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Neutrinoless Double-beta Decay (INT 17-2a)

PARTIAL RESTORATION OF SPIN-ISOSPIN SU(4) SYMMETRY AND DOUBLE BETA DECAY

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Outline

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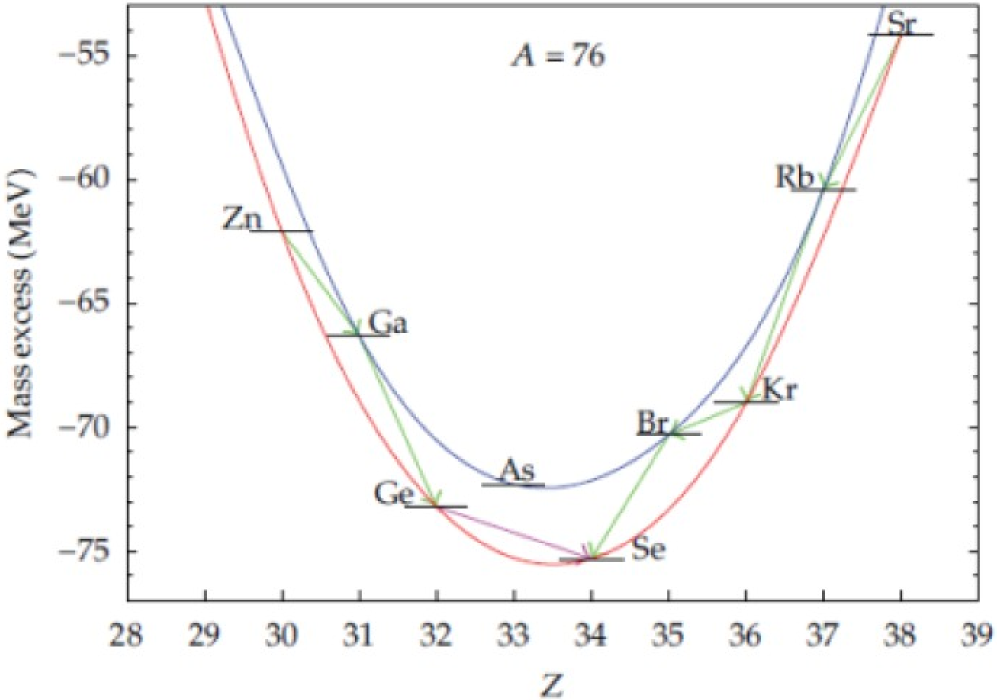
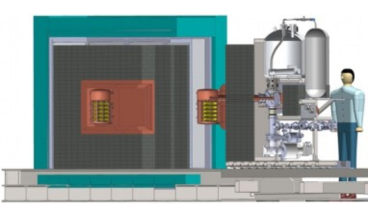
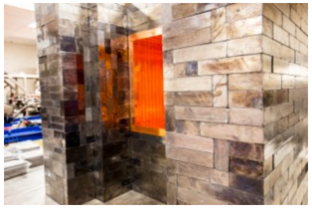
I. Introduction

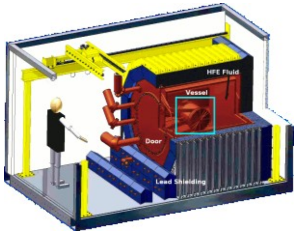


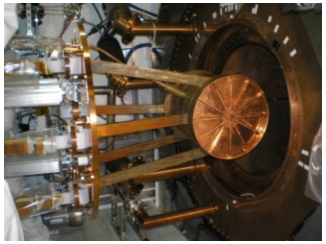
The **MAJORANA Collaboration (USA)0νββ in 76Ge**. MAJORANA plans to collaborate with GERDA for a future tone-scale 76Ge 0νββ search.





EXO-200: 0νββ in Xenon 136. y.



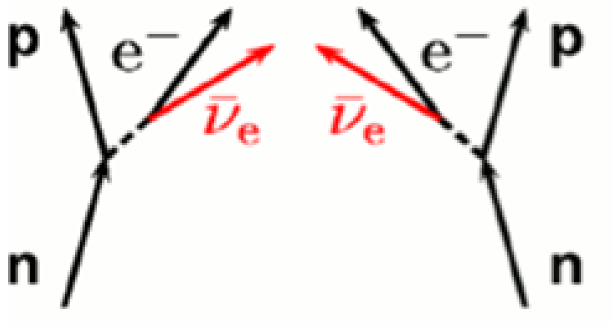


NEMO3 (France) 0νββ in **100Mo**

I. Introduction

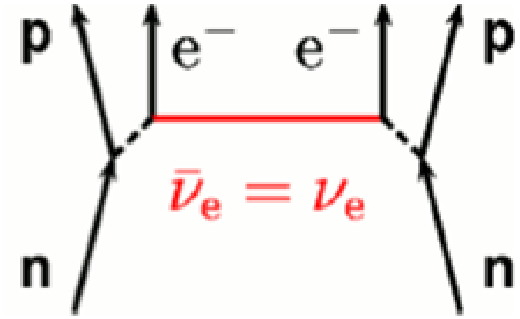
$2\nu\beta\beta,\ (\nu_e \neq \bar{\nu}_e)$

$$(N,Z) \rightarrow (N-2, Z+2) + 2e^- + 2\bar{\nu}_e$$



$0\nu\beta\beta,\ (\nu_e = \bar{\nu}_e)$

$$(N,Z) \rightarrow (N-2, Z+2) + 2e^-$$



$$T_{1/2}^{-1} = G(MF)^2$$

- Kinematic factor (G)
- Nuclear Matrix Elements (M)

$$F = \begin{cases} 1 & ,\ \text{for } 2\nu\beta\beta \\ \dfrac{\langle m_\nu \rangle}{\langle m_e \rangle} & ,\ \text{for } 0\nu\beta\beta \end{cases}$$

I. Introduction

• Nuclear Matrix Elements (M) merges from a microscopic hamiltonian worked in the framework of mean-field theories and often violates the symmetries of hamiltonian.

• BCS theory violates conservation of number of particle and the spin-isospin SU(4) symmetry:

(i) SU(4) is to be restored by the residual interaction,

(ii) This restoration must not be complete to inhibit $\beta\beta$-decay -> Partial SU(4) Symmetry Restoration (PSU4SR)

• Symmetries broken by BCS are restored by QRPA with a special adoption of parameters in the particle-particle (*pp*) and particle-hole (*ph*) channels, for example, in Simkovic et al., PRC 87, 045501 (2013); Fang et al., PRC 92, 044301 (2015); Hyvarinen et al. PRC 91, 024613(2015).

• We present a recipe to implement the PSU4SR based on energetic of F and GT resonances in (*ph*) channel, and on the minima of F and GT β^+-strengths in the (*pp*) channel.

I. Introduction – Main features

• Our physical substratum is the same as in previous QRPA calculations in ββ-decay:

Hirsch & Krmpotic, PRC 41, 792 (1990); Hirsch & Krmpotic, PLB2 46, 5 (1990);
Hirsch, Bauer & Krmpotic, NPA 516, 304 (1990);
Krmpotic, Hirsch & Dias, NPA 542, 85 (1992); Krmpotic, PRC 48, 1452 (1993);
Krmpotic, Mariano, Kuo & Nakayama, PLB 319, 393 (1993);
Krmpotic & Sharma, NPA 572, 329 (1994); Krmpotic, RMF 40, 285 (1994);
Ferreira, Master thesis,UESC-BA, Brazil, (2016),

here we just bring up to date those studies, including the pseudoscalar (*P*) and weak-magnetism (M) matrix elements $M^{0\nu}{}_P$, and $M^{0\nu}{}_M$, as suggested by Simkovic et al. PRC 60, 055502 (1999).

$$J^{\mu\dagger}(\vec{x}) = \overline{\Psi}(\vec{x})\tau^{+}\left[g_V\gamma^{\mu} - g_A\gamma^{\mu}\gamma_5 - ig_M\frac{\sigma^{\mu\nu}q_{\nu}}{2M_N} - g_P q^{\mu}\gamma_5 \right]\Psi(\vec{x})$$

Simkovic et al., PRC 60, 055502 (1999); PRC 77, 045503 (2008).
Krmpotic et al., NPA 612, 223 (1997);
Barbero et al., NPA 628, 170 (1998); PLB 445, 249 (1999).

I. Introduction – Main features

1. Residual interaction is δ-force in units of MeV-fm³ : $-4\pi(v^s P_s + v^t P_t)\delta(r)$,

   spin-singlet parameter in *pp* chanel: $v^s \leftrightarrow g_{pp}^{T=1}$,

   spin-triplet parameter in *pp* chanel: $v^t \leftrightarrow g_{pp}^{T=0}$.

2. We solve the RPA equation only once for intermediate *(N-1,Z+1)* nucleus as in J. Hirsch & F. Krmpotic, PLB 246, 5 (1990) .

3. v^s_{pp} is fixed in the same way as $g^{T=1}_{pp}$.
We require that β⁺ the strength S^+_F becomes minimal when $s=v^s_{pp}/v^s_{pair}$ become $s_{syn}=1$. This is sign of spin restored symmetry leading to

$$S_F^+ \cong 0,\ M_F^{2\nu} \cong 0, M_V^{0\nu}(J^\pi \cong 0^+) \cong 0,$$

and the concentration of S^-_F is in the IAS.

