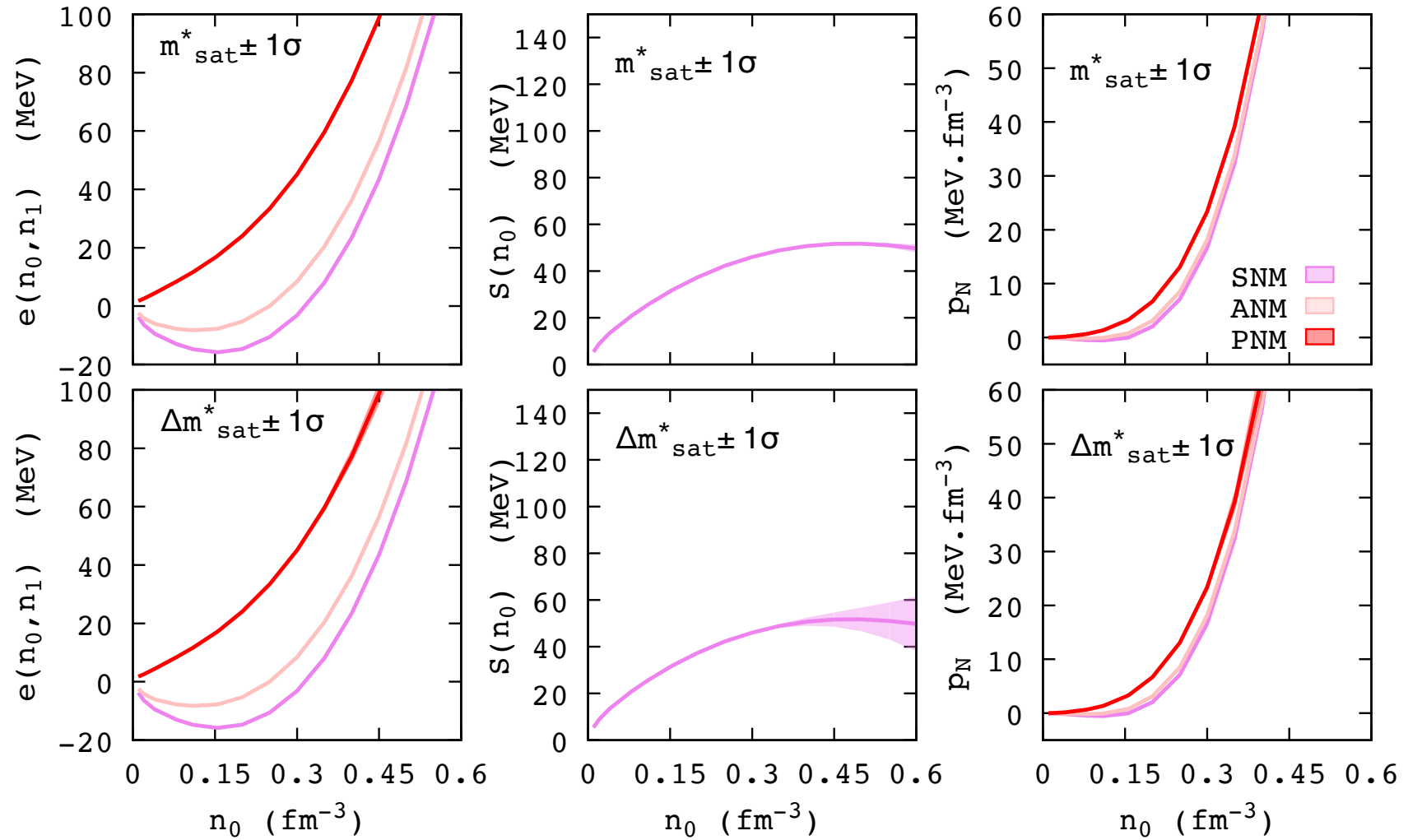
Impact of Q_{sat} & Q_{sym}



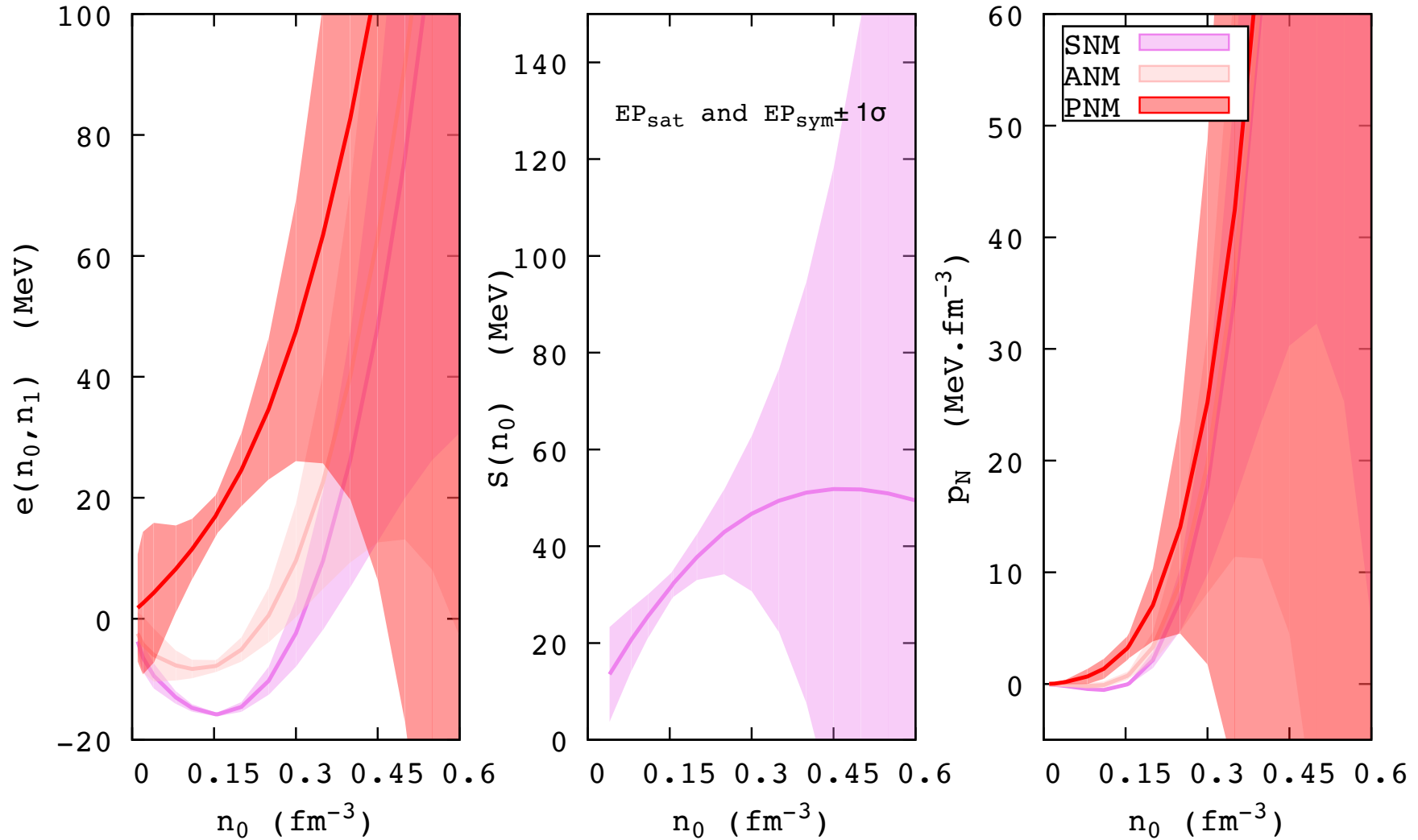
Impact of Z_{sat} & Z_{sym}



Impact of m_{sat}^* & Δm_{sat}^*



Impact of varying all uncertainties $\pm 1\sigma$

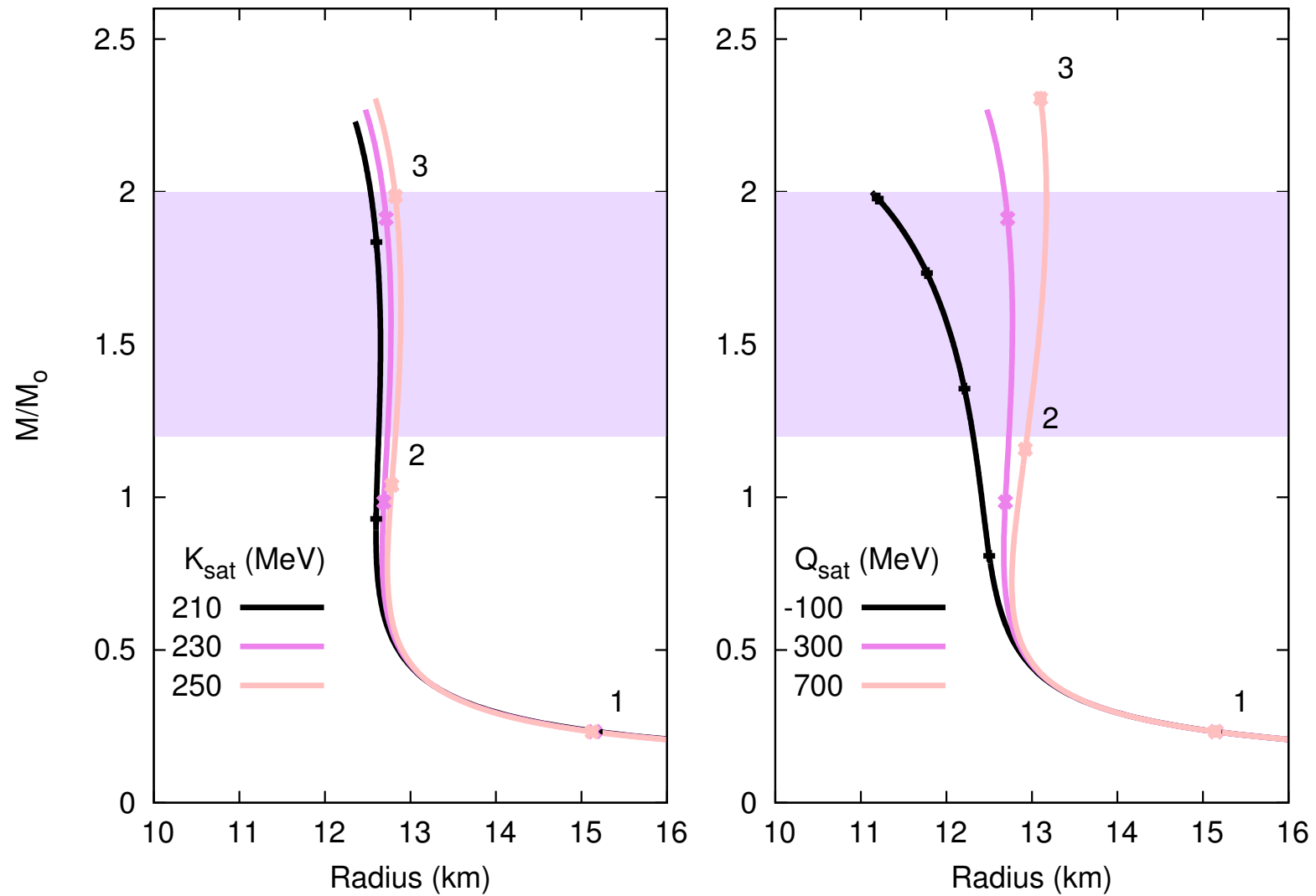


Many EOS does not satisfy basic requirements (stability, causality, ...)

