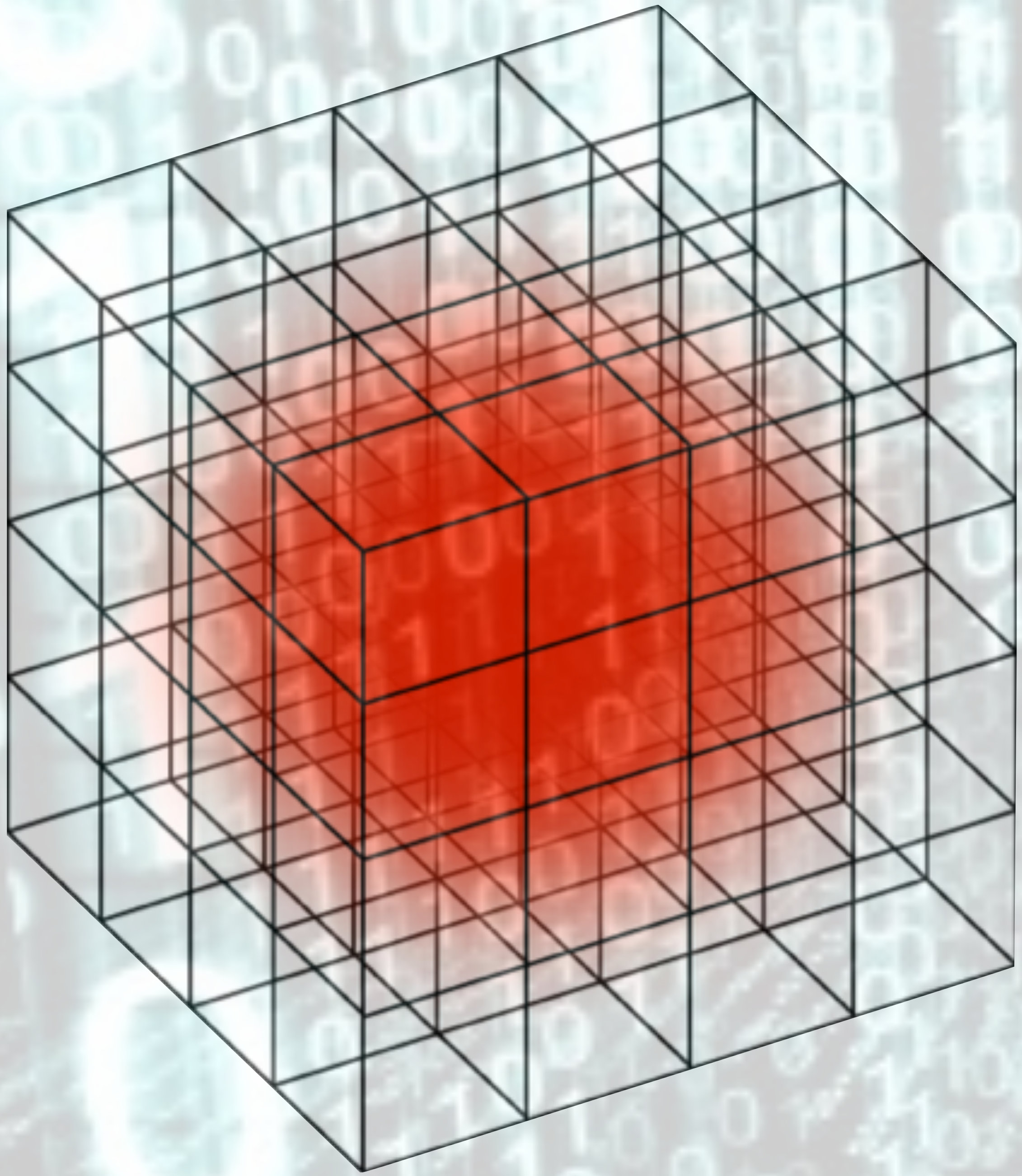
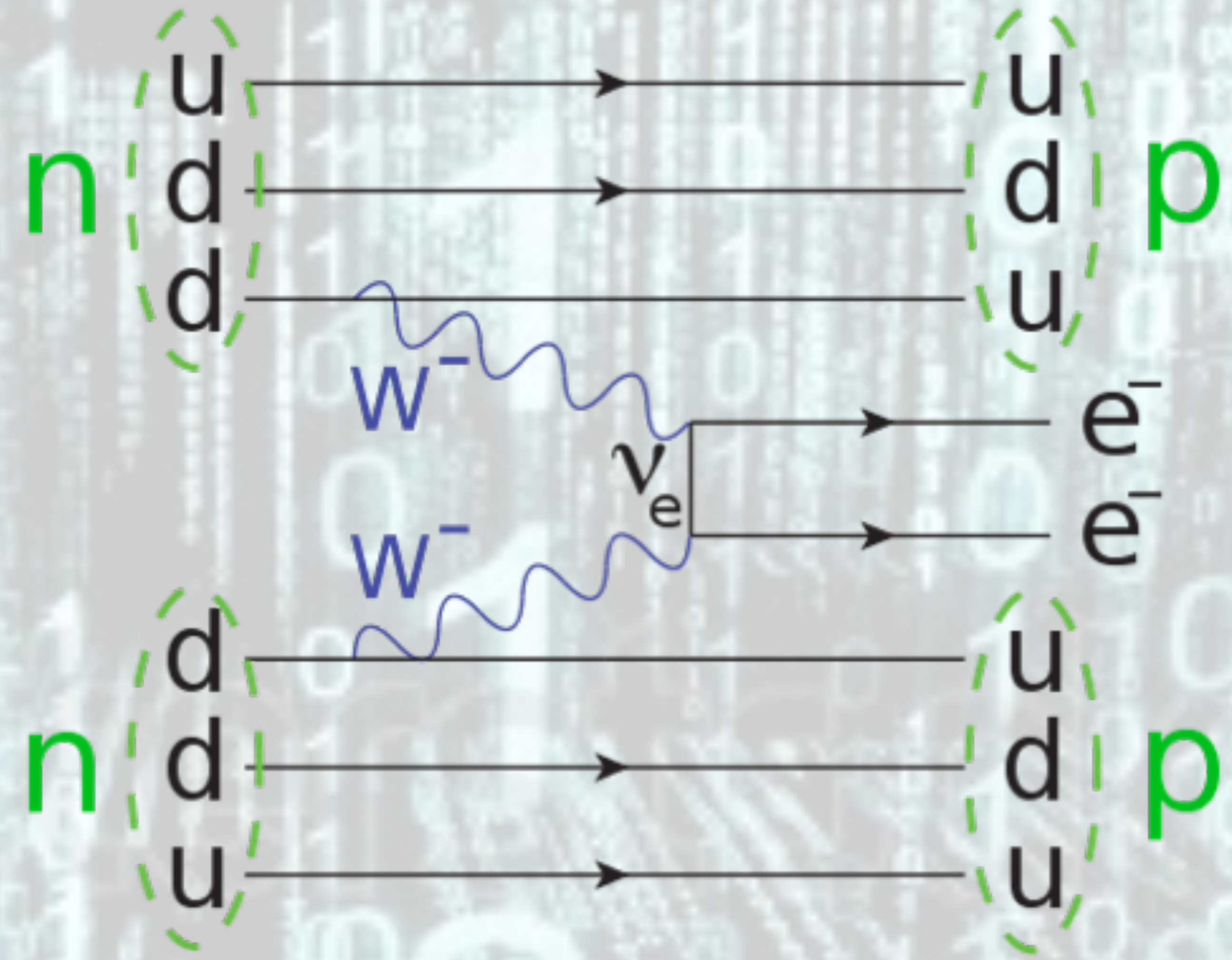


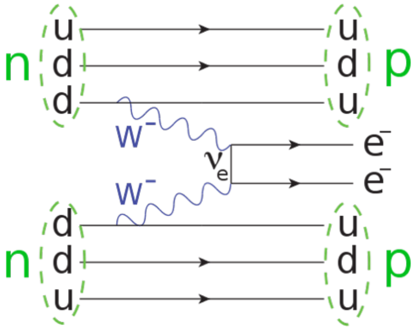


The Axial Structure of Nuclei from Lattice QCD

Martin J Savage

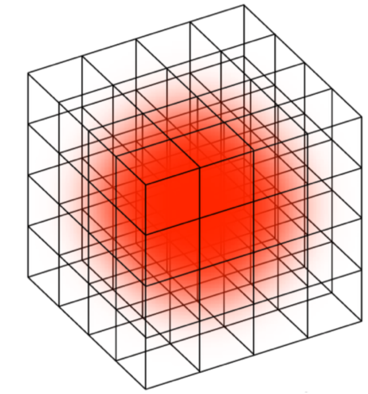
Institute for Nuclear Theory
University of Washington

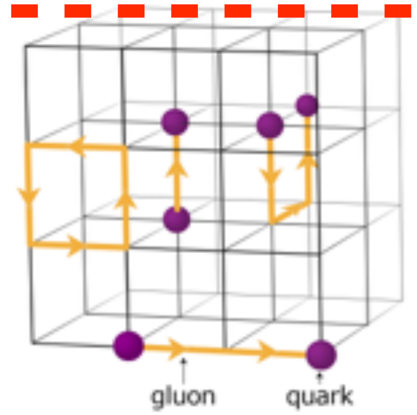
INT, June 14, 2017, Seattle



Energy Scales

Dynamical Degrees of Freedom





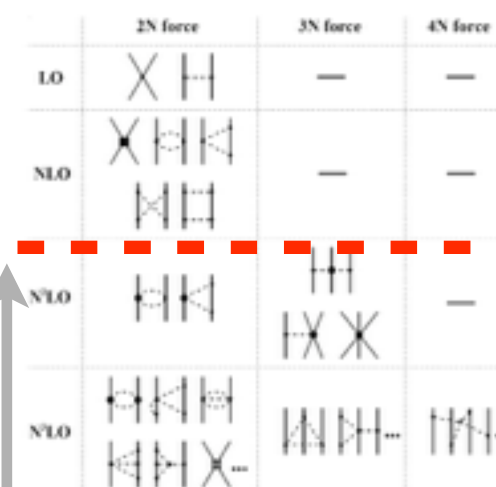
Lattice QCD

≳ 2 GeV

Nucleon

~ 1 GeV

size set by pion



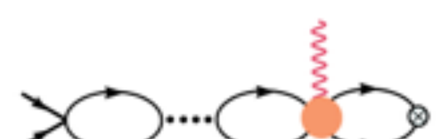
pionful EFT

ρ mass/2

~ 390 MeV

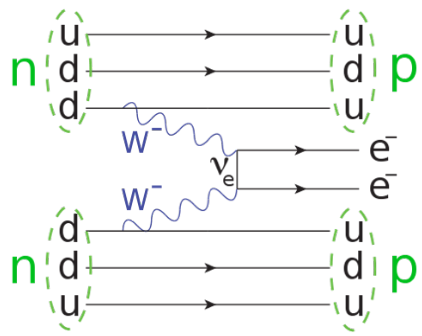
Nucleus

~ 250 MeV

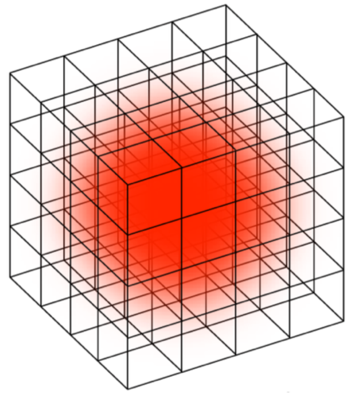


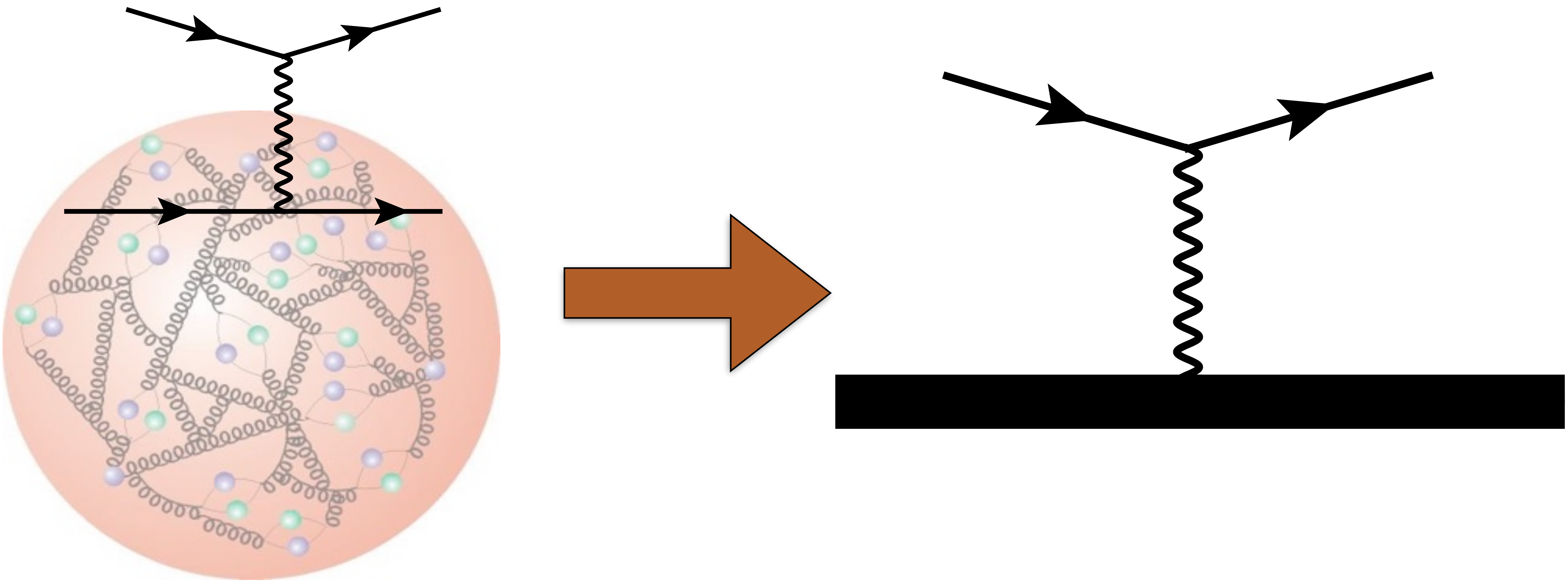
pionless EFT

~ 70 MeV

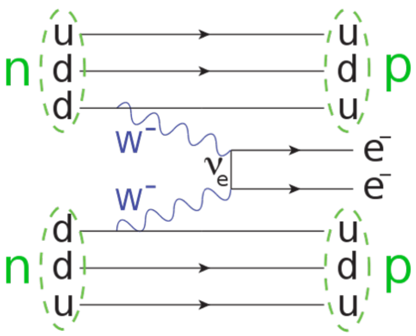


Axial Interactions and Effective Interactions

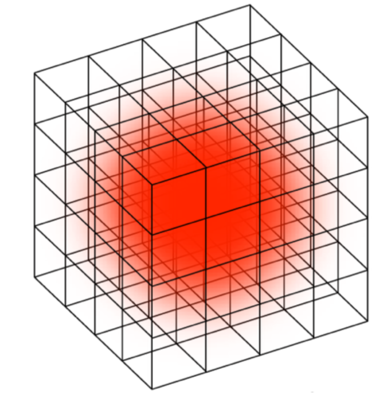


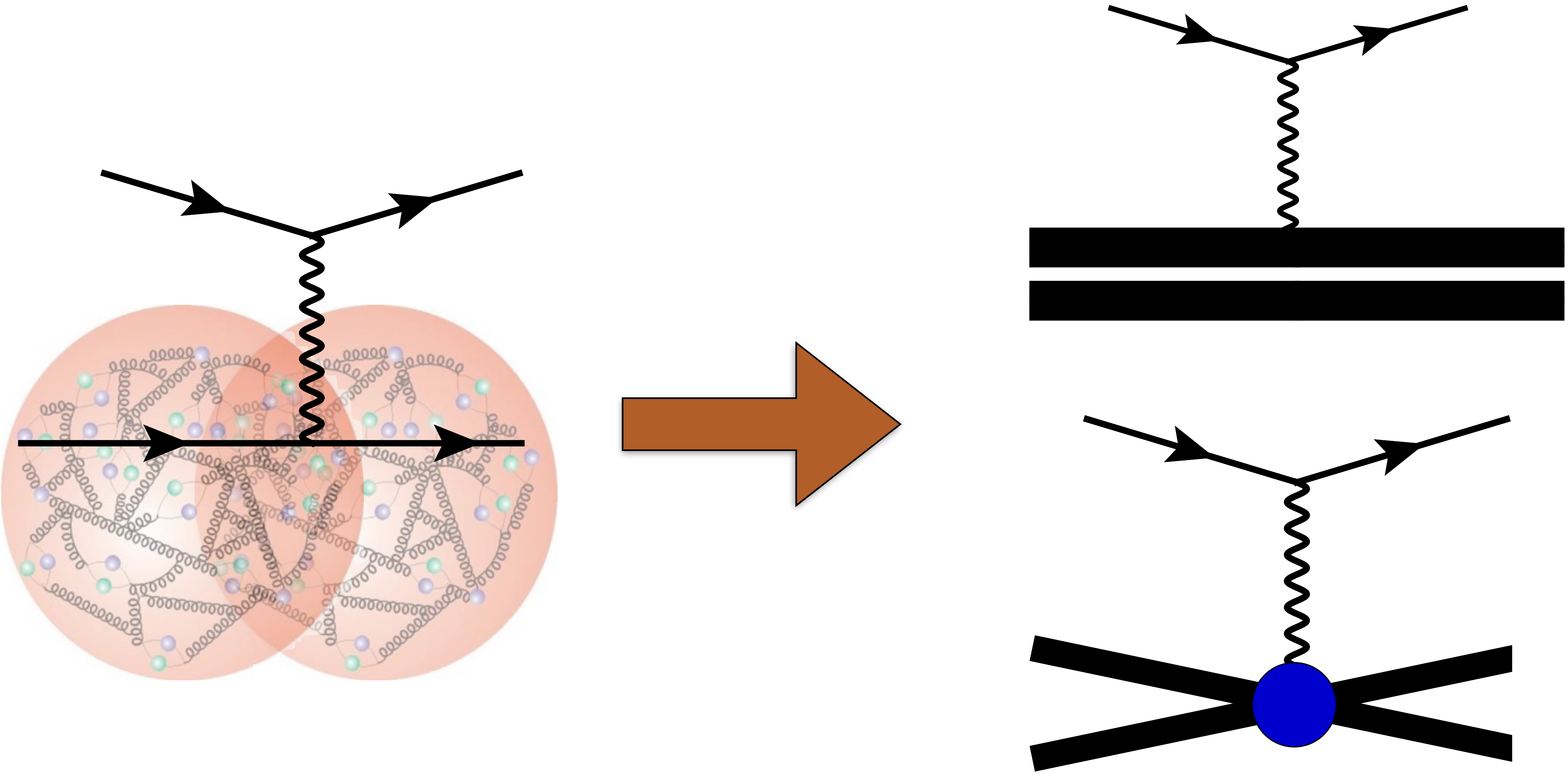


+ ...