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I. Introduction – Main features

4. ν^t_{pp} is fixed following a similar recipe that ν^s_{pp}, we require that GT β^+ strength S^+_{GT} becomes minimal as it was shown in Fig. 2 and 3 of Krmpotic & Sharma. NPA 572. 329 (1994)

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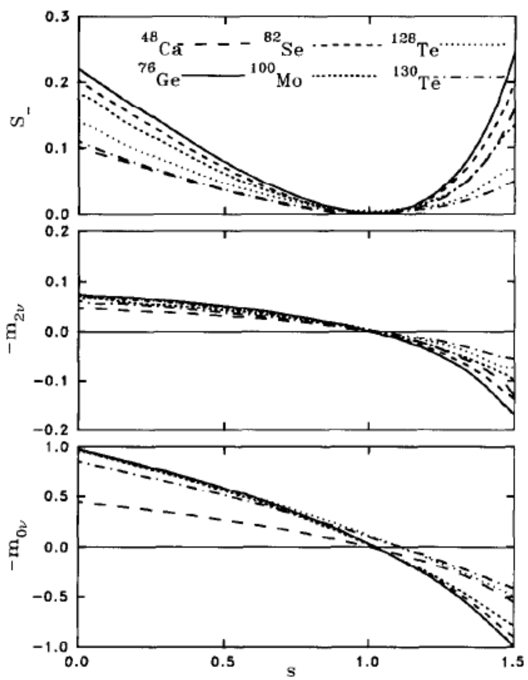

Fig. 1. Fermi observables $\mathscr{S}_-(J^\pi=0^+)$, $m_{2\nu}(J^\pi=0^+)$ (in units of $[\mathrm{MeV}]^{-1}$) and $m_{0\nu}(J^\pi=0^+)$ for the nuclei ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te and ^{130}Te, as a function of particle–particle $S=0$, $T=1$ coupling constant s.

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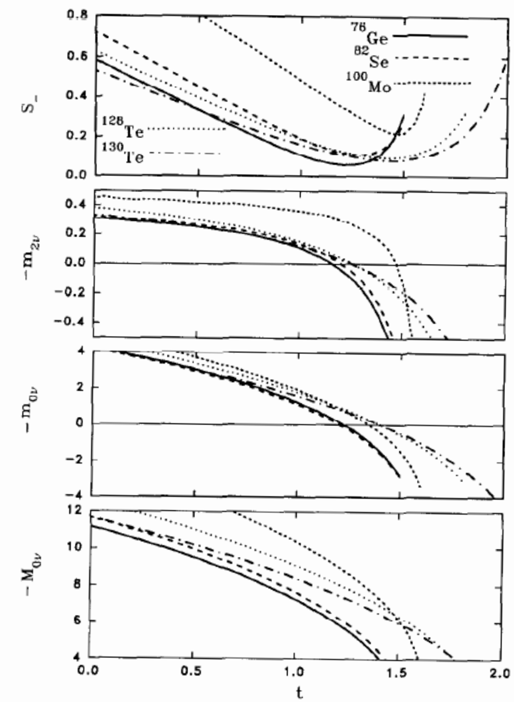


Fig. 3. Gamow–Teller observables $\mathscr{S}_-(J^\pi=1^+)$, $m_{2\nu}(J^\pi=1^+)$ (in units of $[\mathrm{MeV}]^{-1}$), $m_{0\nu}(J^\pi=1^+)$ and the total $\mathscr{M}_{0\nu}$ moment for the nuclei ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te and ^{130}Te, as a function of the particle–particle $S=1$, $T=0$ coupling constant t.

then $t=\nu^t_{pp}/\nu^s_{pair}$ become $t_{sym}\neq 1$ with $S^+_{GT}\neq 0,\ M^{2\nu}_{GT}\neq 0, M^{0\nu}_{A}(J^\pi\cong 1^+)\neq 0.$
and not all concentration S^-_{GT} in the GTR.

I. Introduction – Main features

5.The difference with other studies is that the experimental $2\nu\beta\beta$ moments are not used for gauging the isoscalar *pp parameter t.*
In this way, the QRPA model turns out to be predictable regarding $M^{2\nu}_{GT}$.

6. The restoration of the isospin and SU(4) symmetries, broken in the mean field, are also manifested *ph* channel. We have monitored the ph parameters v^{ph}_s and v^{ph}_t from the experimental energetic of the IAS and the GTR [Nakayama etal, PLB 114, 217 (1982)]:

$$E_{GT} - E_{IAS} = \left(26A^{-\frac{1}{3}} - 18.5\frac{N-Z}{A} \right) \text{MeV}$$

$26A^{-1/3}$ -> from the *SU(4)* symmetry-breaking caused by the LS coupling,

18(N-Z)/A -> symmetry-restoration effect induced by the residual interaction, which displaces the *GT* towards the IAS with increasing *N-Z.*

II. Formalism

A. 0νββ Nuclear Moments

$$(N,Z)\rightarrow(N-2,Z+2)+2e^-$$

$$\begin{cases}|i\rangle=|I\rangle, \quad J^\pi=0^+ \\ |f\rangle=|F;e_1e_2\rangle\end{cases}$$

The 0νββ nuclear moment is

$$M^{0\nu}=\frac{\mathrm{R}}{4\pi}\sum_N\int d\vec{k}v(k;N)\mathrm{M}^{0\nu}(\vec{k};N)$$

with
$$\mathrm{M}^{0\nu}(\vec{k};N)\equiv\langle F|J_\mu^\dagger(-\vec{k})|N\rangle\times\langle N|J^{\mu\dagger}(-\vec{k})|I\rangle$$

and
$$J^{\mu\dagger}(\vec{k})=\int d\vec{x}J^{\mu\dagger}(\vec{x})e^{-i\vec{k}\cdot\vec{x}},$$

is the Fourier transformer of hadronic current and R=$r_0A^{1/3}$, r_0=1.2 fm and the neutrino potential
$$v(k;N)=\frac{2}{\pi}\frac{1}{k(k+\omega_N)}$$

with $\omega_N=E_N-\frac{1}{2}(E_I+E_F)$.

II. Formalism

A. 0νββ Nuclear Moments

Within NRA (when velocity terms are omitted), the hadronic currents:

$$J^{\mu}_{\mathrm{NRA}}(\vec{x}) = \left(\rho(\vec{x}), j(\vec{x})\right)$$

where

$$\rho(\vec{x}) = g_V \sum_n \tau_n^+ \delta(\vec{x} - \vec{r}_n),$$

$$j(\vec{x}) = \frac{g_V}{2M_N} \sum_n \tau_n^+ \delta(\vec{x} - \vec{r}_n)[-g_A \vec{\sigma}_n + f_M^{'} \nabla \times \vec{\sigma}_n - g_P^{'} \nabla\nabla \cdot \vec{\sigma}_n],$$

are one-body density current, with $f_M = g_V + g_M = 4.7$, $f'_M = f_M/(2M_N)$, $g'_P = g_P/(2M_N)$, and $g_V = 1$, $g_A = -1.27$, $g_M = 3.7$.

Using Fourier-Bessel relationship for exponential $e^{i\mathbf{k}.\mathbf{r}}$, spherical vectors and performing the angular integration on Ω_k, multiplying by $\mathrm{R}k^2 v(k;N)/4\pi$ then

$$\mathrm{M}^{0\nu} \equiv \sum_X \mathrm{M}^{0\nu}_X$$

with $X = V, A, P, M$.

II. Formalism

A. 0νββ Nuclear Moments

With:

$$\mathrm{M}_V^{0\nu} = \sum_{J_\alpha^\pi} (-1)^J \sum_{pnp'n'} \rho^{ph}(pnp'n';J_\alpha^\pi) W_{J0J}(pn) W_{J0J}(p'n') R_{JJ}^V (pnp'n';\omega_{J_\alpha^\pi}),$$

$$\mathrm{M}_A^{0\nu} = \sum_{J_\alpha^\pi} (-1)^{L+1} \sum_{pnp'n'} \rho^{ph}(pnp'n';J_\alpha^\pi) W_{L1J}(pn) W_{L1J}(p'n') R_{LL}^A (pnp'n';\omega_{J_\alpha^\pi}),$$

$$\mathrm{M}_P^{0\nu} = -\sum_{J_\alpha^\pi} (-1)^{J+(L+L')/2} \hat{L}\,\hat{L}'(LL'|l)(11|l) \times \sum_{pnp'n'} \rho^{ph}(pnp'n';J_\alpha^\pi) W_{L1J}(pn)$$

$$\times\, W_{L'1J}(p'n') \begin{Bmatrix} L & L' & l \\ 1 & 1 & J \end{Bmatrix} R_{LL'}^P (pnp'n';\omega_{J_\alpha^\pi}),$$

$$\mathrm{M}_M^{0\nu} = -\sum_{J_\alpha^\pi} (-1)^{J+(L+L')/2} \hat{L}\,\hat{L}'(LL'|l)(11|l) \times \sum_{pnp'n'} \rho^{ph}(pnp'n';J_\alpha^\pi) W_{L1J}(pn)$$

$$\times\, W_{L'1J}(p'n') \begin{Bmatrix} L & L' & l \\ 1 & 1 & J \end{Bmatrix} [2 - l(l+1)/2] R_{LL'}^M (pnp'n';\omega_{J_\alpha^\pi}),$$

