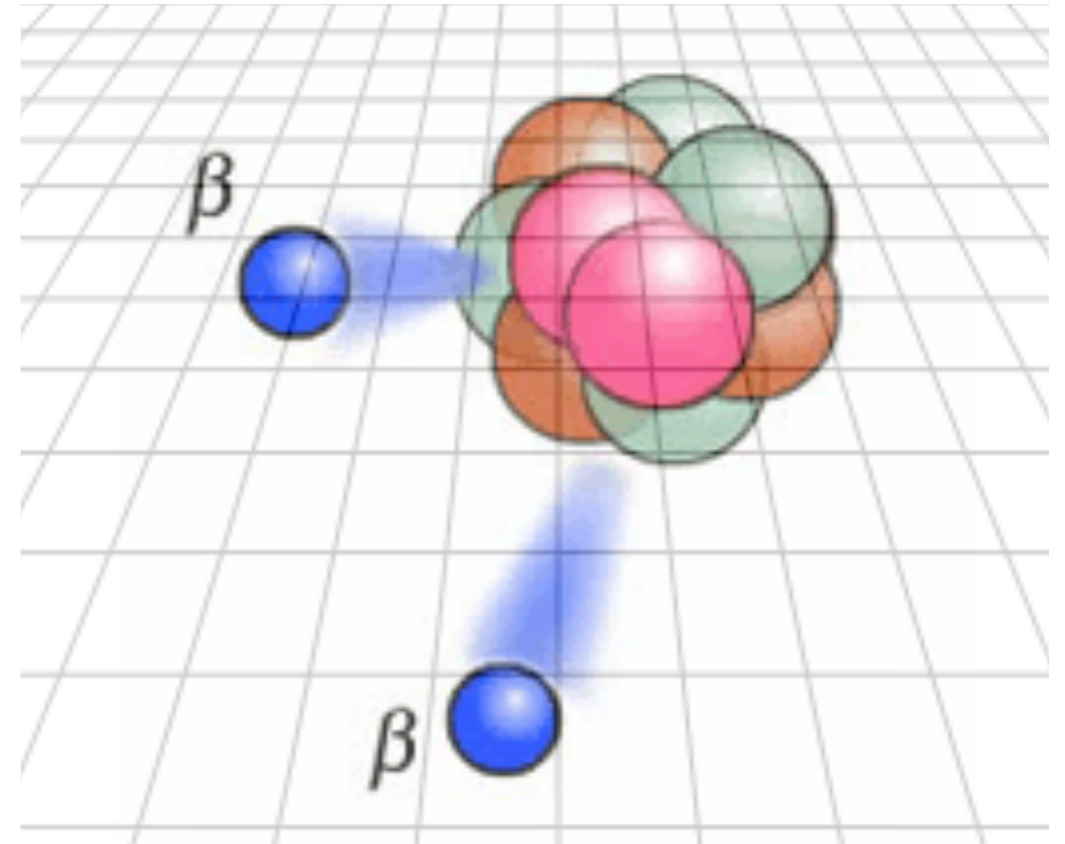


# Neutrinoless Double Beta Decay from Lattice QCD



Amy Nicholson  
UC Berkeley/UNC

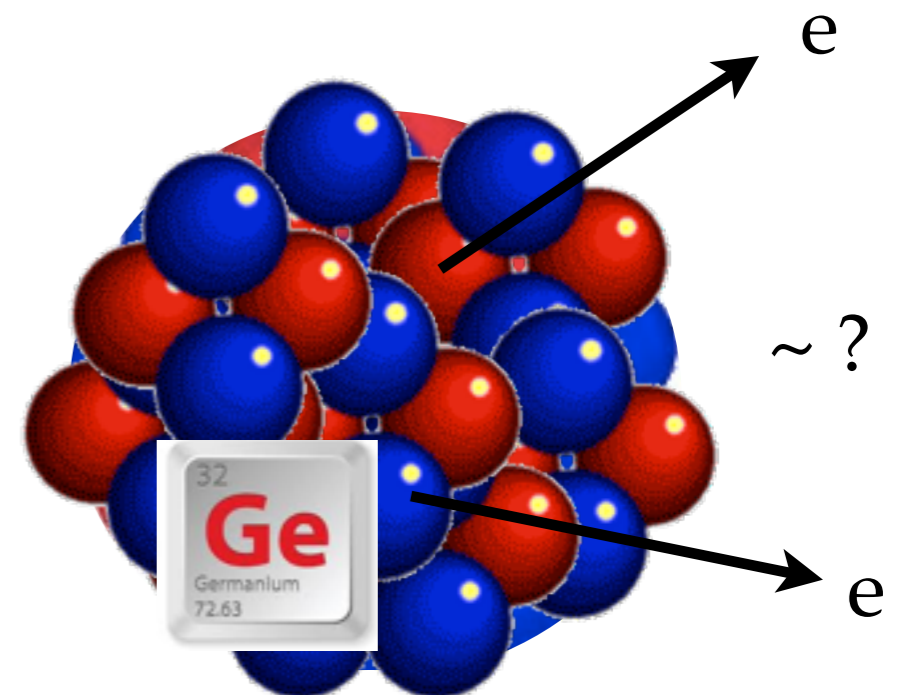
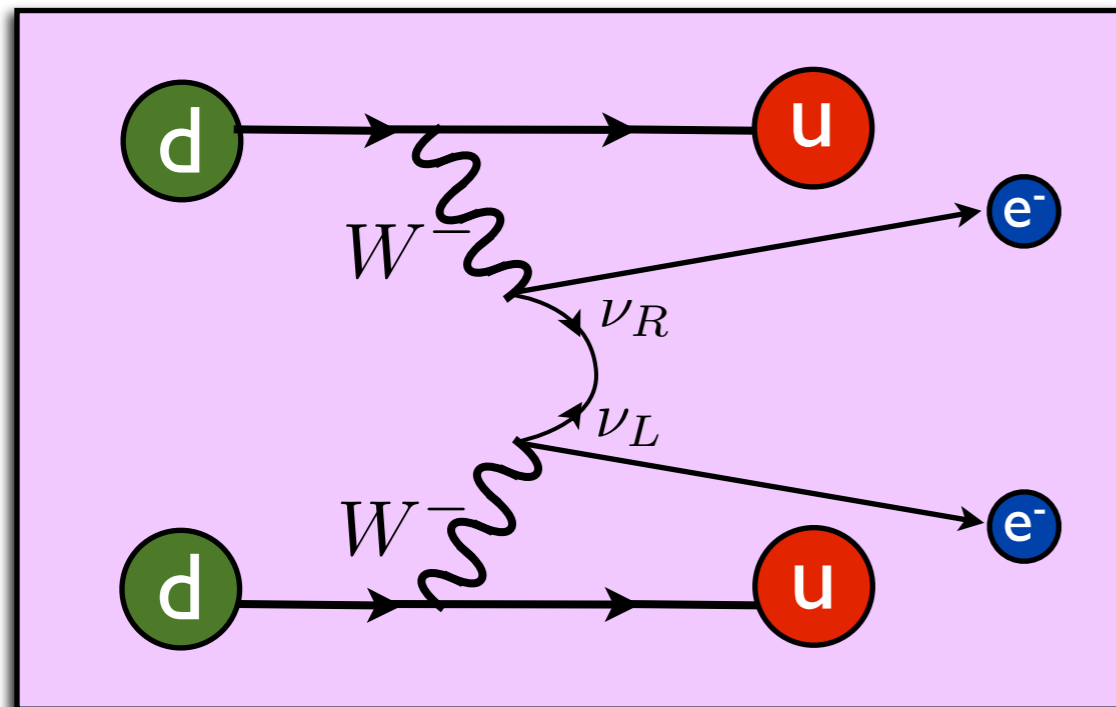
*Institute for Nuclear Theory, Seattle, WA*

*Neutrinoless Double-beta Decay Opening Workshop*

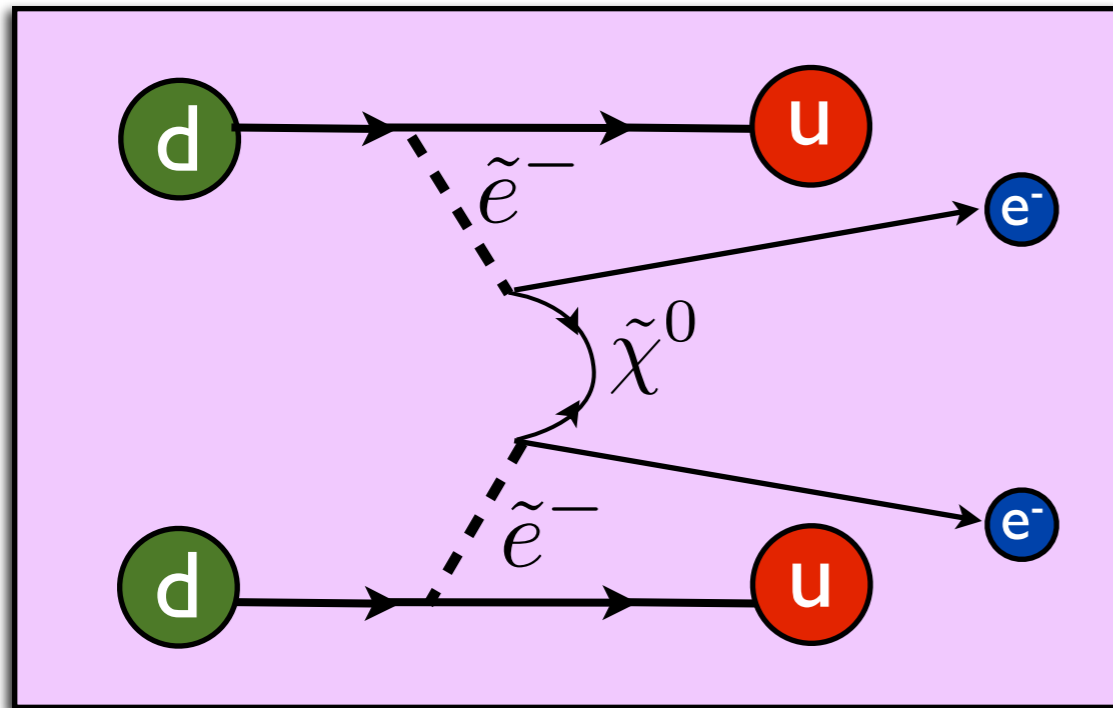
*June 14, 2017*



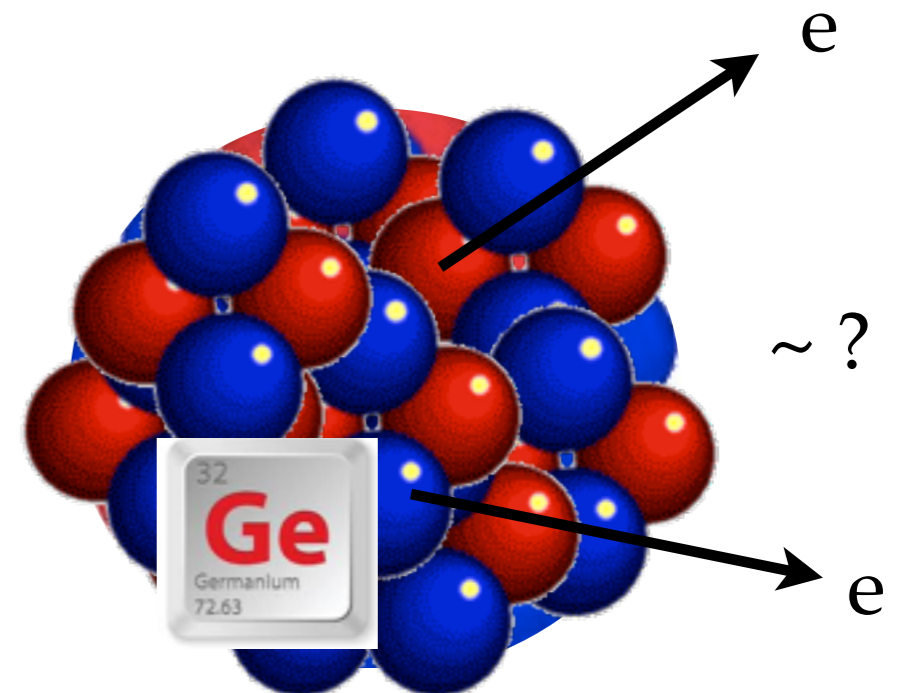
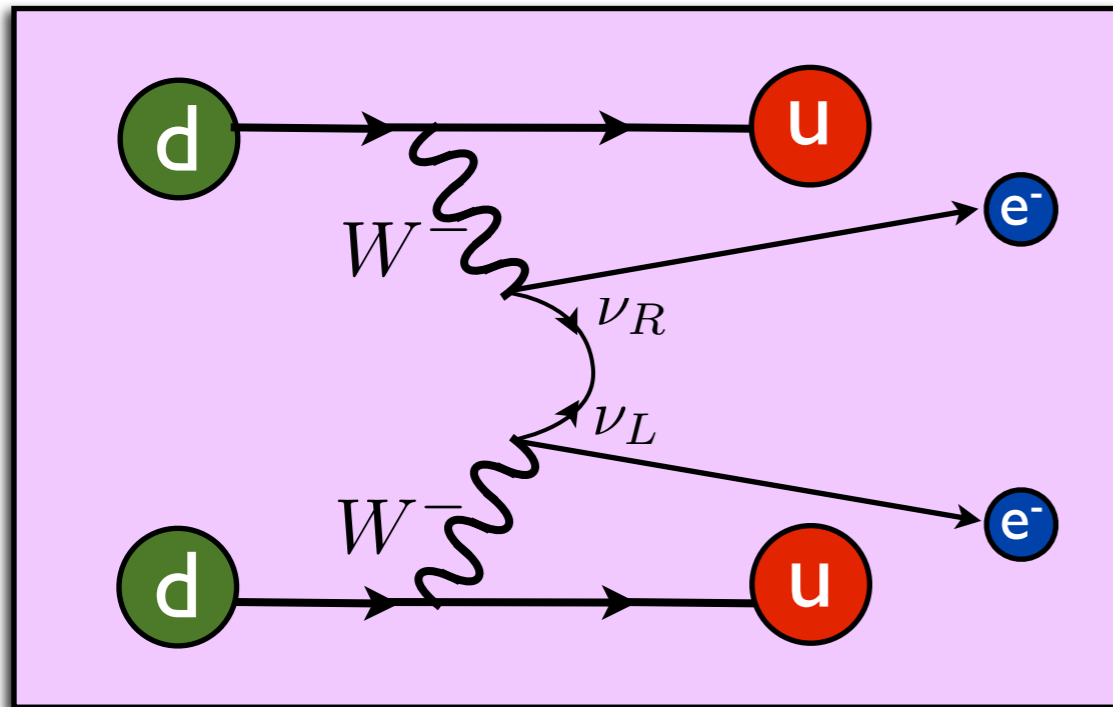
# Relating Theory to Experiment



