Neutrinoless double-beta decay matrix elements with QRPA method

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1. Nuclear matrix elements of $\beta\beta$ decay – originality of my calculation
2. Nonlinear higher RPA by A. Smetana, F. Šimkovic, M. Krivorchenko, and J.T.

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Seattle

Two methods of QRPA approach under closure approx.

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_{\mathrm{f}}^{+}| c_{p'}^{\dagger} c_{n'} \underset{\uparrow\; \overbrace{\sum_{b_{\mathrm{f}}:\mathrm{pnQRPA}} |b_{\mathrm{f}}\rangle\langle b_{\mathrm{f}}| \sum_{b_{\mathrm{i}}:\mathrm{pnQRPA}} |b_{\mathrm{i}}\rangle\langle b_{\mathrm{i}}|}}{} c_{p}^{\dagger} c_{n} \, |0_{\mathrm{i}}^{+}\rangle$$

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_{\mathrm{f}}^{+}| \underbrace{c_{p'}^{\dagger} c_{n'} c_{p}^{\dagger} c_{n}}_{-c_{p'}^{\dagger} c_{p}^{\dagger} \underset{\uparrow\; \overbrace{\sum_{b_{\mathrm{f}}:\mathrm{likeQRPA}} |b_{\mathrm{f}}\rangle\langle b_{\mathrm{f}}| \sum_{b_{\mathrm{i}}:\mathrm{likeQRPA}} |b_{\mathrm{i}}\rangle\langle b_{\mathrm{i}}|}}{} c_{n'} c_{n}} \, |0_{\mathrm{i}}^{+}\rangle$$

The overlap of QRPA states

The QRPA ground state $|0^+_{\mathrm{QRPA},i}\rangle$ is defined as the vacuum of the QRPA quasiboson :

$$O^{\mathrm{i}}_{b}|0^+_{\mathrm{QRPA,i}}\rangle = 0$$

O^{i}_{b} : annihilation operator of QRPA state b

$$|0^+_{\mathrm{QRPA,i}}\rangle = \prod_{K\pi} \frac{1}{\mathcal{N}^{K\pi}_{\mathrm{QRPA,i}}} \exp[v^{(K\pi)}_{\mathrm{i}}]|0^+_{\mathrm{HFB,i}}\rangle\,,$$

$$v^{(K\pi)}_{\mathrm{i}} \cong \sum_{\mu\nu\mu'\nu'} \frac{1}{1+\delta_{K0}} \left(Y^{\mathrm{i},K\pi}\frac{1}{X^{\mathrm{i},K\pi}}\right)^{\dagger}_{\mu\nu,\mu'\nu'} a^{\mathrm{i}\dagger}_{\mu} a^{\mathrm{i}\dagger}_{\nu} a^{\mathrm{i}\dagger}_{\mu'} a^{\mathrm{i}\dagger}_{\nu'}$$

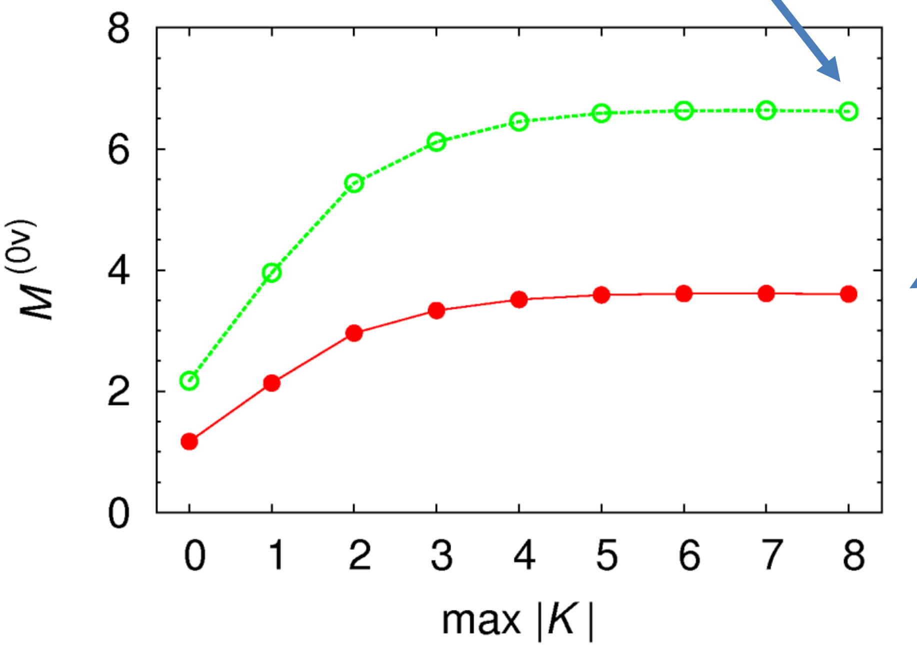
$$O^{\mathrm{i}\dagger}_{b} = \sum_{\mu\nu\mu'\nu'} \left(X^{\mathrm{i},K\pi}_{\mu\nu,b} a^{i\dagger}_{\mu} a^{i\dagger}_{\nu} - Y^{\mathrm{i},K\pi}_{-\mu-\nu,b} a^{\mathrm{i}}_{-\nu} a^{\mathrm{i}}_{-\mu}\right),$$

