THE UNIVERSITY *of* NORTH CAROLINA *at* CHAPEL HILL

Chiral Two-Body Currents for Neutrinoless Double-Beta Decay

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$0\nu\beta\beta$ matrix elements: Uncertainties

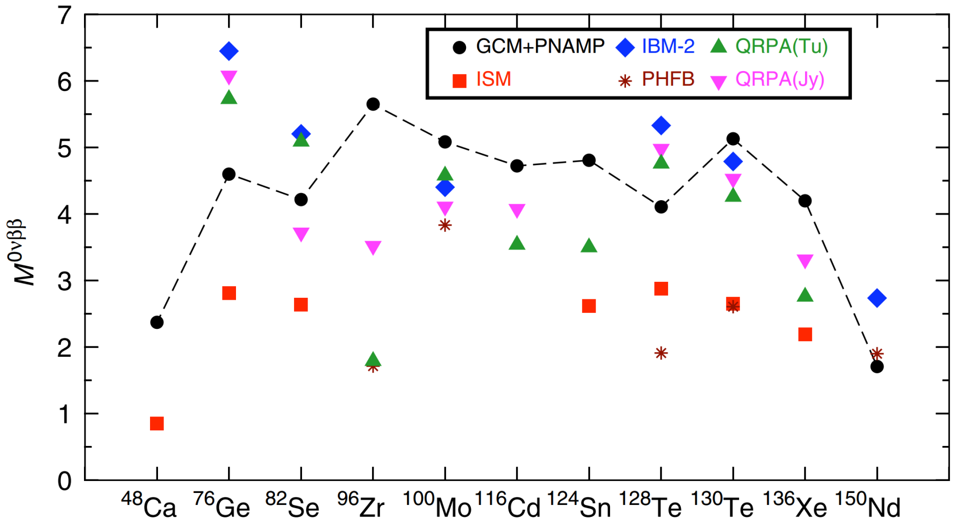
- Neutrinoless double beta decay ($0\nu\beta\beta$) half-life:

$$\left[T_{1/2}^{0\nu\beta\beta}(0_i^+ \to 0_f^+)\right]^{-1} = G^{0\nu\beta\beta}\left|M^{0\nu\beta\beta}\right|^2 \langle m_\nu \rangle^2 \quad (1)$$

- Model dependent

Rodriguez and Martinez-Piendo: PRL (2010)

Vogel: JPG (2012)



- Uncertainties: Wave function vs Transition operator

$$M^{0\nu\beta\beta} = \left\langle 0_f^+ \right| \hat{O}^{0\nu\beta\beta} \left| 0_i^+ \right\rangle \quad (2)$$

✓ Wave function: $\hat{H}$, model space, method → SM vs ImSRG vs GCM vs QRPA ...

✓ Transition operator: two-body currents ...

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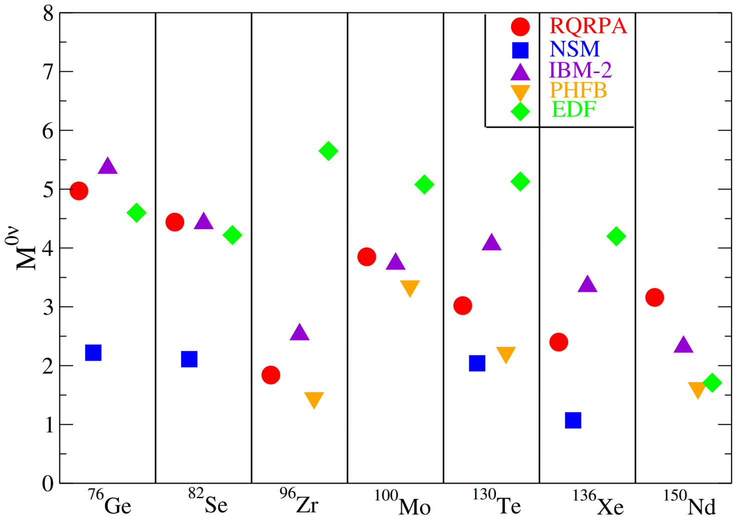

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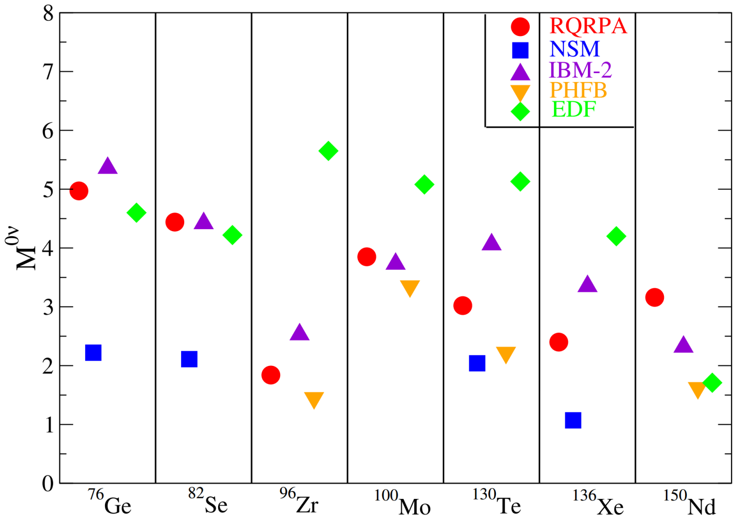

