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0νββ decay nuclear matrix elements with the generator coordinate method

Tomás R. Rodríguez

INT-Program 17-2a

Seattle, June 13-14, 2017

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD
Investigación
Programa Ramón y Cajal

UNIVERSIDAD AUTONOMA DE MADRID

0νββ decay nuclear matrix elements with energy density functional (EDF) methods

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Outline

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Outline

1. Introduction

2. EDF applications

3. GCM-EDF vs. Shell Model

4. Summary and open questions

NME: the problem

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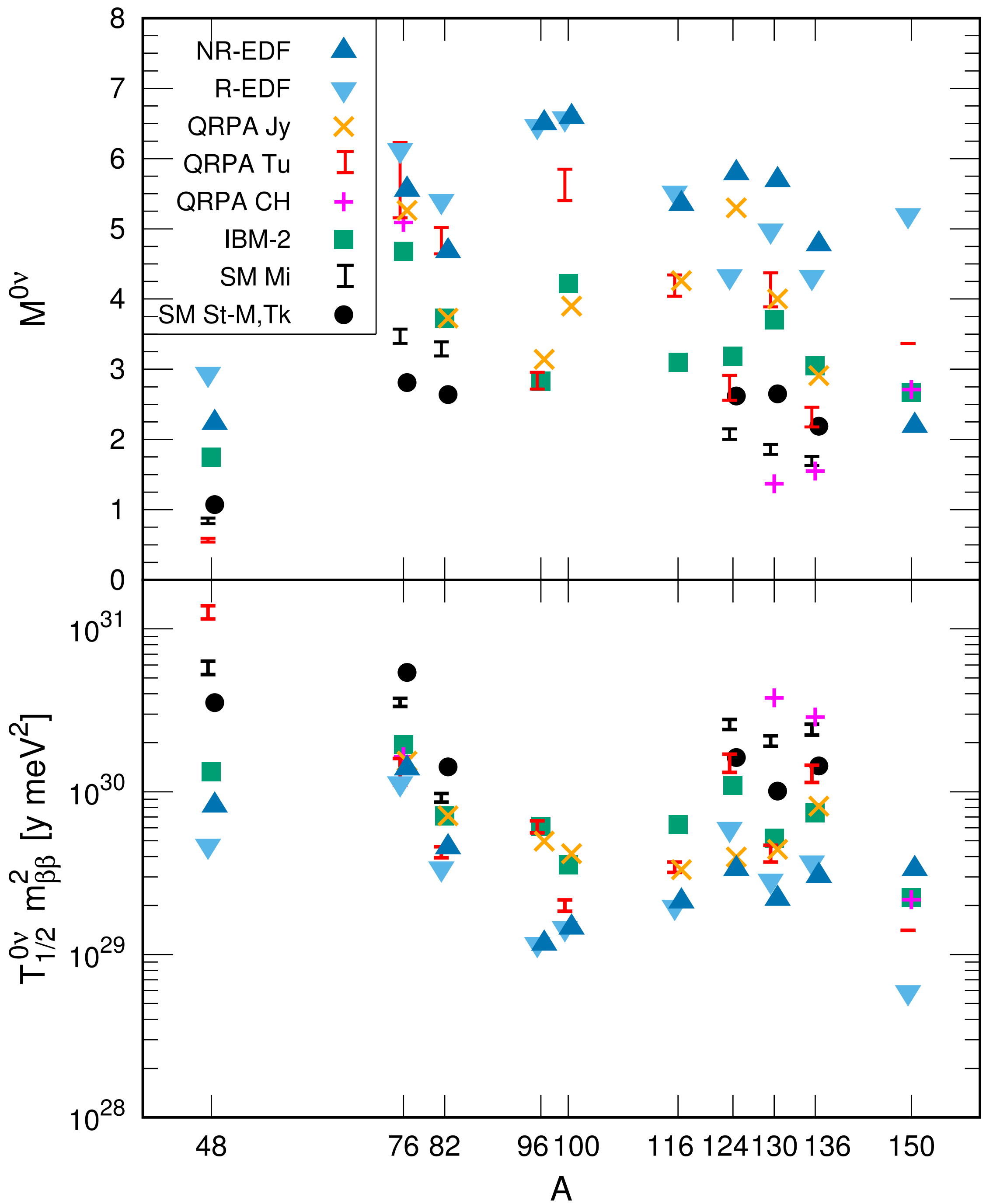

J. Menéndez, J. Engel 2016

➡ Different many-body methods provide different 0νββ NMEs

➡ Where the differences come from?

- ✓ Correlations are not the same.
- ✓ Interactions are different.
- ✓ Valence spaces are different.
- ✓ Transition operator is (sometimes) different.

NME: Starting points

- Leading lepton number violating process contributing to 0νββ decay
  - Exchange of light Majorana neutrino.
  - Exchange of heavy Majorana neutrino.
  - Leptoquarks.
  - Supersymmetric particles.
  - …
- Transition operator connecting initial and final states
  - Relativistic/Non-relativistic.
  - Nucleon size effects.
  - Two-body weak currents.
  - Form factors.
  - Short-range correlations.
  - Closure approximation.
  - …
- Nuclear structure method (fully consistent or not with the operator) for calculating these NME.
  - Correlations.
  - Symmetry conservation.
  - Valence space.
  - …

This is a general method based on the concept of configuration mixing. The wave function that describes the system in this framework can be expressed as:

$$|\Psi^\sigma\rangle = \int f^\sigma(\vec{q})|\Phi(\vec{q})\rangle d\vec{q}$$

$\rightarrow \{|\Phi(\vec{q})\}$ is a set of (in general) non-orthonormal many-body wave functions that depends parametrically on the collective variables $\vec{q}$, called generating coordinates.

$\rightarrow f^\sigma(\vec{q})$ are found by minimizing the energy:

$$E\left[|\Psi^\sigma\right] = \frac{\langle\Psi^\sigma|\hat{H}|\Psi^\sigma\rangle}{\langle\Psi^\sigma|\Psi^\sigma\rangle}$$

Generator Coordinate Method

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$$\delta E\left[|\Psi^{\sigma}\rangle\right] = 0 \Rightarrow \int \mathcal{H}(\vec{q},\vec{q}\,')f^{\sigma}(\vec{q}\,')d\vec{q}\,' = E^{\sigma}\int \mathcal{N}(\vec{q},\vec{q}\,')f^{\sigma}(\vec{q}\,')d\vec{q}\,'$$

Hill-Wheeler-Griffin (HWG) equations

Generator Coordinate Method

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Hill-Wheeler-Griffin (HWG) equations

$\mathcal{N}(\vec{q},\vec{q}\,') = \langle\Phi(\vec{q})|\Phi(\vec{q}\,')\rangle$ norm overlap matrix

$\mathcal{H}(\vec{q},\vec{q}\,') = \langle\Phi(\vec{q})|\hat{H}|\Phi(\vec{q}\,')\rangle$ hamiltonian overlap matrix

How to solve the HGW equations

$$\int \mathcal{H}(\vec{q},\vec{q}\,')f^{\sigma}(\vec{q}\,')d\vec{q}\,' = E^{\sigma}\int \mathcal{N}(\vec{q},\vec{q}\,')f^{\sigma}(\vec{q}\,')d\vec{q}\,'$$

Generator Coordinate Method

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1. Find the eigenvalues and eigenvectors of the norm overlap matrix:

$$\int \mathcal{N}(\vec{q},\vec{q}\,')u_{\Lambda}(\vec{q}\,')d\vec{q}\,' = n_{\Lambda}u_{\Lambda}(\vec{q})$$

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2. From the eigenvalues and eigenvectors of the norm overlap matrix, build an orthonormal basis removing the linear dependencies **(natural basis)**:

$$|\Lambda\rangle = \frac{1}{\sqrt{n_{\Lambda}}}\int u_{\Lambda}(\vec{q})|\Phi(\vec{q})\rangle d\vec{q}\ ; \quad n_{\Lambda} > 0 \qquad\qquad \langle\Lambda|\Lambda'\rangle = \delta_{\Lambda\Lambda'}$$

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3. Re-write the GCM wave functions and the HWG equation in the natural basis:

$$|\Psi^{\sigma}\rangle = \sum_{\Lambda} g^{\sigma}_{\Lambda}|\Lambda\rangle = \int |\Phi(\vec{q})\rangle \sum_{\Lambda}\frac{1}{\sqrt{n_{\Lambda}}}g^{\sigma}_{\Lambda}u_{\Lambda}(\vec{q})d\vec{q}$$

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$f^{\sigma}(\vec{q})$

Generator Coordinate Method

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HGW equations are now regular eigenvalue problems

Generator Coordinate Method

How to solve the HGW equations

$$\Rightarrow \sum_{\Lambda'} \langle\Lambda|\hat{H}|\Lambda'\rangle g^{\sigma}_{\Lambda'} = E^{\sigma} g^{\sigma}_{\Lambda}$$

4. Hamiltonian (or any other operator $\hat{O}$) matrix elements in the natural basis:

$$\langle\Lambda|\hat{O}|\Lambda'\rangle = \int \left(\frac{u_{\Lambda}(\vec{q})}{\sqrt{n_{\Lambda}}}\right)^{*} \langle\Phi(\vec{q})|\hat{O}|\Phi(\vec{q}\,')\rangle \left(\frac{u_{\Lambda'}(\vec{q}\,')}{\sqrt{n_{\Lambda'}}}\right) d\vec{q}d\vec{q}\,'$$

Generator Coordinate Method

How to solve the HGW equations

$$\Rightarrow \sum_{\Lambda'} \langle\Lambda|\hat{H}|\Lambda'\rangle g^{\sigma}_{\Lambda'} = E^{\sigma} g^{\sigma}_{\Lambda}$$

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matrix elements between different “deformations”

How to solve the HGW equations

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5. Hamiltonian (or any other operator $\hat{O}$) expectation values in the GCM states (and transitions):

$$\langle \Psi^{\sigma} | \hat{O} | \Psi^{\sigma} \rangle = \sum_{\Lambda\Lambda'} g^{\sigma *}_{\Lambda} \langle \Lambda | \hat{O} | \Lambda' \rangle g^{\sigma}_{\Lambda'}$$

How to solve the HGW equations

$$\Rightarrow \sum_{\Lambda'}\langle\Lambda|\hat{H}|\Lambda'\rangle g^{\sigma}_{\Lambda'} = E^{\sigma}g^{\sigma}_{\Lambda}$$

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6. Collective wave functions: Weight of the different $\vec{q}$ in the GCM wave function:

$$F^{\sigma}(\vec{q}) = \sum_{\Lambda} g^{\sigma}_{\Lambda}u_{\Lambda}(\vec{q})$$

Generator Coordinate Method

How to solve the HGW equations

$$\Rightarrow \sum_{\Lambda'} \langle \Lambda | \hat{H} | \Lambda' \rangle g^{\sigma}_{\Lambda'} = E^{\sigma} g^{\sigma}_{\Lambda}$$

7. Transition matrix elements in the natural basis:

$$\langle \Lambda_f | \hat{T}^{i\rightarrow f} | \Lambda_i \rangle = \int \left(\frac{u_{\Lambda_f}(\vec{q}_f)}{\sqrt{n_{\Lambda_f}}} \right)^{*} \langle \Phi(\vec{q}_f) | \hat{T}^{i\rightarrow f} | \Phi(\vec{q}_i) \rangle \left(\frac{u_{\Lambda_i}(\vec{q}_i)}{\sqrt{n_{\Lambda_i}}} \right) d\vec{q}_f d\vec{q}_i$$

8. Transition matrix elements between GCM states:

$$\langle \Psi^{\sigma_f}_{f} | \hat{T}^{i\rightarrow f} | \Psi^{\sigma_i}_{i} \rangle = \sum_{\Lambda_f \Lambda_i} g^{\sigma_f *}_{\Lambda_f} \langle \Lambda_f | \hat{T}^{i\rightarrow f} | \Lambda_i \rangle g^{\sigma_i}_{\Lambda_i}$$

Generator Coordinate Method

How to solve the HGW equations

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transition matrix elements between different “deformations”

8. Transition matrix elements between GCM states:

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How to solve the HGW equations

$$\Rightarrow \sum_{\Lambda'}\langle\Lambda|\hat{H}|\Lambda'\rangle g^{\sigma}_{\Lambda'} = E^{\sigma} g^{\sigma}_{\Lambda}$$

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Generator Coordinate Method

REMARKS

- GCM ground states are variational approaches to the exact ground state wave functions.

- The quality of the approximation depends on the sensitivity of the collective coordinates to the nuclear Hamiltonian and/or transition operators.

- Very intuitive physical insight about the role of collective degrees of freedom on $0\nu\beta\beta$ NMEs.

IMPLEMENTATIONS

- Non-relativistic Gogny and Relativistic energy density functionals (EDF).

- SO(8) and Pairing (isoscalar and isovector) plus quadrupole Hamiltonians.

- Shell Model interactions in reduced valence spaces (in progress).

Gogny interaction

Effective nucleon-nucleon interaction: Gogny force (D1S/D1M)

$$V(1,2) = \sum_{i=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2}\left(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau\right)$$

$$+iW_0(\sigma_1+\sigma_2)\vec{k}\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} + V_{\text{Coulomb}}(\vec{r}_1,\vec{r}_2)$$

$$+t_3(1+x_0P^\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left((\vec{r}_1+\vec{r}_2)/2\right)$$

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2-body potential

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Density dependent term

Gogny interaction

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2-body potential

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Density dependent term

- Other alternatives: Skyrme, relativistic Lagrangians, BCPM, …

EDF axial

- Initial intrinsic states: PN-VAP

$$E^{NZ}[|\Phi(\vec{q})\rangle] = \frac{\langle\Phi(\vec{q})|\hat{H}\hat{P}^N\hat{P}^Z|\Phi(\vec{q})\rangle}{\langle\Phi(\vec{q})|\hat{P}^N\hat{P}^Z|\Phi(\vec{q})\rangle} - \vec{\lambda}_{\vec{q}}\left(\langle\Phi(\vec{q})|\hat{\vec{Q}}|\Phi(\vec{q})\rangle - \vec{q}\right)$$

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- Intermediate Particle Number and Angular Momentum Projected states

$$|J;NZ;\vec{q}\rangle = \frac{2J+1}{2}\int_0^\pi d_{00}^{J*}(\beta)e^{-i\beta\hat{J}_y}\hat{P}^N\hat{P}^Z|\Phi(\vec{q})\rangle d\beta$$

EDF axial

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- Final GCM states $|J;NZ;\sigma\rangle = \int f^{J;NZ;\sigma}(\vec{q})|J;NZ;\vec{q}\rangle d\vec{q}$

EDF axial

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- Final GCM states $|J;NZ;\sigma\rangle = \int f^{J;NZ;\sigma}(\vec{q})|J;NZ;\vec{q}\rangle d\vec{q}$

$$\int\left(\mathcal{H}^{J;NZ}(\vec{q},\vec{q}\,') - E^{J;NZ;\sigma}\mathcal{N}^{J;NZ}(\vec{q},\vec{q}\,')\right)f^{J;NZ;\sigma}(\vec{q}\,')d\vec{q}\,' = 0$$

$$\mathcal{H}^{J;NZ}(\vec{q},\vec{q}\,') = \langle J;NZ;\vec{q}|\hat{H}|J;NZ;\vec{q}\,'\rangle$$

$$\mathcal{N}^{J;NZ}(\vec{q},\vec{q}\,') = \langle J;NZ;\vec{q}|J;NZ;\vec{q}\,'\rangle$$

EDF axial

- Initial intrinsic states: ~~PN-VAP~~ plain HFB

$$E^{\cancel{NZ}}[|\Phi(\vec{q})\rangle] = \frac{\langle\Phi(\vec{q})|\hat{H}\cancel{\hat{P}^N}\cancel{\hat{P}^Z}|\Phi(\vec{q})\rangle}{\langle\Phi(\vec{q})|\cancel{\hat{P}^N}\cancel{\hat{P}^Z}|\Phi(\vec{q})\rangle} - \vec{\lambda}_{\vec{q}}\left(\langle\Phi(\vec{q})|\hat{\vec{Q}}|\Phi(\vec{q})\rangle - \vec{q}\right)$$

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$$\mathcal{H}^{J;NZ}(\vec{q},\vec{q}\,') = \langle J;NZ;\vec{q}|\hat{H}|J;NZ;\vec{q}\,'\rangle$$

$$\mathcal{N}^{J;NZ}(\vec{q},\vec{q}\,') = \langle J;NZ;\vec{q}|J;NZ;\vec{q}\,'\rangle$$

EDF axial

1. Axial states $K = 0$
2. Angular momentum $J = 0$
3. Quadrupole deformations $q = q_{20}$
4. Quadrupole and pairing pp/nn correlations $q = (q_{20}, \delta)$
5. Quadrupole and pn correlations $q = (q_{20}, p_0)$
6. Quadrupole and octupole deformations $q = (q_{20}, q_{30})$

$\longrightarrow$

$$|0; N_i Z_i; \sigma\rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0;N_i Z_i;\sigma} |\Lambda_i^{0;N_i Z_i}\rangle$$

$$|0; N_f Z_f; \sigma\rangle = \sum_{\Lambda_f} G_{\Lambda_f}^{0;N_f Z_f;\sigma} |\Lambda_f^{0;N_f Z_f}\rangle$$

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$\longrightarrow$

$$|0; N_iZ_i; \sigma\rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0;N_iZ_i;\sigma} |\Lambda_i^{0;N_iZ_i}\rangle$$

$$|0; N_fZ_f; \sigma\rangle = \sum_{\Lambda_f} G_{\Lambda_f}^{0;N_fZ_f;\sigma} |\Lambda_f^{0;N_fZ_f}\rangle$$

TRANSITIONS:

$$M_\xi^{0\nu\beta\beta} = \langle 0_f^+ | \hat{O}_\xi^{0\nu\beta\beta} | 0_i^+ \rangle = \langle 0; N_fZ_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_iZ_i \rangle =$$

$$\sum_{\Lambda_f \Lambda_i} \left(G_{\Lambda_f}^{0;N_fZ_f} \right)^* \langle \Lambda_f^{0;N_fZ_f} | \hat{O}_\xi^{0\nu\beta\beta} | \Lambda_i^{0;N_iZ_i} \rangle G_{\Lambda_i}^{0;N_iZ_i} = \sum_{q_i q_f; \Lambda_f \Lambda_i}$$

$$\left(\frac{u_{q_f,\Lambda_f}^{0;N_fZ_f}}{\sqrt{n_{\Lambda_f}^{0;N_fZ_f}}} \right)^* \left(G_{\Lambda_f}^{0;N_fZ_f} \right)^* \langle 0; N_fZ_f; q_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_iZ_i; q_i \rangle \left(G_{\Lambda_i}^{0;N_iZ_i} \right) \left(\frac{u_{q_i,\Lambda_i}^{0;N_iZ_i}}{\sqrt{n_{\Lambda_i}^{0;N_iZ_i}}} \right)$$

EDF axial

1. Axial states $K = 0$
2. Angular momentum $J = 0$
3. Quadrupole deformations $q = q_{20}$
4. Quadrupole and pairing pp/nn correlations $q = (q_{20}, \delta)$
5. Quadrupole and pn correlations $q = (q_{20}, p_0)$
6. Quadrupole and octupole deformations $q = (q_{20}, q_{30})$

$\longrightarrow$

$$|0; N_i Z_i; \sigma\rangle = \sum_{\Lambda_i} G^{0;N_i Z_i;\sigma}_{\Lambda_i} |\Lambda^{0;N_i Z_i}_i\rangle$$

$$|0; N_f Z_f; \sigma\rangle = \sum_{\Lambda_f} G^{0;N_f Z_f;\sigma}_{\Lambda_f} |\Lambda^{0;N_f Z_f}_f\rangle$$

TRANSITIONS:

$$M^{0\nu\beta\beta}_{\xi} = \langle 0^+_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0^+_i\rangle = \langle 0; N_f Z_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0; N_i Z_i\rangle =$$

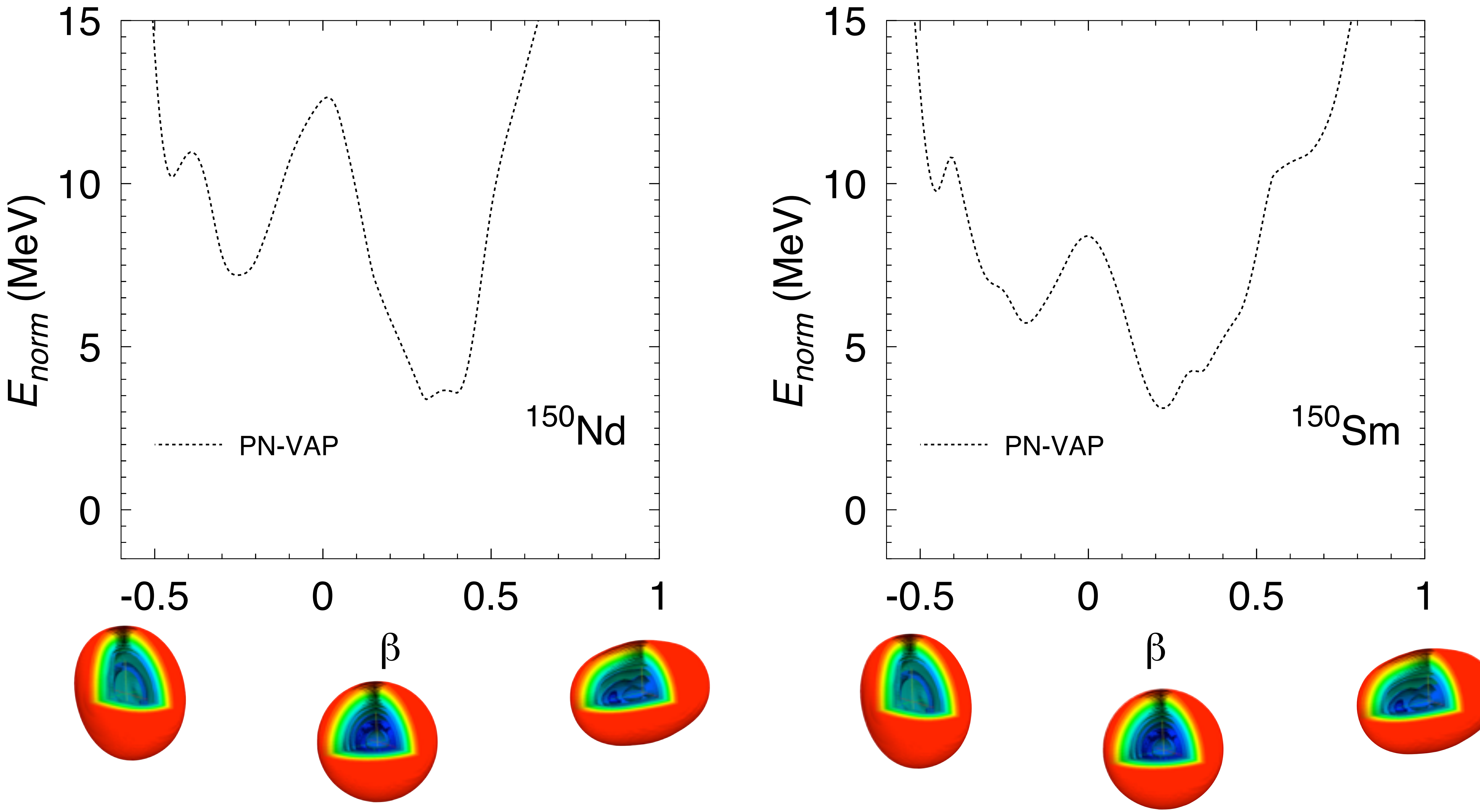
$$\sum_{\Lambda_f \Lambda_i} \left(G^{0;N_f Z_f}_{\Lambda_f}\right)^* \langle \Lambda^{0;N_f Z_f}_f|\hat{O}^{0\nu\beta\beta}_{\xi}|\Lambda^{0;N_i Z_i}_i\rangle G^{0;N_i Z_i}_{\Lambda_i} = \sum_{q_i q_f;\Lambda_f \Lambda_i}$$

$$\left(\frac{u^{0;N_f Z_f}_{q_f,\Lambda_f}}{\sqrt{n^{0;N_f Z_f}_{\Lambda_f}}}\right)^* \left(G^{0;N_f Z_f}_{\Lambda_f}\right)^* \langle 0; N_f Z_f; q_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0; N_i Z_i; q_i\rangle \left(G^{0;N_i Z_i}_{\Lambda_i}\right) \left(\frac{u^{0;N_i Z_i}_{q_i,\Lambda_i}}{\sqrt{n^{0;N_i Z_i}_{\Lambda_i}}}\right)$$

Matrix elements of the double beta transition operators between particle number and angular momentum projected states

