Relativistic nuclear field theory
and
applications to single- and double-beta decay

Caroline Robin, Elena Litvinova

WESTERN MICHIGAN UNIVERSITY

INT Neutrinoless double-beta decay program
Seattle, June 13, 2017

Outline

- **Relativistic Nuclear Field Theory:** connecting the scales of nuclear physics from Quantum Hadrodynamics to emergent collective phenomena
- **Nuclear response to one-body isospin-transfer external field:** Gamow-Teller transitions, beta-decay half-lives and the "quenching" problem
- **Current developments: ground-state correlations in RNFT**
- **Application to double-beta decay: some ideas**
- **Conclusion & perspectives**

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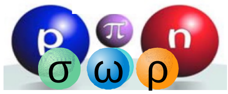
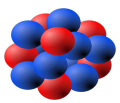
Relativistic Nuclear Field Theory: foundations



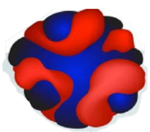
mesons

$m_{\pi,\sigma,\omega,\rho}$ ~140-800 MeV

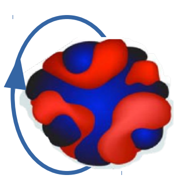


nucleons

S_n ~ 10 MeV

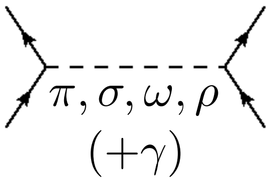


collective vibrations (phonons) ~ few MeV



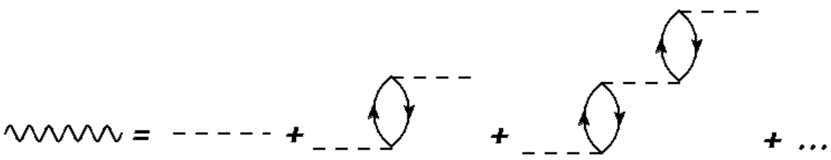
nucleons & phonons

...
more correlations



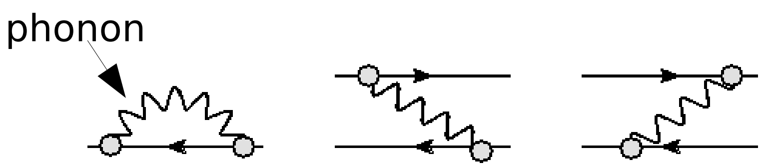
Quantum Hadrodynamics
- Relativistic nucleons

Relativistic mean-field
+ superfluidity



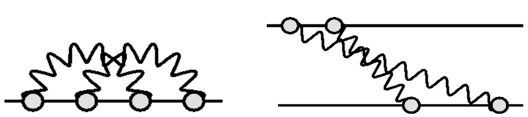
(1p-1h)

Relativistic Random Phase Approximation



(2p-2h)

Particle-Vibration coupling - Nuclear Field theory
– Time-Blocking



(3p-3h)

self-consistent extensions of the Relativistic Mean-Field via Green function techniques

successive corrections in the single-particle motion and effective interaction

...
more correlations

Include complex configurations of nucleons step by step to:
- Keep the advantages of RPA methods: description of collectivity, applicability to many nuclei
- Ultimately achieve a highly-precise description of nuclear phenomena

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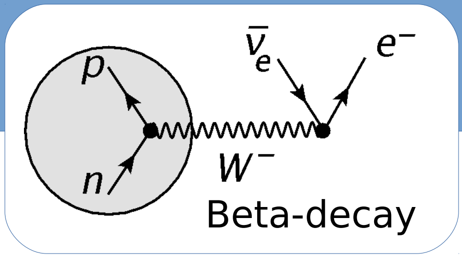
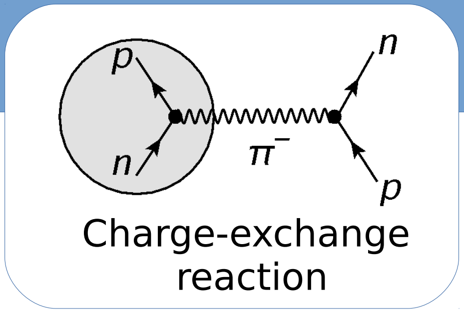
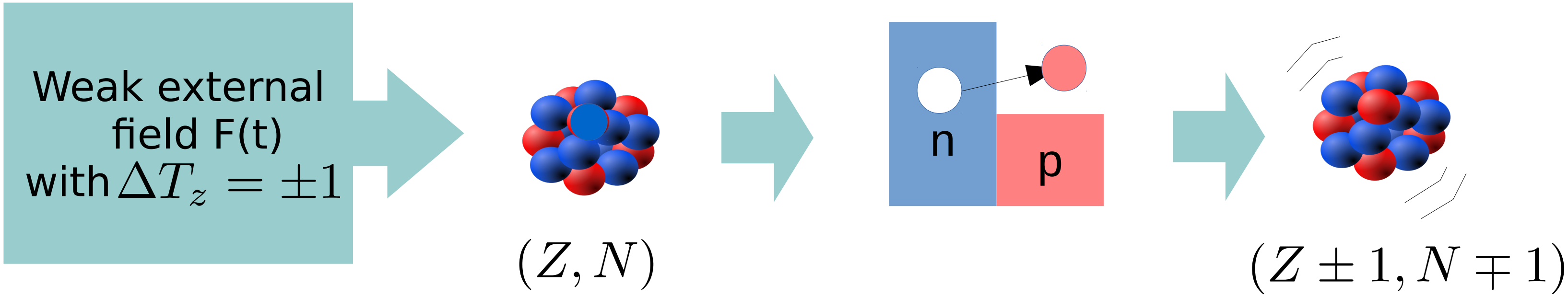
Isospin-transfer modes in nuclei

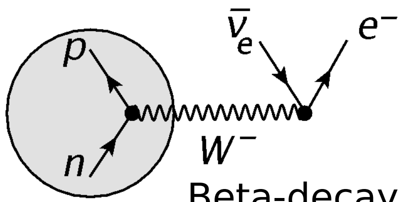


Response to a one-body external field involving a change of the isospin projection:



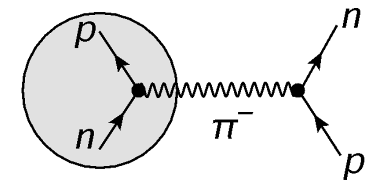


Isospin-transfer modes in nuclei

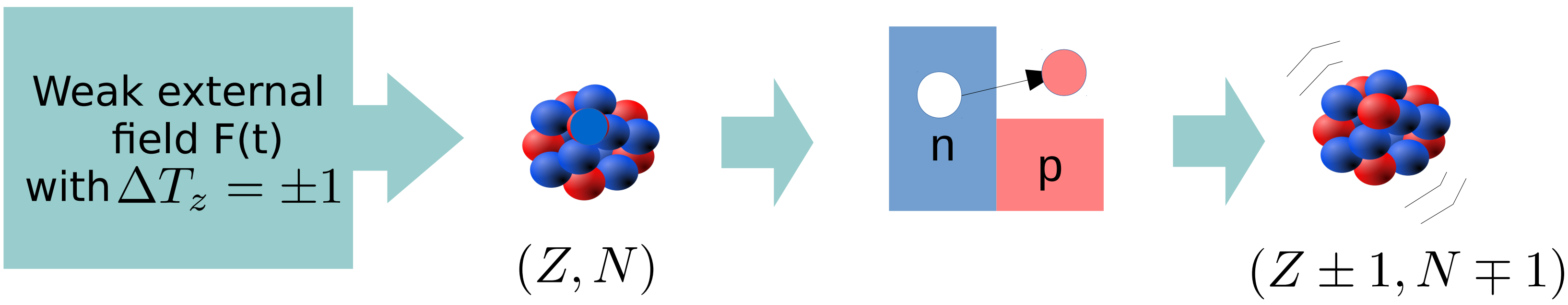


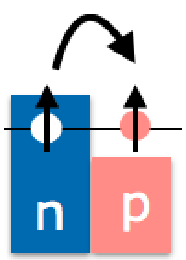
Beta-decay

Response to a one-body external field involving a change of the isospin projection:



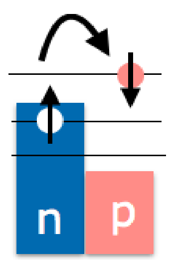
Charge-exchange reaction





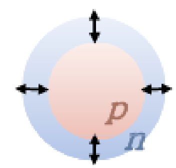
Fermi

$$F_F = \sum_n \tau_\pm^{(n)}$$



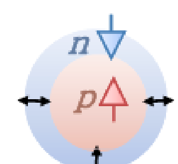
Gamow-Teller

$$F_{GT} = \sum_n \boldsymbol{\sigma}^i_{(n)} \tau_\pm^{(n)}$$



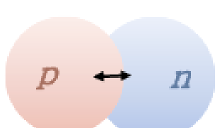
IVGMR

$$F_M = \sum_n r^2_{(n)} \tau_\pm^{(n)}$$



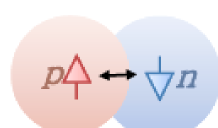
IVSMR

$$F_{SM} = \sum_n r^2_{(n)} \boldsymbol{\sigma}^i_{(n)} \tau_\pm^{(n)}$$



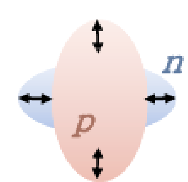
IVGDR

$$F_D = \sum_n r_{(n)} Y_1^{(n)} \tau_\pm^{(n)}$$

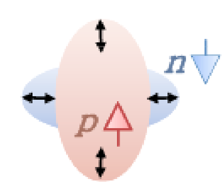


IVSDR

$$F_{SD}^\lambda = \sum_n r_{(n)} [\boldsymbol{\sigma}^i_{(n)} \otimes Y_1^{(n)}]_\lambda \tau_\pm^{(n)}$$



IVGQR



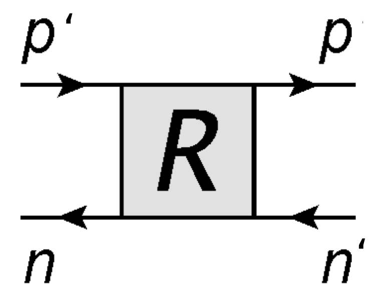
IVSQR

...

Response theory for isospin-transfer modes

★ Theoretically, all the information about these modes is contained in the
proton-neutron response function
= propagator of 2 correlated proton and neutron (in the particle-hole channel)

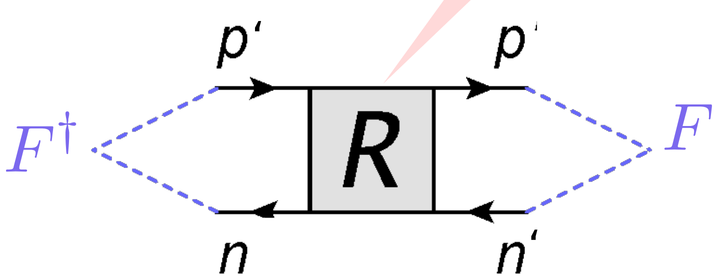
$$R^{ph}_{pn,n'p'}(t-t') = \langle 0|\mathcal{T}\left(\psi_p(t)\bar{\psi}_{n'}(t)\psi_n(t')\bar{\psi}_{p'}(t')\right)|0\rangle$$

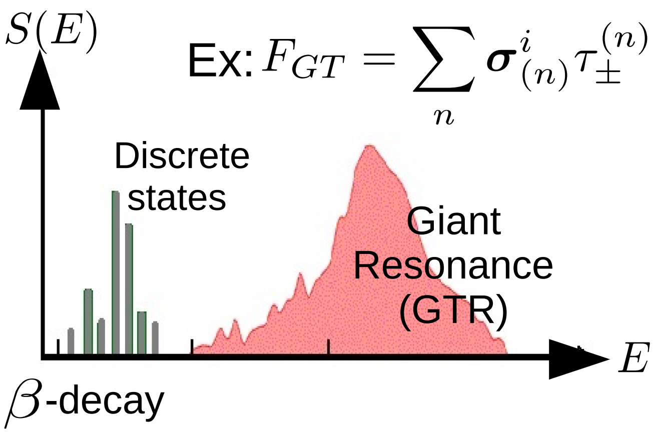


→ For instance, the strength distribution is:

$$S(E) = \sum_f |\langle\Psi_f|\hat{F}|\Psi_i\rangle|^2 \delta(E - E_f + E_i)$$

$$= -\frac{1}{\pi}\lim_{\Delta\to 0^+} \mathrm{Im}\,\langle\Psi_i|\hat{F}^\dagger R(E+i\Delta)\hat{F}|\Psi_i\rangle$$