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✓ **Transition operator: two-body currents ...**

Transition operators

- "g_A-quenching" for single-β decay and $2\nu\beta\beta$-decay
- quenching in magnetic dipole transitions

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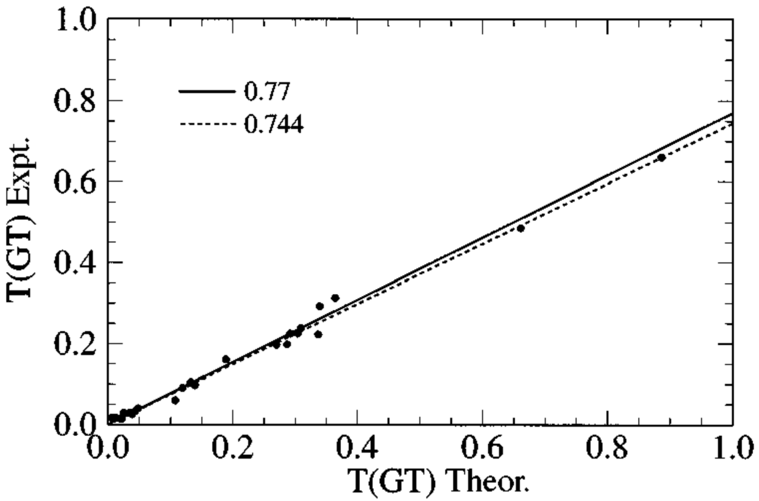
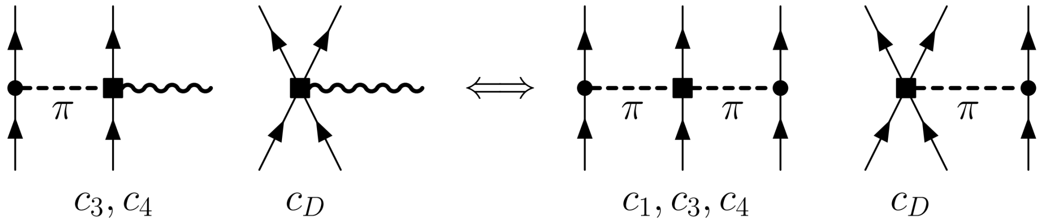
  Caurier et al: RMP (2005); Barea et al: PRC (2015); Ichimura et al: PPNP (2006)



- Similar quenching in $0\nu\beta\beta$ decay ???
- Corrections by Chiral EFT: two-body currents



E. Epelbaum et al: RMP (2009); Menendez, Gazit and Schwenk: PRL (2011)

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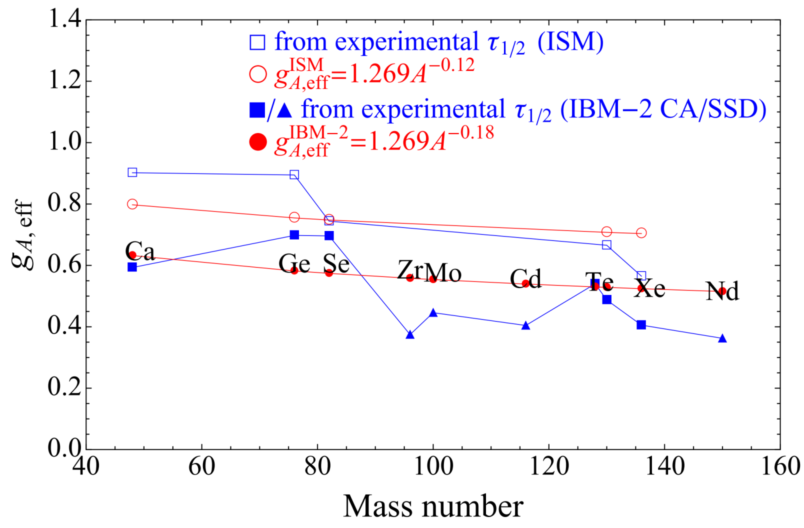