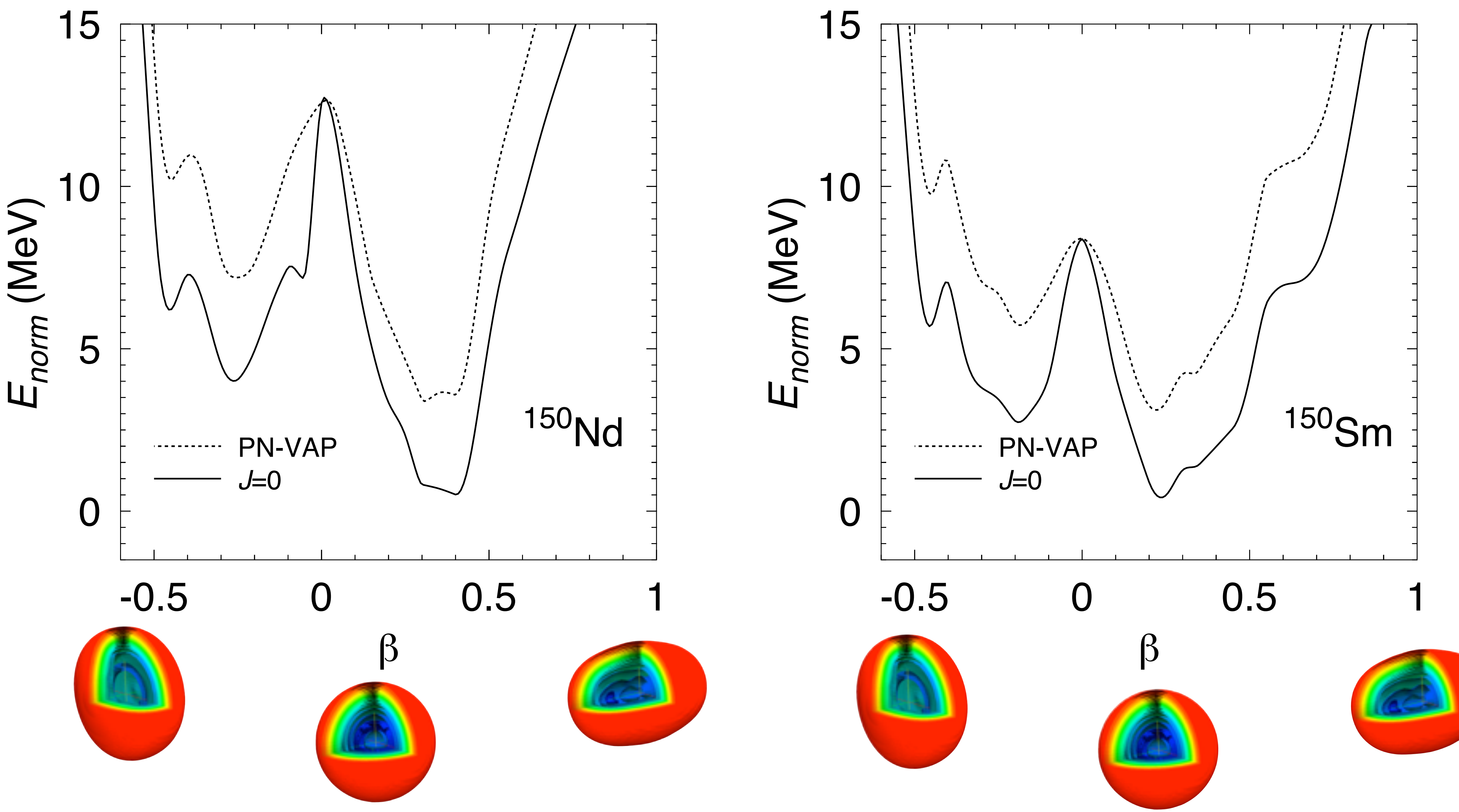
EDF axial

Determination of initial and final states (I)



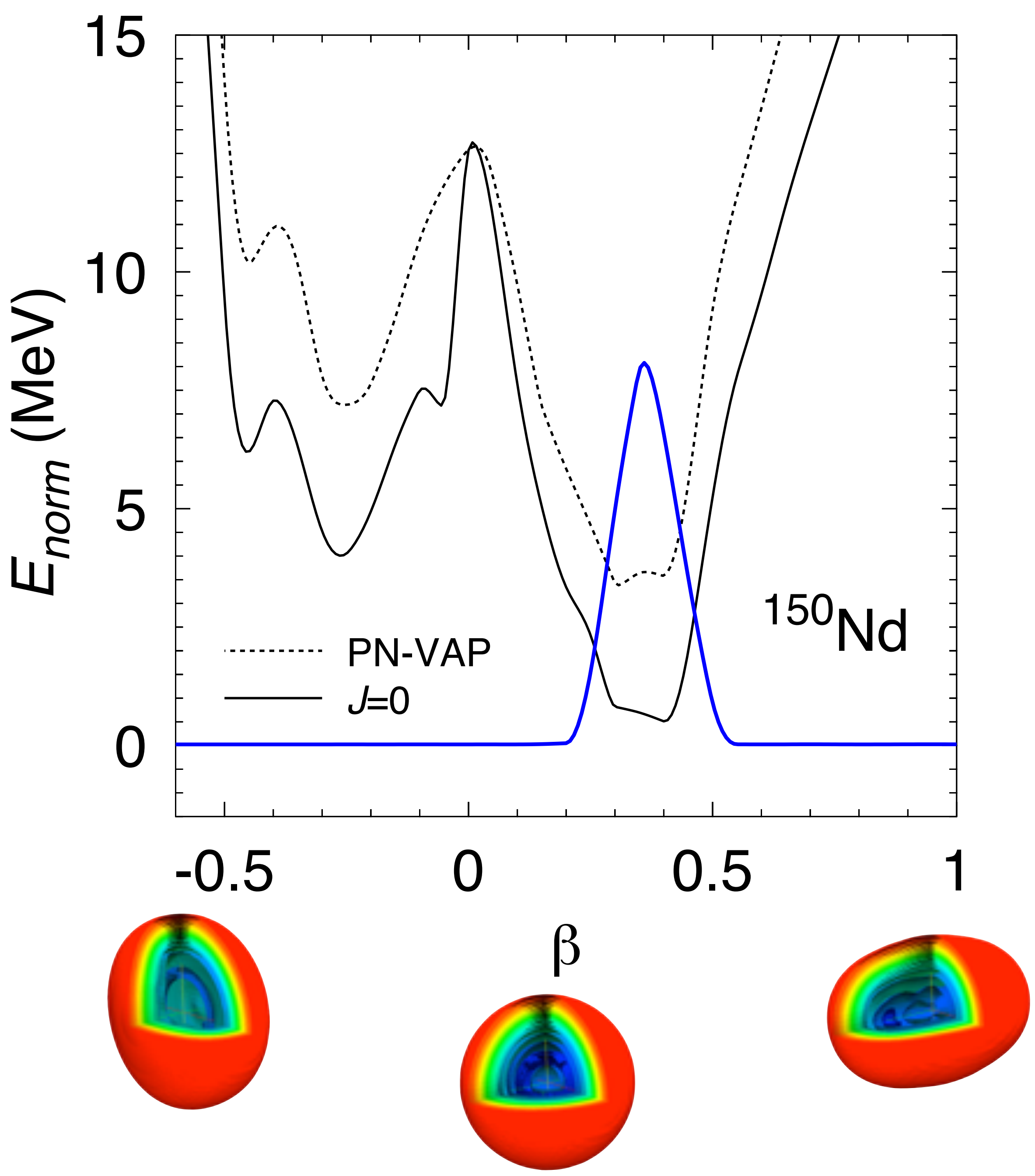
EDF axial

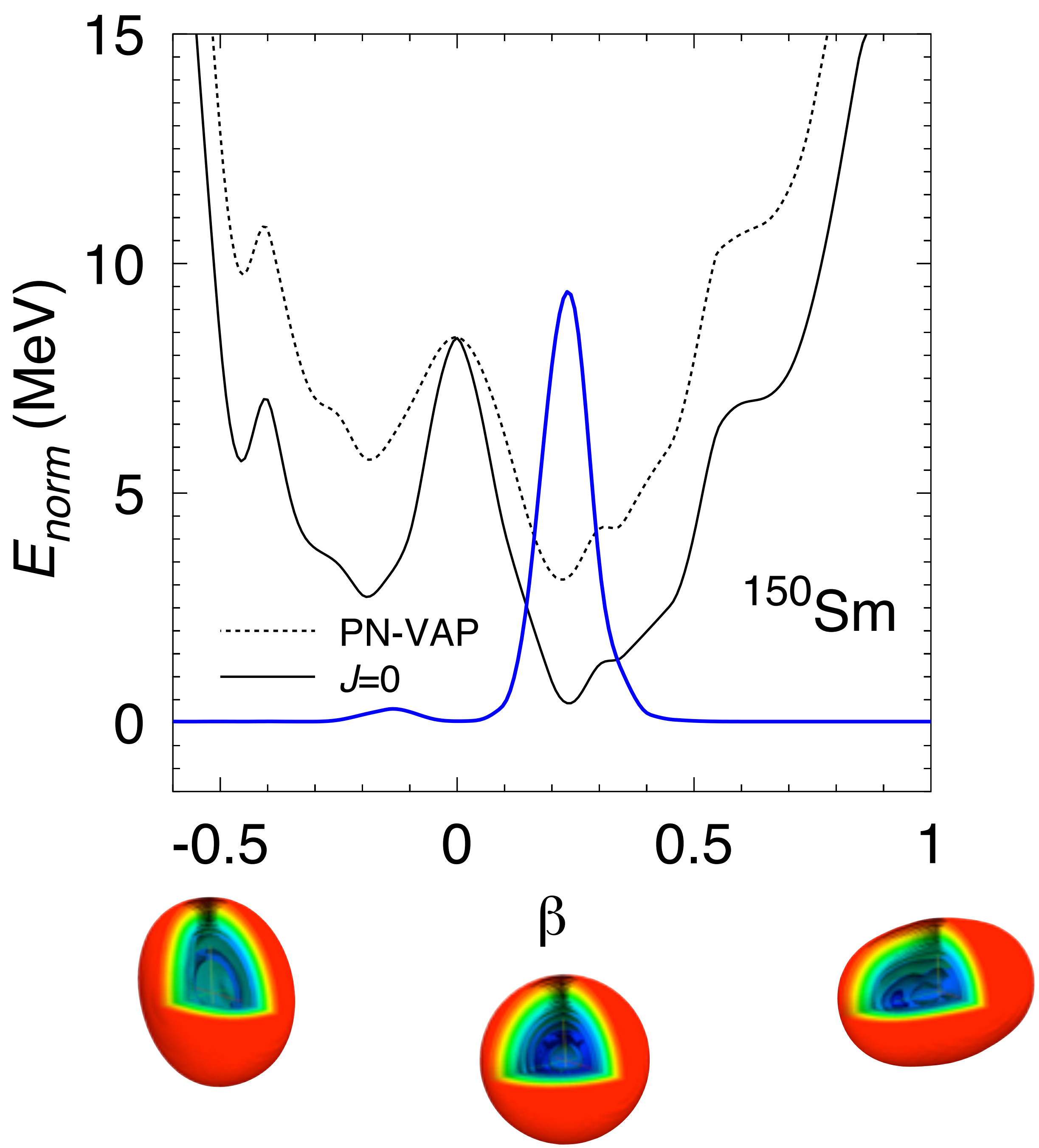
Determination of initial and final states (II)



EDF axial

Determination of initial and final states (and III)





Neutrinoless double beta decay candidates

440 T.R. Rodríguez, G. Martinez-Pinedo / Progress in Particle and Nuclear Physics 66 (2011) 436–440

Table 1
Masses, rms charge radii and total Gamow–Teller strengths $S_{-(+)}$ for mother (granddaughter) calculated with Gogny D1S GCM+PNAMP functional compared to experimental values. Theoretical values for $S_{+/-}$ are quenched by a factor $(0.74)^2$.

Isotope	BEth (MeV)	BE^{exp} (MeV) [27]	Rth (fm)	R^{exp}(fm) [28]	$S^{theo}_{-/+}$	$S^{exp}_{-/+}$
^{48}Ca	420.623	415.991	3.465	3.473	13.55	(14.4 ± 2.2 [29])
^{48}Ti	423.597	418.699	3.557	3.591	1.99	(1.9 ± 0.5 [29])
^{76}Ge	664.204	661.598	4.024	4.081	20.97	(19.89 [30])
^{76}Se	664.949	662.072	4.074	4.139	1.49	(1.45 ± 0.07 [31])
^{82}Se	716.794	712.842	4.100	4.139	23.56	(21.91 [30])
^{82}Kr	717.859	714.273	4.130	4.192	1.24	
^{96}Zr	829.432	828.995	4.298	4.349	27.63	
^{96}Mo	833.793	830.778	4.319	4.384	2.56	(0.29 ± 0.08 [32])
^{100}Mo	861.526	860.457	4.372	4.445	27.87	(26.69 [30])
^{100}Ru	864.875	861.927	4.388	4.453	2.48	
^{116}Cd	988.469	987.440	4.556	4.628	34.30	(32.70 [30])
^{116}Sn	991.079	988.684	4.567	4.626	2.61	($1.09^{+0.13}_{-0.57}$ [33])
^{124}Sn	1051.668	1049.96	4.622	4.675	40.65	
^{124}Te	1051.562	1050.69	4.664	4.717	1.63	
^{128}Te	1082.257	1081.44	4.686	4.735	40.48	(40.08 [30])
^{128}Xe	1080.996	1080.74	4.723	4.775	1.45	
^{130}Te	1096.627	1095.94	4.695	4.742	43.57	(45.90 [30])
^{130}Xe	1097.245	1096.91	4.732	4.783	1.19	
^{136}Xe	1143.333	1141.88	4.756	4.799	46.71	
^{136}Ba	1143.202	1142.77	4.786	4.832	0.96	
^{150}Nd	1234.512	1237.45	5.034	5.041	50.32	
^{150}Sm	1235.936	1239.25	5.041	5.040	1.45	

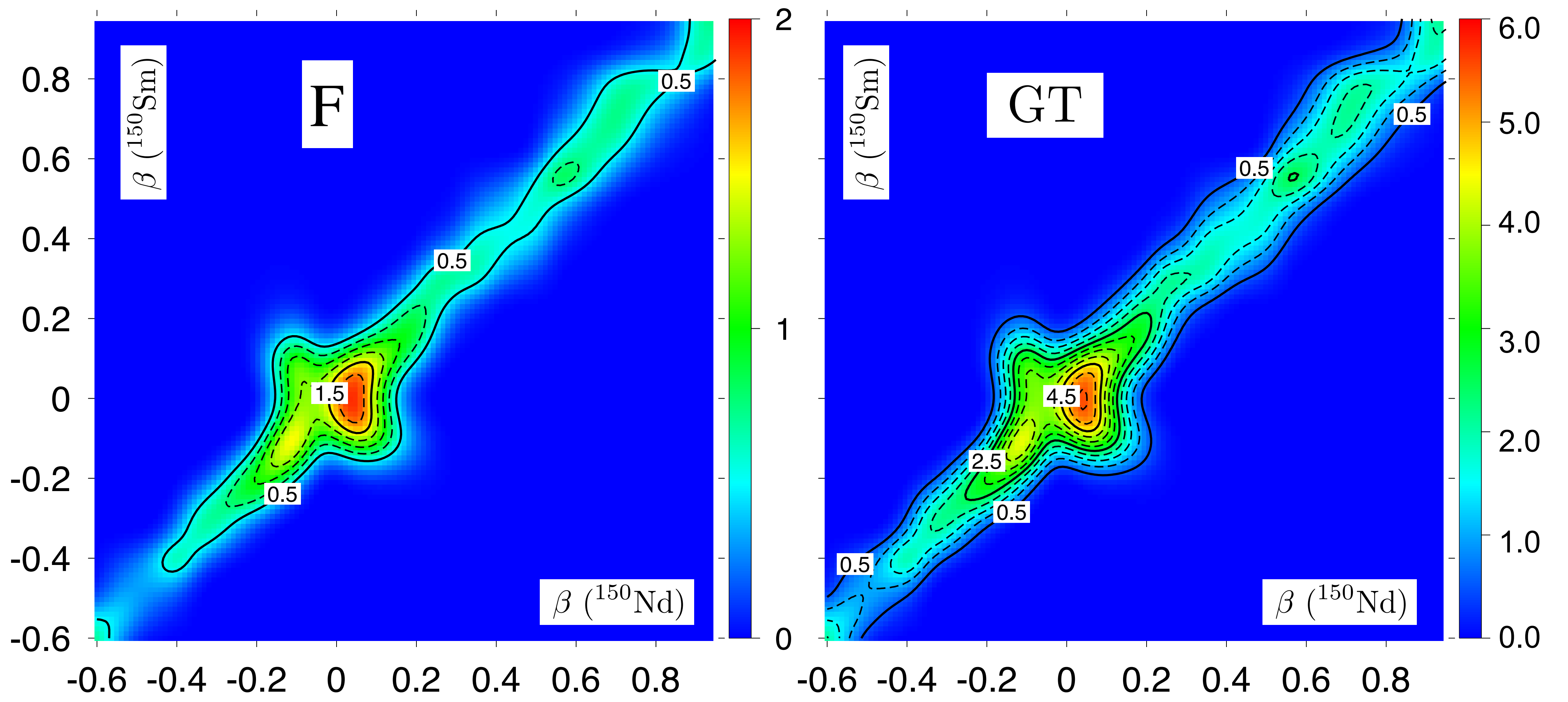
Good agreement between experimental and theoretical Q-values, radii and total strength (quenched)

NME: axial quadrupole deformation

$$\frac{\langle 0;N_fZ_f;q_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0;N_iZ_i;q_i\rangle}{\sqrt{\langle 0;N_fZ_f;q_f|0;N_fZ_f;q_f\rangle\langle 0;N_iZ_i;q_i|0;N_iZ_i;q_i\rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 105, 252503 (2010)

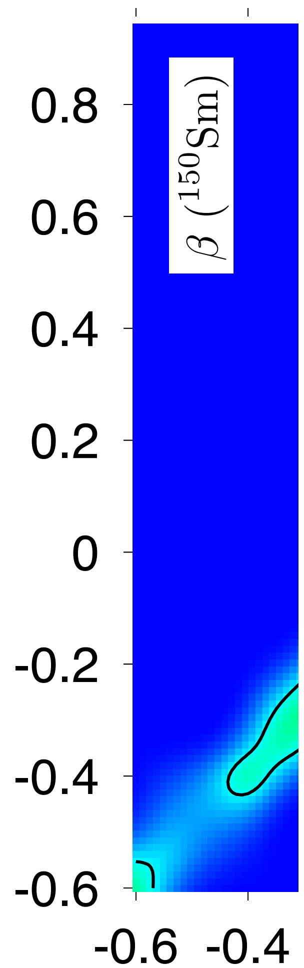


- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found

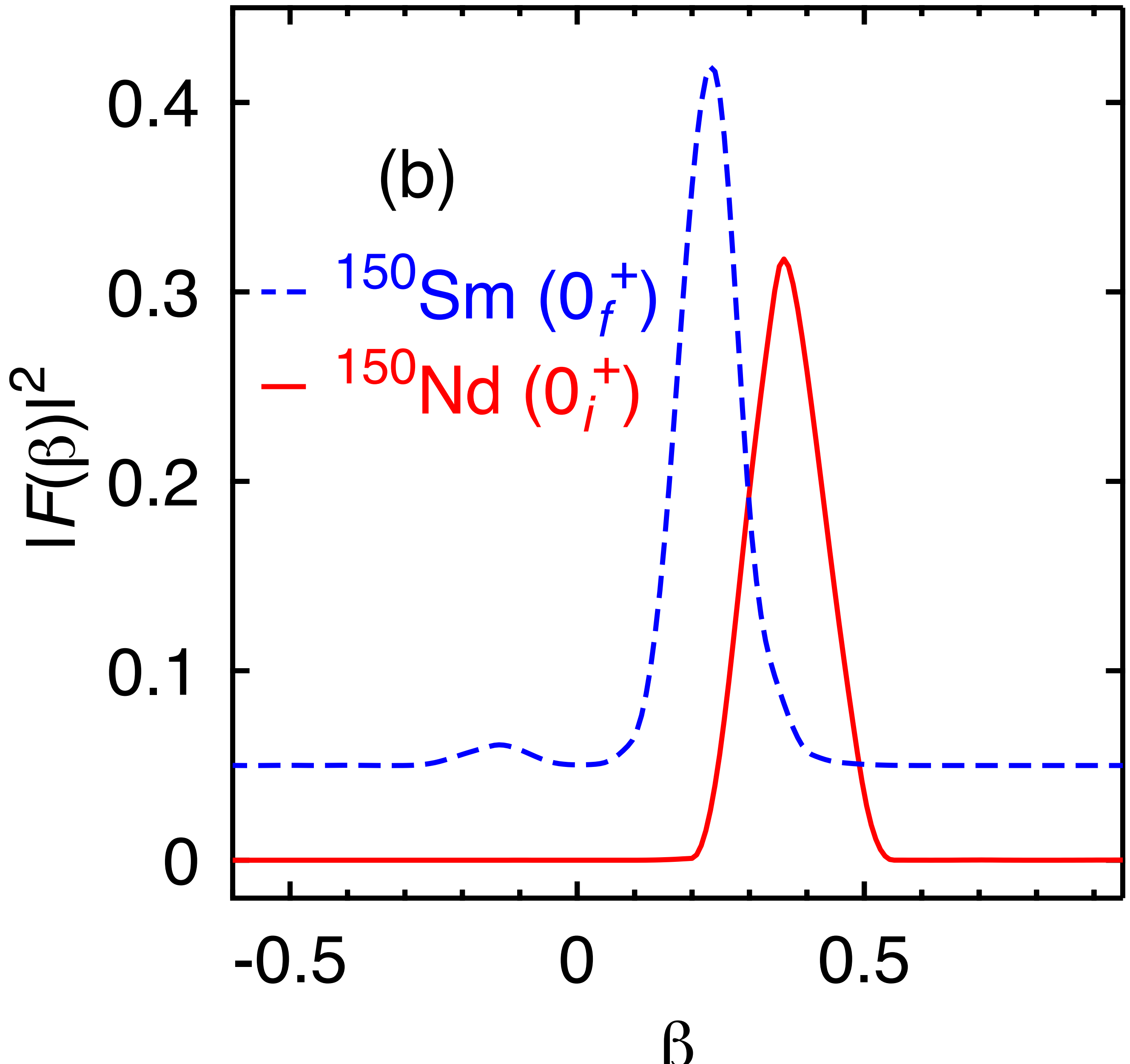
NME: axial quadrupole deformation

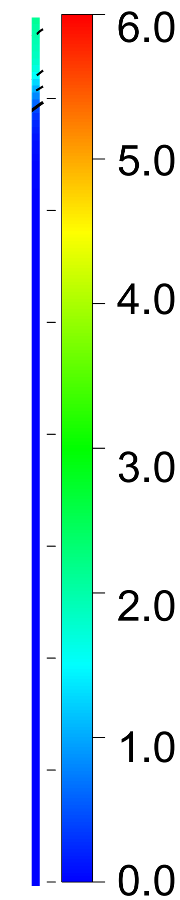
T.R.R., Martínez-Pinedo, PRL 105, 252503 (2010)

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_i Z_i; q_i\rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f\rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i\rangle}}$$



A=150





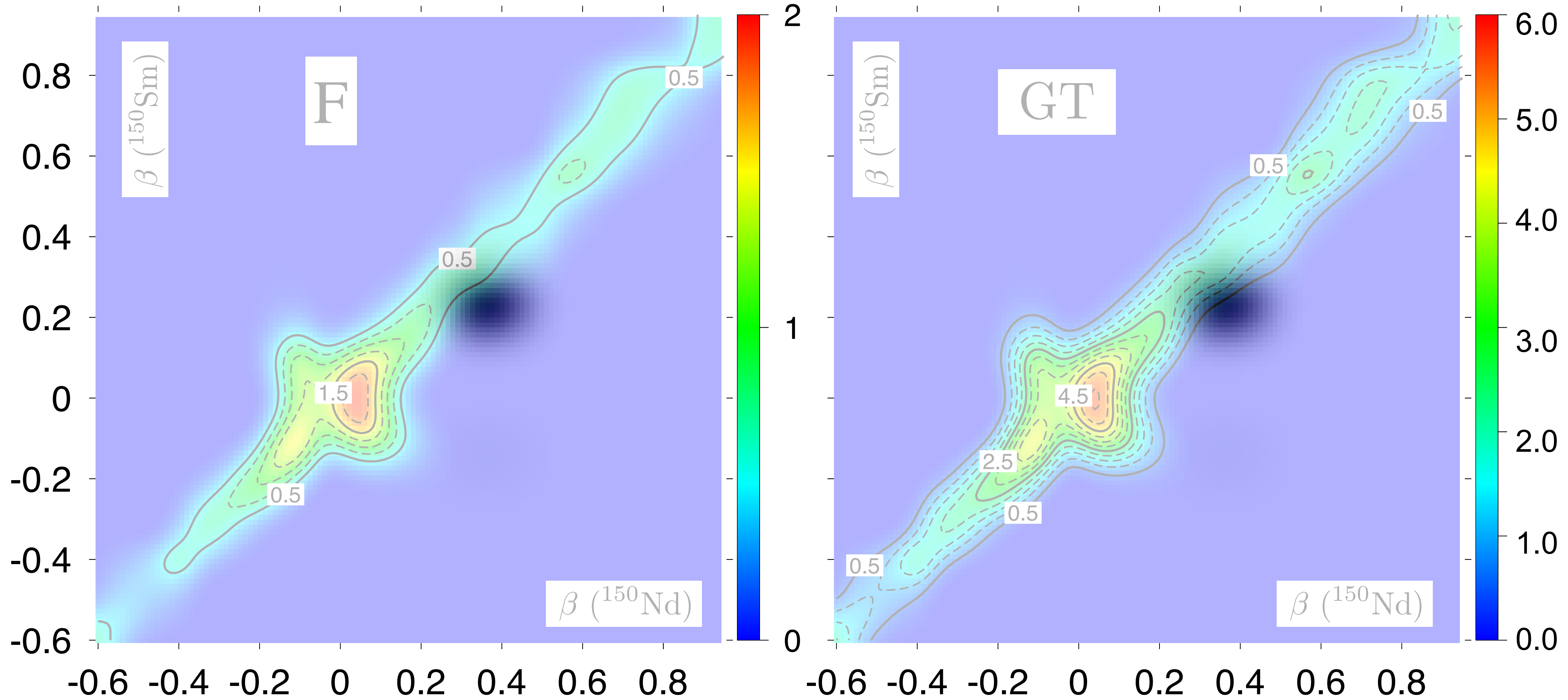
- GT streng
- Similar de
- Maxima a
- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot

NME: axial quadrupole deformation

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 105, 252503 (2010)



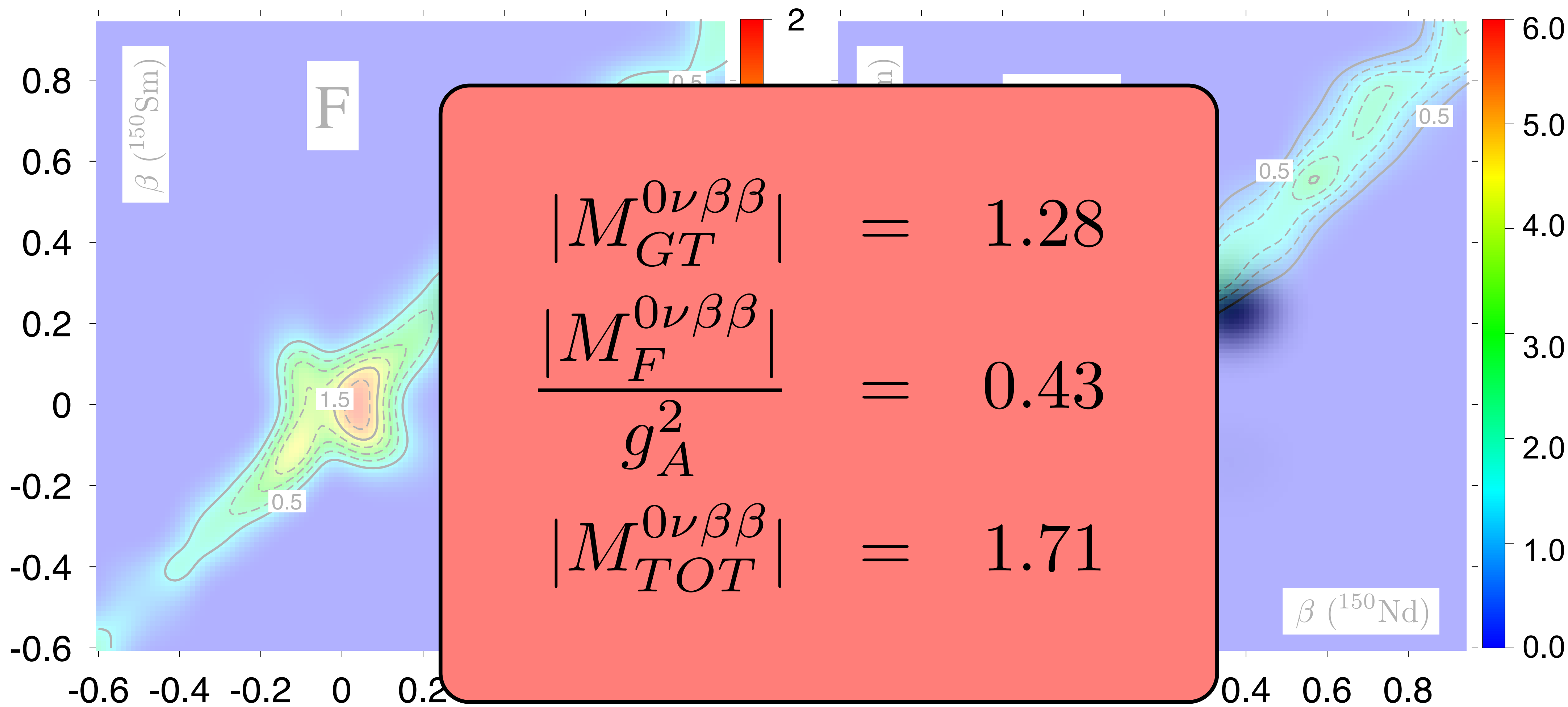
- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found
- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot

NME: axial quadrupole deformation

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}^{0\nu\beta\beta}_{\xi} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 105, 252503 (2010)



$$|M^{0\nu\beta\beta}_{GT}| = 1.28$$

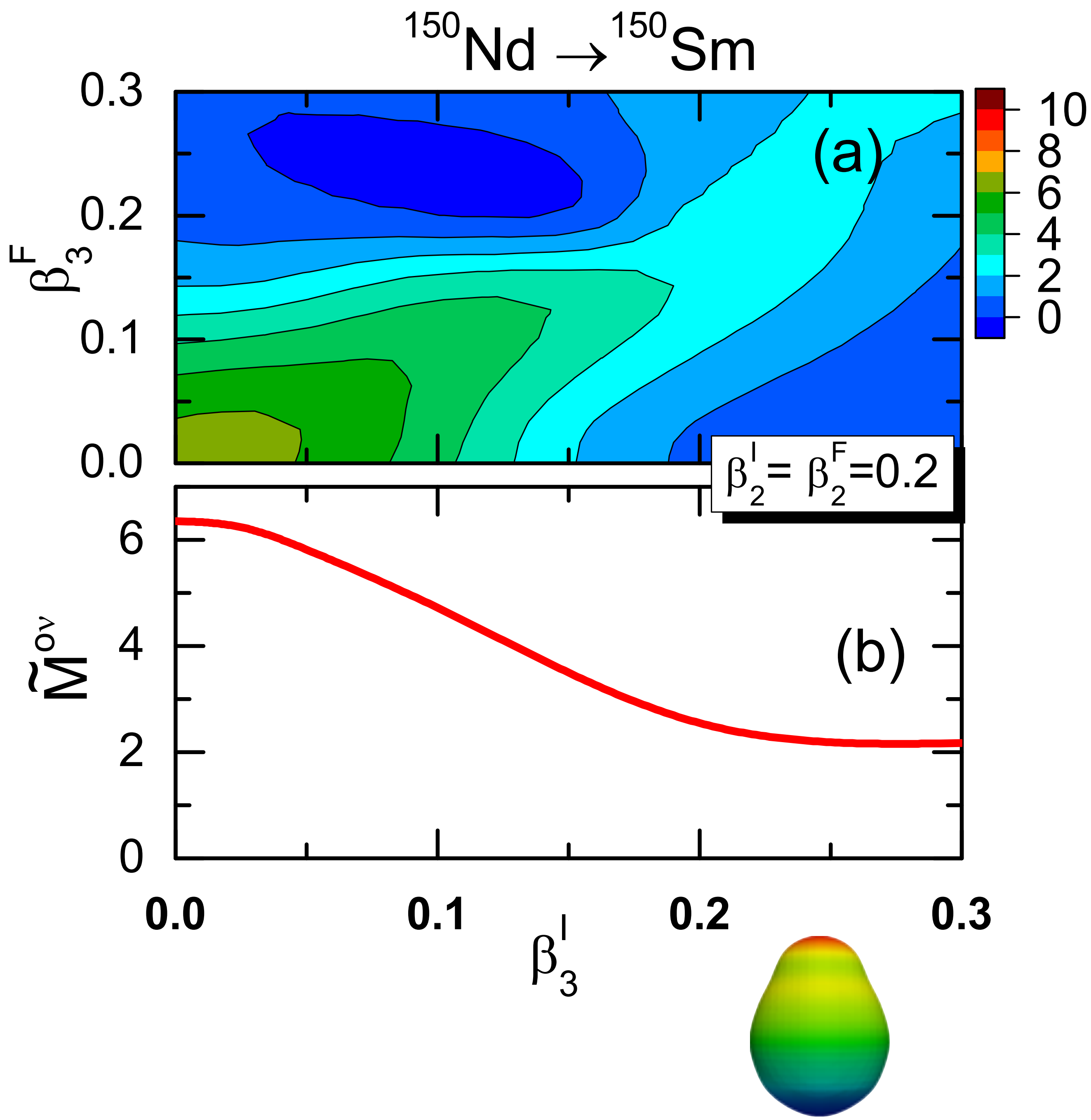
$$\frac{|M^{0\nu\beta\beta}_{F}|}{g_A^2} = 0.43$$

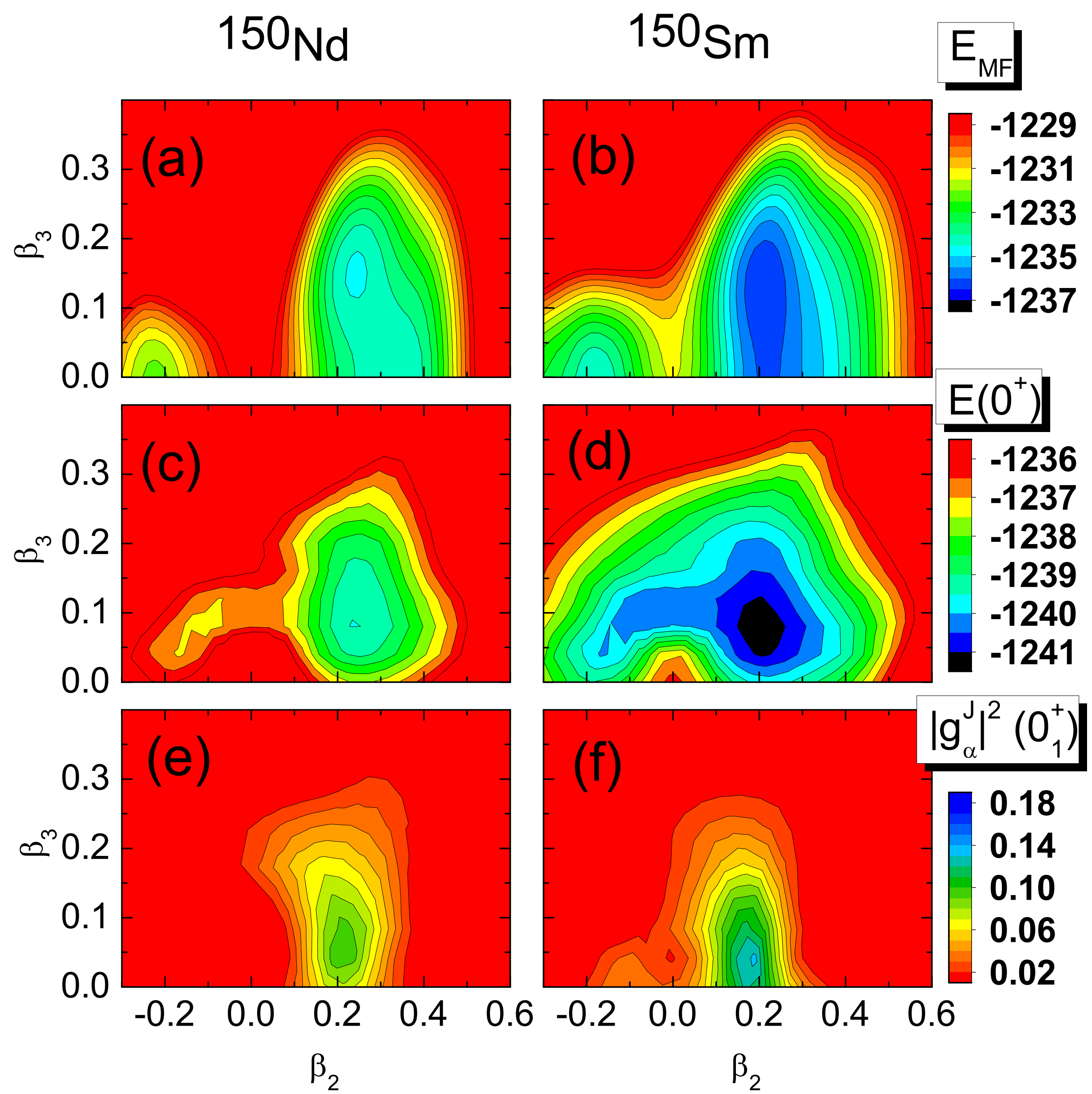
$$|M^{0\nu\beta\beta}_{TOT}| = 1.71$$

- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found
- Final result depends on the distribution of probability of the corresponding initial and final collective states within this plot

NME: axial quadrupole plus octupole deformation

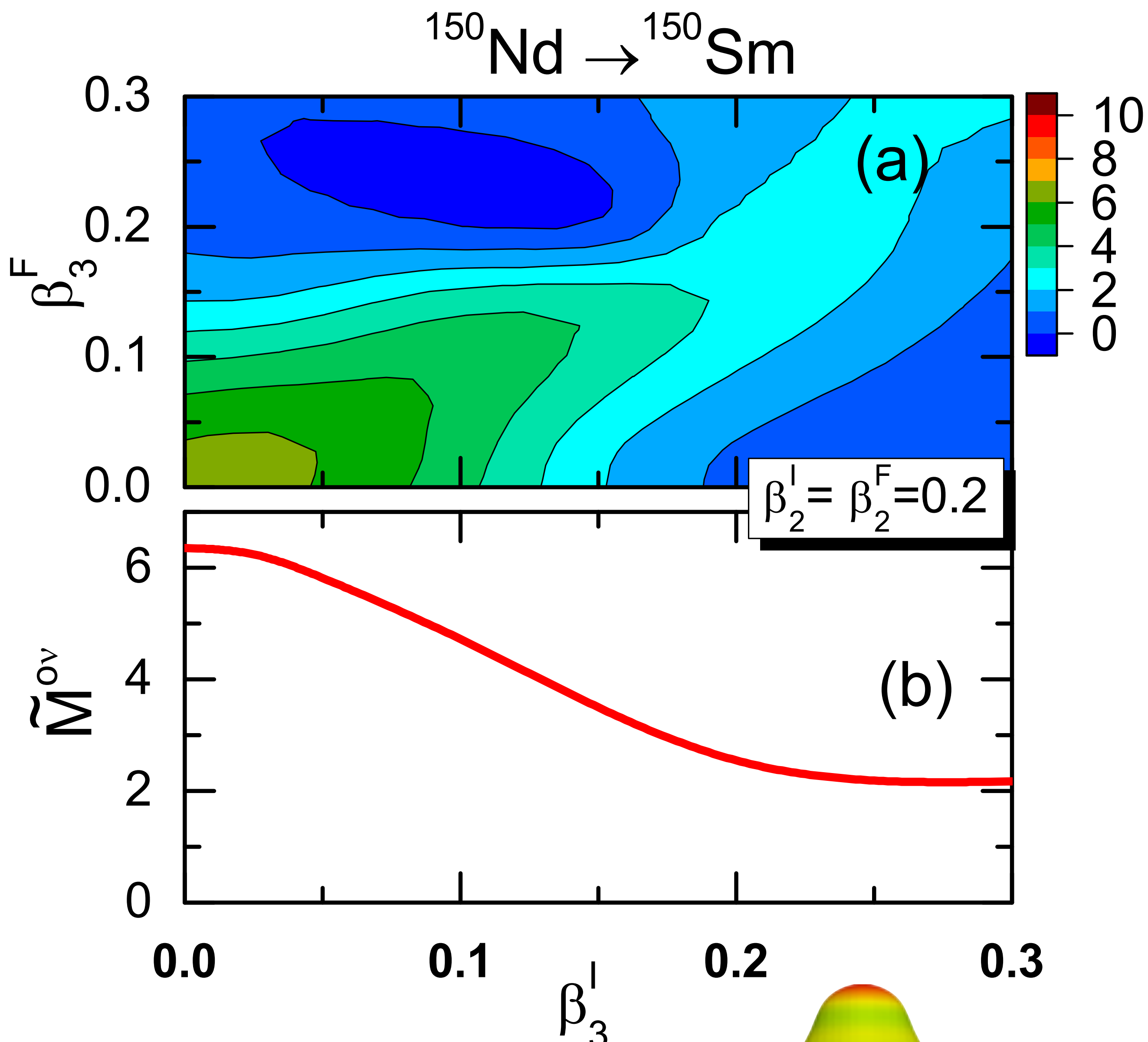
J. M. Yao and J. Engel, PRC 94, 014306 (2016)

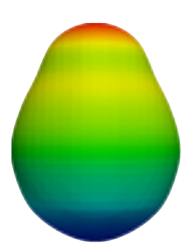




NME: axial quadrupole plus octupole deformation

J. M. Yao and J. Engel, PRC 94, 014306 (2016)





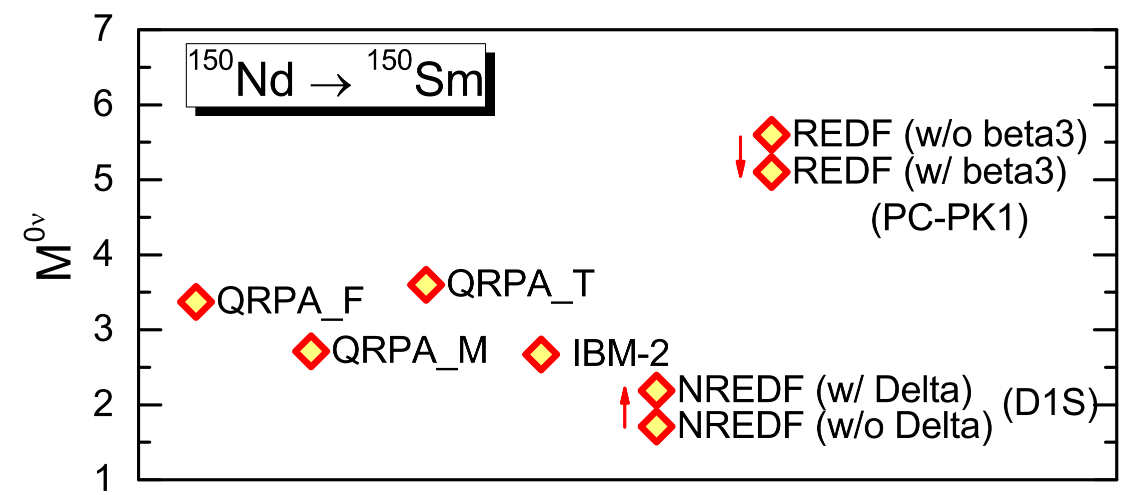
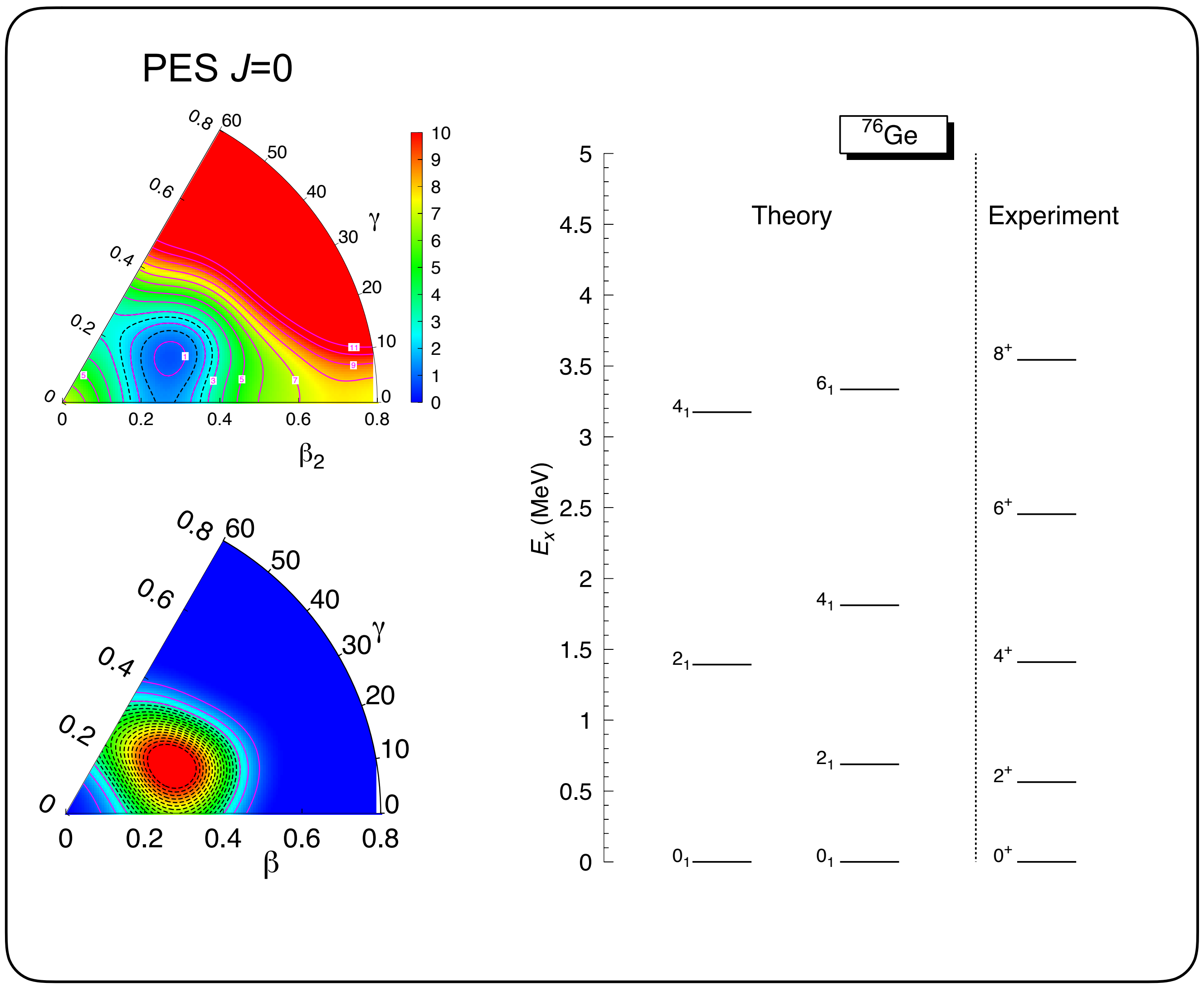


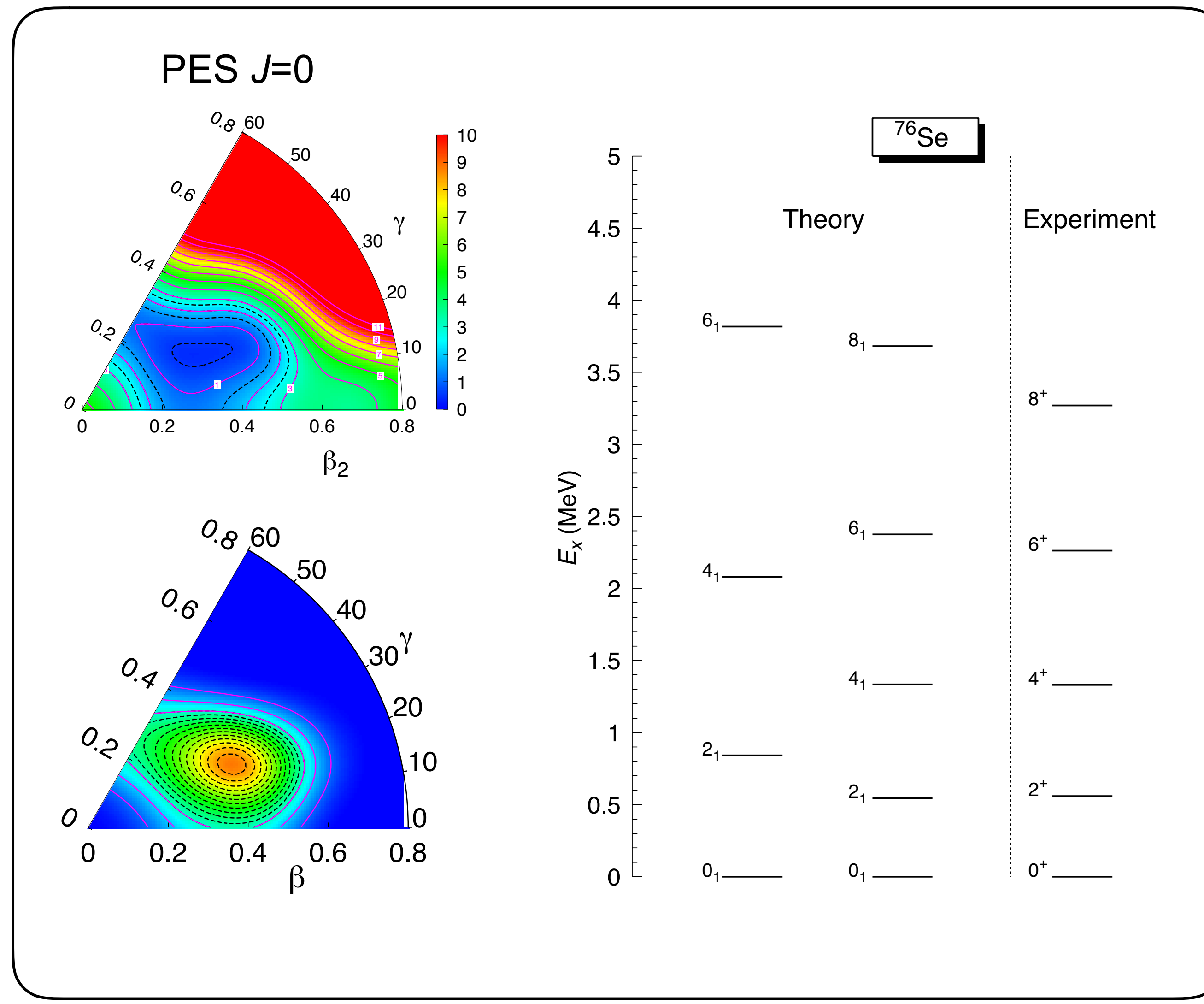
FIG. 5: (Color online) The final matrix element $M^{0\nu}$ from the GCM calculation with and without [46] octupole shape fluctuations (REDF) and those of the QRPA ("QRPA_F" [66], "QRPA_M" [45], "QRPA_T" [47]), the IMB-2 [67], and the non-relativistic GCM, based on the Gogny D1S interaction, with [68] and without [44] pairing fluctuations.