II. Formalism

A. 0νββ Nuclear Moments

The angular parts are: $$W_{LSJ}(pn)=\sqrt{2}\hat{S}\hat{J}\hat{L}\hat{l}_n\hat{j}_n\hat{j}_p(l_nL\,|\,l_p)\begin{Bmatrix} l_p & \frac{1}{2} & j_p \\ L & S & J \\ l_n & \frac{1}{2} & j_n \end{Bmatrix},$$

with the two-body radial integrals

$$R^X_{LL'}(pnp'n';\omega_{J^\pi_\alpha})=\mathrm{R}\int dk\,k^2 v_X(q;\omega_{J^\pi_\alpha})R_L(pn;k)R_{L'}(p'n';k),$$

and the neutrino potentials are

$$v_V(k;\omega_{J^\pi_\alpha})=g_V^2(k^2)v(k;\omega_{J^\pi_\alpha}),\ v_A(k;\omega_{J^\pi_\alpha})=g_A^2(k^2)v(k;\omega_{J^\pi_\alpha}),$$

$$v_M(k;\omega_{J^\pi_\alpha})=k^2 f'^2_M(k^2)v(k;\omega_{J^\pi_\alpha}),$$

$$v_P(k;\omega_{J^\pi_\alpha})=k^2 g'_P(k^2)[2g_A(k^2)-k^2 g'_P(k^2)]v(k;\omega_{J^\pi_\alpha}).$$

with $$R_L(pn;k)=\int_o^\infty u_{n_pl_p}(r)u_{n_nl_n}(r)j_L(kr)r^2dr,$$ and

$$v(k,\omega_{J^\pi_\alpha})=\frac{2}{\pi}\frac{1}{k(k+\omega_{J^\pi_\alpha})},\quad \omega_{J^\pi_\alpha}=E_{J^\pi_\alpha}-E_{0^+_I}+\frac{1}{2}Q_{\beta\beta}.$$

II. Formalism

A. 0νββ Nuclear Moments

One-body state dependent ph density matrices

$$\rho^{ph}(pnp'n';J_\alpha^\pi)=\hat{J}^{-2}\left\langle 0_F^+\left\|(a_{\rm p}^\dagger a_{\bar{n}})_{J^\pi}\right\|J_\alpha^\pi\right\rangle\left\langle J_\alpha^\pi\left\|(a_{\rm p}^\dagger a_{\bar{n}})_{J^\pi}\right\|0_I^+\right\rangle,$$

FNS effects are introduced by

$$g_V\rightarrow g_V(k^2)\equiv g_V\left(\frac{\Lambda_V}{(\Lambda_V^2+k^2)}\right)^2,\quad g_A\rightarrow g_A(k^2)\equiv g_A\left(\frac{\Lambda_A}{(\Lambda_A^2+k^2)}\right)^2.$$

SRC between two nucleon are

$$f^{SRC}(r)=1-j_0(k_C r),\qquad k_C=3.93\,\mathrm{fm}^{-1}$$

Then

$$v_X(q,\omega_J)\rightarrow v_X(q,\omega_J)-\Delta v_X(q,\omega_J),$$

$$\Delta v_X(q,\omega_J)=\frac{1}{2q_C^2}\int_{-1}^{1}dx\int dk\,k^2 v_X(q,\omega_J)\times\delta(\sqrt{q^2+k^2+2xqk}-q_C).$$

II. Formalism

B. 2νββ Nuclear Moments

$$\mathrm{M}^{2\nu} = \mathrm{M}_F^{2\nu} + \mathrm{M}_{GT}^{2\nu},$$

with

$$\mathrm{M}_F^{2\nu} = \sum_\alpha \left[\frac{\langle 0_f^+ \| O_0^{\beta-} \| \lambda_\alpha^+ \rangle \langle \lambda_\alpha^+ \| O_0^{\beta-} \| 0_i^+ \rangle}{D_{0_\alpha^+, f}} \right], \qquad O_0^{\beta-} = \sum_{pn} \langle p \| \hat{1} \| n \rangle \left[a_p^\dagger a_{\bar{n}} \right]_0$$

$$= g_V^2 \sum_{pnp'n'} \rho^{ph}(pnp'n'; 0_\alpha^+) \frac{W_{000}(pn) W_{000}(p'n')}{\omega_{0_\alpha^+}},$$

and

$$\mathrm{M}_{GT}^{2\nu} = \sum_\alpha \left[\frac{\langle 0_f^+ \| O_1^{\beta-} \| \lambda_\alpha^+ \rangle \langle \lambda_\alpha^+ \| O_1^{\beta-} \| 0_i^+ \rangle}{D_{1_\alpha^+, f}} \right], \qquad O_1^{\beta-} = \frac{1}{\sqrt{3}} \sum_{pn} \langle p \| \sigma \| n \rangle \left[a_p^\dagger a_{\bar{n}} \right]_1$$

$$= -g_A^2 \sum_{pnp'n'} \rho^{ph}(pnp'n'; 1_\alpha^+) \frac{W_{011}(pn) W_{011}(p'n')}{\omega_{1_\alpha^+}}.$$

III. Charge-exchange QRPA

• Most used model is charge-exchange QRPA - Halbleib and Sorensen- Nucl. Phys. A 98, 524 (1967) where:

1) BCS equations for the initial even-even nucleus (N,Z) with (v_n, v_p) and $(u_n=(1-v^2_n)^{1/2}, u_p=(1-v^2_p)^{1/2})$, q.p energies $(\varepsilon_n, \varepsilon_p)$ and chem. pot. (λ_n, λ_p), with BCS vaccum

$$|0_I\rangle = \Pi_p (u_p + v_p a_p^\dagger a_{\bar{p}}^\dagger) \Pi_n (u_n + v_n a_n^\dagger a_{\bar{n}}^\dagger),$$

$$\sum_{j_p} (2j_p + 1) v_p^2 = Z, \quad \sum_{j_n} (2j_n + 1) v_n^2 = N.$$

2) Transition β±–densities

$$\rho^+(pn; J_\alpha^\pi) = u_n v_p X_{J_\alpha^\pi}(pn) + u_p v_n Y_{J_\alpha^\pi}(pn),$$

$$\rho^-(pn; J_\alpha^\pi) = u_p v_n X_{J_\alpha^\pi}(pn) + u_n v_p Y_{J_\alpha^\pi}(pn).$$

Excitation energies in neighboring odd-odd $(N \pm 1, Z \mp 1)$

$$E_{J_\alpha^\pi}^{N\pm1, Z\mp1} = E_{0_I^+} + \Omega_{J_\alpha^\pi} \pm \lambda_n \mp \lambda_p,$$

pn-QRPA equation

$$\begin{pmatrix} A_{J_\alpha^\pi} & B_{J_\alpha^\pi} \\ -B_{J_\alpha^\pi} & A_{J_\alpha^\pi} \end{pmatrix} \begin{pmatrix} X_{J_\alpha^\pi} \\ Y_{J_\alpha^\pi} \end{pmatrix} = \Omega_{J_\alpha^\pi} \begin{pmatrix} X_{J_\alpha^\pi} \\ -Y_{J_\alpha^\pi} \end{pmatrix},$$

Ikeda sum-rule

$$S^\beta = S^- - S^+ = N - Z,$$

III. Charge-exchange QRPA

A. Method I

• Vogel and Zirnbauer (PRL57, 3148 (1986)) discovered GSC is suppresing $2\nu\beta\beta$ rates.

•QRPA for both initial and final nuclei and NME averaged. Step 1) and 2) are repeated for *(N, Z)* and *(N-2, Z+2)* gs, and intermediary 1^+_α and $1'^+_\alpha$ in *(N-1, Z+1)*

In the second case the BCS vaccum is

$$|0_F\rangle = \Pi_p(\bar{u}_p + \bar{v}_p a^\dagger_p a^\dagger_{\bar{p}})\Pi_n(\bar{u}_n + \bar{v}_n a^\dagger_n a^\dagger_{\bar{n}}),$$

$$\sum_{j_p}(2j_p+1)\bar{v}_p^2 = Z+2, \quad \sum_{j_n}(2j_n+1)\bar{v}_n^2 = N-2.$$

$$Q_{\beta\beta} = E_{0^+_I} - E_{0^+_F} = 2(\lambda_n - \lambda_p), \quad \mathrm{M}^{2\nu}_{GT} = -\frac{g_A^2}{2}\sum_{pnp'n'} W_{011}(pn)W_{011}(p'n')$$

$$\times\left[\frac{\rho^{ph}(pnp'n';1^+_\alpha)}{\Omega_{1^+_\alpha}} + \frac{\rho^{ph}(pnp'n';\bar{1}^+_\alpha)}{\Omega_{\bar{1}^+_\alpha}}\right].(3.13)$$

$$E_{1^+_\alpha} - E_{0^+_I} = \Omega_{1^+_\alpha} - \frac{1}{2}Q_{\beta\beta},$$

$$\omega_{1^+_\alpha} = \Omega_{1^+_\alpha}.$$

III. Charge-exchange QRPA

B. Method II

• Civitarese, Faessler and Tomoda [PLB 194, 11(1987)], repeating steps 1) and 2) for g.s. of *(N, Z)* and *(N-2, Z+2)*, and intermediary 1^+_α and $1^+_{\alpha'}$ in *(N-1, Z+1)*.