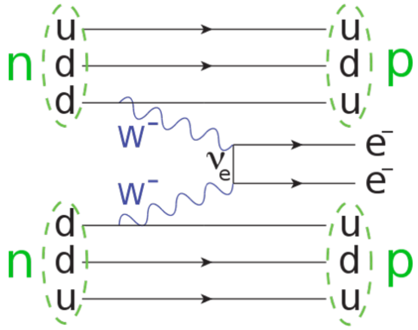


Axial Interactions and Effective Interactions

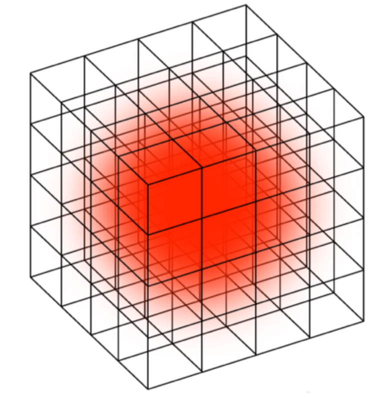


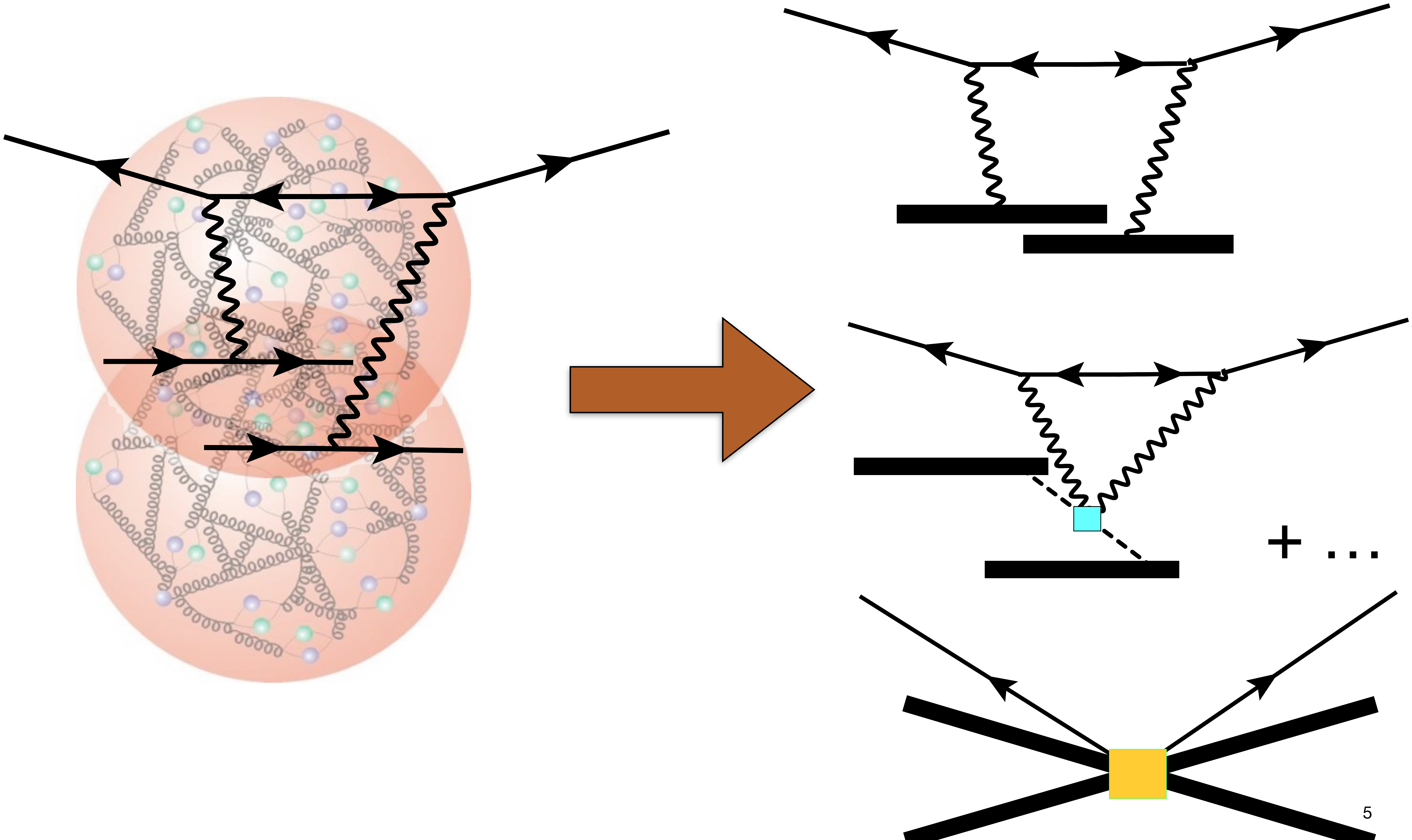


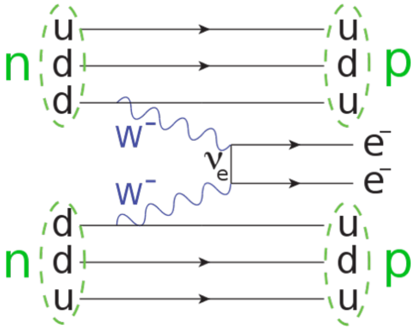
+ . . .



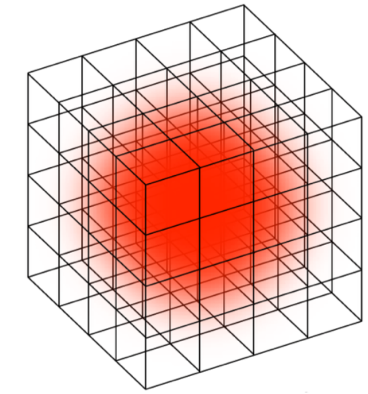
Axial Interactions and Effective Interactions

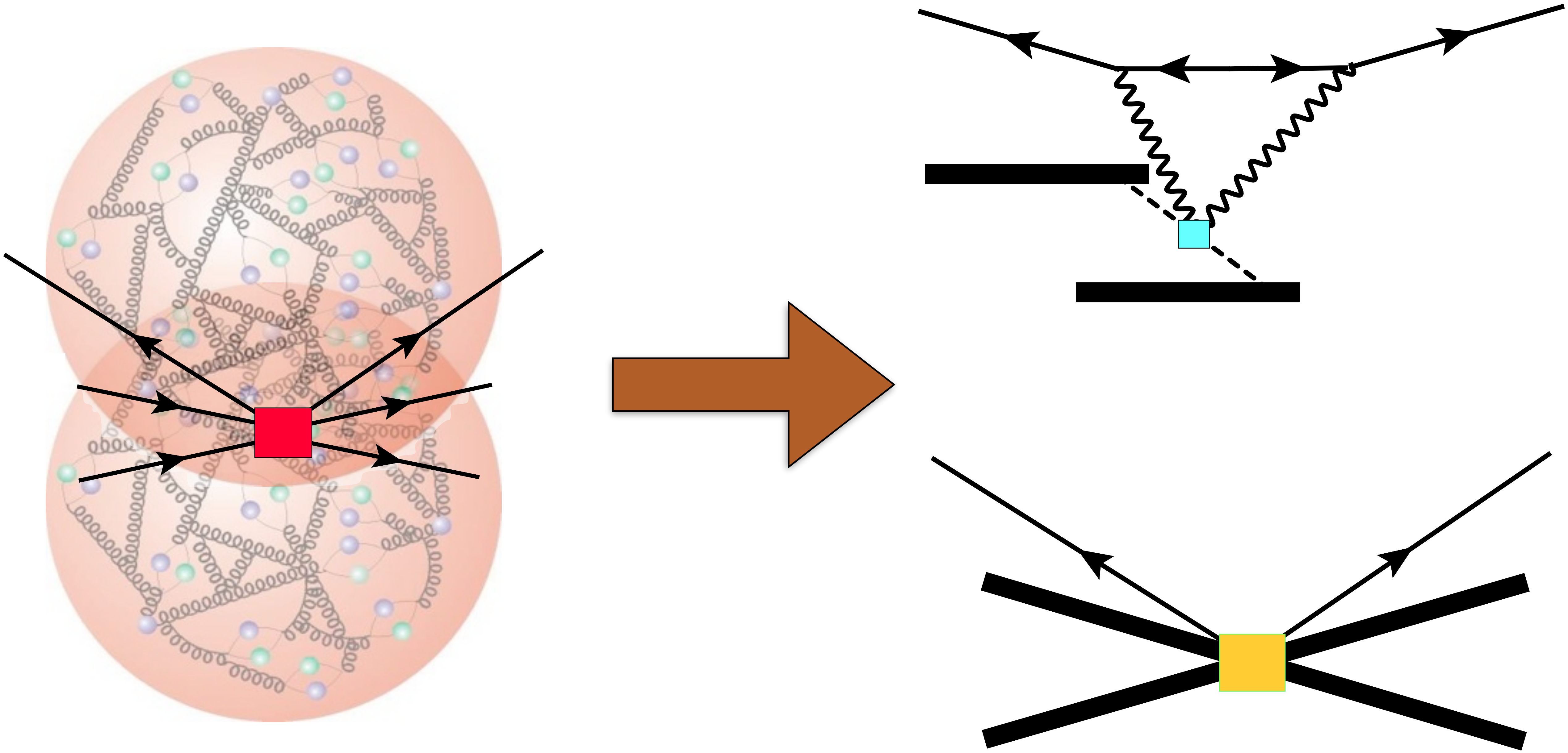




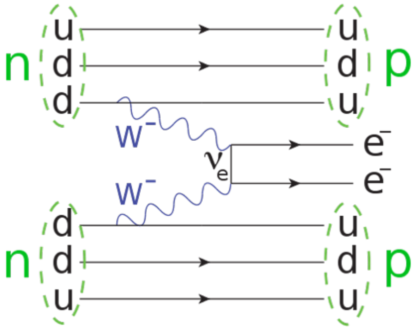


Axial Interactions and Effective Interactions

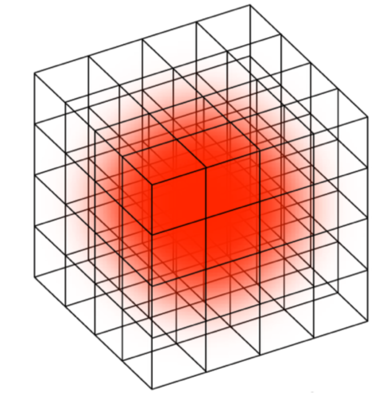


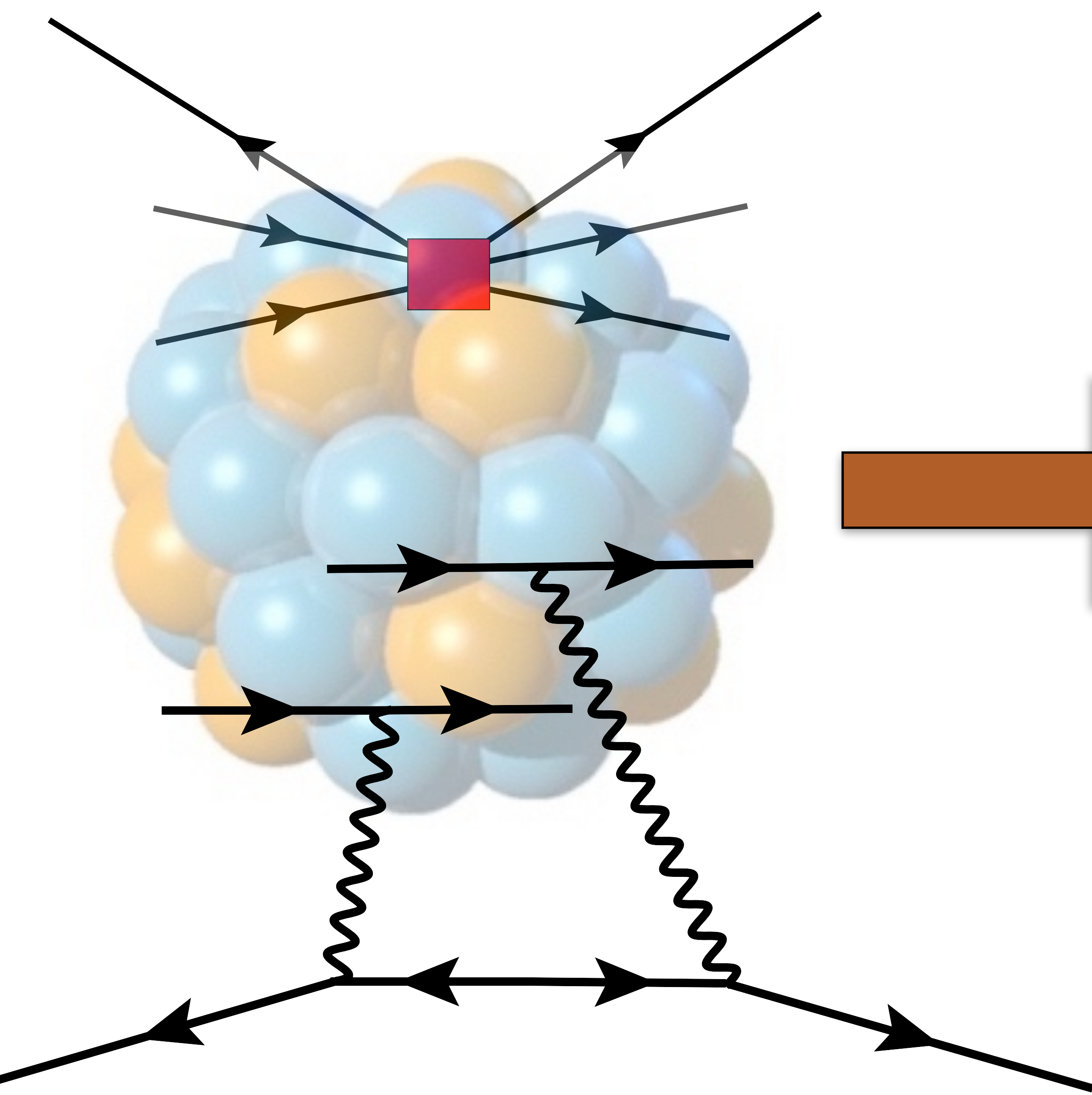


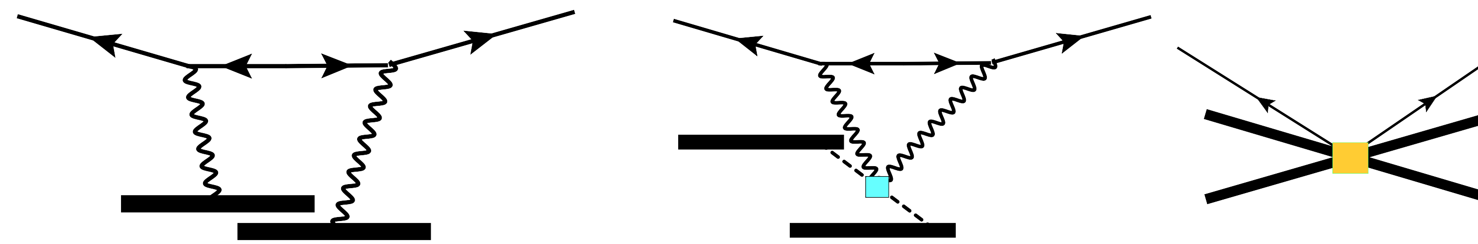
+ . . .



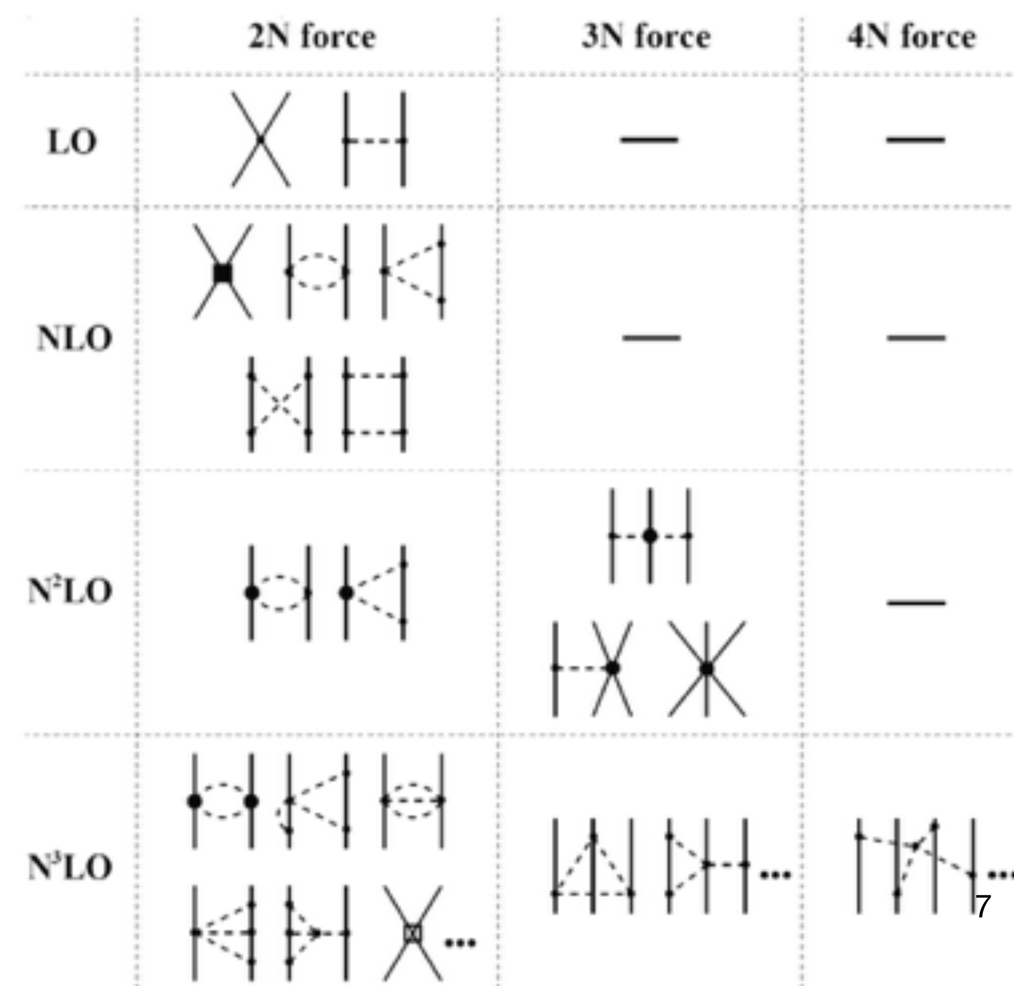
Axial Interactions and Effective Interactions





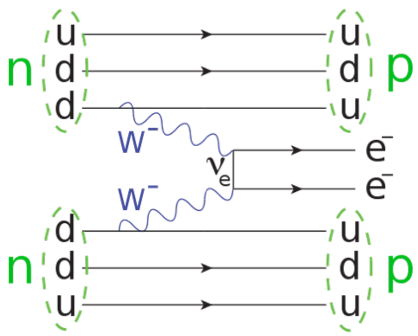


+ …

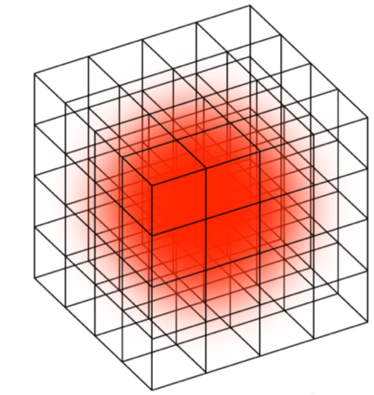


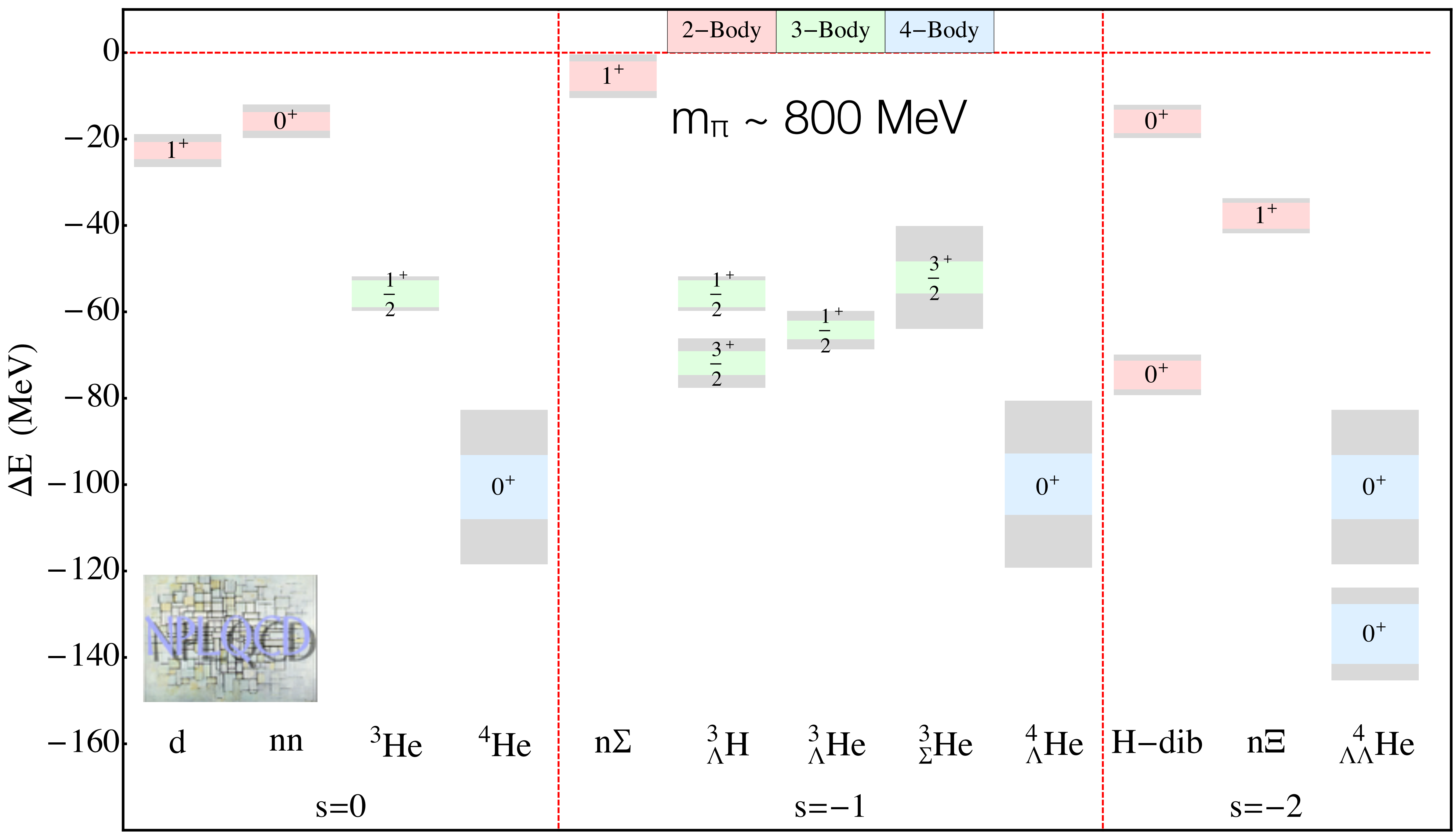
work to consistent orders, etc …

Review: Nuclear Physics from Lattice QCD , S.R. Beane, W. Detmold, K. Orginos, M.J. Savage, Prog.Part.Nucl.Phys. 66 (2011)