NME: triaxial quadrupole deformation

A=76

T. R. R., J. Phys. G 44, 034002 (2017)



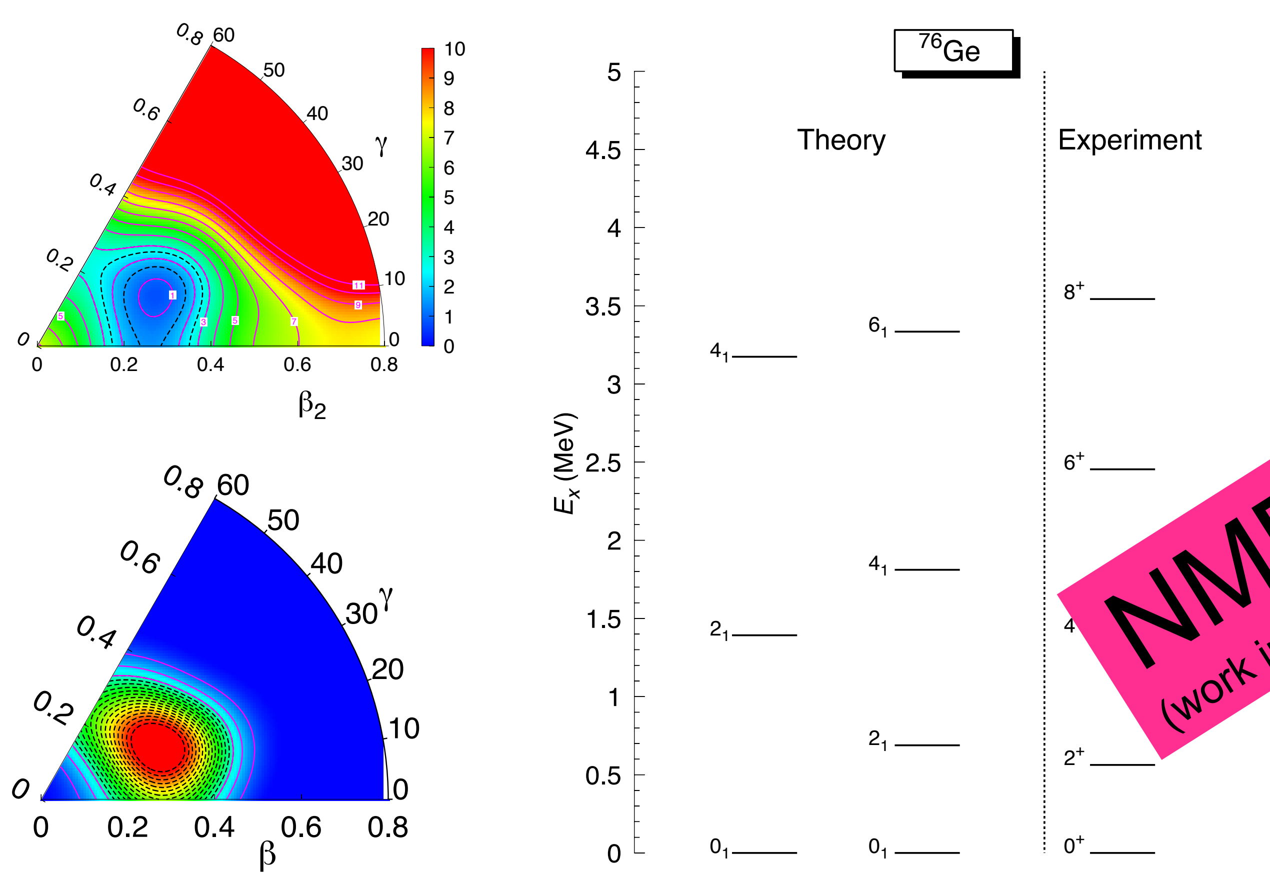


NME: triaxial quadrupole deformation

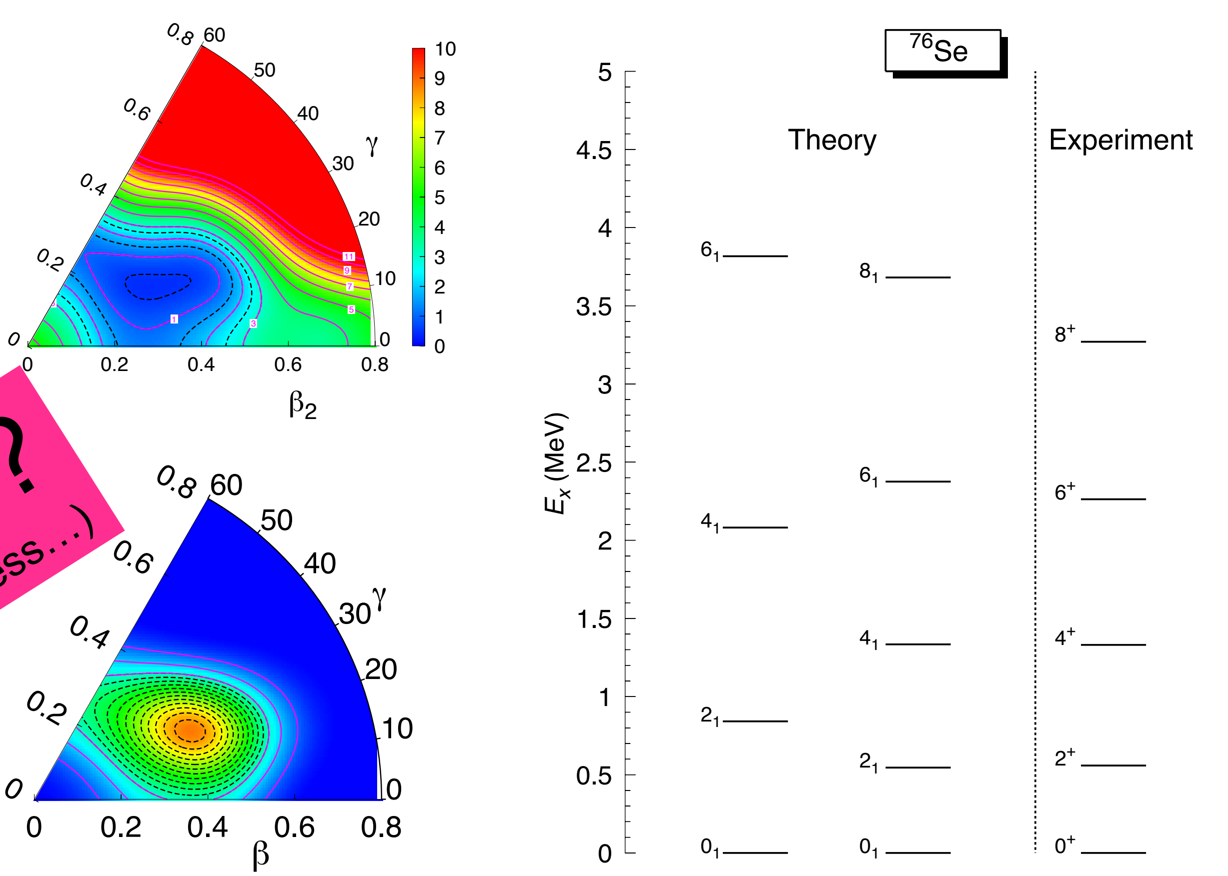
A=76

T. R. R., J. Phys. G 44, 034002 (2017)

PES *J*=0



PES *J*=0

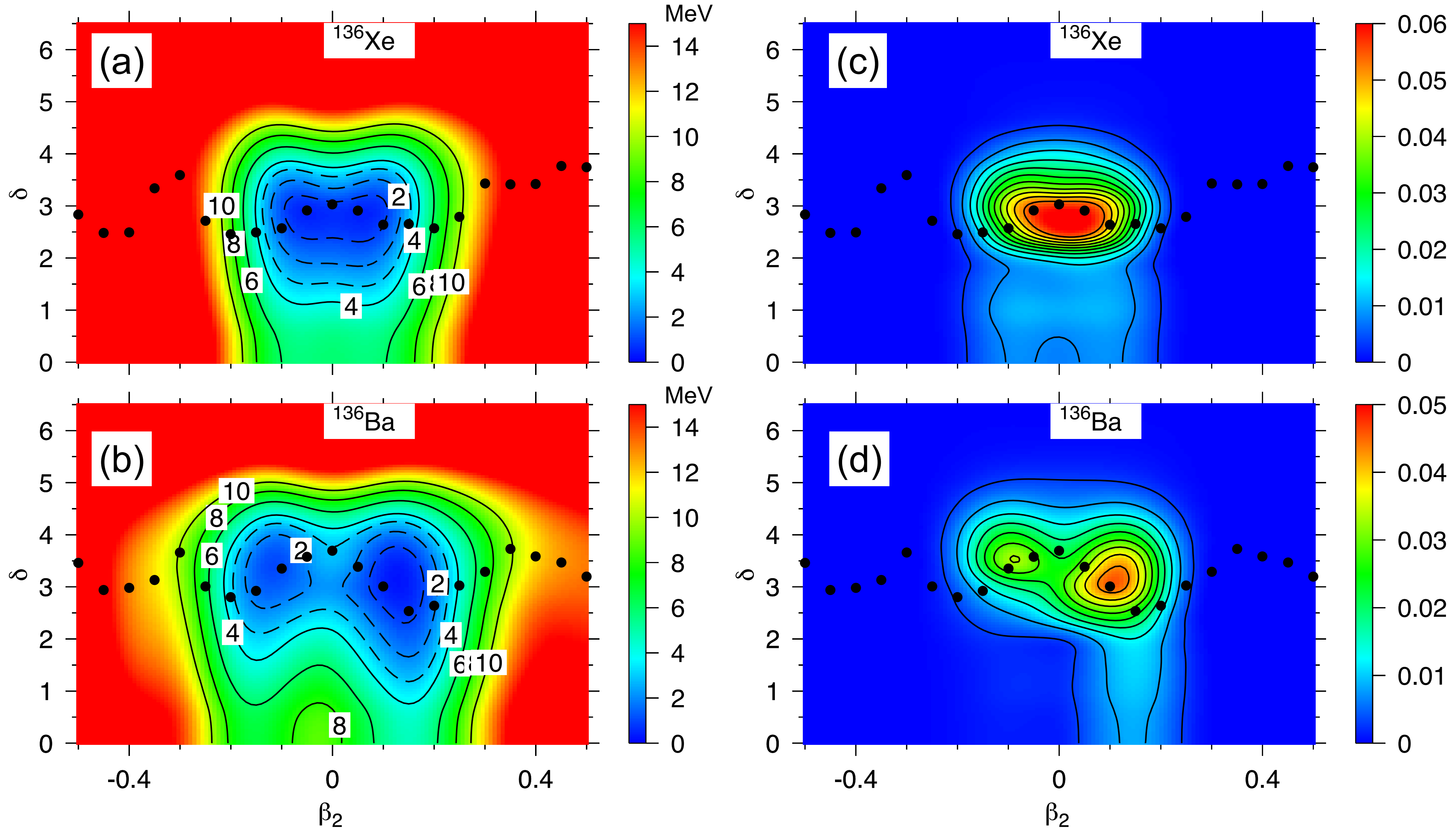


NME??
(work in progress...)

NME: Shape and pp/nn pairing fluctuations

Angular momentum projected potential energy surfaces

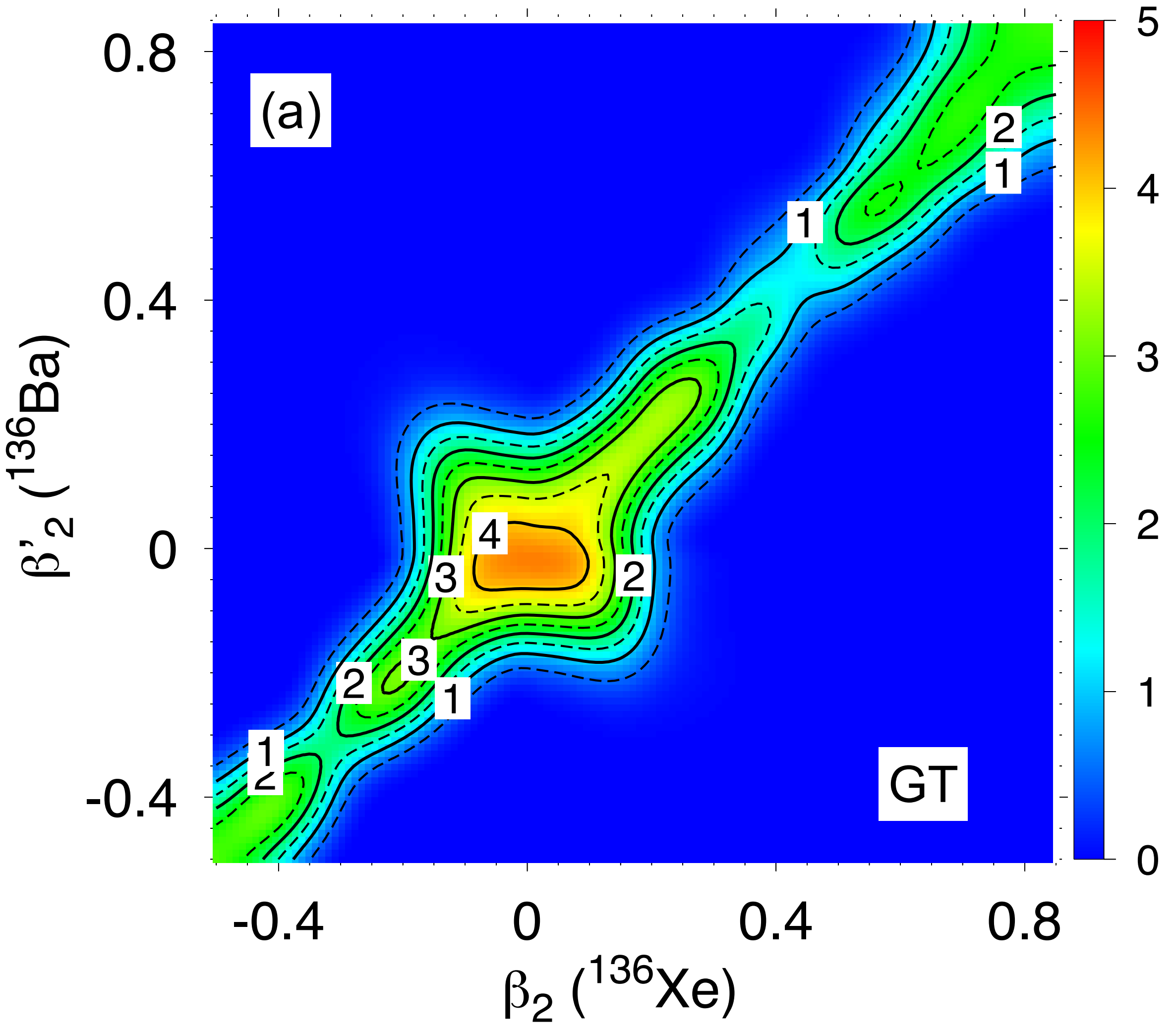
Collective ground state wave functions



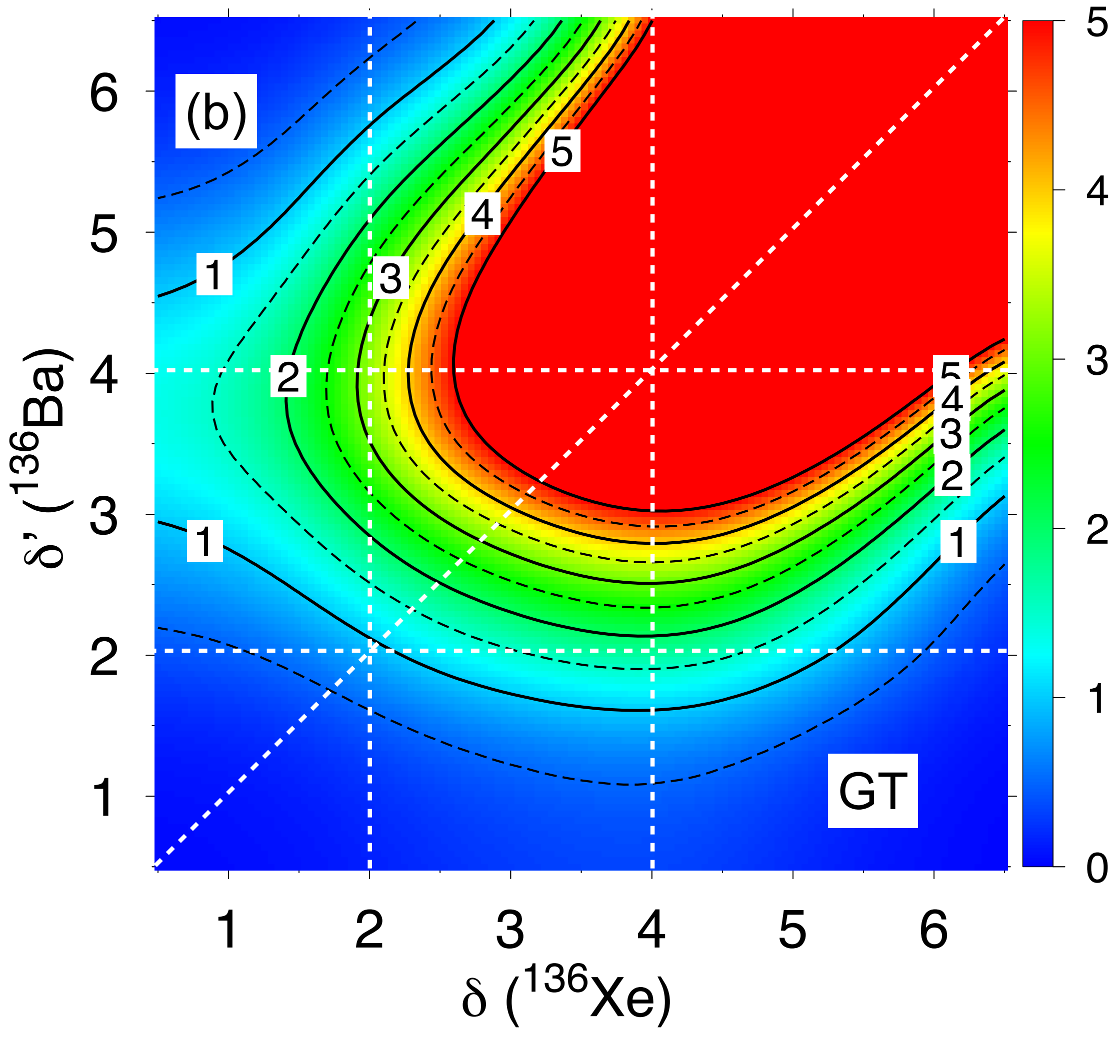
N. López-Vaquero, T.R.R., J.L. Egido, PRL 111, 142501 (2013)

NME: Shape and pp/nn pairing fluctuations

Dependence on deformation



Dependence on pp/nn pairing



N. López-Vaquero, T.R.R., J.L. Egido, PRL 111, 142501 (2013)

NME: Shape and pp/nn pairing fluctuations

Isotope	$\Delta Q(\beta_2)$	$\Delta Q(\beta_2,\delta)$	$M^{0\nu}(\beta_2)$	$M^{0\nu}(\beta_2,\delta)$	Var (%)	$\frac{T_{1/2}(\beta_2,\delta)}{T_{1/2}(\beta_2)}$
^{48}Ca	0.265	0.131	$2.370^{1.914}_{0.456}$	$2.229^{1.797}_{0.431}$	-6	1.13
^{76}Ge	0.271	0.190	$4.601^{3.715}_{0.886}$	$5.551^{4.470}_{1.082}$	21	0.69
^{82}Se	-0.366	-0.246	$4.218^{3.381}_{0.837}$	$4.674^{3.743}_{0.931}$	11	0.81
^{96}Zr	2.580	2.628	$5.650^{4.618}_{1.032}$	$6.498^{5.296}_{1.202}$	15	0.76
^{100}Mo	1.879	1.757	$5.084^{4.149}_{0.935}$	$6.588^{5.361}_{1.227}$	30	0.60
^{116}Cd	1.365	1.337	$4.795^{3.931}_{0.864}$	$5.348^{4.372}_{0.976}$	12	0.80
^{124}Sn	-0.830	-0.687	$4.808^{3.893}_{0.916}$	$5.787^{4.680}_{1.107}$	20	0.69
^{128}Te	-0.564	-0.594	$4.107^{3.079}_{1.027}$	$5.687^{4.255}_{1.432}$	38	0.52
^{130}Te	-0.348	-0.628	$5.130^{4.141}_{0.989}$	$6.405^{5.161}_{1.244}$	25	0.64
^{136}Xe	-1.027	-0.787	$4.199^{3.673}_{0.526}$	$4.773^{4.170}_{0.604}$	14	0.77
^{150}Nd	-0.380	-0.282	$1.707^{1.278}_{0.429}$	$2.190^{1.639}_{0.551}$	29	0.61

N. López-Vaquero, T.R.R., J.L. Egido, PRL 111, 142501 (2013)

NME: Shape and pn pairing fluctuations

$$H = h_0 - \sum_{\mu=-1}^{1} g_\mu^{T=1}\, S_\mu^\dagger S_\mu - \frac{\chi}{2}\sum_{K=-2}^{2} Q_{2K}^\dagger Q_{2K}$$
$$- g^{T=0}\sum_{\nu=-1}^{1} P_\nu^\dagger P_\nu + g_{ph}\sum_{\mu,\nu=-1}^{1} F_\nu^{\mu\dagger} F_\nu^\mu\,, \qquad (2)$$

where h_0 contains spherical single particle energies, Q_{2K} are the components of a quadrupole operator defined in Ref. [15], and

$$S_\mu^\dagger = \frac{1}{\sqrt{2}}\sum_l \hat{l}[c_l^\dagger c_l^\dagger]^{001}_{00\mu}\,, \quad P_\mu^\dagger = \frac{1}{\sqrt{2}}\sum_l \hat{l}[c_l^\dagger c_l^\dagger]^{010}_{0\mu 0}\,,$$
$$F_\nu^\mu = \frac{1}{2}\sum_i \sigma_i^\mu \tau_i^\nu = \sum_l \hat{l}[c_l^\dagger \bar{c}_l]^{011}_{0\mu\nu}\,. \qquad (3)$$

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_Q Q_{20} - \frac{\lambda_P}{2}\left(P_0 + P_0^\dagger\right)\,, \quad (6)$$

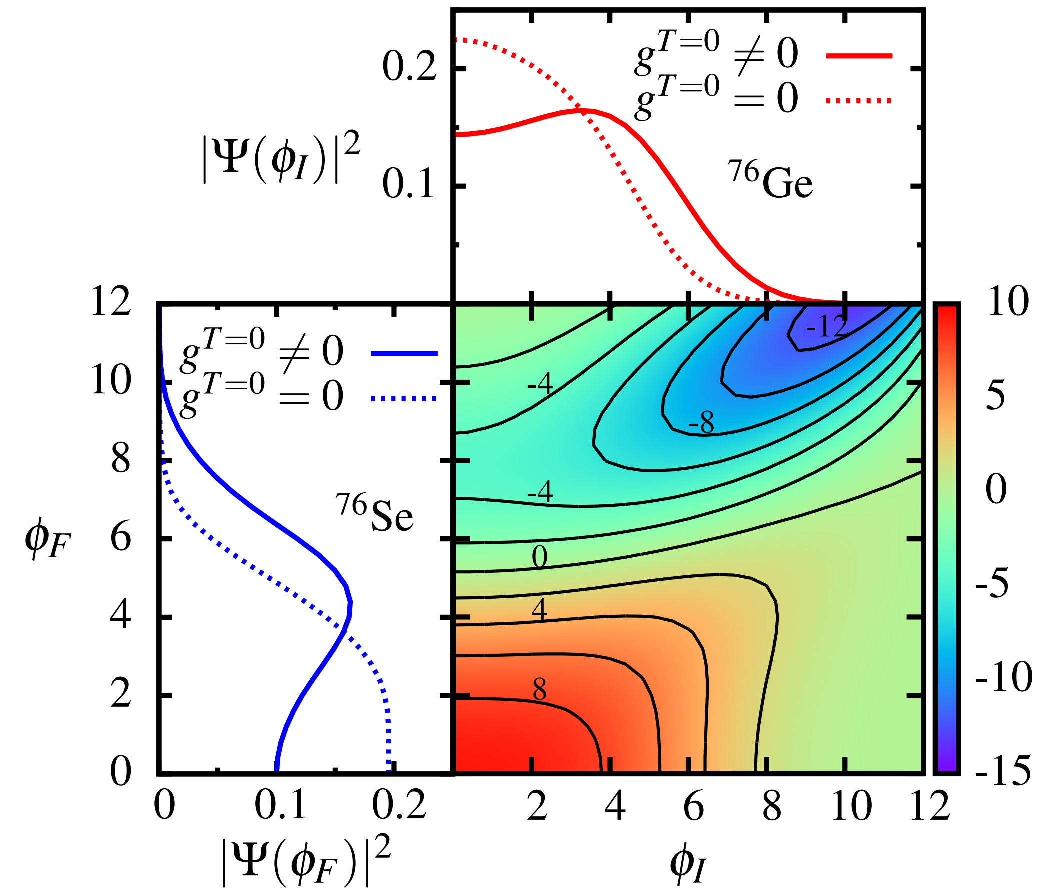


FIG. 3. (Color online.) **Bottom right:** $\mathcal{N}_{\phi_I}\mathcal{N}_{\phi_F}\langle\phi_F|\mathcal{P}_F\hat{M}_{0\nu}\mathcal{P}_I|\phi_I\rangle$ for projected quasiparticle vacua with different values of the initial and final isoscalar pairing amplitudes ϕ_I and ϕ_F, from the SkO′-based interaction (see text). **Top and bottom left:** Square of collective wave functions in ^{76}Ge and ^{76}Se.