Their $2\nu\beta\beta$ moment reads

$$M^{2\nu}_{GT} = -g_A^2 \sum_{pnp'n'\alpha\alpha'} W_{011}(pn) W_{011}(p'n') \times \left[\frac{\rho^{ph}(pnp'n';1^+_\alpha)\langle 1^+_\alpha | \bar{1}^+_{\alpha'} \rangle \rho^{ph}(pnp'n';\bar{1}^+_{\alpha'})}{m_e c^2 + \frac{1}{2} Q_{\beta\beta} + E_{1^+_\alpha} - E_{0^+_i}} \right].$$

where the overlap is

$$\langle 1^+_\alpha | \bar{1}^+_{\alpha'} \rangle = \sum_{pn} [X_{1^+_\alpha}(pn) X_{\bar{1}^+_{\alpha'}}(pn) - Y_{1^+_\alpha}(pn) Y_{\bar{1}^+_{\alpha'}}(pn)]. \qquad (3.14)$$

In recent applications of Method II, the denominator is replaced by

$$M^{2\nu}_{GT} = -2g_A^2 \langle 0^+_I | 0^+_F \rangle \sum_{pnp'n'\alpha\alpha'} W_{011}(pn) W_{011}(p'n') \times \left[\frac{\rho^+(p'n';\bar{1}^+_{\alpha'})\langle \bar{1}^+_{\alpha'} | 1^+_\alpha \rangle \rho^-(pn;1^+_\alpha)}{\Omega_{1^+_\alpha} - \Omega_{\bar{1}^+_{\alpha'}}} \right].$$

where $\langle 0^+_I | 0^+_F \rangle = \Pi_p (u_p \bar{u}_p + v_p \bar{v}_p) \Pi_n (u_n \bar{u}_n + v_n \bar{v}_n).$

III. Charge-exchange QRPA

C. Method III

• Eqs. (3.13) and (3.14) are physically sound ansatz for HS eqs. Then in Hirsch & Krmpotic PLB 246, 5 (1990) , is proposed to solve only one QRPA equation

$$\begin{pmatrix} \tilde{A}_{J^\pi_\alpha} & \tilde{B}_{J^\pi_\alpha} \\ -\tilde{B}_{J^\pi_\alpha} & \tilde{A}_{J^\pi_\alpha} \end{pmatrix} \begin{pmatrix} \tilde{X}_{J^\pi_\alpha} \\ \tilde{Y}_{J^\pi_\alpha} \end{pmatrix} = \tilde{\Omega}_{J^\pi_\alpha} \begin{pmatrix} \tilde{X}_{J^\pi_\alpha} \\ -\tilde{Y}_{J^\pi_\alpha} \end{pmatrix},$$

solved for the vacuum $\left|\tilde{0}\right\rangle = \Pi_p(u_p + \bar{v}_p c_p^\dagger c_{\bar{p}}^\dagger)\Pi_n(\bar{u}_n + v_n c_n^\dagger c_{\bar{n}}^\dagger).$

and BCS eqs. were solved for even nuclei. The GT moment is

$$\mathrm{M}^{2\nu}_{GT} = -\mathrm{g}_A^2 \sum_{pnp'n'\alpha} W_{011}(pn) W_{011}(p'n') \times \left[\frac{\tilde{\rho}^-(pn;1^+_\alpha)\tilde{\rho}^+(p'n';\bar{1}^+_\alpha)}{\tilde{\Omega}_{1^+_\alpha}} \right].$$

where

$$\tilde{\rho}^-(pn;J^\pi_\alpha) = \sqrt{\sigma_p\sigma_n}\left(u_p v_n \tilde{X}_{J^\pi_\alpha}(pn) + \bar{u}_n \bar{v}_p \tilde{Y}_{J^\pi_\alpha}(pn)\right),$$

$$\tilde{\rho}^+(pn;J^\pi_\alpha) = \sqrt{\sigma_p\sigma_n}\left(\bar{v}_p \bar{u}_n \tilde{X}_{J^\pi_\alpha}(pn) + u_p v_n \tilde{Y}_{J^\pi_\alpha}(pn)\right),$$

$$\sigma_\mathrm{p}^{-1} = u_p^2 + \bar{v}_p^2, \quad \sigma_\mathrm{n}^{-1} = u_n^2 + \bar{v}_n^2.$$

and GT strengths

$$\tilde{S}^\pm_{GT} = \sum_{pn\alpha} |\tilde{\rho}^\pm(pn,1^+_\alpha) W_{011}(pn)|^2,$$

$$\tilde{S}^\beta = \tilde{S}^- - \tilde{S}^+ = N - Z - 2.$$

III. Charge-exchange QRPA

D. Method IV

Cha [PRC 27,2268 (1983)] in studies of single β-decay: *"because the intersection between two-qp's takes place in a residual nucleus, we should calculate ε's, u's, and v's in the daughter nucleus."* Then, instead of dealing with two-vacua QRPA, BCS eqs. are solved only for intermediary nucleus, with vacuum

$$|0_{\rm int}\rangle = \Pi_p (u'_p + v'_p c_p^\dagger c_{\bar{p}}^\dagger) \Pi_n (u'_n + v'_n c_n^\dagger c_{\bar{n}}^\dagger),$$

and *u's, and v's:*

$$\sum_p (2j_p+1) v'^2_p = Z+1, \quad \sum_n (2j_n+1) v'^2_n = N-1,$$

Sum rule:

$$S'^{\beta-} = N - Z - 2.$$

The GT moment is

$$M_{GT}^{2\nu} = -g_A^2 \sum_{pnp'n'\alpha} W_{011}(pn) W_{011}(p'n') \times \left[\frac{\rho'^{-}(pn;1^+_\alpha)\, \rho'^{+}(p'n';1^+_\alpha)}{\Omega'_{1^+_\alpha}} \right].$$

and the 0νββ NME are evaluated with the one-body densities

$$\rho^{-}(pnp'n';J^\pi_\alpha) = \rho'^{-}(pn;J^\pi_\alpha)\ \rho'^{+}(p'n';J^\pi_\alpha)$$

III. Charge-exchange QRPA

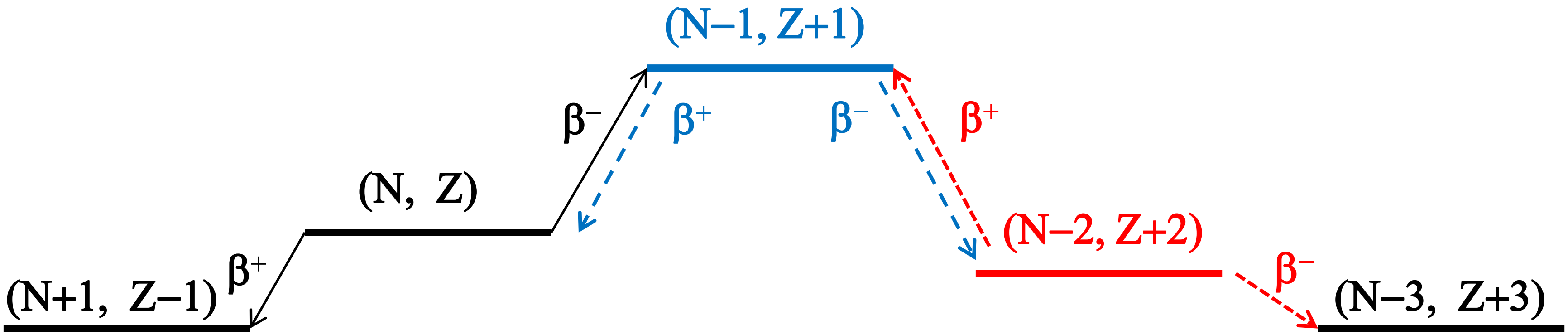
Method I and II (*(N, Z)* and *(N-2, Z+2)*, and *(N-1, Z+1)*) involves also the nuclei *(N+1, Z-1)* and *(N-3, Z+3)*. This is because GSC for

$$(N,Z)\underset{\beta^-}{\rightarrow}(N-1,Z+1),\ \text{and}\ (N-1,Z+1)\underset{\beta^-}{\rightarrow}(N-2,Z+2)\cong(N-2,Z+2)\underset{\beta^+}{\rightarrow}(N-1,Z+1)$$

correspond respectively, to

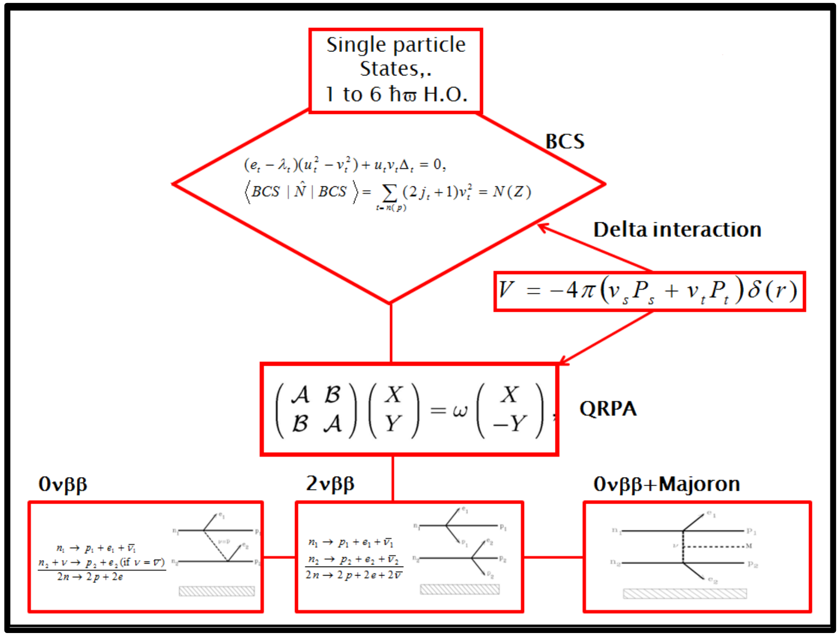
$$(N,Z)\underset{\beta^+}{\rightarrow}(N+1,Z-1),\ \text{and}\ (N-2,Z+2)\underset{\beta^-}{\rightarrow}(N-3,Z+3).$$

On the contrary, Method III and IV only involve nuclei within the isobaric triplet *(N, Z)*, *(N-1, Z+1)* and *(N-2, Z+2)* where ββ-decay occurs.