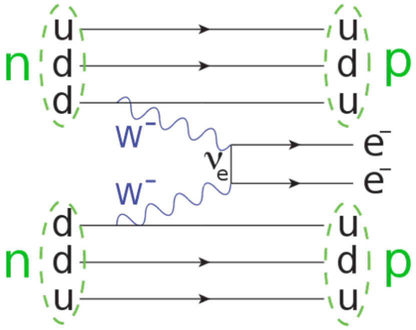


Nuclei from QCD



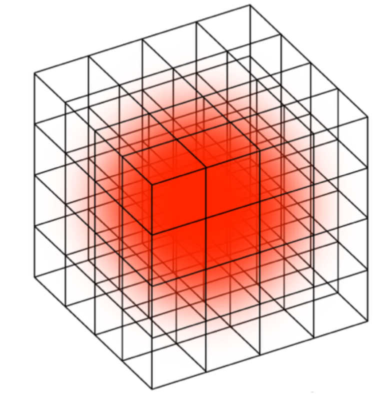


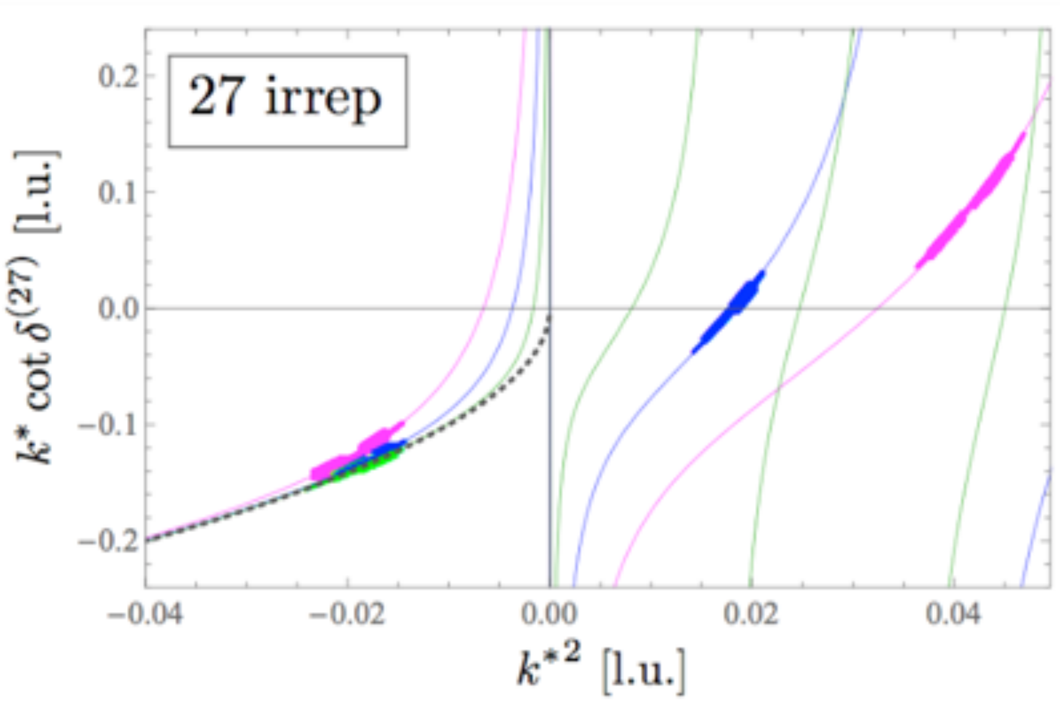
Beane *et al* (NPLQCD), Phys.Rev. D87 (2013) 3, 034506, Phys.Rev. C88 (2013) 2, 024003



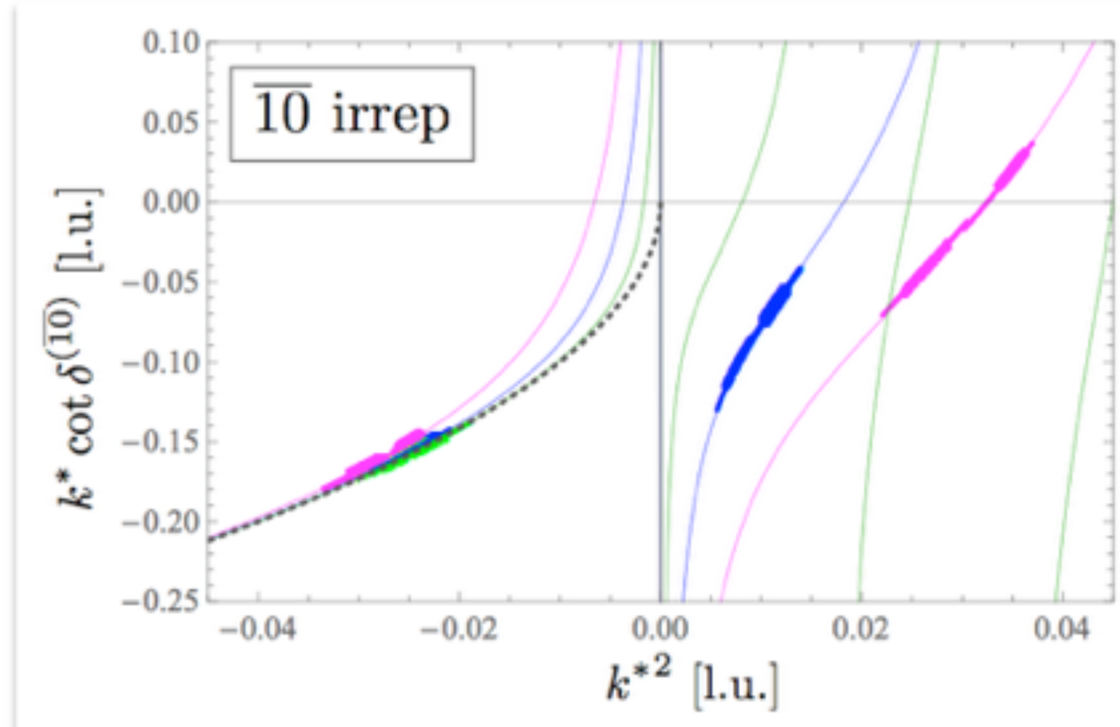
Luscher's method
Baryon-Baryon Scattering

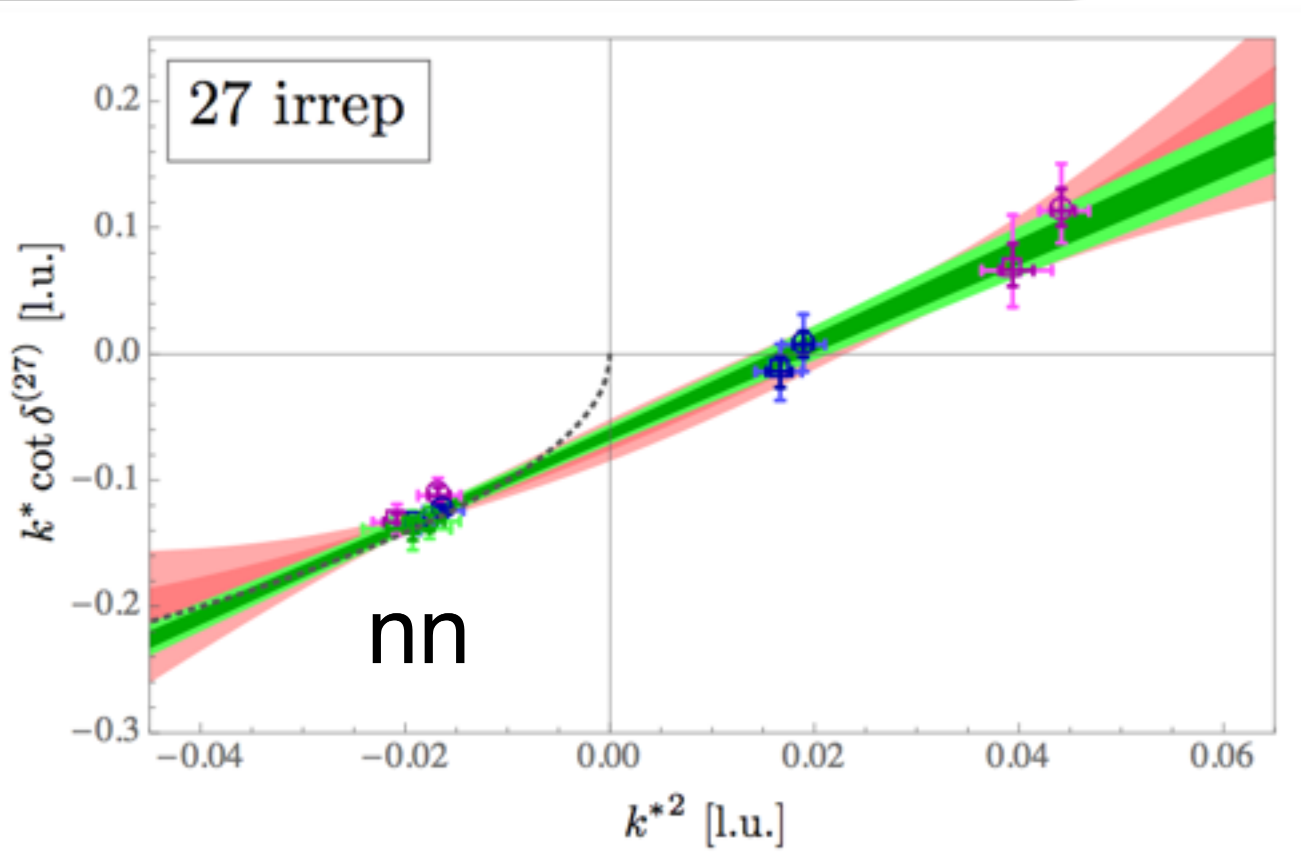
(2012)

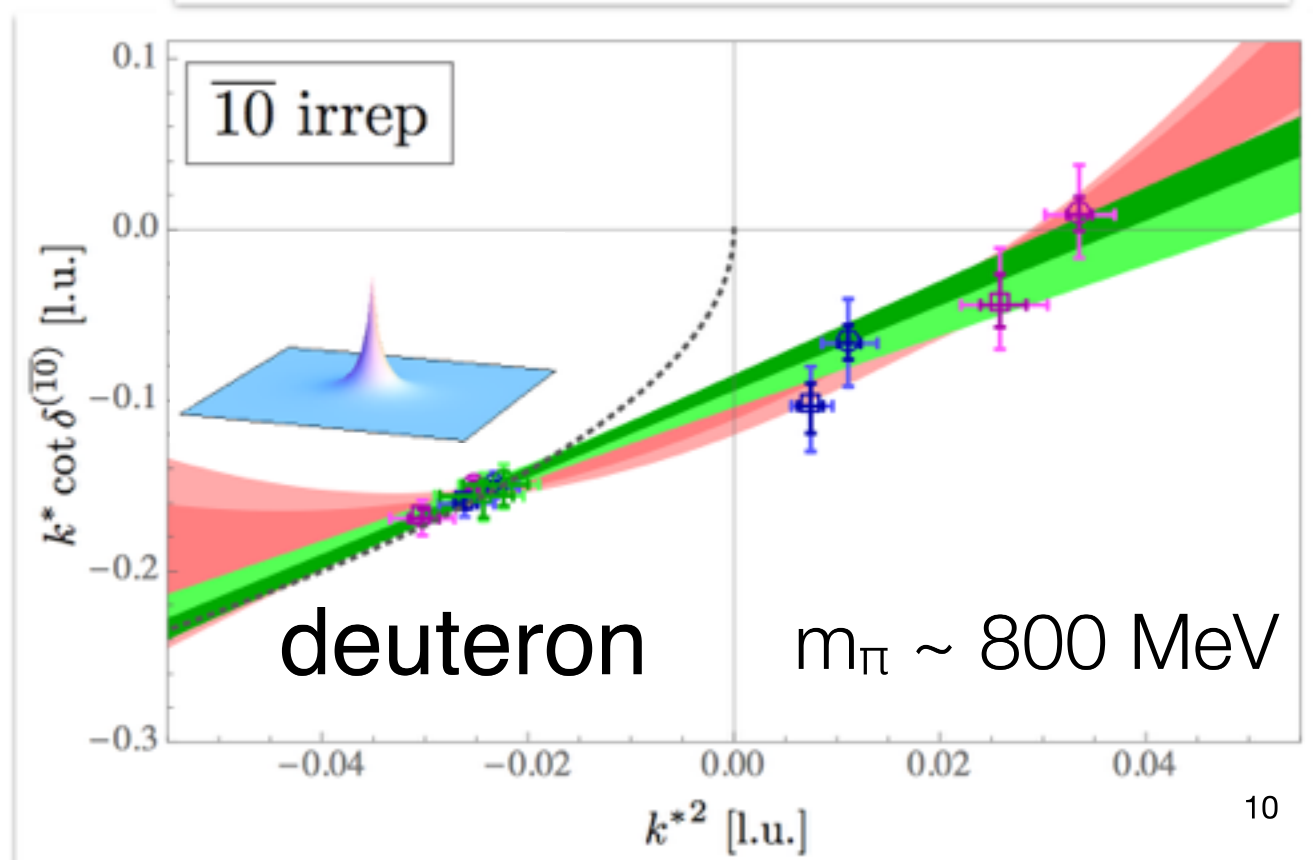


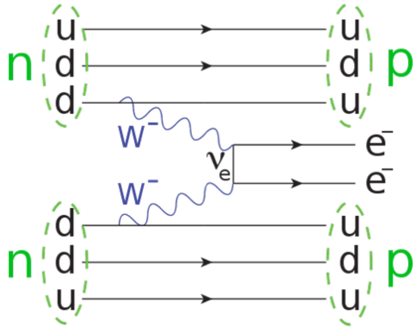


NPLQCD

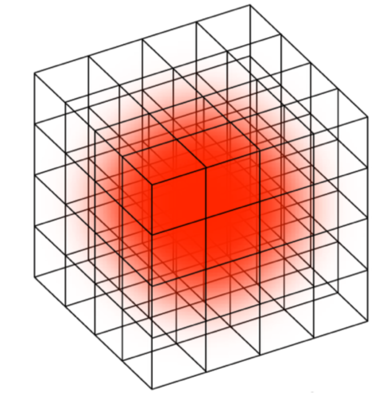


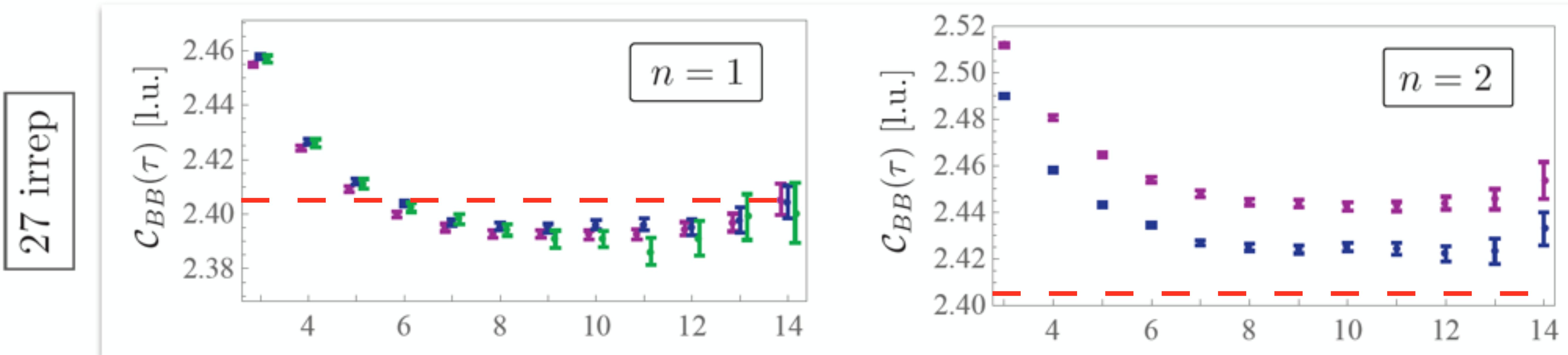






Energy Eigenvalues
A direct output from LQCD

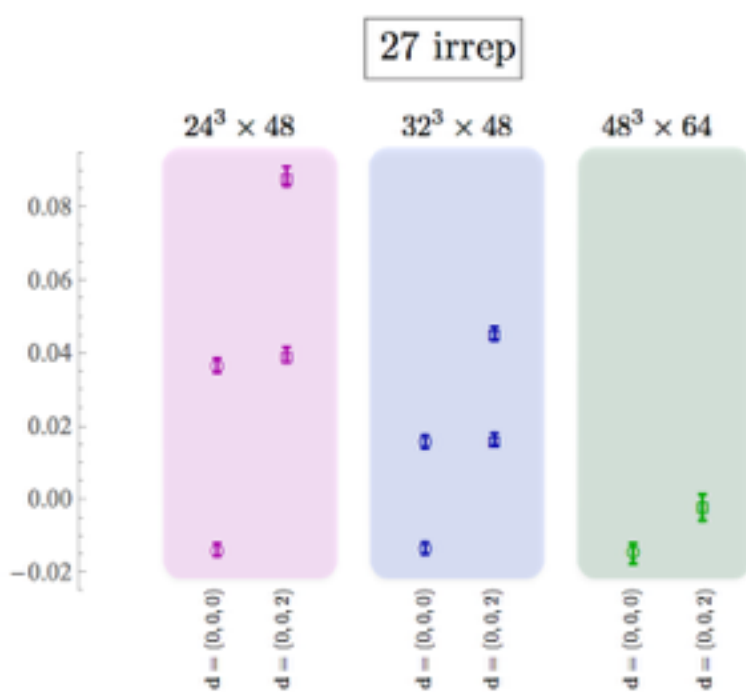


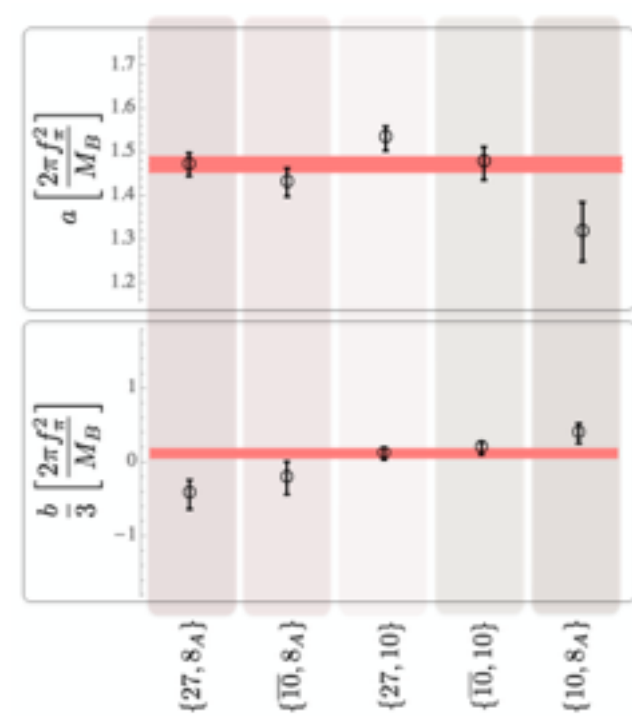


L = 24
L = 32
L = 48

~volume-independent -ve energies
bound state , exp(- kappa L)

