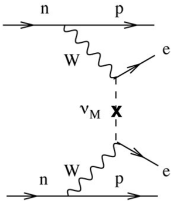
Multi-Reference In-Medium SRG for Neutrinoless Double Beta Decay

Jiangming Yao (UNC)
J. Engel (UNC), H. Hergert (MSU), C.F. Jiao (CMU), L.J. Wang (UNC)

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

INT Program 17-2a, Neutrinoless Double-beta Decay, June 20, 2017, Seattle

Matrix elements for the $0\nu\beta\beta$ decay



Half-life of neutrinoless DBD(exchange Majorana ν)

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} g_A^4 \left| |\langle m_{\nu_L} \rangle| m_e^{-1} M_{\nu_L}^{0\nu} + |\langle m_{\nu_H}^{-1} \rangle| m_p M_{\nu_H}^{0\nu} \right|^2$$

- The transition matrix element

$$M_i^{0\nu} = \langle 0_F^+ | \hat{O}_i^{0\nu} | 0_I^+ \rangle$$

- The transition operator

$$\hat{O}_i^{0\nu} = \frac{4\pi R}{g_A^2} \iint d^3x_1 d^3x_2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)}}{A_i} \mathcal{J}_\mu^\dagger(\boldsymbol{x}_1) \mathcal{J}^{\mu\dagger}(\boldsymbol{x}_2)$$

where $\mathcal{J}_\mu^\dagger$ is the charge-exchange current operator and $R = 1.2A^{1/2}$.

$\rightarrow$ $i = \nu_L$: $A_i \simeq q(q + E_d)$, where $E_d \simeq \langle E_m \rangle - (E_I + E_F)/2$ (closure approx).

$\rightarrow$ $i = \nu_H$: $A_i \simeq m_p m_e$

Matrix elements for the $0\nu\beta\beta$ decay

- Decomposition of the *current-product* operator

$$\left[\mathcal{J}^\dagger\mathcal{J}^\dagger\right]_{VV} = g_V^2(\boldsymbol{q}^2)\,(\bar\psi\gamma_\mu\tau_-\psi)_1\,(\bar\psi\gamma^\mu\tau_-\psi)_2\,,$$

$$\left[\mathcal{J}^\dagger\mathcal{J}^\dagger\right]_{AA} = g_A^2(\boldsymbol{q}^2)\,(\bar\psi\gamma_\mu\gamma_5\tau_-\psi)_1\,(\bar\psi\gamma^\mu\gamma_5\tau_-\psi)_2\,,$$

$$\left[\mathcal{J}^\dagger\mathcal{J}^\dagger\right]_{AP} = 2g_A(\boldsymbol{q}^2)g_P(\boldsymbol{q}^2)\,(\bar\psi\boldsymbol{\gamma}\gamma_5\tau_-\psi)_1\,(\bar\psi\boldsymbol{q}\gamma_5\tau_-\psi)$$

$$\left[\mathcal{J}^\dagger\mathcal{J}^\dagger\right]_{PP} = g_P^2(\boldsymbol{q}^2)\,(\bar\psi\boldsymbol{q}\gamma_5\tau_-\psi)_1\,(\bar\psi\boldsymbol{q}\gamma_5\tau_-\psi)_2\,,$$

$$\left[\mathcal{J}^\dagger\mathcal{J}^\dagger\right]_{MM} = g_M^2(\boldsymbol{q}^2)\left(\bar\psi\frac{\sigma_{\mu i}}{2m_p}q^i\tau_-\psi\right)_1\left(\bar\psi\frac{\sigma^{\mu j}}{2m_p}q_j\tau_-\psi\right)$$

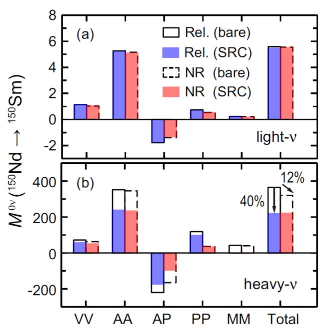
Non-Relativistic Reduction:

$$\left[-h_F(\boldsymbol{q}^2) + h_{GT}(\boldsymbol{q}^2)\sigma_{12} + h_T(\boldsymbol{q}^2)S^q_{12}\right]\tau_-^{(1)}\tau_-^{(2)}$$

with $\sigma_{12} = \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$ and
$S^q_{12} = 3(\boldsymbol{\sigma}^{(1)}\cdot\hat{\boldsymbol{q}})(\boldsymbol{\sigma}^{(2)}\cdot\hat{\boldsymbol{q}}) - \sigma_{12}$.