$$a^{\mathrm{i}}_{\mu}|0^+_{\mathrm{HFB,i}}\rangle = 0.$$

J. Terasaki, PRC **87**, 024316 (2013)

Result for ^{150}Nd→^{150}Sm

HFB gs is used instead of QRPA gs in the overlap calculations.



The value of my method

The product of the QRPA ground-state normalization factors=**1.84**

$$M^{(0\nu)} = \sum_{K'=-\max K}^{\max K} \sum_{\pi} M^{(0\nu)}(K'\pi)$$

Comparison ($^{150}\mathrm{Nd} \rightarrow {}^{150}\mathrm{Sm}$, g_A=1.25)

UNDER DICUSSION

	J. T.	Fang et al. (Tübingen)
$M^{(0v)}$	3.60	3.34
Method	Like-particle QRPA	PnQRPA
Residual interaction	Skyrme + volume pairing, no pn pairing	G matrix (CD Bonn) +pn pairing
Overlap calculation	1/normalization factors = 0.54	1/normalization factors = 1

The pn pairing interaction has an effect to reduce the NME.

D.-L. Fang et al., PRC **83**, 034320 (2011)
J. Terasaki, PRC **91**, 034318 (2015)

Two paths in QRPA approach under closure approx.

$$\sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \sum_{d_\mathrm{f} d_\mathrm{i}:\mathrm{pnQRPA}} \langle 0_\mathrm{f}^+|c_{p'}^\dagger c_{n'}|d_\mathrm{f}\rangle\langle d_\mathrm{f}|d_\mathrm{i}\rangle\langle d_\mathrm{i}|c_p^\dagger c_n\ |0_\mathrm{i}^+\rangle$$

$$\rotatebox{90}{$=$}$$

$$\sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \sum_{b_\mathrm{f} b_\mathrm{i}:\mathrm{likeQRPA}} \langle 0_\mathrm{f}^+|c_{p'}^\dagger c_p^\dagger|b_\mathrm{f}\rangle\langle b_\mathrm{f}|b_\mathrm{i}\rangle\langle b_\mathrm{i}|c_n\ c_{n'}|0_\mathrm{i}^+\rangle$$

Pn-pairing int. is important for β decay.
Like-particle pairing int. is important for two-particle transfer.

Dependence on residual interaction is small

$$\text{Overlap of QRPA states} = \frac{\text{Unnormalized overlap}}{\prod_{\mathrm{f}\ \mathrm{i}} (\text{Normalization factors of the QRPA g.s.})}$$

The equivalence of the two different paths provides us with a constraint on the strengths of the effective interactions having different roles in the QRPA.

This principle → the strength of the *T*=0 pn-pairing int.

J.T. PRC **93**, 024317 (2016)

Other interactions used:
Skyrme SkM*, like-particle pairing, and Coulomb interaction

Pairing int. (MeV fm^3)	150Nd		150Sm	
	Proton	Neutron	Proton	Neutron
Like-ptcl.	-218.52	-176.36	-218.52	-181.65
T=0 (pn)	-197.44		-200.09	

	$^{150}\mathrm{Nd} \rightarrow {}^{150}\mathrm{Sm}$	**2νββ nuclear matrix element**
g_A=1.254 (bare value)	My cal.	0.0816
	Semiexp.	0.0368
g_A=1.000 (effective value)	My cal.	0.0849
	Semiexp.	0.0579

