

Neutron Star Constraints From Mergers and Cold Atoms

J. M. Lattimer

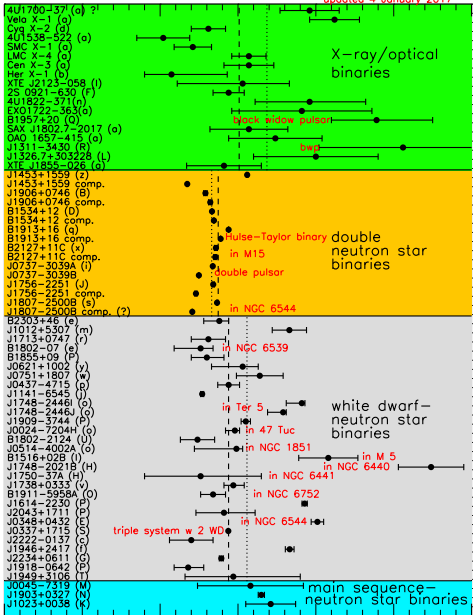
Department of Physics & Astronomy



Observational Signatures of Nucleosynthesis in Neutron Star Mergers
INT, Seattle, Washington
31 July - 4 August, 2017

- ▶ The Unitary Gas Constraint on the Nuclear Symmetry Energy
- ▶ Neutron Star Universal Structure Relations
- ▶ Using Tidal Deformabilities to Infer the Equation of State
- ▶ Universal Structure Relations for Hybrid Stars

updated 4 January 2017



0.0 0.5 1.0 1.5 2.0 2.5 3.0
Neutron star mass (M_{\odot})

vanKerkwijk 2010
Romani et al. 2012

Although simple average mass of w.d. companions is $0.23 M_{\odot}$ larger, weighted average is $0.07 M_{\odot}$ larger

Demorest et al. 2010
Fonseca et al. 2016
Antoniadis et al. 2013
Barr et al. 2016

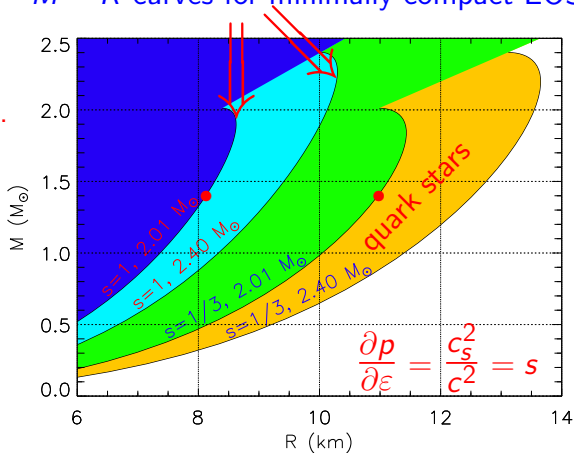
Causality + GR Limits and the Maximum Mass

A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precision upper limit to R , with a well-measured mass, sets an upper limit to the maximum mass.

$$R_{1.4} > 8.15 \text{ km}$$
$$M_{\text{max}} \geq 2.01 M_{\odot}$$

$M - R$ curves for minimally compact EOS



If quark matter exists in the interior, the minimum radii are substantially larger.

The Unitary Gas

The **unitary gas** is an idealized system consisting of fermions interacting via a pairwise zero-range s-wave interaction with an infinite scattering length:

As long as the scattering length $a \gg k_F^{-1}$ (interparticle spacing), and the range of the interaction $R \ll k_F^{-1}$, the properties of the gas are universal in the sense they don't depend on the details of the interaction.

The sole remaining length scale is $k_F = (3\pi^2 n)^{1/3}$, so the unitary gas energy is a constant times the Fermi energy $\hbar^2 k_F^2 / (2m)$:

$$E_{\text{UG}} = \xi_0 \frac{3\hbar^2 k_F^2}{10m}.$$

$\xi_0 \simeq 0.37$ is known as the **Bertsch** parameter, measured in cold-atom experiments.

The Unitary Gas as Analogue of the Neutron Gas

A pure neutron matter (PNM) gas differs from the unitary gas:

- ▶ $|a| \simeq 18.5 \text{ fm}$; $|ak_F|^{-1} \simeq 0.03$ for $n = n_s$.
- ▶ $R \simeq 2.7 \text{ fm}$; $Rk_F \approx 4.5$ for $n = n_s$.
- ▶ Repulsive 3-body interactions are additionally necessary for neutron matter to fit the energies of light nuclei.
- ▶ Neutron matter has potentially attractive p-wave and higher-order interactions.

The first three imply $E_{\text{PNM}} > E_{\text{UG}}$:

- ▶ $\xi \simeq \xi_0 + 0.6|ak_F|^{-1} + \dots$ $|ak_F|^{-1} \ll 1$
- ▶ $\xi \simeq \xi_0 + 0.12Rk_F + \dots$ $Rk_F \ll 1$

A reasonable conjecture would appear to be ($u = n/n_s$)

$$E_{\text{PNM}}(u) = E(u, Y_p = 0) \geq E_{\text{UG},0} u^{2/3} \simeq 12.6 u^{2/3} \text{ MeV}$$

Consequences for the Nuclear Symmetry Energy

$$S(u) = E_{\text{PNM}} - E(u, Y_p = 1/2).$$

A good approximation for the Y_p -dependence of E is

$$S(u) \simeq \frac{1}{8} \frac{\partial^2 E(u, Y_p)}{\partial Y_p^2}.$$

Near n_s ,

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_{\text{sym}}}{18}(u-1)^2 + \dots$$

$$E(u, Y_p = 1/2) \simeq -B + \frac{K_s}{18}(u-1)^2 + \dots$$

In this case, the unitary gas conjecture is

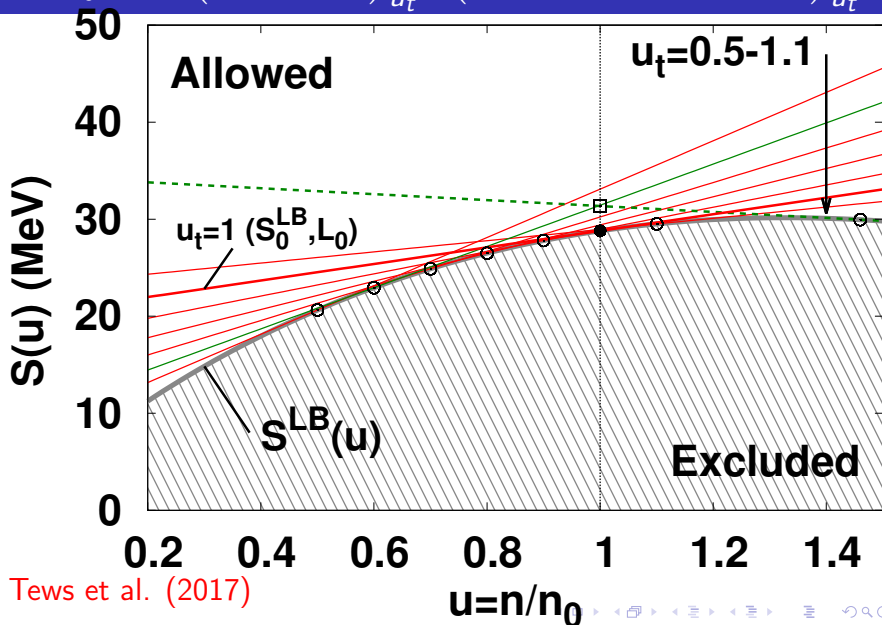
$$S(u) > E_{\text{UG},0} u^{2/3} - \left[-B + \frac{K_s}{18}(u-1)^2 + \dots \right] \equiv S^{\text{LB}}(u)$$