The Non-Rel. reduction is safe for computing $M^{0\nu}_{\nu_L}$ regardless of whether the SRCs are included.

NR reduction and SRC effect

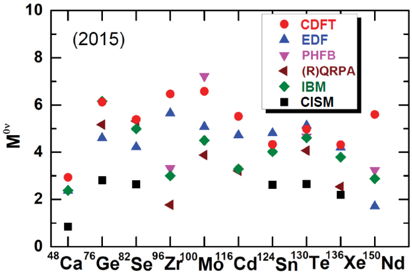


GCM+CDFT: L.S. Song, JMY, P. Ring, and J. Meng, PRC(2017)

See J. Meng's talk for a detailed introduction.

Matrix elements for the $0\nu\beta\beta$ decay

- Many-body approaches for nuclear structure and the matrix elements (exchange light Majorana neutrino)



GCM+CDFT: JMY, L.S Song, K. Hagino, P. Ring, J. Meng, PRC (2015)

GCM+EDF(Gogny): T. R. Rodriguez and G. Martinez-Pinedo, PRL(2010)

PHFB: P. K. Rath et al., PRC(2010)

(R)QRPA: A. Faessler et al., JPG(2012)

IBM: J. Barea and F. Iachello, PRC(2009)

CISM: J. Menendez et al., NPA(2009)

...

Systematic uncertainties

- Different approximation: Shell-Model, GCM, QRPA, IBM, PHFB
- Different model space: one-shell or full shell
- Different correlation: np pairing, collective v.s. non-collective

$\Rightarrow$ A factor of 2-3. J. Engel, JPG(2015)

Towards ab-inition calculations for the $0\nu\beta\beta$ decay

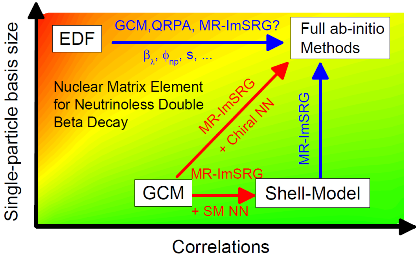
Our goal

A full *ab-initio* calculation of

- structure properties of medium-heavy (deformed) nuclei
- and the nuclear matrix elements for the $0\nu\beta\beta$ decay

with a Multi-Reference In-Medium Similarity Renormalization Group approach.

Hergert, Bogner, Morris, Schwenk, & Tsukiyama, Phys. Rep.(2016)

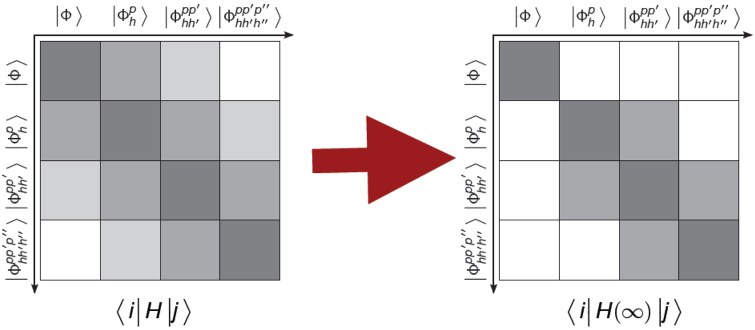


√ GCM v.s. SM based on a H_{SM}.
→ See T. R. Rodríguez and C.F. Jiao's talks.

- IMSRG+GCM v.s. SM based a H_{SM}.
- IMSRG+GCM based on a $H_{\chi EFT}$.

IMSRG: a tool to tackle the missing correlations in GCM and the small model-space problem in Shell-Model.

IMSRG: decoupling in A-body space



aim: decouple reference state $|\Phi\rangle$ from excitations

See H. Hergert's talk for the details.