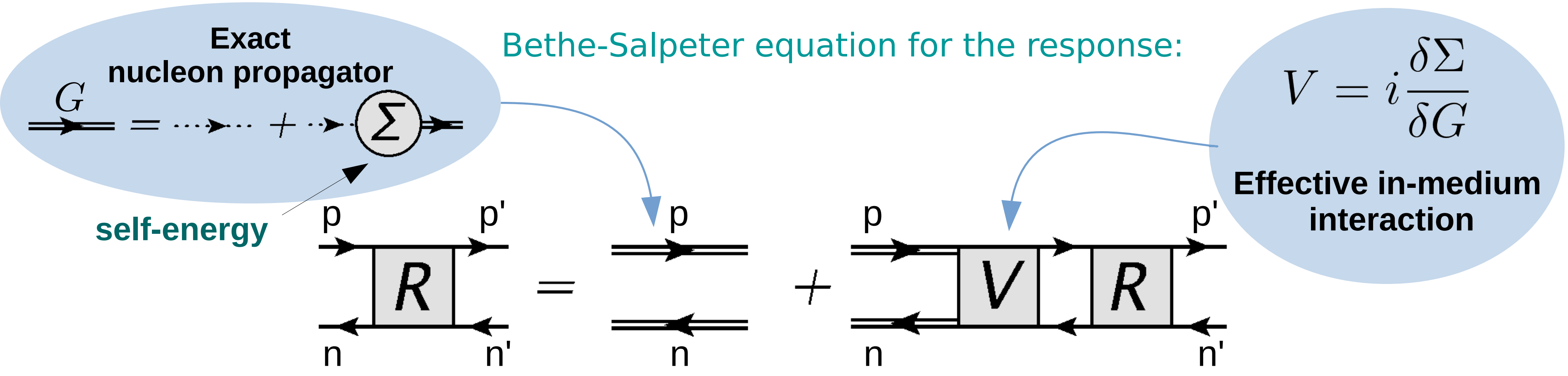




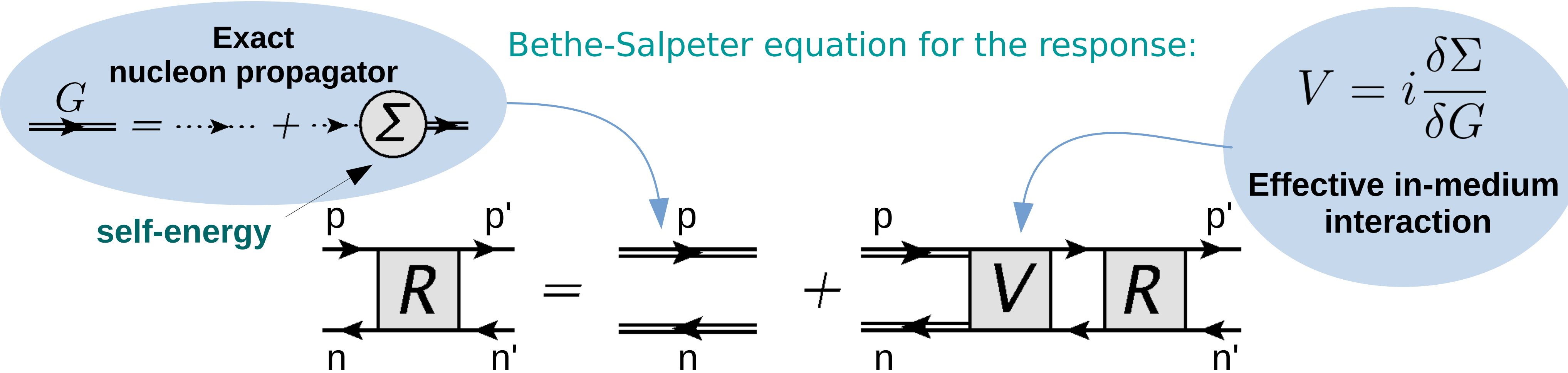
→ the response of the mother nucleus (N,Z) gives information
about the states of the daughter (N+1,Z-1) or (N-1,Z+1) nucleus

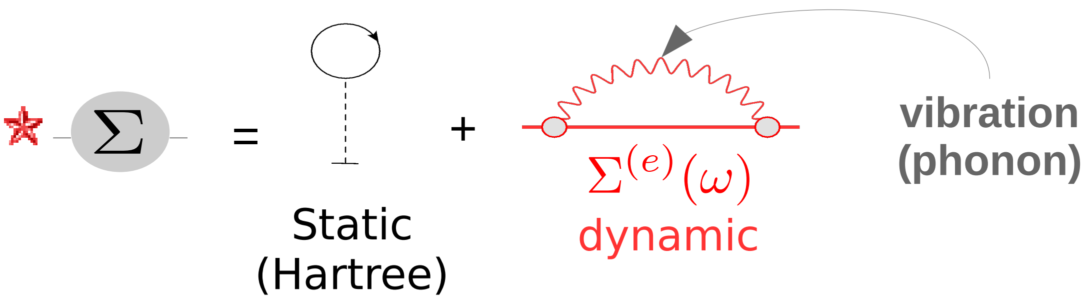
Response theory for isospin-transfer modes

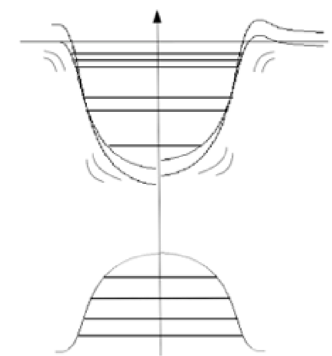
Bethe-Salpeter equation for the response:

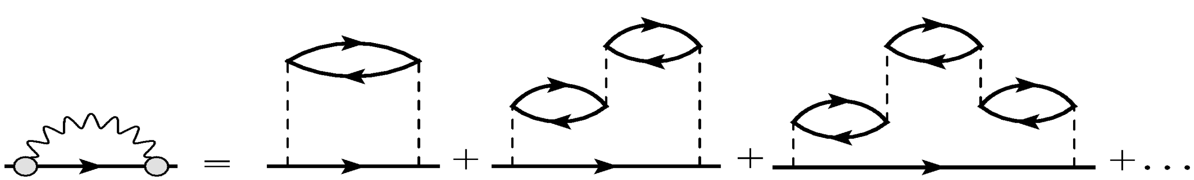


Response theory for isospin-transfer modes

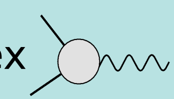






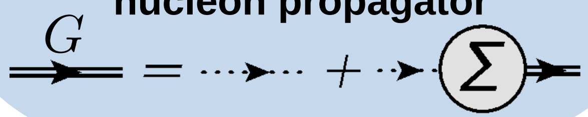


→ New-order parameter
= PVC vertex



Response theory for isospin-transfer modes

Exact nucleon propagator

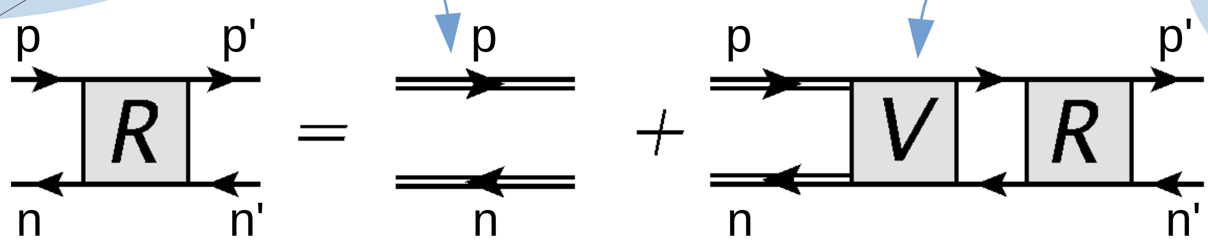


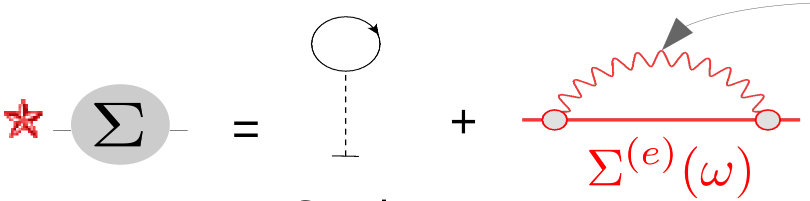
self-energy

Bethe-Salpeter equation for the response:

$$V = i\frac{\delta\Sigma}{\delta G}$$

Effective in-medium interaction

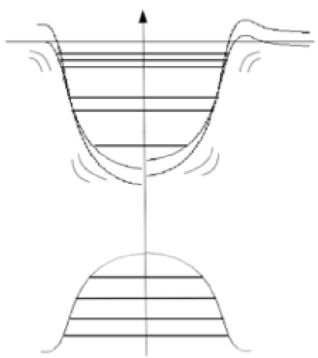


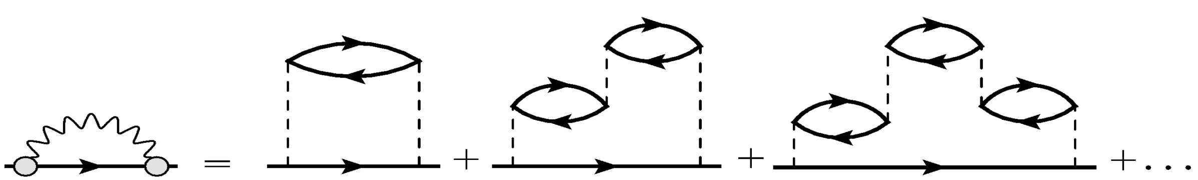


Static (Hartree)

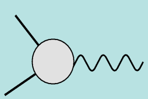
$\Sigma^{(e)}(\omega)$ dynamic

vibration (phonon)



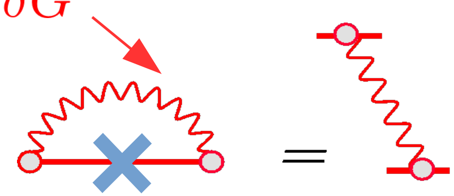


→ New-order parameter = PVC vertex



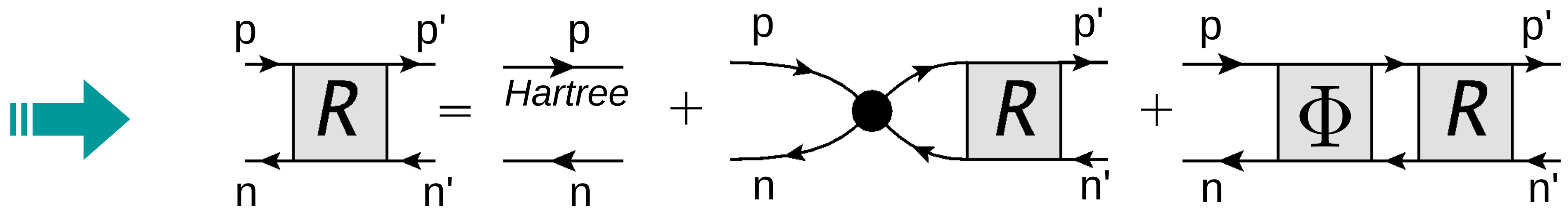
$$V(\omega) = i\frac{\delta\widetilde{\Sigma}_{RHB}}{\delta G} + i\frac{\delta\Sigma^{e}(\omega)}{\delta G}$$

Static meson-exchange



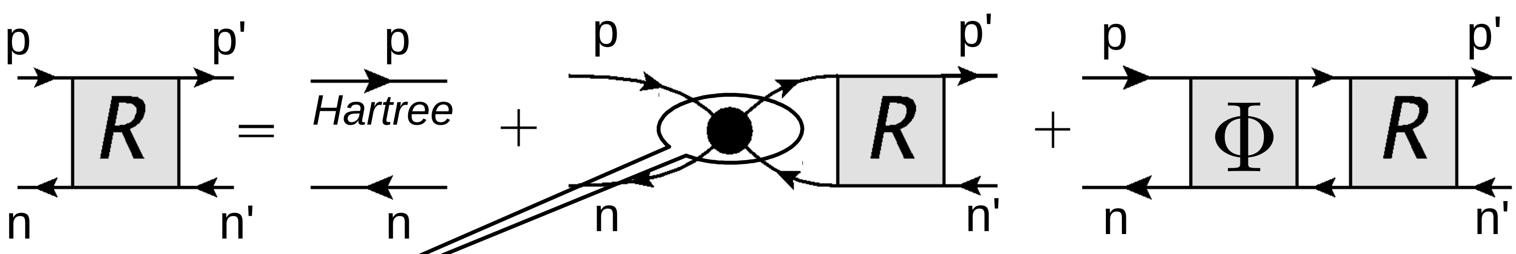
Energy-dependent phonon exchange

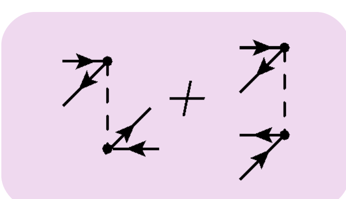
Response theory for isospin-transfer modes