Thus, the symmetry energy parameters S_0 and L must satisfy

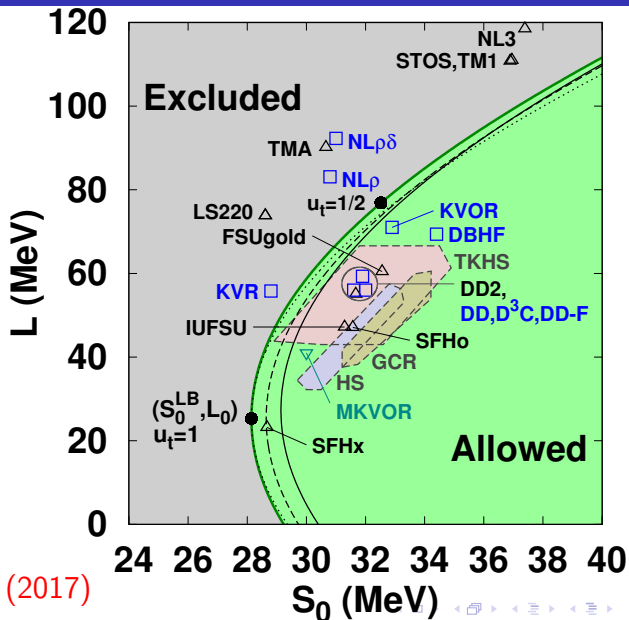
$$S(u=1) = S_0 \geq S_0^{\text{LB}} = E_{\text{UG},0} + B \simeq 28.5 \text{ MeV}$$

$$L(u=1) = L_0 = 3 (udS/du)_{u=1} = 2E_{\text{UG},0} \simeq 25.2 \text{ MeV}$$

$$S_0 > S_0^{\text{LB}} : (S = S^{\text{LB}})_{u_t}, (dS/du = dS^{\text{LB}}/du)_{u_t}$$



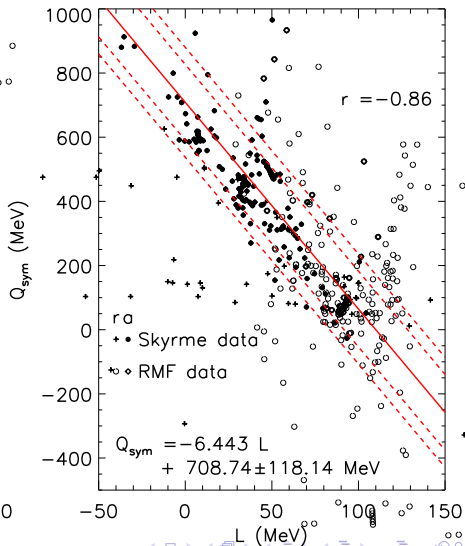
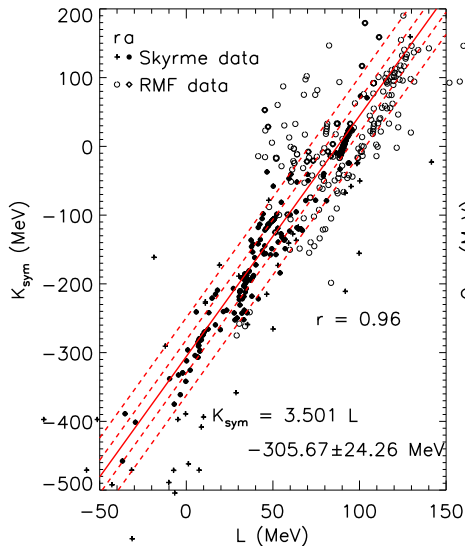
Symmetry Parameter Exclusions



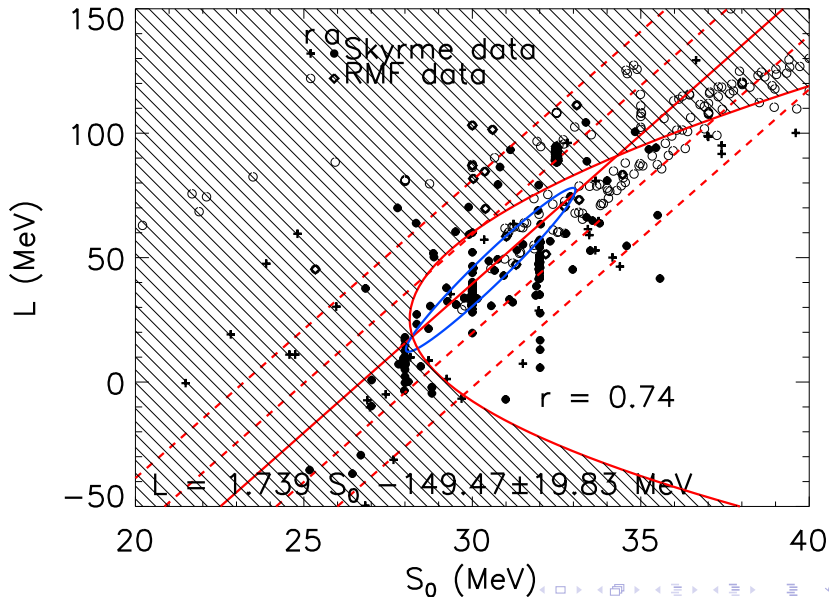
Tews et al. (2017)

Symmetry Parameter Correlations

Compilations from Dutra et al. (2012, 2014)⁺



More Realistic Exclusion Region



Analytic Approximation for the Boundary

$$S(u_t) = S^{\text{LB}}(u_t), \quad \left(\frac{dS}{du}\right)_{u_t} = \left(\frac{dS^{\text{LB}}}{du}\right)_{u_t}$$

gives

$$S_0 + \frac{L}{3}(u_t - 1) + \frac{K_{\text{sym}}}{18}(u_t - 1)^2 = E_{\text{UG},0}u_t^{2/3} + B - \frac{K_s}{18}(u_t - 1)^2$$

$$L + \frac{K_{\text{sym}}}{3}(u_t - 1) = 2E_{\text{UG},0}u_t^{-1/3} - \frac{K_s}{3}(u_t - 1)$$

Assume $K_n = 3L$ (i.e., $K_{\text{sym}} \approx 3L - K_s$). Then

$$S_0 = \frac{E_{\text{UG},0}}{3u_t^{4/3}}(1 + 2u_t^2) - E_0, \quad L = \frac{2E_{\text{UG},0}}{u_t^{4/3}}$$

or after eliminating u_t ,

$$S_0 = \frac{L}{6} \left[1 + 2 \left(\frac{2E_{\text{UG},0}}{L} \right)^{3/2} \right] - E_0$$