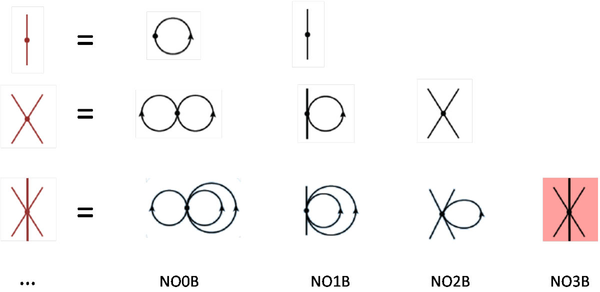
MR-IMSRG(2) for ^{48}Ca-Ti

1. Generate a symmetry-conserved HFB (GCM) state $|\Phi\rangle$ and the density matrix elements (see L.J. Wang's talk)

$$\rho^{nB} = \langle\Phi|A^{i_1\cdots i_n}_{j_1\cdots j_n}|\Phi\rangle$$

2. Normal-order all the operators $\hat{O}$ w.r.t the Ref. state $|\Phi\rangle$

$$\hat{O} = \langle\hat{O}\rangle + \sum_{i_1 j_1} O^{i_1}_{j_1}\tilde{A}^{i_1}_{j_1} + \frac{1}{4}\sum_{i_1 i_2 j_1 j_2} O^{i_1 i_2}_{j_1 j_2}\tilde{A}^{i_1 i_2}_{j_1 j_2} + \cdots$$



MR-IMSRG(2) for ^{48}Ca-Ti

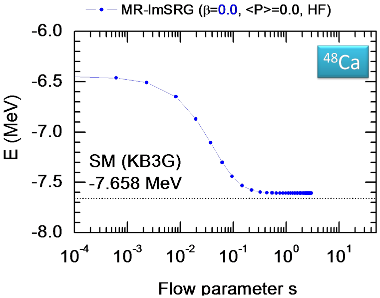
3. Decouple the reference state from excitations through the flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)],$$

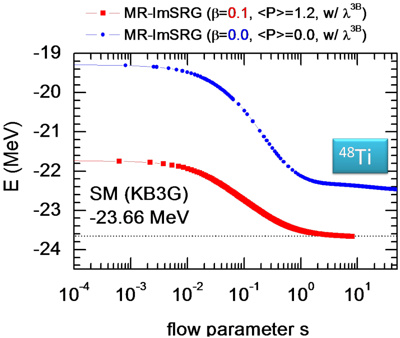
where $\hat{H}(s) = e^{\hat{\Omega}(s)}\hat{H}_0 e^{-\hat{\Omega}(s)}$. The man-body operator $\eta(s)$ (generator) and $\Omega(s)$ are anti-hermitian operators.

4. Compute observables (energy, transition)

$$\langle \hat{O}(\infty)\rangle = \langle \Phi | e^{\hat{\Omega}(\infty)}\hat{O}_0 e^{-\hat{\Omega}(\infty)} | \Phi \rangle.$$



MR-IMSRG(2) for ^{48}Ca-Ti



- Spherical HFB state w/o np iso-scalar pairing (projected onto $N\&Z$)
  - $\rightarrow$ $s = 0$: $E_{np} = -0.648$ MeV
  - $\rightarrow$ $s = \infty$: $E_{np} = -2.032$ MeV
- Deformed HFB state with np iso-scalar pairing (projected onto J, $N\&Z$)
  - $\rightarrow$ $s = 0$: $E_{np} = -1.345$ MeV
  - $\rightarrow$ $s = \infty$: $E_{np} = -2.023$ MeV

$\Rightarrow$ A deformed Ref. state is necessary for reproducing the total energy of ^{48}Ti.

MR-IMSRG(2) for the total GT strength in ^{48}Ti

$$S_{GT}^{\beta^+} \equiv \sum_m \left| \langle {}^{48}\mathrm{Sc}(1_m^+) | \sigma\tau_+ | {}^{48}\mathrm{Ti}(0^+) \rangle \right|^2$$
$$= \sum_{ijkl} \langle j | \sigma\tau_- | i \rangle \langle l | \sigma\tau_+ | k \rangle \langle 0^+ | a^\dagger_{j_p} a_{i_n} a^\dagger_{l_n} a_{k_p} | 0^+ \rangle .$$

Sph.HFB as the Ref. State

$$\begin{aligned} & \langle 0^+ | e^{\hat\Omega(\infty)} \hat S_{GT} e^{-\hat\Omega(\infty)} | 0^+ \rangle \\ = & \langle 0^+ | \hat S_{GT} + [\hat\Omega, \hat S_{GT}] + \dots | 0^+ \rangle \\ = & 4.048 - 3.693 + 0.299 + \dots \\ = & 0.801 \end{aligned}$$

Def.HFB as the Ref. State

$$\begin{aligned} & \langle 0^+ | e^{\hat\Omega(\infty)} \hat S_{GT} e^{-\hat\Omega(\infty)} | 0^+ \rangle \\ = & \langle 0^+ | \hat S_{GT} + [\hat\Omega, \hat S_{GT}] + \dots | 0^+ \rangle \\ = & 2.616 - 1.774 + 0.015 + \dots \\ = & 0.908 \end{aligned}$$