→ We need to remove them

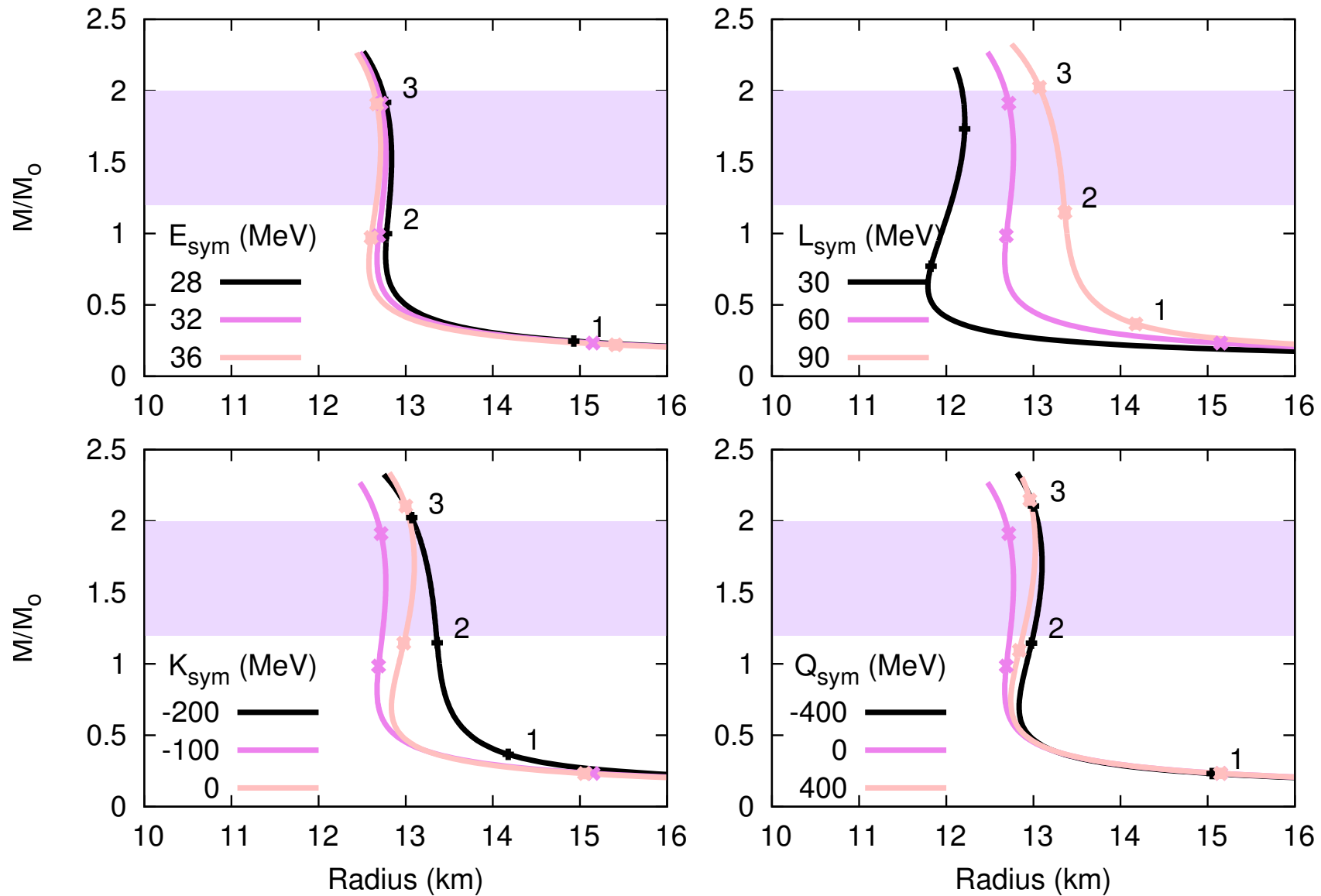
Application to neutron star

Impact of the isoscalar empirical parameters



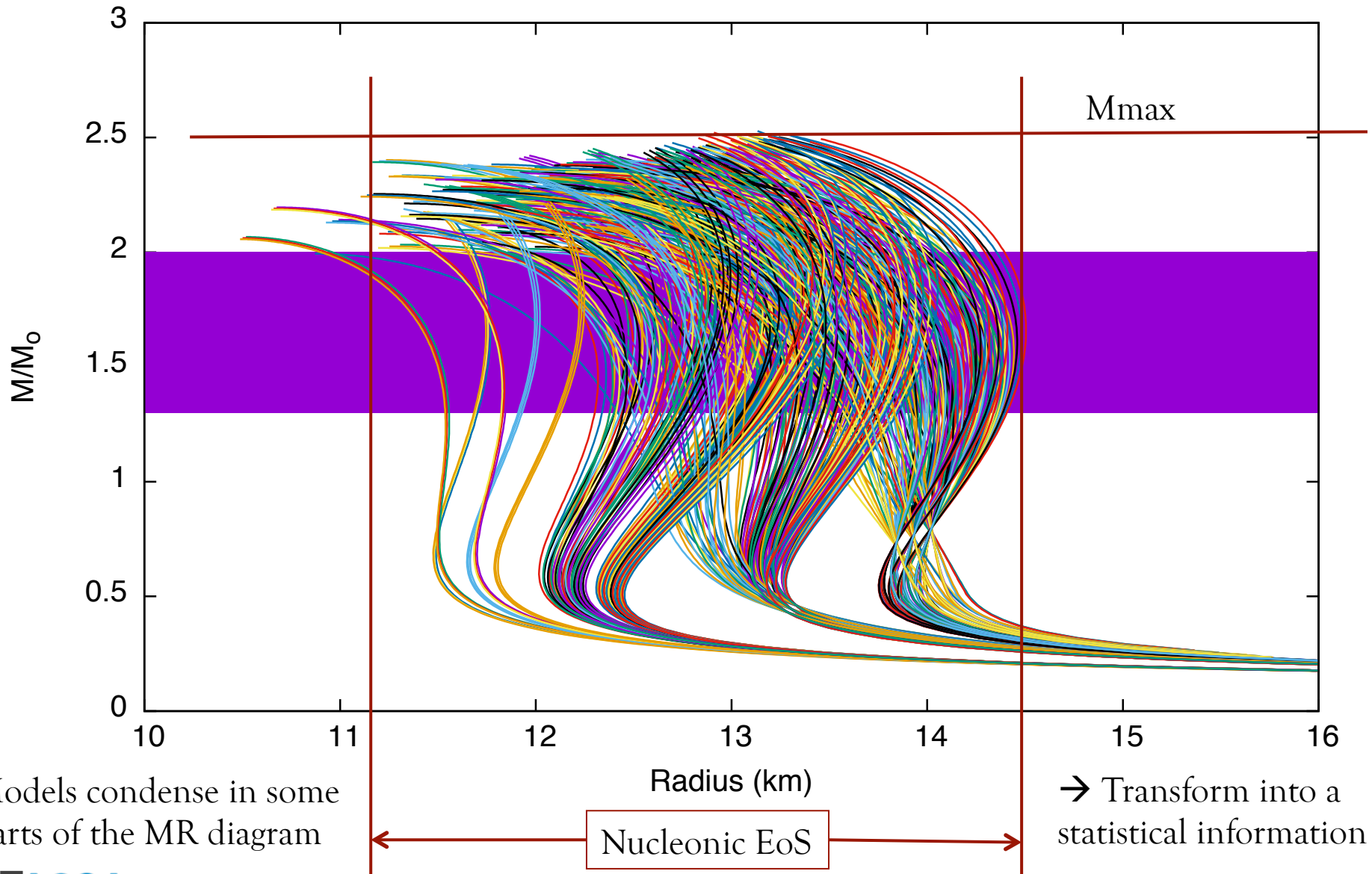
Small impact of these parameters

Impact of the isovector empirical parameters



INT Largest source of uncertainty: L_{sym} and K_{sym}

Impact of the “exp” unknown on the Mass/Radius relation



Bayesian analysis

Prior probability distribution: $g_{P_{\alpha,1}, P_{\alpha,2}}(P_{\alpha}) = \frac{1}{\sqrt{2\pi}P_{\alpha,2}} \exp -\frac{1}{2} \left(\frac{P_{\alpha} - P_{\alpha,1}}{P_{\alpha,2}} \right)^2$

P_{α}	E_{sym} MeV	L_{sym} MeV	K_{sar} MeV	K_{sym} MeV	Q_{sar} MeV	Q_{sym} MeV	Z_{sar} MeV	Z_{sym} MeV
$P_{\alpha,1}$	32	60	230	-100	300	0	-500	-500
$P_{\alpha,2}$	2	15	20	100	400	400	1000	1000
Min	26	20	190	-400	-1300	-2000	-4500	-5500
Max	38	90	270	200	1900	2000	3500	4500
step	2	10	20	75	400	400	1000	1000
N	7	8	5	9	9	11	9	11

Likelihood probability: filter imposing general physical constraints

$$p_{lik}(\{P_{\alpha}\}_i) = \frac{1}{N_{lik}} w_{\text{filter}}(\{P_{\alpha}\}_i) \prod_{\alpha=1}^8 g_{P_{\alpha}^1, P_{\alpha}^2}(P_{\alpha})$$

← prior

Physical filters: Stability: $\Delta P > 0$ Causality : $v_s^2 < c^2$
 Positiveness of the symmetry energy
 $M_{\text{max}} > 2M_{\odot}$

3 hypothesis for the cooling of NS: some NS may need fast cooling

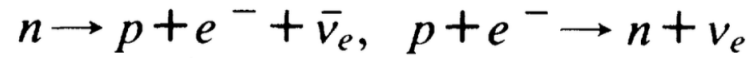
DURCA-0: No fast (dUrca) cooling for all observed NS

DURCA-1: Fast cooling only for NS with $M > 1.8M_{\odot}$

DURCA-2: Fast cooling only for NS with $1.8M_{\odot} > M > 1.6M_{\odot}$

DURCA: Impact on the proton fraction $x_p = n_p/n_b$

The direct URCA process



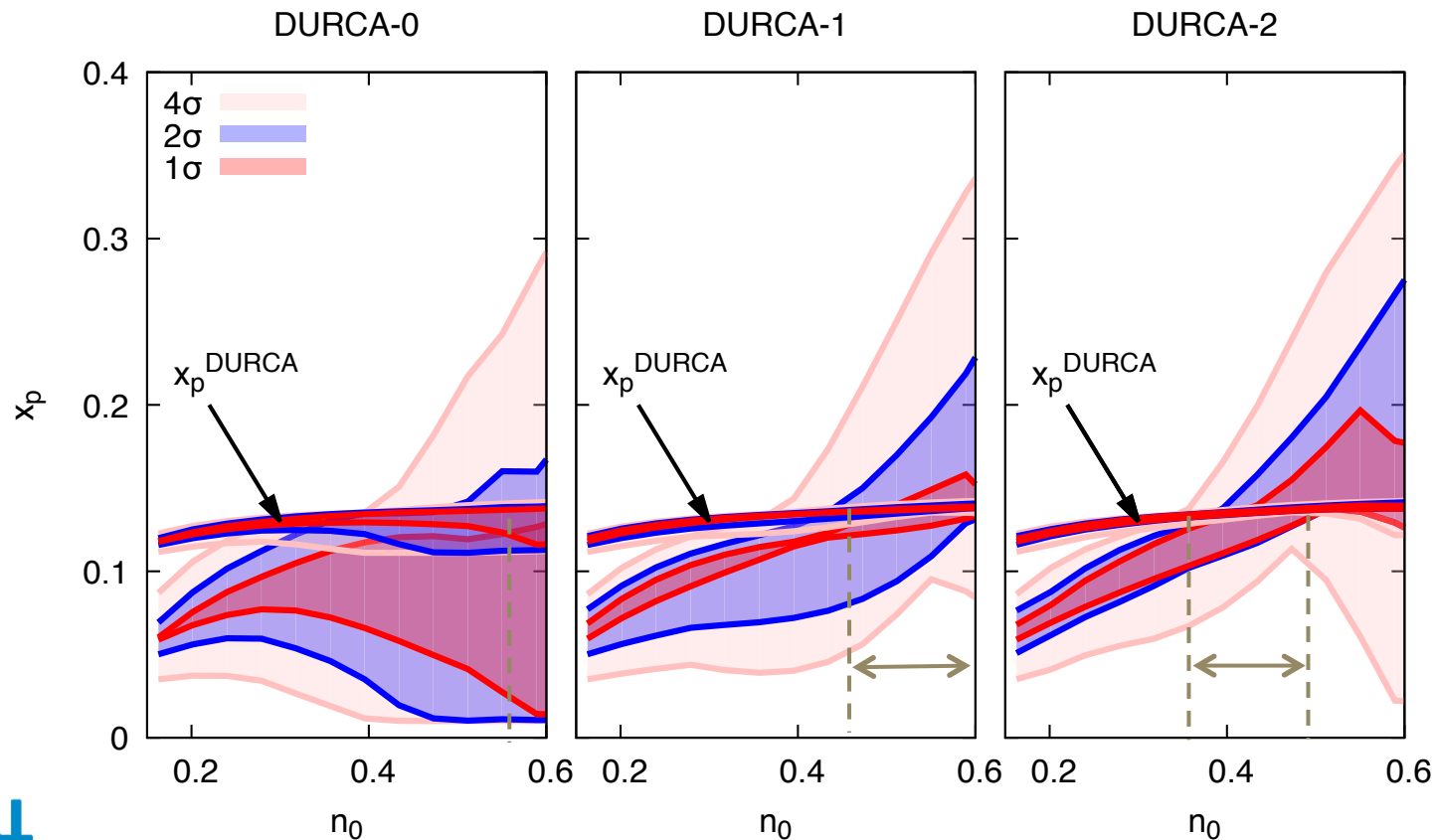
Energy and momentum conservation:

$$p_{Fp} + p_{Fe} > p_{Fn} \rightarrow x_p > x_{DU} \approx 1/9$$

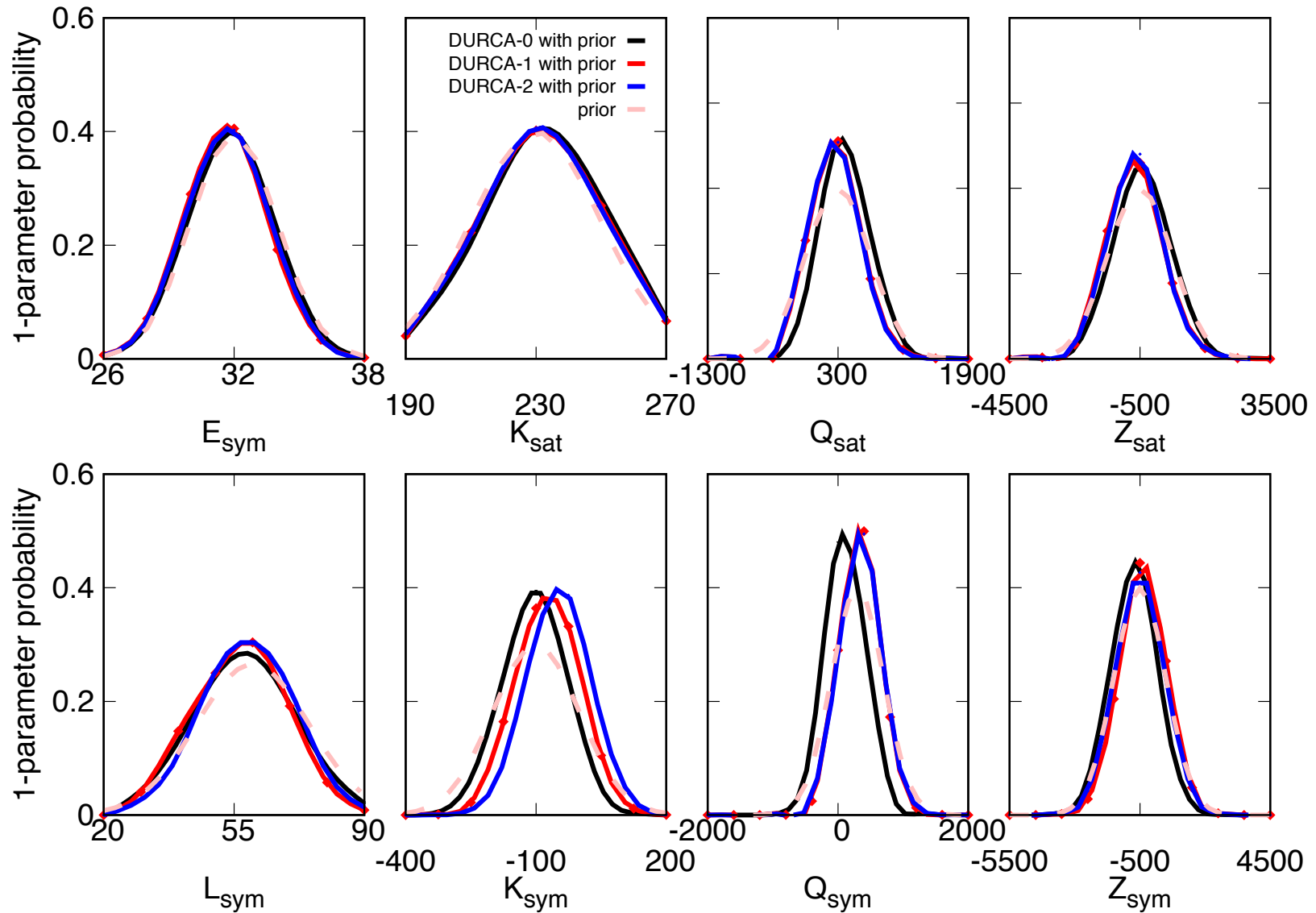
For n, p, e⁻, μ⁻ matter:
$$x_{DU} = \left[1 + (1 + x_{ep}^{1/3})^3 \right]^{-1}$$

with
$$x_{ep} = n_e/n_p = n_e/(n_e + n_\mu)$$

Gamow-Shoenberg 1941,
Lattimer et al., PRL 1991



Posterior probabilities



Correlation matrix

$$\text{corr}(P_\alpha, P_\beta) = \frac{\text{cov}(P_\alpha, P_\beta)}{\sigma_\alpha \sigma_\beta}$$

DURCA-0

Z _{sym}	0.0	-0.1	-0.1	-0.0	-0.0	-0.1	-0.3	1.0
Q _{sym}	-0.0	-0.1	-0.1	-0.0	-0.2	-0.5	1.0	-0.3
K _{sym}	-0.0	-0.0	0.1	-0.1	-0.3	1.0	-0.5	-0.1
L _{sym}	0.0	0.0	0.0	-0.0	1.0	-0.3	-0.2	-0.0
E _{sym}	0.0	0.0	0.0	1.0	-0.0	-0.1	0.0	-0.0
Z _{sat}	-0.0	-0.3	1.0	0.0	0.0	0.1	-0.1	-0.1
Q _{sat}	-0.1	1.0	-0.3	0.0	0.0	-0.0	-0.1	-0.1
K _{sat}	1.0	-0.1	-0.0	0.0	0.0	-0.0	-0.0	0.0
	K _{sat}	Q _{sat}	Z _{sat}	E _{sym}	L _{sym}	K _{sym}	Q _{sym}	Z _{sym}

DURCA hypothesis

causality

Low order empirical parameters are very weakly impacted.

Correlations remain weak.



DURCA-0

Z _{sym}	0.0	-0.1	-0.1	-0.0	-0.0	-0.1	-0.3	1.0
Q _{sym}	-0.0	-0.1	-0.1	-0.0	-0.2	-0.5	1.0	-0.3
K _{sym}	-0.0	-0.0	0.1	-0.1	-0.3	1.0	-0.5	-0.1
L _{sym}	0.0	0.0	0.0	-0.0	1.0	-0.3	-0.2	-0.0
E _{sym}	0.0	0.0	0.0	1.0	-0.0	-0.1	-0.0	-0.0
Z _{sat}	-0.0	-0.3	1.0	0.0	0.0	0.1	-0.1	-0.1
Q _{sat}	-0.1	1.0	-0.3	0.0	0.0	-0.0	-0.1	-0.1
K _{sat}	1.0	-0.1	-0.0	0.0	0.0	-0.0	-0.0	0.0
	K _{sat}	Q _{sat}	Z _{sat}	E _{sym}	L _{sym}	K _{sym}	Q _{sym}	Z _{sym}