# Relating Theory to Experiment

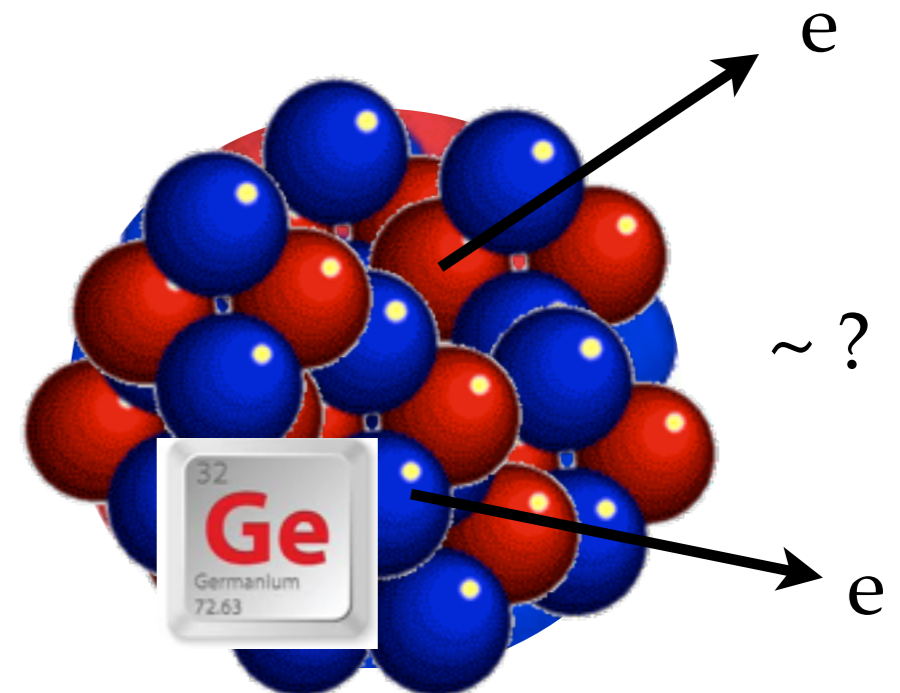
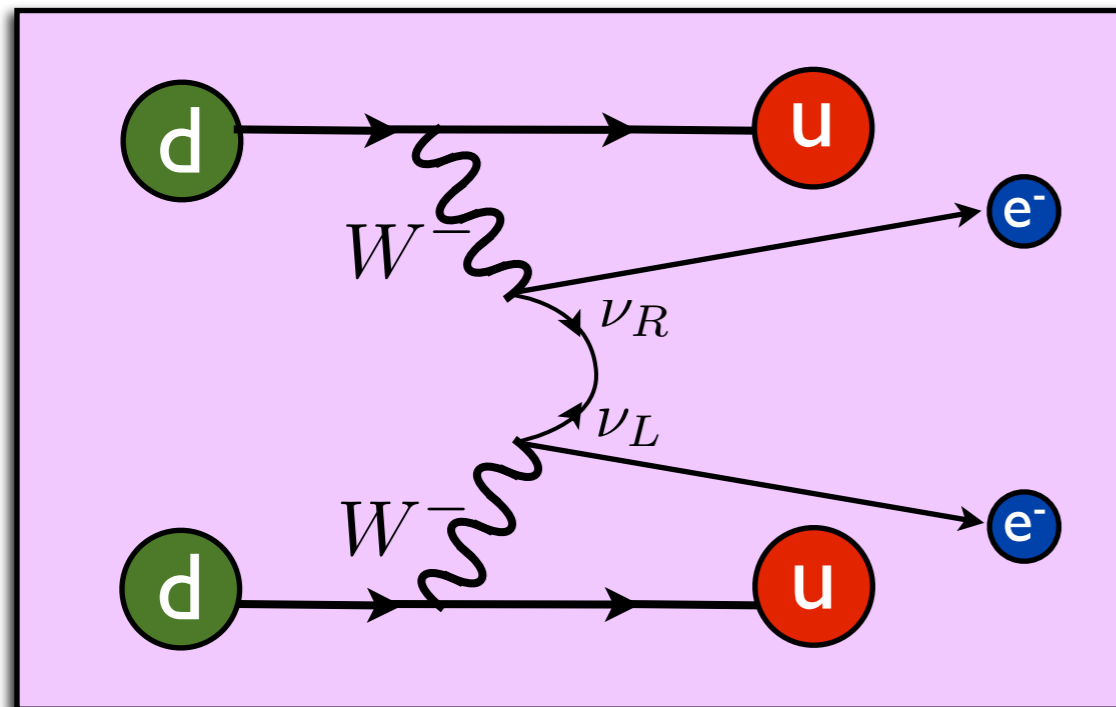


$0\nu\beta\beta$  experiments  
could help  
constrain R-parity  
violating  
coefficients

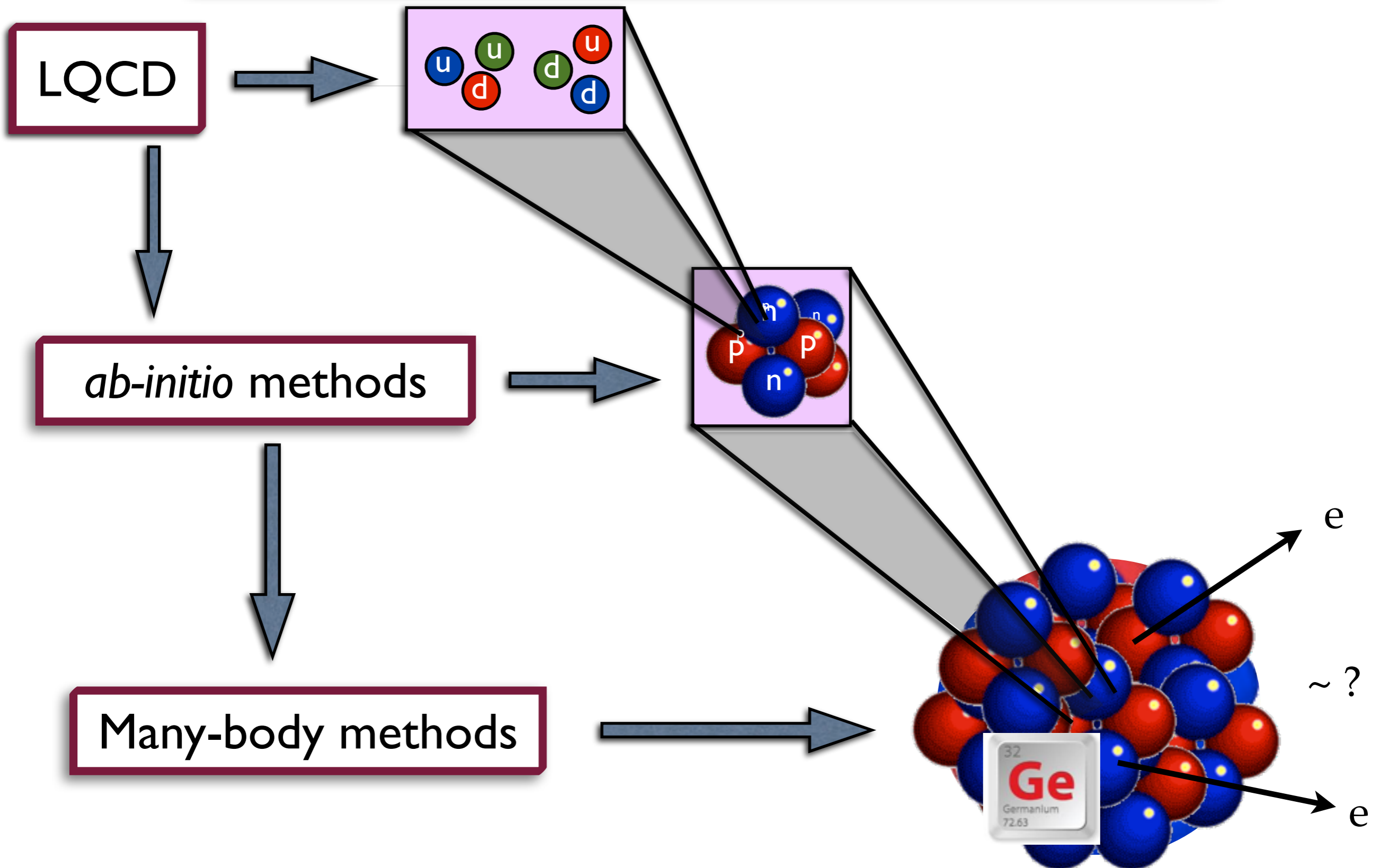


# Relating Theory to Experiment

- LQCD: formulation of QCD in discretized, finite spacetime
- All errors are quantifiable and may be systematically removed
  - Extrapolations to continuum, infinite volume, physical quark mass
- LQCD will never calculate your favorite  $0\nu\beta\beta$  isotope
  - Monte Carlo noise (sign) problem, quark contractions, large range of scales,....



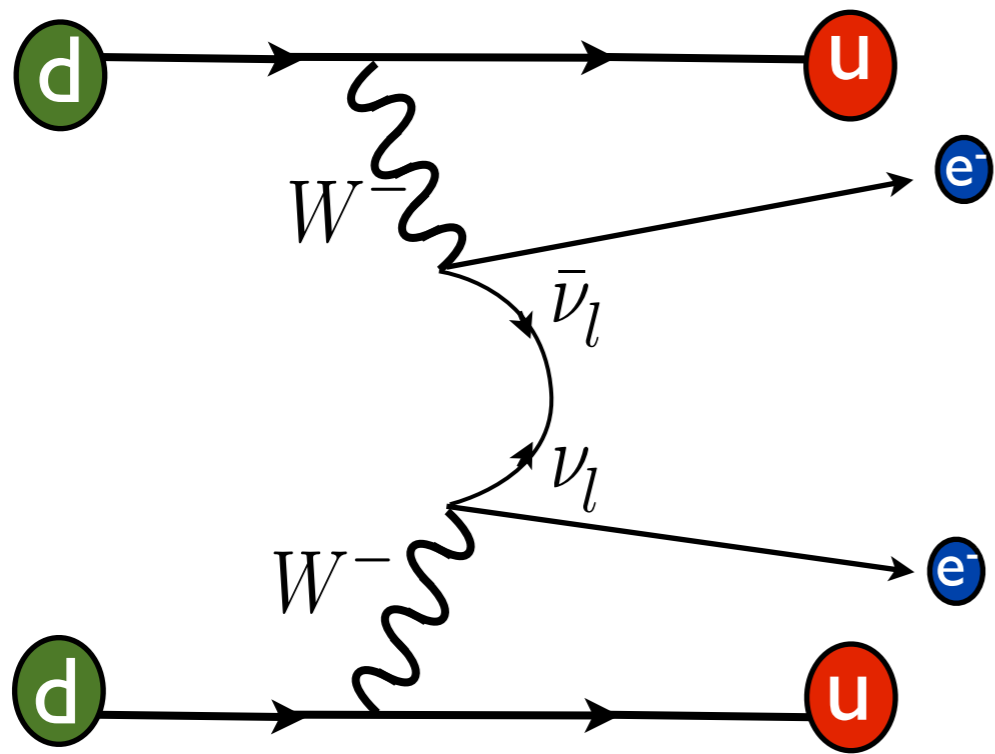
# Relating Theory to Experiment



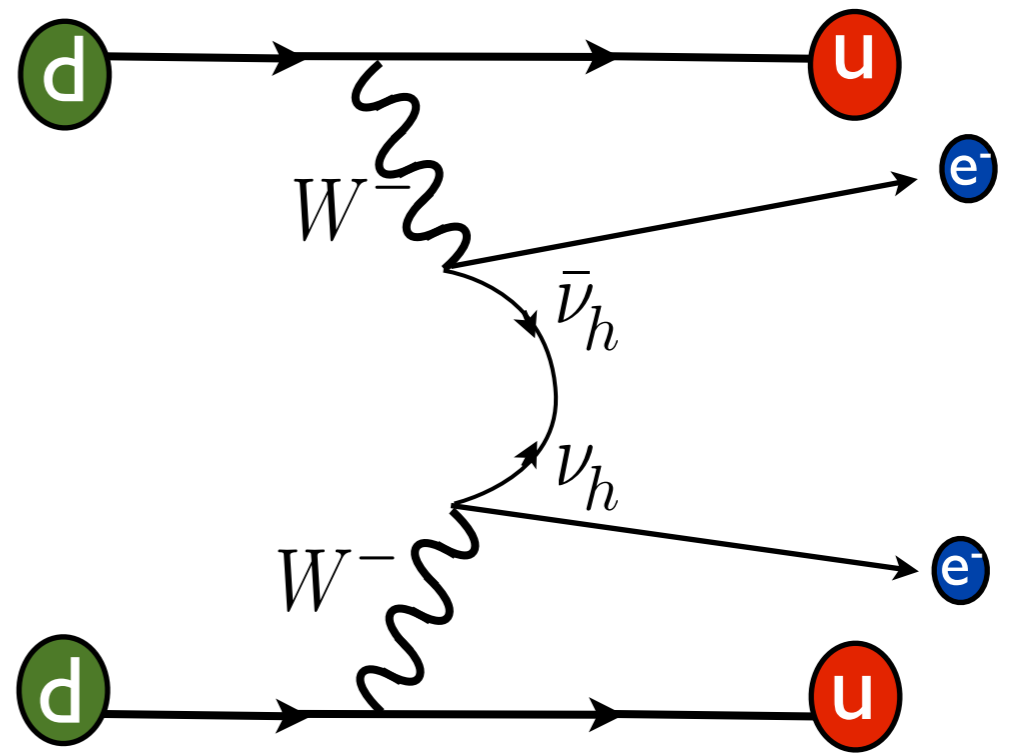
# Lattice QCD

## contributions to $0\nu\beta\beta$

- Long-range
  - Axial charge of the nucleon
- Short-range
  - Leading order single pion exchange contribution
  - Two-nucleon matrix elements