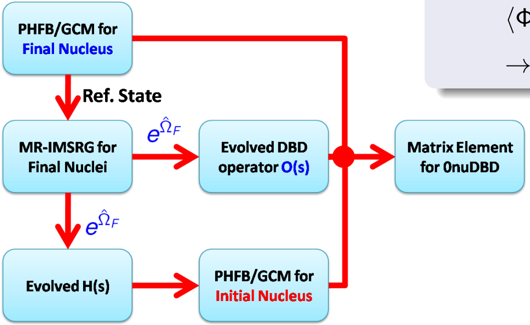
Shell-model (KB3G: 1.213)

MR-IMSRG(2) for the nuclear matrix elements of $0\nu\beta\beta$

$$M^{0\nu} = \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu} e^{-\hat{\Omega}_I(\infty)} | \Phi_I \rangle$$

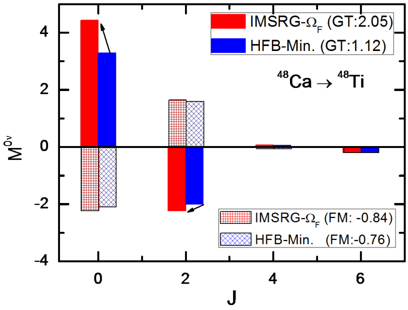
- Difficult to treat the operator with $\hat{\Omega}_F(\infty) \neq \hat{\Omega}_I(\infty)$.
- Strategy:

$$\langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu} e^{-\hat{\Omega}_I(\infty)} | \Phi_I \rangle$$
$$\rightarrow \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu} e^{-\hat{\Omega}_F(\infty)} | \overline{\Phi_I} \rangle$$



MR-IMSRG(2) for ^{48}Ca-Ti

- A spherical HF state for $|\Phi_I\rangle$.
- Operators are truncated up to NO2B level.



GT

$$
\begin{aligned}
& \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu}_{GT} e^{-\hat{\Omega}_F(\infty)} | \Phi_I \rangle \\
= & \langle \Phi_F | \hat{O}^{0\nu}_{GT} + [\hat{\Omega}_F, \hat{O}^{0\nu}_{GT}] + \ldots | \Phi_I \rangle \\
= & 1.118 + 0.870 + 0.061 + \ldots \\
= & 2.051
\end{aligned}
$$

SM (KB3G): 0.868

Fermi

$$
\begin{aligned}
& \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu}_{FM} e^{-\hat{\Omega}_F(\infty)} | \Phi_I \rangle \\
= & \langle \Phi_F | \hat{O}^{0\nu}_{FM} + [\hat{\Omega}_F, \hat{O}^{0\nu}_{FM}] + \ldots | \Phi_I \rangle \\
= & -0.763 - 0.061 - 0.015 + \ldots \\
= & -0.840
\end{aligned}
$$

SM (KB3G): -0.243

MR-IMSRG(2) for ^{48}Ca-Ti

Dominate correction to the matrix element: $[\hat{\Omega}_F, \hat{O}^{0\nu}_{GT}]$

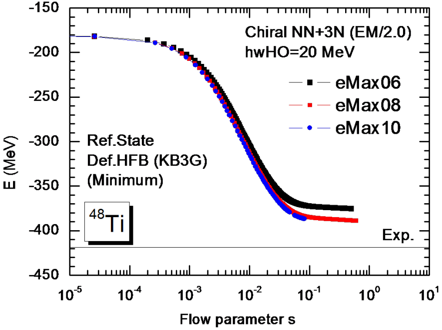
In the MR-IMSRG(2) calc. with the KB3G, $\Omega^{(1)} = 0$. The dominant terms are

$$[\hat{\Omega}, \hat{O}^{0\nu}_{GT}]^J_{KL34} = \frac{1}{2}\sum_{CD} \Omega^J_{KLCD} O^J_{CD34}(1 - n_C - n_D) - \frac{1}{2}\sum_{12} O^J_{KL12}\Omega^J_{1234}(1 - n_1 - n_2).$$

where K, L, C, D for protons and $1, 2, 3, 4$ for neutrons.