IV. Numerical results

Code QRAP2DB



$$2(e_t-\lambda_t)u_tv_t=(u_t^2-v_t^2)\Delta_t$$

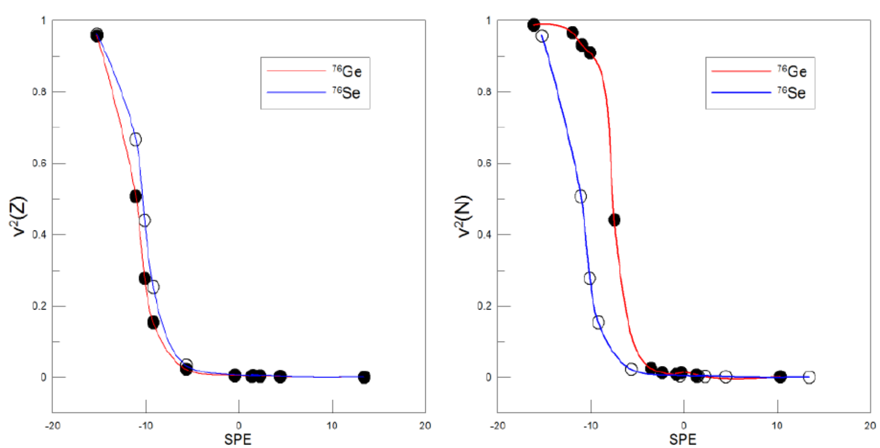
$$\Delta_t=-\hat{j}_{t'}^{-1}\sum_t\hat{j}_tu_tv_t\langle tt;J=0|V|t't',J=0\rangle$$

$$\Delta^N=-\frac{1}{2}[B(Z,N-1)-2B(Z,N)+B(Z,N+1)]$$

$$\Delta^Z=-\frac{1}{2}[B(Z-1,N)-2B(Z,N)+B(Z+1,N)]$$

* s.p.e in 76Ge

Notação	Camada	n	l	$j+1/2$	spe(N)	spe(Z)
155	$1h_{9/2}$	1	5	5	10.320	13.446
156	$1h_{11/2}$	1	5	6	1.330	4.456
144	$1g_{7/2}$	1	4	4	-0.310	1.508
222	$2d_{3/2}$	2	2	2	-0.860	2.266
301	$3s_{1/2}$	3	0	1	-2.340	1.382
223	$2d_{5/2}$	2	2	3	-3.570	-0.444
145	$1g_{9/2}$	1	4	5	-7.510	-5.692
211	$2p_{1/2}$	2	1	1	-10.110	-9.220
133	$1f_{5/2}$	1	3	3	-10.970	-10.126
212	$2p_{3/2}$	2	1	2	-11.980	-11.090
134	$1f_{7/2}$	1	3	4	-16.150	-15.306



IV. Numerical results

* *ph*-channel

•Systematic study of GT ressonancies
Nakayama et al. PLB 114, 217 (1982).

$$E_{GT} - E_{IAS=F} = \left(26A^{-\frac{1}{3}} - 18.5\frac{N-Z}{A} \right) \text{MeV}$$

- For all nuclei (not ^{48}Ca) were adopted:

$$v_{s}^{PH} = 55 \text{ MeV.fm}^{-3}, \quad v_{t}^{PH} = 92 \text{ MeV.fm}^{-3}$$

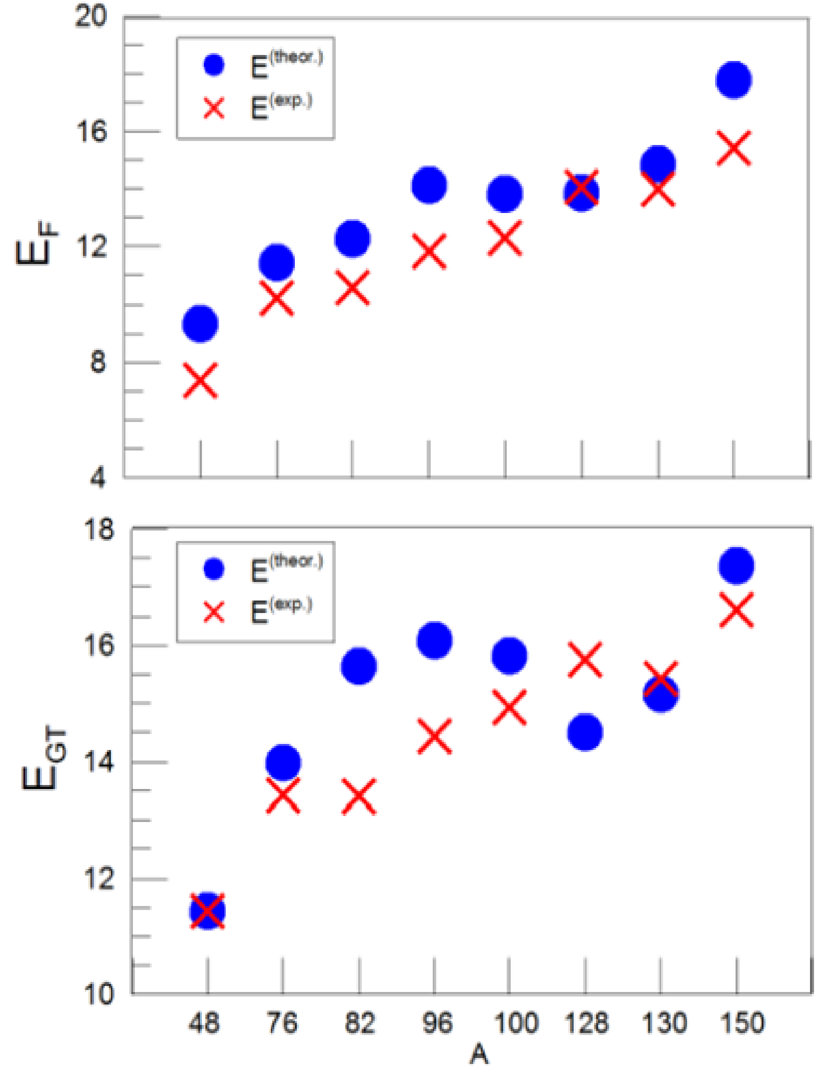
- Fermi & Gamow-Teller theoretical

$$E_F = \sum_{pn\alpha} \frac{\left|\rho^{'-}(pn\,0^{+}_{\alpha})\right|^2 \Omega_{0^{+}_{\alpha}}}{\sum_{pn\alpha}\left|\rho^{'-}(pn\,0^{+}_{\alpha})\right|^2}, \quad E_{GT} = \frac{\sum_{pn\alpha}\left|\rho^{'-}(pn1^{+}_{\alpha})\right|^2 \Omega_{1^{+}_{\alpha}}}{\sum_{pn\alpha}\left|\rho^{'-}(pn1^{+}_{\alpha})\right|^2} > 10MeV.$$

- Fermi experimental and Coulomb energies

$$E_{IAS} = \varepsilon_{Coul}(Z+1, A) - \varepsilon_{Coul}(Z+1, A),$$

$$\varepsilon_{Coul}(Z, A) = 0.70\frac{Z^2}{A^{1/3}}\left(1. - 0.76Z^{-\frac{2}{3}}\right)$$