Long-range

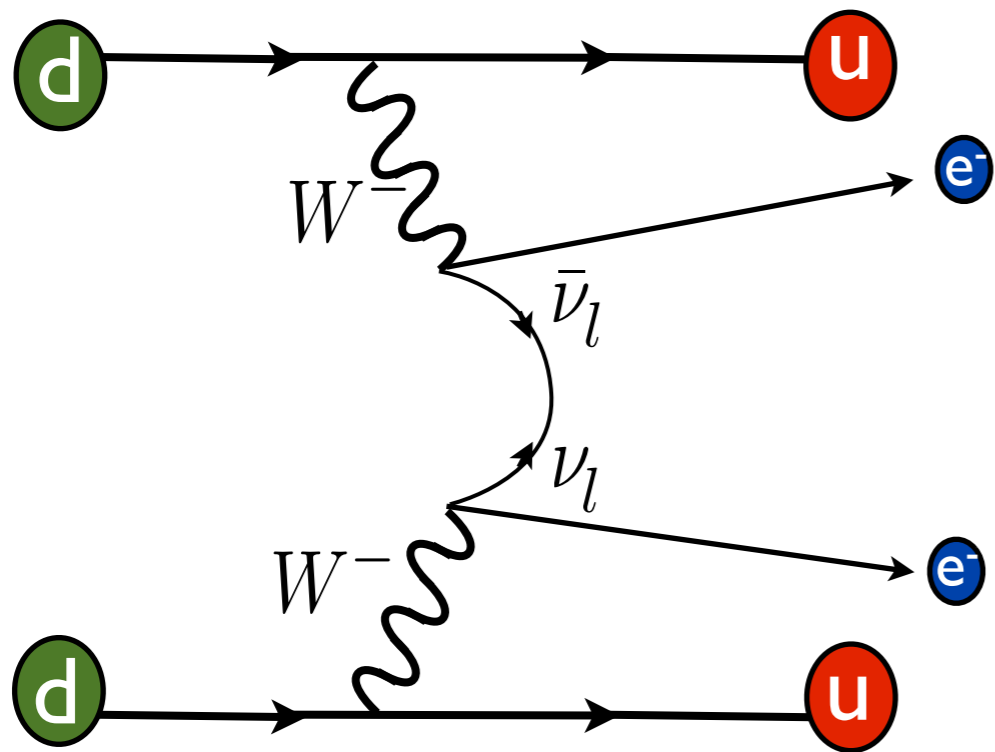


Short-range

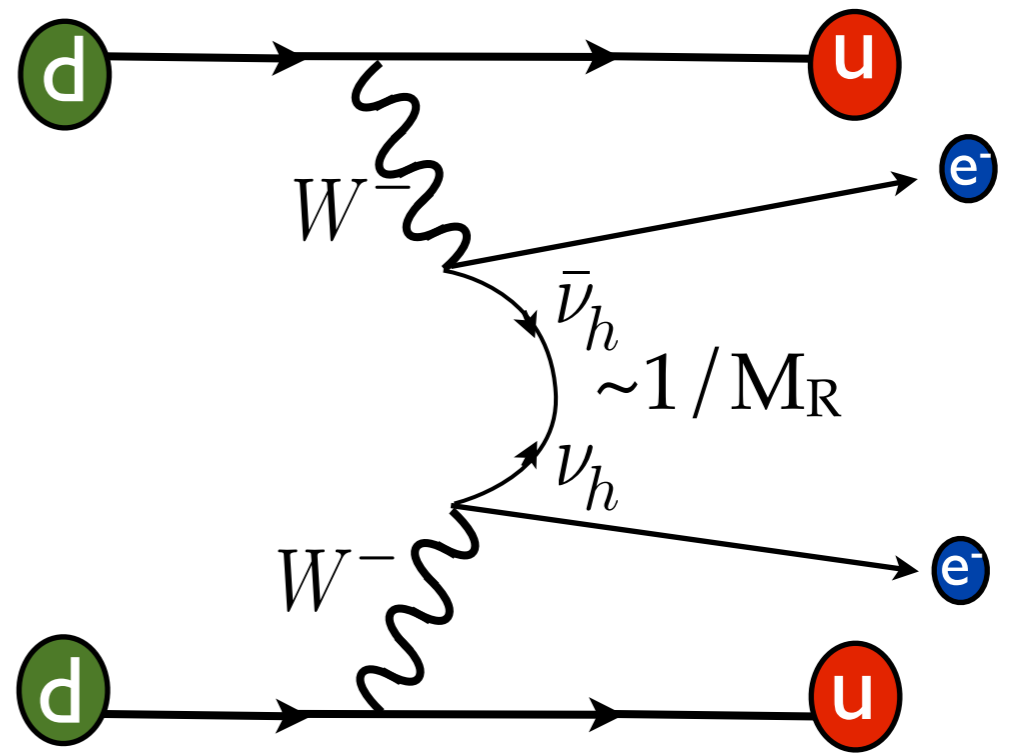


$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$



Long-range



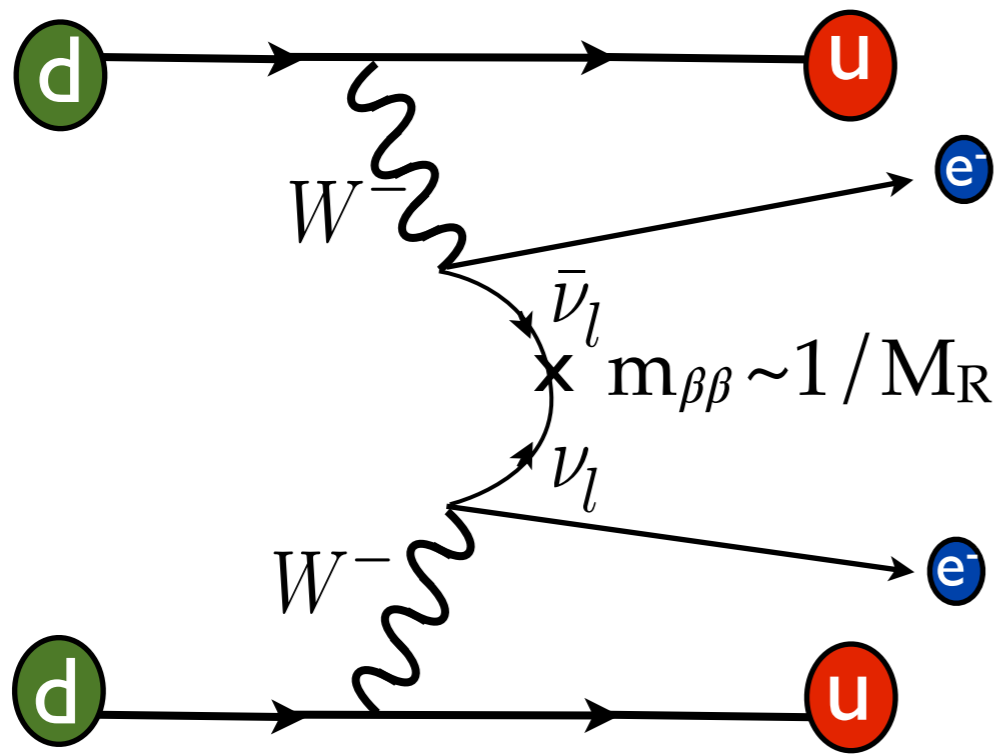
Short-range



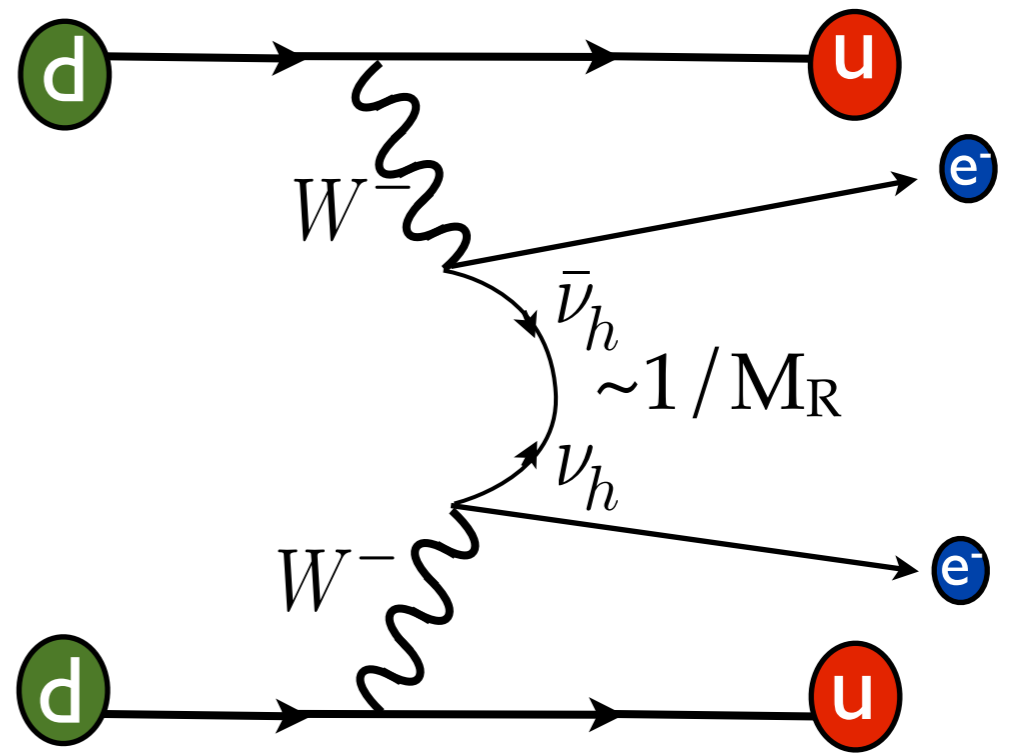
$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$





Long-range

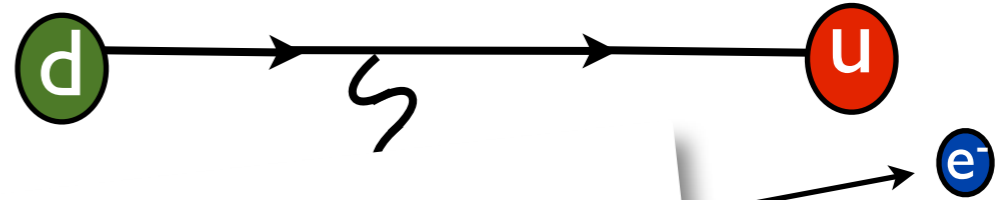
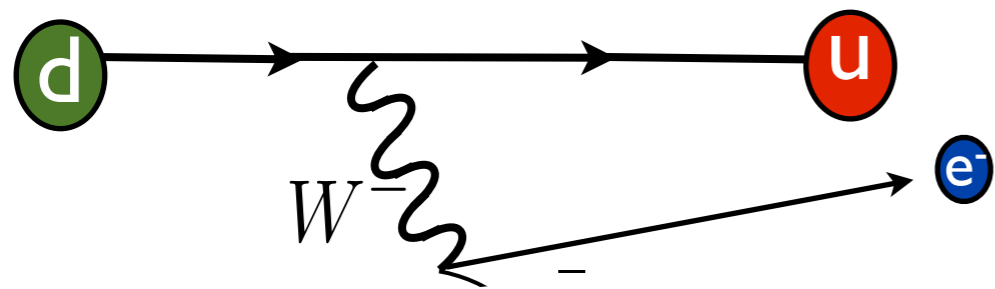


Short-range



$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$



Which type dominates depends on details of BSM model



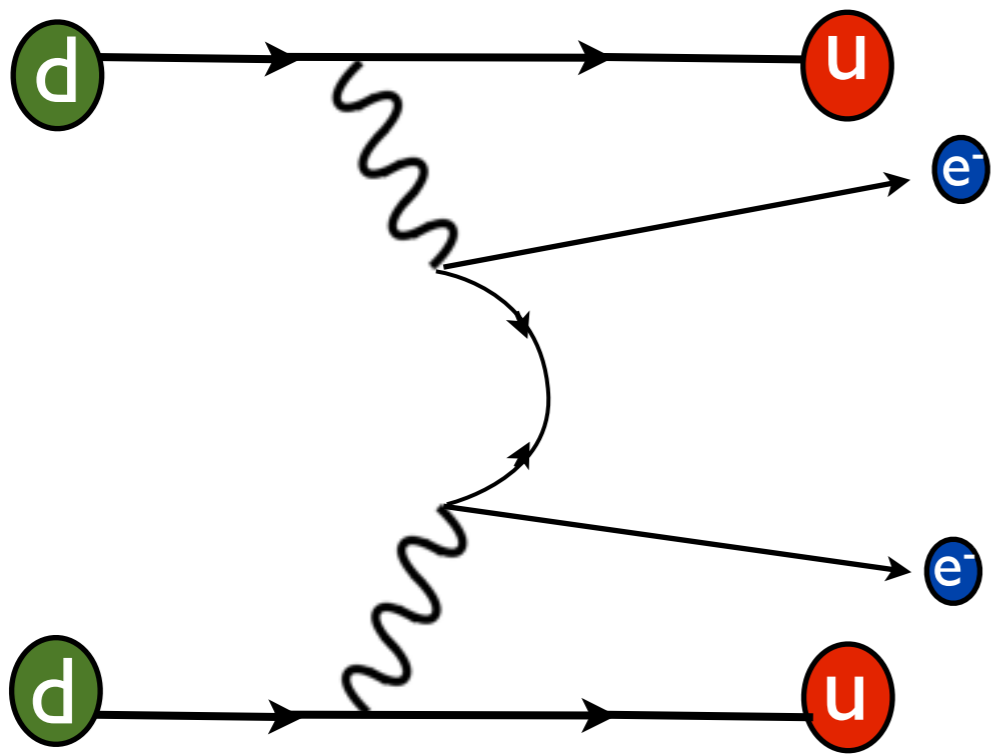
Long-range

Short-range

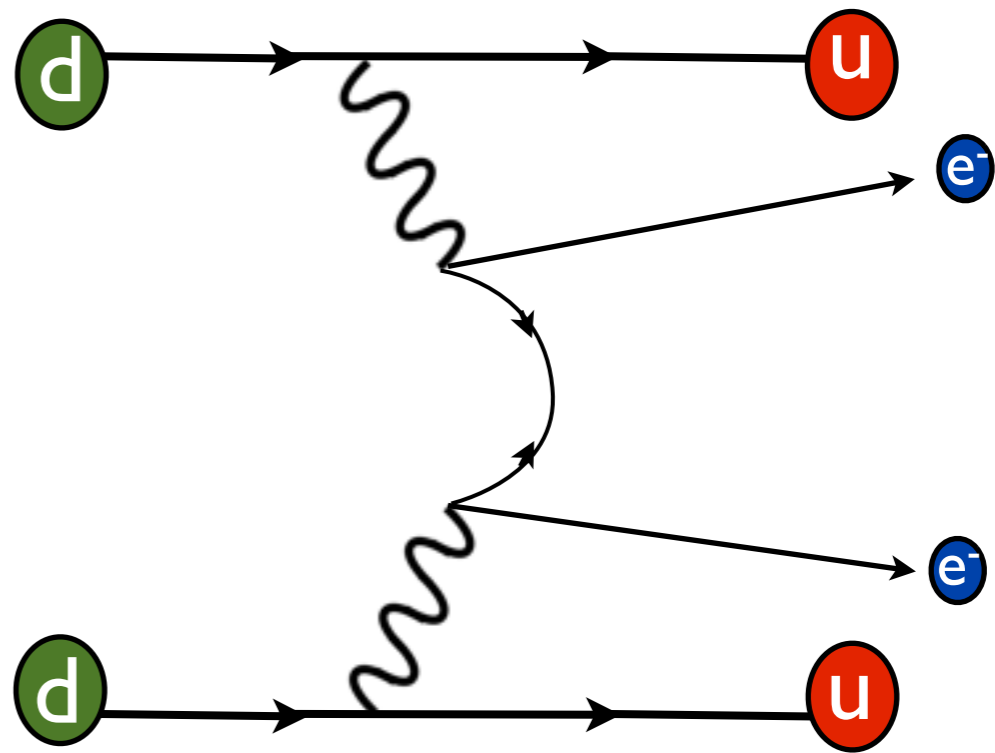


$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

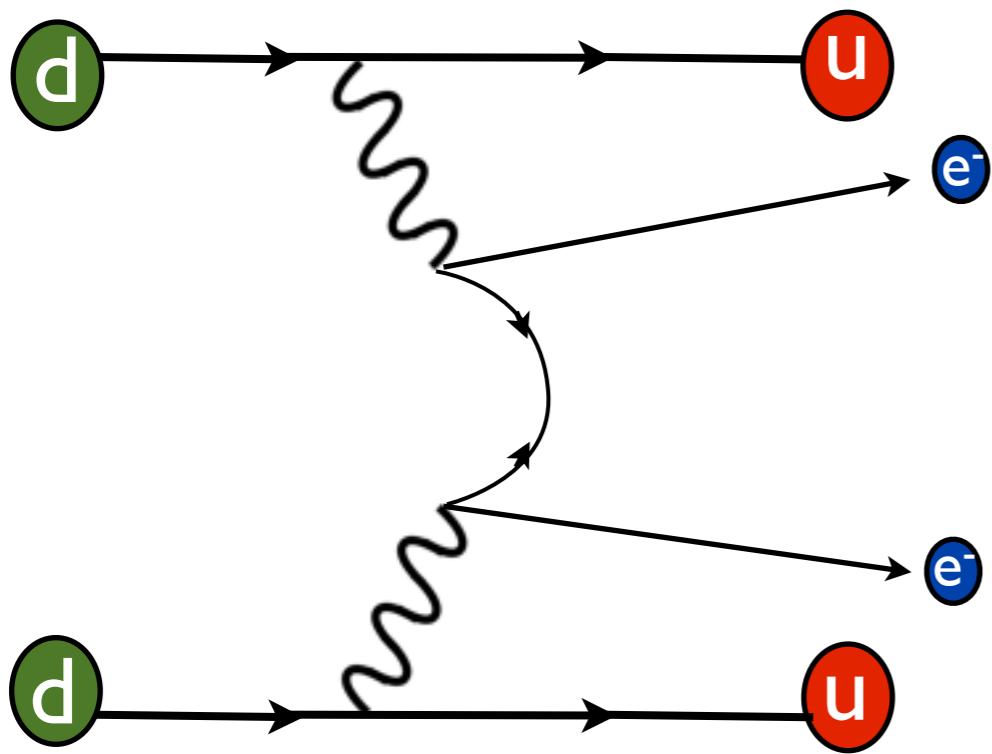
$$m_l \sim M_D^2 / M_R \quad m_h \sim M_R$$