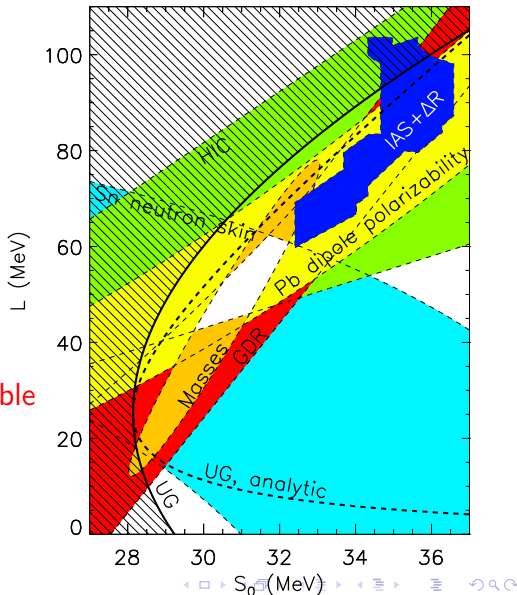
Experimental Constraints

Isvector Skins and
Isobaric Analog States
from Danielewicz et al. (2017)

Other experimental constraints
from Lattimer & Lim (2013)

Unitary gas constraints from
Tews et al. (2017)

Experimental and neutron
matter constraints are compatible
with unitary gas bounds.



Piecewise Polytopes

Crust EOS is known: $n < n_0 = 0.4n_s$.

Read, Lackey, Owen & Friedman (2009) found high-density EOS can be modeled as piecewise polytopes with 3 segments.

They found universal break points ($n_1 \simeq 1.85n_s$, $n_2 \simeq 3.7n_s$) optimized fits to a wide family of modeled EOSs.

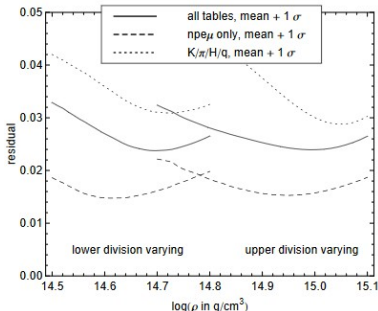
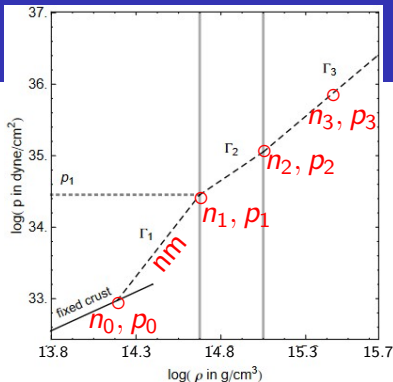
For $n_0 < n < n_1$, assume neutron matter EOS. Arbitrarily choose $n_3 = 7.4n_s$.

For a given p_1 (or Γ_1):

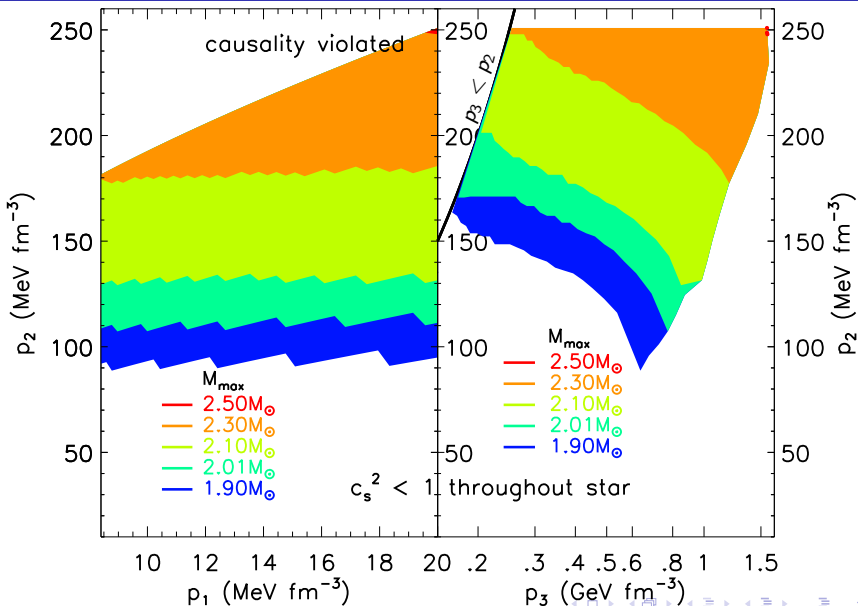
$0 < \Gamma_2 < \Gamma_{2c}$ or $p_1 < p_2 < p_{2c}$.

$0 < \Gamma_3 < \Gamma_{3c}$ or $p_2 < p_3 < p_{3c}$.

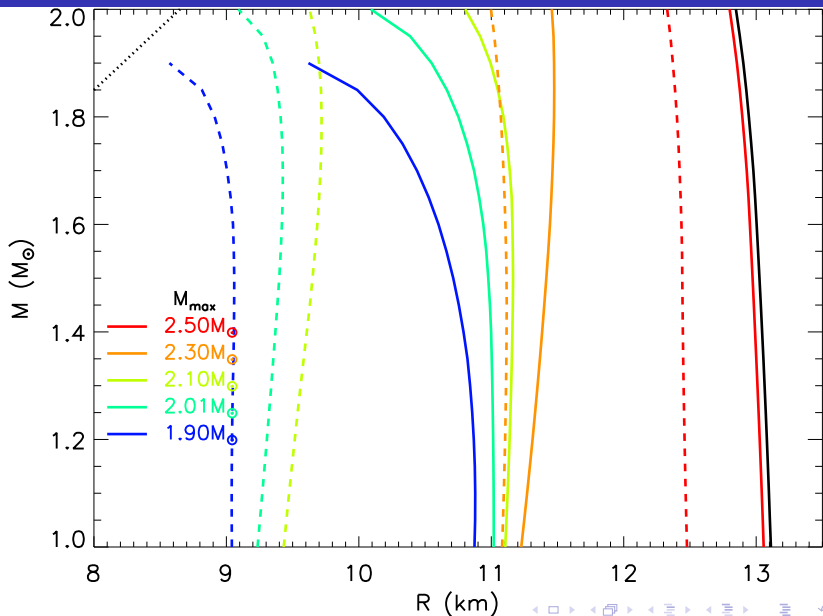
Minimum values of p_2, p_3 set by M_{max} ; maximum values set by causality.