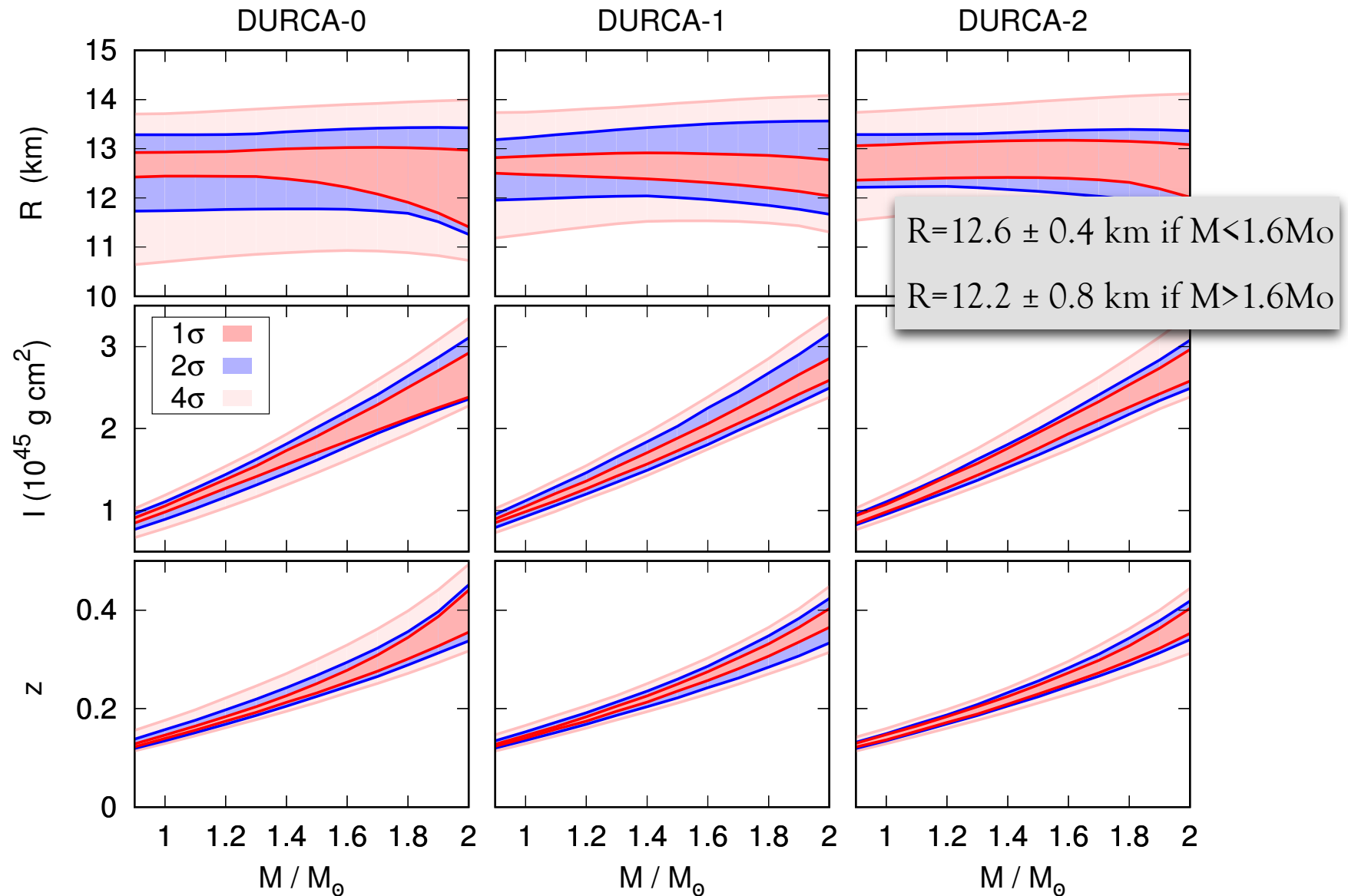
DURCA-1

Z _{sym}	-0.0	-0.1	-0.0	0.0	0.0	-0.2	-0.4	1.0
Q _{sym}	-0.0	0.0	-0.0	-0.0	-0.1	-0.6	1.0	-0.4
K _{sym}	0.1	0.3	0.1	-0.1	-0.4	1.0	-0.6	-0.2
L _{sym}	0.0	0.1	0.0	-0.1	1.0	-0.4	-0.1	0.0
E _{sym}	0.0	0.0	0.0	1.0	-0.1	-0.1	-0.0	0.0
Z _{sat}	0.0	-0.4	1.0	0.0	0.0	0.1	-0.0	-0.0
Q _{sat}	-0.1	1.0	-0.4	0.0	0.1	0.3	0.0	-0.1
K _{sat}	1.0	-0.1	0.0	0.0	0.0	0.1	-0.0	-0.0
	K _{sat}	Q _{sat}	Z _{sat}	E _{sym}	L _{sym}	K _{sym}	Q _{sym}	Z _{sym}

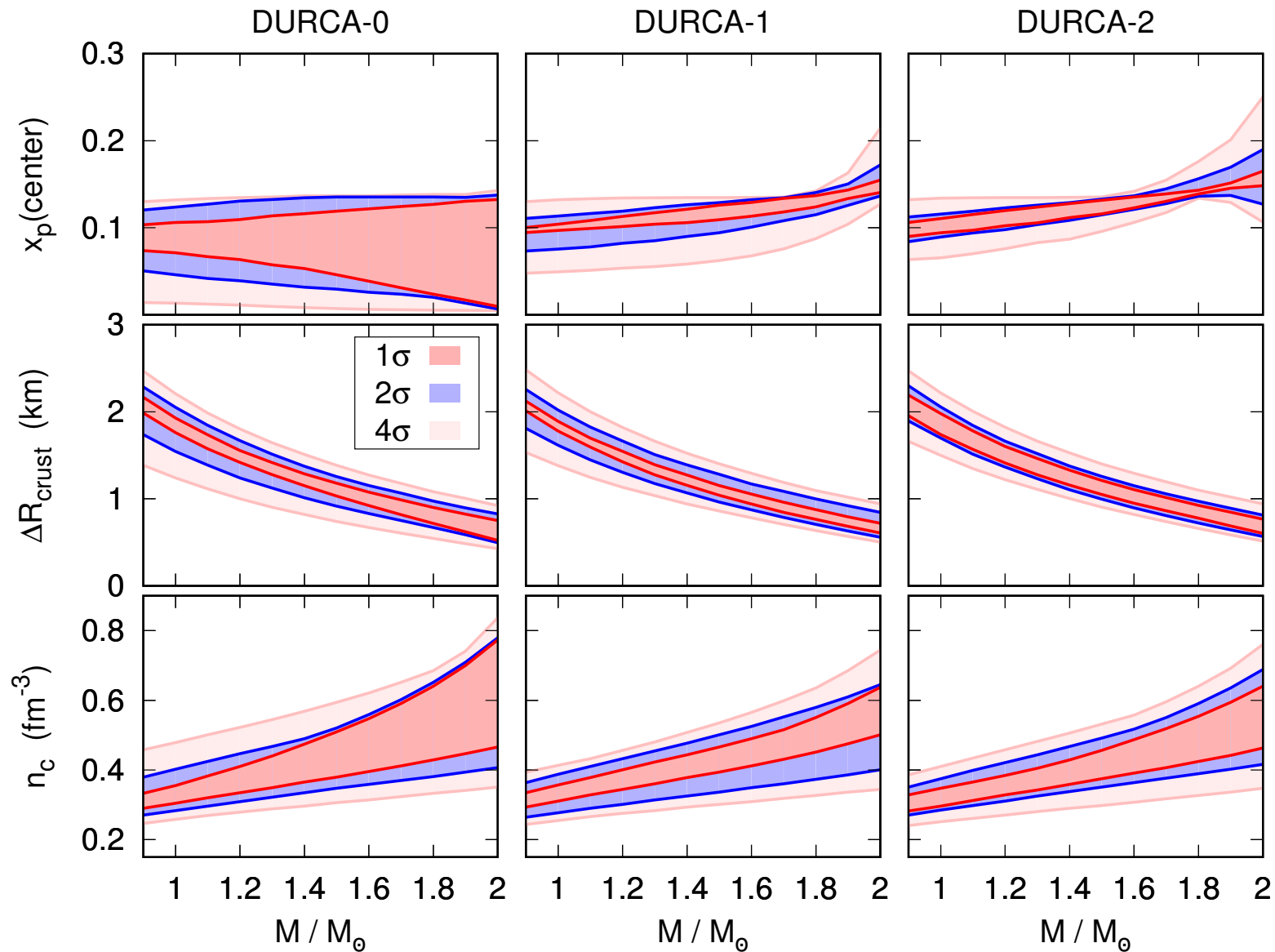
DURCA-2

Z _{sym}	-0.0	-0.1	-0.0	0.0	0.0	-0.2	-0.3	1.0
Q _{sym}	0.0	-0.0	-0.0	-0.0	-0.1	-0.5	1.0	-0.3
K _{sym}	0.1	0.3	0.0	-0.2	-0.5	1.0	-0.5	-0.2
L _{sym}	0.1	0.2	0.0	-0.1	1.0	-0.5	-0.1	0.0
E _{sym}	0.0	0.0	0.0	1.0	-0.1	-0.2	-0.0	0.0
Z _{sat}	-0.0	-0.4	1.0	0.0	0.0	0.0	-0.0	-0.0
Q _{sat}	-0.0	1.0	-0.4	0.0	0.2	0.3	-0.0	-0.1
K _{sat}	1.0	-0.0	-0.0	0.0	0.1	0.1	0.0	-0.0
	K _{sat}	Q _{sat}	Z _{sat}	E _{sym}	L _{sym}	K _{sym}	Q _{sym}	Z _{sym}