Long-range

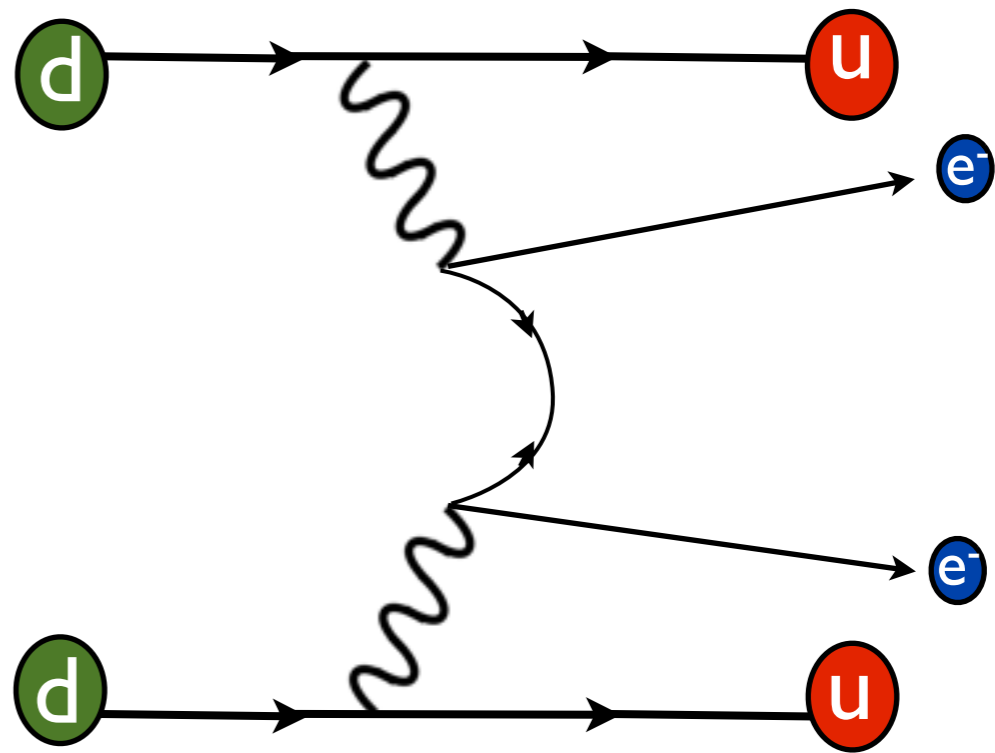


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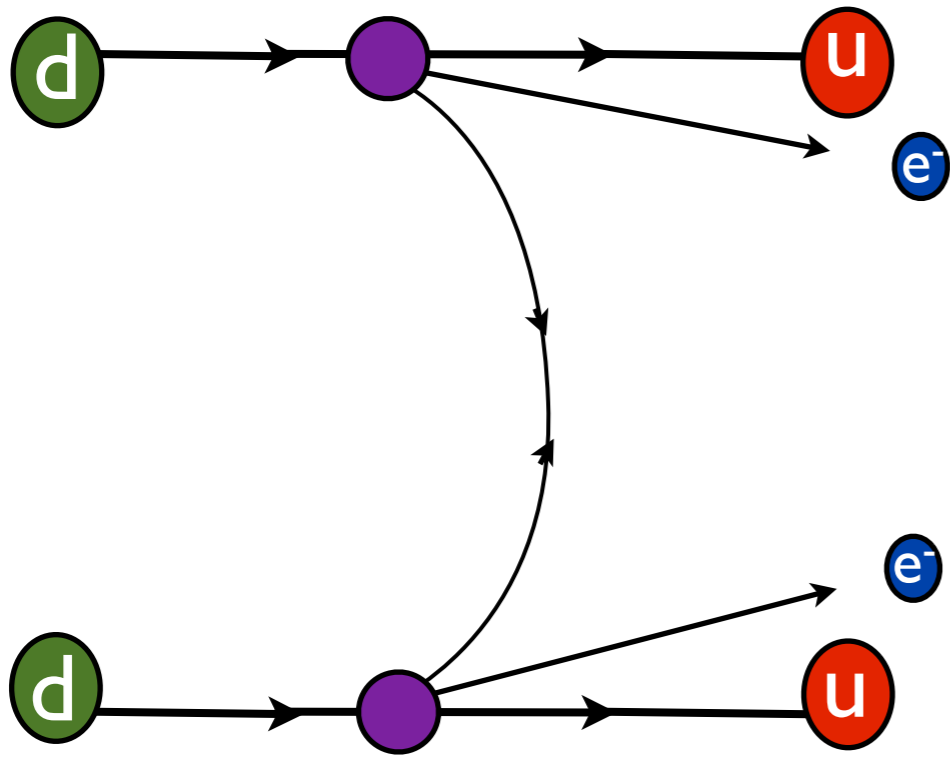


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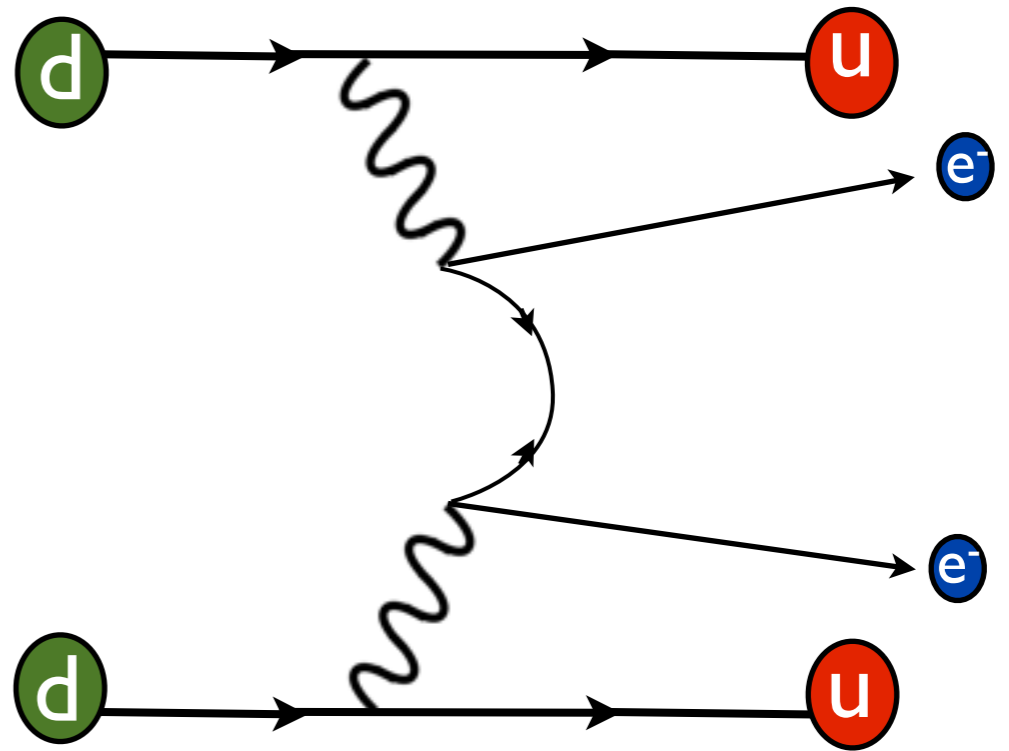
$$\Lambda \ll M_W$$



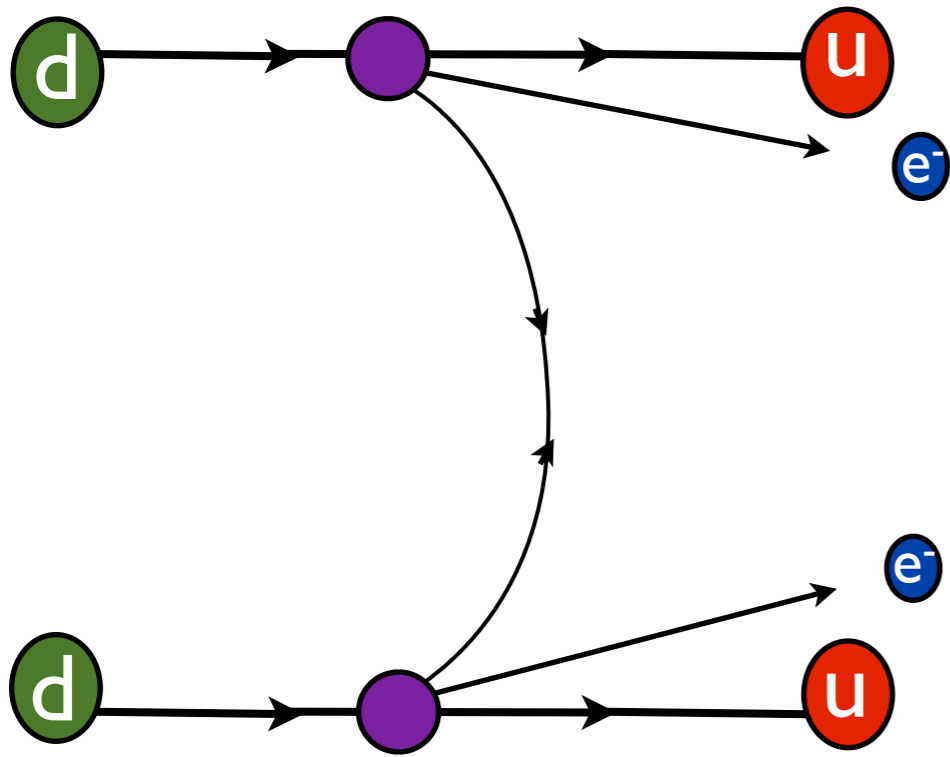
Short-range



Long-range

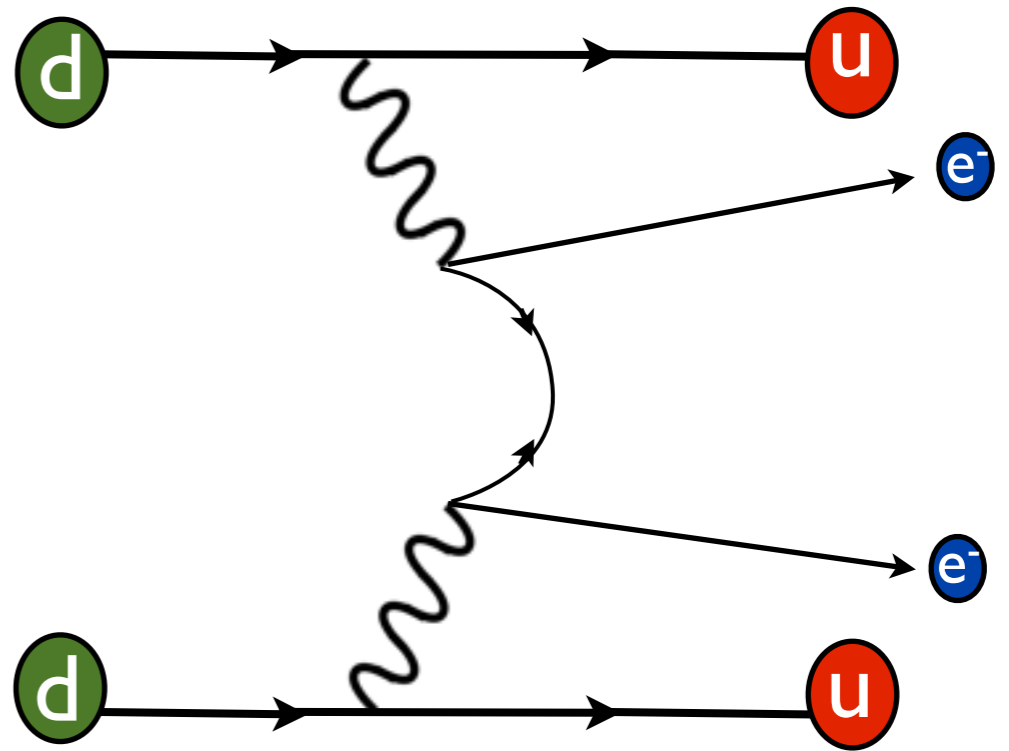


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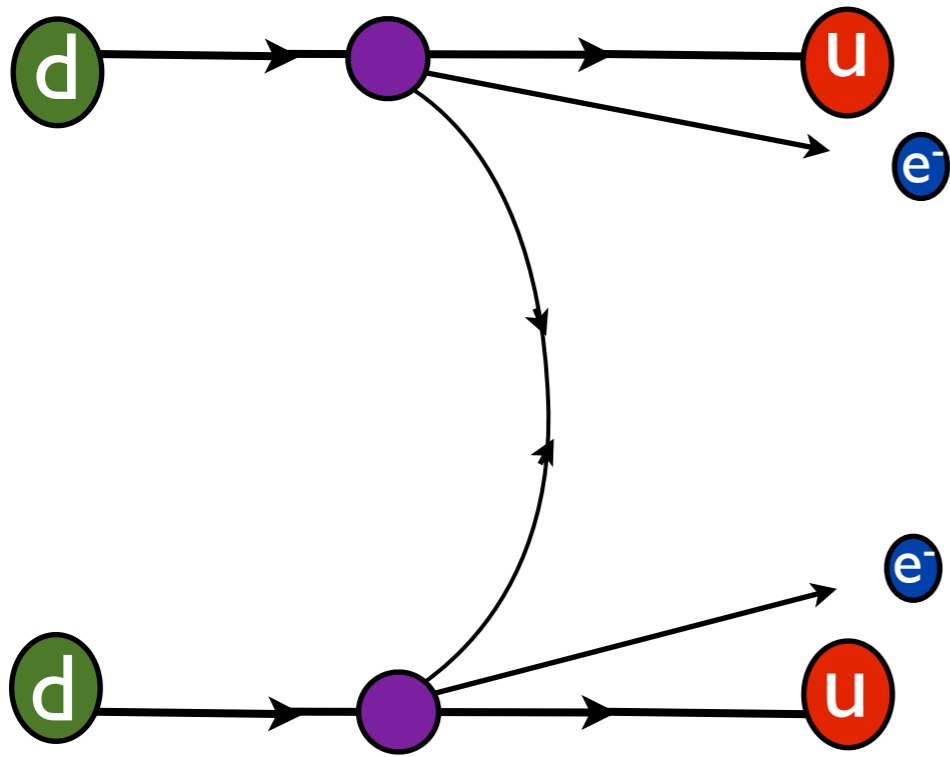