Response theory for isospin-transfer modes

time





Isovector static meson exchange

$$\widetilde{V} = V_\pi + V_\rho + V_{\delta\pi}$$

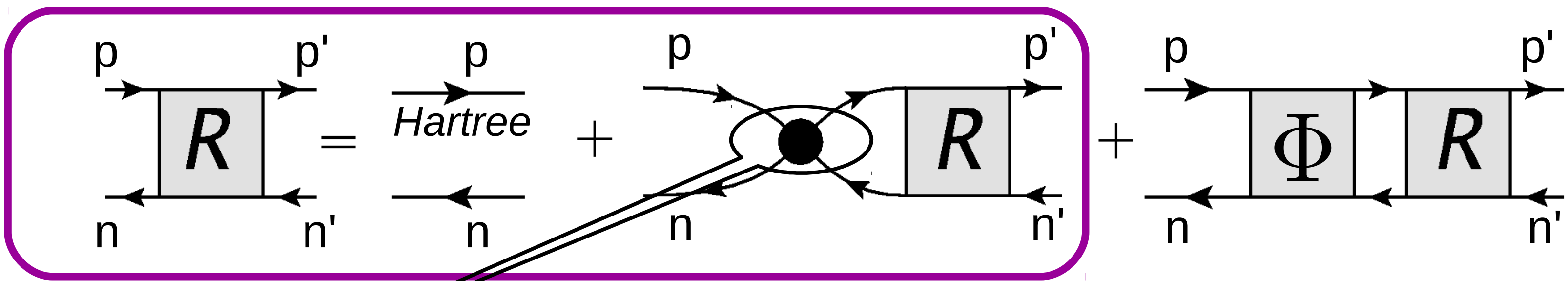
With free-space coupling constant

Landau-Migdal contact term

g'=0.6

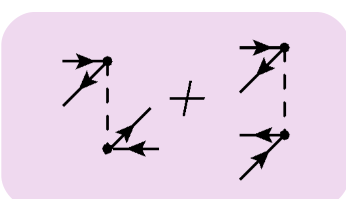
Response theory for isospin-transfer modes

time



pn-RRPA

accounts for 1p1h configurations
(on correlated ground state)



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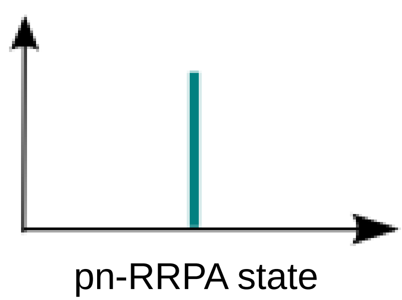
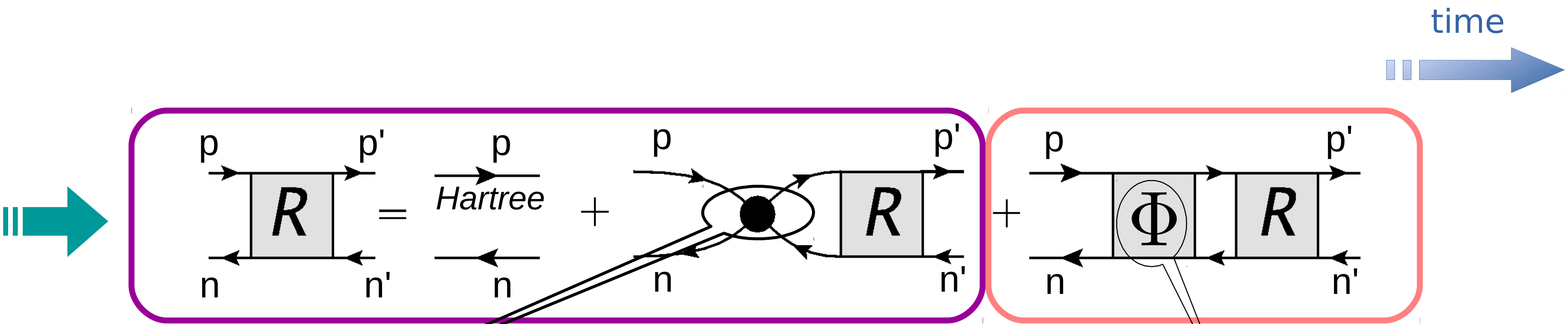


Response theory for isospin-transfer modes

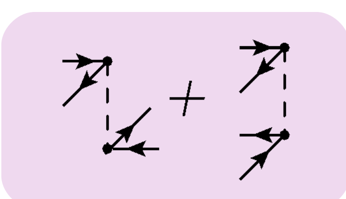


pn-RRPA

accounts for 1p1h configurations
(on correlated ground state)

+PVC

energy-dependent interaction:



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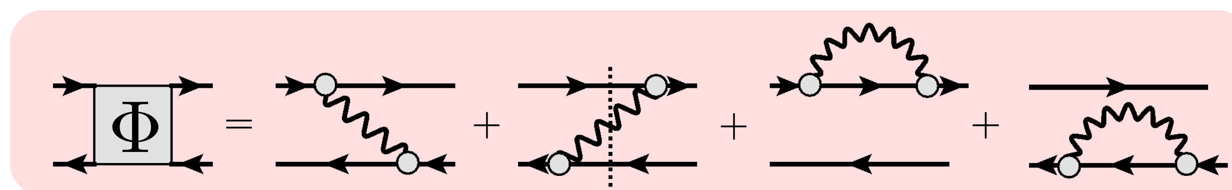
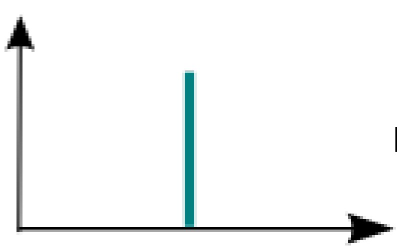
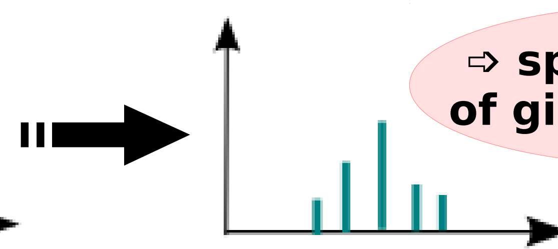


accounts for
1p1h ⊗ 1phonon = 2p2h
configurations



pn-RRPA state



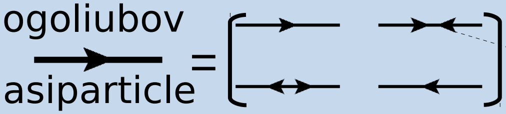
pn-RRPA + PVC

⇨ **spreading width**
of giant resonances

Response theory for isospin-transfer modes

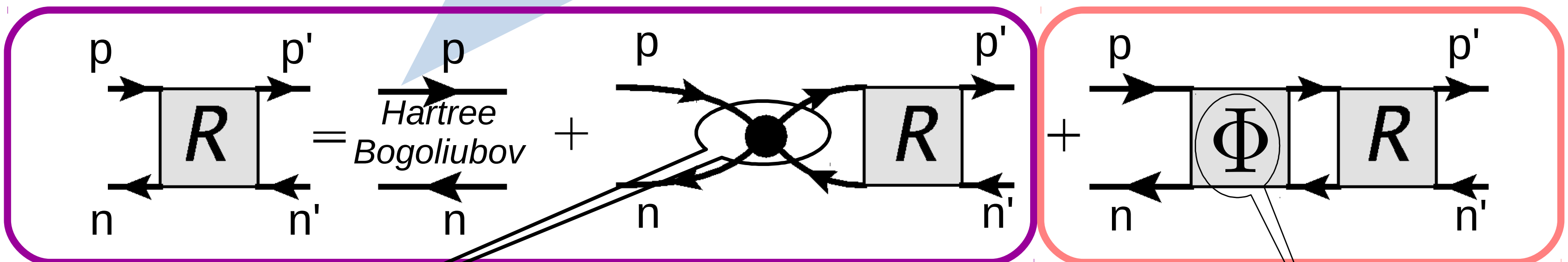
In open-shell nuclei:

Bogoliubov Quasiparticle



→ spinors of dimensions 16 (spin,isospin,relativity,pairing)

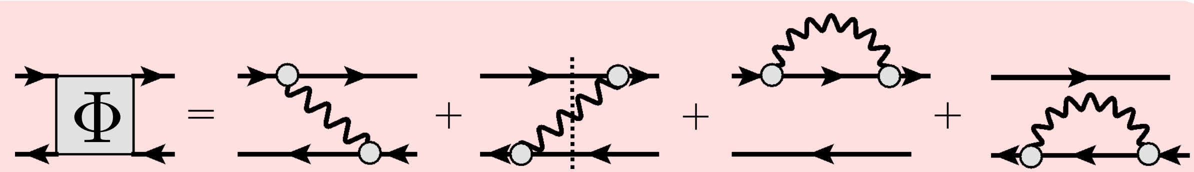
Gorkov propagator



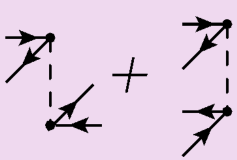
pn-RQRPA accounts for 2qp configurations (on correlated ground state)

+QVC

energy-dependent interaction:



accounts for 2qp ⊗ 1phonon = 4qp configurations



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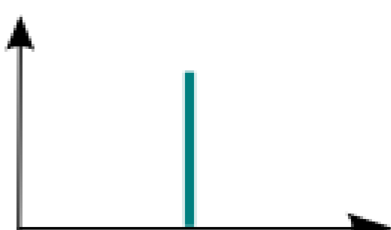
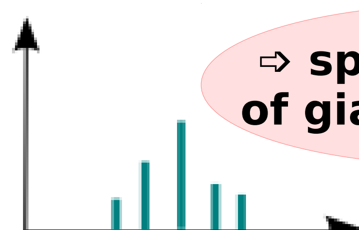
With free-space coupling constant

Landau-Migdal contact term

g’=0.6



pn-RQRPA state



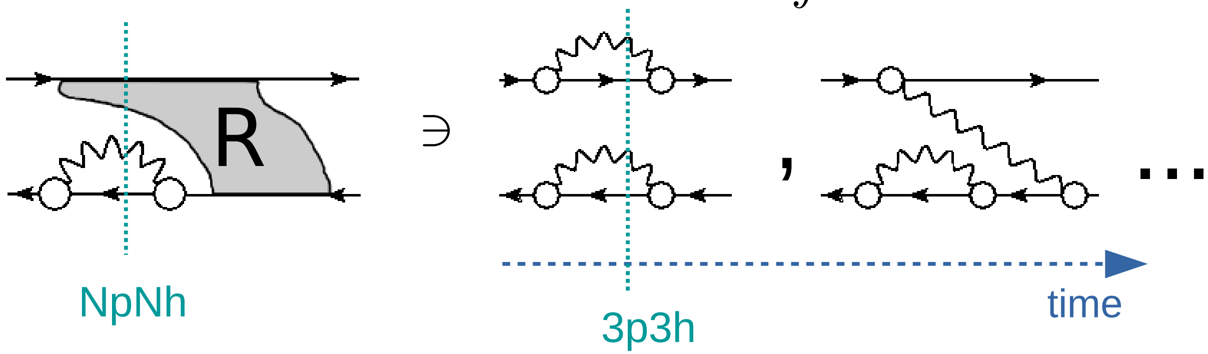
pn-RQRPA + QVC

⇨ **spreading width of giant resonances**

Response theory for isospin-transfer modes

Problem: Integration over all intermediate times ⇒ complicated BSE, NpNh configurations:

$$R(\omega,\varepsilon) = G(\omega+\varepsilon)G(\varepsilon) - iG(\omega+\varepsilon)G(\varepsilon)\int d\varepsilon_1 d\varepsilon_2 V(\varepsilon,\varepsilon_2,\omega)R(\varepsilon_2,\varepsilon_1,\omega)$$