- The 1st term (protons): 0.243
- The 2nd term (neutrons): 0.602

MR-IMSRG(2) for ^{48}Ca-Ti



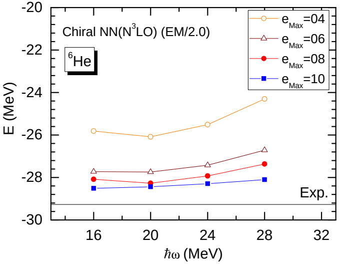
GT (eMax06)

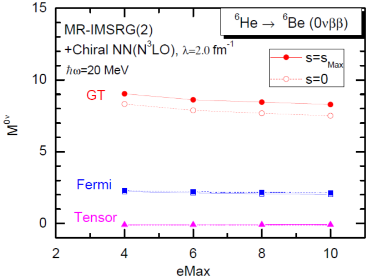
$$
\begin{aligned}
& \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu}_{GT} e^{-\hat{\Omega}_F(\infty)} | \Phi_I \rangle \\
= \; & \langle \Phi_F | \hat{O}^{0\nu}_{GT} + [\hat{\Omega}_F, \hat{O}^{0\nu}_{GT}] + \ldots | \Phi_I \rangle \\
= \; & 1.618 + 0.791 + 0.094 + \ldots \\
= \; & 2.505
\end{aligned}
$$

Fermi (eMax06)

$$
\begin{aligned}
& \langle \Phi_F | e^{\hat{\Omega}_F(\infty)} \hat{O}^{0\nu}_{FM} e^{-\hat{\Omega}_F(\infty)} | \Phi_I \rangle \\
= \; & \langle \Phi_F | \hat{O}^{0\nu}_{FM} + [\hat{\Omega}_F, \hat{O}^{0\nu}_{FM}] + \ldots | \Phi_I \rangle \\
= \; & -0.682 - 0.011 - 0.027 + \ldots \\
= \; & -0.720
\end{aligned}
$$

NLDBD between isospin multiples: ^{6}He $\rightarrow ^6$Be



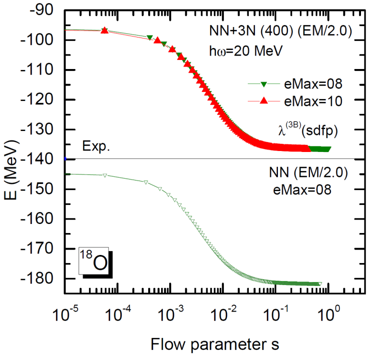


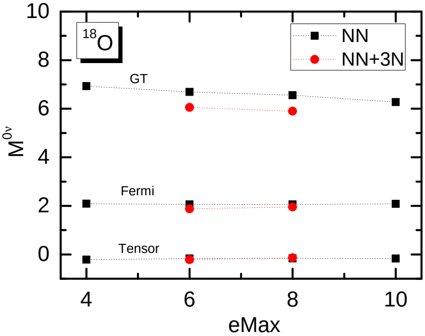
- No problem of $\Omega_I \neq \Omega_F$.

$$M^{0\nu} \equiv \langle TT_z - 2|[\hat{O}^{0\nu}]^{2-2}|TT_z\rangle$$
$$\rightarrow \langle TT_z||[\hat{O}^{0\nu}]^{20}|TT_z\rangle$$

- A benchmark for other ab-initio methods.

NLDBD between isospin multiples: $^{18}O \rightarrow {}^{18}Ne$





Effects of 3NF at the NO2B level

Increases the energy (about 45 MeV) and decreases the $M^{0\nu}$ (about 10%).

Summary and outlook

- Summary:
  - Different choice of reference state affects the solution of the flow equation.
  - An ab-intio calculation of the $M^{0\nu}$ for isospin multiples (^{6}He and ^{18}O) has been performed, which provides a test ground for other ab-initio methods.
  - The effect of 3NF at the NO2B level decreases the $M^{0\nu}_{GT}$.
- Next:
  - The reason for the enhanced transition matrix elements.
  - Application to the matrix elements for the $0\nu\beta\beta$ candidates.
  - Extension of the MR-IMSRG(2) to MR-IMSRG(3).

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  - The effect of 3NF at the NO2B level decreases the $M^{0\nu}_{GT}$.
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  - Application to the matrix elements for the $0\nu\beta\beta$ candidates.
  - Extension of the MR-IMSRG(2) to MR-IMSRG(3).

Thanks for your attention!