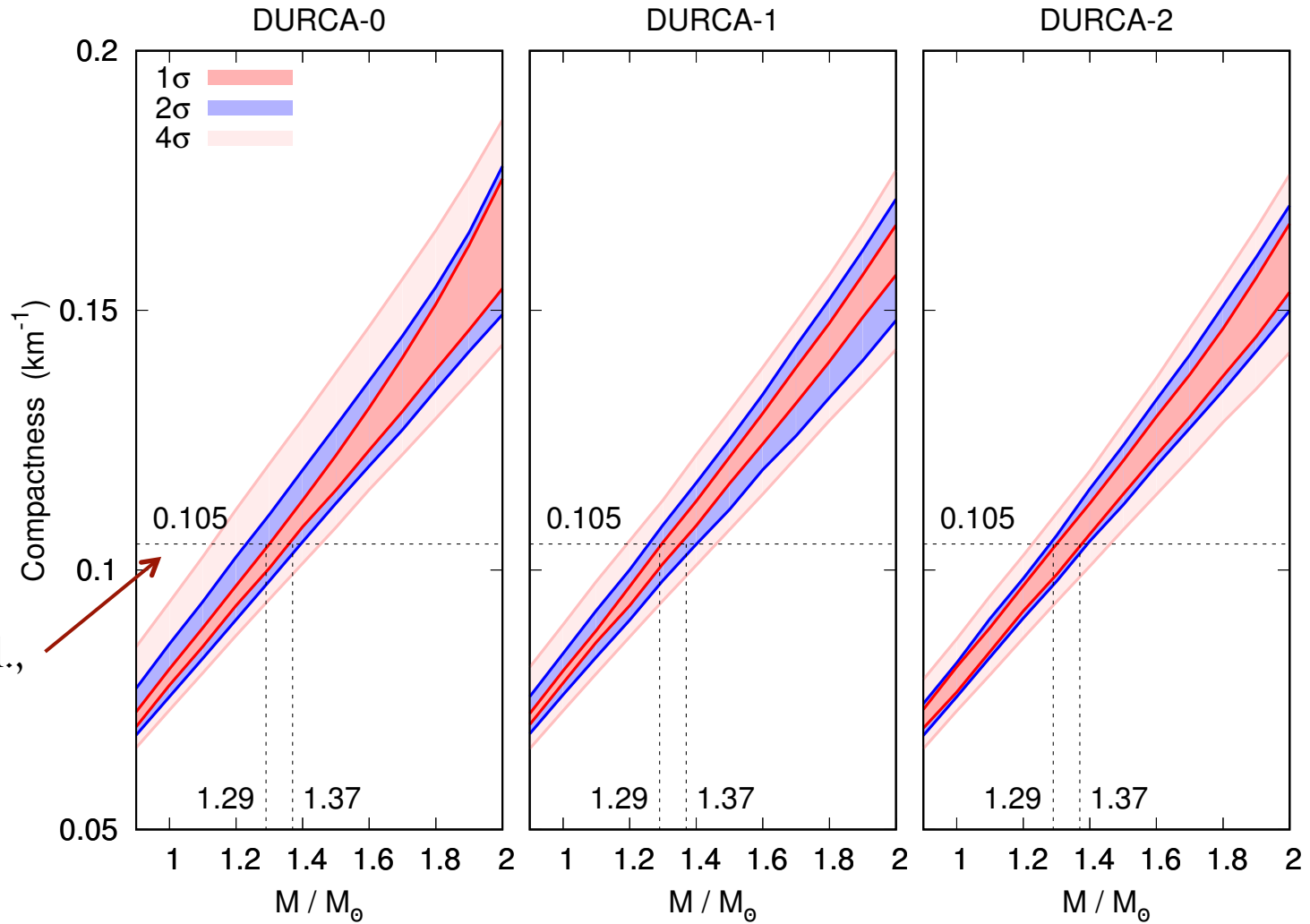
Predictions of general properties (M)



Predictions of general properties (M)