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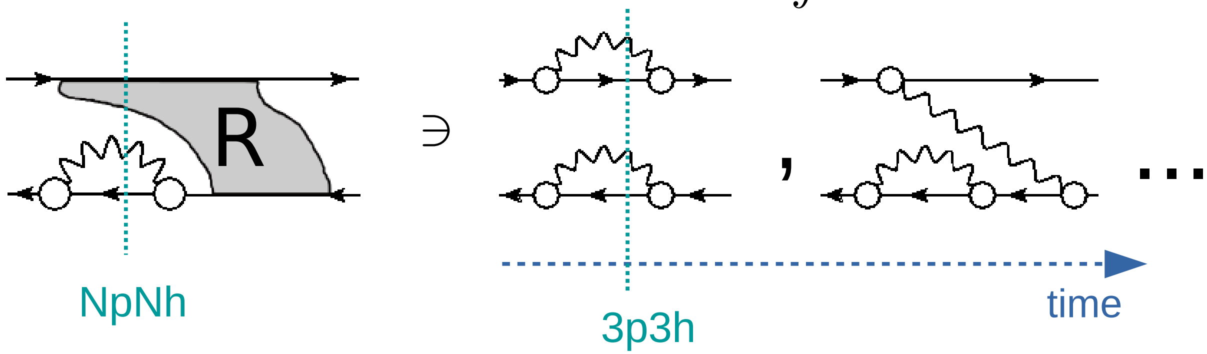
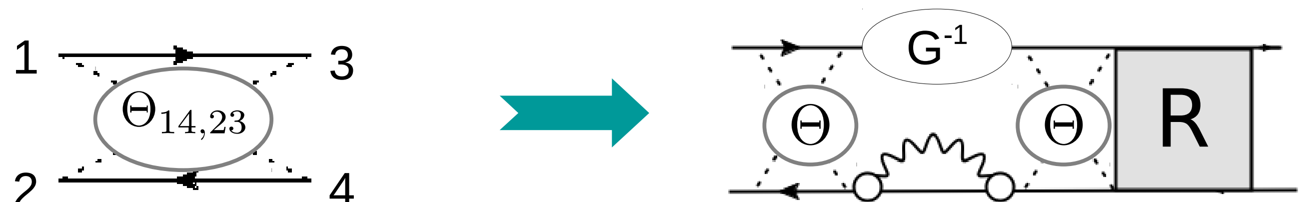


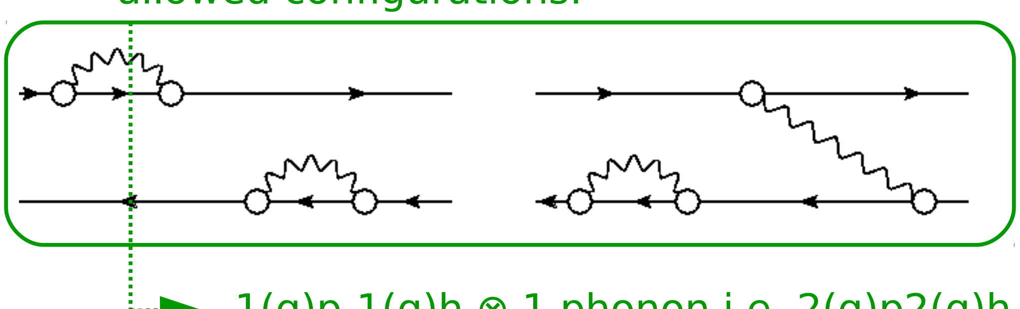
Solution: Time-Blocking Approximation [V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)]



$\Theta_{14,23} = \theta(t_4 - t_1)\theta(t_3 - t_2)$

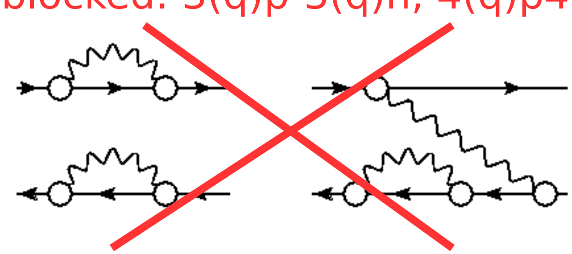
$$R(\omega) = R^0(\omega) - iR^0(\omega)(\tilde{V} + \Phi(\omega))R(\omega)$$

→ allowed configurations:



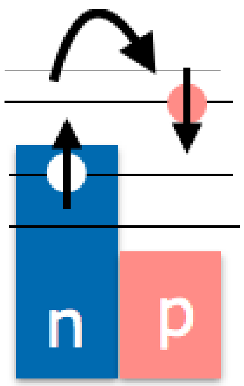
→ 1(q)p-1(q)h ⊗ 1 phonon i.e. 2(q)p2(q)h

→ blocked: 3(q)p-3(q)h, 4(q)p4(q)h...



... but can be included in a next step (under development)

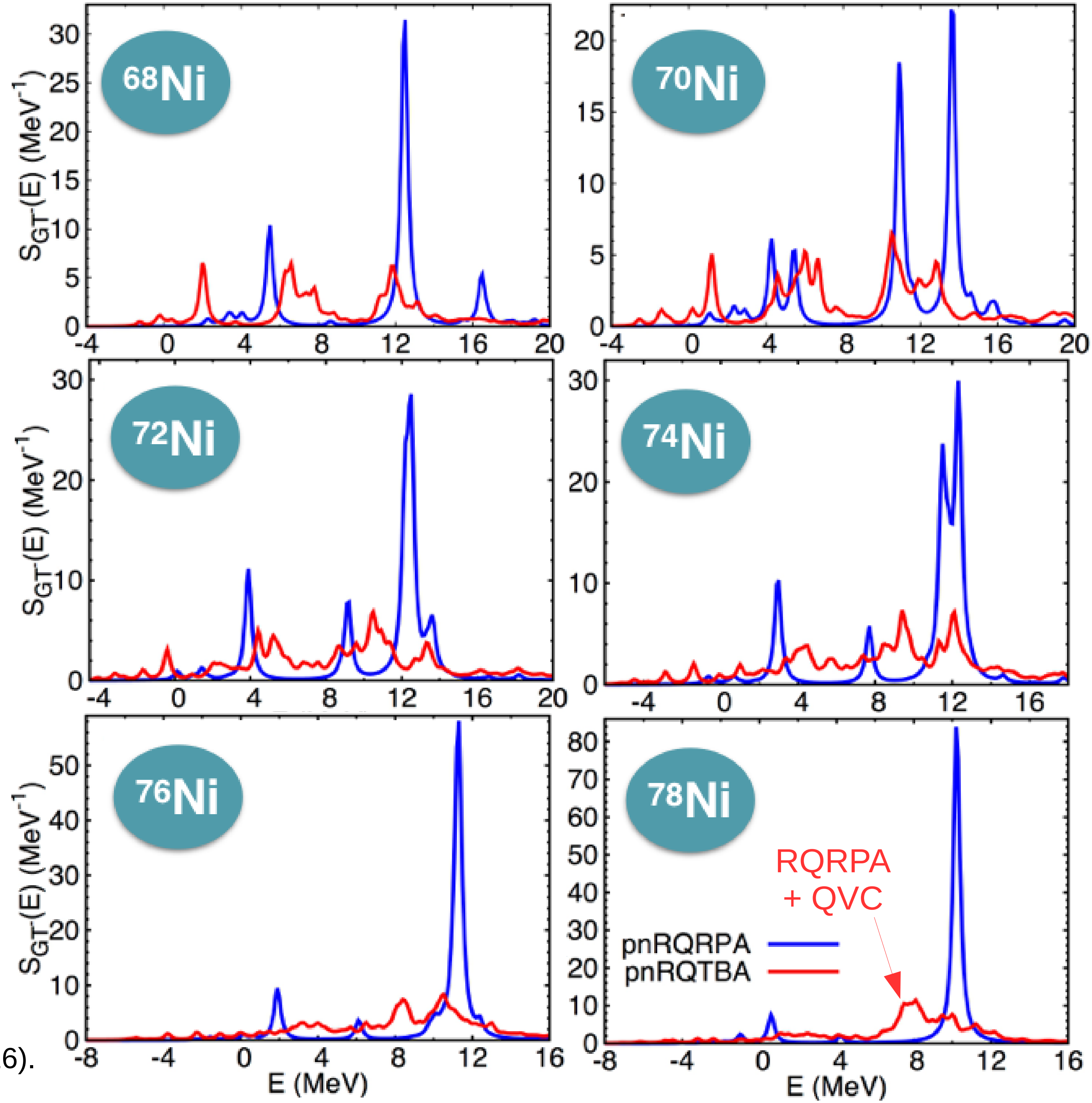
Gamow-Teller transitions in Nickel isotopes (Ni → Cu)



$$F_{GT^-} = \sum_n \begin{pmatrix} \sigma^i_{(n)} & 0 \\ 0 & \sigma^i_{(n)} \end{pmatrix} \tau_-^{(n)}$$

$$\Delta T_z = -1$$
$$\Delta S = 1$$
$$\Delta L = 0$$

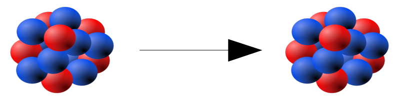
QVC brings fragmentation of the strength and spreading over a larger energy range



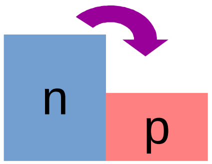
(Smearing Δ= 200 keV)

C. R. and E. Litvinova EPJA 52, 205 (2016).

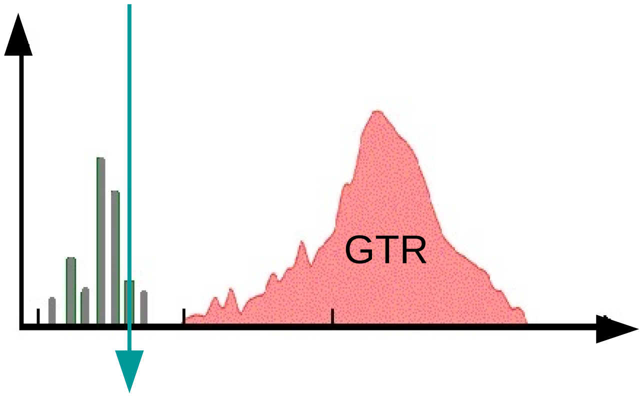
Low-energy GT strength and beta-decay half-lives

- ß⁻-decay:  $+e^- + \bar{\nu}_e$

  (Z,N) → $(Z+1,N-1)$



In the allowed GT approximation, it is determined by the low-lying GT strength:



Maximal energy release = Q_β = $M_{at}(Z,n) – M_{at}(Z+1,N-1)$

→ beta-decay half-lives: $$\frac{1}{T_{1/2}} = \frac{g_a^2}{D}\int^{Q_\beta} f(Z,Q_\beta - E)S(E)dE$$

Leptonic phase-space factor

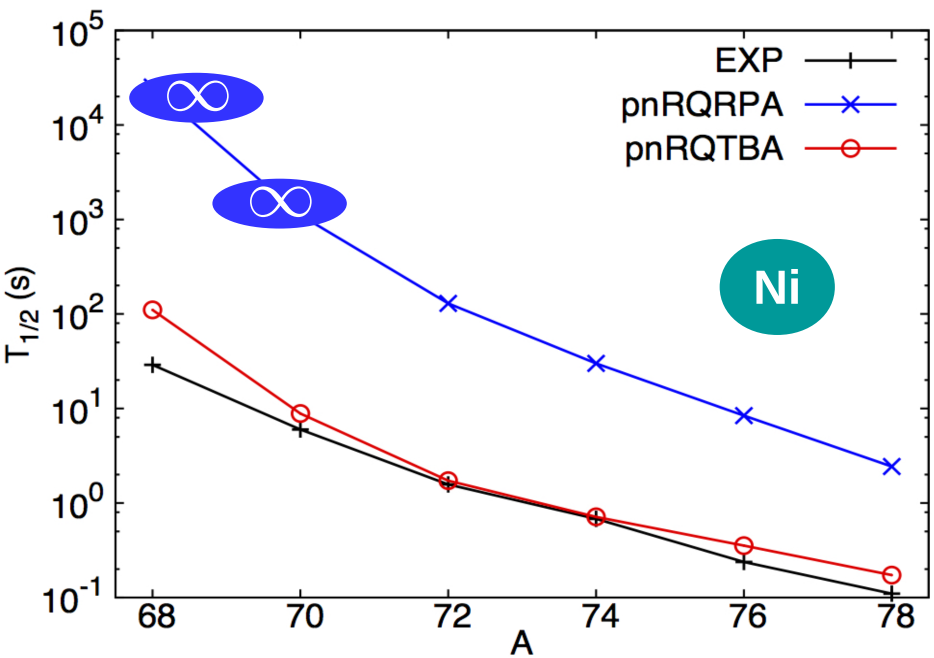
Low-energy GT strength and beta-decay half-lives