Development of IMSRG approaches

- IMSRG for spherical closed (open)-shell nuclei

  K. Tsukiyama, S. K. Bogner, A. Schwenk, PRL106, 222502 (2011); PRC 85, 061304(R) (2012)

- IMSRG+PNP (Sph.HFB) for spherical open-shell nuclei

  H. Hergert, S. Binder, A. Calci, J. Langhammer, R. Roth, PRL 110, 242501 (2013)

  H. Hergert, S. K. Bogner, T. D. Morris, S. Binder, A. Calci, J. Langhammer, R. Roth, PRC90, 041302(R) (2014)

- Valence-Space Shell-Model based on the IMSRG decoupled chiral Hamiltonian

  S. K. Bogner, H. Hergert, J. D. Holt, A. Schwenk, S. Binder, A. Calci, J. Langhammer, R. Roth, PRL 113, 142501 (2014)

  S. R. Stroberg, H. Hergert, J. D. Holt, S. K. Bogner, A. Schwenk, PRC93, 051301 (2016)

  S. R. Stroberg, A. Calci, H. Hergert, J. D. Holt, S. K. Bogner, R. Roth, A. Schwenk, PRL 118, 032502 (2017)

- No-Core Shell-Model based on the MR-IMSRG decoupled chiral Hamiltonian

  E. Gebrerufael, K. Vobig, H. Hergert, and R. Roth, arXiv: 1610.05254v1 [nucl-th] 17 Oct 2016

- IMSRG+EOM(TDA) for excited states

  N. M. Parzuchowski, T. D. Morris, S. K. Bogner, PRC95, 044304 (2017)

- IMSRG+GCM (Multi-Reference State) for deformed/transitional nuclei

  ...

MR-IMSRG(2*) for oxygen isotopes

Induced-NO3B in H at each s

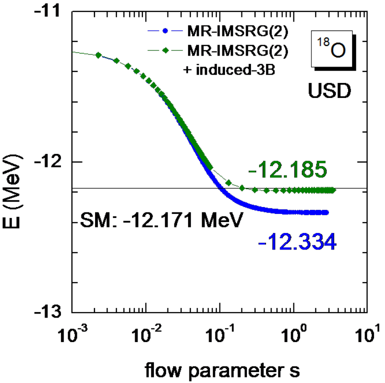
BCH expansion for $H(s)$:

$$e^{\hat{\Omega}(s)} H_0 e^{-\hat{\Omega}(s)} = H_0 + \sum_{n=1} \frac{1}{n!} \tilde{H}^{(n)}(s),$$

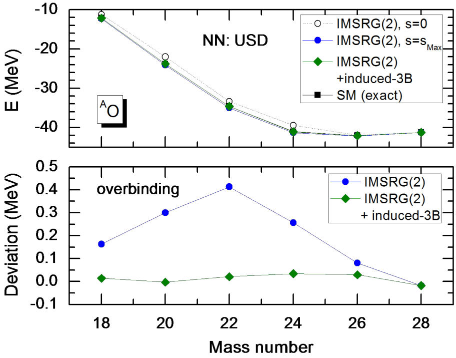
with

$$\tilde{H}^{(n)} = H^{(n)}_{NO2B} + [\Omega(s), \tilde{W}^{(n-1)}] + [\Omega(s), h^{(n-1)}]$$

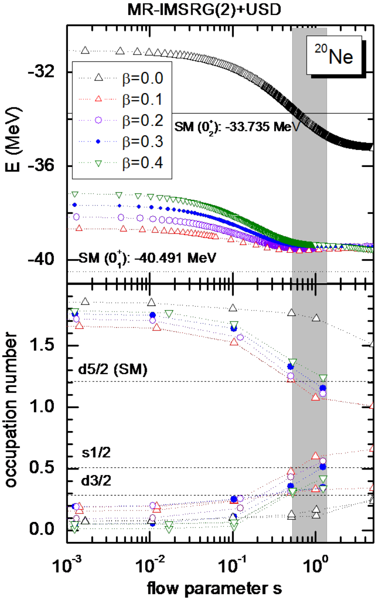
- $H^{(n)}_{NO2B} = [\Omega(s), H^{(n-1)}_{NO2B}]$
- $\tilde{W}^{(n-1)} \equiv [\Omega(s), \tilde{H}^{(n-2)}]^{3B}$
- $h^{(n-1)}$ for the difference between $\tilde{H}^{(n-1)}$ and $H^{(n-1)}_{NO2B}$