N. Hinohara and J. Engel, PRC 031031(R) (2014)

NME: Shape and pn pairing fluctuations

$$H = h_0 - \sum_{\mu=-1}^{1} g_\mu^{T=1} S_\mu^\dagger S_\mu - \frac{\chi}{2}\sum_{K=-2}^{2} Q_{2K}^\dagger Q_{2K}$$
$$- g^{T=0}\sum_{\nu=-1}^{1} P_\nu^\dagger P_\nu + g_{ph}\sum_{\mu,\nu=-1}^{1} F_\nu^{\mu\dagger} F_\nu^\mu\,, \qquad (2)$$

where h_0 contains spherical single particle energies, Q_{2K} are the components of a quadrupole operator defined in Ref. [15], and

$$S_\mu^\dagger = \frac{1}{\sqrt{2}}\sum_l \hat{l}[c_l^\dagger c_l^\dagger]_{00\mu}^{001}\,, \quad P_\mu^\dagger = \frac{1}{\sqrt{2}}\sum_l \hat{l}[c_l^\dagger c_l^\dagger]_{0\mu0}^{010}\,,$$
$$F_\nu^\mu = \frac{1}{2}\sum_i \sigma_i^\mu \tau_i^\nu = \sum_l \hat{l}[c_l^\dagger \bar{c}_l]_{0\mu\nu}^{011}\,. \qquad (3)$$

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_Q Q_{20} - \frac{\lambda_P}{2}\left(P_0 + P_0^\dagger\right), \quad (6)$$

Very difficult (perhaps impossible) to implement with current EDFs!

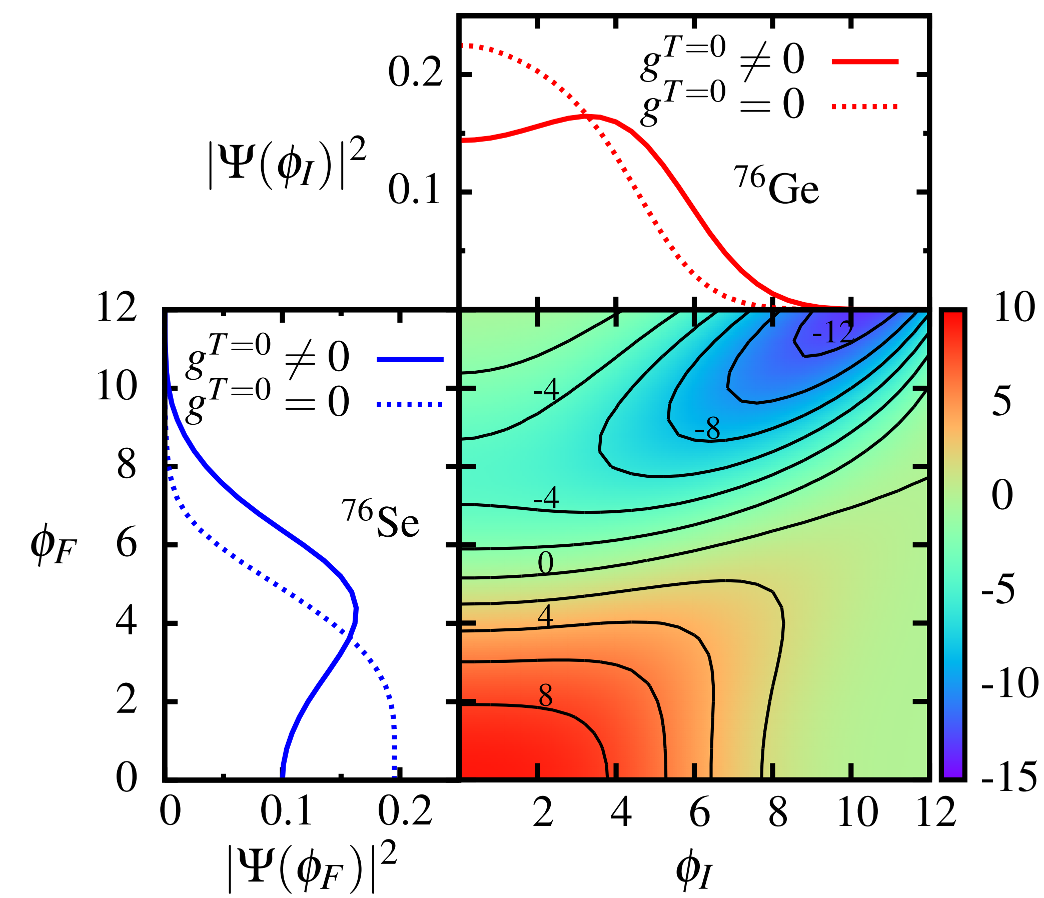


FIG. 3. (Color online.) **Bottom right:** $\mathcal{N}_{\phi_I}\mathcal{N}_{\phi_F}\langle\phi_F|\hat{P}_F M_{0\nu}\hat{P}_I|\phi_I\rangle$ for projected quasiparticle vacua with different values of the initial and final isoscalar pairing amplitudes ϕ_I and ϕ_F, from the SkO′-based interaction (see text). **Top and bottom left:** Square of collective wave functions in ^{76}Ge and ^{76}Se.

N. Hinohara and J. Engel, PRC 031031(R) (2014)

NME in isotopic chains

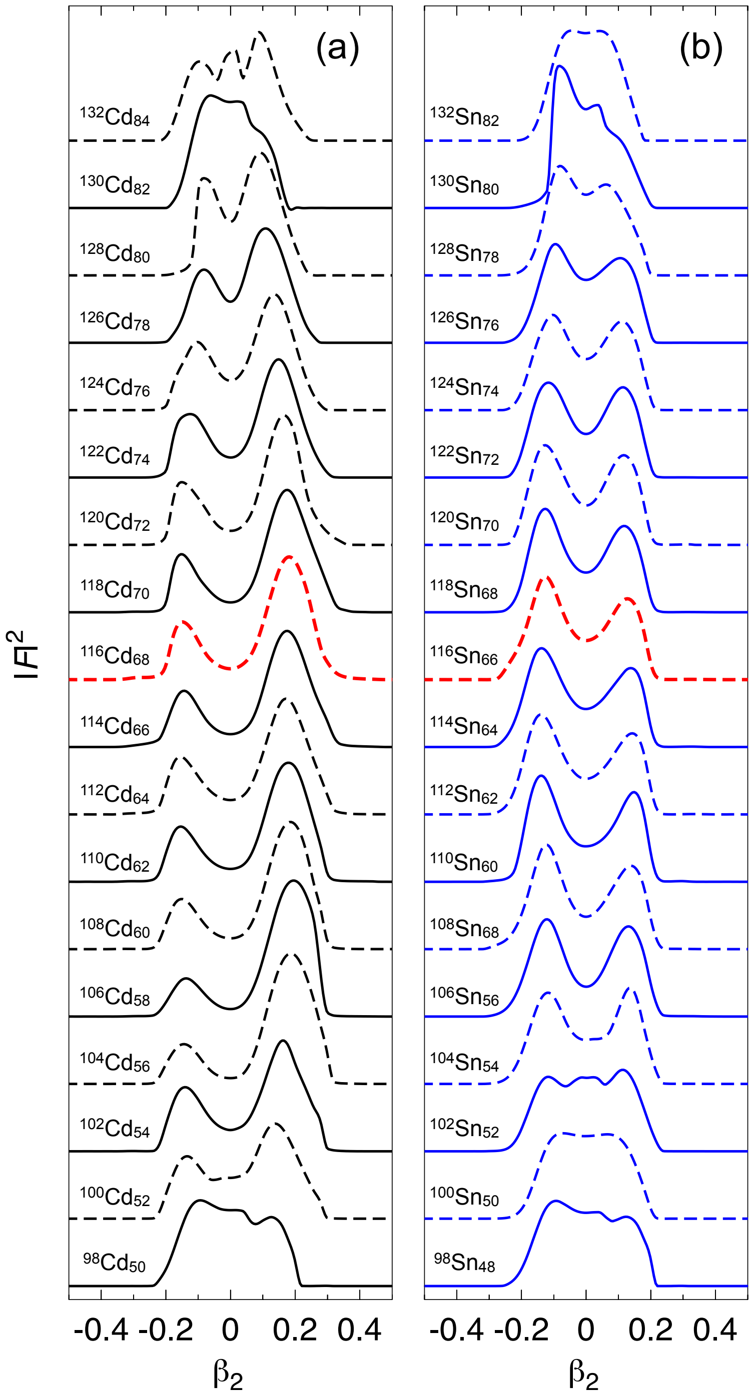
We want to study the role of

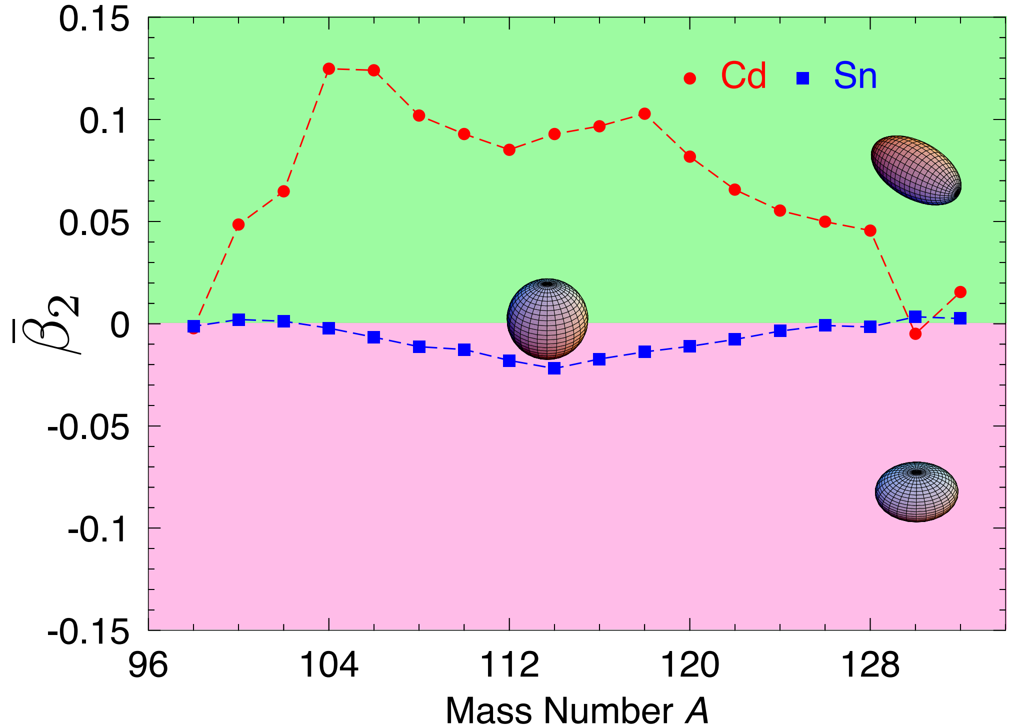
- Pairing pp/nn correlations.
- Deformation.
- Shell effects.
- Spatial dependence of the neutrino potentials.

in the nuclear matrix elements in a whole isotopic chain using state-of-the-art energy density functional methods.

Ground state properties

Collective wave functions for Cd and Sn



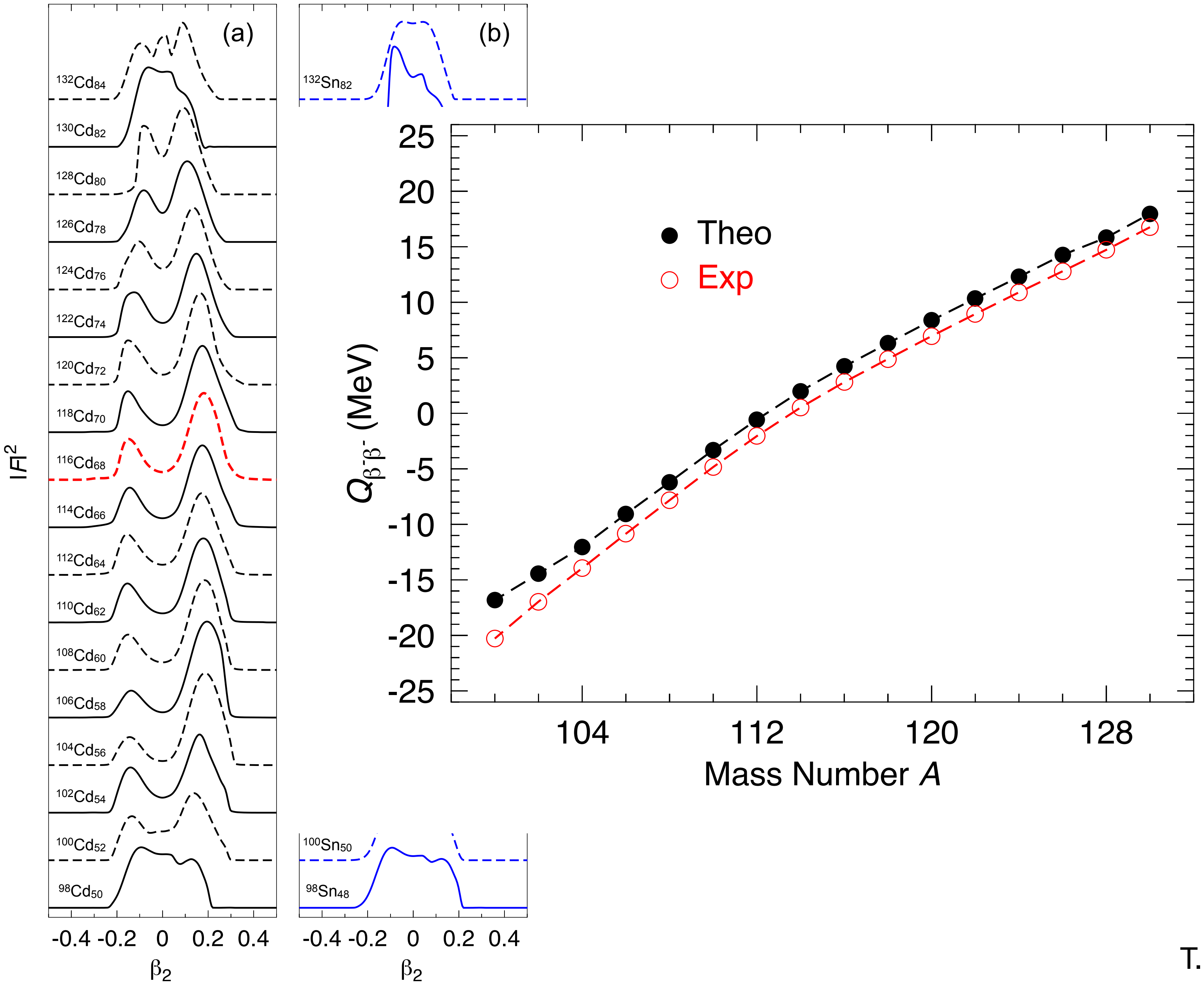


- Sn isotopes are spherical and Cd slightly prolate deformed when beyond mean field correlations are included.

T.R.R., Martínez-Pinedo, PLB 719, 174 (2013)

Ground state properties

Collective wave functions for Cd and Sn

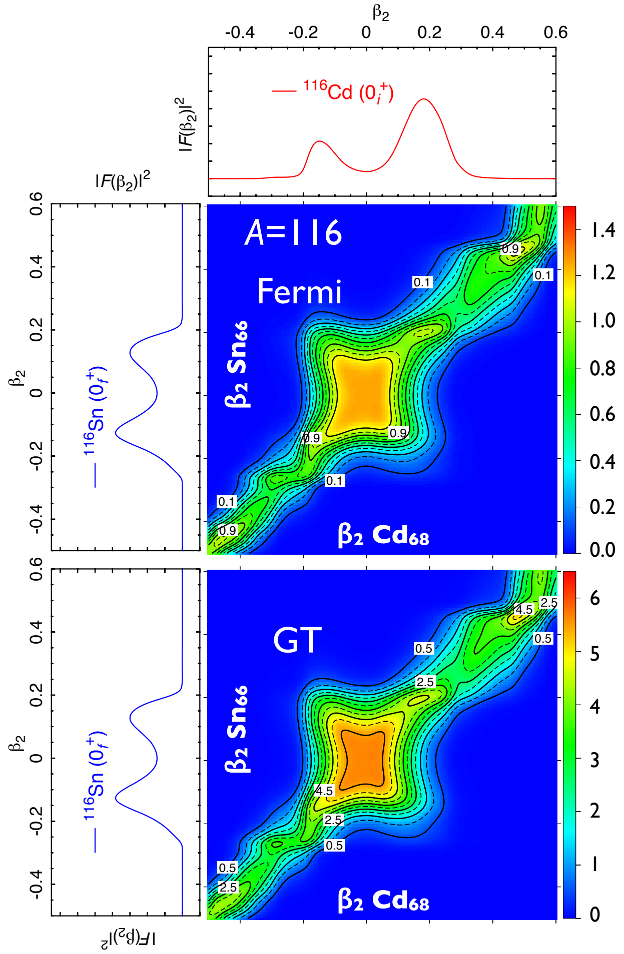


- Sn isotopes are spherical and Cd slightly prolate deformed when beyond mean field correlations are included.

- Good agreement between experimental and theoretical *Q*-values within the accuracy of the force (Gogny D1S).

T.R.R., Martínez-Pinedo, PLB 719, 174 (2013)

A=116 (possible candidate for detection)



NME: $^{116}Cd \rightarrow ^{116}Sn$

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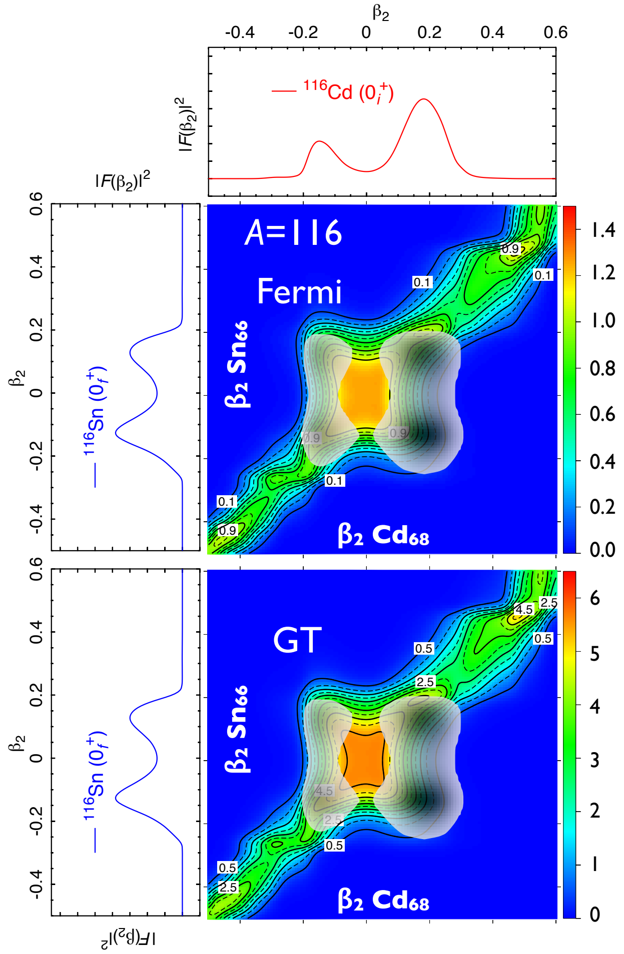

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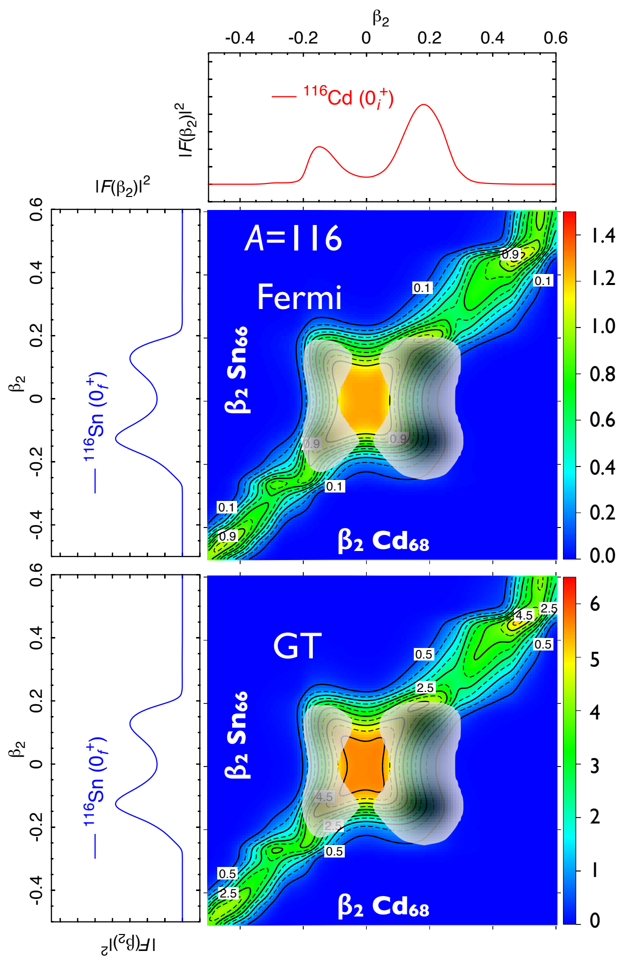

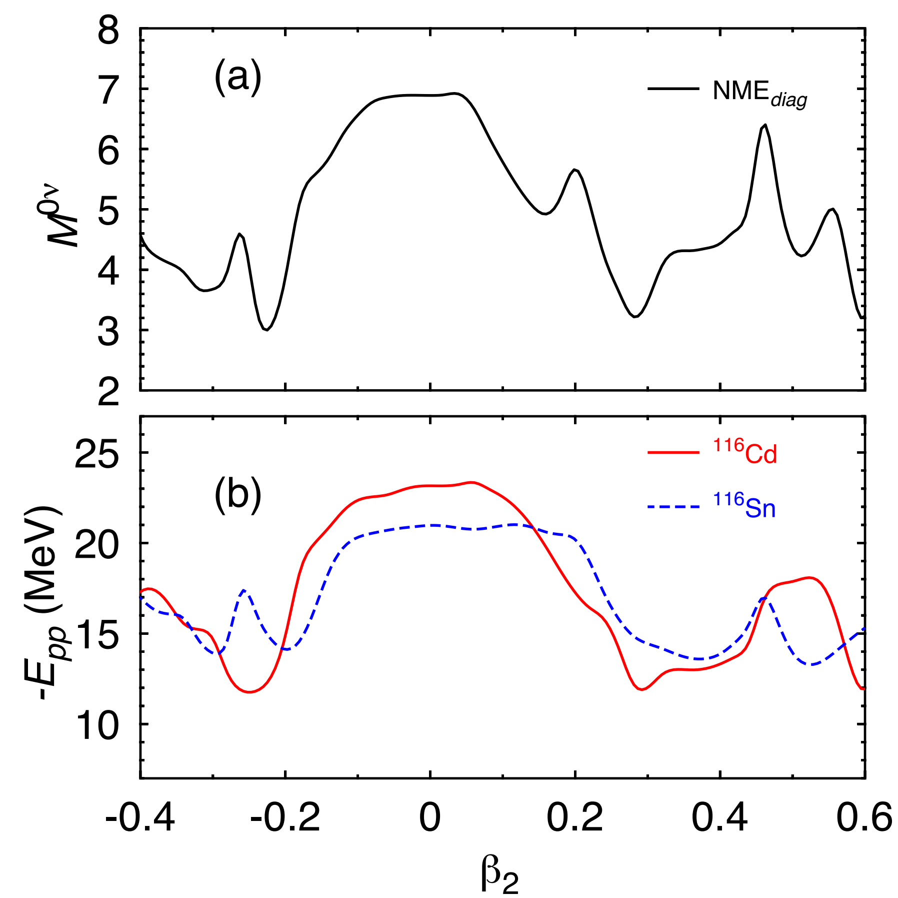

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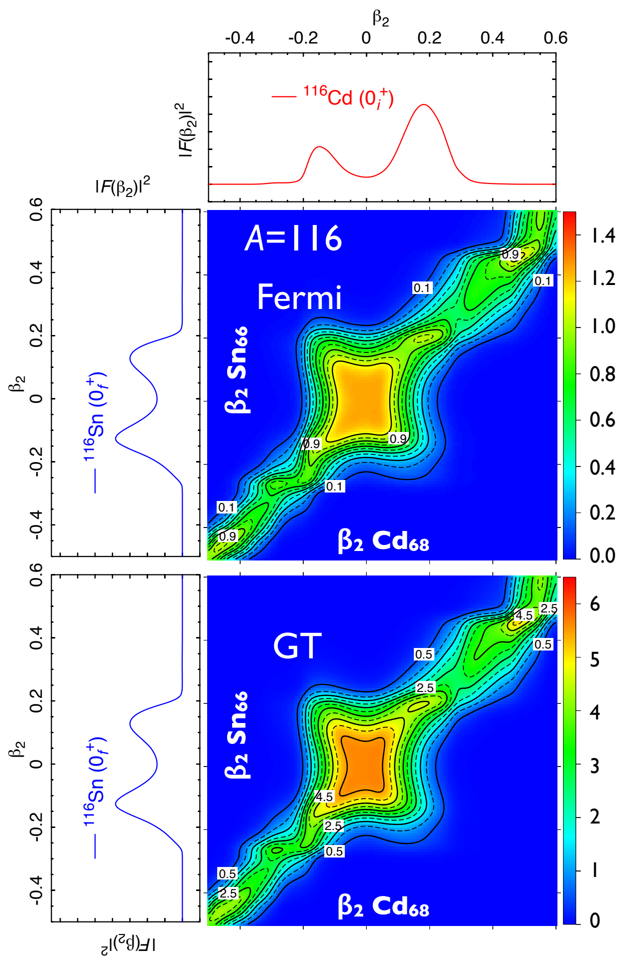