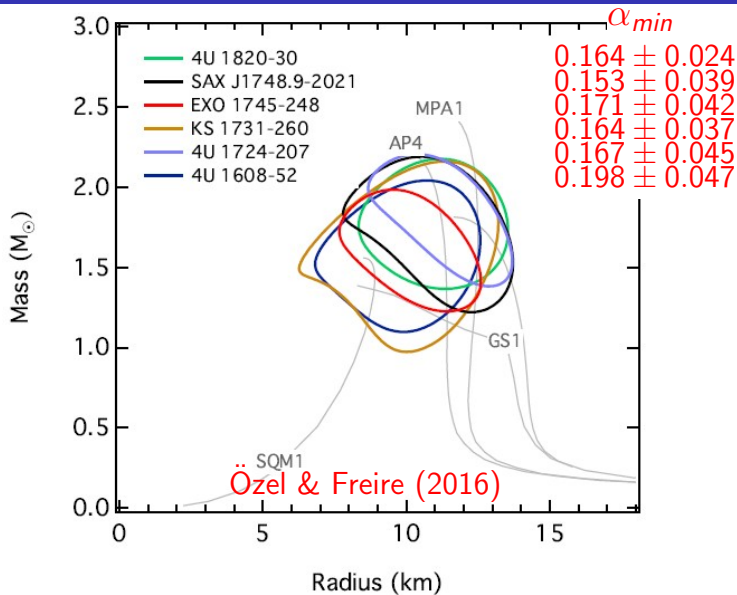
Maximum Mass and Causality Constraints



Mass-Radius Constraints from Causality



PRE $M - R$ Estimates



QLMXB $M - R$ Estimates

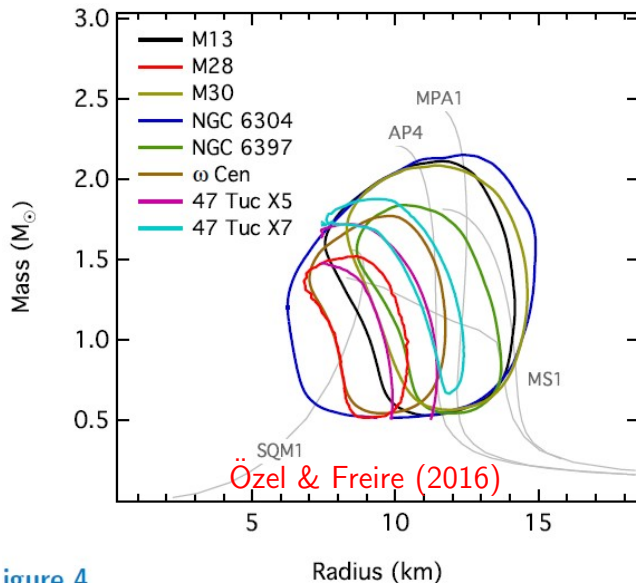
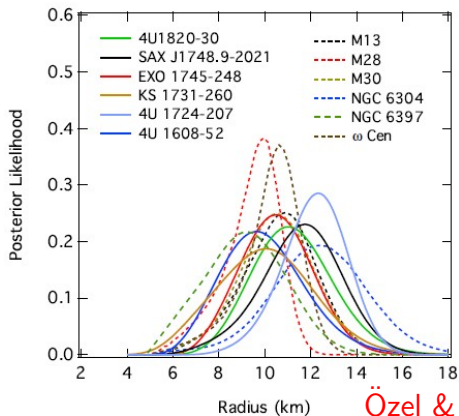


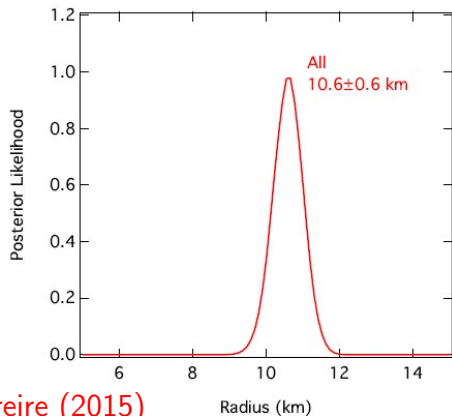
Figure 4

Combined R fits

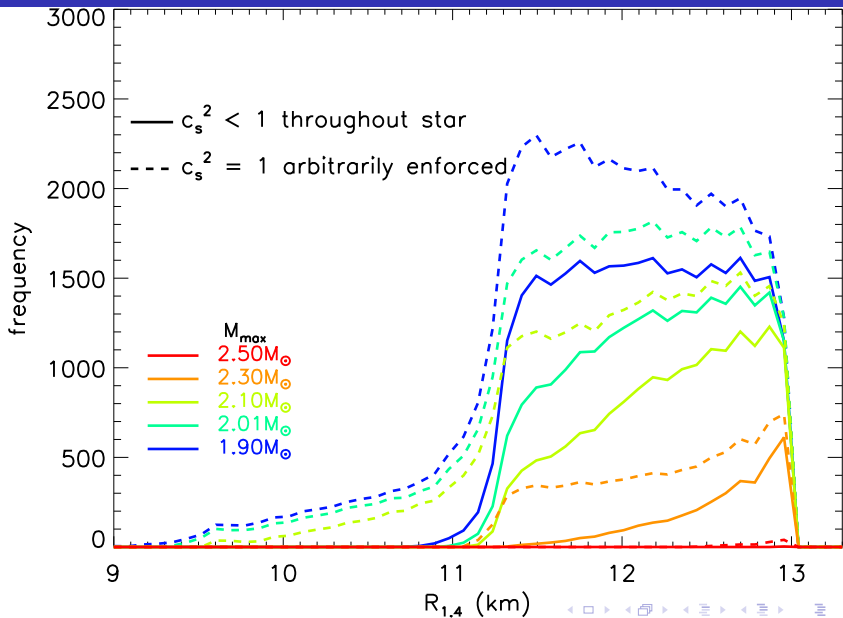
Assume $P(M)$ is that measured from pulsar timing
($\bar{M} = 1.4M_{\odot}$).



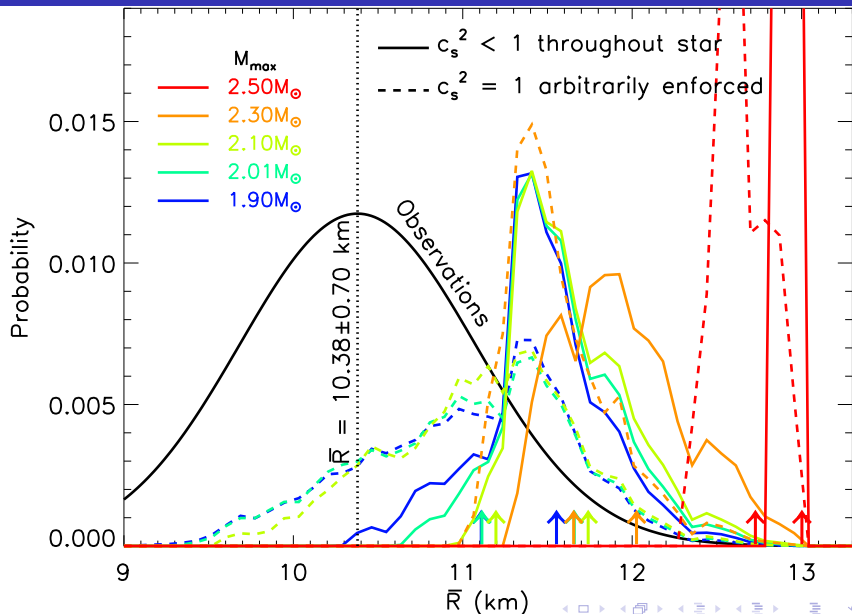
Özel & Freire (2015)