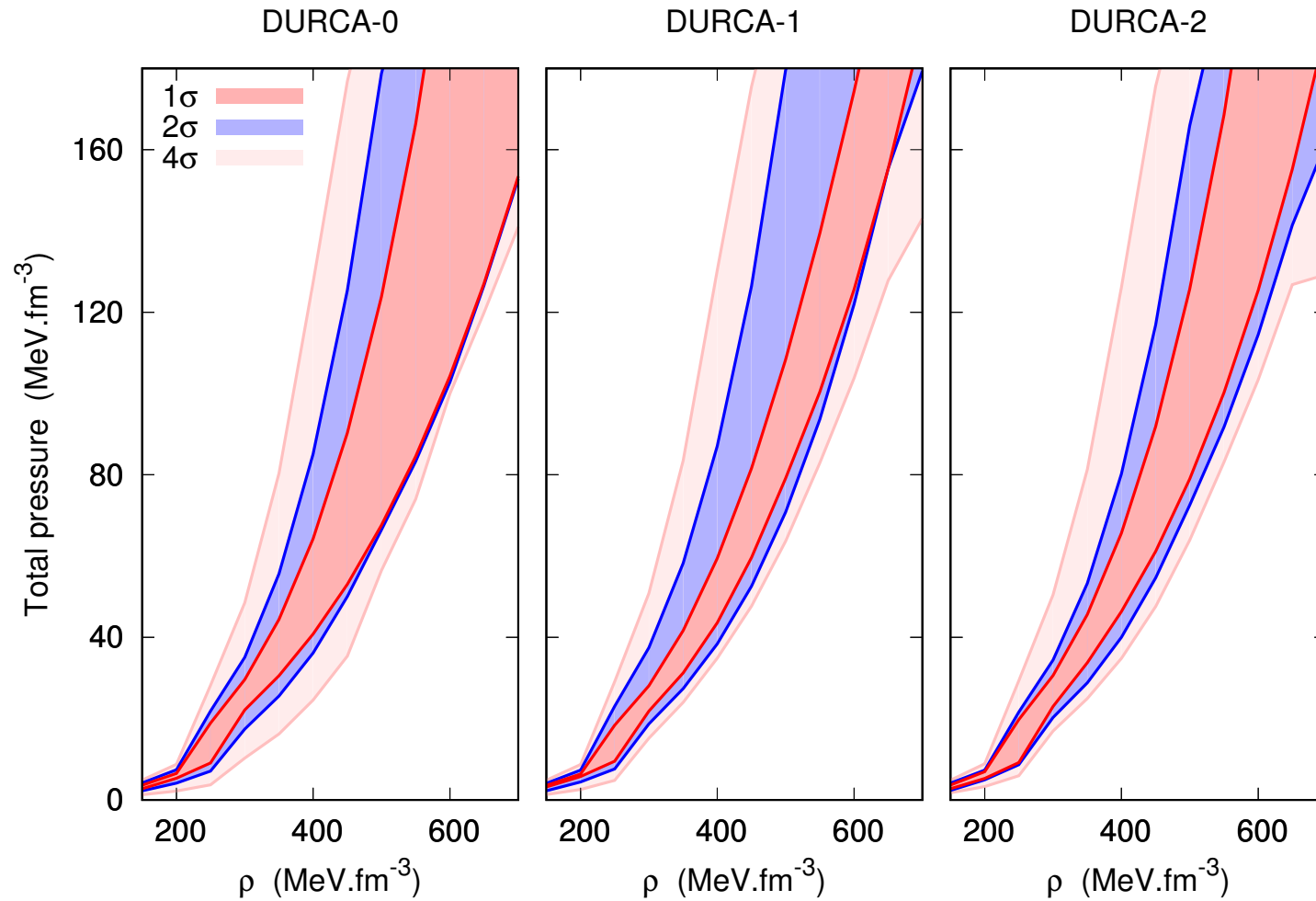


Compactness of RX J0720.4-3125

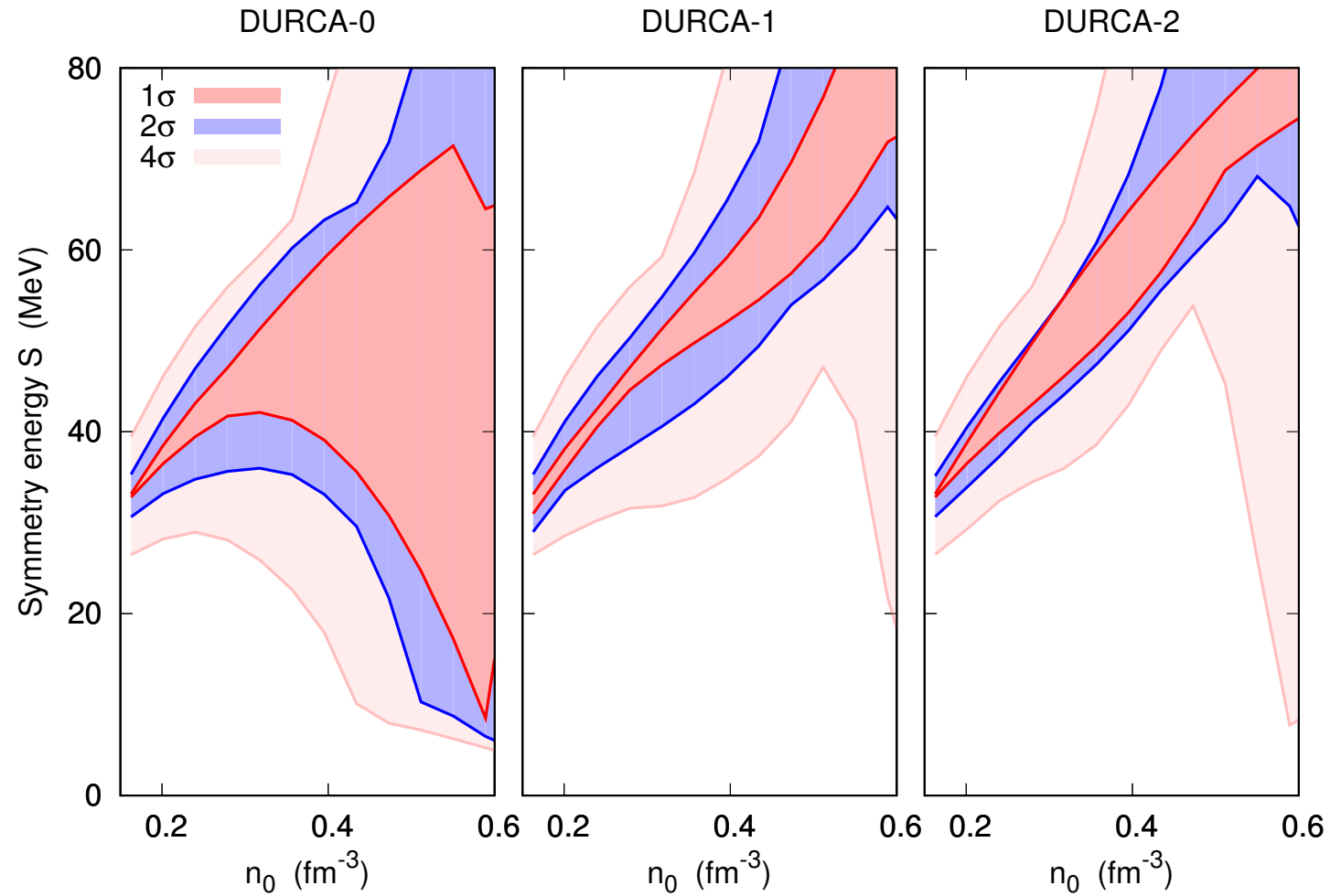


Hambaryan et al.,
A&A 2017