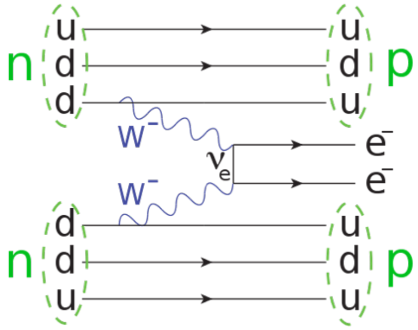
volume-dependent, +ve or small -ve, energies
continuum (scattering) states



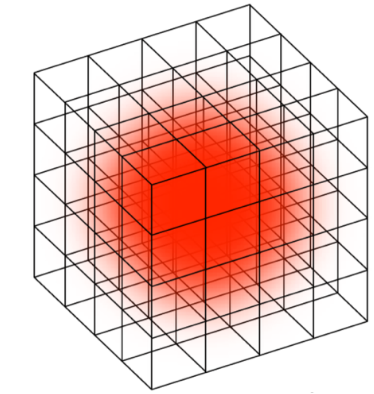


$\kappa L \sim 6$

$\delta_{\rm vol} E \sim 0.2\%$



Exposing Luscher's method for S-D Coupled Channels - 2013



Two-Nucleon Systems in a Finite Volume: (II) $^3S_1$-$^3D_1$ Coupled Channels and the Deuteron

Raúl A. Briceño[a],[1] Zohreh Davoudi[b],[2,3] Thomas Luu[c],[4,5] and Martin J. Savage[d][2,3]

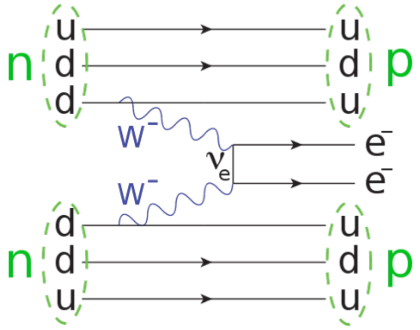
$$\left[\delta\mathcal{G}^V\right]_{JM_J,LS;J'M_J',L'S'} = \frac{iMk^*}{4\pi}\delta_{S1}\delta_{S'1}\left[\delta_{JJ'}\delta_{M_JM_J'}\delta_{LL'} + i\sum_{l,m}\frac{(4\pi)^{3/2}}{k^{*l+1}}c^{\mathbf{d}}_{lm}(k^{*2};\mathrm{L})\right.$$
$$\left.\times\sum_{M_L,M_L',M_S}\langle JM_J|LM_L1M_S\rangle\langle L'M_L'1M_S|J'M_J'\rangle\int d\Omega\; Y^*_{LM_L}Y^*_{lm}Y_{L'M_L'}\right]$$

$$c^{\mathbf{d}}_{lm}(k^{*2};\mathrm{L}) = \frac{\sqrt{4\pi}}{\mathrm{L}^3}\left(\frac{2\pi}{\mathrm{L}}\right)^{l-2}\mathcal{Z}^{\mathbf{d}}_{lm}[1;(k^*\mathrm{L}/2\pi)^2]$$
$$\mathcal{Z}^{\mathbf{d}}_{lm}[s;x^2] = \sum_{\mathbf{n}}\frac{|\mathbf{r}|^l Y_{l,m}(\mathbf{r})}{(r^2-x^2)^s} \quad ,$$

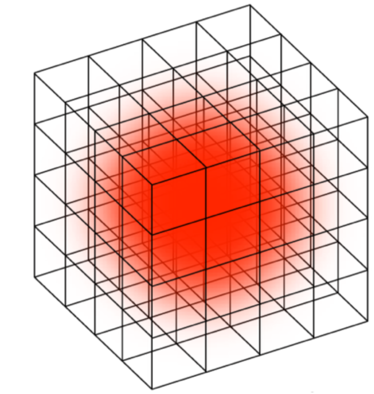
$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{1,S} & \mathcal{M}_{1,SD} & 0 & 0 \\ \mathcal{M}_{1,SD} & \mathcal{M}_{1,D} & 0 & 0 \\ 0 & 0 & \mathcal{M}_{2,D} & 0 \\ 0 & 0 & 0 & \mathcal{M}_{3,D} \end{pmatrix}$$

$$S_{(J=1)} = \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix}\begin{pmatrix} e^{2i\delta_{1\alpha}} & 0 \\ 0 & e^{2i\delta_{1\beta}} \end{pmatrix}\begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix}$$

Luscher Quantization Condition : $\det[\mathcal{M}^{-1} + \delta\mathcal{G}^V] = 0$

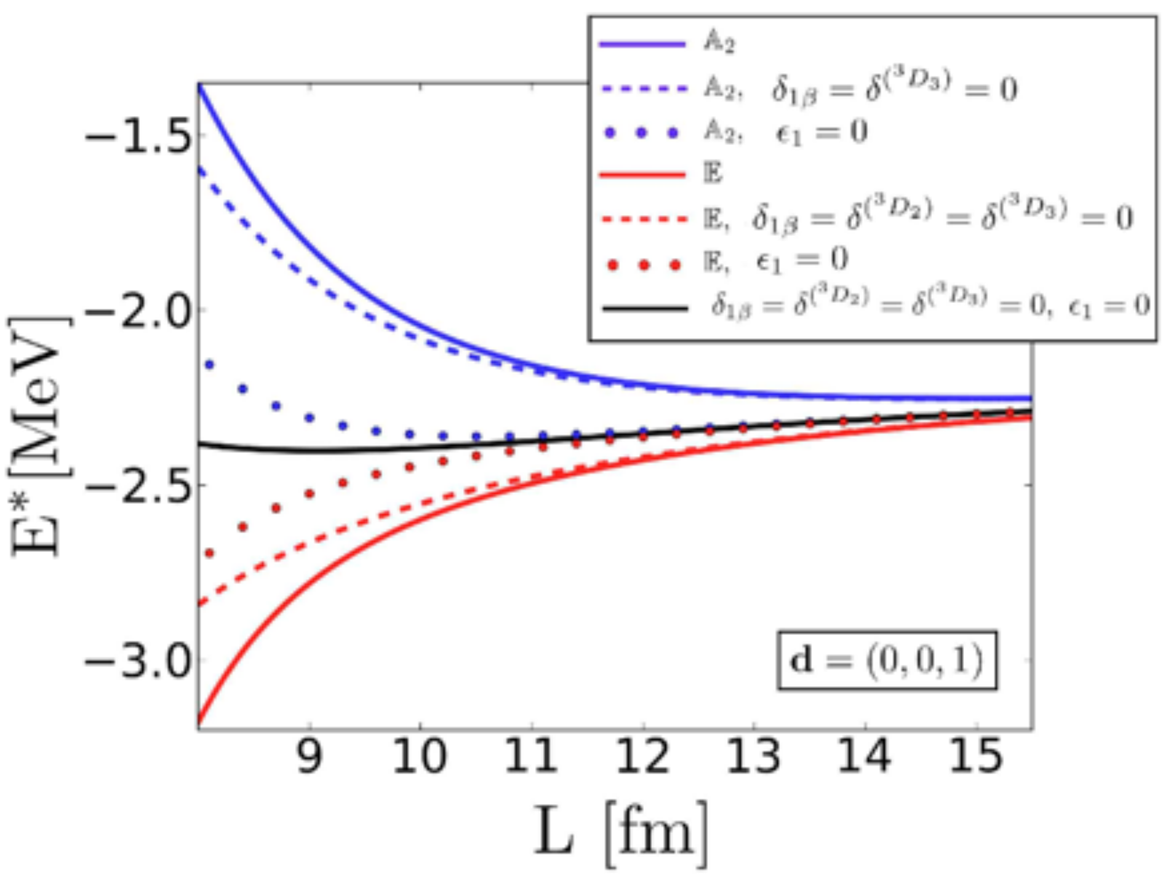


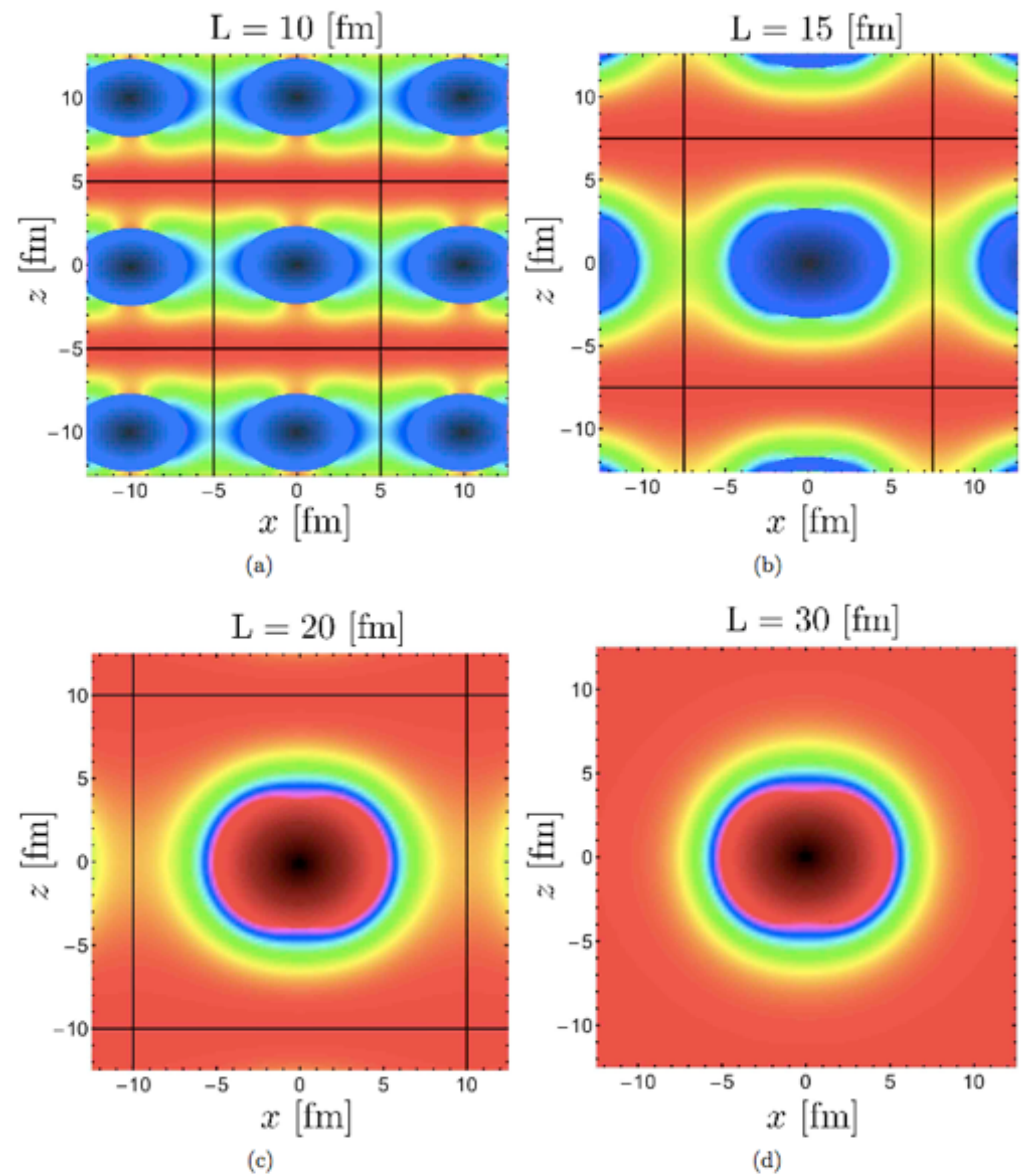
Exposing Luscher's method for S-D Coupled Channels - 2013

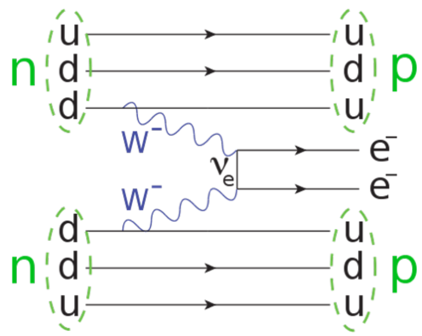


Two-nucleon systems in a finite volume. II.
3S1−3D1coupled channels and the deuteron
Raul A. Briceño, Zohreh Davoudi, Thomas Luu, Martin J. Savage,
Phys.Rev. D88 (2013) no.11, 114507

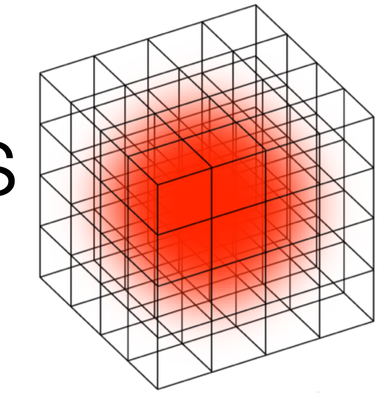
Mass density of boosted deuteron d=(0,0,1)

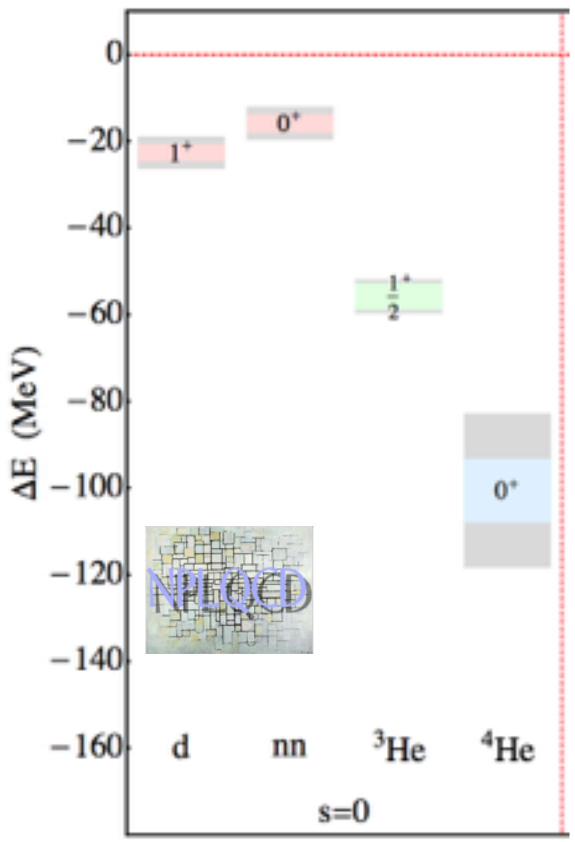


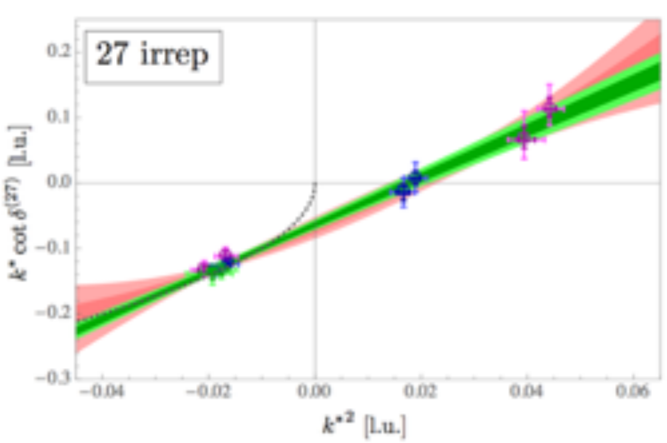


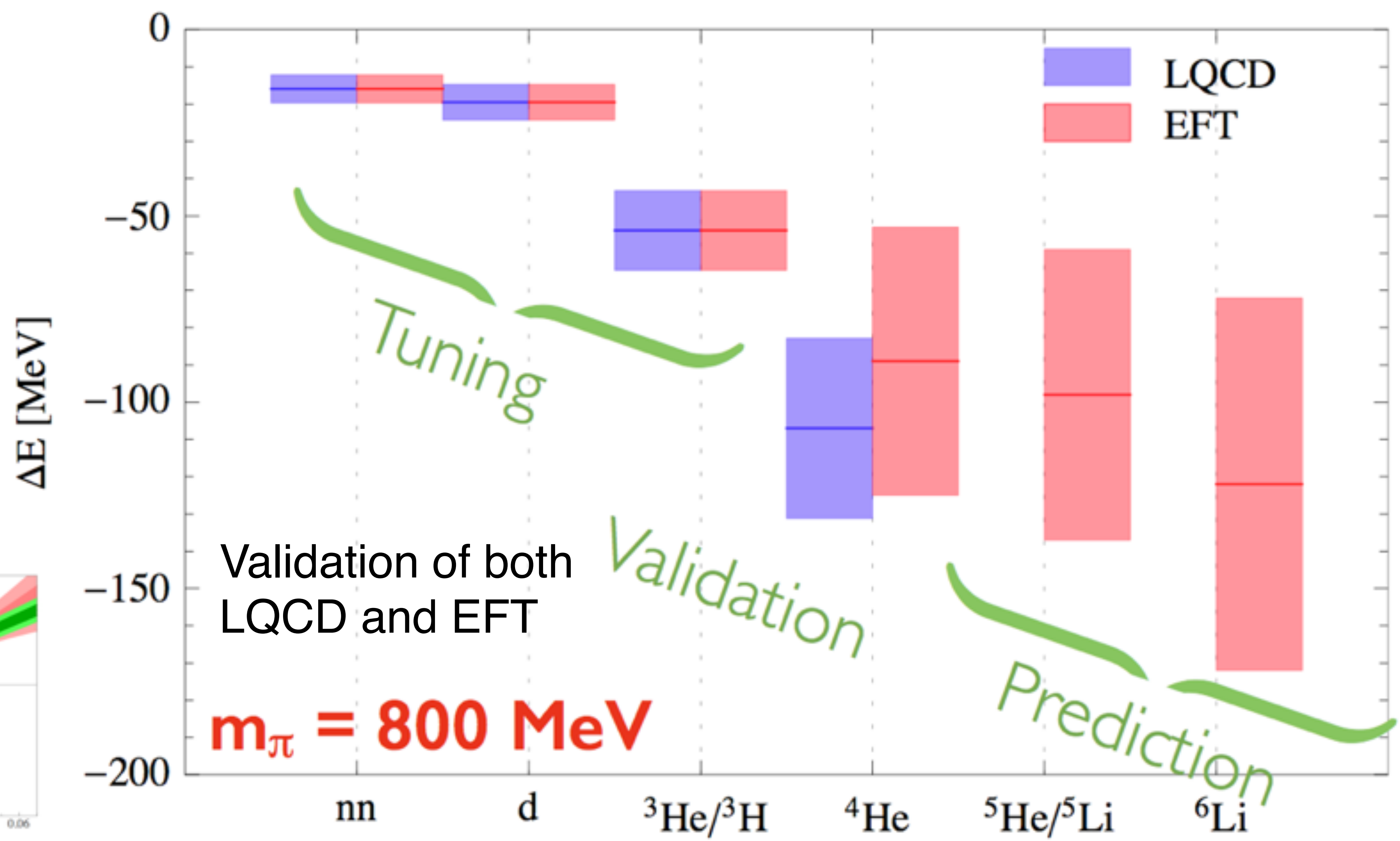


Predictions Beyond the LQCD calculations
First Realization of the Dream !!