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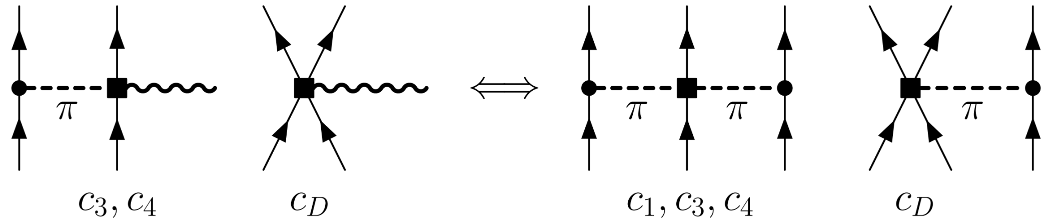

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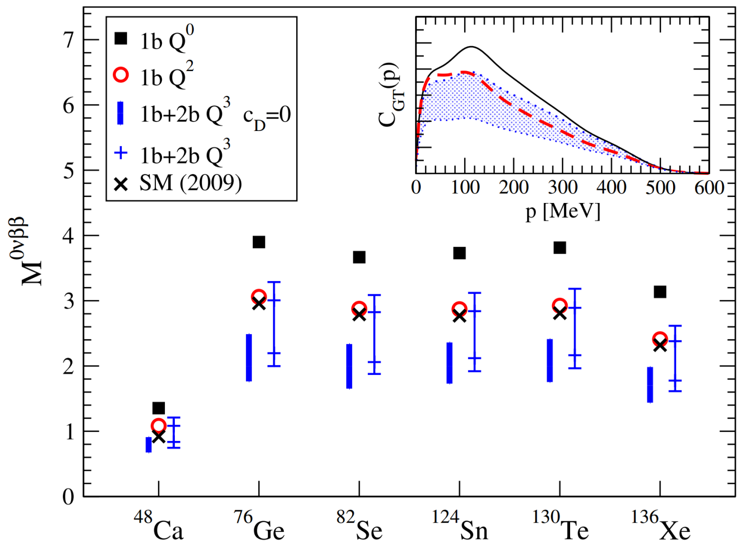
Two-body currents for $0\nu\beta\beta$: normal-ordering

Used to reduce two-body operator to effective one-body operator

Prior work: *Menendez, Gazit and Schwenk: PRL (2011); Engel, Simkovic and Vogel: PRC (2014)*

- ✓ Fermi-gas approximation (analytically)
- ✓ Neglected momentum transfer $\boldsymbol{q} \neq 0$ terms
- ✓ Neglected tensor-like terms of two-body currents
- ✓ Without regulators



- Contribute from -35% to 10% to matrix element (SM)
- Should be included in all calculations
- Need to investigate effects of normal ordering and $\boldsymbol{q} \neq 0$ terms !!!

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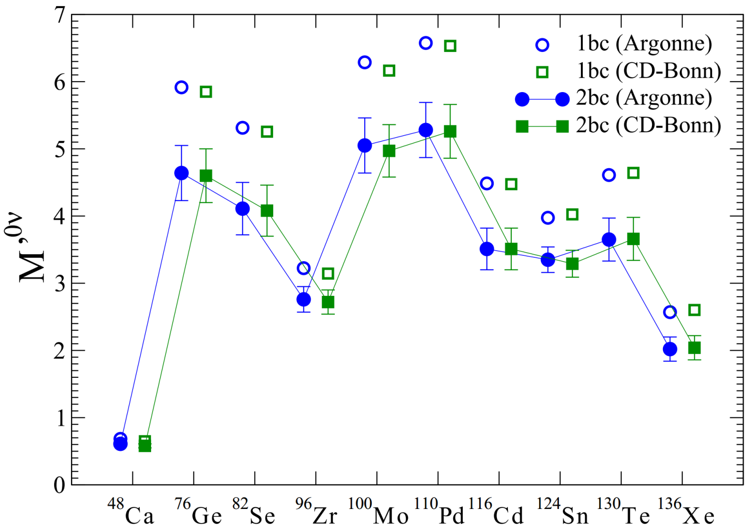
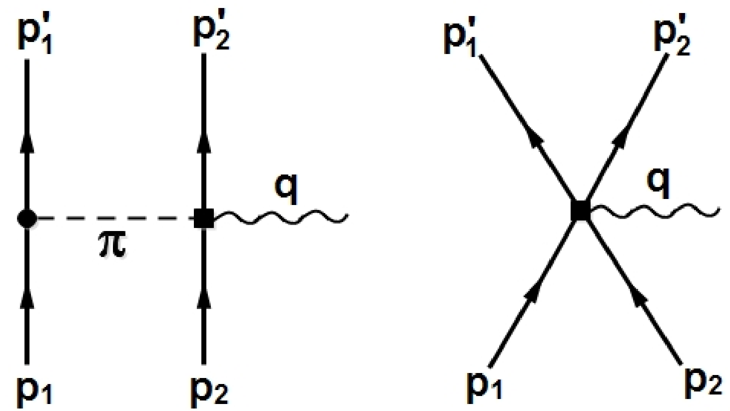


- Contribute about -20% to matrix element (QRPA)
- Should be included in all calculations
- Need to investigate effects of normal ordering and $\boldsymbol{q} \neq 0$ terms !!!