- Usually the semiexp. 2νββ nuclear matrix element is fitted by adjusting the strength of the pn pairing interaction in the QRPA approach.
- In my cal. that interaction strength is determined by an original theoretical method.
- Semiexp. value is obtained by the exp. half-life and phase-space factor including $g_{A.}$

Second part: extension of RPA – under development

We aim at solving

the discrepancy problem of the nuclear matrix elements between the different methods

One of what we can do is

extension of RPA to higher-order particle-hole correlations

Our choice of method for the extension

Nonlinear higher RPA (nhRPA)

including the 2p-2h, … for expressing the excitations on top of the ground state

NhRPA equation

arXiv:1701.08368

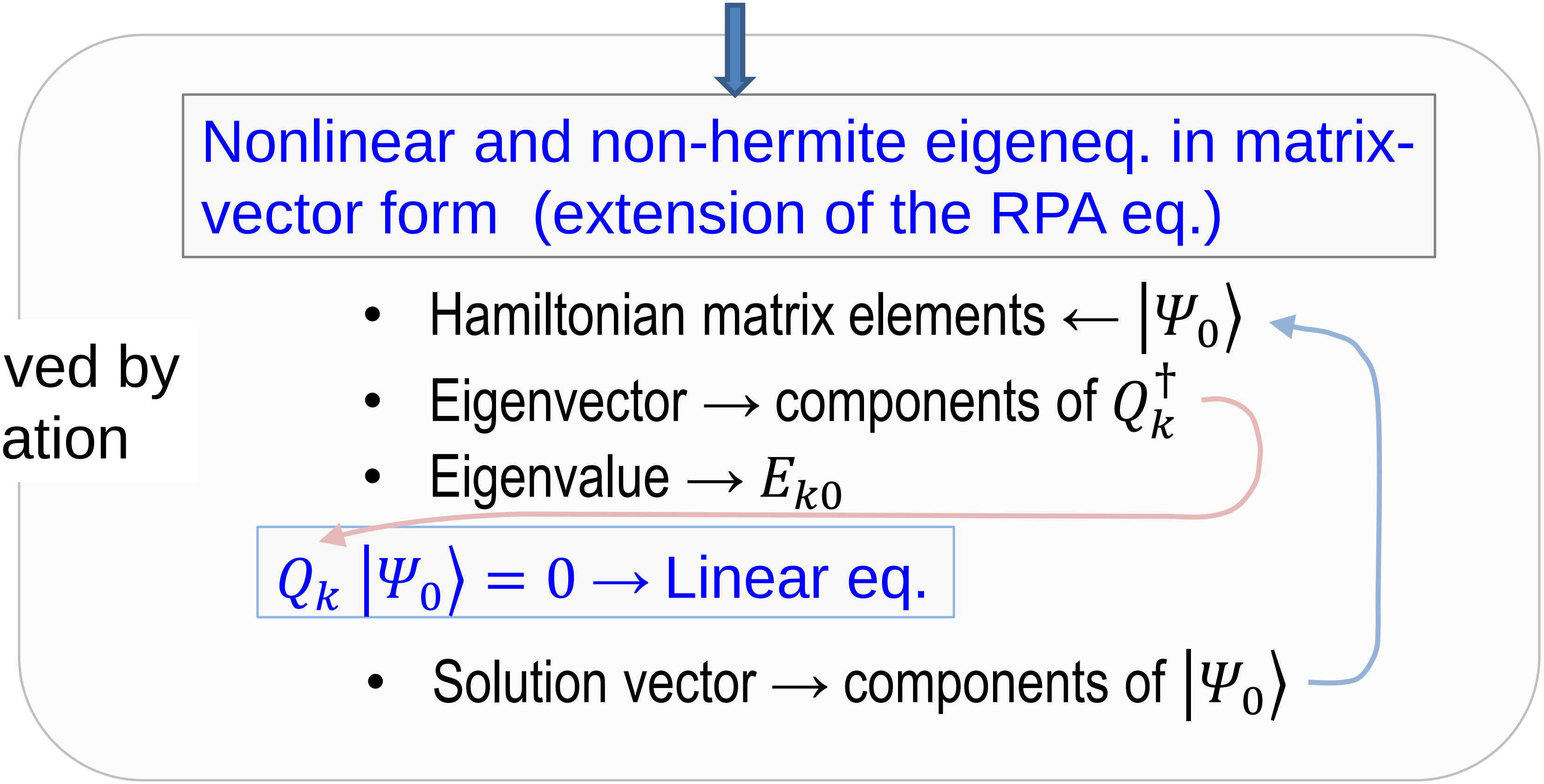
Express excited state $|\Psi_k\rangle$ as