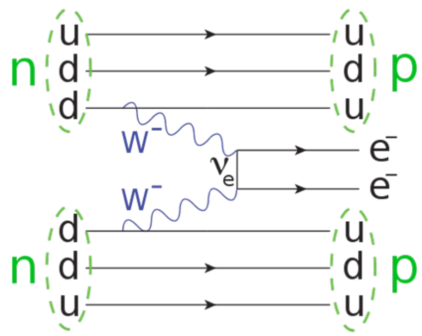






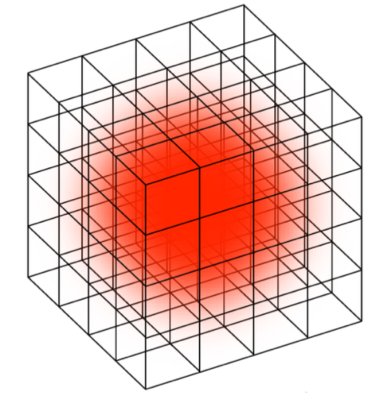
Effective Field Theory for Lattice Nuclei
N. Barnea et al, Phys.Rev.Lett. 114 (2015) no.5, 052501

Ground-State Properties of 4He and 16 O Extrapolated from Lattice QCD with Pionless EFT
L. Contessi et al, e-Print: arXiv:1701.06516

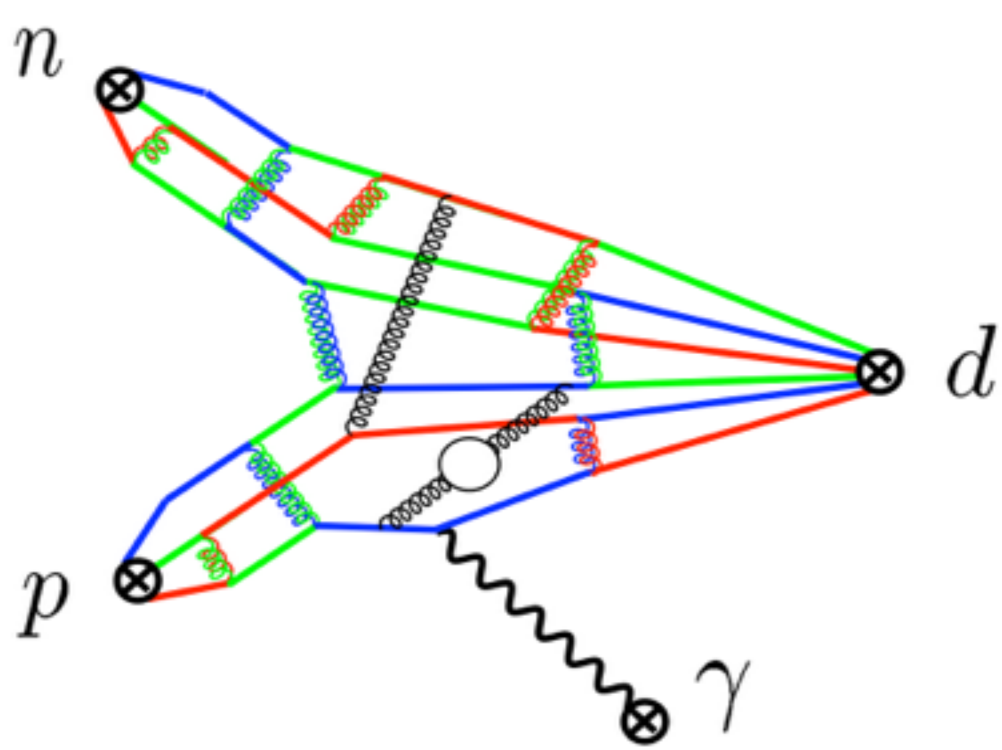


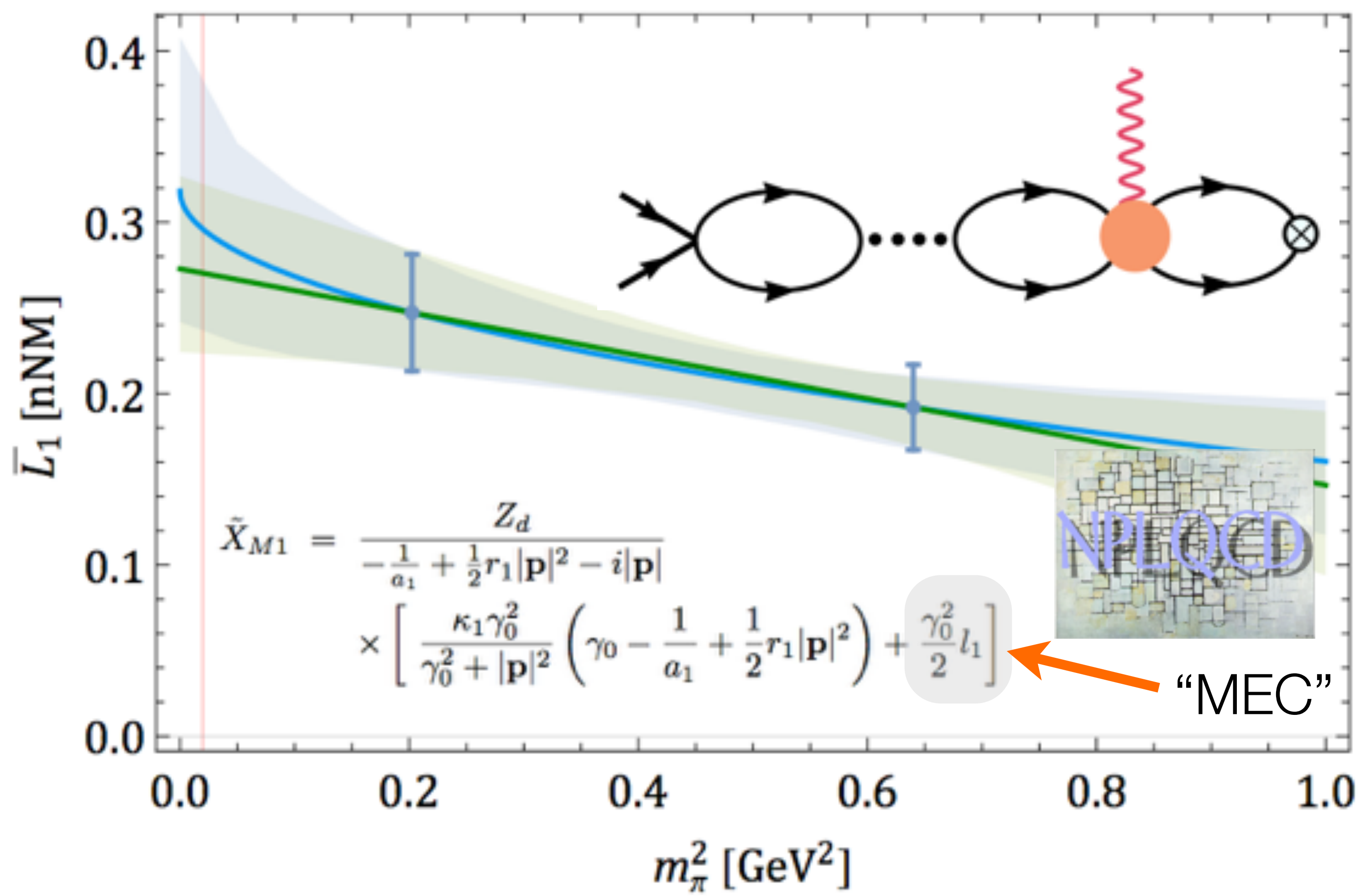
Radiative Capture :

$$np \rightarrow d\gamma$$



Ab Initio Calculation of the np → dγ Radiative Capture Process
Beane et al., NPLQCD, Phys. Rev. Lett. 115 (2015) 13, 132001



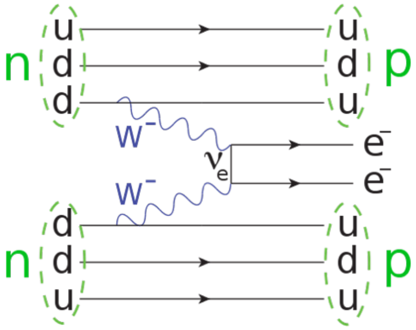


physical point:

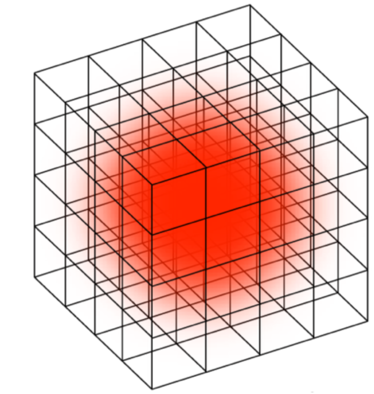
σ^{LQCD} = 334.9(5.3) mb $\quad v = 2,200$ m/s

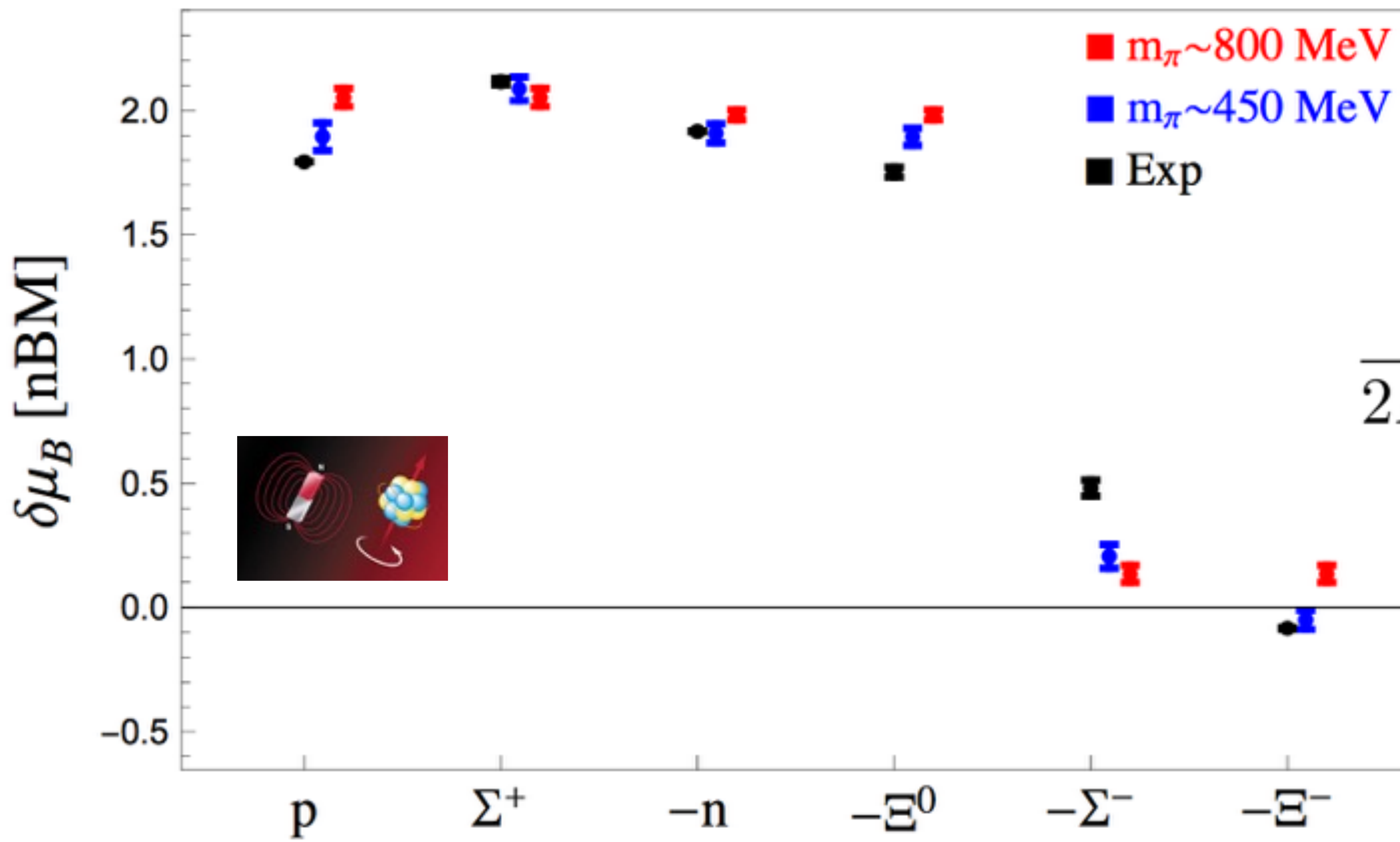
σ^{expt} = 334.2(0.5) mb

[306 mb single nucleons alone]

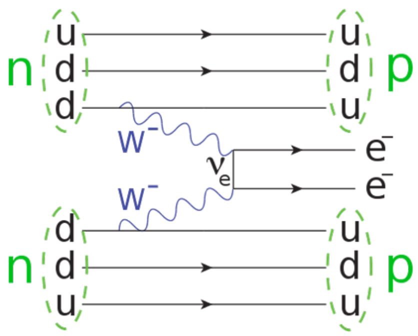


Features of Nucleons/Baryons

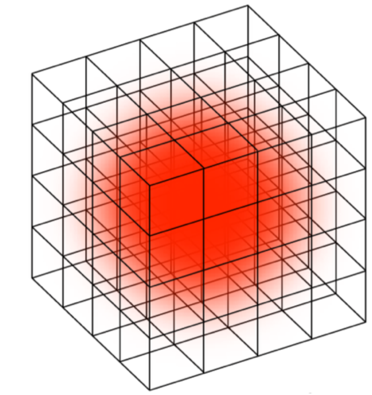




$$\frac{e}{2M(m_\pi)}$$



Axial Interactions

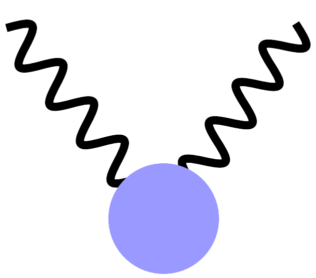


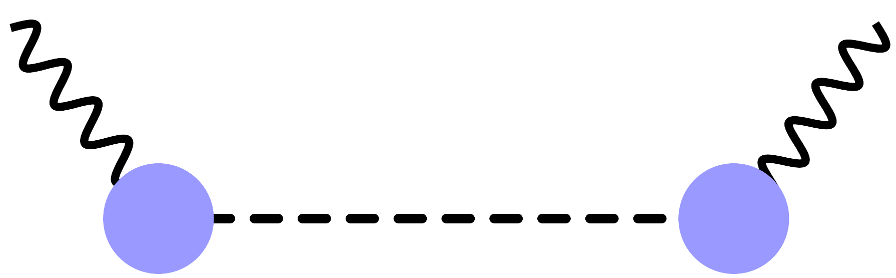
LO ChiPT

vacuum matrix elements - long range part of nuclear force

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left[\, D_\mu \Sigma D^\mu \Sigma^\dagger \right] + \lambda \mathrm{Tr}\left[\, m_q \Sigma^\dagger + m_q^\dagger \Sigma \,\right]$$

$$D_\mu = \partial_\mu + i\left\{a_\mu, \Sigma\right\}$$





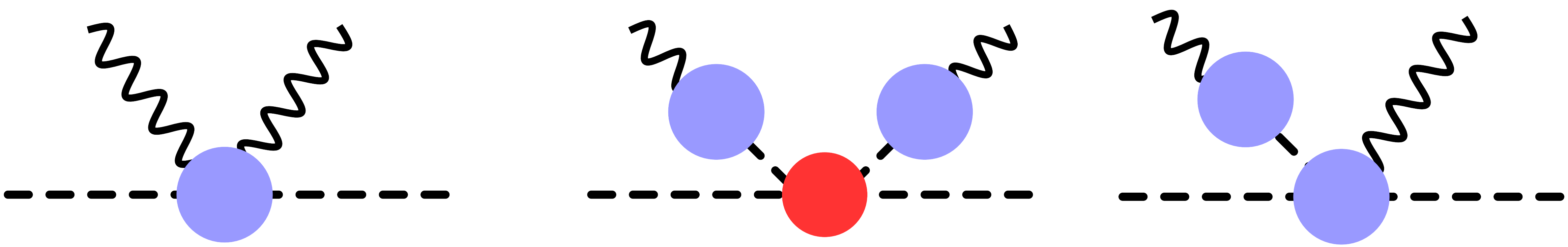
$$\Gamma^{\mu\nu} = i\frac{f^2}{2}\delta^{ab}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2 - m_\pi^2}\right]$$

Central Force and Tensor Force

$$q_\mu \Gamma^{\mu\nu} \sim m_\pi^2$$

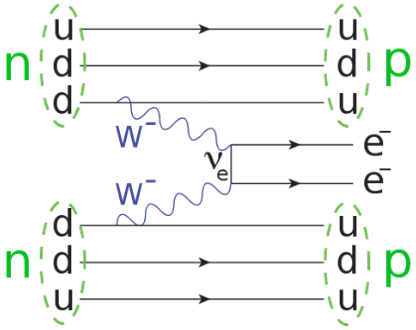
LO ChiPT

single pion matrix elements

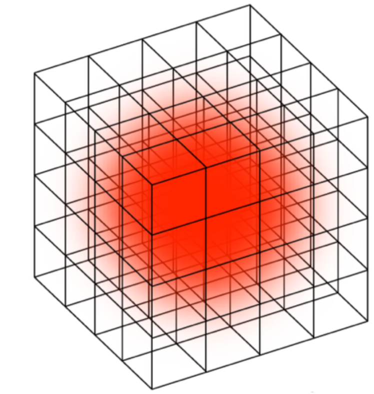


$g_{\mu\nu}$

q_μ



Double Beta Decay and The Isotensor Polarizability



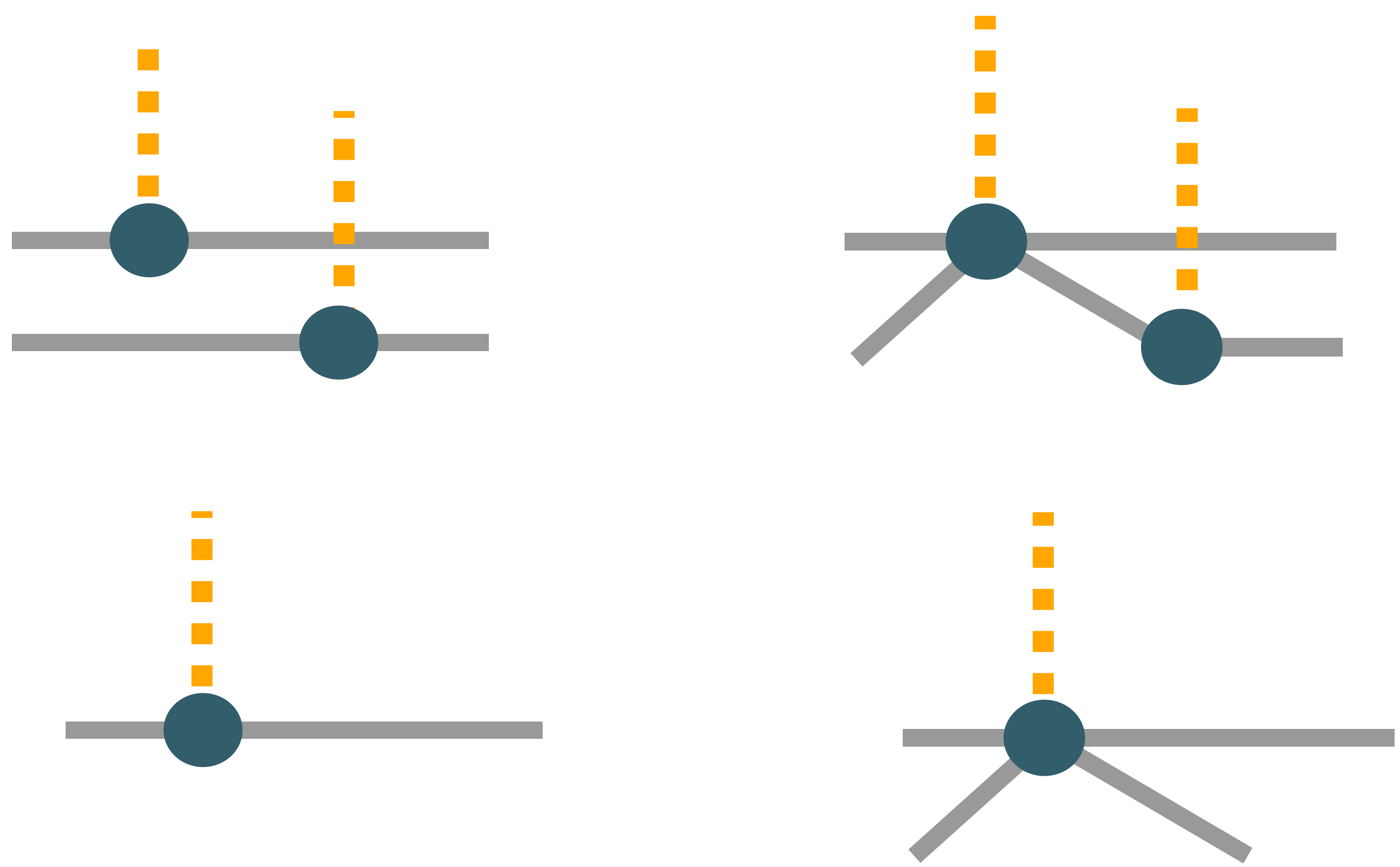
Double-ββ-Decay Matrix Elements from Lattice Quantum Chromodynamics
Brian C. Tiburzi, Michael L. Wagman, Frank Winter, Emmanuel Chang, Zohreh Davoudi, William Detmold, Kostas Orginos, Martin J. Savage, Phiala E. Shanahan.
arXiv:1702.02929 [hep-lat]