Oxygen isotopes



MR-IMSRG for deformed ^{20}Ne with SM Hamiltonian



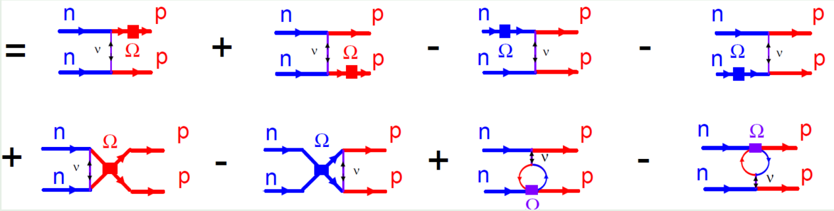
- Clustering structure in ^{20}Ne: a challenge for SM
- MR state: PNAMP+HFB with different deformation
- The spherical (reference) state fails to evolve to the deformed g.s.
- About 1 MeV discrepancy: something missing in the MR-IMSRG(2)?

Computing the NME for the neutrinoless DBD

If $\Omega_I = \Omega_F = \Omega$,

$$
\begin{aligned}
& M^{0\nu} \\
= \quad & \langle \Phi_F | e^{\hat{\Omega}(\infty)} \hat{O}^{0\nu} e^{-\hat{\Omega}(\infty)} | \Phi_I \rangle \\
= \quad & \langle \Phi_F | \hat{O}^{0\nu} + \sum_{n=1}^{\infty} \frac{1}{n!} [\hat{\Omega}(\infty), \hat{O}^{[n-1]}] | \Phi_I \rangle .
\end{aligned}
$$

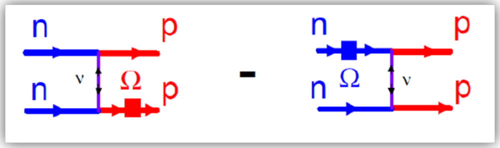
Diagrams for $[\hat{\Omega}, \hat{O}^{0\nu}]$



Expression for $[\Omega, O^{0\nu}]$ (1B)

$$[\hat{\Omega}^{(1)}, \hat{O}^{0\nu}]^{J}_{KL34} = \sum_{A} \left[1 + (-1)^{J-j_K-j_L+1}\right] \Omega^{(1)}_{KA} O_{AL34} - \sum_{1} \left[1 + (-1)^{J-j_3-j_4+1}\right] \Omega^{(1)}_{13} O_{KL14}. \quad (2)$$

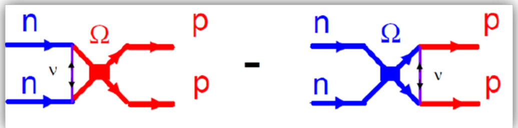
where K, L, C, D for protons and $1, 2, 3, 4$ for neutrons.



Expression for $[\Omega, O^{0\nu}]$ (pp)

$$[\Omega^{(2)}, O]^J_{KL34}(pp) = \frac{1}{2}\sum_{CD}\Omega^J_{KLCD}O^J_{CD34}(1 - n_C - n_D) - \frac{1}{2}\sum_{12}O^J_{KL12}\Omega^J_{1234}(1 - n_1 - n_2).$$

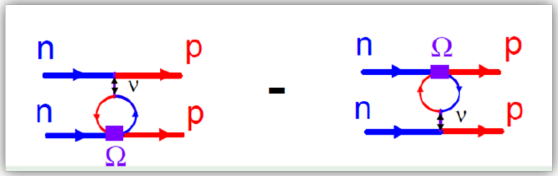
where K, L, C, D for protons and $1, 2, 3, 4$ for neutrons.



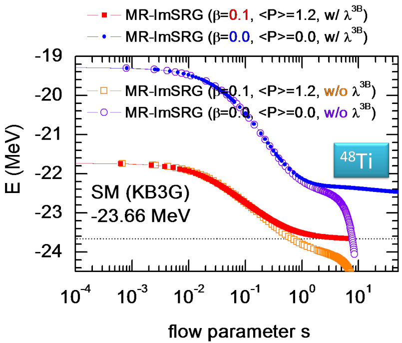
Expression for $[\Omega, O^{0\nu}]$ (ph)