Ground state

$$|\Psi_k\rangle = Q_k^\dagger|\Psi_0\rangle$$

$$[H, Q_k^\dagger]|\Psi_0\rangle = E_{k0}Q_k^\dagger|\Psi_0\rangle$$

D.J.Rowe,
Rev.Mod.Phys. **40**,
153 (1968)



Nonlinear and non-hermite eigeneq. in matrix-vector form (extension of the RPA eq.)

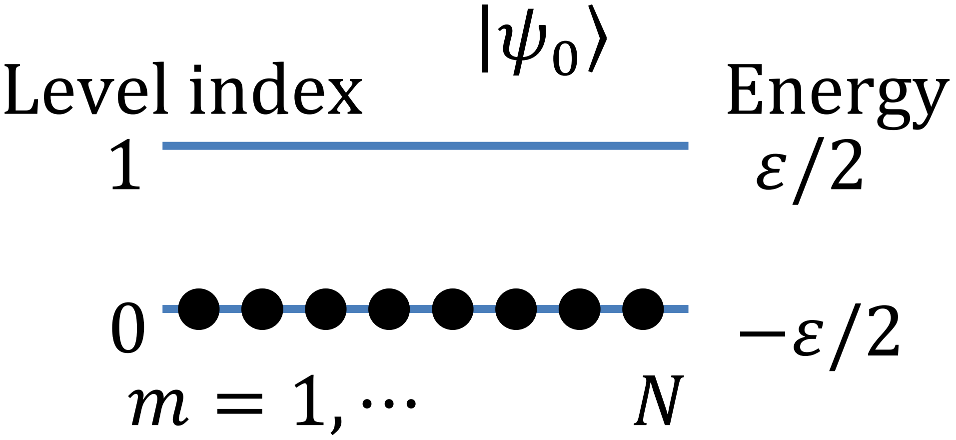
Solved by iteration

- Hamiltonian matrix elements ← $|\Psi_0\rangle$
- Eigenvector → components of $Q_k^\dagger$
- Eigenvalue → E_{k0}

$Q_k|\Psi_0\rangle = 0 \rightarrow$ Linear eq.

- Solution vector → components of $|\Psi_0\rangle$

Lipkin model



Useful for test of theory, often used.

H.J. Lipkin et al., N.P. **62**, 188 (1965)

$$H = \varepsilon J_z + \frac{V}{2}(J_+^2 + J_-^2)$$

$$J_z = \frac{1}{2}\sum_{m=1}^{N}\left(a_{1m}^\dagger a_{1m} - a_{0m}^\dagger a_{0m}\right)$$

$$J_+ = \sum_{m=1}^{N} a_{1m}^\dagger a_{0m}\,, \qquad J_- = J_+^\dagger$$

Two subspace

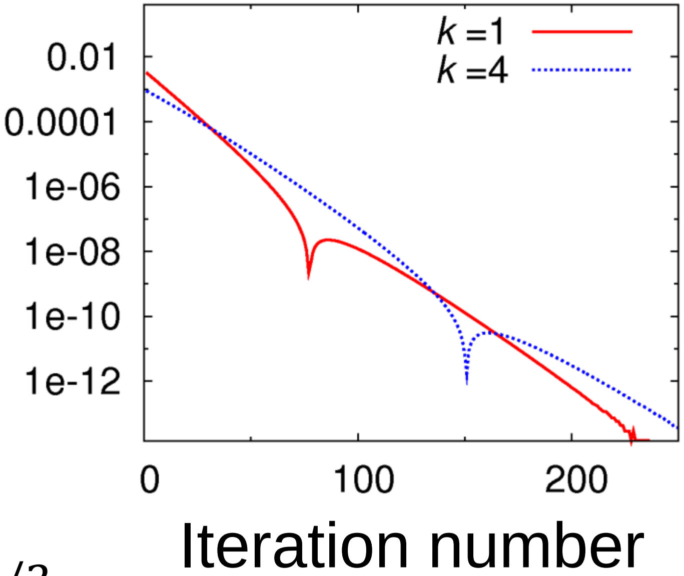
$$\{\, |\psi_0\rangle,\ J_+^2 |\psi_0\rangle, \cdots,\ J_+^N |\psi_0\rangle \,\}$$

decoupled

$$\{\, J_+ |\psi_0\rangle\,, \cdots,\ J_+^{N-1} |\psi_0\rangle \,\}$$

Achievement 1

We found that nhRPA is equivalent to exact Schrödinger eq. by solving the equations for the first time.