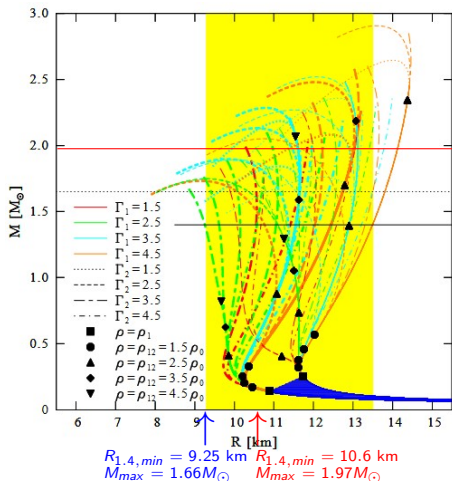
Piecewise-Polytrope Average Radius Distributions



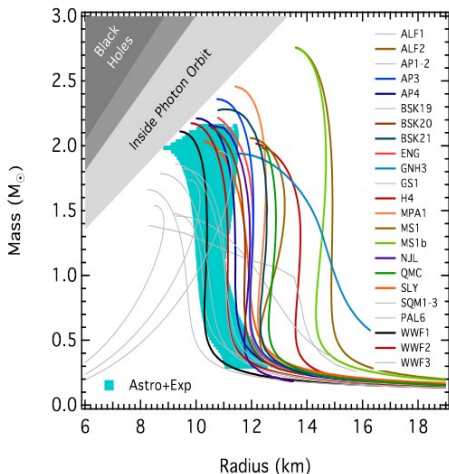
Folding Observations with Piecewise Polytropes