Chiral 2-body currents: momentum space

$$\langle \boldsymbol{p}_1'\boldsymbol{p}_2' | \hat{\boldsymbol{J}}_{2b}(\boldsymbol{q}) | \boldsymbol{p}_1\boldsymbol{p}_2 \rangle = \sum_{1<2}^{A} \boldsymbol{A}_{12}\, \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{q}) \quad (3)$$



where

$$\boldsymbol{A}_{12}(1\pi) = \frac{g_A}{2m_N f_\pi^2}\Big[\frac{i}{2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^-\bar{\boldsymbol{p}}_1 + 2c_3\boldsymbol{\tau}_2^-\boldsymbol{k}_2 + \left(c_4+\frac{1}{4}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^-\boldsymbol{\sigma}_1\times\boldsymbol{k}_2 + \frac{1+c_6}{4}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^-\boldsymbol{\sigma}_1\times\boldsymbol{q}\Big]\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{k}_2}{\boldsymbol{k}_2^2+m_\pi^2} + (1\leftrightarrow 2), \quad (4)$$

$$\boldsymbol{A}_{12}(2\pi) = \frac{g_A}{m_N f_\pi^2}\Big[d_1(\boldsymbol{\tau}_1^-\boldsymbol{\sigma}_1 + \boldsymbol{\tau}_2^-\boldsymbol{\sigma}_2) + d_2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^-\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\Big], \quad (5)$$

T.-S. Park et al: PRC, 67, 055206 (2003)

Chiral 2-body currents: coordinate space

$$\hat{\boldsymbol{J}}_{2b}(\boldsymbol{x}) = \sum_{k<l}^{A} \boldsymbol{J}_{kl}(\boldsymbol{x}) \tag{6}$$

$$\begin{aligned}
\boldsymbol{J}_{kl}(\boldsymbol{x}) = & \frac{g_A}{8m_N f_\pi^2}\left\{\hat{\boldsymbol{p}}_k,\ \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\frac{Y_1}{r}\right\}\delta(\boldsymbol{x}-\boldsymbol{r}_k) + \frac{ig_A}{4m_N f_\pi^2}\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\frac{Y_1}{r}\nabla_{\boldsymbol{x}}\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& + \frac{ig_A}{8m_N f_\pi^2}\boldsymbol{\tau}_\times^a\left[\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} m_\pi^2\left(1+\frac{2}{m_\pi r}+\frac{2}{m_\pi^2 r^2}\right)Y_0 + \frac{\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}}{r^2}Y_1 - \frac{\boldsymbol{\sigma}_l}{r^2}Y_1\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{c_3 g_A}{m_N f_\pi^2}\boldsymbol{\tau}_l^a\left[m_\pi^2\left(\left(\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_l}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_l}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_l}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{g_A}{2m_N f_\pi^2}\left(c_4+\frac{1}{4}\right)\boldsymbol{\tau}_\times^a\left[m_\pi^2\left(\left(\boldsymbol{\sigma}_k \times \hat{\boldsymbol{r}}\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_\times}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_\times}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_\times}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{g_A}{2m_N f_\pi^2}\left(\frac{1+c_6}{4}\right)\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \times \nabla_{\boldsymbol{x}}\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\frac{Y_1}{r}\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& + \frac{g_A}{8m_N f_\pi^2}\left\{\hat{\boldsymbol{p}}_l,\ \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\frac{Y_1}{r}\right\}\delta(\boldsymbol{x}-\boldsymbol{r}_l) + i\frac{g_A}{4m_N f_\pi^2}\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\frac{Y_1}{r}\nabla_{\boldsymbol{x}}\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& - \frac{ig_A}{8m_N f_\pi^2}\boldsymbol{\tau}_\times^a\left[\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} m_\pi^2\left(1+\frac{2}{m_\pi r}+\frac{2}{m_\pi^2 r^2}\right)Y_0 + \frac{\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}}{r^2}\hat{\boldsymbol{r}}Y_1 - \frac{\boldsymbol{\sigma}_k}{r^2}Y_1\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& - \frac{g_A c_3}{m_N f_\pi^2}\boldsymbol{\tau}_k^a\left[m_\pi^2\left(\left(\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_k}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_k}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_k}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& + \frac{g_A}{2m_N f_\pi^2}\left(c_4+\frac{1}{4}\right)\boldsymbol{\tau}_\times^a\left[m_\pi^2\left(\left(\boldsymbol{\sigma}_l \times \hat{\boldsymbol{r}}\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}} + \frac{\boldsymbol{\sigma}_\times}{3}\right)Y_2 - \frac{\boldsymbol{\sigma}_\times}{3}Y_0\right) + \frac{\boldsymbol{\sigma}_\times}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
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& + \frac{g_A}{m_N f_\pi^2}\left[\hat{d}_1(\boldsymbol{\tau}_k^a \boldsymbol{\sigma}_k + \boldsymbol{\tau}_l^a \boldsymbol{\sigma}_l) + \hat{d}_2 \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_\times\right]\delta(\boldsymbol{r})\delta(\boldsymbol{x}-\boldsymbol{r}_k)
\end{aligned} \tag{7}$$