Long-range

$$\Lambda \ll \Lambda_{\text{QCD}}$$

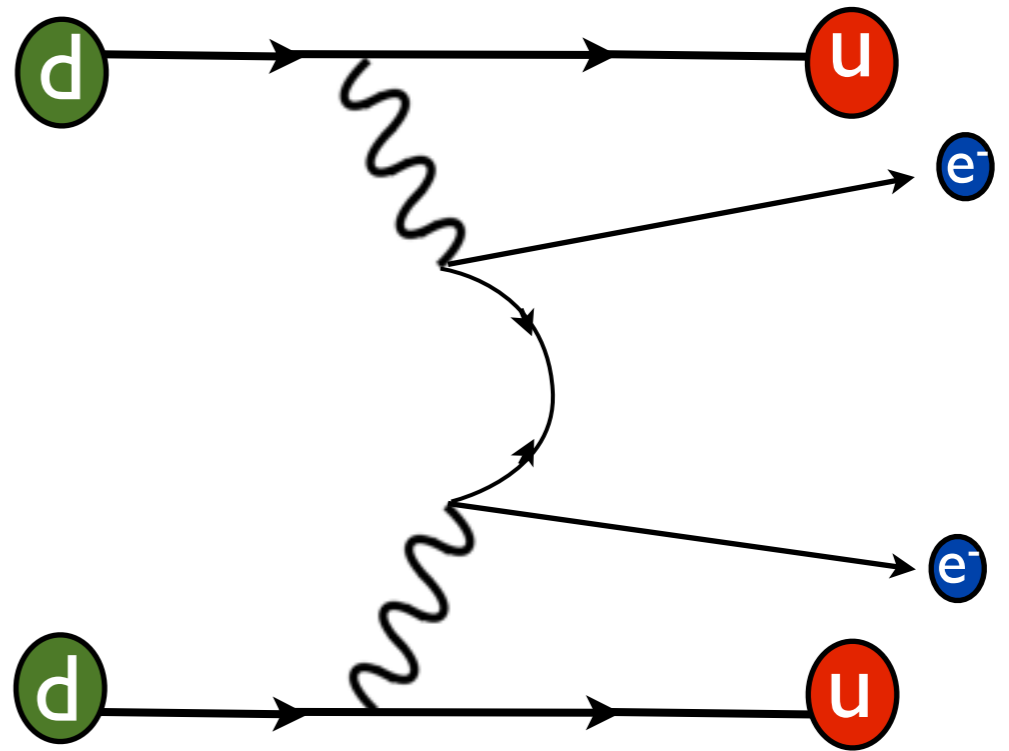


Short-range

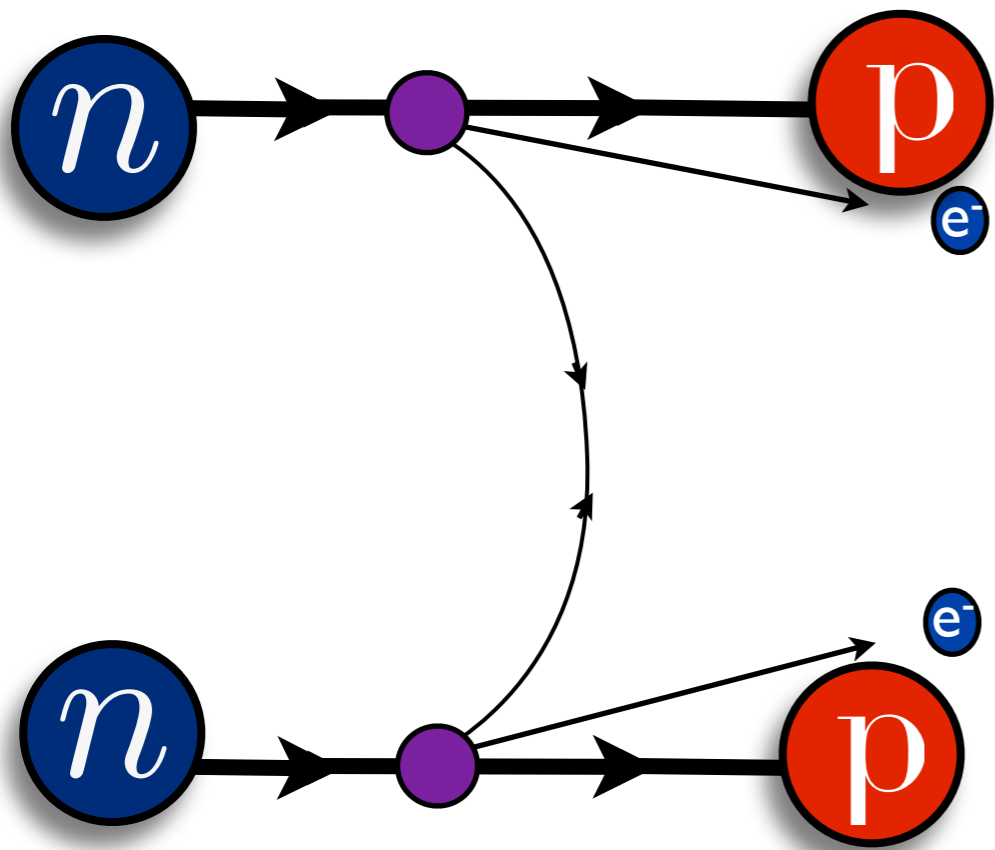


Long-range

$\Lambda \ll \Lambda_{\text{QCD}}$

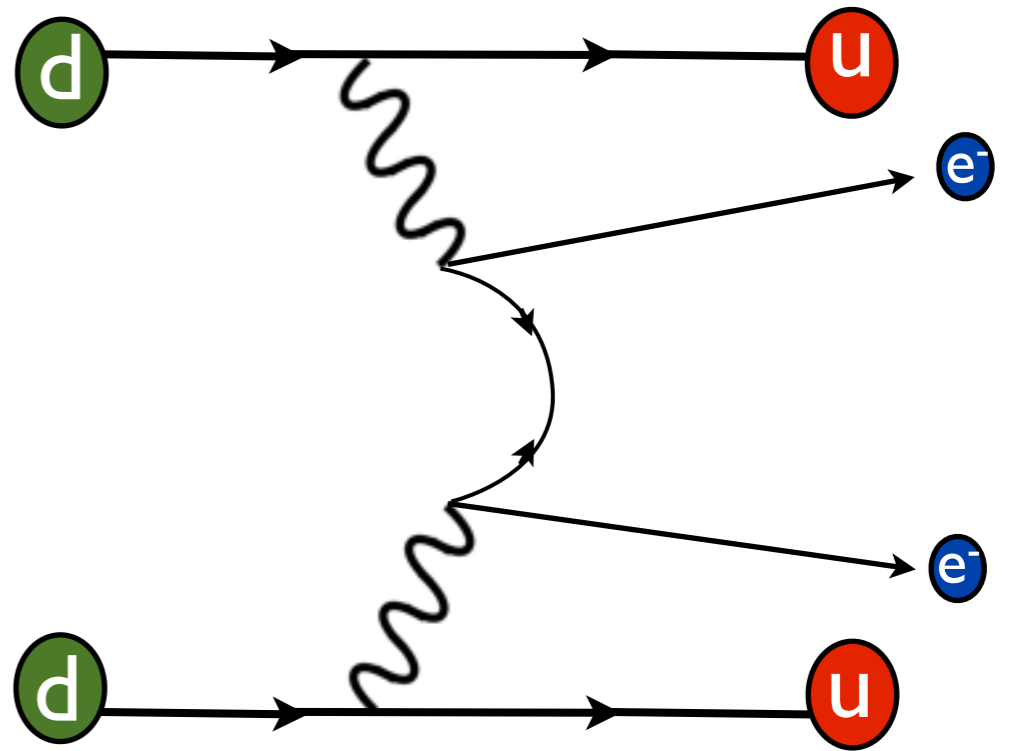


Short-range



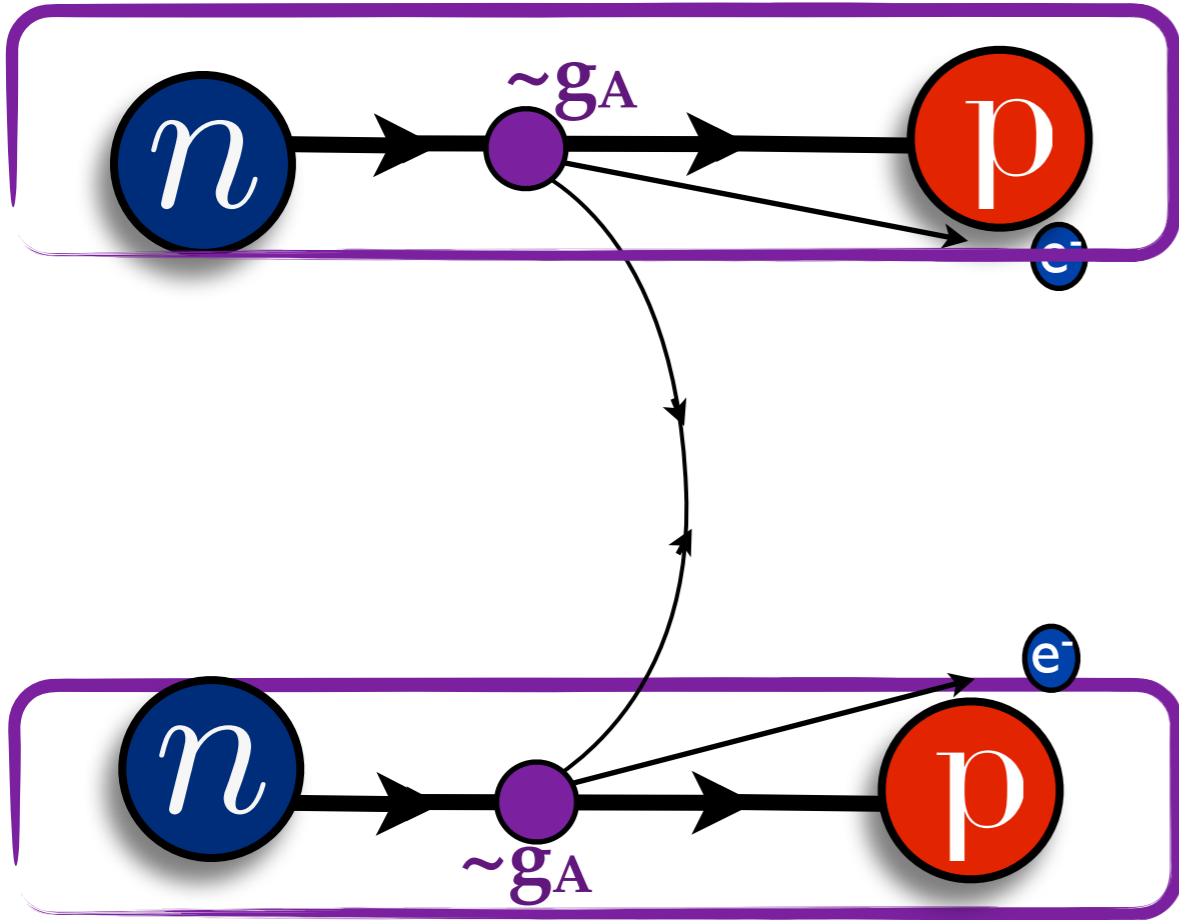
Long-range

$$\Lambda \ll \Lambda_{\text{QCD}}$$

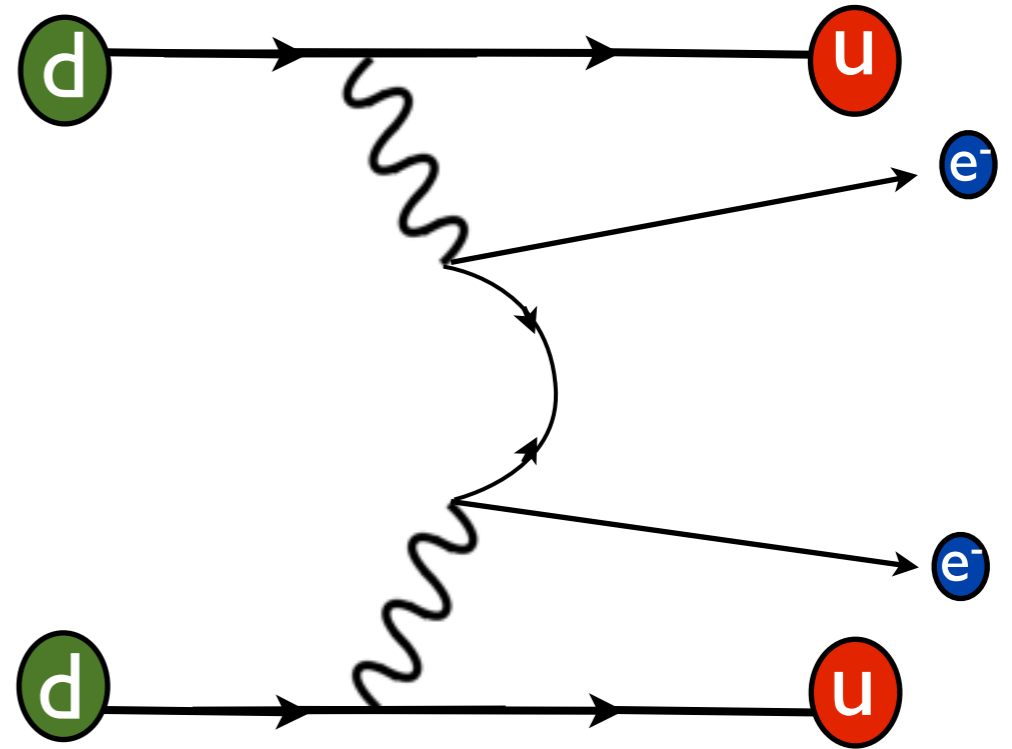


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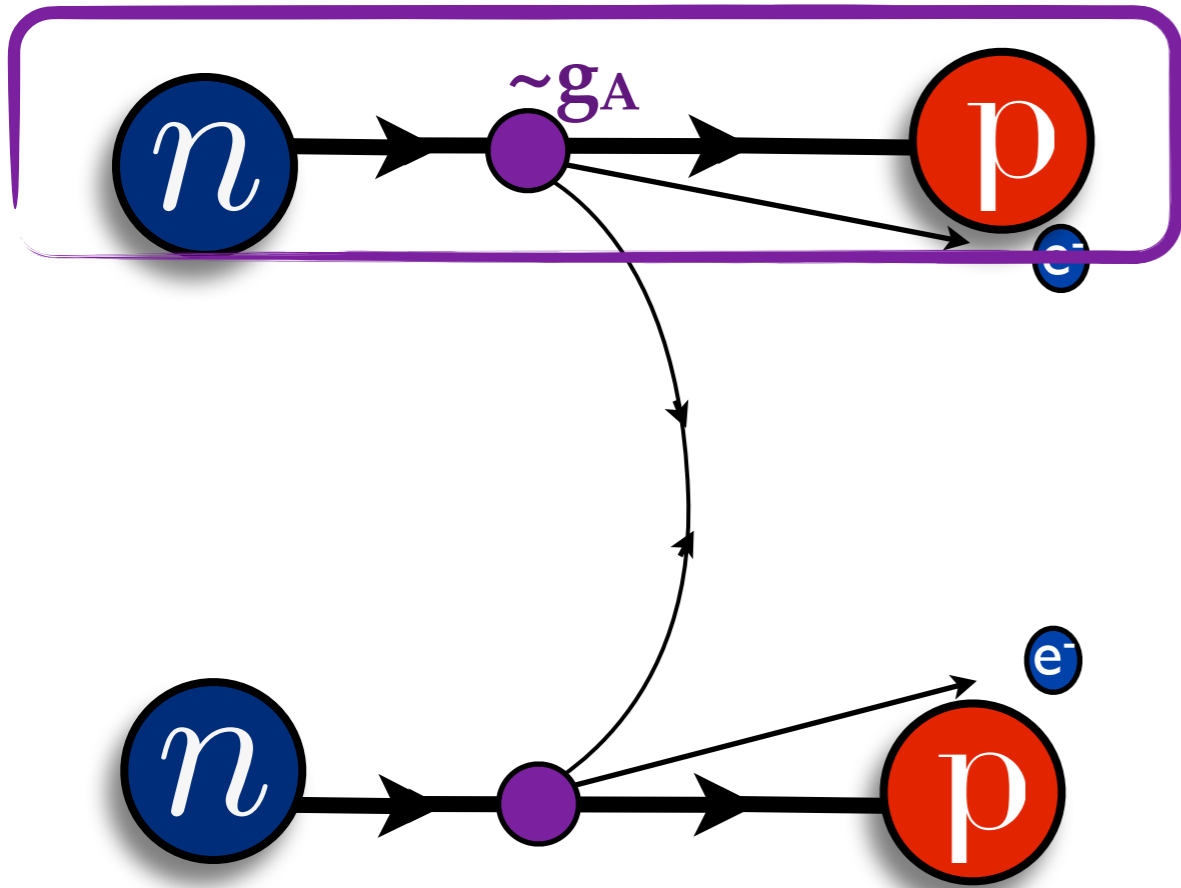