Other Studies

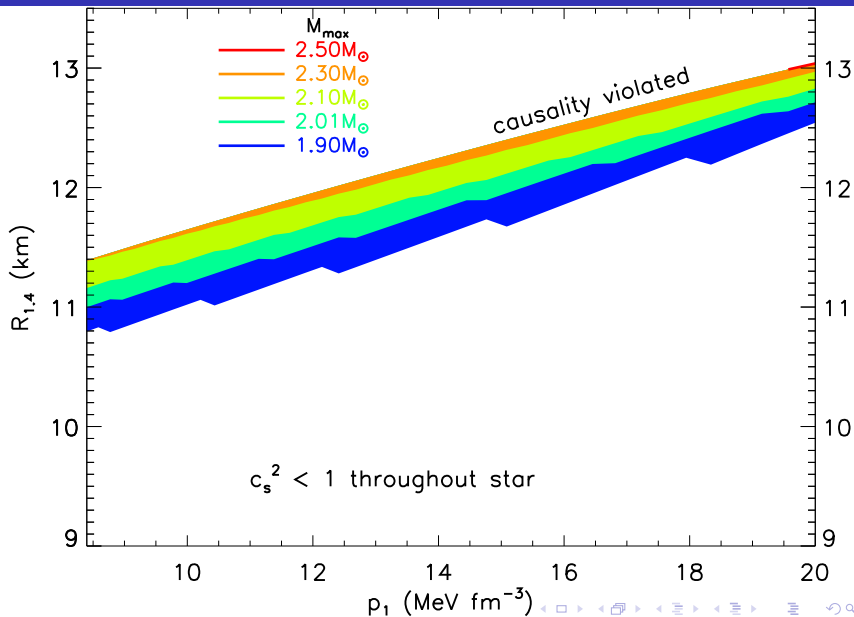


Hebeler, Lattimer, Pethick & Schwenk 2010

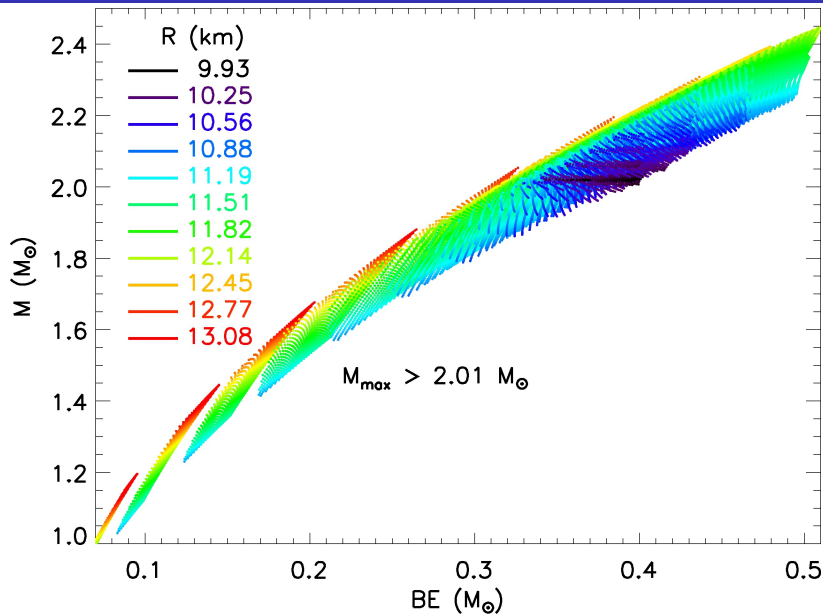


Özel & Freire 2016

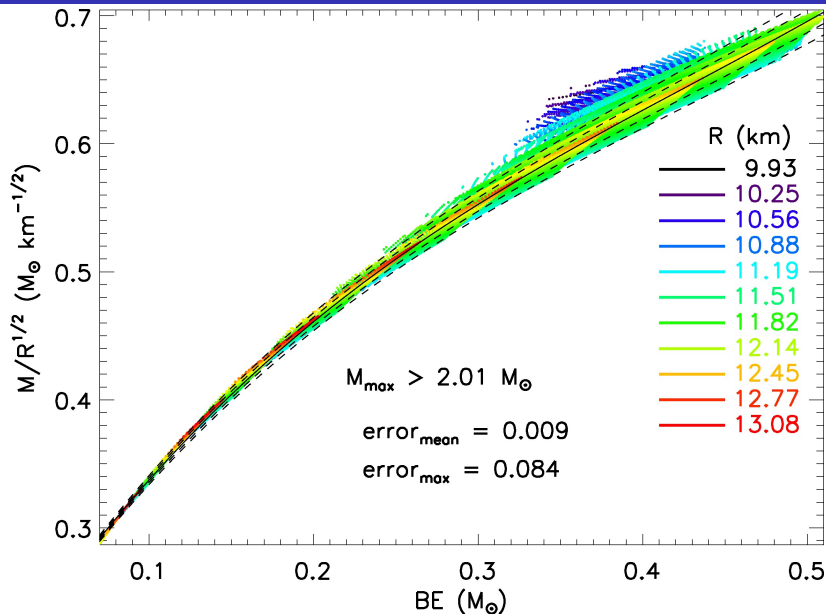
Radius - ρ_1 Correlation



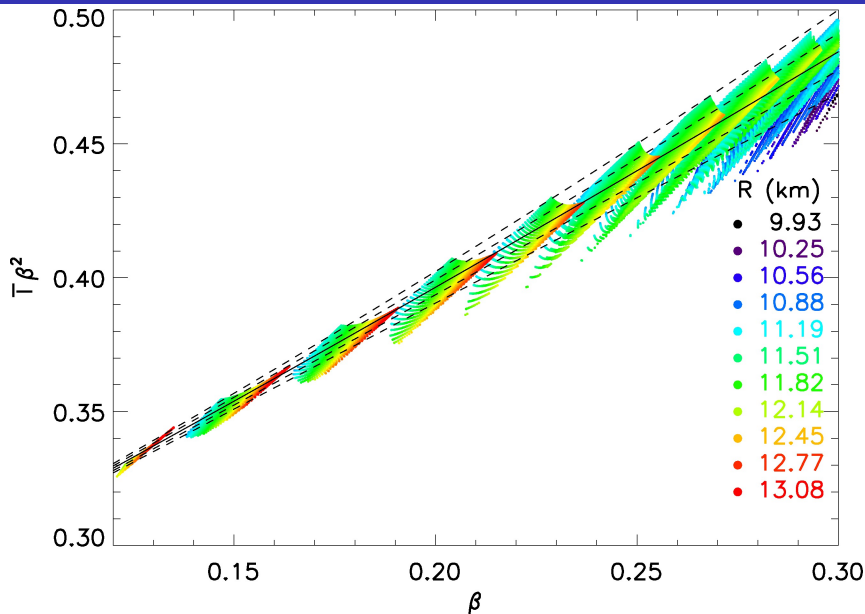
Binding Energy - Mass Correlations



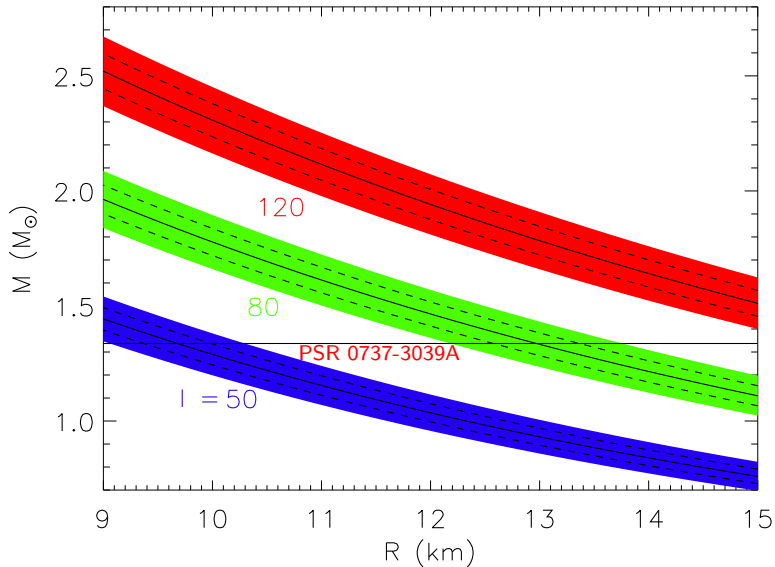
Binding Energy - Mass - Radius Correlations



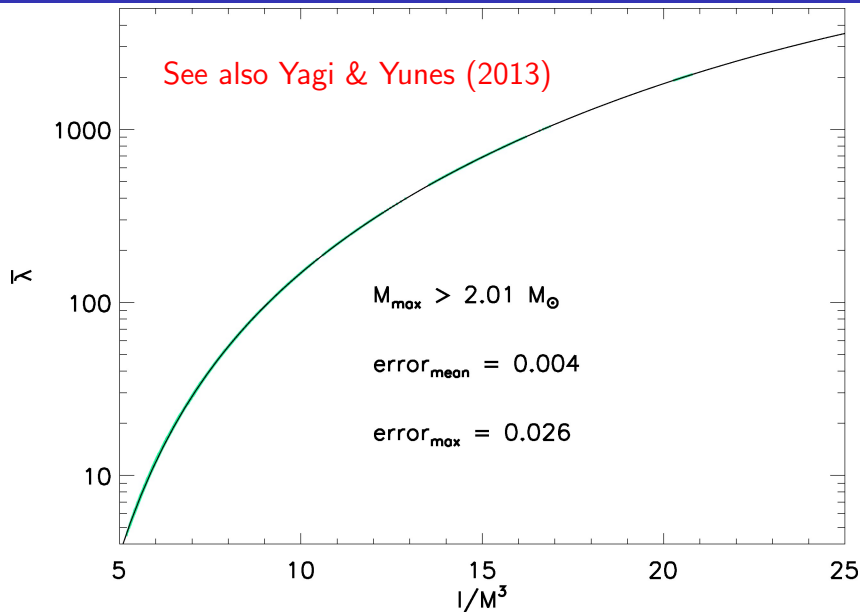
Moment of Inertia - Mass - Radius Correlations