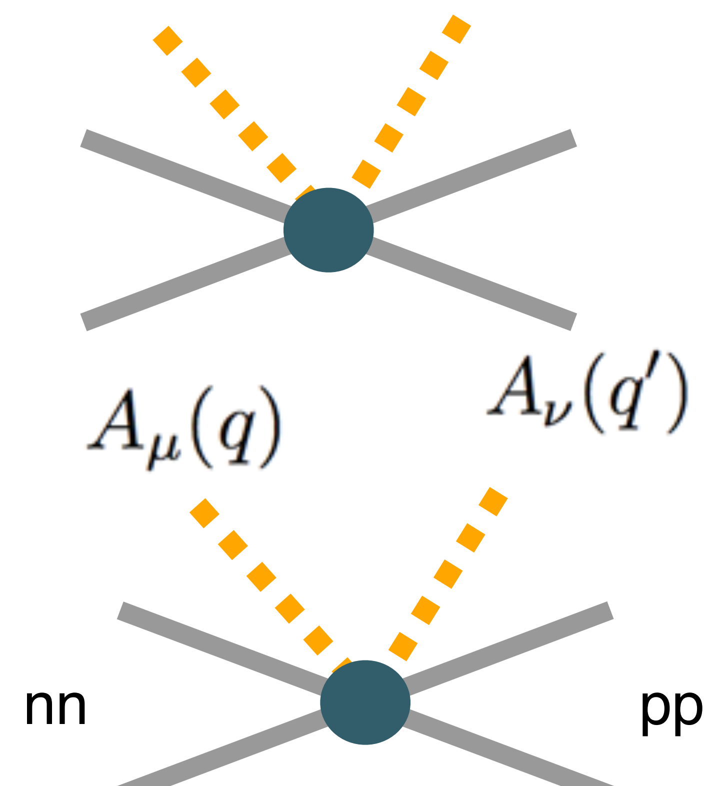
The isotensor axial polarisability and lattice QCD input for nuclear double-ββ-decay phenomenology
Phiala E. Shanahan, Brian C. Tiburzi, Michael L. Wagman, Frank Winter, Emmanuel Chang, Zohreh Davoudi, William Detmold, Kostas Orginos, Martin J. Savage,
arXiv:1701.03456 [hep-lat]

Proton-proton fusion and tritium β-decay from lattice quantum chromodynamics
Martin J. Savage, Phiala E. Shanahan, Brian C. Tiburzi, Michael L. Wagman, Frank Winter, Silas R. Beane, Emmanuel Chang, Zohreh Davoudi, William Detmold, Kostas Orginos, arXiv:1610.04545

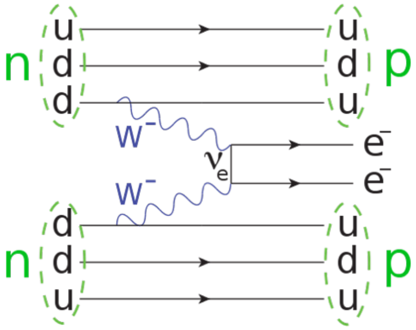
$$nn \to pp$$

[Weinberg power-counting fails in this channel while KSW power-counting converges]

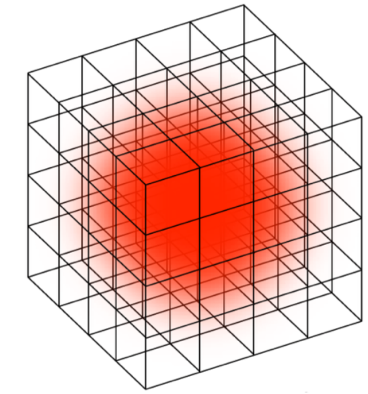


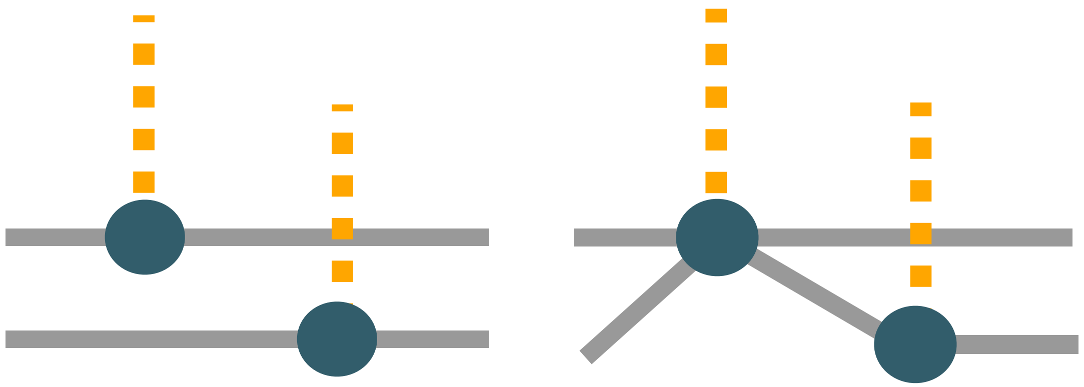


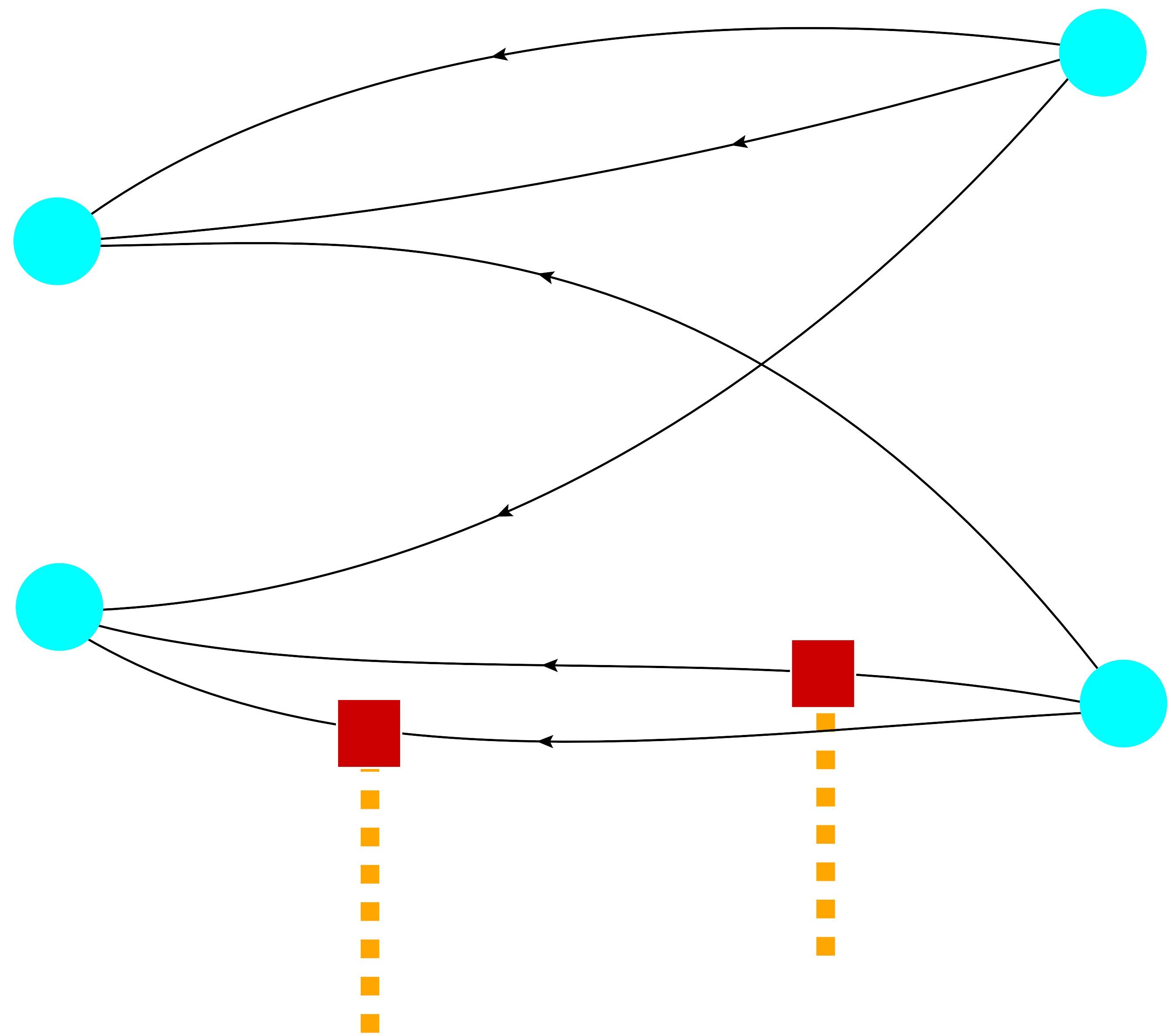
isotensor polarizability

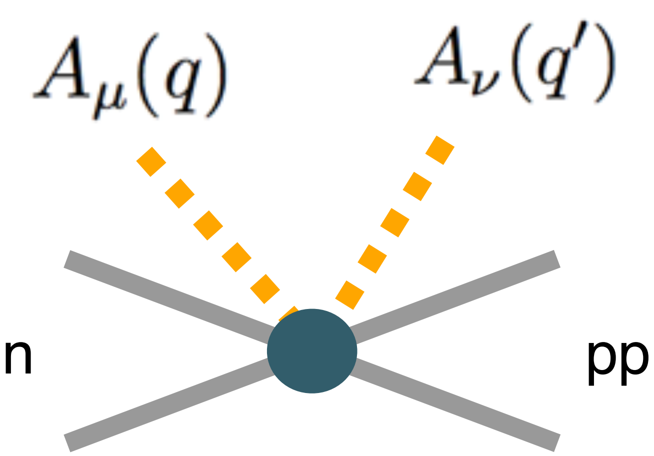


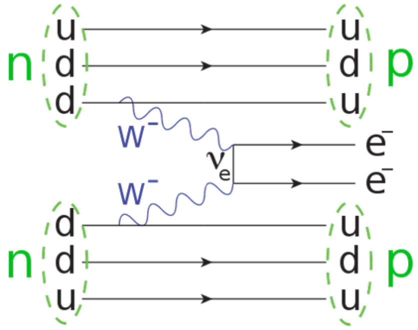
Double Beta Decay and The Isotensor Polarizability



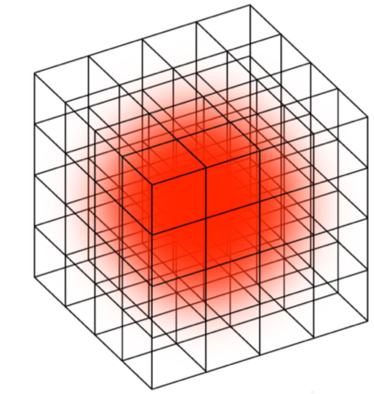






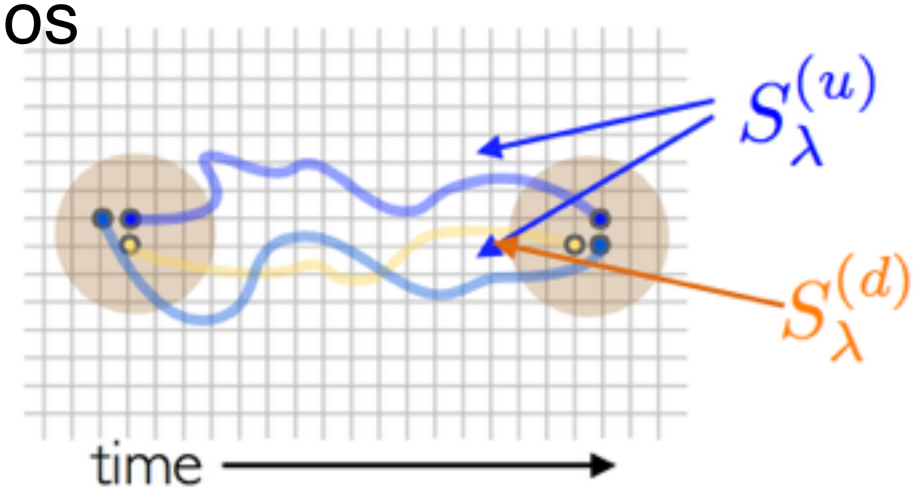


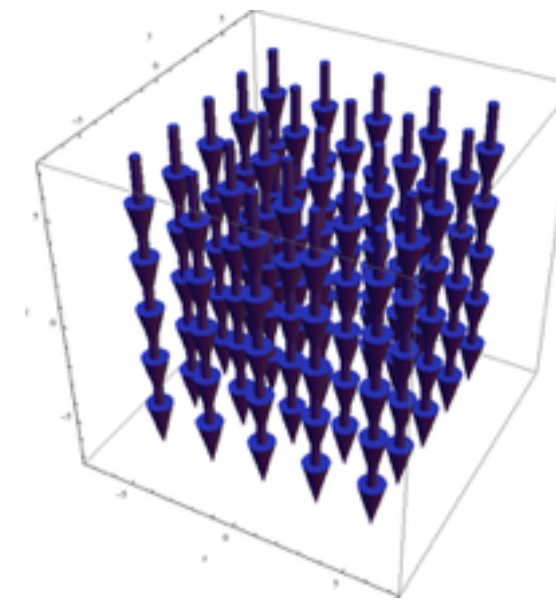
Axial-Current Matrix Elements



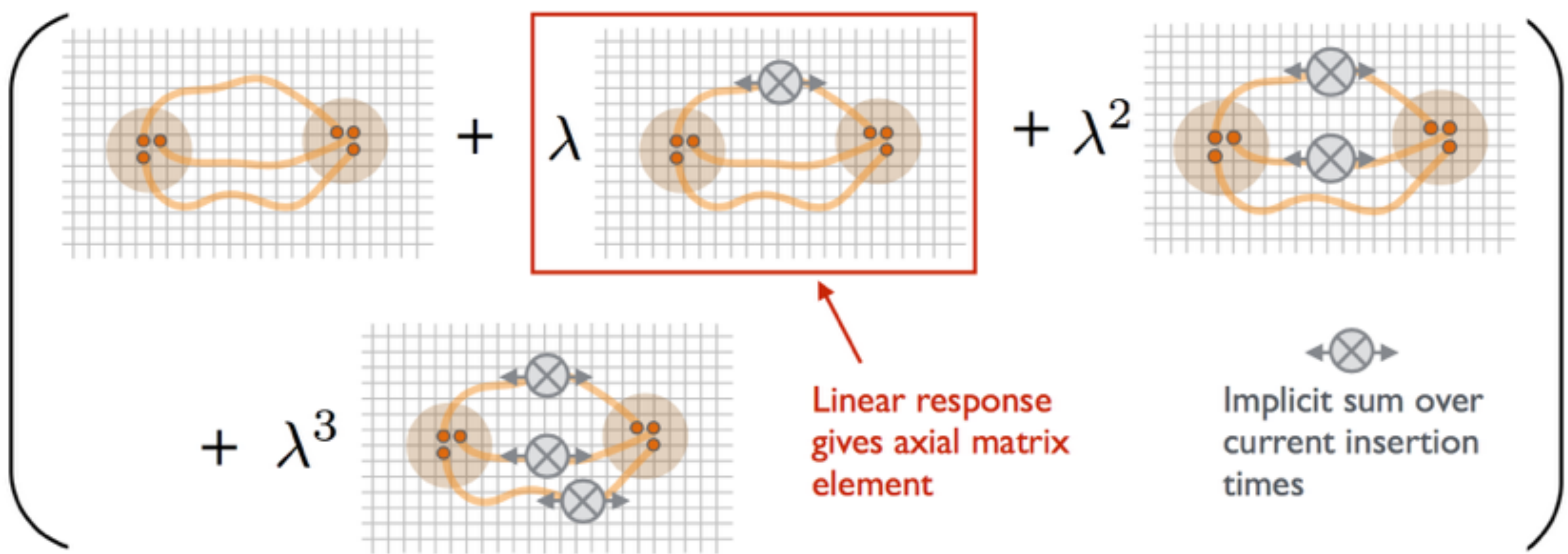
New algorithm by Kostas Orginos

$$C_{\lambda_u;\lambda_d}(t) =$$



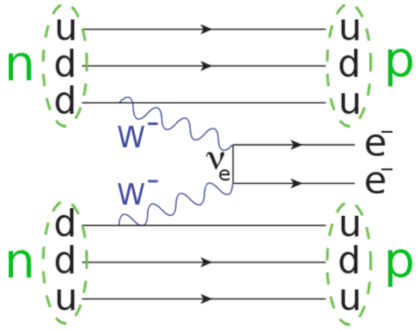


$$C_{\lambda_u;\lambda_d}(t) =$$

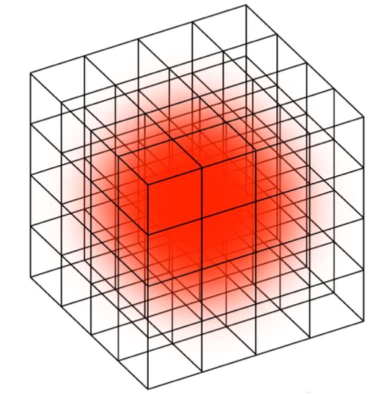


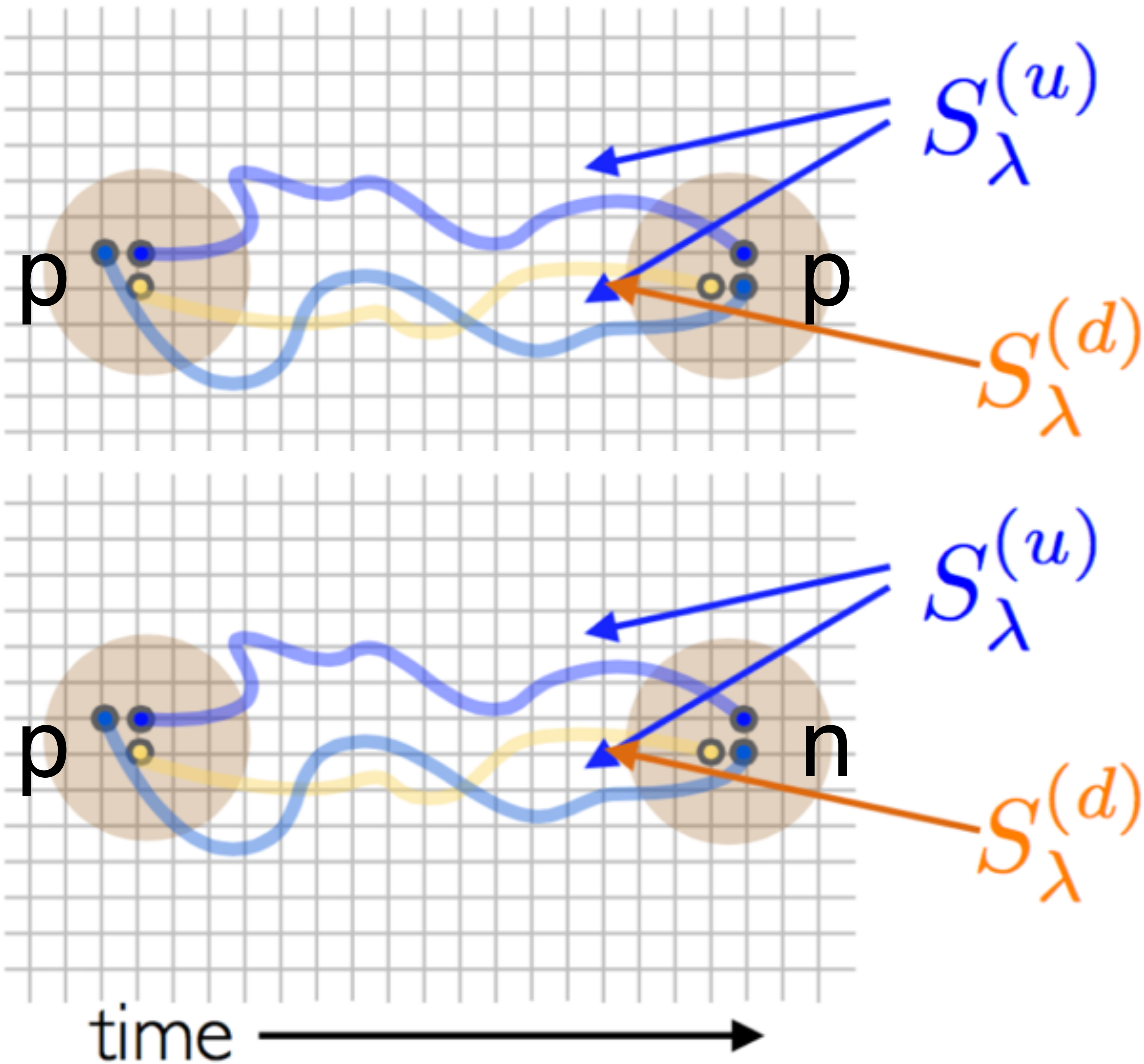
$$S_{\{\Lambda_1,\Lambda_2,\ldots\}}(x,y) = S(x,y) + \int dz\, S(x,z)\Lambda_1(z)S(z,y)$$

slide created by Phiala Shanahan



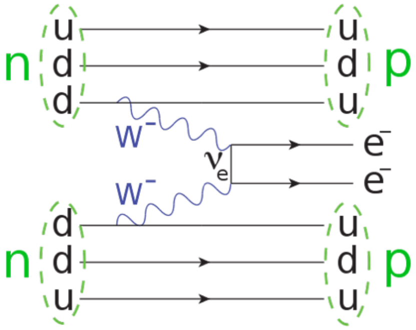
Axial-Current Matrix Elements
pp fusion



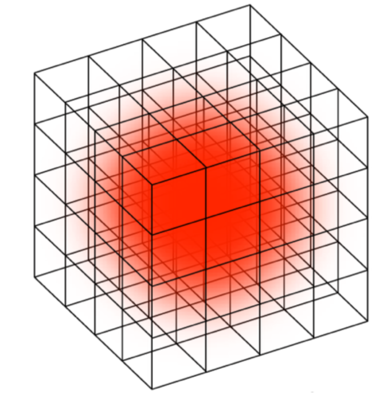


pp -> np

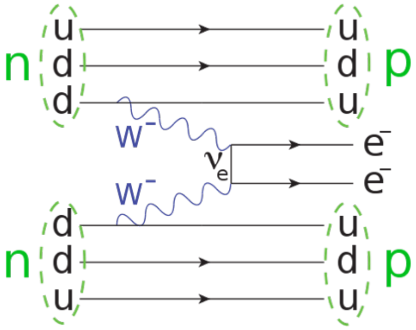
correlation function linear in A(x)