★ Half-lives and low-energy strength:

$$\frac{1}{T_{1/2}} = \frac{g_a^2}{D} \int^{Q_\beta} f(Z, Q_\beta - E) S(E) dE$$

With g_a=1

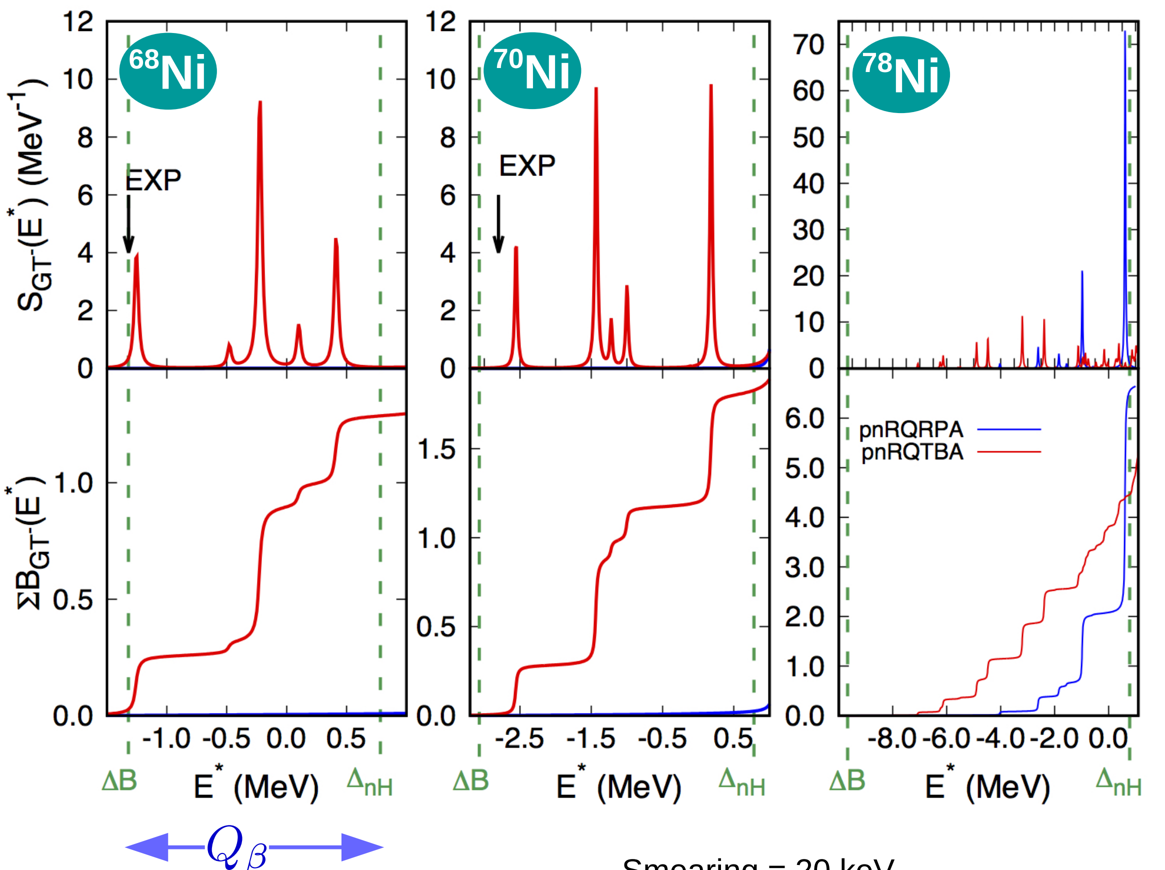
Leptonic phase-space factor



→ big improvement due to QVC!

exp data from nndc.bnl.gov

C.R. and E. Litvinova EPJA 52, 205 (2016).



Smearing = 20 keV

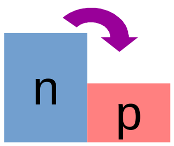
- ^{68}Ni and ^{70}Ni : appearance of strength in the Q_β window due to QVC → finite lifetime
- ^{78}Ni: more strength with RQRPA but located at higher energies → smaller lifetime with QVC due to phase space factor

Gamow-Teller transitions and the “quenching” problem

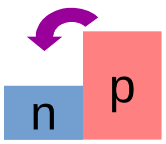
- “Quenching problem”:

The observed GT strength (~up to the GR region) in nuclei is ~30-40% less than the model independent Ikeda sum rule: $S_- - S_+ = 3(N-Z)$

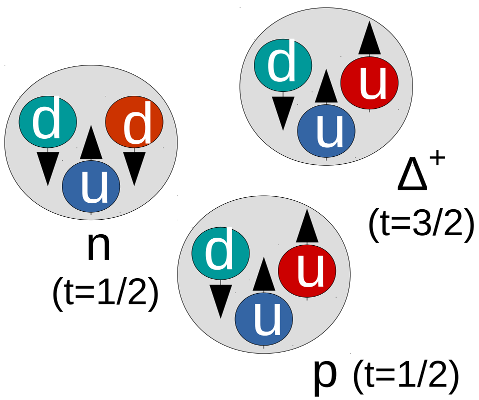
$$S_- = \sum B(GT^-)$$



$$S_+ = \sum B(GT^+)$$



⇒ some strength is pushed at high energies → possible mechanisms?



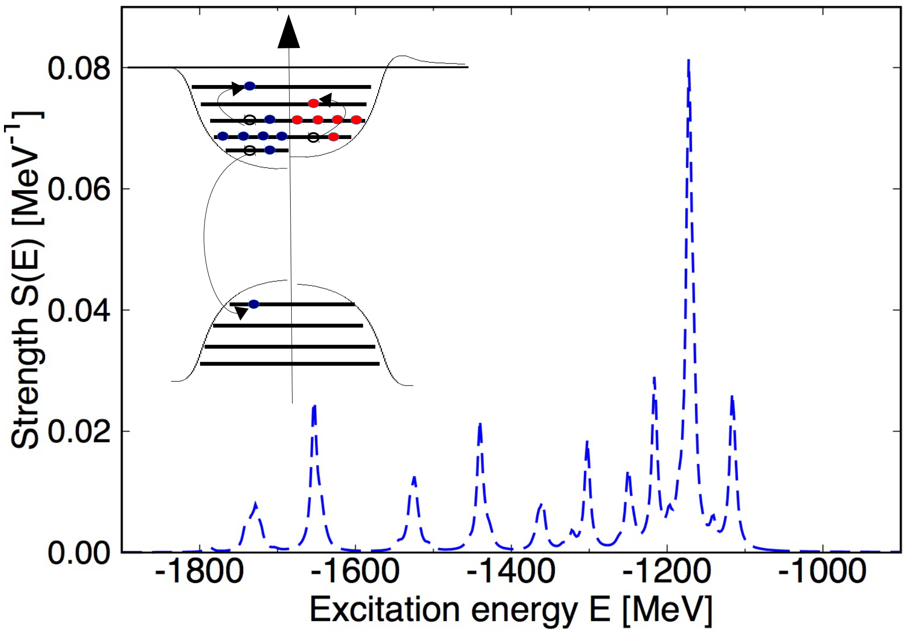
- Coupling of 1p1h to Δ baryon (not done here)

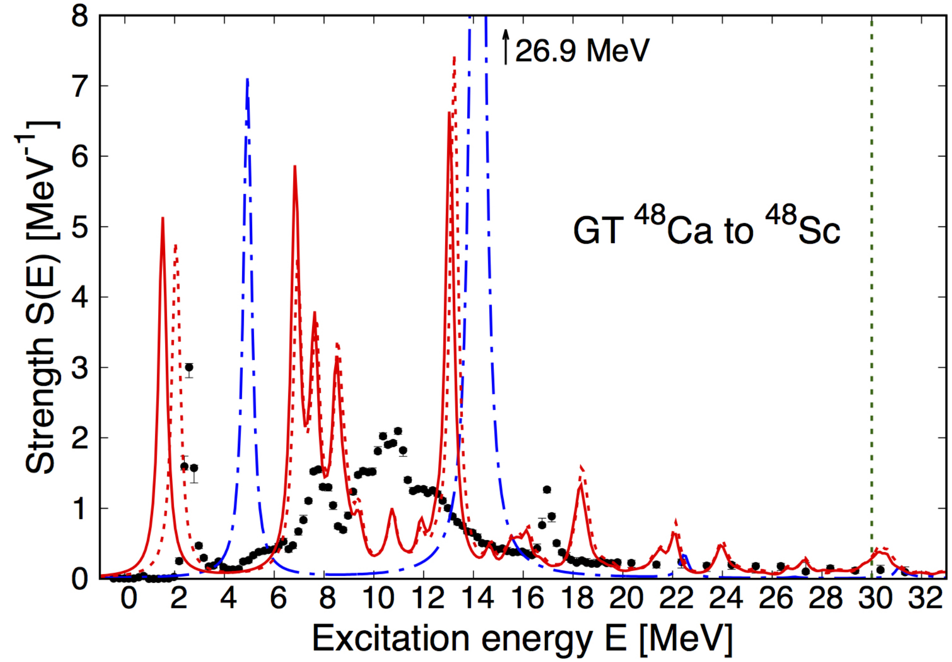
- Coupling of 1p1h to higher-order configurations such as 2p2h, 3p3h…
⇒ important to introduce complex configurations in large model spaces

At present with RNFT+TBA:

- ✔2(q)p-2(q)h configurations
- ✔in an energy window from 30 MeV up to ~100 MeV in light or doubly magic nuclei

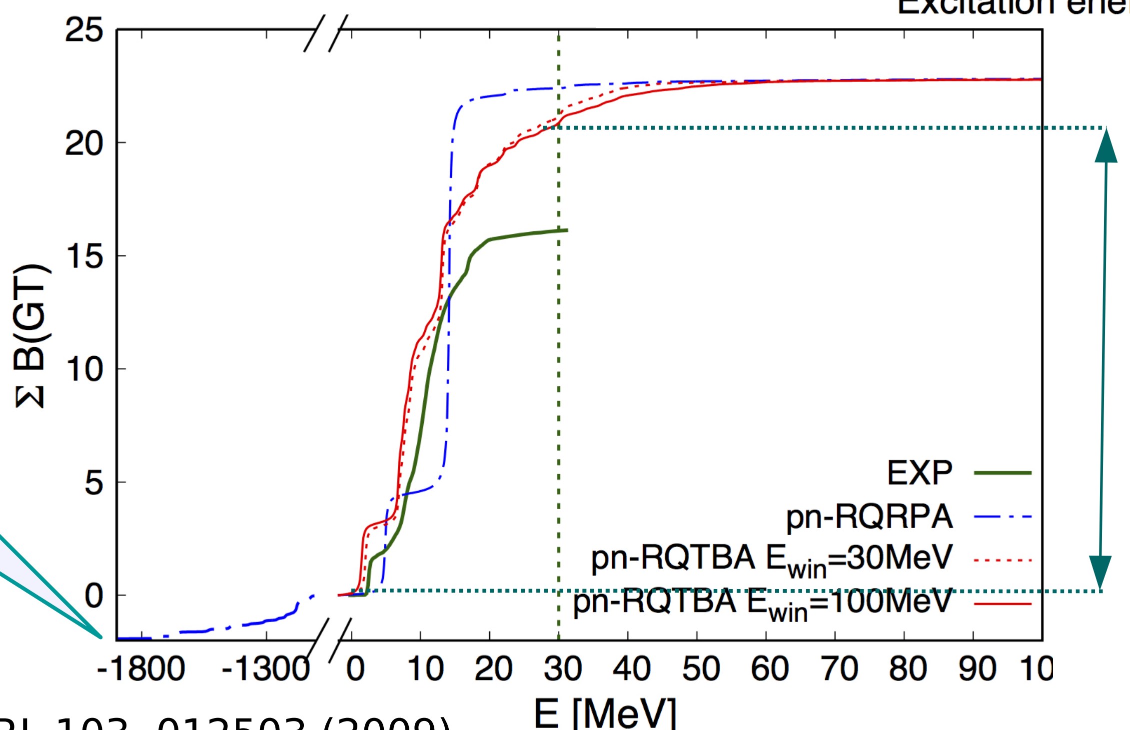
Gamow-Teller transitions and the “quenching” problem





+ transitions from the Fermi sea to the Dirac sea (~8%)

[N. Paar et al., PRC 69, 054303]



Up to 30 MeV: **~91%** (vs 98% in RQRPA) of the total GT_ strength

→ RQRPA strength naturally “quenched” due to complex configurations

But not enough... (exp: 71%)