IV. Numerical results

* *pp*-channel parameters from P-SU4-SR

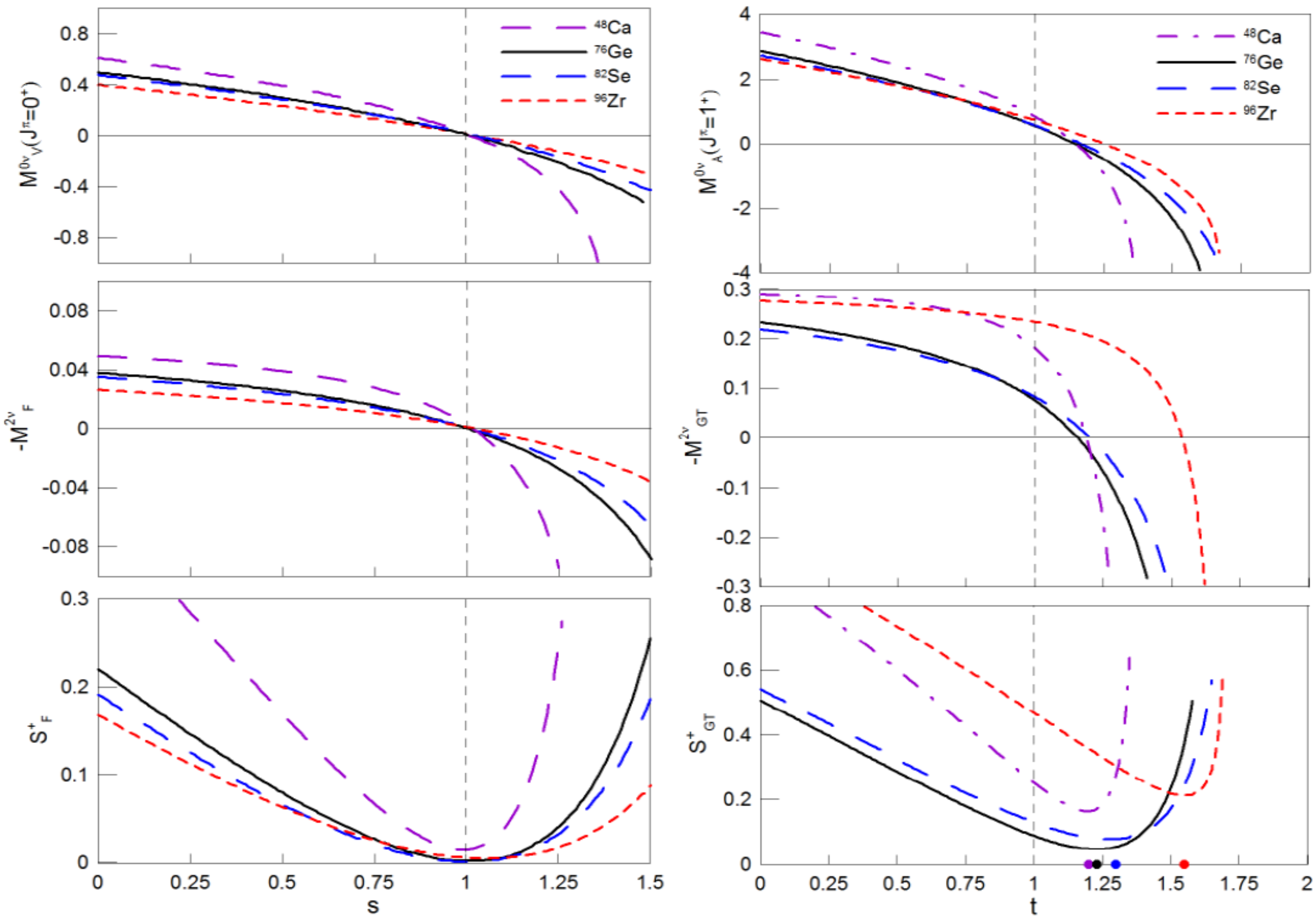


FIG. 1: (Color online) β^+-decay transition strengths, 2ν NM given in natural units, and 0ν NM normalized to g_A^2. We show vector observables, as a function of the ratio $s = v^s_{pp}/\overline{v}^s_{pair}$, on the left side, and axial-vector ones, as a function of the ratio $t = v^t_{pp}/\overline{v}^s_{pair}$, on the right side. The values of t_{sym} on the axis t are indicated by points.

IV. Numerical results

* *pp*-channel parameters from P-SU4-SR

$$s = \frac{v_s^{PP}}{v_s^{pair}}, \quad t = \frac{v_t^{PP}}{v_s^{pair}}$$

$$s \approx 1$$

$$v_s^{pair} = \frac{v_s^{pairN} + v_s^{pairZ}}{2}$$

The values exhibited in Table I are very close to those obtained previously in [NPA 572, 329 (2014), Table 4], where Method III was used to calculate the NM.

The above similarity is the main reason for associating P-SU4-SR in ββ-decay.

TABLE I: Values of the parameters s_{sym} and t_{sym}, and experimental and calculated energies of the IAS and GTR in the initial nucleus. The energies are given in units of MeV.

^AZ	s_{sym}	t_{sym}	E^{cal}_{IAS}	E^{exp}_{IAS}	E^{cal}_{GTR}	E^{exp}_{GTR}
^{48}Ca	1.00	1.20	8.70	7.36	13.66	11.43
^{76}Ge	1.00	1.23	11.47	10.21	13.92	13.42
^{82}Se	1.00	1.30	12.25	10.59	15.59	13.41
^{96}Zr	1.00	1.55	14.18	11.85	16.10	14.45
^{100}Mo	1.00	1.49	13.70	12.29	15.83	14.93
^{128}Te	1.00	1.41	13.74	14.06	14.36	15.75
^{130}Te	1.00	1.45	14.71	13.98	14.95	15.42
^{150}Nd	1.00	1.29	20.21	15.42	18.46	16.61

IV. Numerical results

P-SU4-SR effects

Table II: ββ2ν-decay moments evaluated within the BCS (unperturbed) and QRPA (perturbed) approximations are compared with the experimental results recommended by Barabash [NPA 935, 52(2015)]. All the quantities are given in natural units.

	BCS			QRPA			
^{A}Z	$M_F^{2\nu}$	$M_{GT}^{2\nu}$	$M^{2\nu}$	$M_F^{2\nu}$	$M_{GT}^{2\nu}$	$\|M^{2\nu}\|$	$\|M_{exp}^{2\nu}\|$
^{48}Ca	-0.148	-0.545	-0.693	-0.004	0.022	$0.018^{+0.110}_{-0.035}$	0.038 ± 0.003
^{76}Ge	-0.193	-0.693	-0.886	-0.000	0.051	$0.051^{+0.035}_{-0.030}$	0.113 ± 0.006
^{82}Se	-0.217	-0.686	-0.903	-0.001	0.062	$0.062^{+0.033}_{-0.029}$	0.083 ± 0.004
^{96}Zr	-0.107	-0.878	-0.985	-0.001	0.024	$0.023^{+0.157}_{-0.036}$	0.080 ± 0.004
^{100}Mo	-0.126	-1.213	-1.339	-0.001	0.035	$0.034^{+0.182}_{-0.115}$	0.185 ± 0.005
^{128}Te	-0.296	-1.174	-1.470	-0.003	0.086	$0.083^{+0.029}_{-0.026}$	0.046 ± 0.006
^{130}Te	-0.263	-1.025	-1.288	-0.002	0.083	$0.081^{+0.022}_{-0.020}$	0.031 ± 0.004
^{150}Nd	-0.057	-0.887	-0.944	-0.001	0.067	$0.067^{+0.011}_{-0.011}$	0.058 ± 0.004

IV. Numerical results

TABLE III: $\beta\beta0\nu$-decay moments $M^{0\nu}_X$, as well the total moments $M^{0\nu}=\sum_X M^{0\nu}_X$ (normalized to g^2_A, with g_A = 1,27), evaluated within the BCS (unperturbed) and QRPA (perturbed) approximations, are shown. In both cases the FNS and SRC effects are included. At the bottom of the table are shown the 76Ge results: i) without SRC, in the row labeled as ^{76}Ge*, ii) the bare values of moments, i.e., without the FNS and SRC effects, in the row labeled as ^{76}Ge** , and iii) the moments obtained in Ref. [10] and derived from relations (4.5).

	BCS					QRPA				
^AZ	$M^{0\nu}_V$	$M^{0\nu}_A$	$M^{0\nu}_P$	$M^{0\nu}_M$	$M^{0\nu}$	$M^{0\nu}_V$	$M^{0\nu}_A$	$M^{0\nu}_P$	$M^{0\nu}_M$	$M^{0\nu}$
^{48}Ca	1.91	9.10	-1.54	0.49	9.96	0.58	2.57	-0.76	0.33	$2.72^{-0.40}_{+0.32}$
^{76}Ge	2.52	12.35	-2.15	0.71	13.42	0.64	3.02	-0.86	0.40	$3.19^{-0.24}_{+0.46}$
^{82}Se	2.61	12.58	-2.21	0.72	13.70	0.65	2.76	-0.84	0.39	$2.96^{-0.23}_{+0.22}$
^{96}Zr	2.43	12.70	-2.15	0.71	13.70	0.70	1.89	-0.74	0.38	$2.22^{-0.42}_{+0.35}$
^{100}Mo	2.85	15.17	-2.51	0.84	16.35	0.82	2.48	-0.90	0.45	$2.85^{-0.43}_{+0.42}$
^{128}Te	2.78	13.55	-2.13	0.66	14.87	0.84	3.31	-0.97	0.41	$3.59^{-0.19}_{+0.19}$
^{130}Te	2.48	12.12	-1.91	0.60	13.29	0.75	2.81	-0.84	0.36	$3.07^{-0.16}_{+0.16}$
^{150}Nd	2.02	10.94	-1.75	0.57	11.77	0.77	3.95	-0.93	0.37	$4.16^{-0.12}_{+0.11}$
^{76}Ge*	2.54	12.54	-2.21	0.71	13.57	0.65	3.14	-0.90	0.40	3.29
^{76}Ge**	2.90	13.72	-2.55	1.08	15.14	0.85	3.83	-1.11	0.65	4.22
^{76}Ge [10]						1.74	5.48	-1.60	0.29	5.26

Actual Hyvärinen & Suhonen, PRC 91, 024613 (2015).

$$M^{0\nu}_V \to M^{VV}_F,$$
$$M^{0\nu}_A \to M^{AA}_{GT},$$
$$M^{0\nu}_M \to M^{MM}_{GT} + M^{MM}_T,$$
$$M^{0\nu}_P \to M^{PP}_{GT} + M^{PP}_T + M^{AP}_{GT} + M^{AP}_T.$$

IV. Numerical results

i) The residual interaction, through the PSU4SR, is critical in reducing the nuclear moments. The reduction for the neutrinoless ββ0ν-decay NM is less pronounced than in the case of ββ2ν-decay.

ii) This quenching effect is smaller on induced current moments $M^{0\nu}_P$ and $M^{0\nu}_M$ than on $M^{0\nu}_V$ and $M^{0\nu}_A$, which results from the standard V-A weak current.

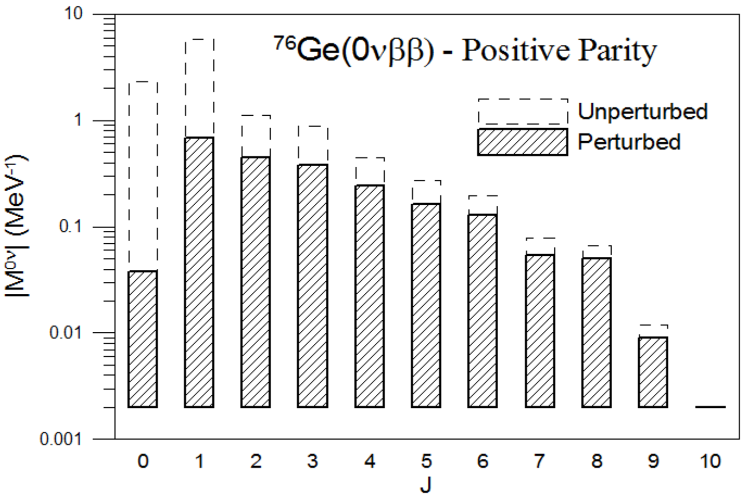
iii) Our $M^{0\nu}_M$ are, in principle, larger than in other calculations by the factor $(f_M/g_M)^2$ = 1.61, since we include the term $g_V = 2M_N$ in the NRA of the weak Hamiltonian.