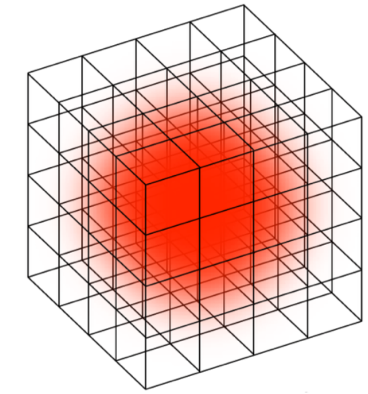
Axial-Current Matrix Elements
pp fusion

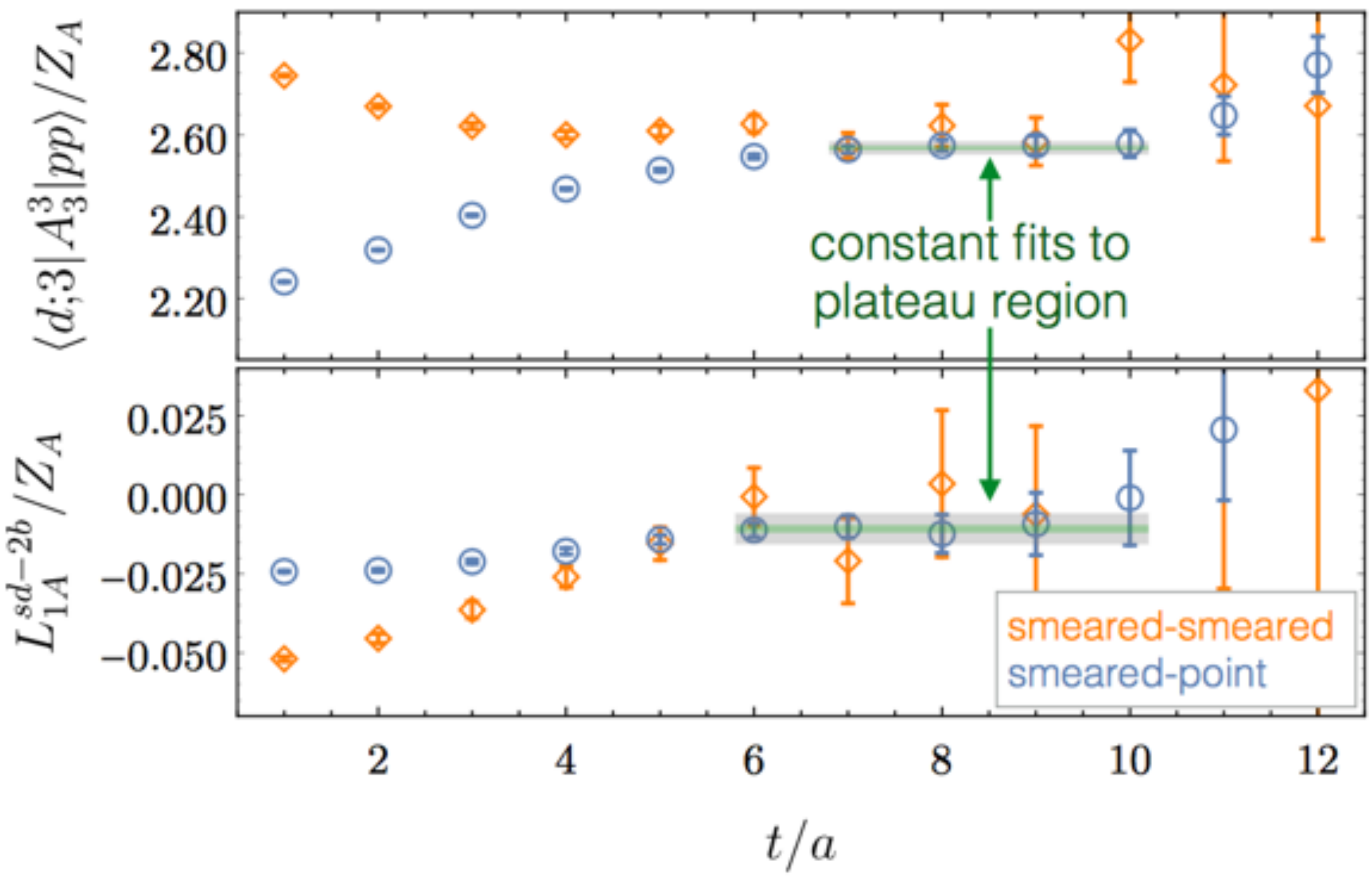


$$
\begin{aligned}
C^{({}^3S_1,{}^1S_0)}_{\lambda_u;\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} &= \sum_{t_1=0}^{t}\sum_{\mathfrak{n},\mathfrak{m}} Z_{\mathfrak{n}'} Z^{\dagger}_{\mathfrak{m}} e^{-E_{\mathfrak{n}'}(t-t_1)} e^{-E_{\mathfrak{m}} t_1} \langle \mathfrak{n}' | \tilde{J}_3^{(u)} | \mathfrak{m} \rangle \\
&= \sum_{\mathfrak{n},\mathfrak{m}} Z_{\mathfrak{n}'} Z^{\dagger}_{\mathfrak{m}} \frac{e^{-E_{\mathfrak{n}'} t} - e^{-E_{\mathfrak{m}} t}}{aE_{\mathfrak{m}} - aE_{\mathfrak{n}'}} \langle \mathfrak{n}' | \tilde{J}_3^{(u)} | \mathfrak{m} \rangle ,
\end{aligned}
$$



Axial-Current Matrix Elements
pp fusion



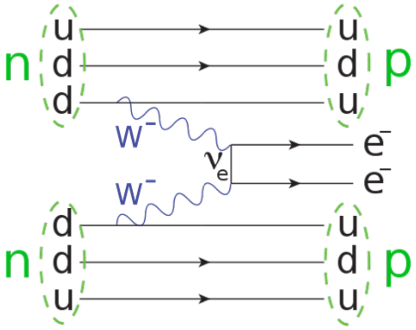


EFT parameter dictating fusion rate

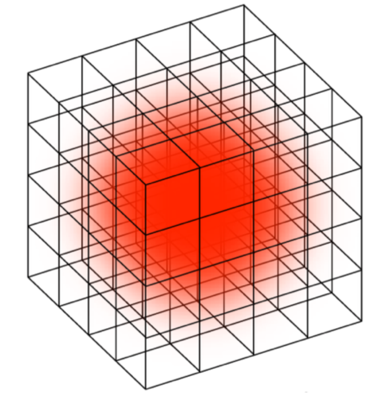
$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.011(1)(15)$$

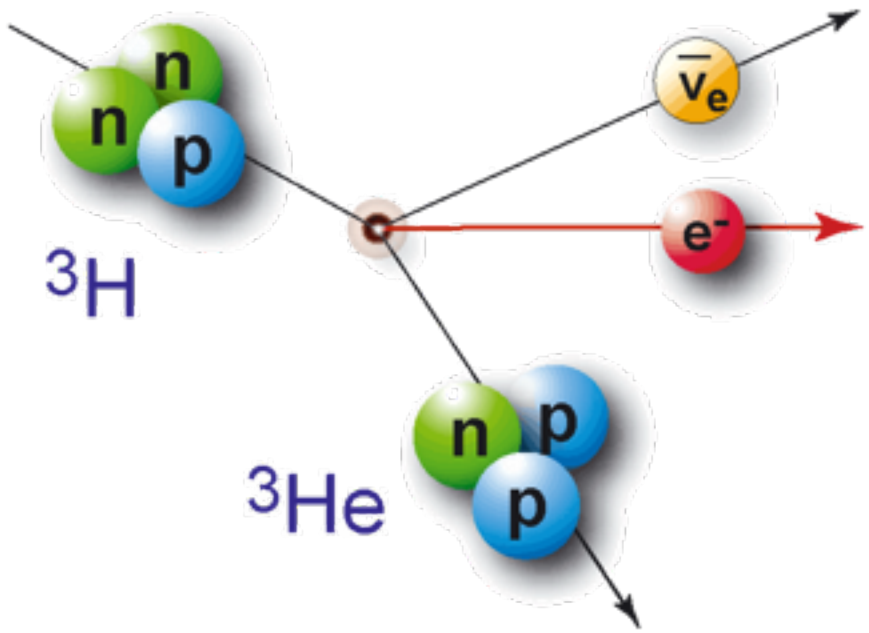
Extrapolate, predict physical cross-section

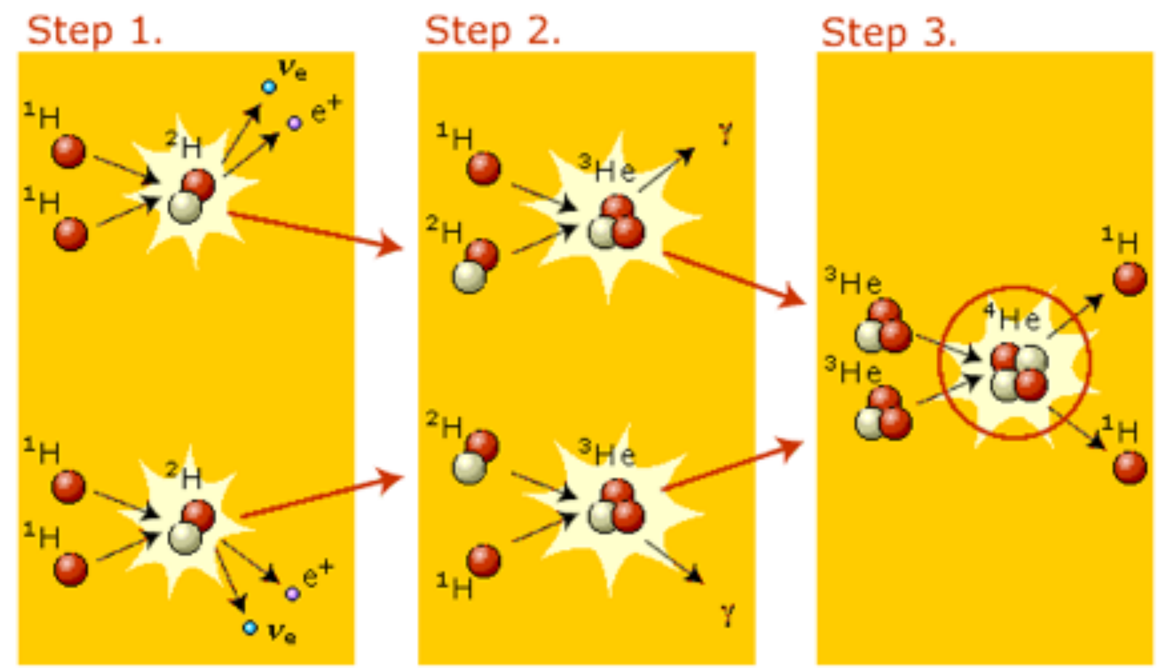
slide created by Phiala Shanahan



Proton-Proton Fusion and Tritium β-Decay



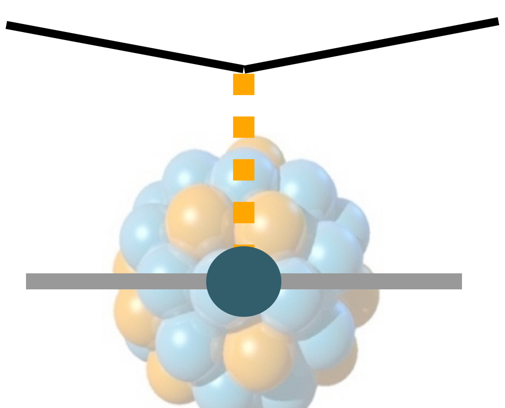


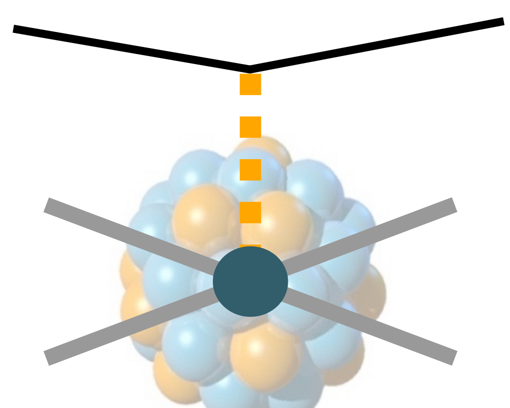


$$\delta\mathcal{L} = -gW\,\frac{g_A}{2}\,N^\dagger\sigma^z\tau^3N - \frac{gW\,l_{1,A}}{2M\sqrt{r_1r_3}}\left[t_3^\dagger s_3 + \text{h.c.}\right] + \ldots$$

single-nucleon interaction

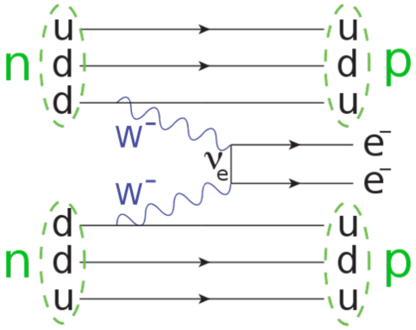
leading two-nucleon interaction



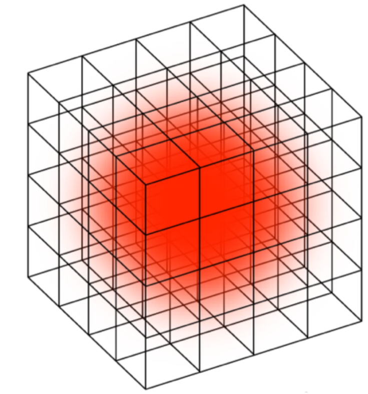


$$\frac{g_A(^3\text{H})}{Z_A} = 1.272(6)(22), \quad \frac{g_A(^3\text{H})}{g_A} = 0.979(3)(10)$$

$\Lambda(0) = 2.6585(06)(72)(25)$ and $L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \text{fm}^3$

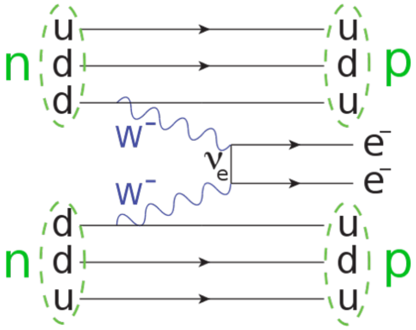


Double Beta Decay and The Isotensor Polarizability

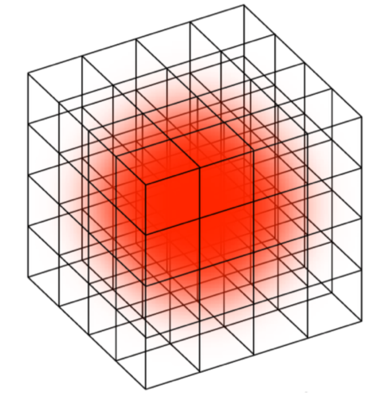


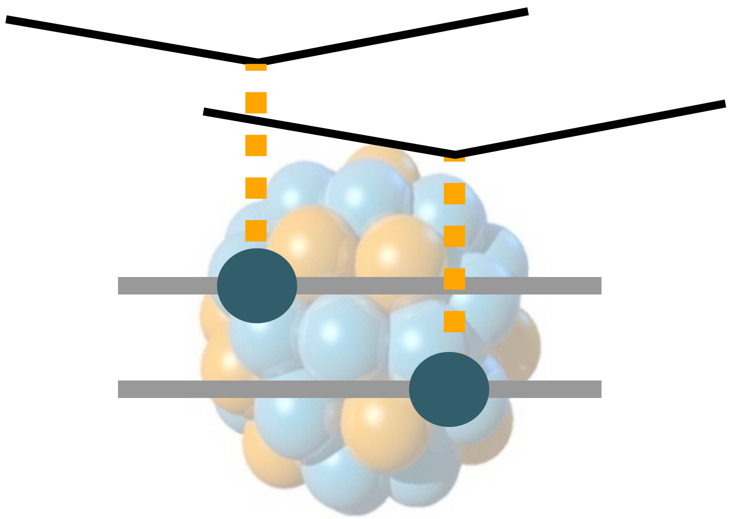
$$C_{nn\to pp}(t) = \sum_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \sum_{t_1=0}^{t} \sum_{t_2=0}^{t} \langle 0|\chi_{pp}(\boldsymbol{x},t) T\left\{J_3^+(\boldsymbol{y},t_1) J_3^+(\boldsymbol{z},t_2)\right\} \chi_{nn}^\dagger(0)|0\rangle$$

$$= \frac{2}{a^2} \sum_{\mathfrak{n},\mathfrak{m},\mathfrak{l}'} Z_{\mathfrak{n}} Z_{\mathfrak{m}}^\dagger e^{-E_{\mathfrak{n}} t} \frac{\langle \mathfrak{n}|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|\mathfrak{m}\rangle}{E_{\mathfrak{l}'} - E_{\mathfrak{m}}} \left(\frac{e^{-(E_{\mathfrak{l}'} - E_{\mathfrak{n}})t} - 1}{E_{\mathfrak{l}'} - E_{\mathfrak{n}}} + \frac{e^{(E_{\mathfrak{n}} - E_{\mathfrak{m}})t} - 1}{E_{\mathfrak{n}} - E_{\mathfrak{m}}} \right)$$