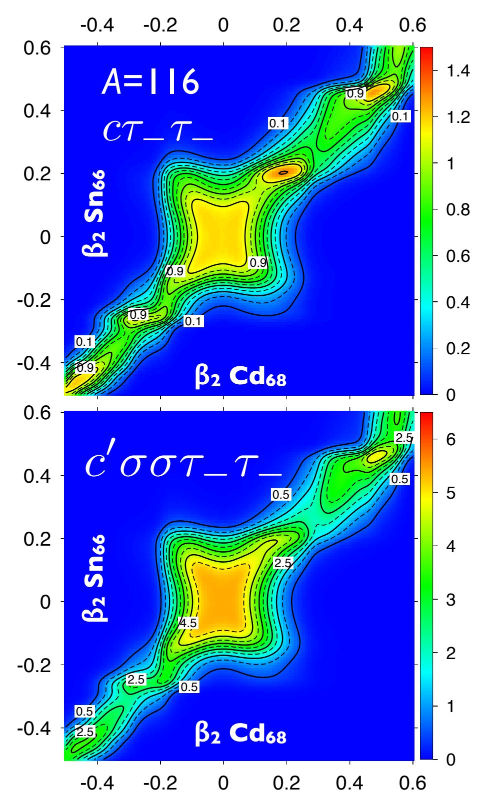

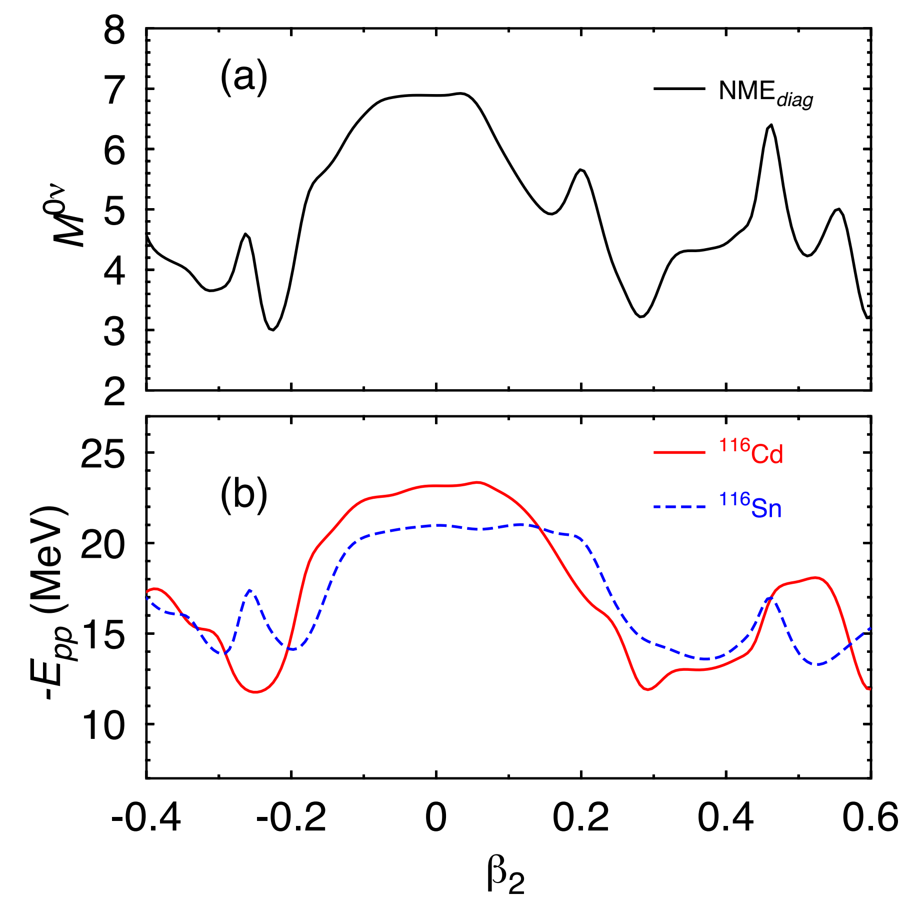

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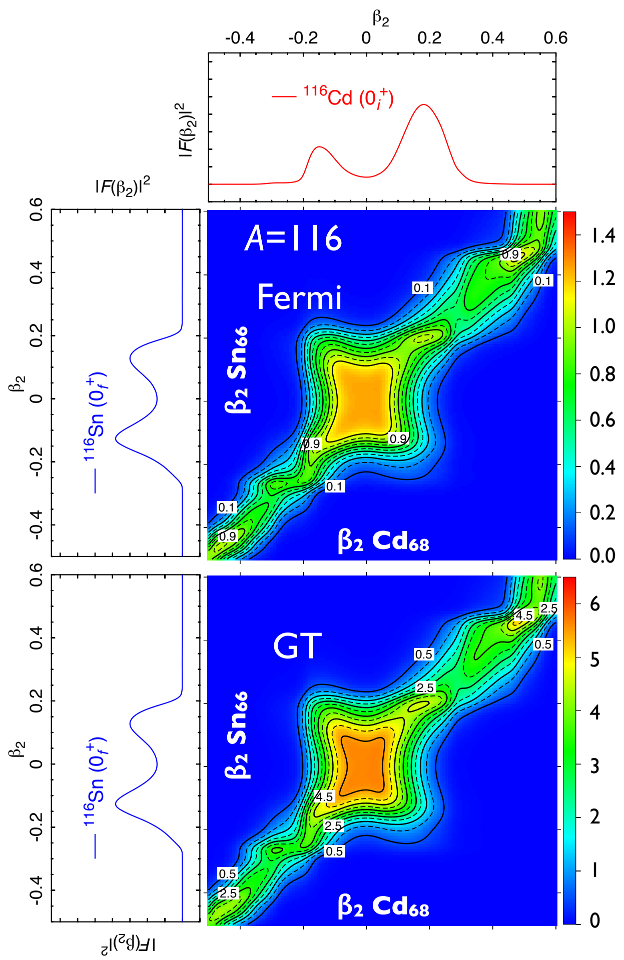

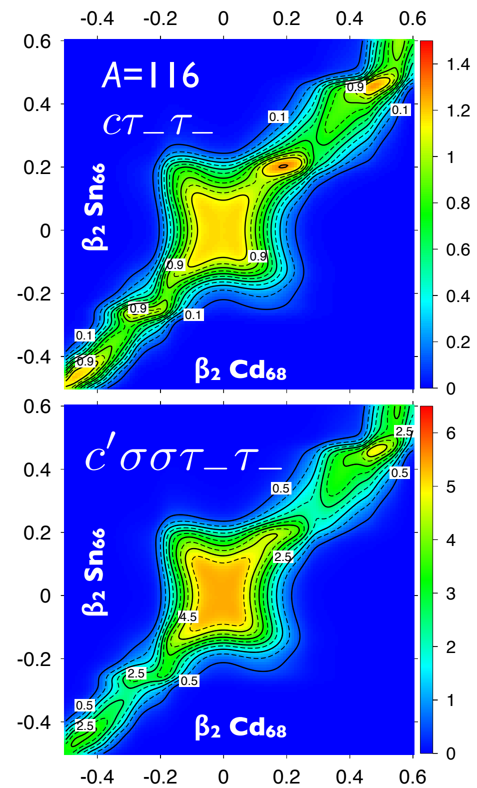

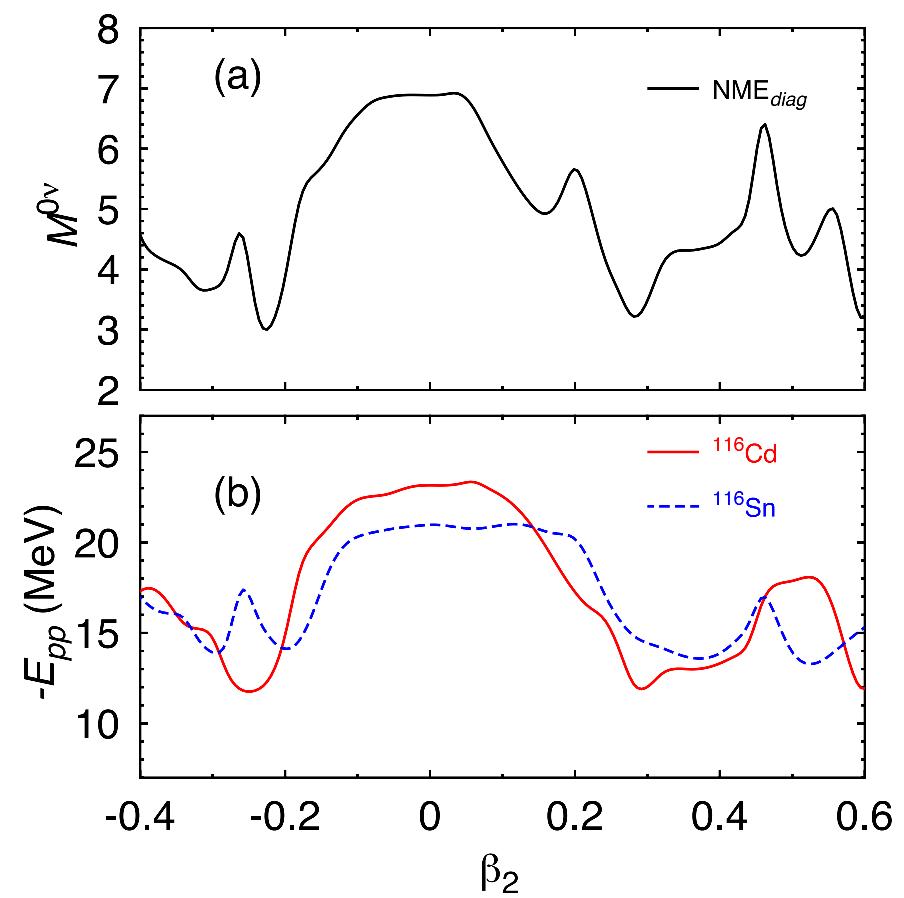

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NME: $^{A}Cd \rightarrow {}^{A}Sn$ Shell Effects

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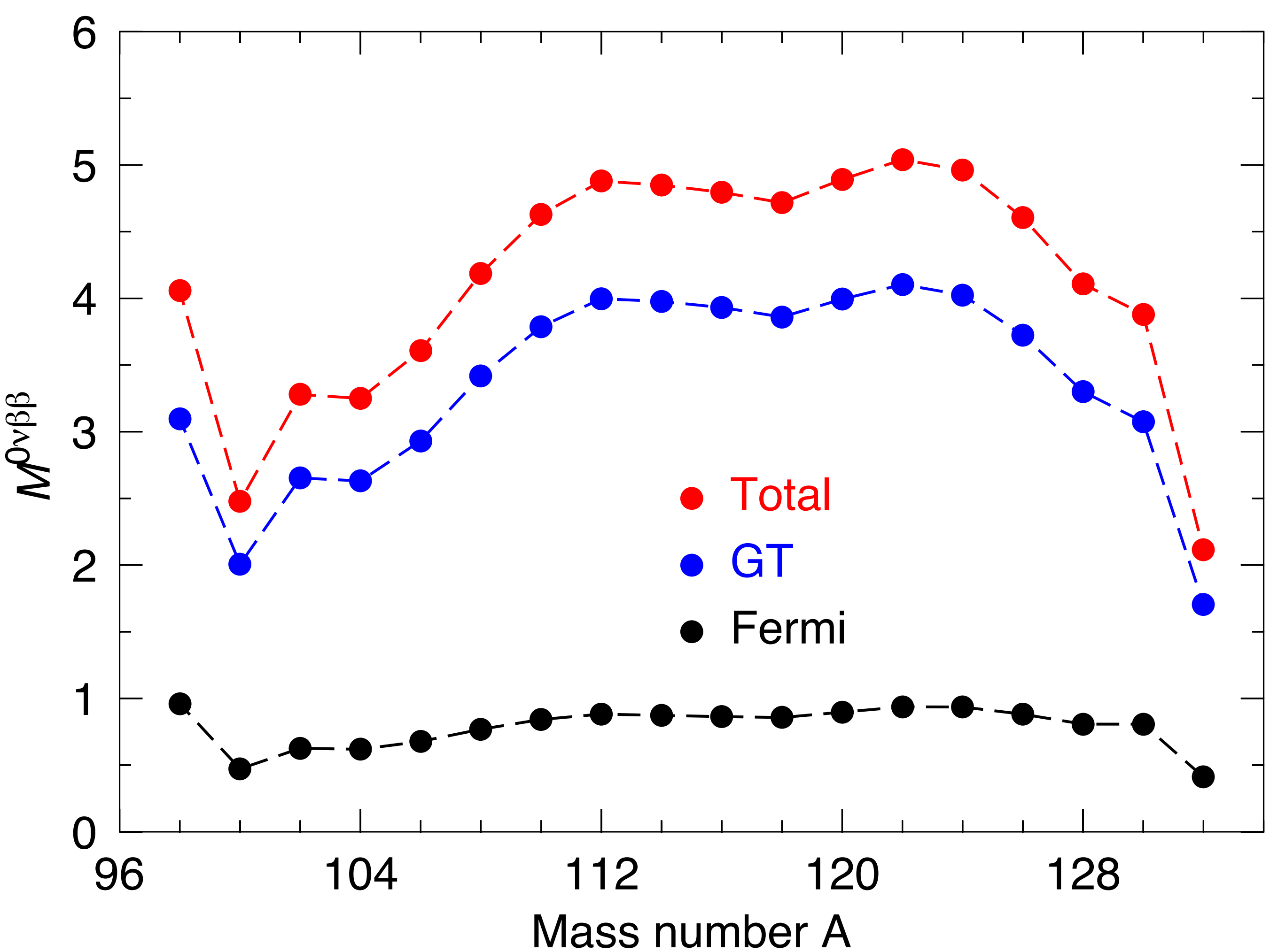

T.R.R., Martínez-Pinedo, PLB 719, 174 (2013)

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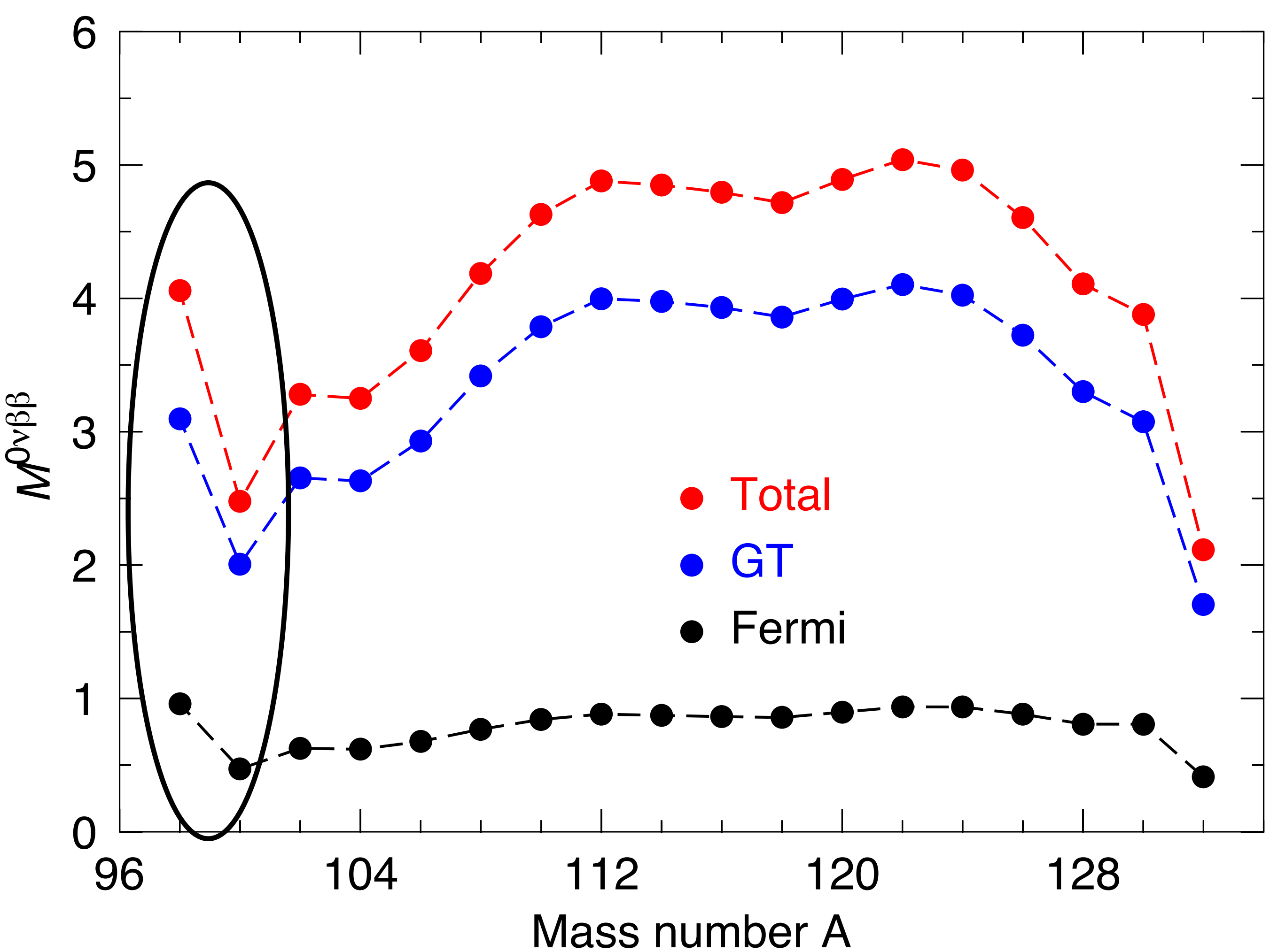

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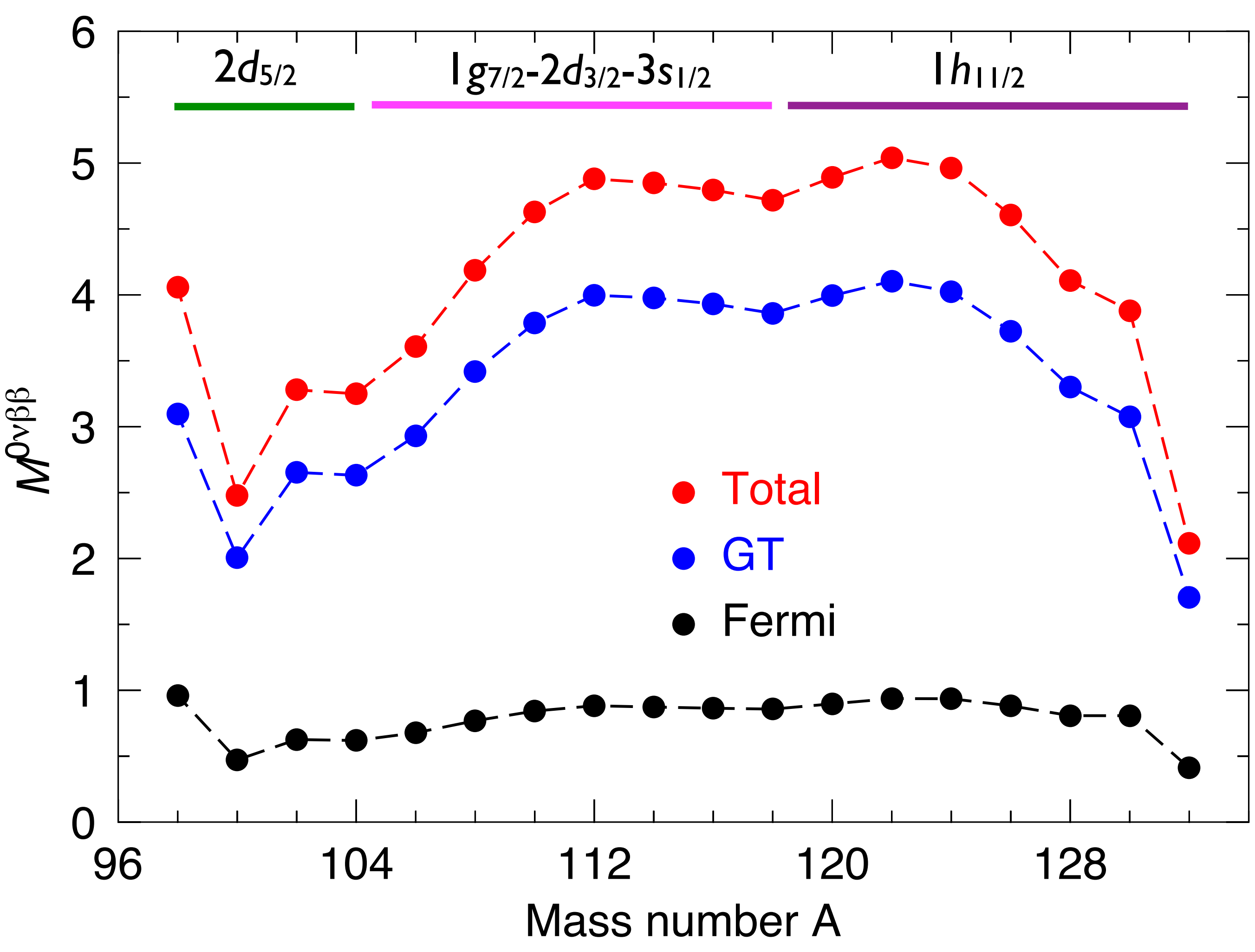

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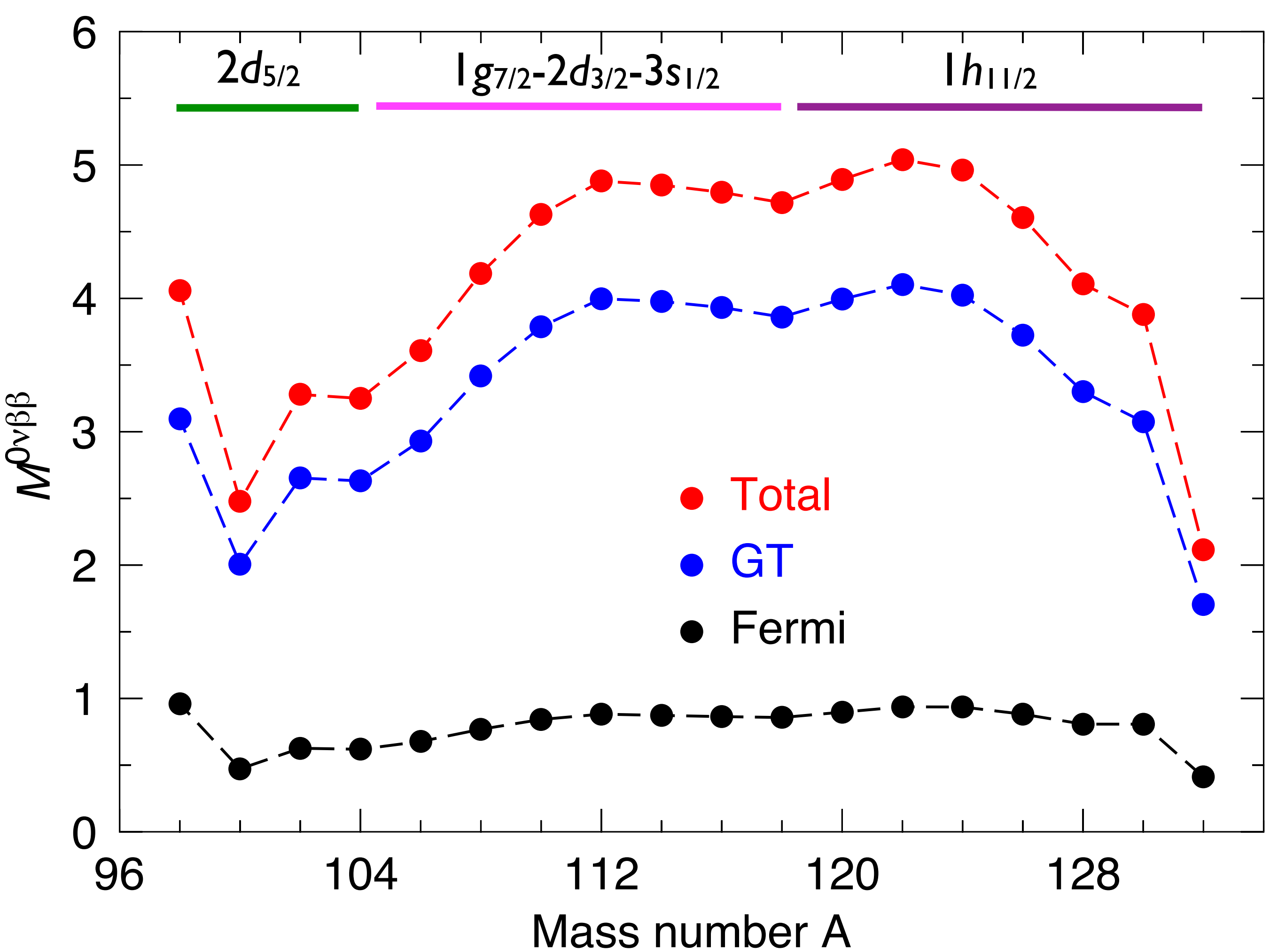