Chiral 2-body currents: regulators

- Different regulators for cutoff Λ

$$\delta_\Lambda(\boldsymbol{r}) = \int \frac{d\boldsymbol{k}_2}{(2\pi)^3} S_\Lambda^2(\boldsymbol{k}_2^2) e^{i\boldsymbol{k}_2 \cdot \boldsymbol{r}} \tag{8}$$

$$Y_{0\Lambda}(r) = \int \frac{d\boldsymbol{k}_2}{(2\pi)^3} S_\Lambda^2(\boldsymbol{k}_2^2) \frac{e^{i\boldsymbol{k}_2 \cdot \boldsymbol{r}}}{\boldsymbol{k}_2^2 + m_\pi^2} \tag{9}$$

$$Y_{1\Lambda}(r) = -r \frac{\partial}{\partial r} Y_{0\Lambda}(r) \tag{10}$$

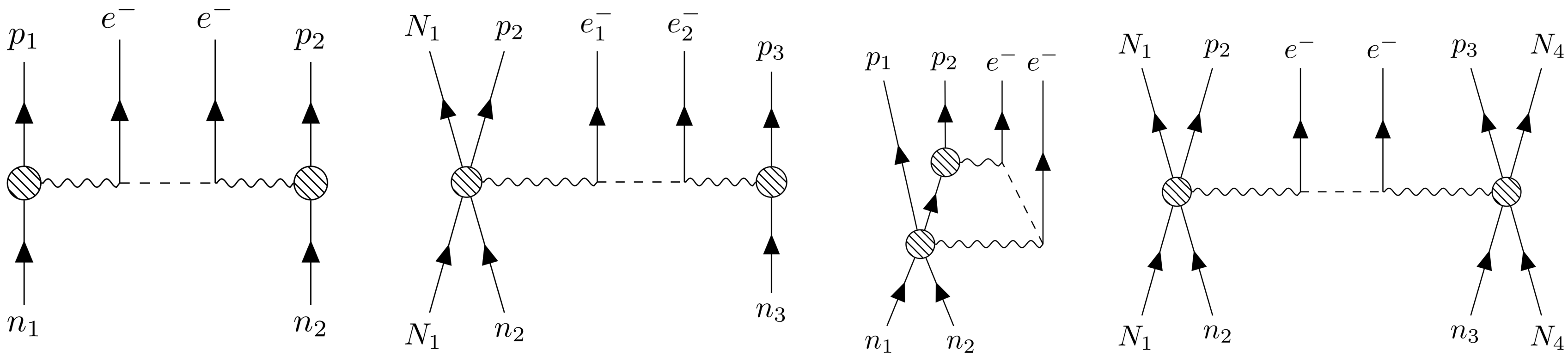
$$Y_{2\Lambda}(r) = \frac{1}{m_\pi^2} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} Y_{0\Lambda}(r) \tag{11}$$

where

$$S_\Lambda(\boldsymbol{k}_2^2) = \exp\left(- \frac{\boldsymbol{k}_2^2}{2\Lambda^2} \right) \tag{12}$$

$0\nu\beta\beta$ matrix element: 1-body+2-body

$$\mathcal{M}^{0\nu} = \frac{1}{(2\pi)^3}\int d\boldsymbol{x}_1 d\boldsymbol{x}_2 \int d\boldsymbol{q}\frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)}}{q(q+E_d)}\left\langle N_f\middle|\hat{J}^{\mu}(\boldsymbol{x}_1)\hat{J}_{\mu}(\boldsymbol{x}_2)\middle|N_i\right\rangle \tag{13}$$