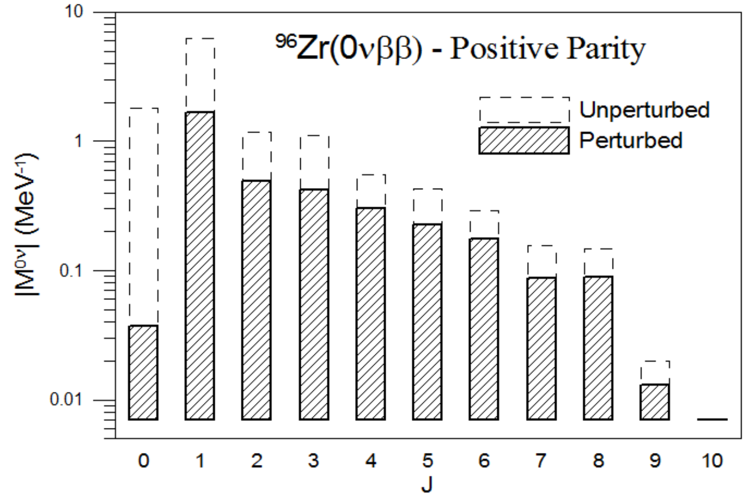
iv) Compared to the role played by the residual interaction in the pp channel, the FNS and SRC effects are relatively small. FNS in bare are ~15-20% and SRC are ~3-5% similar results in Simkovic PRC 79,055501(2009)

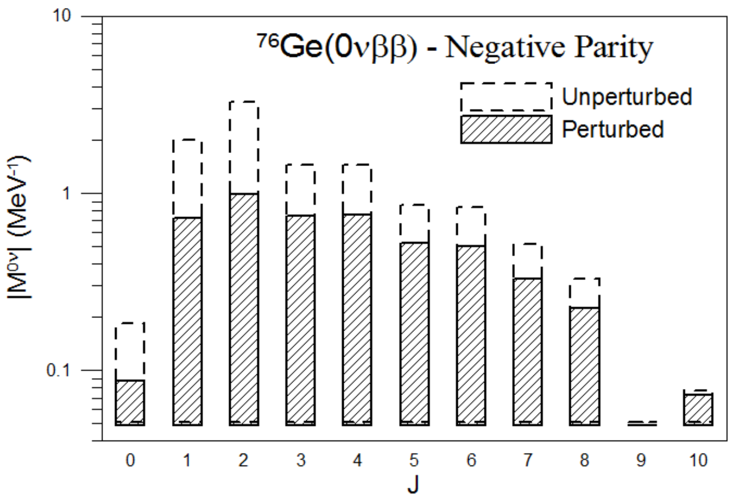
TABLE IV: Fine structure of $M^{0\nu}$ moments (normalized to g^2_A, with g_A = 1.27) for ^{76}Ge.

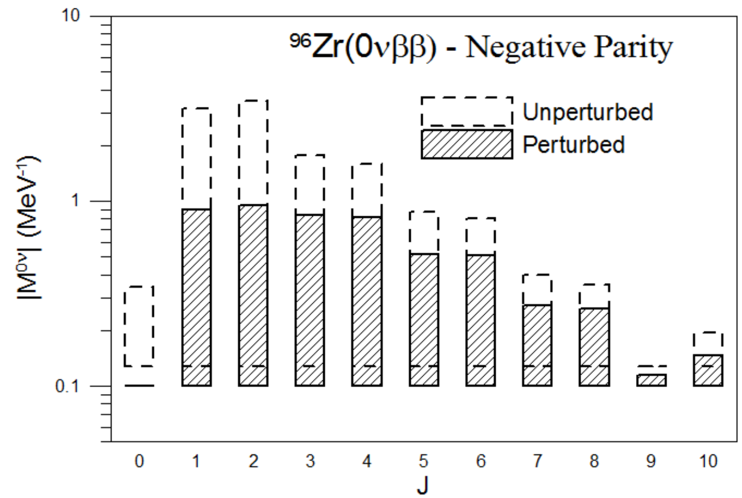
	BCS					QRPA				
J^π	$M_V^{0\nu}$	$M_A^{0\nu}$	$M_P^{0\nu}$	$M_M^{0\nu}$	$M^{0\nu}$	$M_V^{0\nu}$	$M_A^{0\nu}$	$M_P^{0\nu}$	$M_M^{0\nu}$	$M^{0\nu}$
0^+	1.06	0.00	0.00	0.00	1.06	0.02	0.00	0.00	0.00	0.02
1^+	0.00	4.75	-0.48	0.05	4.33	0.00	-0.39	-0.05	0.01	-0.43
2^+	0.36	0.54	0.00	0.05	0.95	0.14	0.24	0.00	0.03	0.40
3^+	0.00	1.01	-0.35	0.06	0.72	0.00	0.45	-0.16	0.03	0.32
4^+	0.14	0.23	0.00	0.04	0.42	0.08	0.14	0.00	0.03	0.24
5^+	0.00	0.40	-0.18	0.04	0.27	0.00	0.24	-0.11	0.03	0.16
6^+	0.06	0.10	0.00	0.03	0.18	0.04	0.07	0.00	0.02	0.13
7^+	0.00	0.15	-0.07	0.02	0.11	0.00	0.11	-0.05	0.02	0.08
8^+	0.02	0.03	0.00	0.01	0.06	0.01	0.02	0.00	0.01	0.05
9^+	0.00	0.06	-0.03	0.01	0.05	0.00	0.04	-0.02	0.01	0.03
10^+	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\pi = +$	1.64	7.27	-1.11	0.33	8.15	0.29	0.92	-0.39	0.19	1.00
0^-	0.00	0.15	-0.07	0.00	0.08	0.00	0.07	-0.04	0.00	0.03
1^-	0.47	0.62	0.00	0.03	1.12	0.15	0.24	0.00	0.01	0.40
2^-	0.00	2.26	-0.47	0.06	1.85	0.00	0.66	-0.16	0.02	0.52
3^-	0.24	0.43	0.00	0.06	0.72	0.11	0.23	0.00	0.03	0.37
4^-	0.00	0.80	-0.29	0.06	0.57	0.00	0.40	-0.15	0.03	0.28
5^-	0.12	0.21	0.00	0.05	0.38	0.06	0.13	0.00	0.03	0.23
6^-	0.00	0.36	-0.15	0.05	0.26	0.00	0.21	-0.09	0.03	0.15
7^-	0.05	0.10	0.00	0.04	0.19	0.03	0.07	0.00	0.03	0.12
8^-	0.00	0.10	-0.05	0.02	0.08	0.00	0.07	-0.03	0.01	0.05
9^-	0.00	0.01	0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.02
10^-	0.00	0.02	-0.01	0.00	0.01	0.00	0.02	-0.01	0.00	0.01
$\pi = -$	0.88	5.06	-1.04	0.37	5.28	0.35	2.11	-0.48	0.19	2.18