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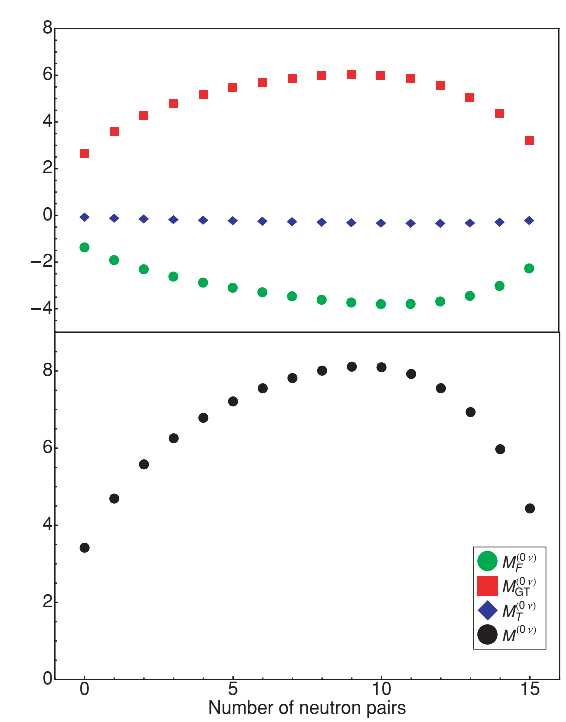

J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009)

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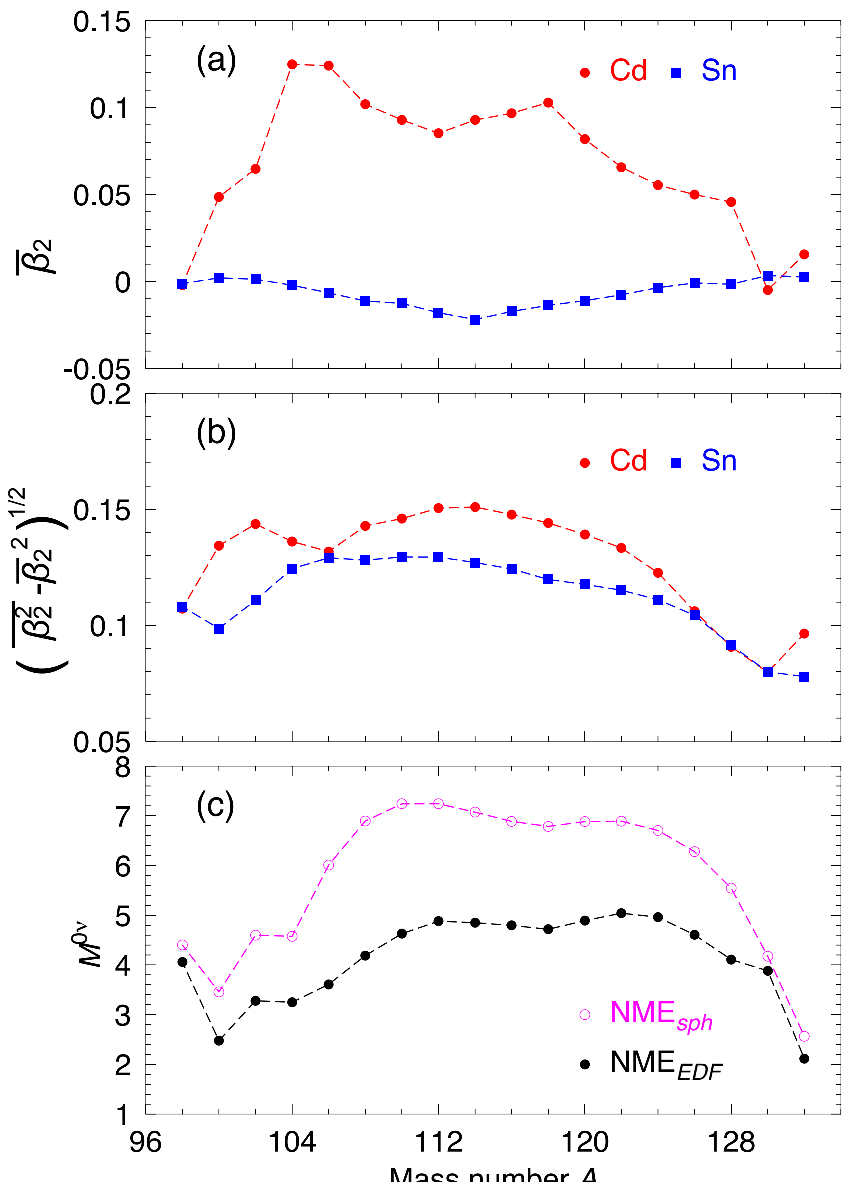
- Reduction of the NME with respect to the spherical value when shape mixing is included
- Larger reduction when the difference in deformation is larger



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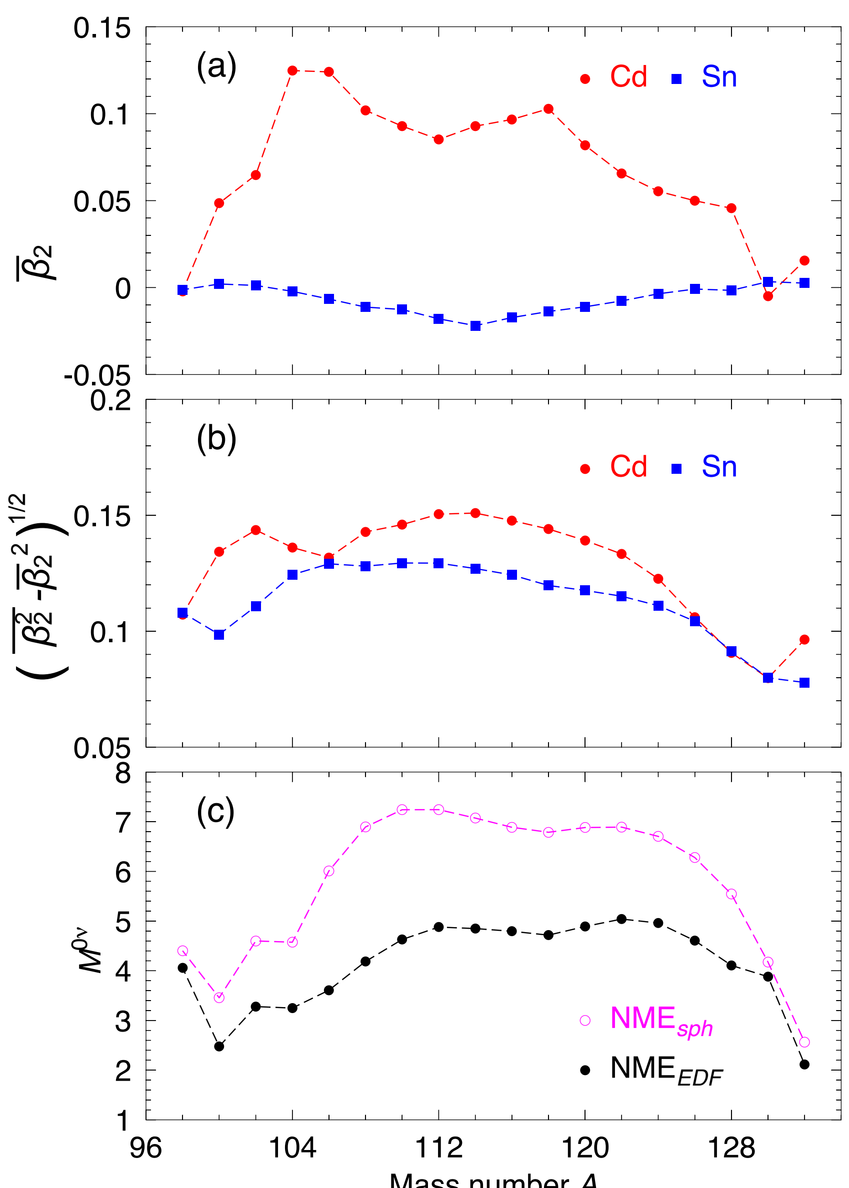
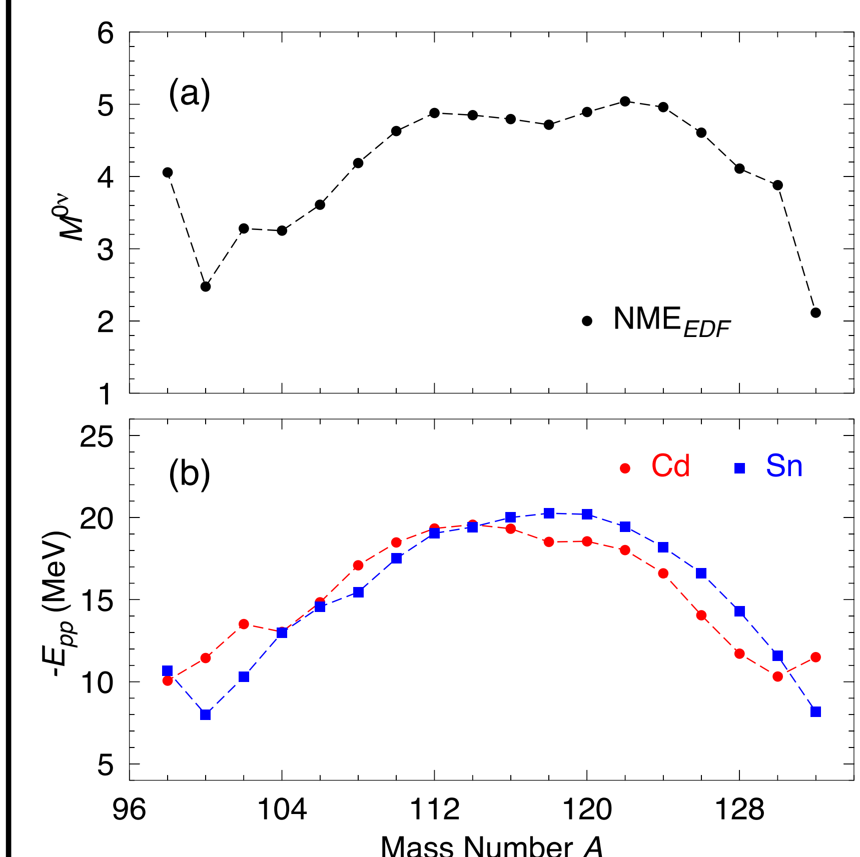


- Larger pairing correlations in mother/daughter nuclei produces larger NMEs.

- Closely related to shell effects



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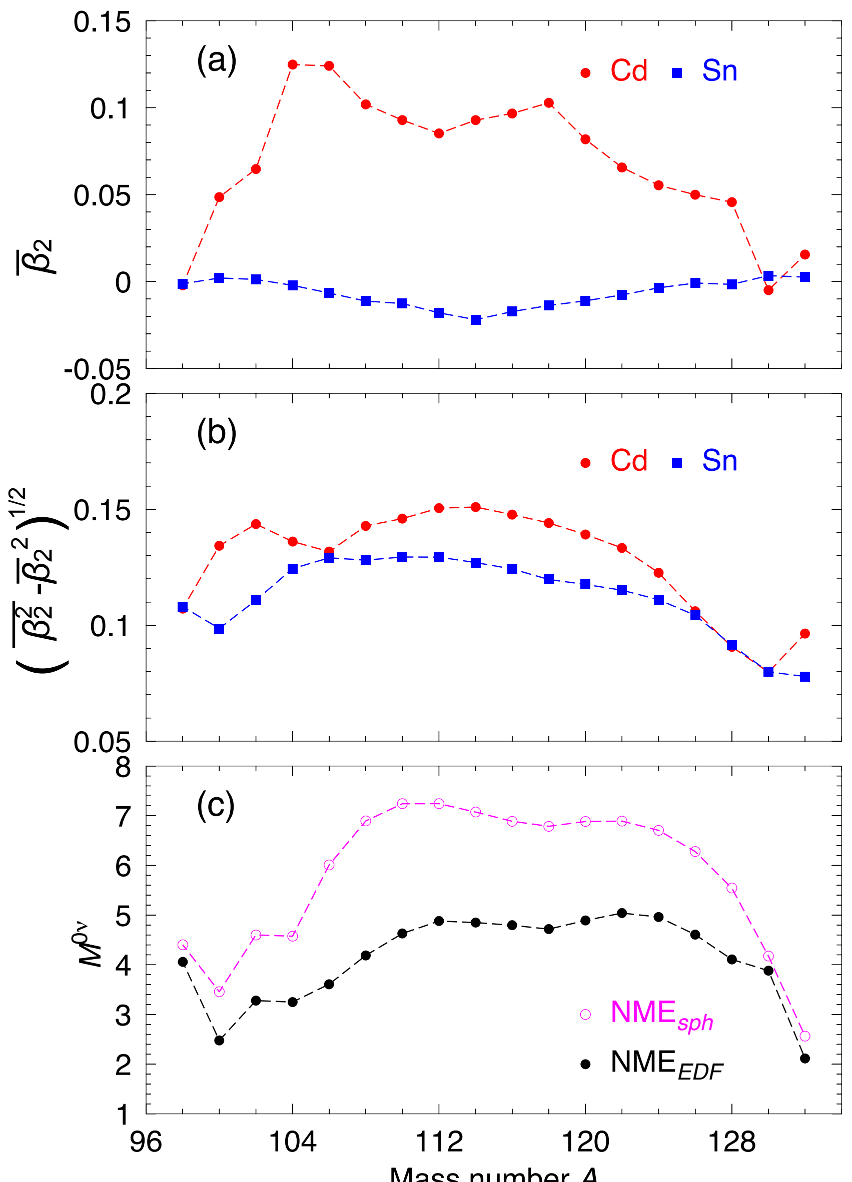
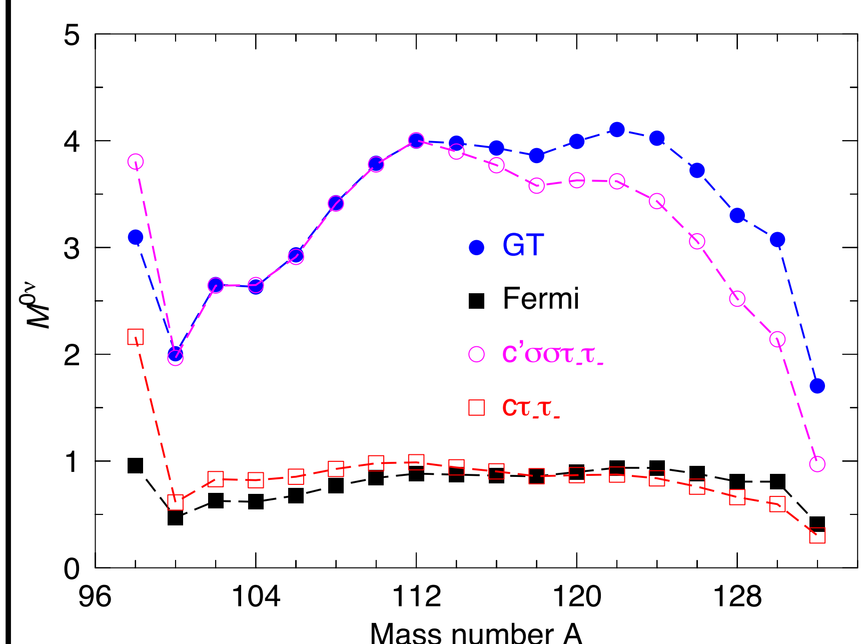


- NMEs almost proportional to the ones found with using constant neutrino potentials.
- The agreement is worse when $1h_{11/2}$ starts to be filled in. Parity? Multipoles?



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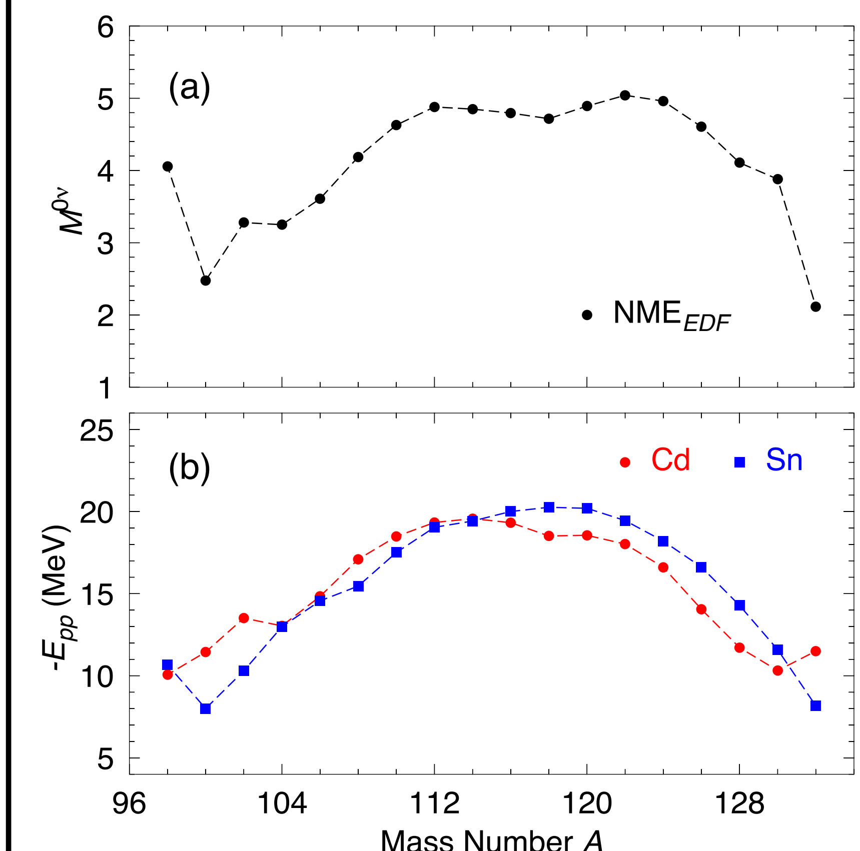
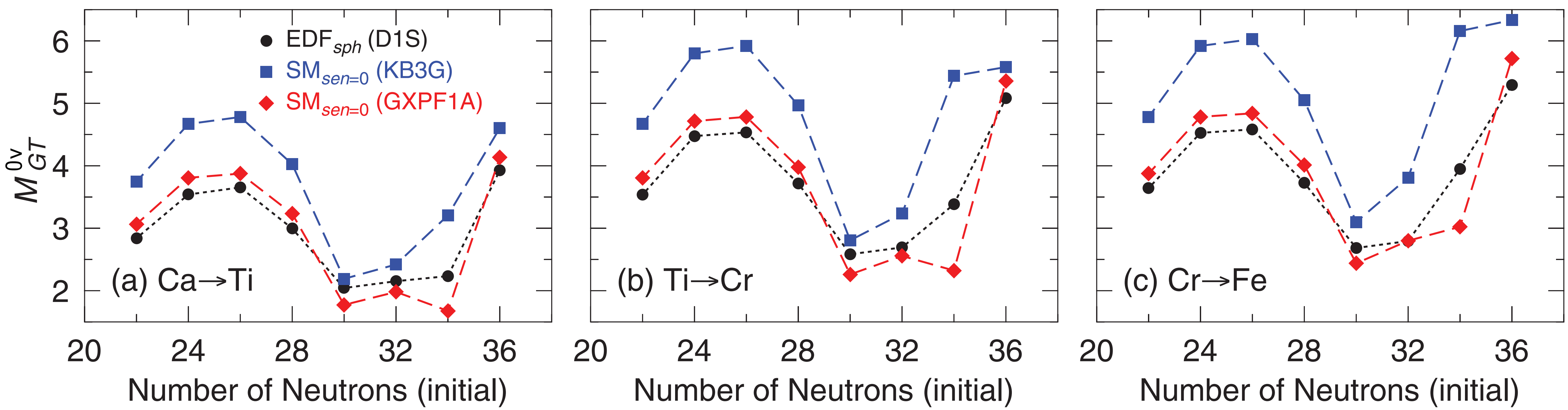
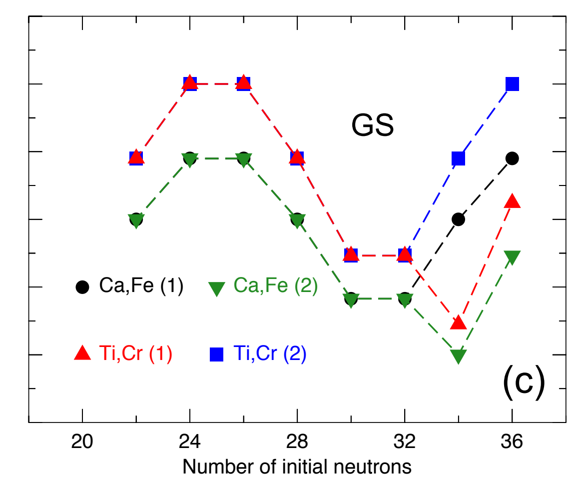


T.R.R., Martínez-Pinedo, PLB 719, 174 (2013)

NME: *pf*-shell

Where do the differences between SM and GCM come from?





J. Menéndez, T. R. R., A. Poves, G. Martínez-Pinedo, PRC 90, 024311 (2014).

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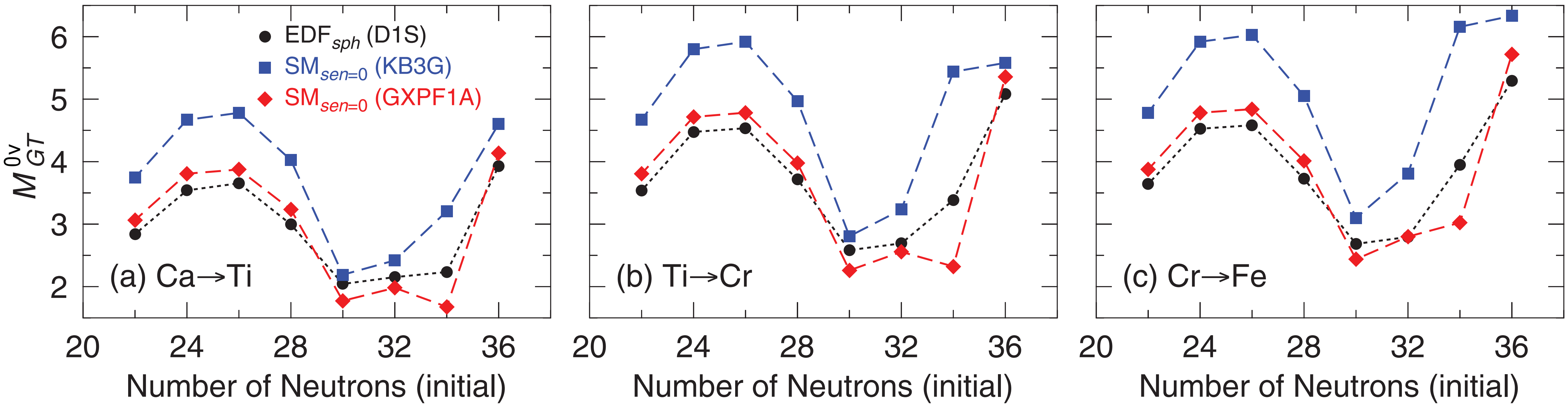
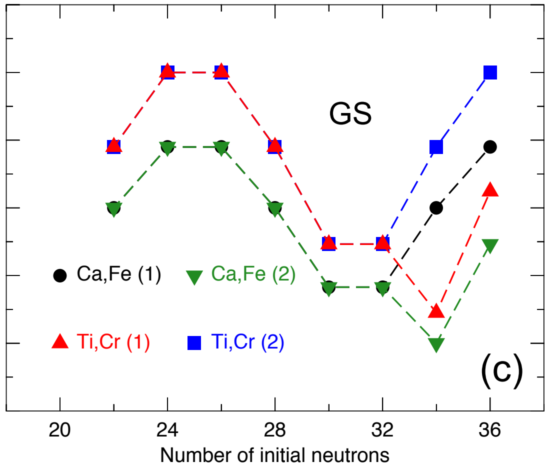


- Same pattern in spherical EDF, seniority 0 Shell Model, and Generalized Seniority model (overall scale?)

- What is the effect of including more **correlations**?