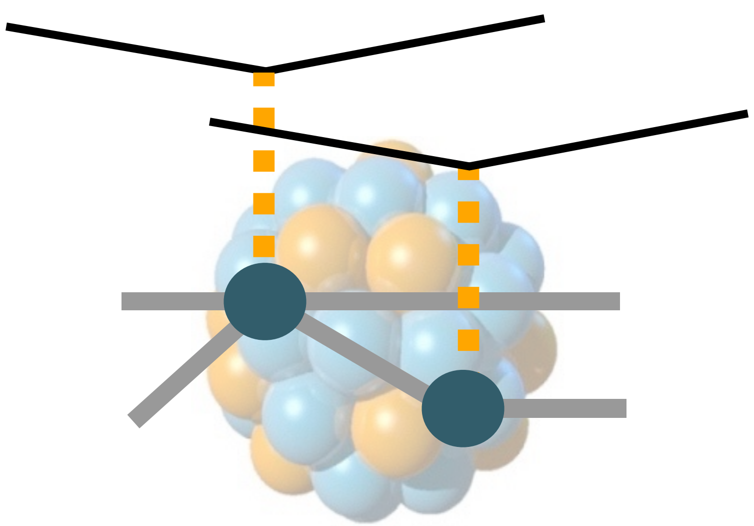


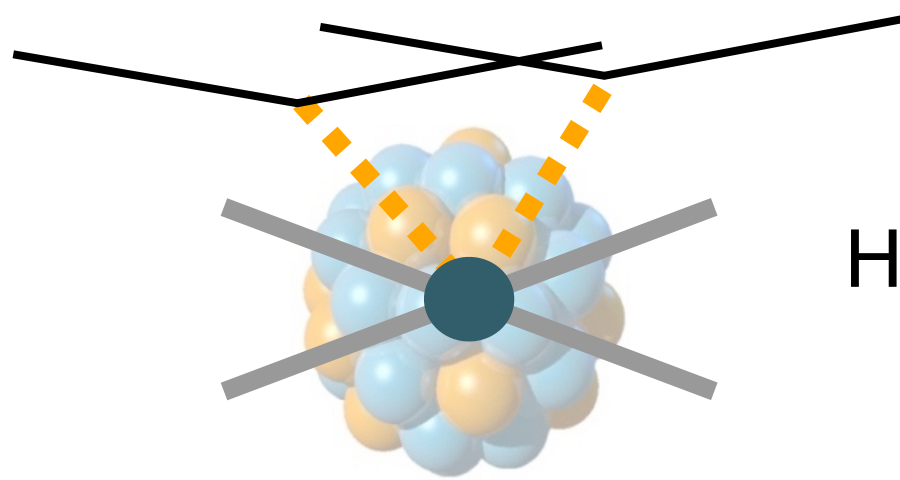
Double Beta Decay and The Isotensor Polarizability





two single-nucleon interactions
with one axial current

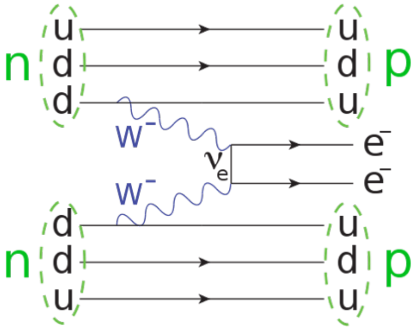




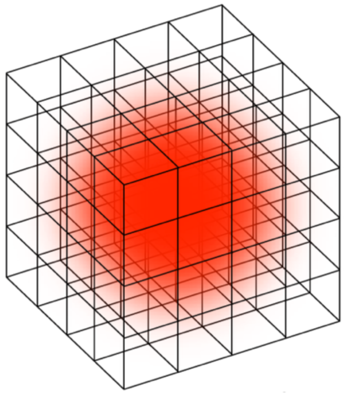
$H_{2,S}$ = 4.7(1.3)(1.8) fm
Lattice QCD, 800MeV

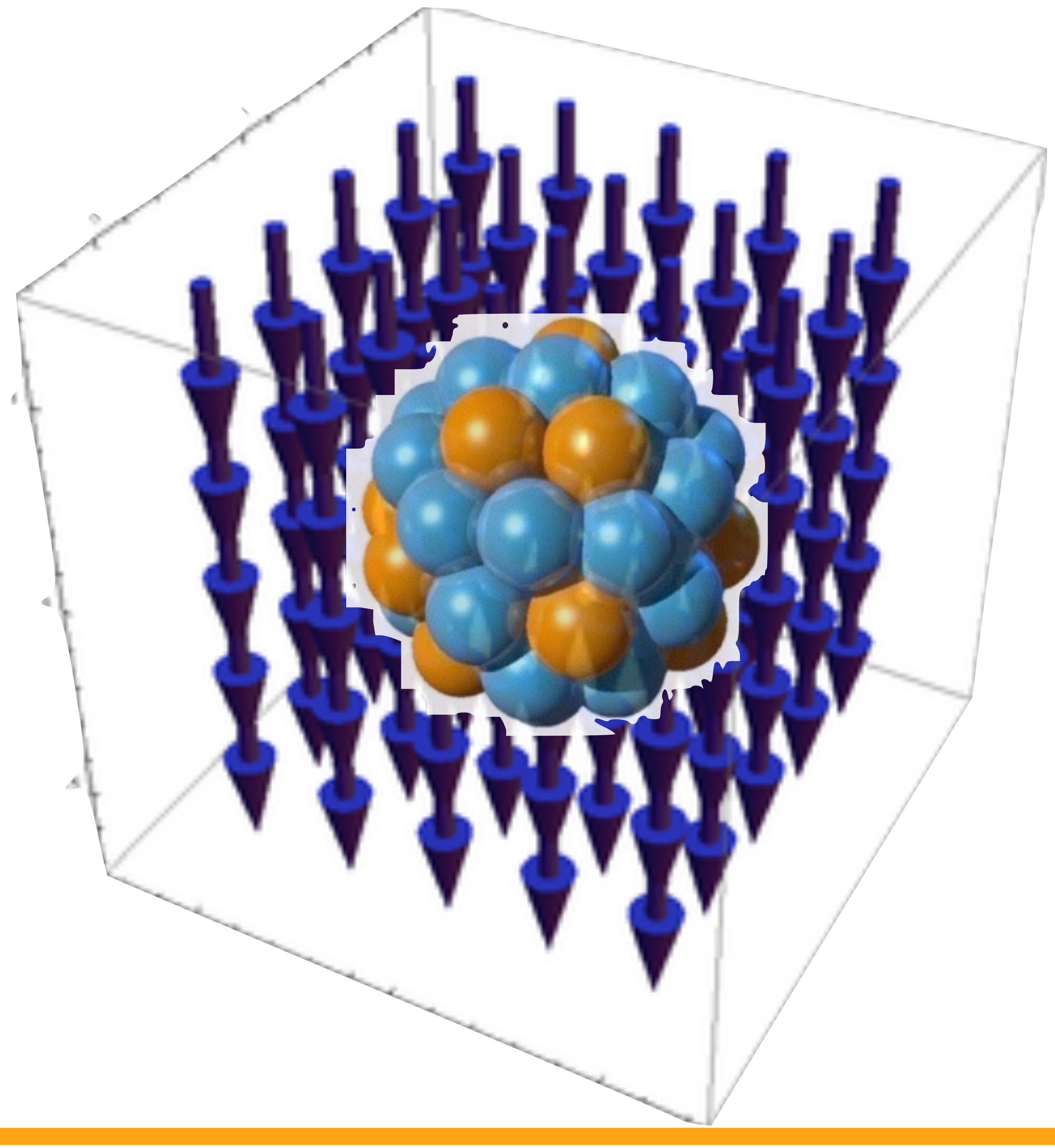
correlated two-nucleon interaction
with one axial current

correlated two-nucleon interaction
with two axial currents

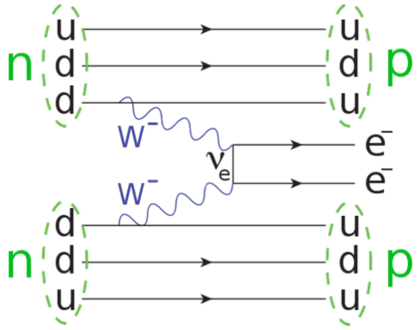


To Do :
Many-Body Calculations

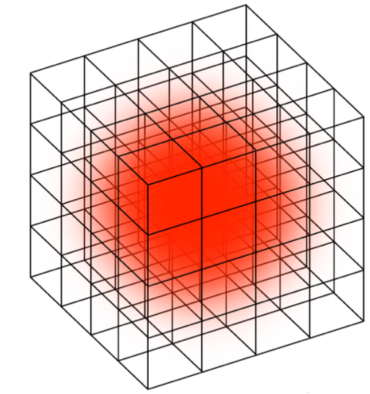




Matching Observables >> Matching to EFT >> Matching to Model

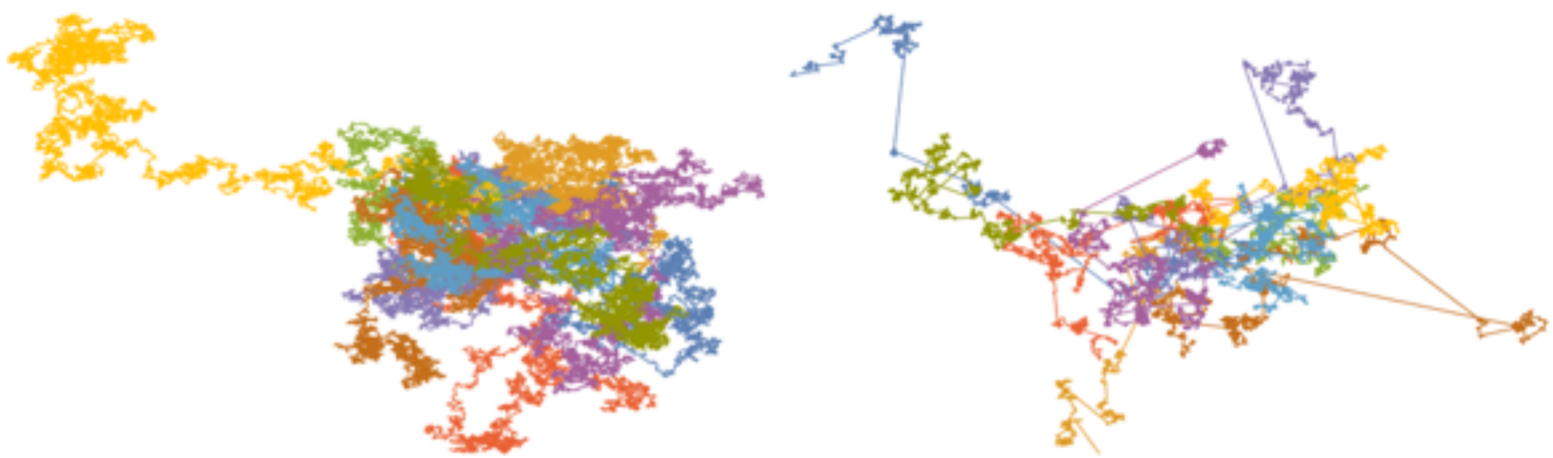


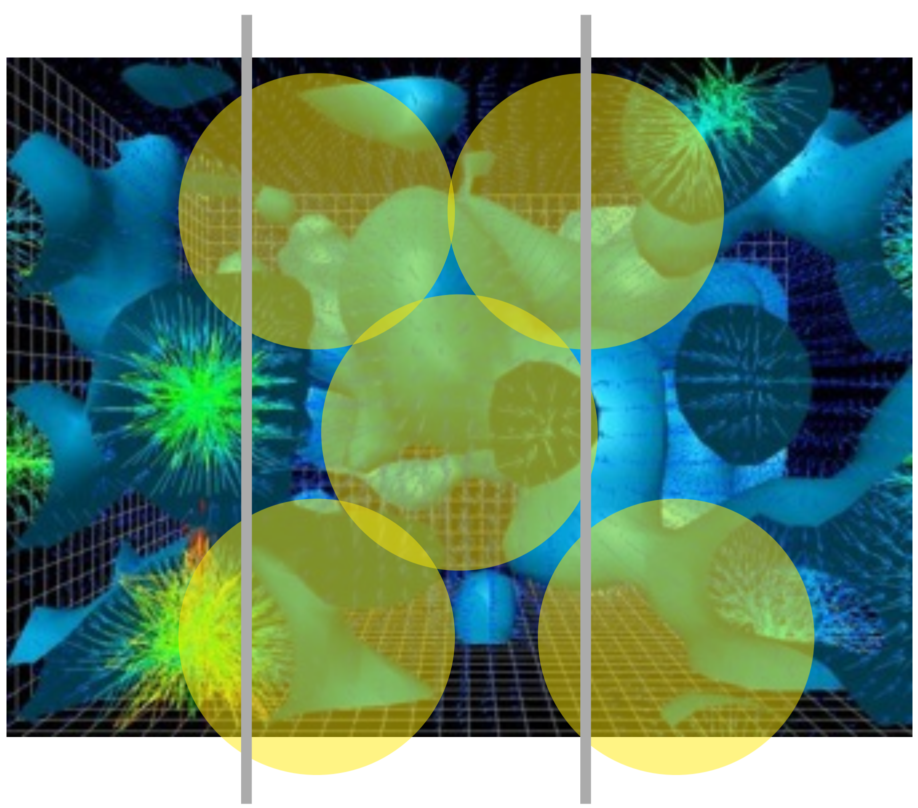
Statistics and the Sign Problem

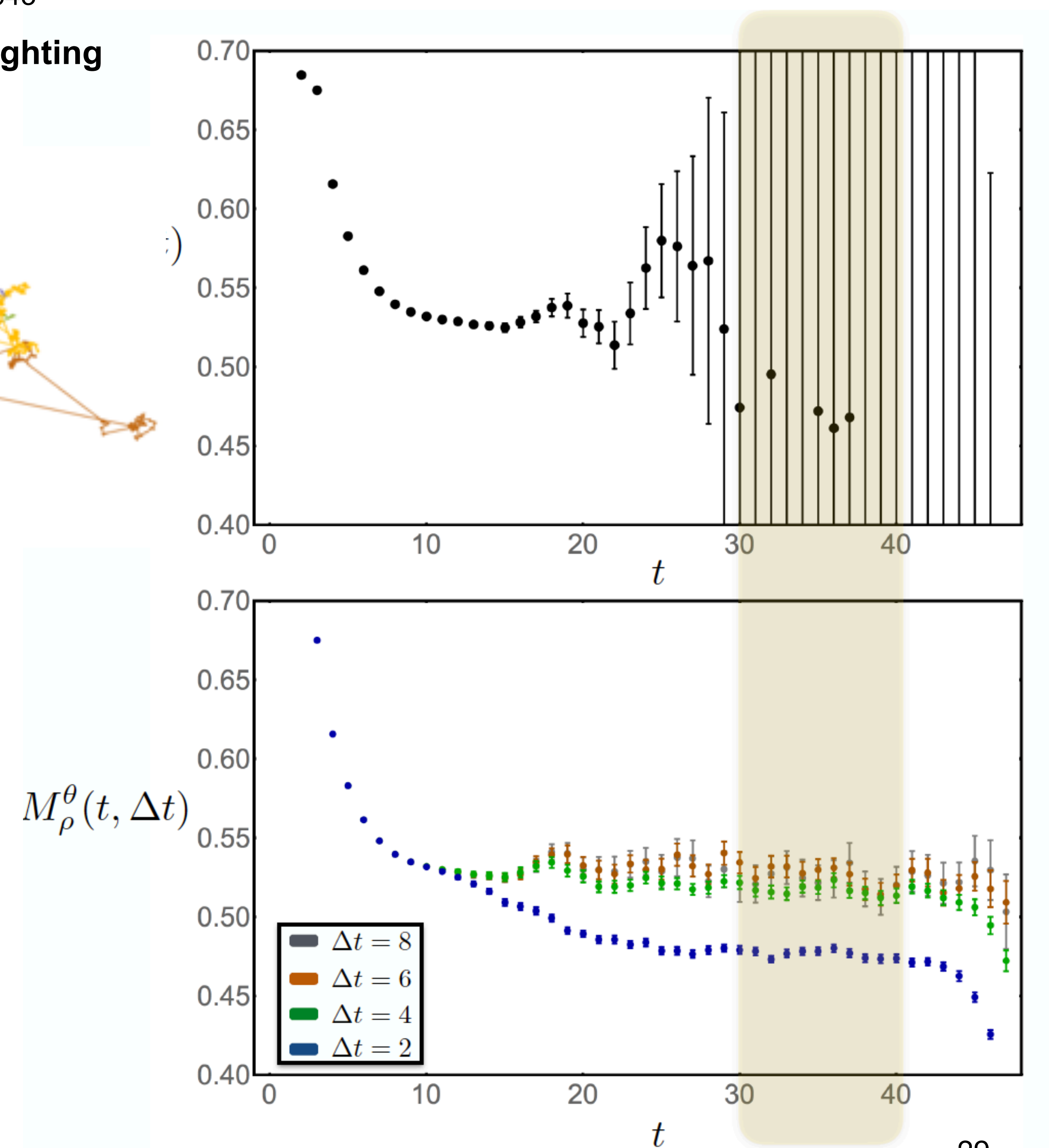


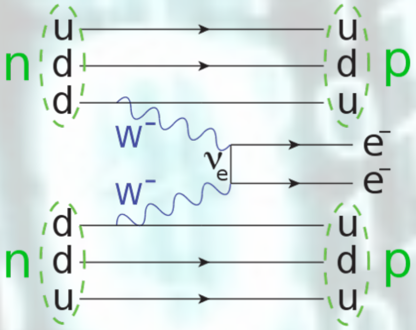
On the Statistics of Baryon Correlation Functions in Lattice QCD
Michael L. Wagman, Martin J. Savage (Washington U., Seattle). e-Print: arXiv:1611.07643

Taming the Signal-to-Noise Problem in Lattice QCD by Phase Reweighting
Michael L. Wagman, Martin J. Savage, e-Print: arXiv:1704.07356 [hep-lat]

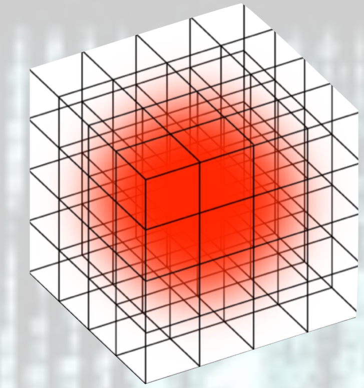








Closing Remarks

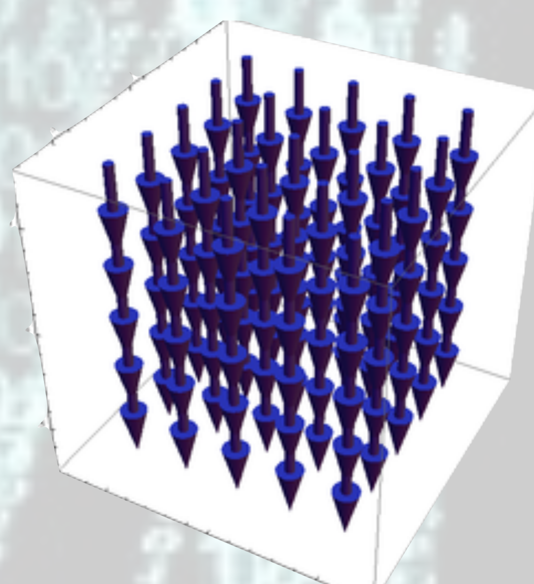


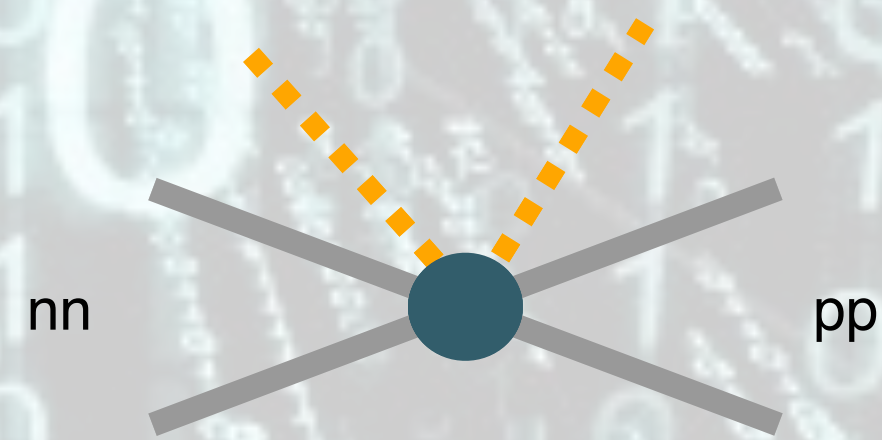
Lattice QCD combined with nuclear many-body techniques is beginning to provide first principles predictive capabilities for nuclear physics.

One and two axial current matrix elements have been sucessfully extracted in light nuclear systems at unphysical light-quark masses.

``Effective” gA quenching results from multi-nucleon operators, and does not provide a complete prescription for double-beta decay MEs.

In progress:
Extending to other quantities.
Reducing uncertainties.





isotensor polarizability

FIN