Relative error of E_{k0}

$$\frac{|E^o_{k0} - E^o_{k0}(\text{exact})|}{E^o_{k0}(\text{exact})}$$

N = 4, $\varepsilon = 1$

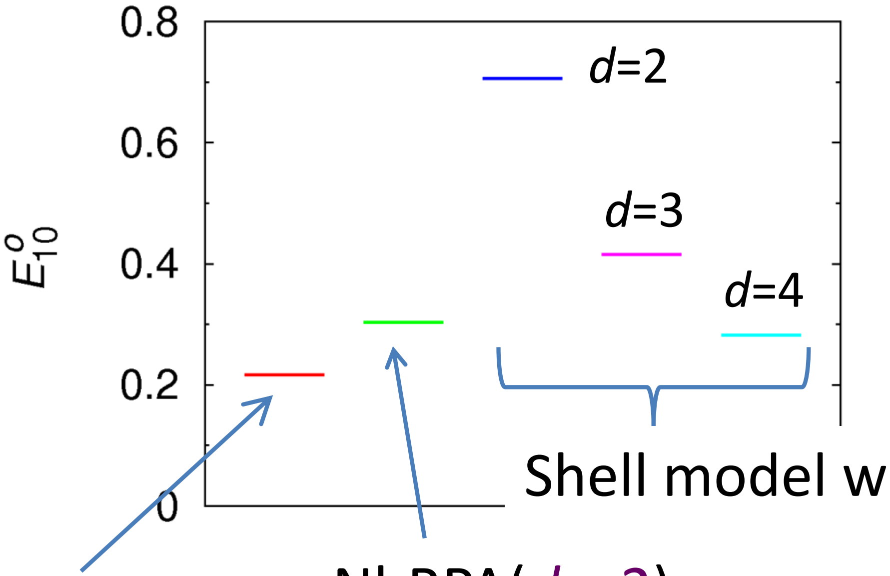
$$Q^{e\dagger}_k = c_k + \sum_{l=1}^{N/2} \left(X^k_{2l} J^{2l}_+ + Y^k_{2l} J^{2l}_- \right)$$

This term has been overlooked by other groups years.

Necessary for the subspace including the ground state.

Achievement 2

Comparison with shell model *under truncation of dimension of matrix used in calculation*



Shell model with truncation

Exact (shell model, *d* =10)

NhRPA(*d* = 2)

d: dimension of the matrix used in the calculation
d of exact cal. = *N/2* = 10

Reason

Unperturbed ground state

$$Q_k^\dagger|\Psi_0\rangle = \overbrace{\left[\underbrace{\sum_{l=1}^{d}\left(X_{2l}^k J_+^{2l} + Y_{2l}^k J_-^{2l}\right)}_{} + c_k\right]}^{Q_k^\dagger} \underbrace{\sum_{i=0}^{d}\beta_{2i} J_+^{2i}|\psi_0\rangle}_{}$$

Ground state

Eigeneq. with matrix of dimension *d*

Linear eq. with matrix of dimension *d*

0th component

P-h component

$$\boxed{Q_k|\Psi_0\rangle = 0}$$

- The highest order of J_+^{2l} of excited state = 4*d*
- Corresponding order of shell model = 2*d*

Summary

1. Three originalities in calculation of $\beta\beta$ NME presented:

   i. Like-particle QRPA
   ii. Accurate overlap calculation
   iii. Theoretical determination of the strength of T=0 pairing interaction

   For $2\nu\beta\beta$ NME of ^{150}Nd, Cal./semiexp = 1.47, (g_A = 1.0).

2. Extension of RPA presented: nonlinear higher RPA

   i. Equivalent to exact Schrödinger eq.
   ii. High performance under truncation of wavefunction space
   iii. Iteration necessary.