EXP: K. Yako et al., PRL 103, 012503 (2009)

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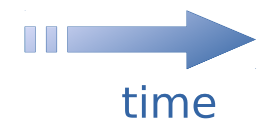
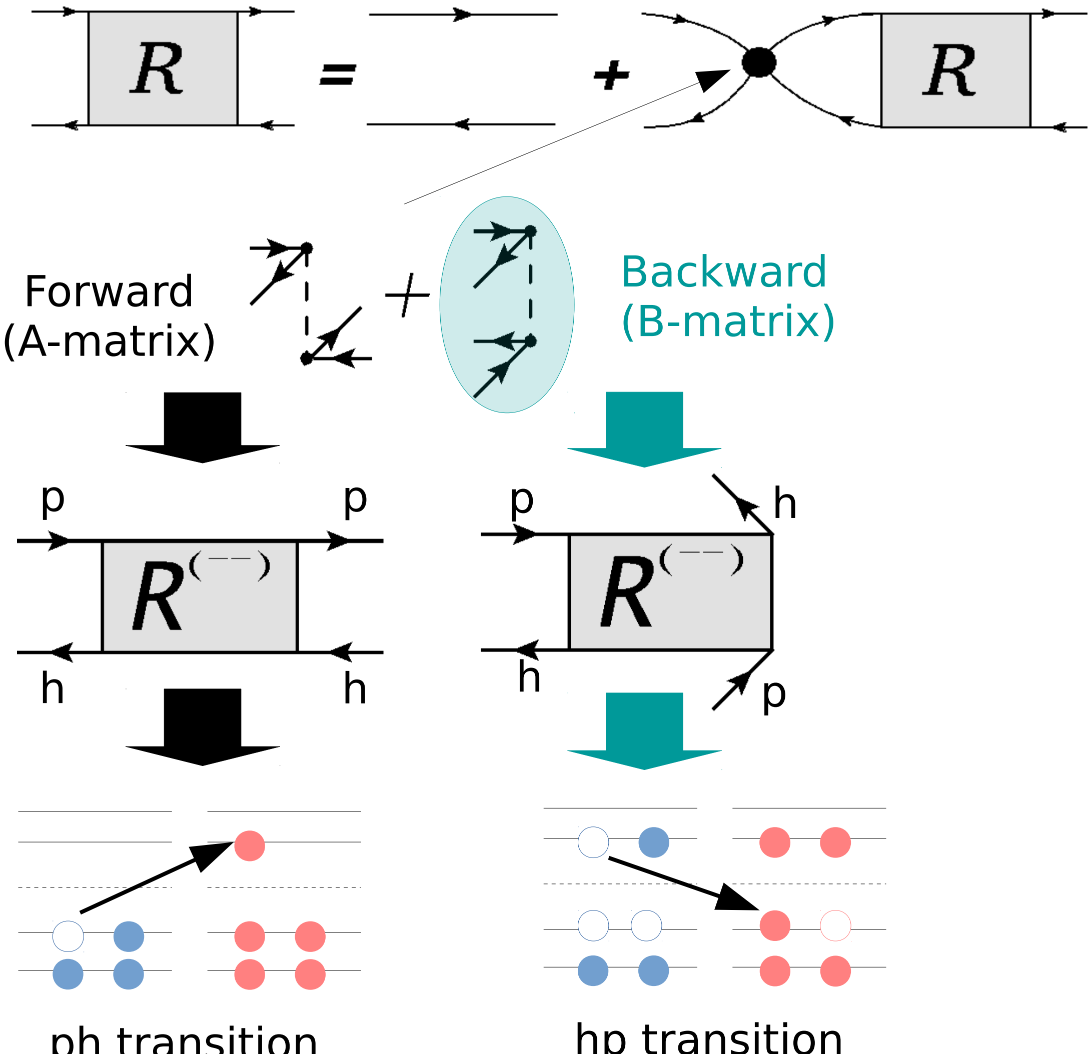
- **Current developments: ground-state correlations in RNFT**
- **Application to double-beta decay: some ideas**
- **Conclusion & perspectives**

Ground-state correlations in RNFT

Ground-state correlations (GSC) in the Green's functions formalism are generated by the so-called "backward-going diagrams":

★ In R(Q)RPA:

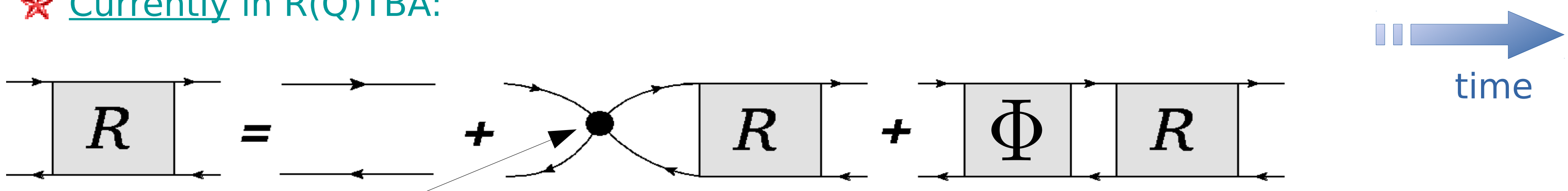


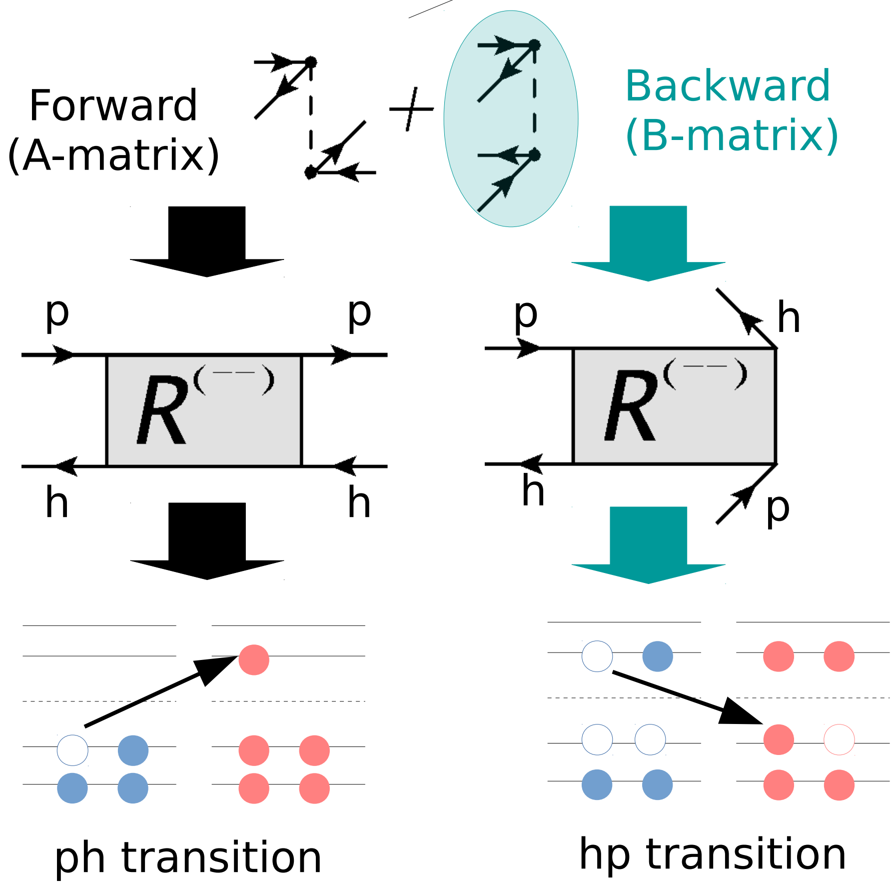


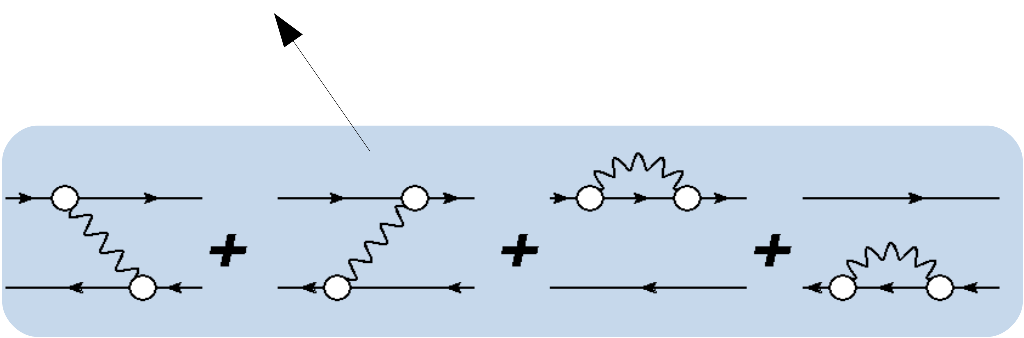
Ground-state correlations in RNFT

Ground-state correlations (GSC) in the Green's functions formalism are generated by the so-called "backward-going diagrams":

☆ <u>Currently</u> in R(Q)TBA:





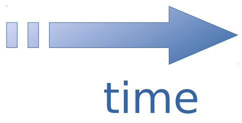


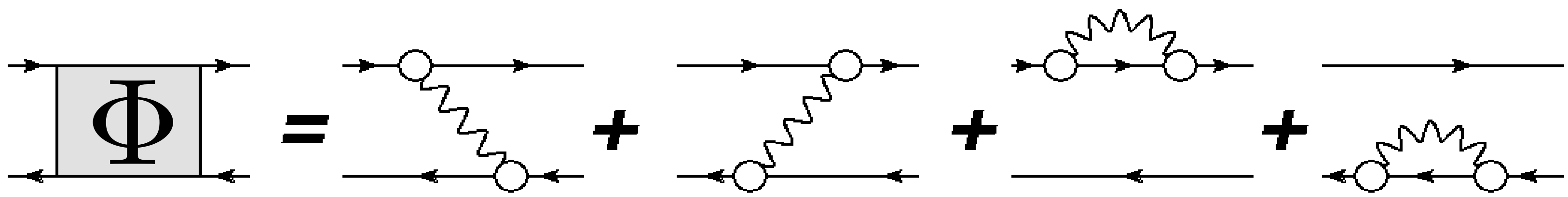
Only forward-going diagrams

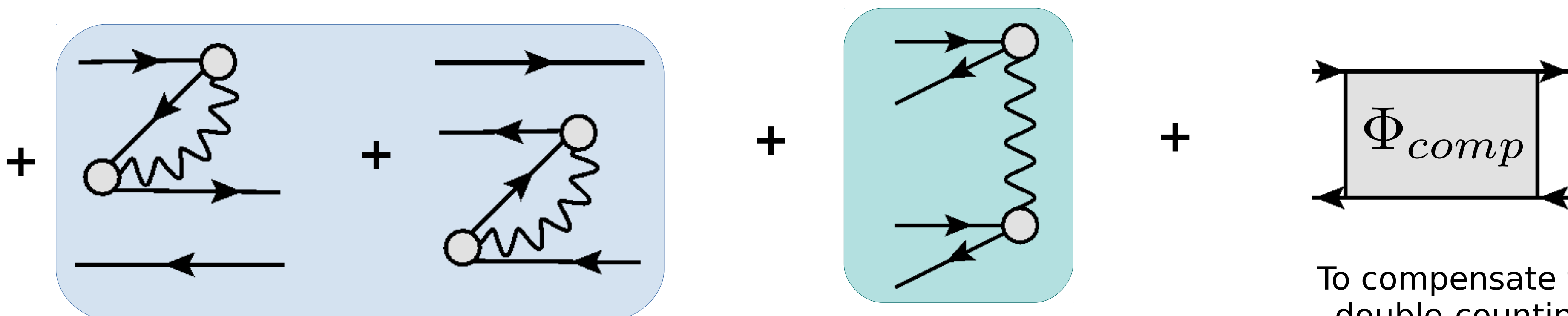
⇒ no GSC induced by
(quasi)vibration coupling

Ground-state correlations in RNFT

- When GSC induced by QVC are included in the TBA, the component $R^{(--)}$ of the response are modified by the following diagrams:







Add to the A-matrix
-
Dynamic but do not introduce new poles

Adds to the B-matrix
-
Static

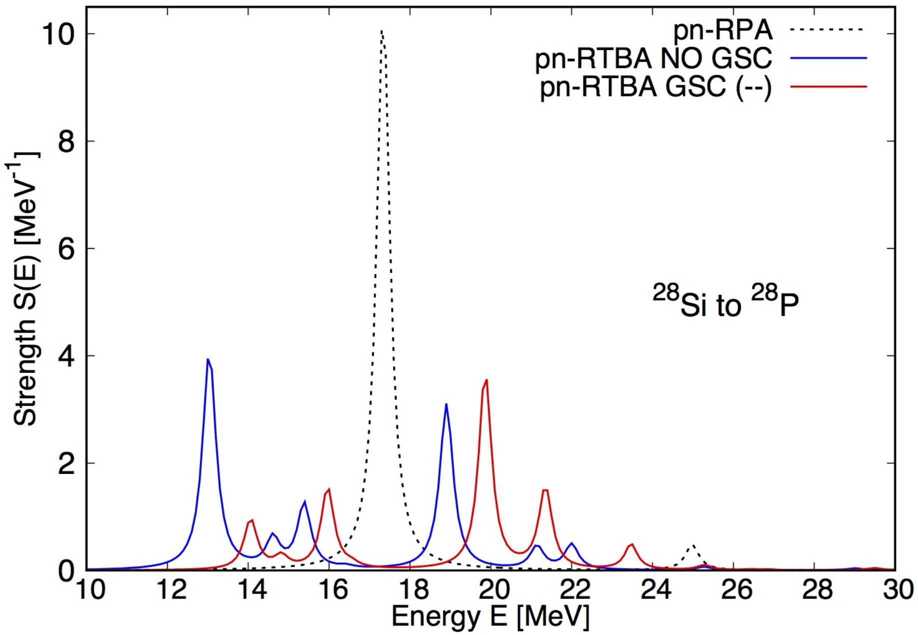
To compensate for double-counting of double self-energy insertions

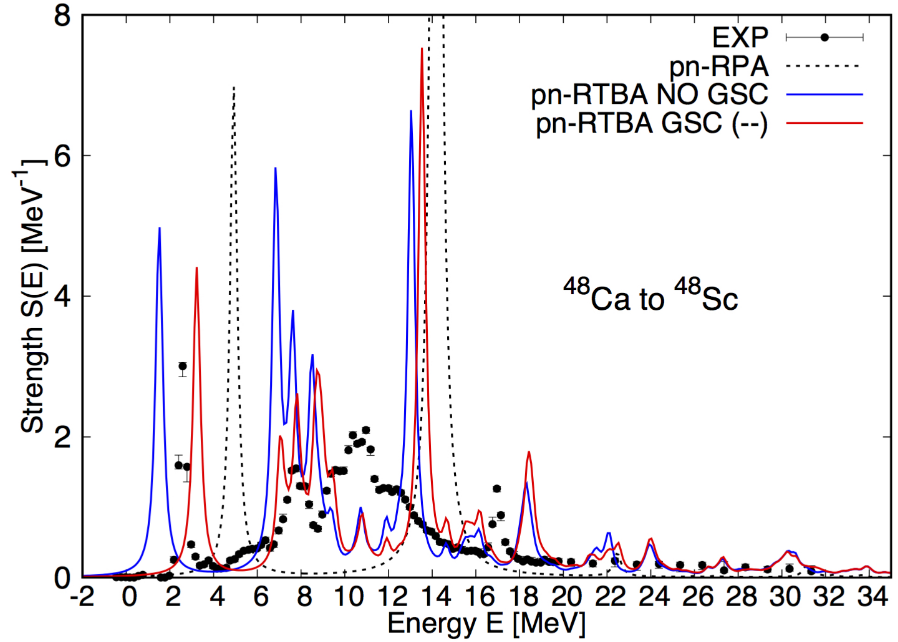
No new states → these diagrams only shift the previous R(Q)TBA poles

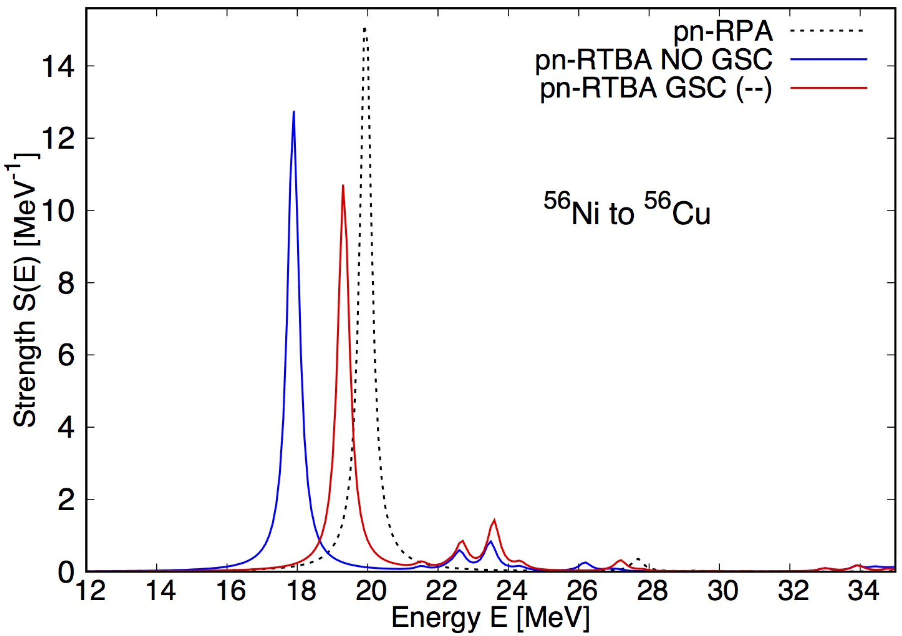
S.P. Kamerdzhiev, G.Ya. Tertychny, V.I. Tselyaev, Fiz. Elem. Chastits At. Yadra 28, 333–390 (1997)

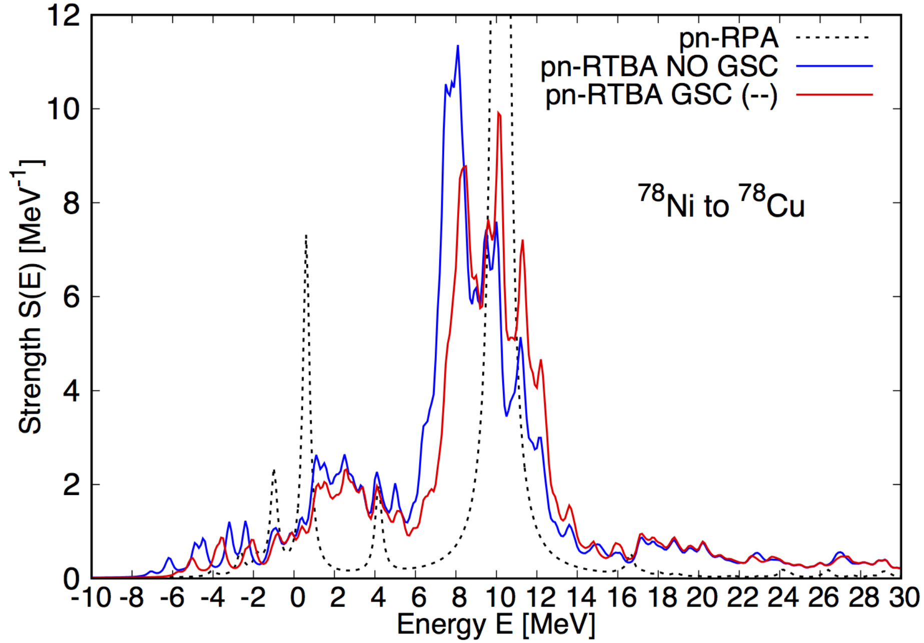
Ground-state correlations in RNFT

→ Very preliminary results:

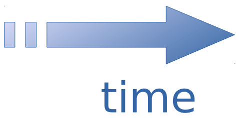




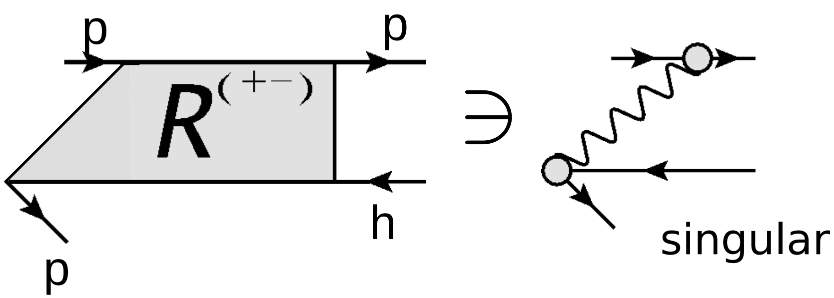


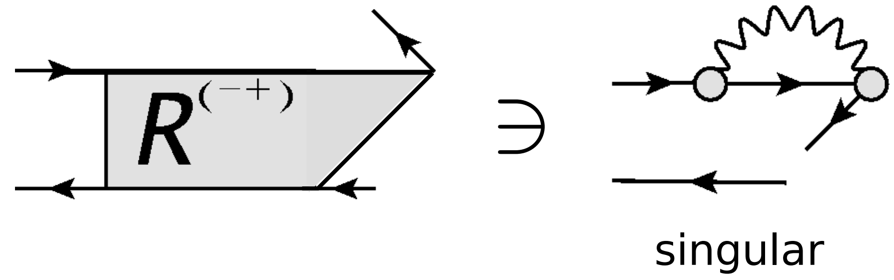


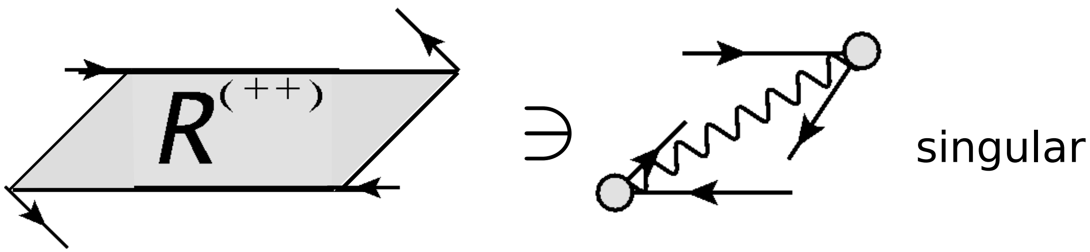
Ground-state correlations in RNFT



- Additionally, new components of the response appear:







⇒ New poles are generated

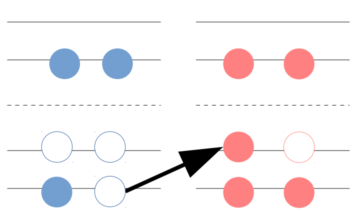
new feature
compared to (Q)RPA

- These components are related to R$^{(--)}$ through:

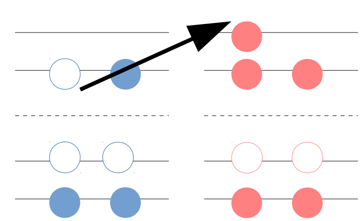
$$R(\omega) = \left(1 + Q^{(+-)}(\omega)\right) R^{(--)}(\omega) \left(1 + Q^{(-+)}(\omega)\right) + P^{(++)}(\omega)$$

→ the dimensions of the problem remain the same!