IV. Numerical results









IV. Numerical results

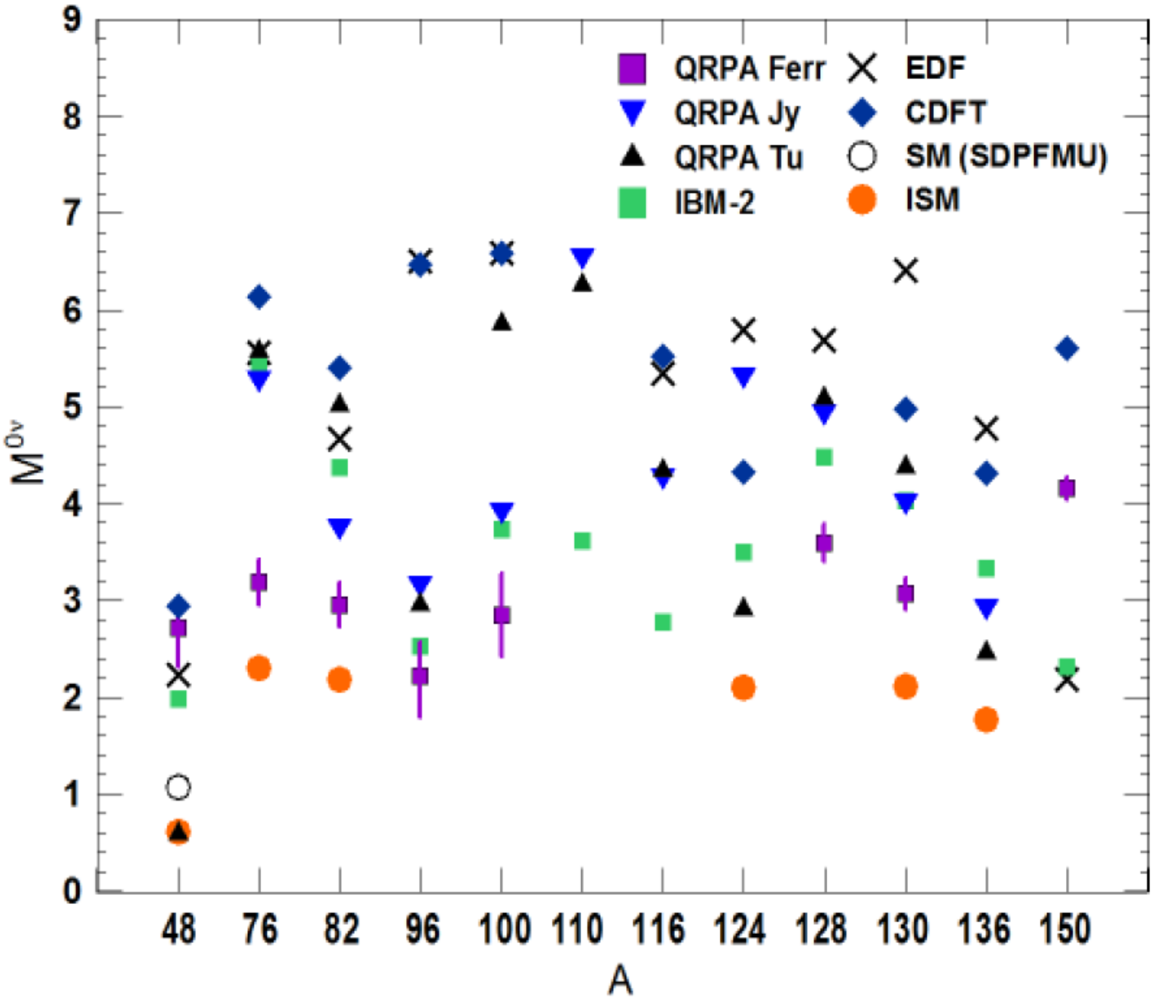


FIG. 3: ββν0 nuclear moments evaluated with several nuclear structure model calculations:
i) QRPA by Tubingen (QRPA Tu) **[8]** (g_A = 1.27), Jyvaskyla (QRPA Jy) **[10]** (g_A = 1.26) groups, and our results from Table III (QRPA Ferr) (g_A = 1.27),
ii) interacting shell model (ISM) **[52]** (g_A =1.25), Large-scale shell model (SM (SDPFMU)) **[60]**,
iii) interacting boson model (IBM2) **[61]** (g_A = 1.269),
vi) energy density functional method (EDF) **[62]** (g_A = 1.25), and covariant density functional theory (CDFT) **[63]** (g_A = 1.254).
All results are normalized to g_A^2.

[8] Simkovic et.al, PRC 87, 045501 (2013). [10] Hyvarinen & Suhonen, PRC 91, 024613 (2015).
[52] Menendez et.al, NPA 818, 139 (2009). [60] Iwata et.al, PRL 116, 112502 (2016) .
[61] Barea, PRC 87, 014315 (2013). [62] Vaquero et.al, PRL 111, 142501 (2013).
[63] Song et.al, PRC 90, 054309 (2014).

V. Final remarks

- The one-QRPA method is used for the first time to $\beta\beta$–decay.
- Stress once again the strong bonding between the residual interaction, GSC, PSU4SR and quenching of the $\beta\beta$–decay NM.
- To implement PSU4SR , we resort to energetic of GT resonances and minima of single β^{+}.
- The residual proton-neutron interaction plays a fundamental role in the PSU4SR, both ph and pp channels.
- We find Method IV preferable over Method II, basically because it only involves the nuclei within the isobaric triplet (N, Z), (N-1, Z + 1), (N-2, Z + 2) where the $\beta\beta$ decay occurs, while the last one involves also the nuclei (N + 1, Z-1) and (N-3, Z + 3).
- Our results for $\beta\beta0\nu$ NM are lower on average by 40%, attributing this difference to employ one-QRPA method instead usual two-QRPA-method.
- It is hard to say which is the best way to the way of restoration of symmetry, since $\beta\beta0\nu$ NM are not experimentally measurable.

Acknowlegments

IN ALTUM

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IV. Numerical results

Table V: $M^{2\nu}$ and $M^{0\nu}$ NM within Method II and Method IV :
(i) t_{sym} from P-U4SR

(ii) t↑
(iii) t↓ from $|M^{2\nu}_{exp}|^2$
Results for g_A=1.26 from Simkovic PRC 91, 024613 (2016) should be compared with ours t↓ of Method II.

		Method II			Method IV		
Nuclei	App	t	$M^{2\nu}$	$M^{0\nu}$	t	$M^{2\nu}$	$M^{0\nu}$
^{48}Ca	sym	1.200	0.124	3.66	1.200	0.018	2.72
	↑	1.186	0.040	4.08	1.209	0.038	2.64
	↓	1.168	-0.039	4.50	1.170	-0.038	2.96
^{76}Ge	sym	1.230	0.052	4.63	1.230	0.051	3.19
	↑	1.280	0.113	4.27	1.296	0.113	2.79
	↓	1.005	-0.113	5.81	0.887	-0.113	4.79
	Ref. [10]			5.26			
^{82}Se	sym	1.300	0.051	3.35	1.300	0.062	2.96
	↑	1.359	0.083	3.08	1.326	0.083	2.81
	↓	0.906	-0.083	4.70	1.003	-0.083	4.37
	Ref. [10]			4.69			
^{96}Zr	sym	1.550	0.014	4.89	1.550	0.023	2.22
	↑	1.573	0.081	4.60	1.573	0.081	2.04
	↓	1.506	-0.080	5.35	1.481	-0.080	2.68
	Ref. [10]			3.14			
^{100}Mo	sym	1.490	0.173	4.45	1.490	0.034	2.85
	↑	1.495	0.186	4.39	1.525	0.186	2.48
	↓	1.229	-0.185	6.37	1.347	-0.185	3.92
	Ref. [10]			3.90			
^{128}Te	sym	1.410	0.073	3.14	1.410	0.083	3.59
	↑	1.354	0.046	3.32	1.351	0.046	3.86
	↓	1.119	-0.046	4.04	1.165	-0.046	4.64
	Ref. [10]			4.92			
^{130}Te	sym	1.450	0.119	3.77	1.450	0.081	3.07
	↑	1.302	0.031	4.34	1.343	0.031	3.48
	↓	1.192	-0.031	4.78	1.175	-0.031	4.07
	Ref. [10]			4.00			
^{150}Nd	sym	1.290	-0.084	4.66	1.290	-0.067	4.16
	↑	1.636	0.058	3.71	1.637	0.058	3.10
	↓	1.365	-0.058	4.47	1.324	-0.058	4.06

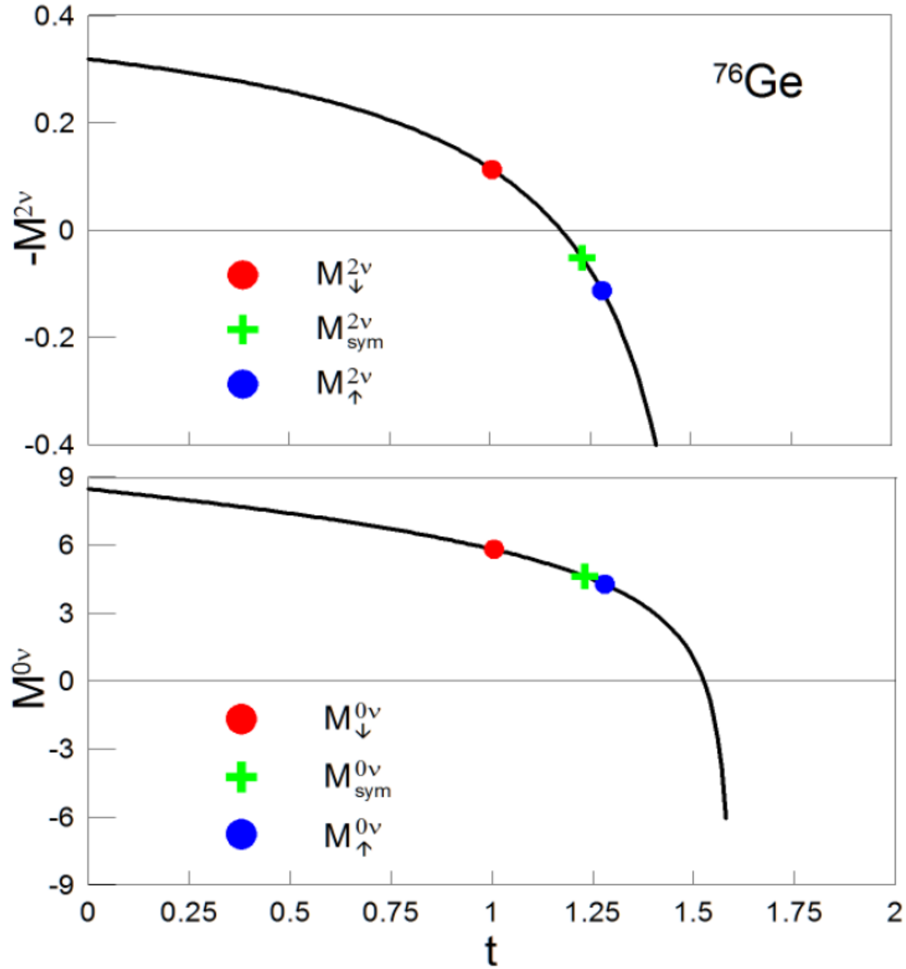

FIG. 2: Isoscalar parameters t in ^{76}Ge within the Method II for $|M^{2\nu}_{exp}|$=0.113. The NM $M^{2\nu}$ is given in natural units, while $M^{0\nu}$ is dimensionless. It should be noted that M2n is negative at t = 0.