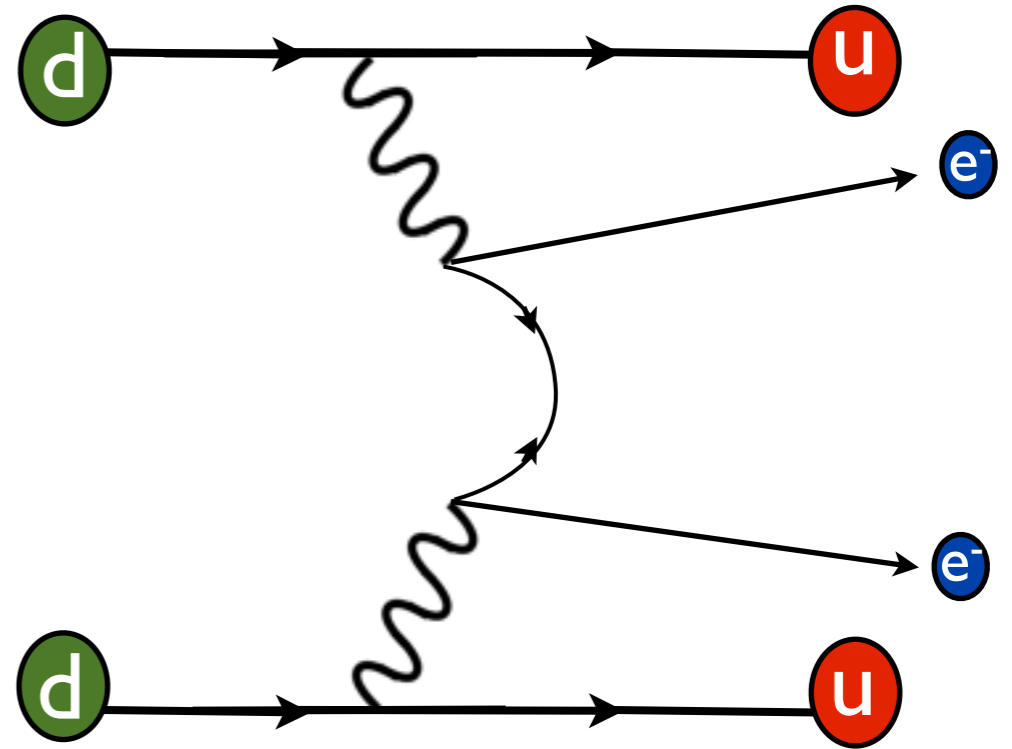
Long-range



Short-range

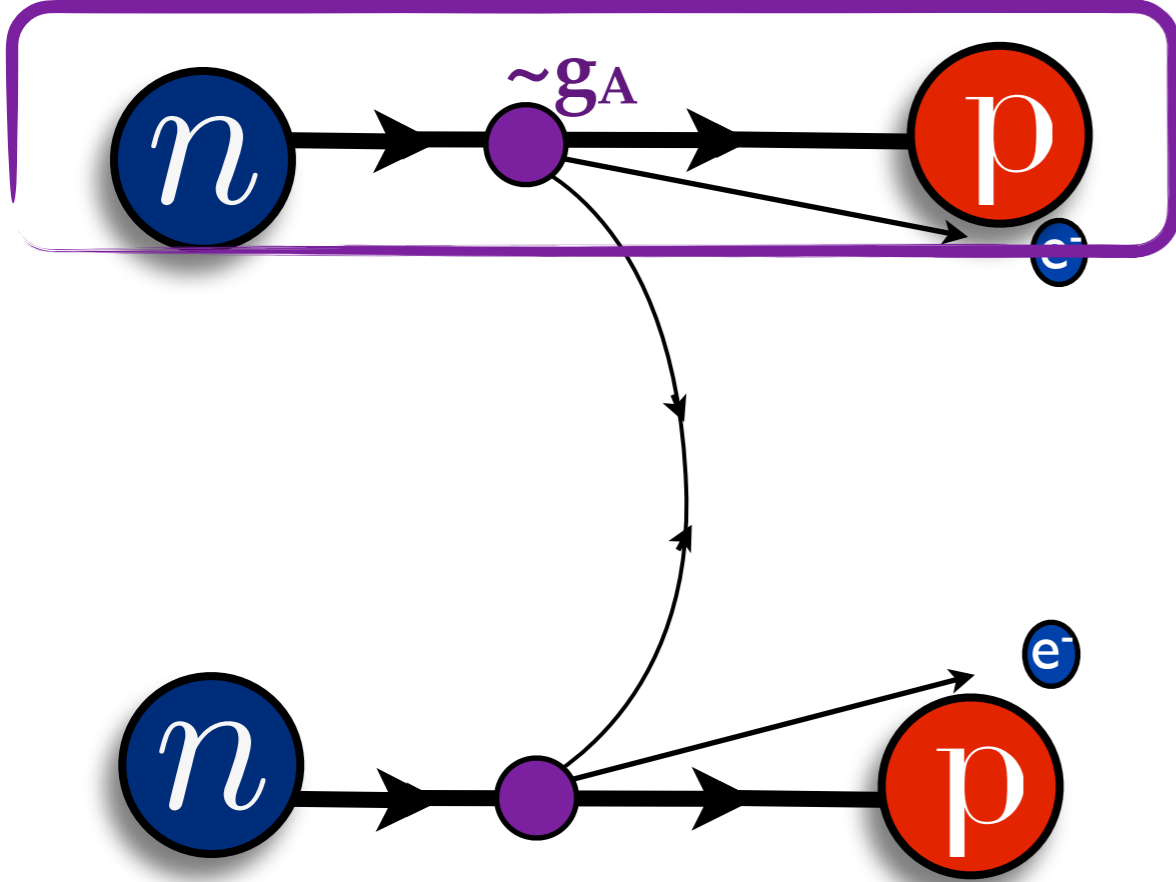


Long-range

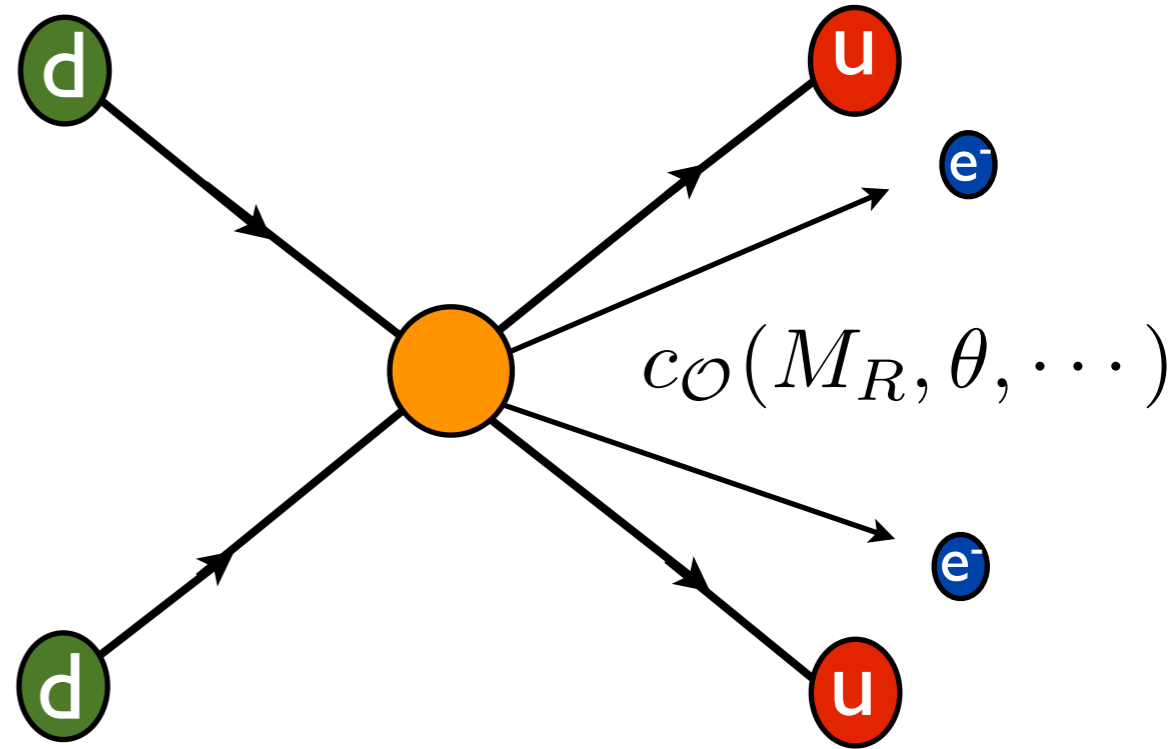


Short-range

$$\Lambda \ll M_W$$



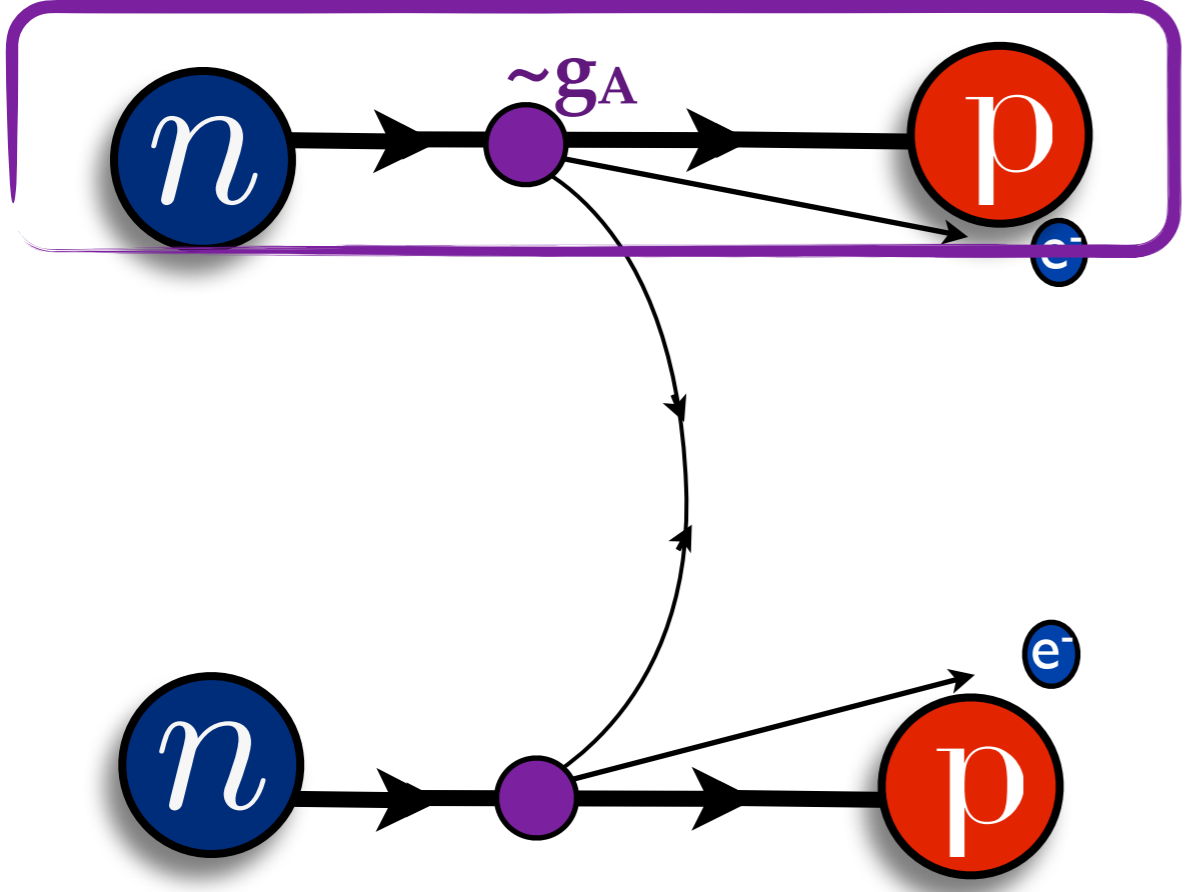
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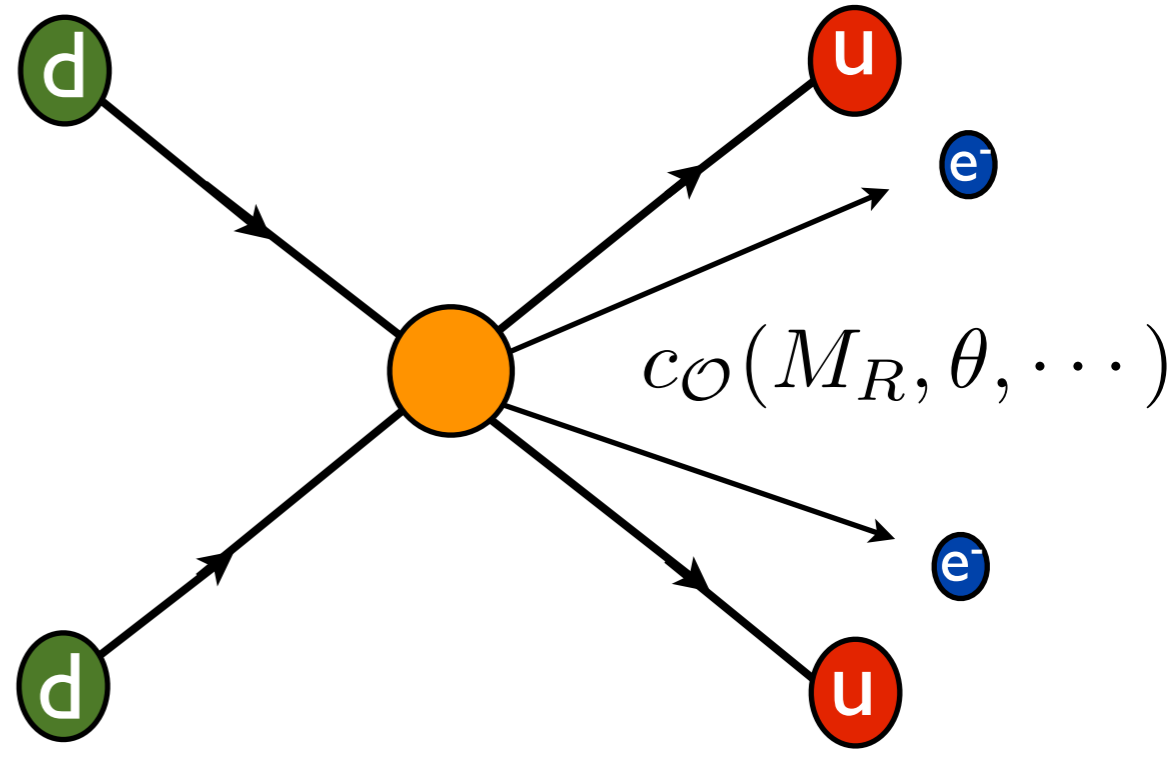
Short-range

$$\begin{aligned} \mathcal{O}_{1+}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{2\pm}^{ab} &= (\bar{q}_R \tau^a q_L) (\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R) (\bar{q}_L \tau^b q_R), \\ \mathcal{O}_{3\pm}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{4\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L), \\ \mathcal{O}_{5\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L). \end{aligned}$$

Prezeau, Ramsey-Musolf, Vogel (2003)