(a) Double 1B (b) 1B plus 2B (c) 1B then 2B (d) Double 2B

taken from K. A. Wendt's notes (2015)

$$\hat{J}^{\mu}_{1b}(\boldsymbol{x}) = \sum_{n=1}^{A}\left[g_{\mu 0}J_0(q^2) + g_{\mu j}\boldsymbol{J}_{n,j}(q^2,\boldsymbol{q},\boldsymbol{\sigma}_n)\right]\hat{\boldsymbol{\tau}}_-(n)\delta(\boldsymbol{x}-\boldsymbol{x}_n), \tag{14}$$

$$J_0(q^2) = g_V(q^2), \tag{15}$$

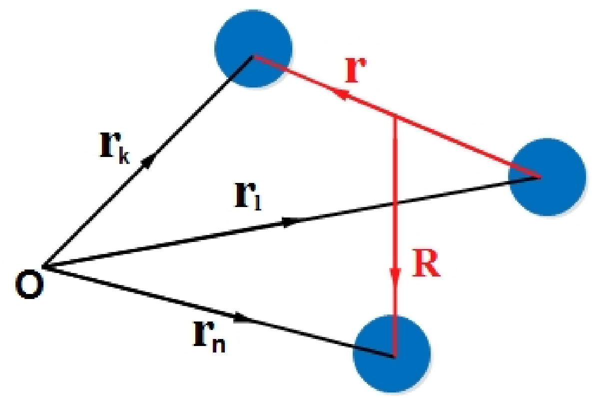
$$\boldsymbol{J}_n(q^2,\boldsymbol{q},\boldsymbol{\sigma}_n) = g_M(q^2)i\frac{\boldsymbol{\sigma}_n\times\boldsymbol{q}}{2m_p} + g_A(q^2)\boldsymbol{\sigma}_n - g_P(q^2)\frac{\boldsymbol{q}\boldsymbol{\sigma}_n\cdot\boldsymbol{q}}{2m_p} \tag{16}$$

$0\nu\beta\beta$ matrix element: 3-body operators

✓ Neglect 3-body tensor-like terms:

$$\big(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}_{12}\big)\big(\hat{\boldsymbol{r}}_{12} \cdot \hat{\boldsymbol{r}}_{13}\big)\big(\boldsymbol{\sigma}_3 \cdot \hat{\boldsymbol{r}}_{13}\big),$$
$$\big(\boldsymbol{\sigma}_1 \times \hat{\boldsymbol{r}}_{12}\big) \cdot \hat{\boldsymbol{r}}_{13}\big(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}_{12}\big)\big(\boldsymbol{\sigma}_3 \cdot \hat{\boldsymbol{r}}_{13}\big), \cdots$$

✓ Matrix elements of 3-body operators via Jacobi coordinates



About 20 terms:

$$\mathcal{M}_{1-2}^{0\nu(2b+1b)} = \sum_{k<l,n}^{A} \frac{-1}{4\pi m_N f_\pi^2} \int dq \frac{q \cdot g_A^2(q^2)}{(q+E_d)} \Big\langle N_F \Big| \sum_{LM} j_L(q\frac{r}{2}) j_L(qR) \times Y_{LM}(\hat{\boldsymbol{r}}) Y_{LM}^*(\hat{\boldsymbol{R}}) \Big\{ \hat{\boldsymbol{p}}_k,\ \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r} \Big\} \cdot \boldsymbol{\sigma}_n \hat{\boldsymbol{\tau}}_n^- \Big| N_I \Big\rangle \quad (17)$$

$$\mathcal{M}_{2-2}^{0\nu(2b+1b)} = \sum_{k<l,n}^{A} \frac{-i}{4\pi m_N f_\pi^2} \int dq \frac{q \cdot g_A^2(q^2)}{(q+E_d)} \Big\langle N_F \Big| \sum_{LM} j_L(q\frac{r}{2}) j_L(qR) Y_{LM}(\hat{\boldsymbol{r}}) Y_{LM}^*(\hat{\boldsymbol{R}}) \times \boldsymbol{\tau}_\times^a \Big[\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} m_\pi^2 \Big(1 + \frac{2}{m_\pi r} + \frac{2}{m_\pi^2 r^2}\Big) Y_0 + \frac{\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}}{r^2} Y_1 - \frac{\boldsymbol{\sigma}_l}{r^2} Y_1 \Big] \cdot \boldsymbol{\sigma}_n \hat{\boldsymbol{\tau}}_n^- \Big| N_I \Big\rangle \quad (18)$$

$$\cdots\ \cdots\ \cdots \quad (19)$$

What we are doing:

- From full three-body operators

$$\mathcal{M}^{0\nu}{}^{(2b+1b)}_{each} = \sum_{k<l,n}^{A} \left\langle N_F \middle| \hat{O}_{3b} \middle| N_I \right\rangle$$
$$= -\frac{1}{2} \sum_{abcdef} \sum_{J_{ab} J_{de} J} \hat{J} \left\langle \left[[ab]^{J_{ab}} c \right]^{J0} \middle| \hat{O}_{3b} \middle| \left[[de]^{J_{de}} f \right]^{J0} \right\rangle$$
$$\times \underbrace{\left\langle N_F \middle| \left[\left[\left[\hat{c}_a^{(\tau_a)\dagger} \hat{c}_b^{(\tau_b)\dagger} \right]^{J_{ab}} \hat{c}_c^{(\tau_c)\dagger} \right]^{J} \left[\left[\hat{c}_{\tilde{d}}^{(\tau_d)} \hat{c}_{\tilde{e}}^{(\tau_e)} \right]^{J_{de}} \hat{c}_{\tilde{f}}^{(\tau_f)} \right]^{J} \right]^{00} \middle| N_I \right\rangle}_{\text{For PHFB or GCM states with np-pairing}} \quad (20)$$

where $a = \{\tau_a, n_a, l_a, j_a\}$ etc. and $\tilde{d} = \{\tau_d, n_d, l_d, j_d, (-m_d)\}$ etc.

- From normal-ordering with realistic core

$$\propto \sum_{ijkl} \left(\sum_{ab} \langle ija | \hat{O}_{3b} | klb \rangle \rho_b^a \right) \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_l \hat{c}_k \quad (21)$$

- From analytic normal-ordering (Fermi-gas): *Menendez, Gazit and Schwenk: PRL (2011)*

Definition and Importance

- Definition (J-scheme and M-scheme)

$$\rho_{3b}^{J} \equiv \left\langle 0_f^+ \middle| \left[\left[\left[\hat{c}_{j_1}^{(\tau_1)\dagger} \, \hat{c}_{j_2}^{(\tau_2)\dagger} \right]^{J_{12}} \hat{c}_{j_3}^{(\tau_3)\dagger} \right]^{J} \cdot \left[\left[\hat{c}_{\tilde{j}_4}^{(\tau_4)} \, \hat{c}_{\tilde{j}_5}^{(\tau_5)} \right]^{J_{45}} \hat{c}_{\tilde{j}_6}^{(\tau_6)} \right]^{J} \right]^{0} \middle| 0_i^+ \right\rangle \tag{22}$$

$$\rho_{3b}^{M} \equiv \langle 0_f^+ | \hat{c}_{\tau_1 j_1 m_1}^\dagger \hat{c}_{\tau_2 j_2 m_2}^\dagger \hat{c}_{\tau_3 j_3 m_3}^\dagger \hat{c}_{\tau_4 j_4 m_4} \hat{c}_{\tau_5 j_5 m_5} \hat{c}_{\tau_6 j_6 m_6} | 0_i^+ \rangle \tag{23}$$

- Important for $0\nu\beta\beta$ and ImSRG

$$A_{l\cdots q}^{a\cdots k} = c_a^\dagger \cdots c_k^\dagger c_q \cdots c_l$$

$$\rho_r^k = \langle \Psi | A_r^k | \Psi \rangle = \rho_{rk}$$
$$\rho_{rs}^{kl} = \langle \Psi | A_{rs}^{kl} | \Psi \rangle = \rho_{rs,kl}^{(2)}$$
$$\rho_{rst}^{klm} = \langle \Psi | A_{rst}^{klm} | \Psi \rangle = \rho_{rst,klm}^{(3)}$$

$$\rho_r^k \equiv \lambda_r^k$$
$$\rho_{rs}^{kl} \equiv \lambda_{rs}^{kl} + \mathcal{A}(\lambda_r^k \lambda_s^l)$$
$$\rho_{rst}^{klm} \equiv \lambda_{rst}^{klm} + \mathcal{A}(\lambda_r^k \lambda_s^l \lambda_t^m + \lambda_r^k \lambda_{st}^{lm})$$

where antisymmetrizer $\mathcal{A}$ generates all unique permutations of the indices of the product of tensors it is applied to.

From M- to J-scheme

- Rotated matrix elements in signature basis

$$\langle\phi_f|\hat{d}_i^{(\tau_1)\dagger}\hat{d}_j^{(\tau_2)\dagger}\hat{d}_k^{(\tau_3)\dagger}\hat{d}_l^{(\tau_4)}\hat{d}_m^{(\tau_5)}\hat{d}_n^{(\tau_6)}|\tilde{\phi}_i\rangle = \langle\phi_f|\tilde{\phi}_i\rangle\langle\tilde{\phi}_i|\bar{\alpha}_i^{(\tau_1)}\bar{\alpha}_j^{(\tau_2)}\bar{\alpha}_k^{(\tau_3)}\alpha_l^{(\tau_4)}\alpha_m^{(\tau_5)}\alpha_n^{(\tau_6)}|\tilde{\phi}_i\rangle \quad (24)$$

$$\bar{\alpha}_k^{(\tau)} = e^{\hat{Z}^\dagger}\hat{d}_k^{(\tau)\dagger}e^{-\hat{Z}^\dagger}, \quad \alpha_m^{(\tau)} = e^{\hat{Z}^\dagger}\hat{d}_m^{(\tau)}e^{-\hat{Z}^\dagger}; \qquad \hat{Z} = \sum_{\mu<\nu} Z_{\mu\nu}\hat{a}_\mu^\dagger\hat{a}_\nu^\dagger, \quad Z = (VU^{-1})^*$$