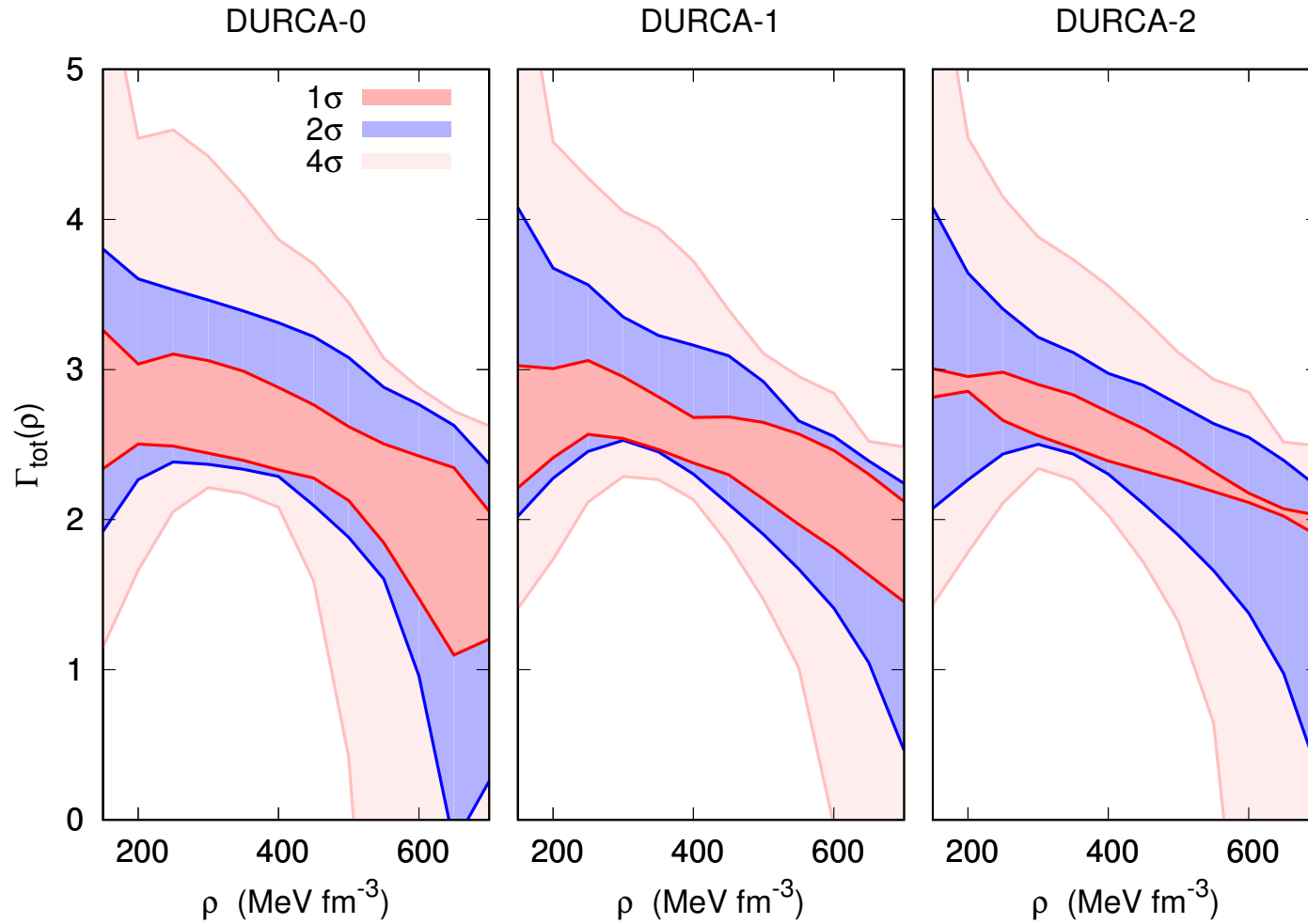
Prediction for the nuclear EOS at β -equilibrium



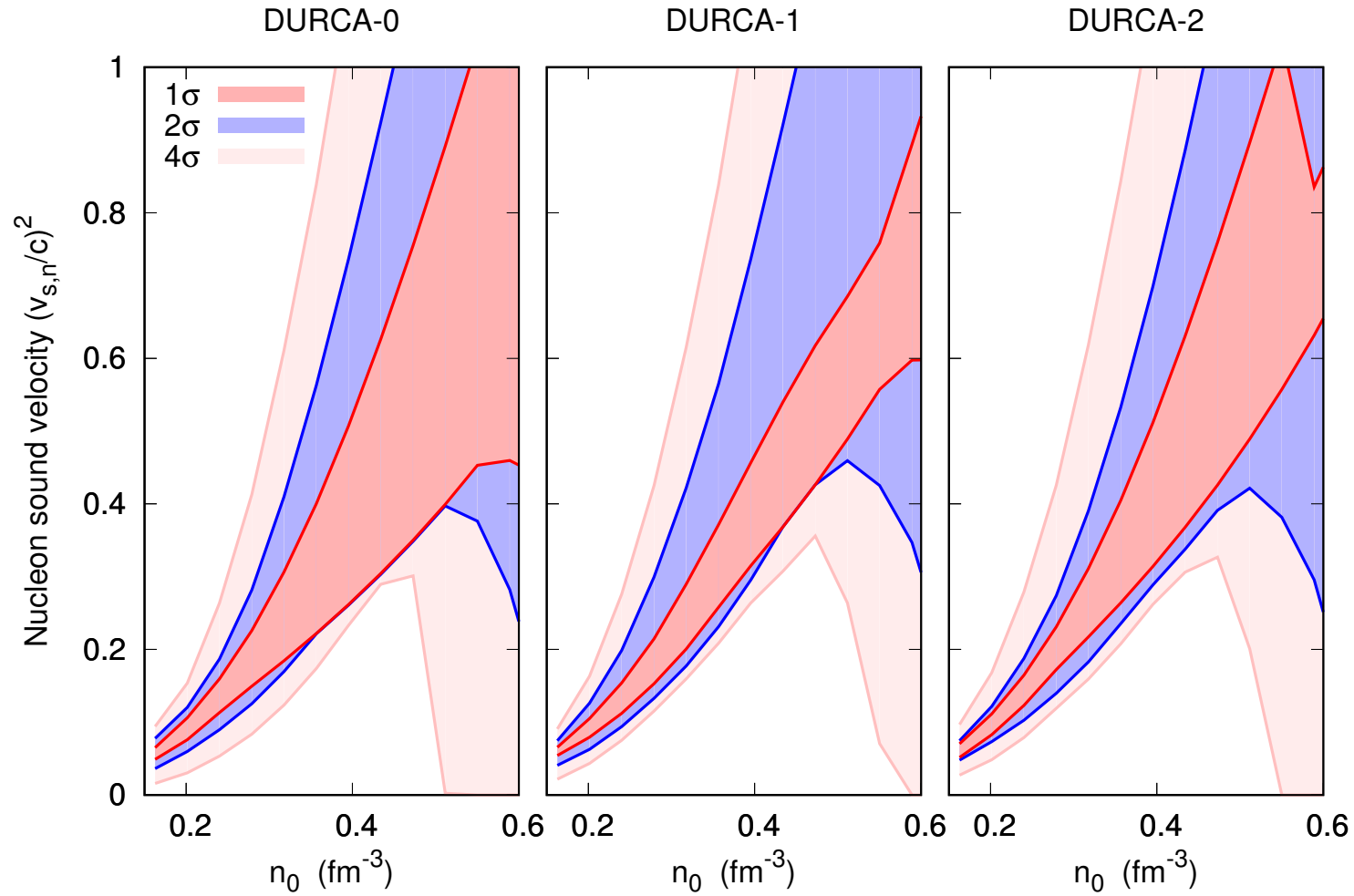
Prediction for the symmetry energy $S(n)$