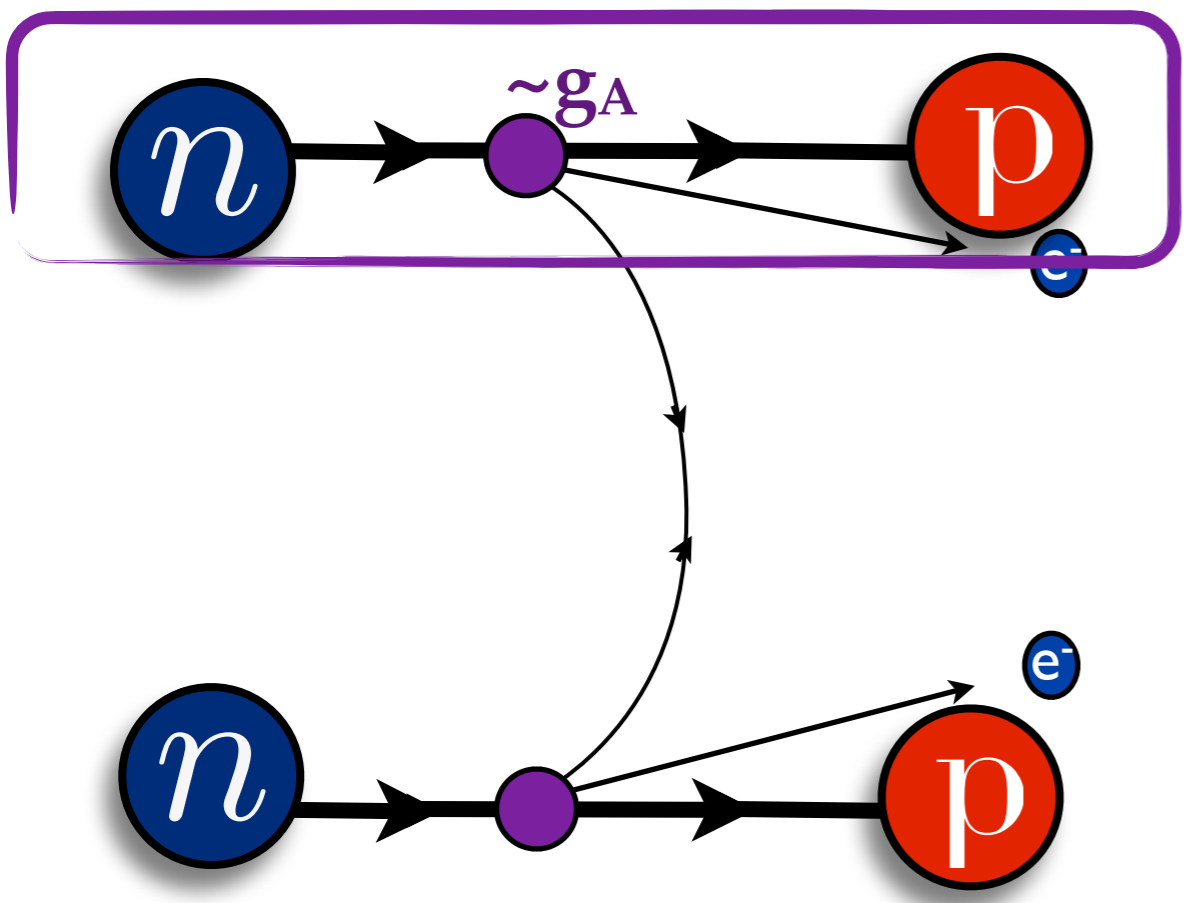


Long-range

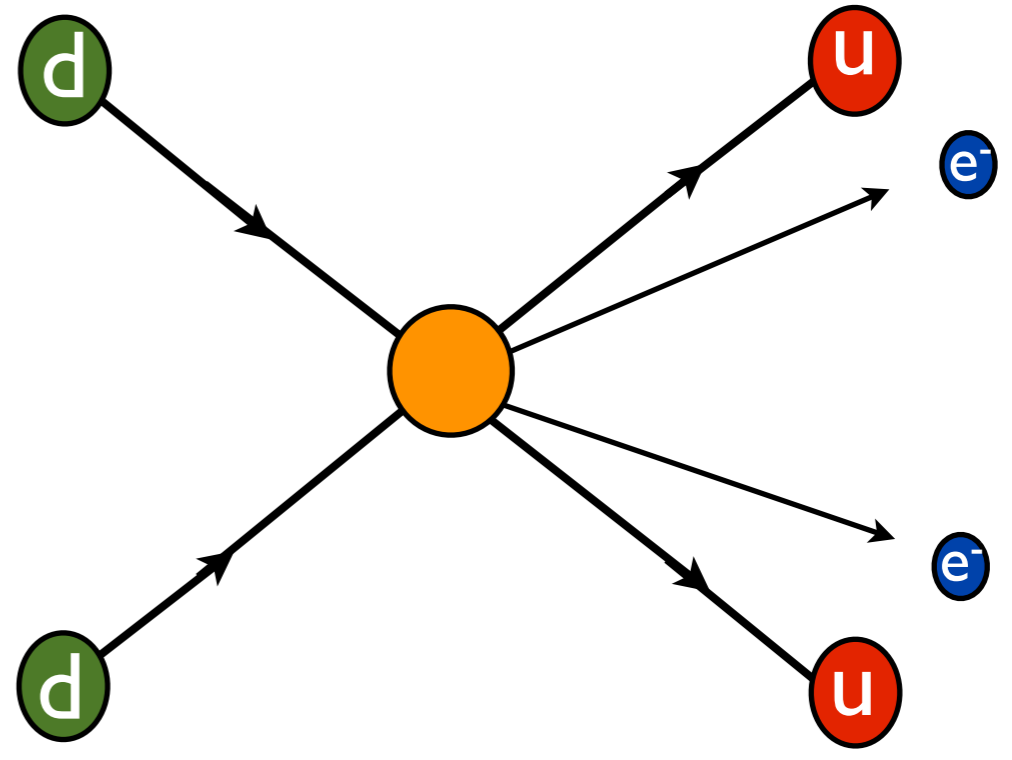


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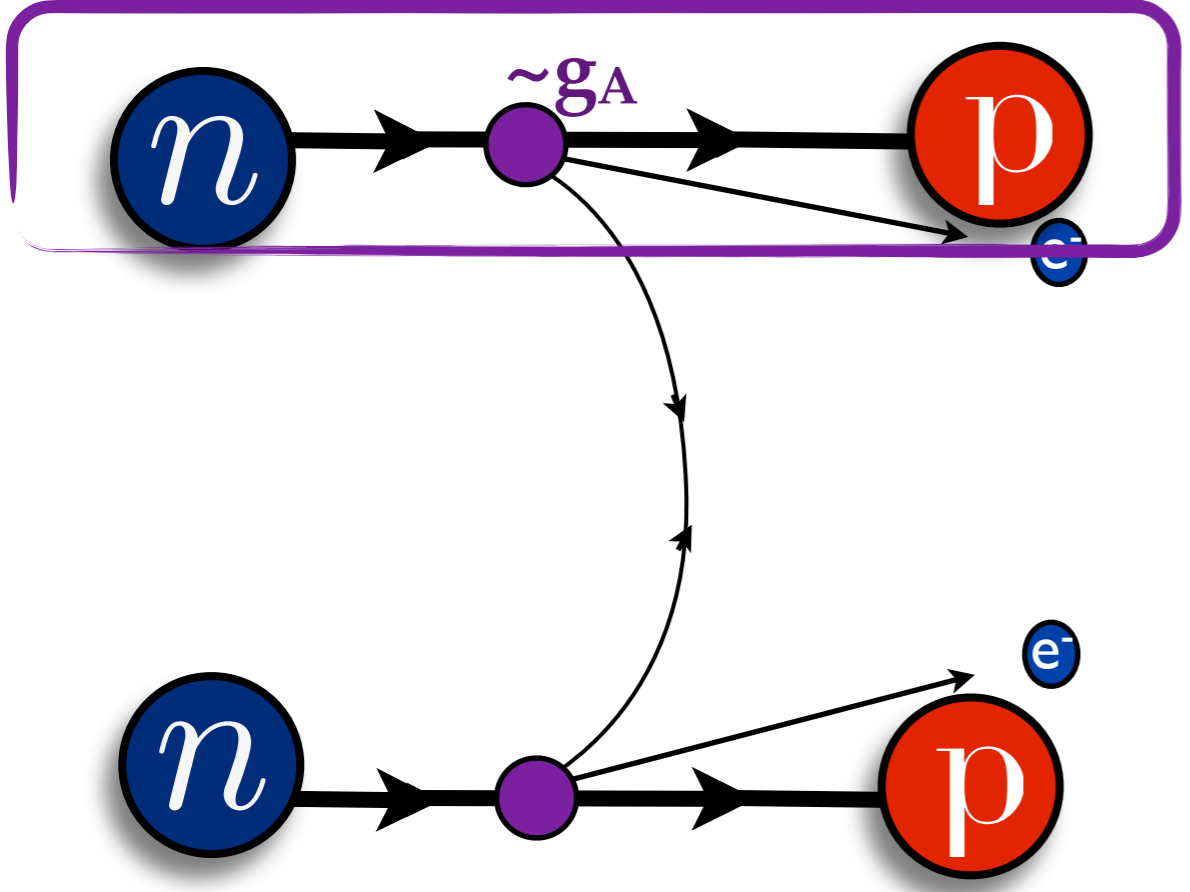
$\Lambda \ll \Lambda_{\text{QCD}}$



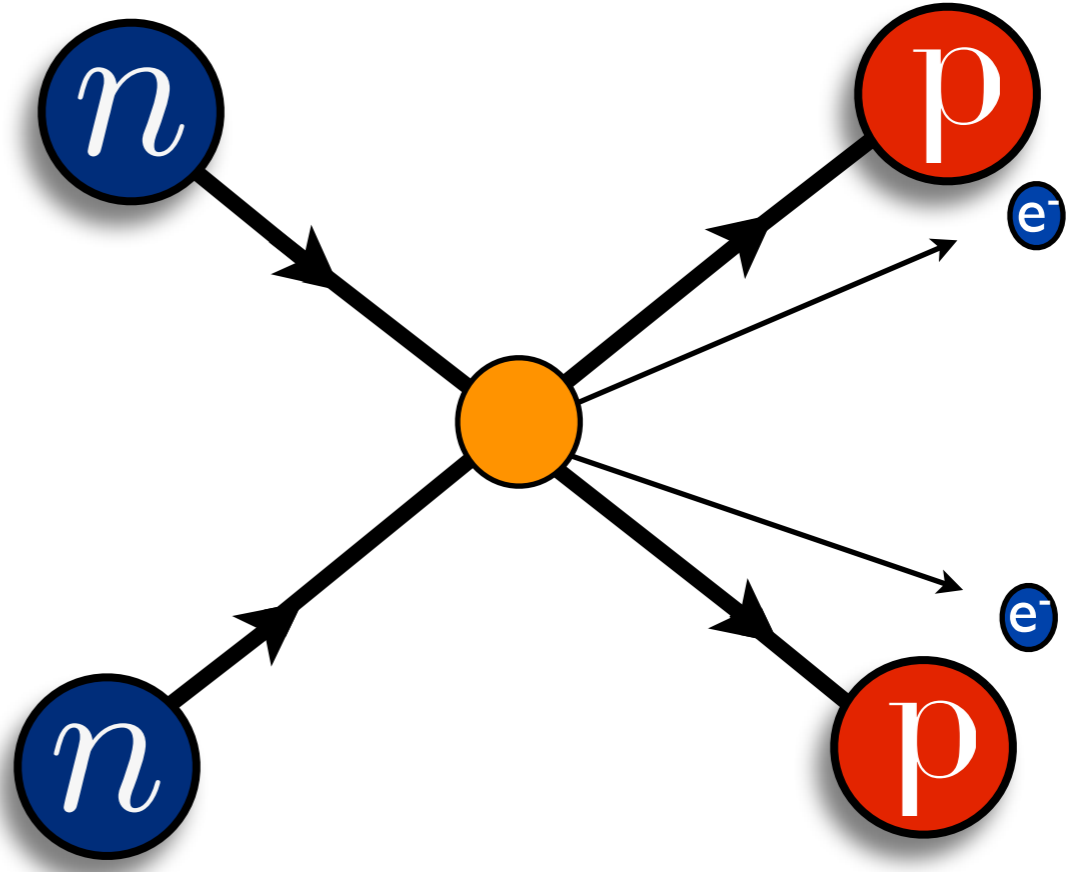
Long-range



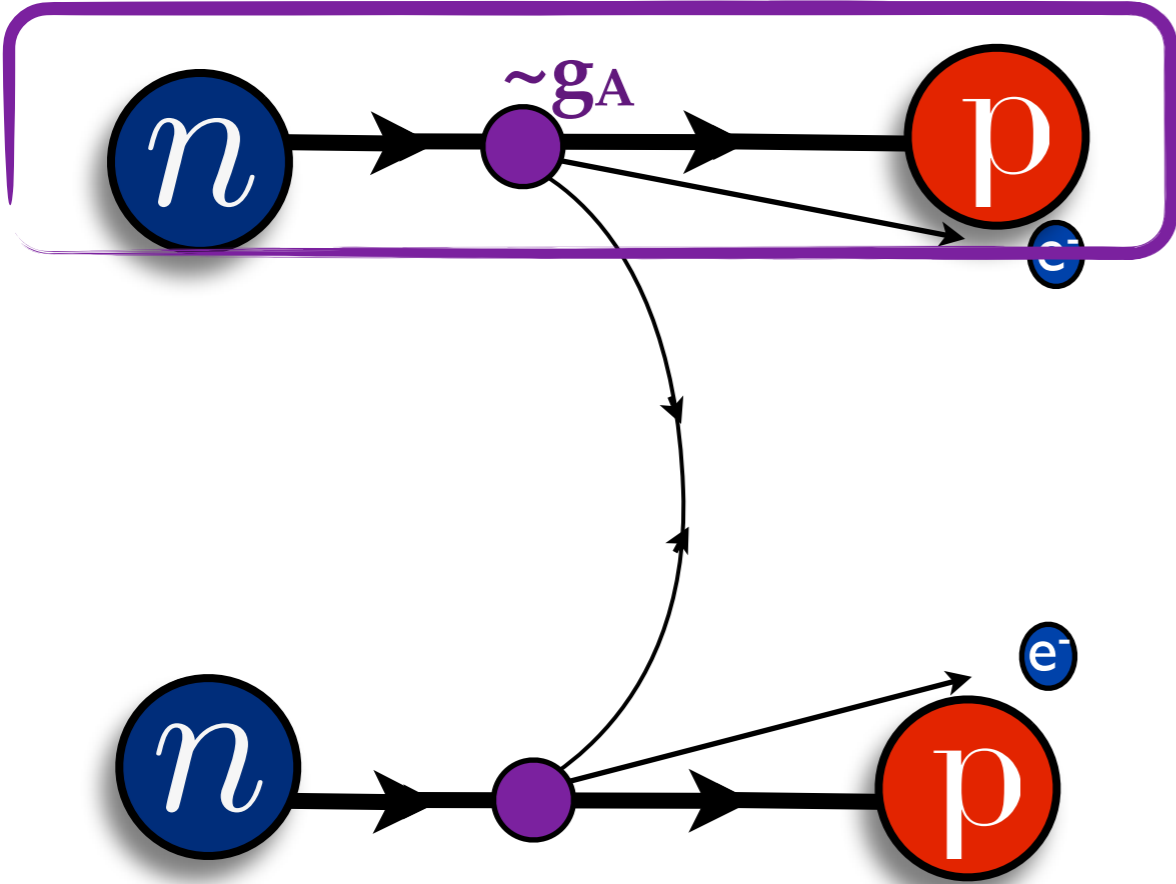
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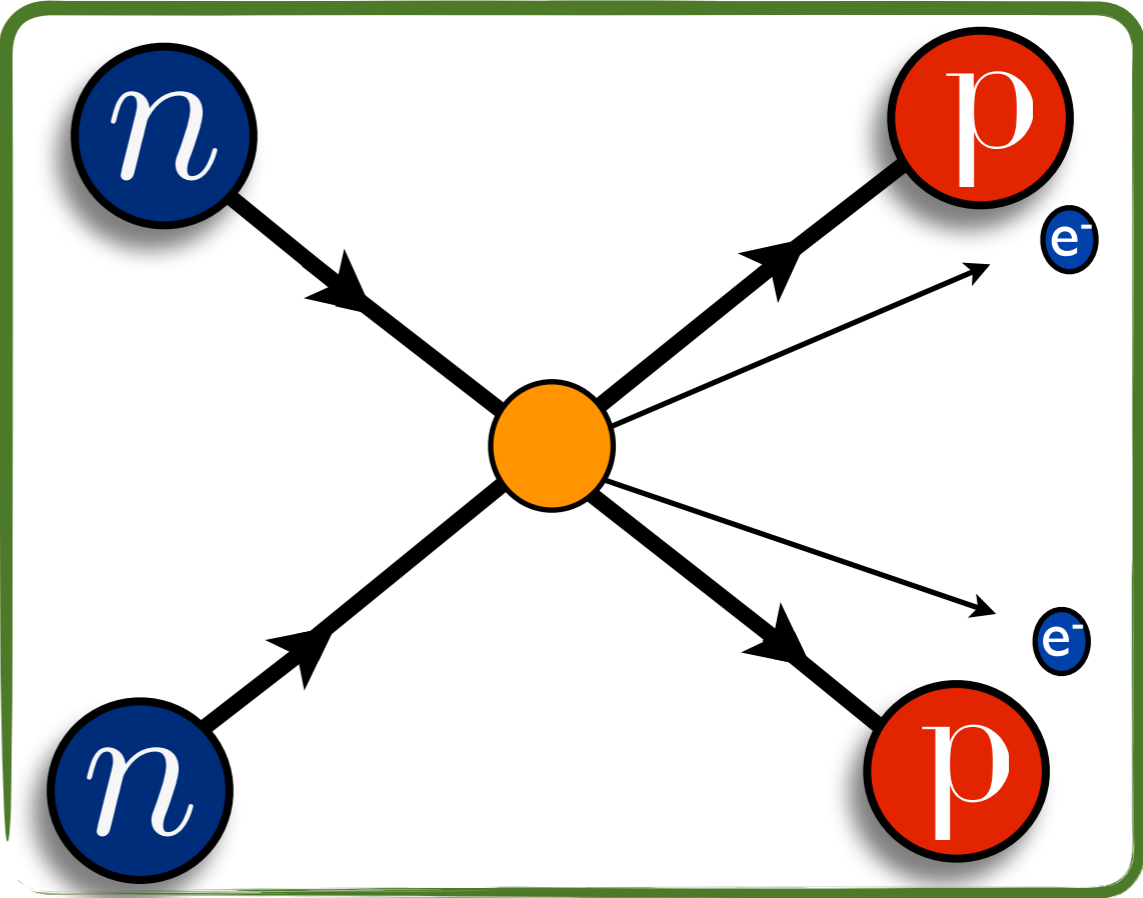
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Short-range