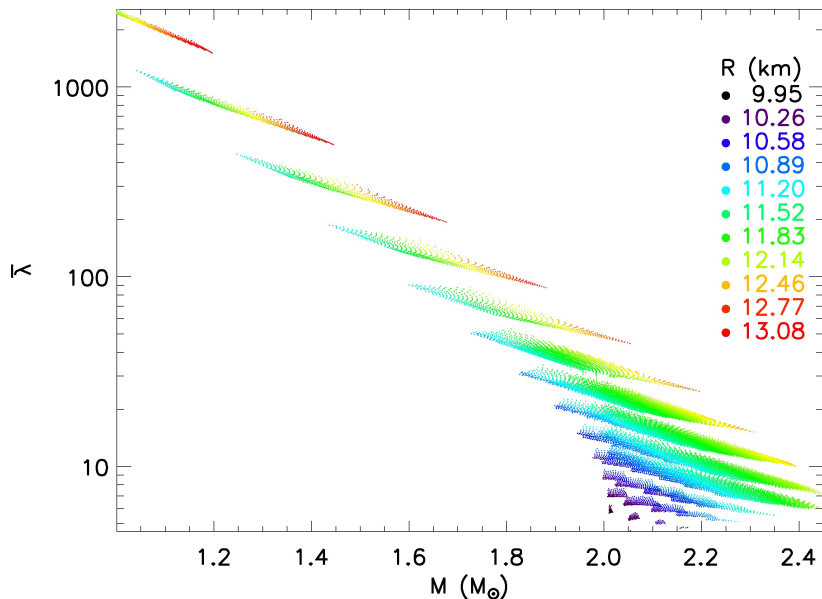
Moment of Inertia - Radius Constraints



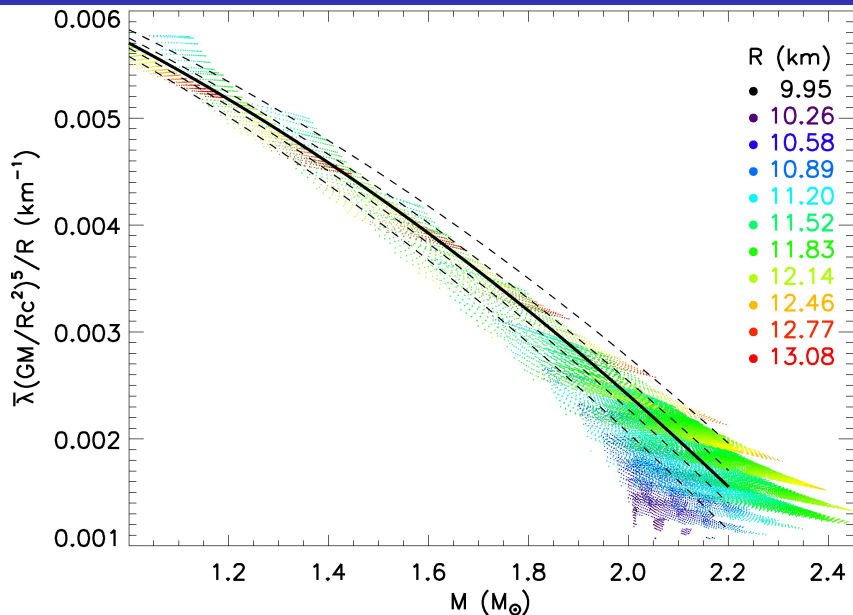
Tidal Deformatibility - Moment of Inertia



Tidal Deformatibility - Mass



Tidal Deformatibility - Mass - Radius



Binary Tidal Deformability

In a neutron star merger, both stars are tidally deformed. The most accurately measured deformability parameter is

$$\bar{\Lambda} = \frac{16}{13} [\bar{\lambda}_1 q^4 (12q + 1) + \bar{\lambda}_2 (1 + 12q)]$$

where

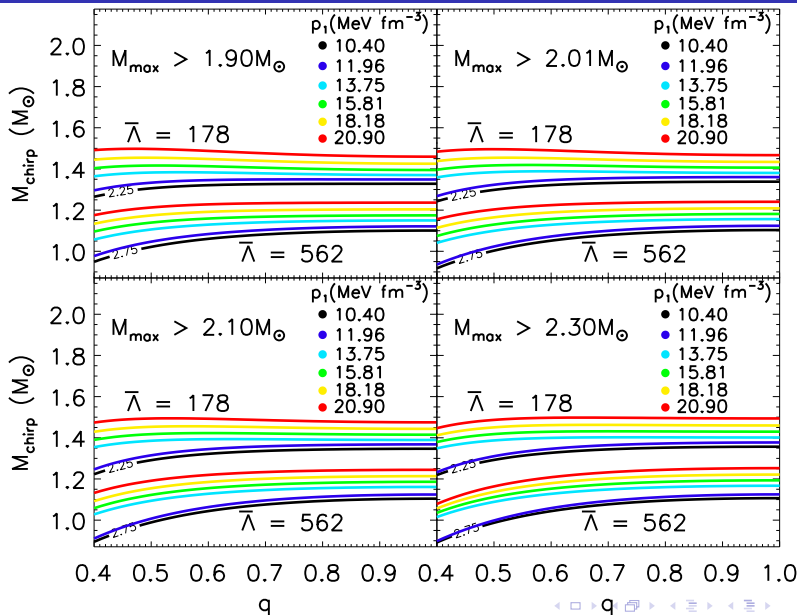
$$q = \frac{M_1}{M_2} < 1$$

For $S/N \approx 20 - 30$, typical measurement accuracies are expected to be (Rodriguez et al. 2014; Wade et al. 2014):

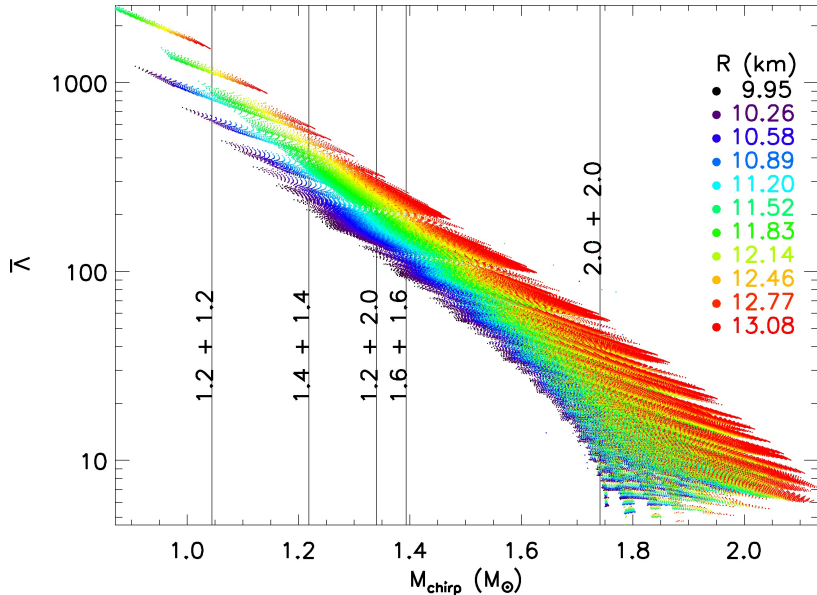
$$\Delta M_{chirp} \sim 0.01 - 0.02\%, \quad \Delta \bar{\Lambda} \sim 20 - 25\%$$

$$\Delta(M_1 + M_2) \sim 1 - 2\%, \quad \Delta q \sim 10 - 15\%$$

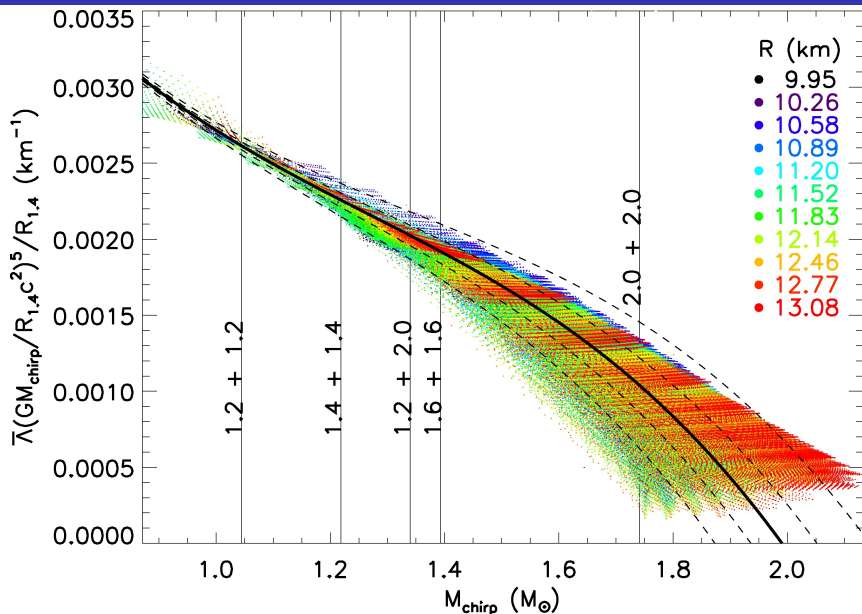
Binary Tidal Deformability - $\bar{\lambda}$