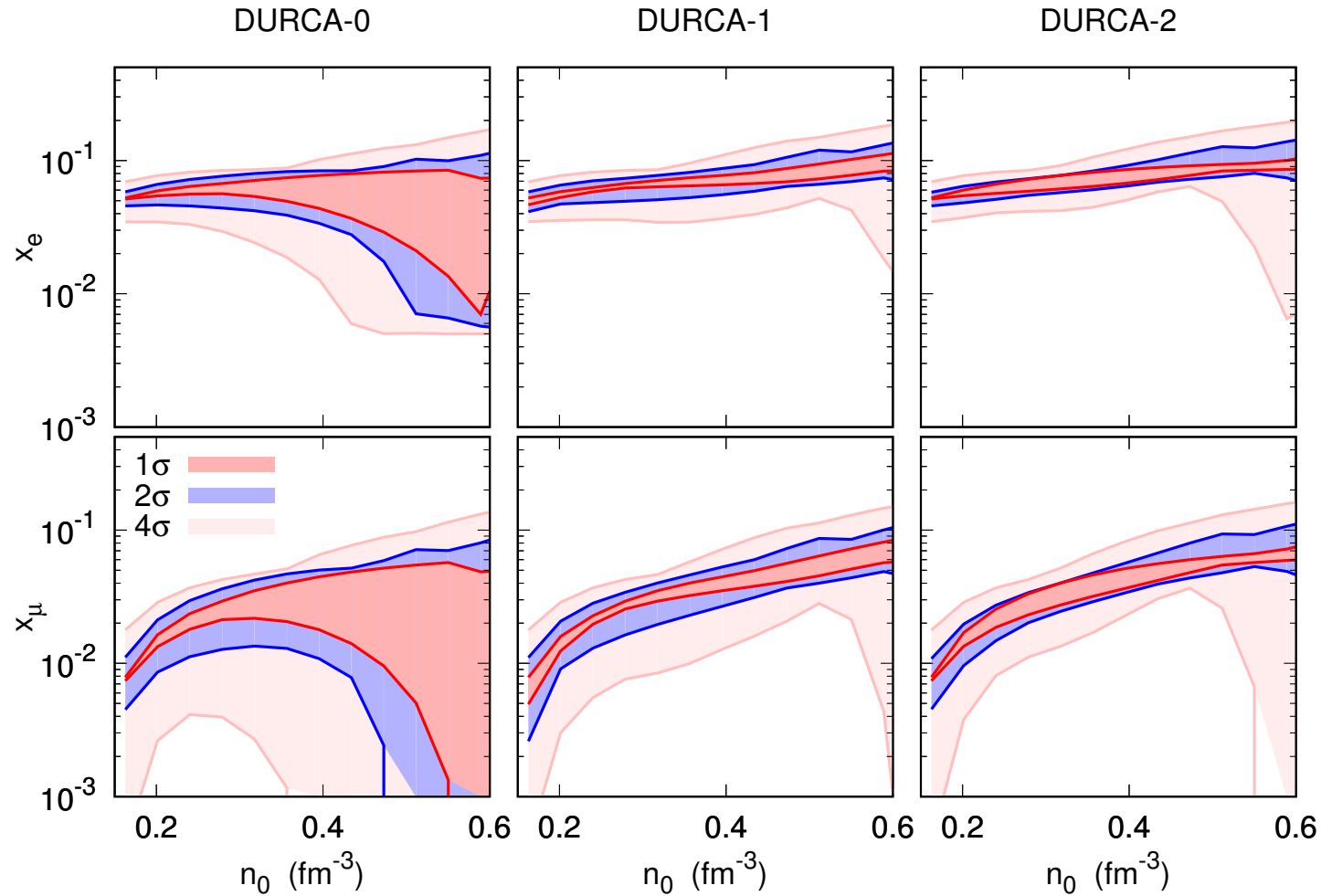
Prediction for the adiabatic index $\Gamma(n)$



Prediction for sound velocity $v_s(n)$



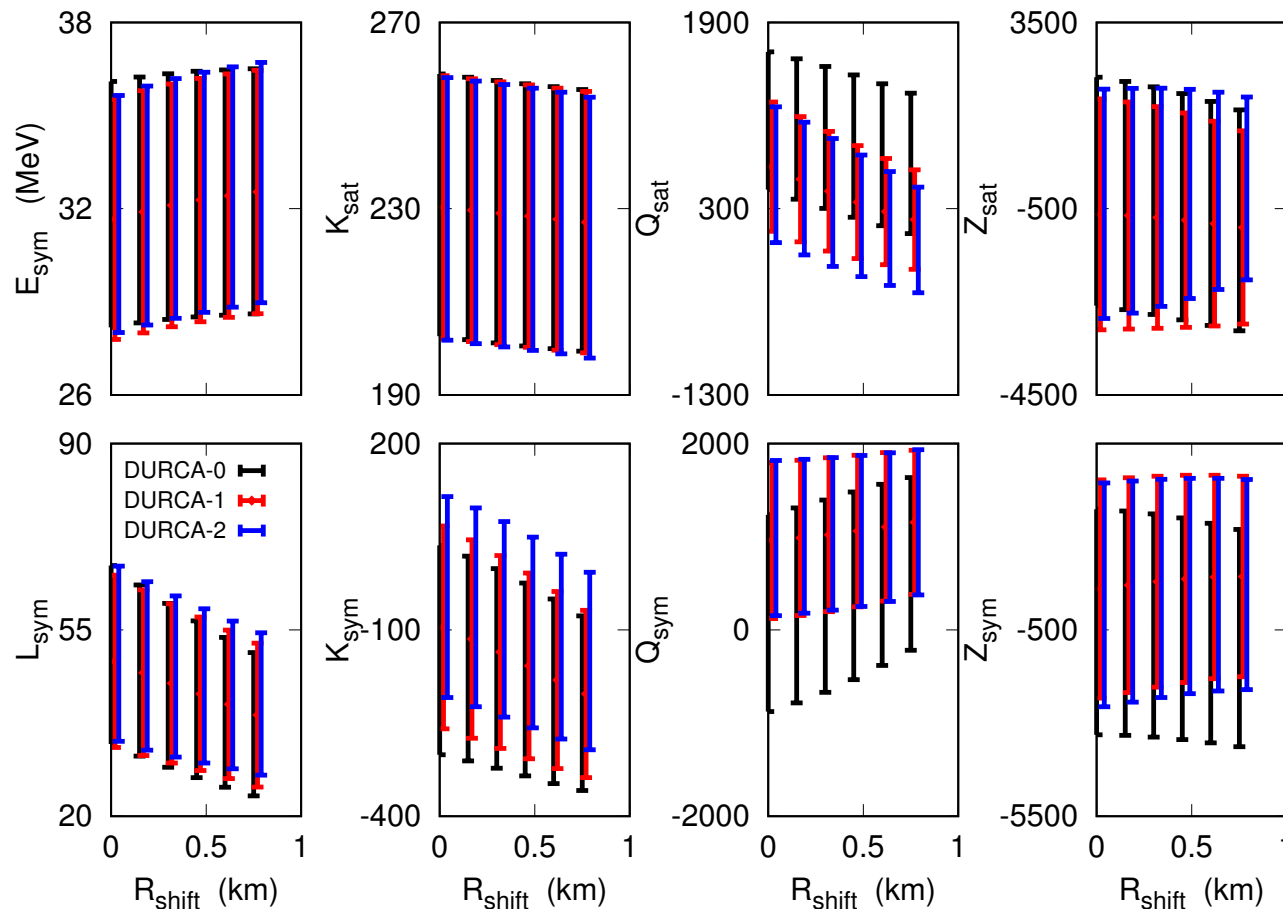
Prediction for particle fractions



If it can be proved that dUrca exist \rightarrow possible tight constraint for the nuclear EOS

Inversion problem: what is the impact of the empirical parameters of the measure of MR relation?

$R_{\text{best}}(M)$ is our most probable MR relation.
 Suppose it is shifted as $R_{\text{best}}(M) - R_{\text{shift}}$



Conclusions

- We propose a flexible form of the EoS which can mimic most of existing nucleonic EoS.
- Information from nuclear physics can be easily encoded, but are not enough and astrophysical information are also needed.
- **Most important parameters are L_{sym} and K_{sym} , as well as $Q_{\text{sat/sym}}$**
- We can predict universal features for nuclear EoS (MR diagram):
 - R=12-13 km
 - P(ρ)
- If it can be proven that dUrca exists \rightarrow possible tight constraint for the nuclear EOS.
- At β -equilibrium, melting temperature of clusters is expected to be quite low (<5 MeV)
 - \rightarrow The impact of clusters on ν propagation remains quite weak.