- They induce new types of transitions:



hh transition



pp transition

→ These effects should be very important for (p,n) strength in n-rich nuclei & (n,p) strength in p-rich nuclei

S.P. Kamerdzhiev, G.Ya. Tertychny, V.I. Tselyaev, Fiz. Elem. Chastits At. Yadra 28, 333–390 (1997)

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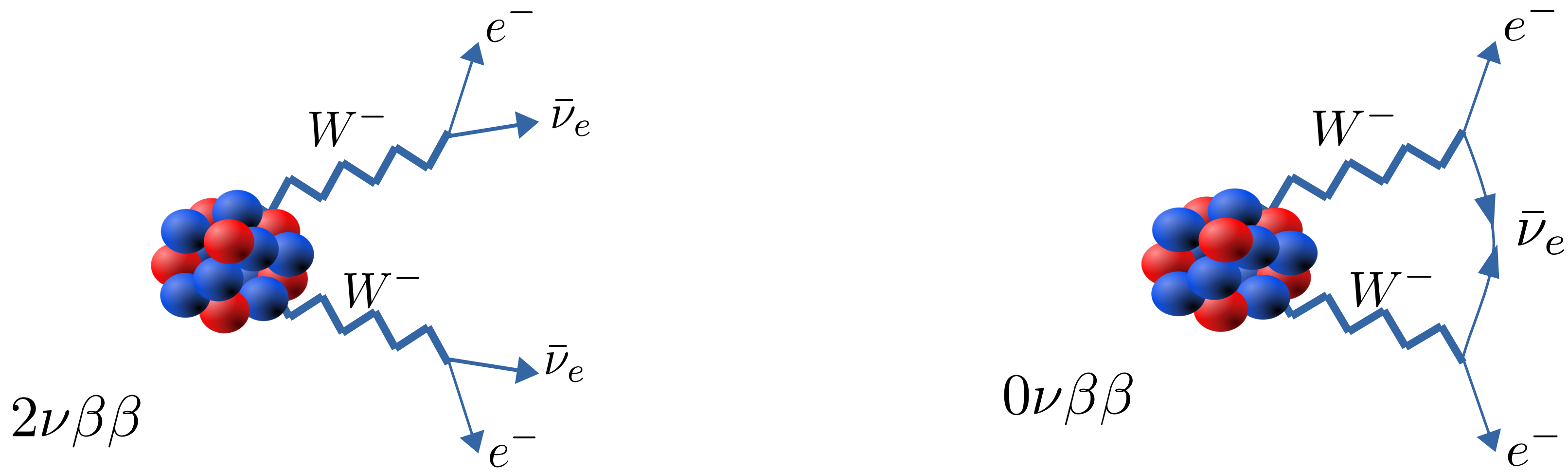
Application to double-beta decay: some ideas



- Two-neutrino double-beta decay amplitude:

$$A_{i\to f}^{2\nu\beta\beta} = -\frac{1}{2}\int d^4x_1 d^4x_2$$

$$\times \langle \Psi_f; (\mathbf{p_1}, s_1); (\mathbf{p_2}, s_2); (\mathbf{q_1}, \sigma_1); (\mathbf{q_2}, \sigma_2) | \mathcal{T}\left(\mathcal{H}_{weak}(x_1)\mathcal{H}_{weak}(x_2)\right) |\Psi_i\rangle$$

(N-2,Z+2) $\quad e^- \quad \bar{\nu}_e \quad$ (N,Z)

$$\mathcal{H}_{weak}(x) = \frac{G_F}{\sqrt{2}} J_\mu(x) L^{\dagger\mu}(x)$$

Application to double-beta decay: some ideas

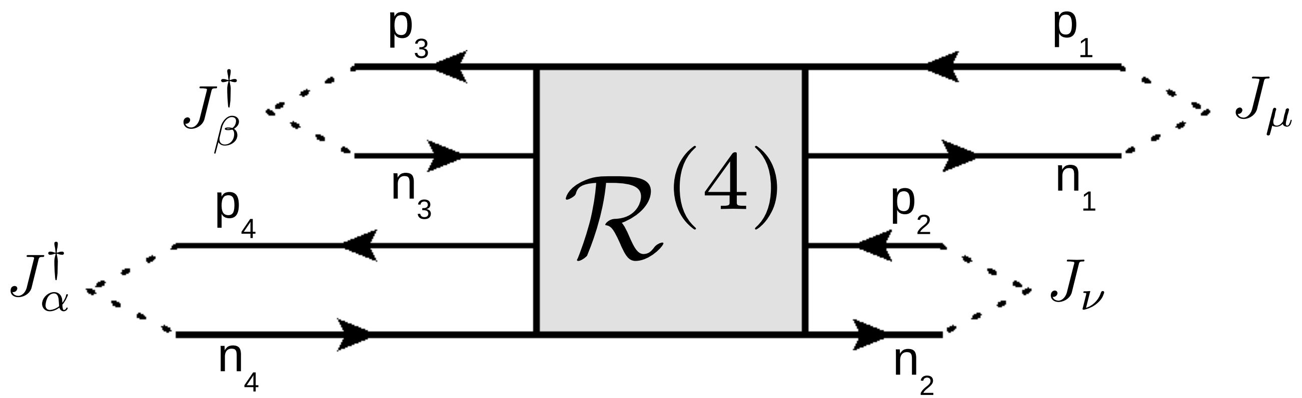
[…] → Inclusive probability for double-beta decay (after summation over final states):

$$P^{(2\nu\beta\beta)} \sim G_F^4 \int d^3p_1\, d^3p_2\, d^3q_1\, d^3q_2\, dx_1^0\, dx_2^0\, dy_1^0\, dy_2^0$$
$$\times e^{i(p_1^0+q_1^0)(x_1^0-y_1^0)} e^{i(p_2^0+q_2^0)(x_2^0-y_2^0)}$$
$$\times \mathcal{W}_{\alpha\beta\mu\nu}(x_1^0, x_2^0, y_1^0, y_2^0) \mathcal{L}^{\alpha\beta\mu\nu}(p_1, p_2, q_1, q_2)$$

Hadronic tensor

Leptonic tensor

$$\mathcal{W}_{\alpha\beta\mu\nu}(x_1^0, x_2^0, y_1^0, y_2^0) = \sum_{p_1\ldots p_4, n_1\ldots n_4} \langle n_4|J_\alpha^\dagger|p_4\rangle\langle n_3|J_\beta^\dagger|p_3\rangle$$
$$\times \mathcal{R}^{(4)}_{n_4p_4,n_3p_3,p_1n_1,p_2n_2}(y_2^0, y_1^0, x_1^0, x_2^0)\ \langle p_1|J_\mu|n_1\rangle\langle p_2|J_\nu|n_2\rangle$$



Application to double-beta decay: some ideas

- Decomposition of the four-nucleon Green's function:

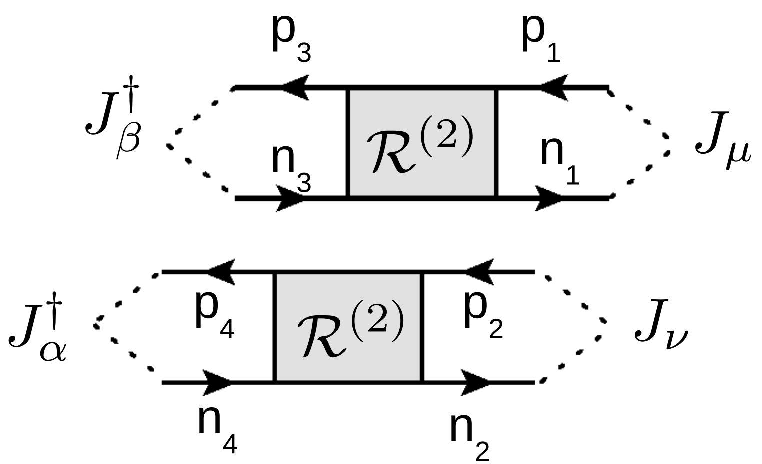
$$\mathcal{R}^{(4)} = \sum \mathcal{R}^{(2)}\mathcal{R}^{(2)} + \mathcal{R}^{(3)C}\mathcal{R}^{(1)} + \mathcal{R}^{(4)C}$$

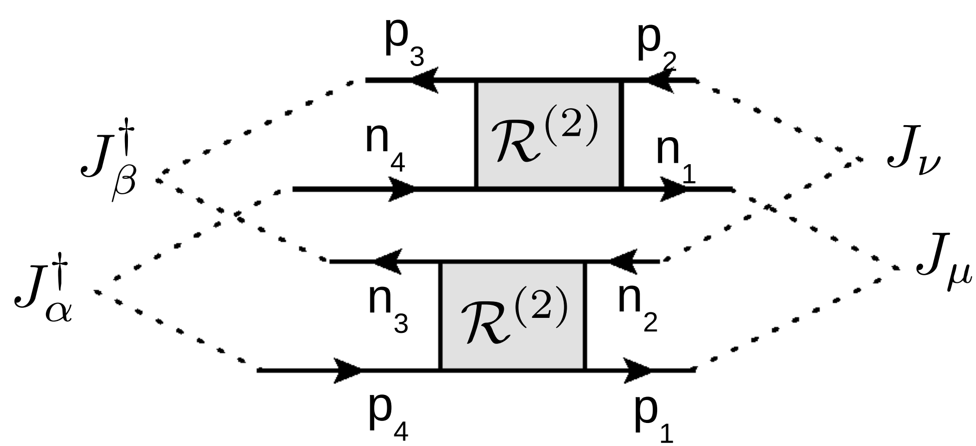
Application to double-beta decay: some ideas

- Decomposition of the four-nucleon Green's function:

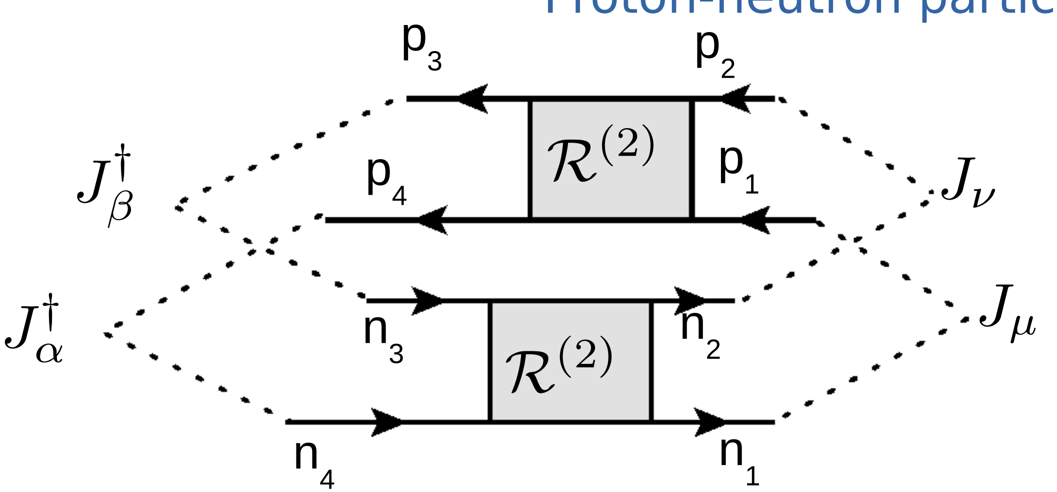
  → Possible approximation: neglect pure three- and four-body correlations

$$\mathcal{R}^{(4)} = \sum \mathcal{R}^{(2)}\mathcal{R}^{(2)} + \cancel{\mathcal{R}^{(3)C}}\mathcal{R}^{(1)} + \cancel{\mathcal{R}^{(4)C}}$$

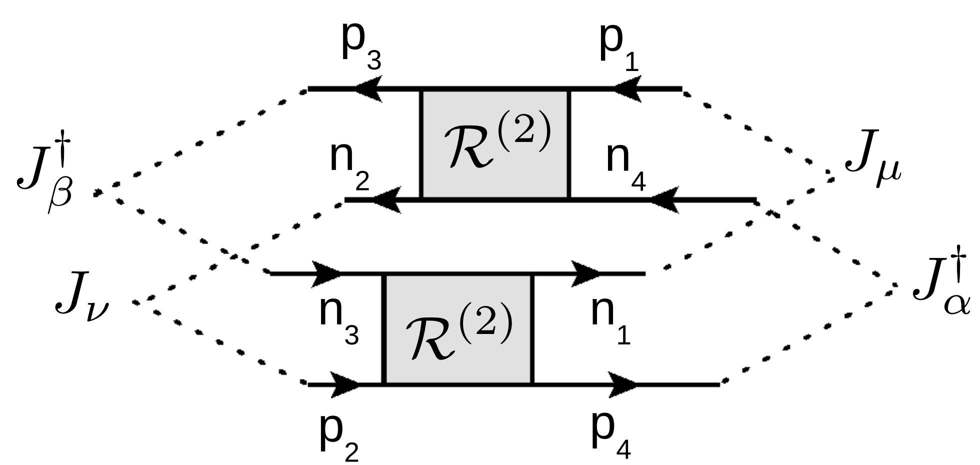




Proton-neutron particle-hole channels



Neutral particle-particle



Proton-neutron particle-particle

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Conclusion, perspectives

→ Conclusions/Perspectives:

- The RNFT appears as a powerful framework for the microscopic description of mid-mass to heavy nuclei, which allows the account for complex configurations of nucleons in a large model space.

- So far encouraging applications to single Gamow-Teller/beta-decay. RNFT can tackle the challenge of describing both the low-energy strength and overall distribution to higher excitation energy.

- Current extensions to higher-order correlations in the ground state appear promising. Also ongoing: Inclusion of Np-Nh configurations in the response via iterative techniques.

- Ongoing extensions to double-charge exchange and double-beta decay ($2\nu\beta\beta$ and $0\nu\beta\beta$)

- Long-term goals: inclusion of the Fock term, inclusion of two-body currents and Delta resonance, start from bare interaction.

Support: US-NSF Grants
PHY-1404343 and PHY-1204486

Conclusion, perspectives

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- So far encouraging ap… RNFT can tackle the challenge of descri… tribution to higher excitation energy…

Thank you!

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