- Wick's theorem or Pfaffian

$$\langle\tilde{\phi}_I|\bar{\alpha}_i^{(\tau_i)}\bar{\alpha}_j^{(\tau_j)}|\tilde{\phi}_I\rangle = A_{ij}, \quad \langle\tilde{\phi}_I|\bar{\alpha}_i^{(\tau_i)}\alpha_j^{(\tau_j)}|\tilde{\phi}_I\rangle = C_{ij}, \quad \langle\tilde{\phi}_I|\alpha_i^{(\tau_i)}\alpha_j^{(\tau_j)}|\tilde{\phi}_I\rangle = B_{ij} \quad (25)$$

$$\langle\tilde{\phi}_I|\bar{\alpha}_i^{(\tau_1)}\bar{\alpha}_j^{(\tau_2)}\bar{\alpha}_k^{(\tau_3)}\alpha_l^{(\tau_4)}\alpha_m^{(\tau_5)}\alpha_n^{(\tau_6)}|\tilde{\phi}_I\rangle = A_{ij}B_{lm}C_{kn} - A_{ij}C_{km}B_{ln} + A_{ij}C_{kl}B_{mn} + \cdots \quad (26)$$

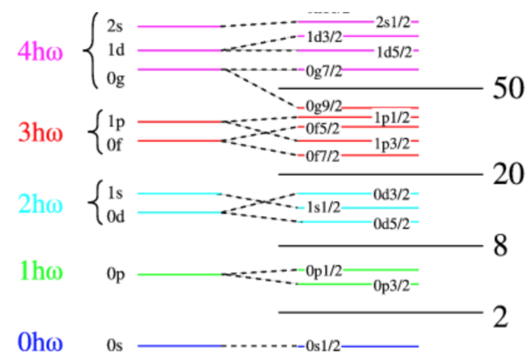
$$\langle\tilde{\phi}_I|\bar{\alpha}_i^{(\tau_1)}\bar{\alpha}_j^{(\tau_2)}\bar{\alpha}_k^{(\tau_3)}\alpha_l^{(\tau_4)}\alpha_m^{(\tau_5)}\alpha_n^{(\tau_6)}|\tilde{\phi}_I\rangle = Pf\begin{pmatrix} A & C \\ -C^T & B \end{pmatrix} \quad (27)$$

where

$$Pf(\mathcal{A}) \equiv \frac{1}{2^n n!}\sum_{\sigma\in S_{2n}} \text{sgn}(\sigma)\prod_{i=1}^{n} a_{\sigma(2i-1)\sigma(2i)}, \text{ and } Pf(X) = \sqrt{\det X} \quad (28)$$

Numerical details

Nuclei	Model space	Computation time (each HFB with PNP)
^{20}Ne	$1s_{1/2}, 0d_{5/2}, 0d_{3/2}$	$\sim$ 3 CPU hours
^{48}Ca	$1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0f_{7/2}$	$\sim$ 22 CPU hours
^{76}Ge	$1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2}$	$\sim$ 24 CPU hours



Numerical check

- ✓ Trace of $\rho^{(3)}$: $\mathrm{Tr}\left(\rho_{ij}^{(3)}\right) = -N(N-1)(N-2)$
- ✓ Anti-symmetry of $\rho^{(3)}$ in M-scheme:

$$\rho^{(3)^M} \equiv \left\langle 0_f^+ \middle| \hat{c}^\dagger_{\tau_1 j_1 m_1} \hat{c}^\dagger_{\tau_2 j_2 m_2} \hat{c}^\dagger_{\tau_3 j_3 m_3} \hat{c}_{\tau_4 j_4 m_4} \hat{c}_{\tau_5 j_5 m_5} \hat{c}_{\tau_6 j_6 m_6} \middle| 0_i^+ \right\rangle \qquad (29)$$

- ✓ Symmetry in J-scheme: $\rho_{ij}^{(3)^J} = \rho_{ji}^{(3)^J}, \quad \lambda_{ij}^{(3)^J} = \lambda_{ji}^{(3)^J}$

Summary

- Transition operator: uncertainty of $0\nu\beta\beta$ matrix elements.
- Pioneering work of chiral 2b currents: Fermi-gas approximation, no $q \neq 0$ term: -35% to 10% correction.
- Effects of 2b currents: full 3b-operators vs normal-ordering

Outlook

- Different regulators ...

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Thank you for your attention!