Binary Tidal Deformatibility - M_{chirp}

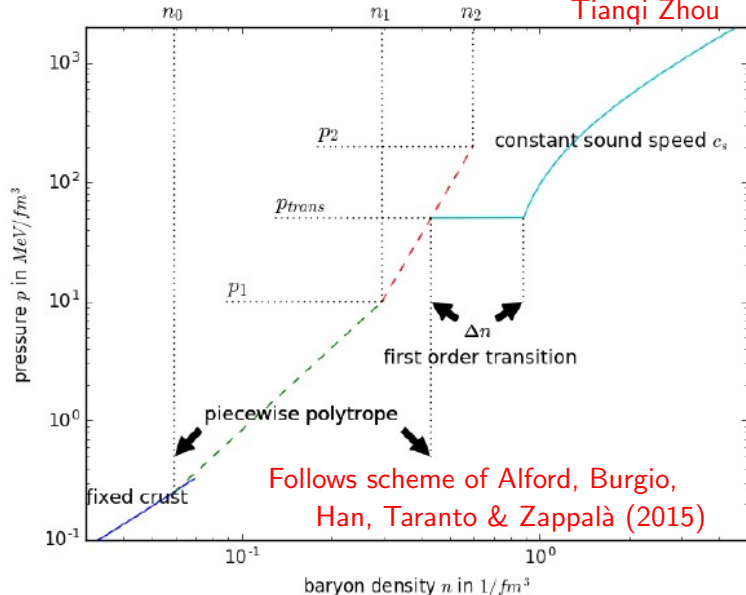


Binary Tidal Deformability - $M_{\text{chirp}} - R_{1.4}$



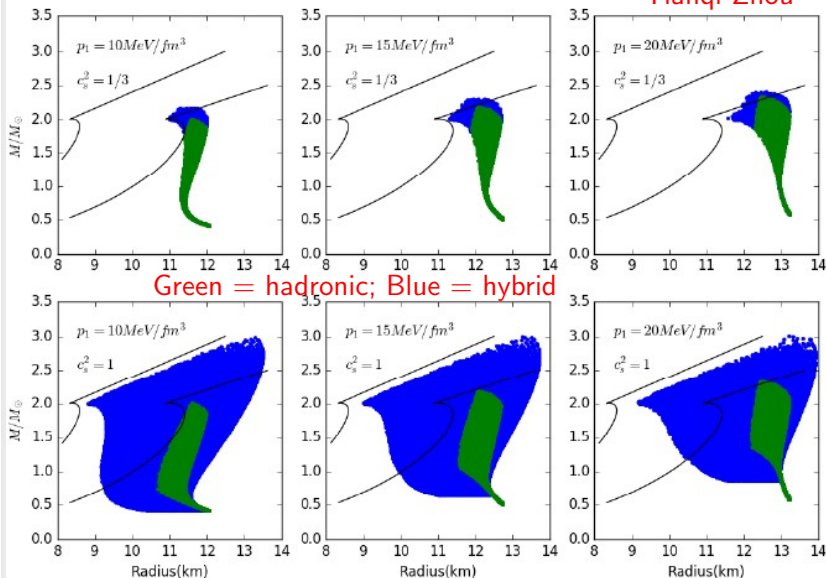
Hybrid Stars With First-Order Phase Transition

Tianqi Zhou



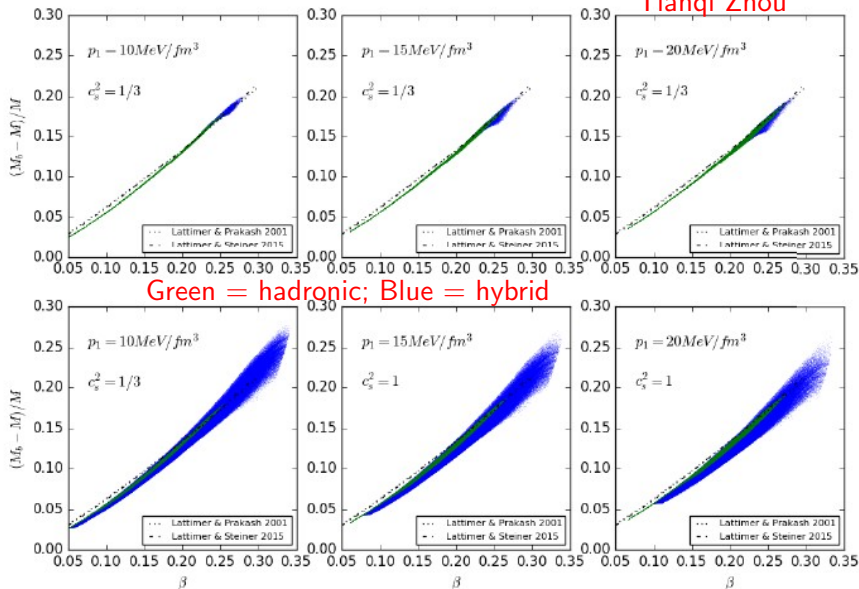
Mass-Radius Comparisons

Tianqi Zhou



Binding Energy - Compactness Comparisons

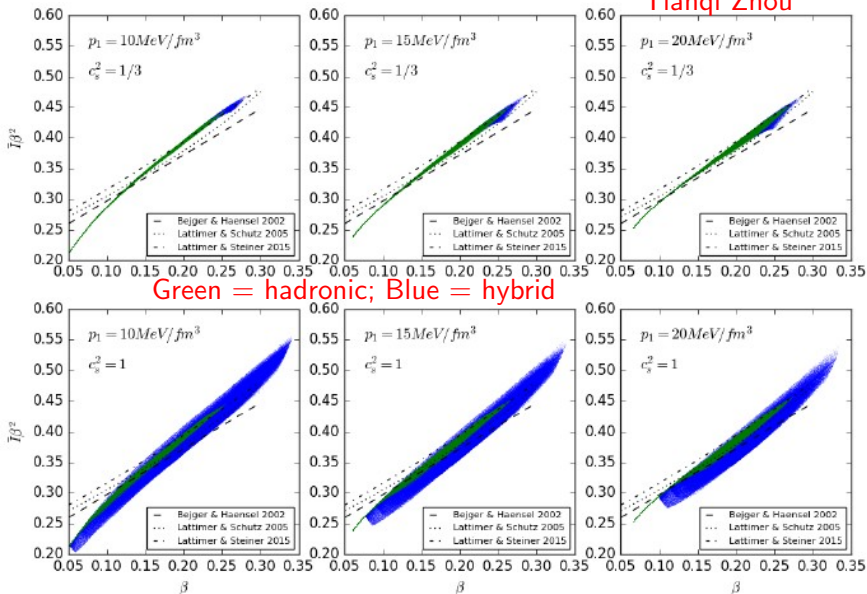
Tianqi Zhou



Green = hadronic; Blue = hybrid

Moment of Inertia - Compactness Comparisons

Tianqi Zhou



Tidal Deformability Comparisons