$$
\begin{aligned}
& [\Omega^{(2)}, O]^J_{kl34}(ph) \\
= \; & -\sum_{J'} \hat{J'}^2 \left\{ \begin{array}{ccc} j_k & j_l & J \\ j_3 & j_4 & J' \end{array} \right\} \sum_{a6} \langle (k\bar{4})J'|O|(6\bar{a})J'\rangle (n_6 - n_a) \langle (6\bar{a})J'|\Omega|(3\bar{l})J'\rangle \\
& -(-1)^{j_k+j_l+J+1} \sum_{J'} \hat{J'}^2 \left\{ \begin{array}{ccc} j_l & j_k & J \\ j_3 & j_4 & J' \end{array} \right\} \sum_{a6} \langle (l\bar{4})J'|O|(6\bar{a})J'\rangle (n_6 - n_a) \langle (6\bar{a})J'|\Omega|(3\bar{k})J'\rangle \\
& +\sum_{J'} \hat{J'}^2 \left\{ \begin{array}{ccc} j_k & j_l & J \\ j_3 & j_4 & J' \end{array} \right\} \sum_{a6} \langle (k\bar{4})J'|\Omega|(a\bar{6})J'\rangle (n_a - n_6) \langle (a\bar{6})J'|O|(3\bar{l})J'\rangle \\
& +(-1)^{j_k+j_l+J+1} \sum_{J'} \hat{J'}^2 \left\{ \begin{array}{ccc} j_l & j_k & J \\ j_3 & j_4 & J' \end{array} \right\} \sum_{a6} \langle (l\bar{4})J'|\Omega|(a\bar{6})J'\rangle (n_a - n_6) \langle (a\bar{6})J'|O|(3\bar{k})J'\rangle
\end{aligned}
\tag{3}
$$

where k, l, c, d, a for protons and $1, 2, 3, 4, 6$ for neutrons.



MR-IMSRG(2) for ^{48}Ca-Ti



Irreducible three-body density

$$\lambda^{pqr}_{stu} \equiv \rho^{pqr}_{stu} - \hat{\mathcal{A}}(\lambda^{p}_{s}\lambda^{qr}_{tu}) - \hat{\mathcal{A}}(\rho^{p}_{s}\rho^{q}_{t}\rho^{r}_{u}),$$

important for decoupling the $H(s)$, where $\lambda^{qr}_{tu} \equiv \rho^{qr}_{tu} - \hat{\mathcal{A}}(\rho^{q}_{t}\rho^{r}_{u})$.

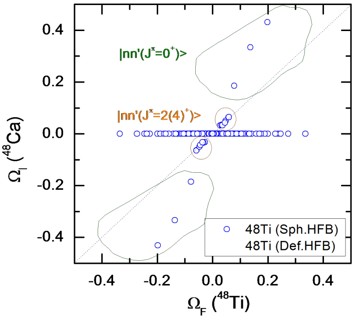
MR-IMSRG for ^{48}Ca-Ti: comparison of $\Omega^{(2)}$

The Ω at NO2B level

$$\Omega_{I/F}(s) = \sum_{ij} \Omega_{ij}(s)\tilde{A}^i_j + \frac{1}{4}\sum_{ijkl} \Omega_{ijkl}(s)\tilde{A}^{ij}_{kl} \quad (4)$$

where

- $\Omega_{ij}(s)$: zero by using the H_{SM}.
- The two-body M.E. in J-scheme: $\langle (ij)JM|\Omega|(kl)JM\rangle$.



If $\Omega_F \simeq \Omega_I$, expansion in terms of $\Omega_F - \Omega_I$

$$\begin{aligned} e^{\Omega_F} O^{0\nu} e^{-\Omega_I} &= e^{\Omega_F} e^{-\Omega_I} e^{\Omega_I} O^{0\nu} e^{-\Omega_I} \\ &\simeq e^{\Omega_I}\left[1 + (\Omega_F - \Omega_I) - \frac{1}{2}[\Omega_I, \Omega_F]\right] O^{0\nu} e^{-\Omega_I} \end{aligned} \quad (5)$$

IMSRG-Ω_I: $M^{0\nu}_{GT} = 2.169 \rightarrow 2.449$(NO2B), even larger?

MR-IMSRG for ^{48}Ca-Ti: comparison of $\Omega^{(2)}$

The Ω at NO2B level

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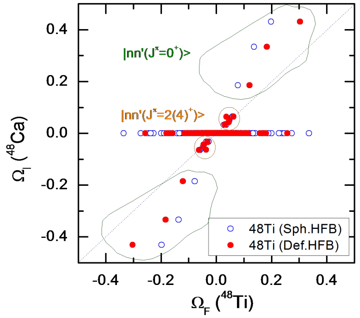

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IMSRG-Ω_I: $M^{0\nu}_{GT} = 2.169 \to 2.449$(NO2B), even larger?