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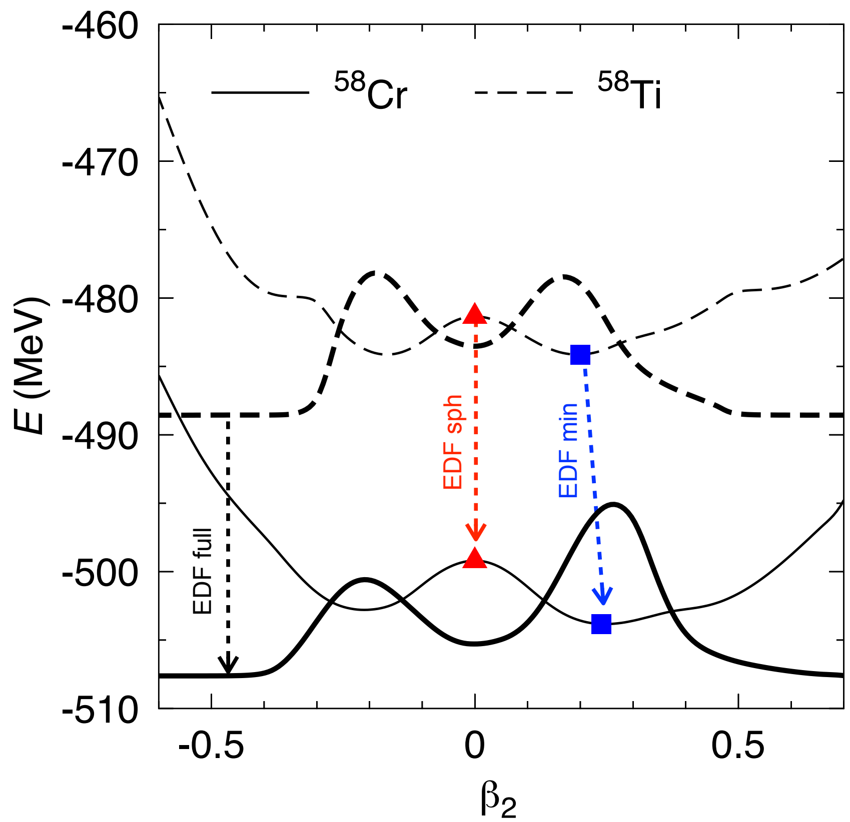

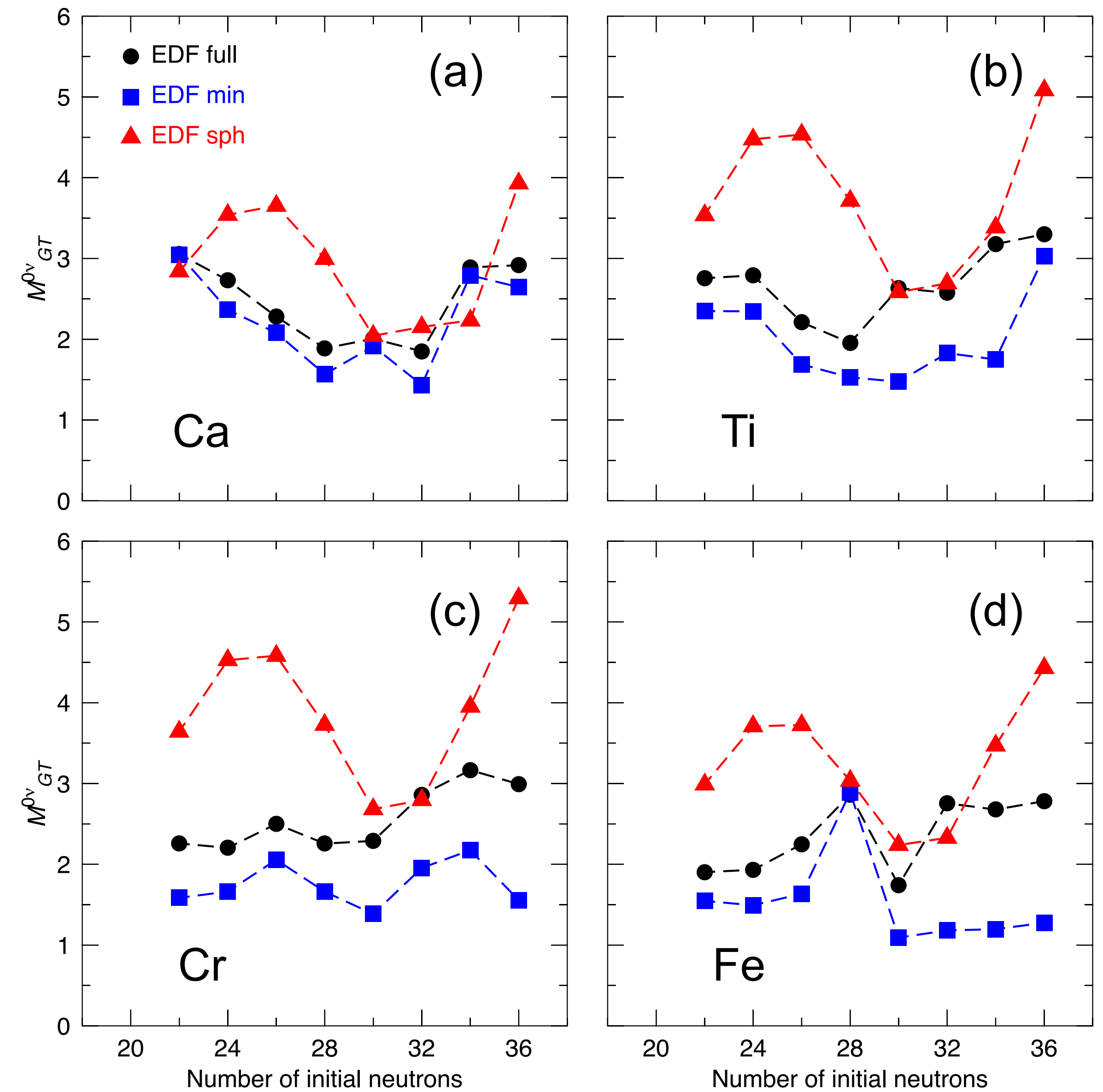

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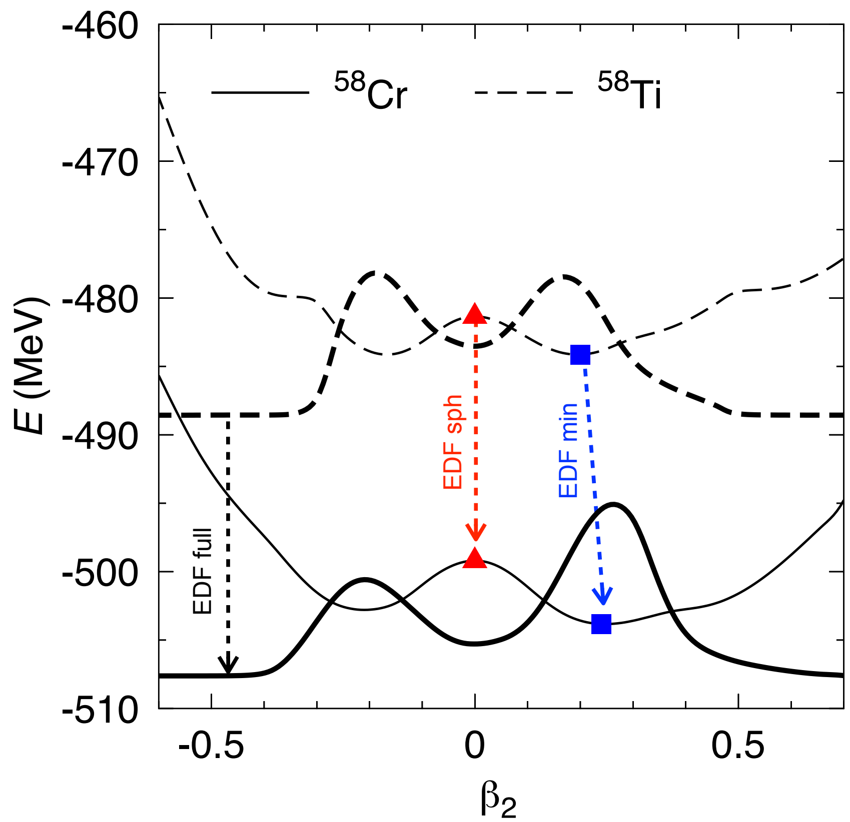

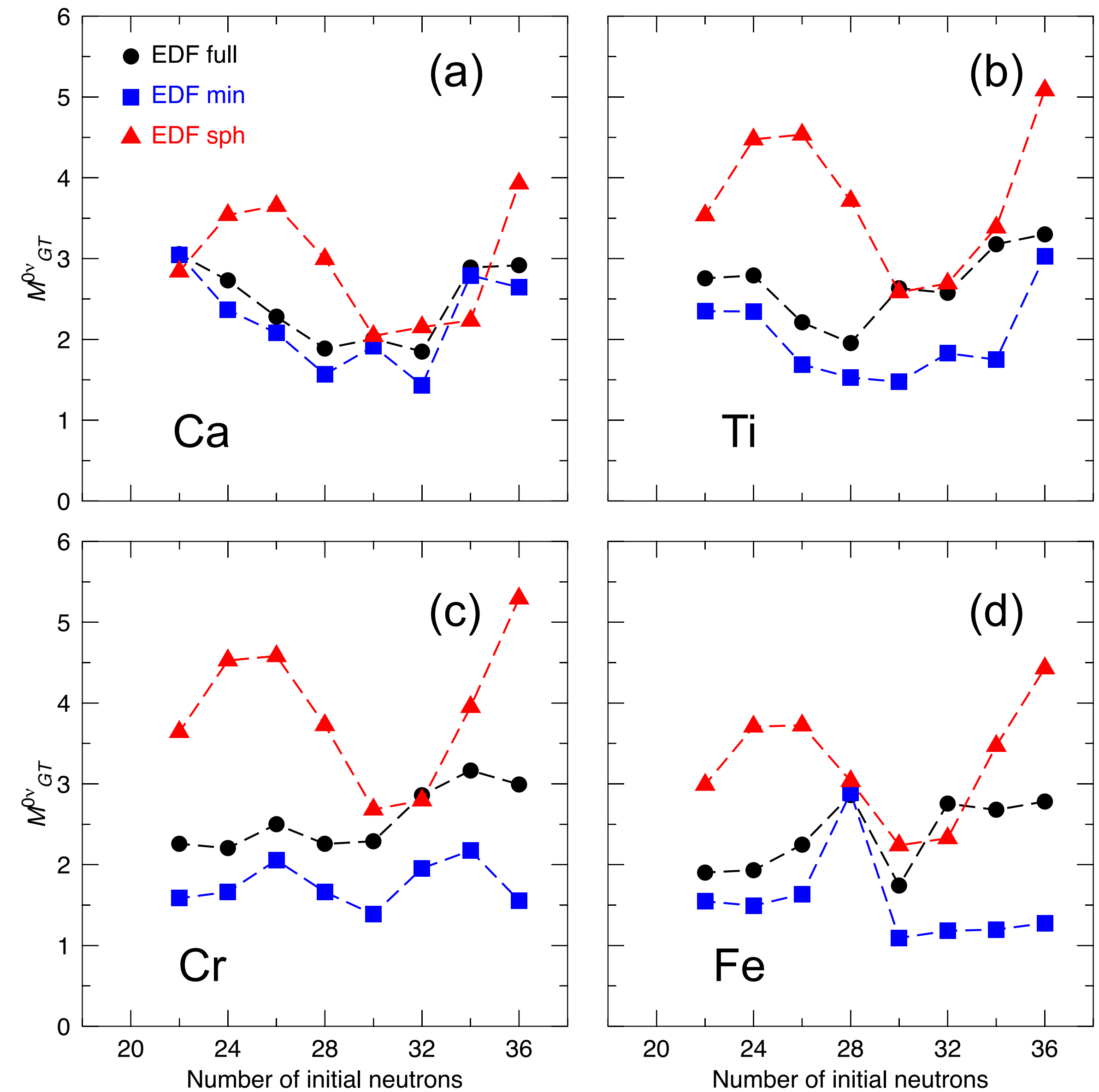

- NMEs are reduced with respect to the spherical value when correlations are included.

- The biggest reduction is produced by angular momentum restoration and configuration mixing produces an increase of the NME.

- Cross-check nuclei: ^{42}Ca, ^{50}Ca, ^{56}Fe

J. Menéndez, T. R. R., A. Poves, G. Martínez-Pinedo, PRC 90, 024311 (2014).

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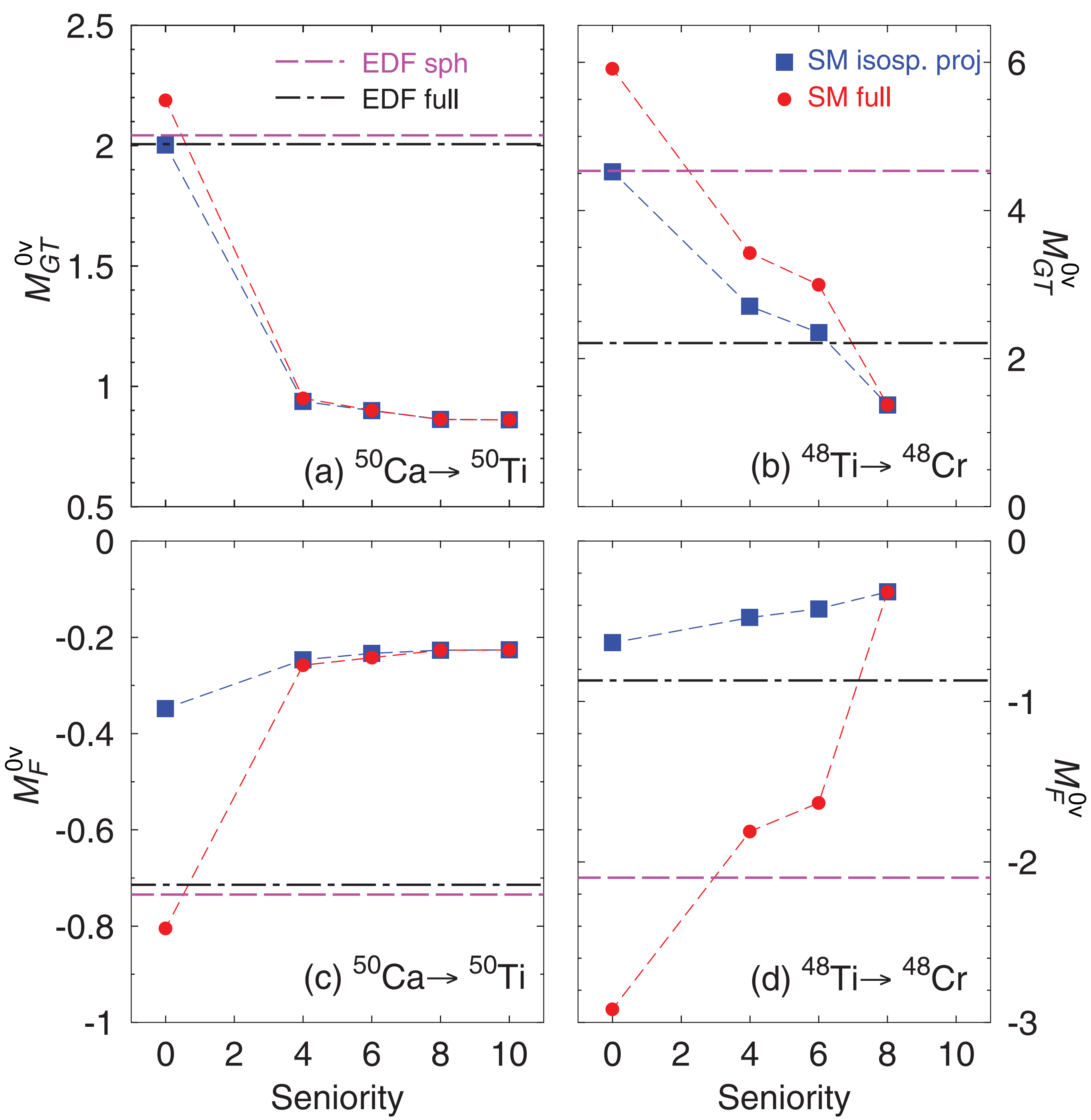

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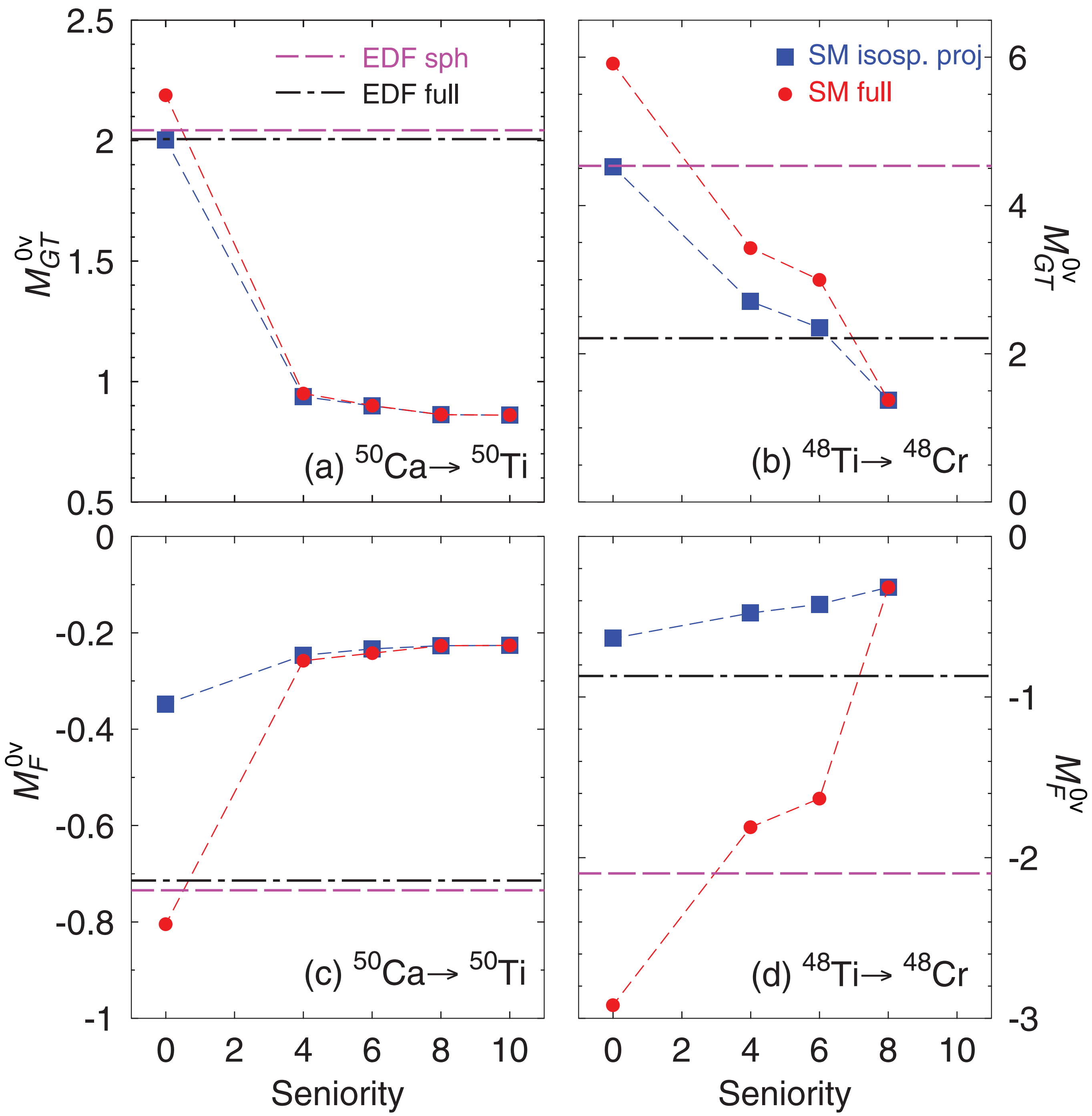

- The biggest reduction (in Shell model calculations) is produced by including higher seniority components in the nuclear wave functions.
- Isospin projection is relevant for the Fermi part of the NME and less important for the Gamow-Teller part.
- EDF does not include properly those higher seniority components, specially in spherical nuclei.
- p-n pairing effects could also be important in the reduction of the NME.

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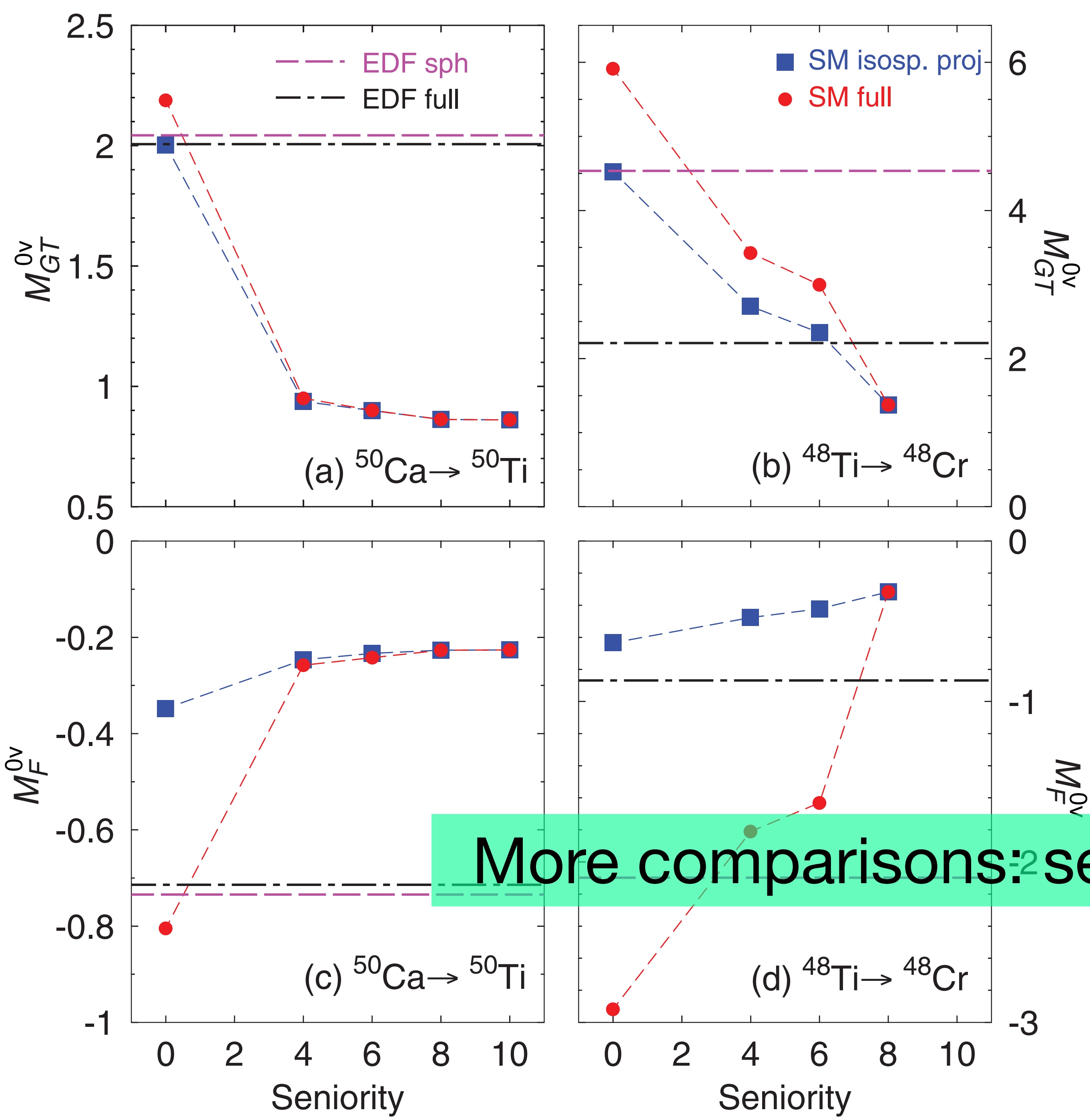

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More comparisons: see Nobuo's talk!

J. Menéndez, T. R. R., A. Poves, G. Martínez-Pinedo, PRC 90, 024311 (2014).

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	^{48}Ca	^{48}Ti	NME (F/GT/T)
spherical	-7.558	-20.497	-2.276/4.736/0.116
GCM:Q_{20}	-7.670	-23.556	in progress
GCM:Q_{20}+T=1	-7.855	-24.198	in progress
GCM:Q_{20}+T=1+T=0	-	-24.467	in progress
SM seniority 0	-7.578	-20.507	-2.287/4.783/0.116
SM full	-7.959	-24.896	-0.234/0.886/0.057

preliminary

- GCM and Shell Model calculations have been performed in the *pf*-shell with KB3G interactions both!
- Variational approach to SM results with GCM approaches is evident.
- Almost perfect agreement between SM seniority 0 and PN-VAP spherical calculations both for energies **and** NMEs!

T. R. R., J. Menéndez, … in progress

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T. R. R., J. Menéndez, … in progress

Summary

- NMEs with EDF methods have been implemented exploring many degrees of freedom so far (axial quadrupole and octupole deformations, axial pp/nn pairing). Transitions between spherical and superfluid nuclei are the most favored ones.

- Inclusion of proton-neutron pairing reduces the NMEs but it is difficult to implement in actual EDF applications.

- Relativistic effects and tensor terms are small in the EDF framework

- Systematic comparisons between ISM/EDF methods have been performed. Striking similarity between EDF spherical and SM seniority zero calculations is found. Is it confirmed by GCM calculations with SM interactions?

Some open questions

- **Isospin mixing has to be done in the future. However, it is very involved (perhaps impossible) with the current Gogny EDFs?**
- **Triaxiality has to be taken into account in *A*=76 decay (at least).**
- **How relevant is the proper description of the spectra in 0νββ NMEs?**
- **Odd-odd nuclei is still a major challenge for GCM calculations.**
- **Computational time?!?**