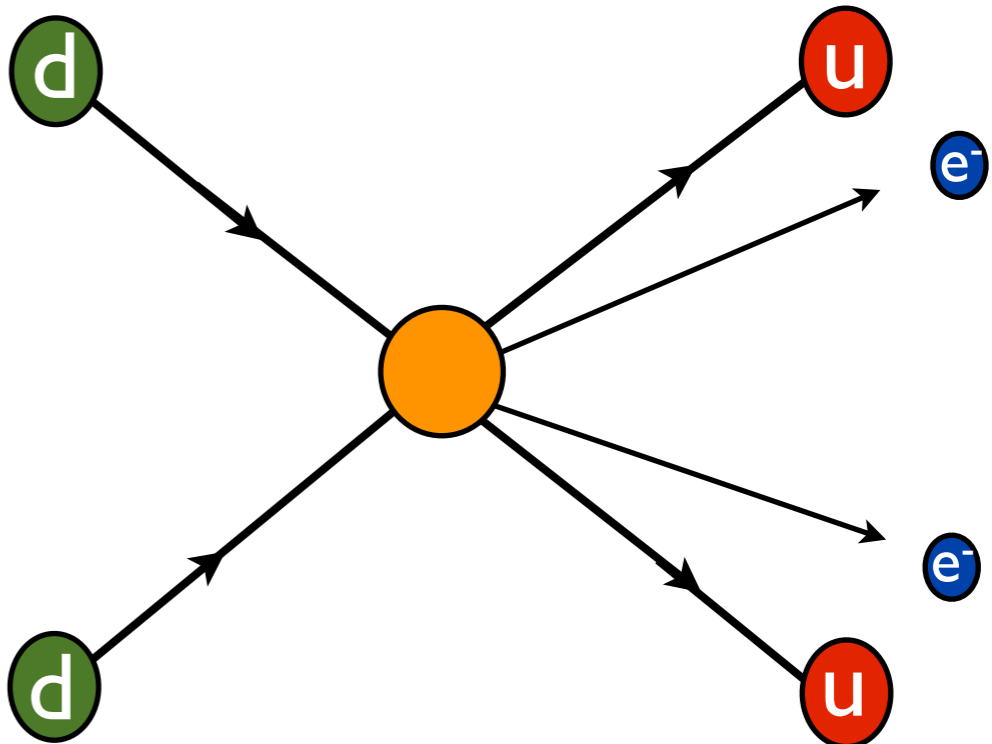
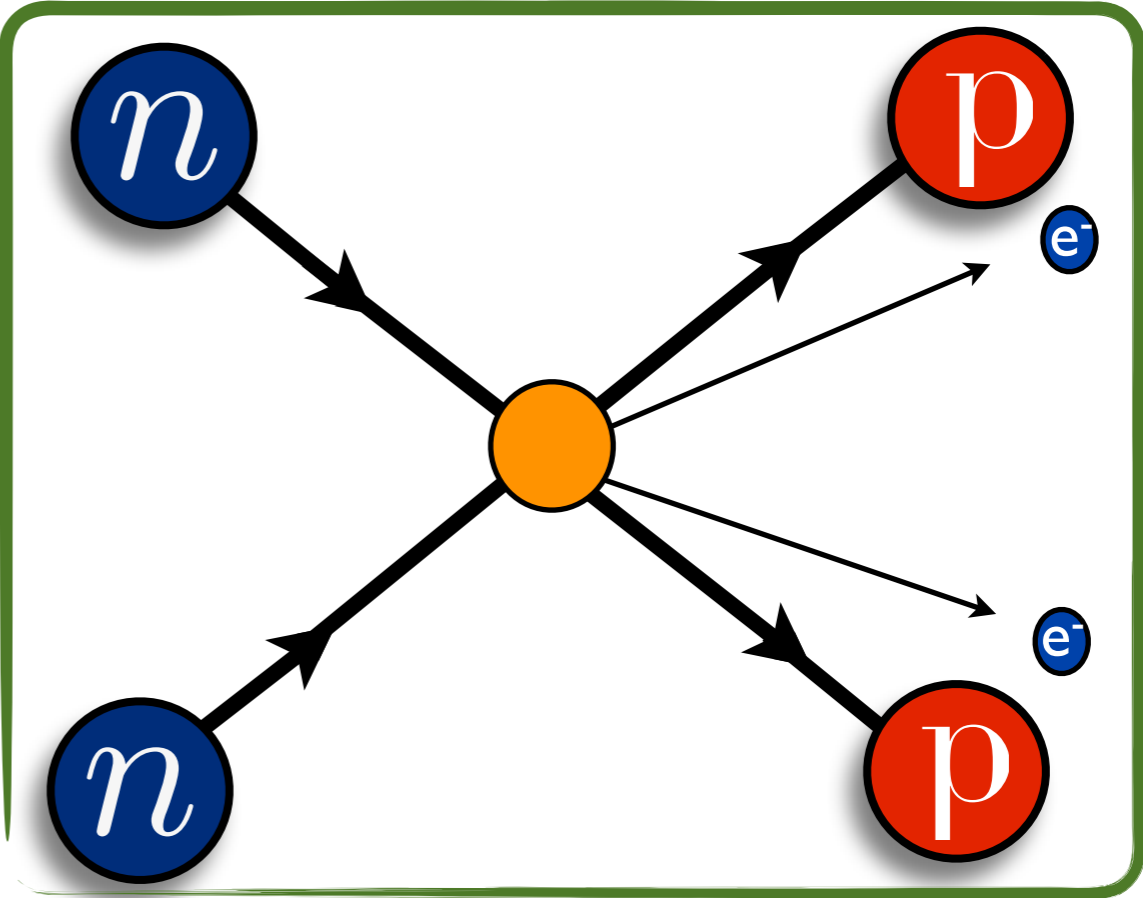
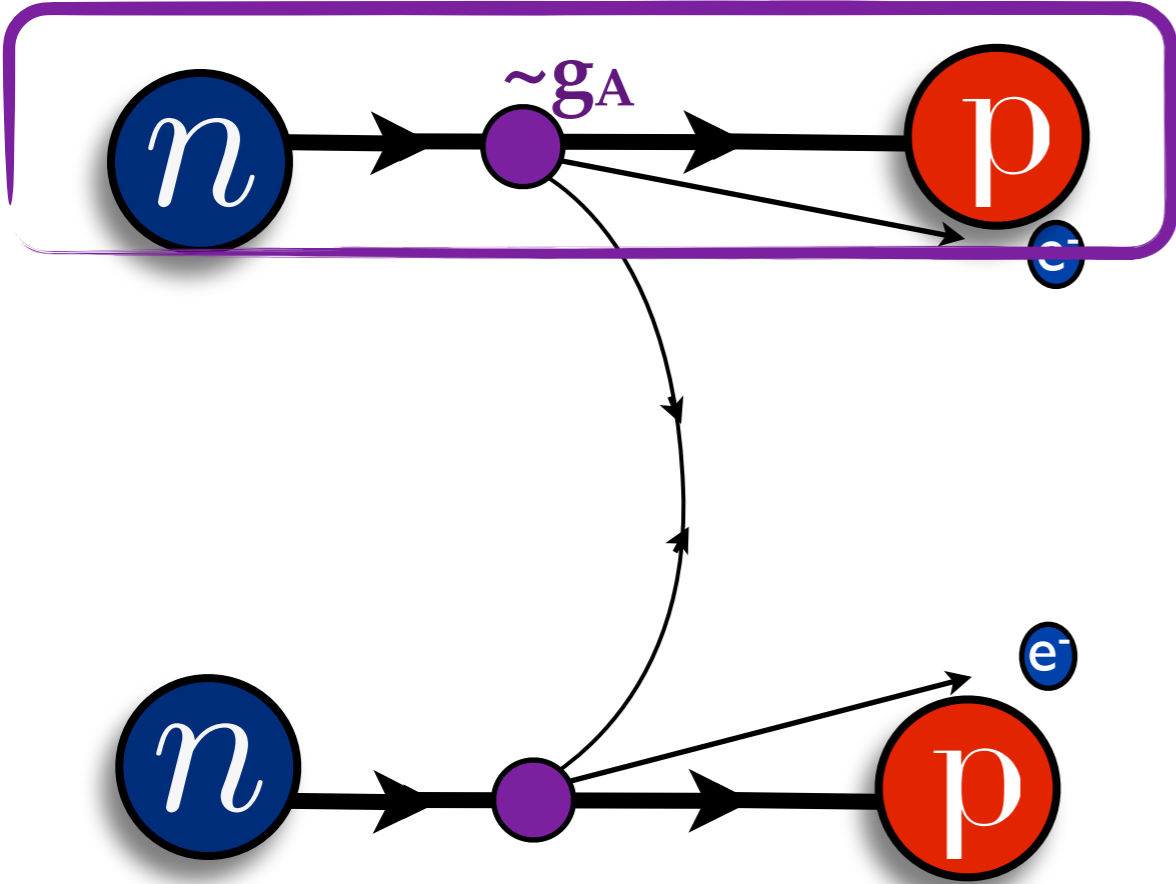


Long-range

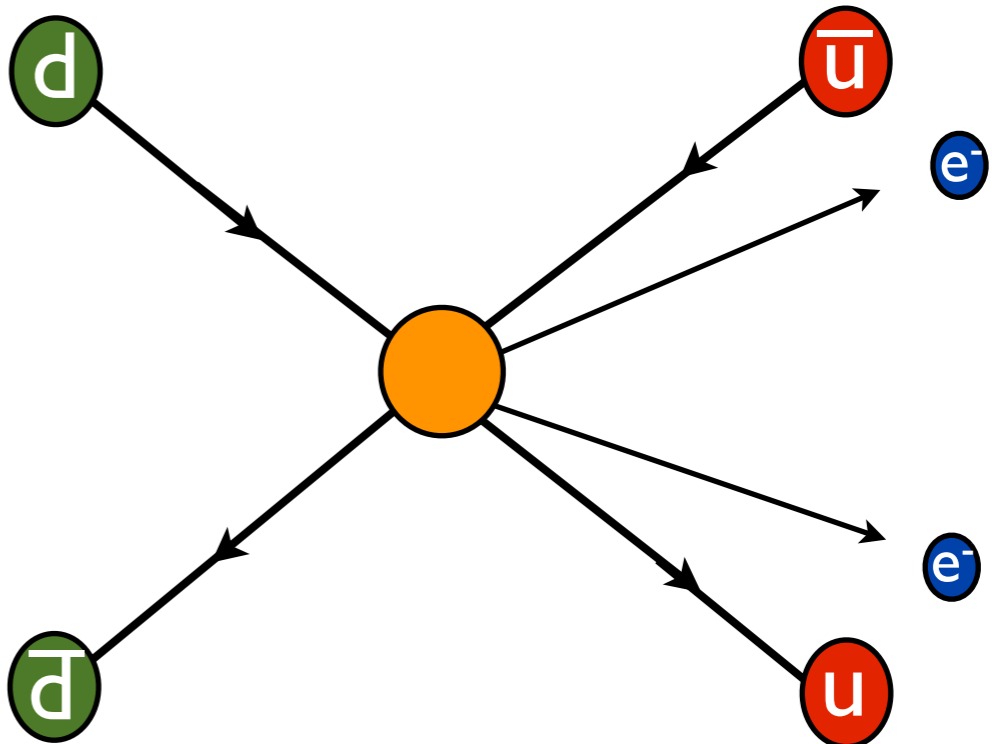
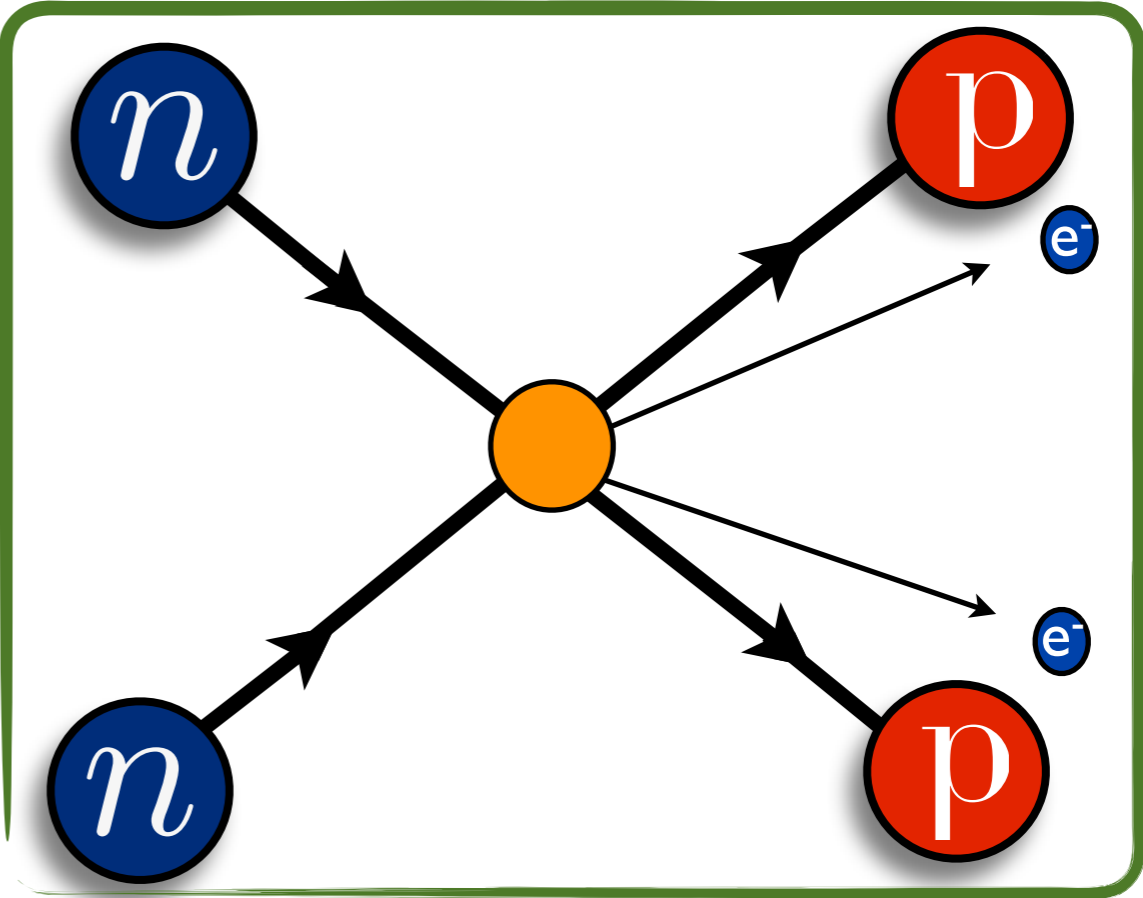
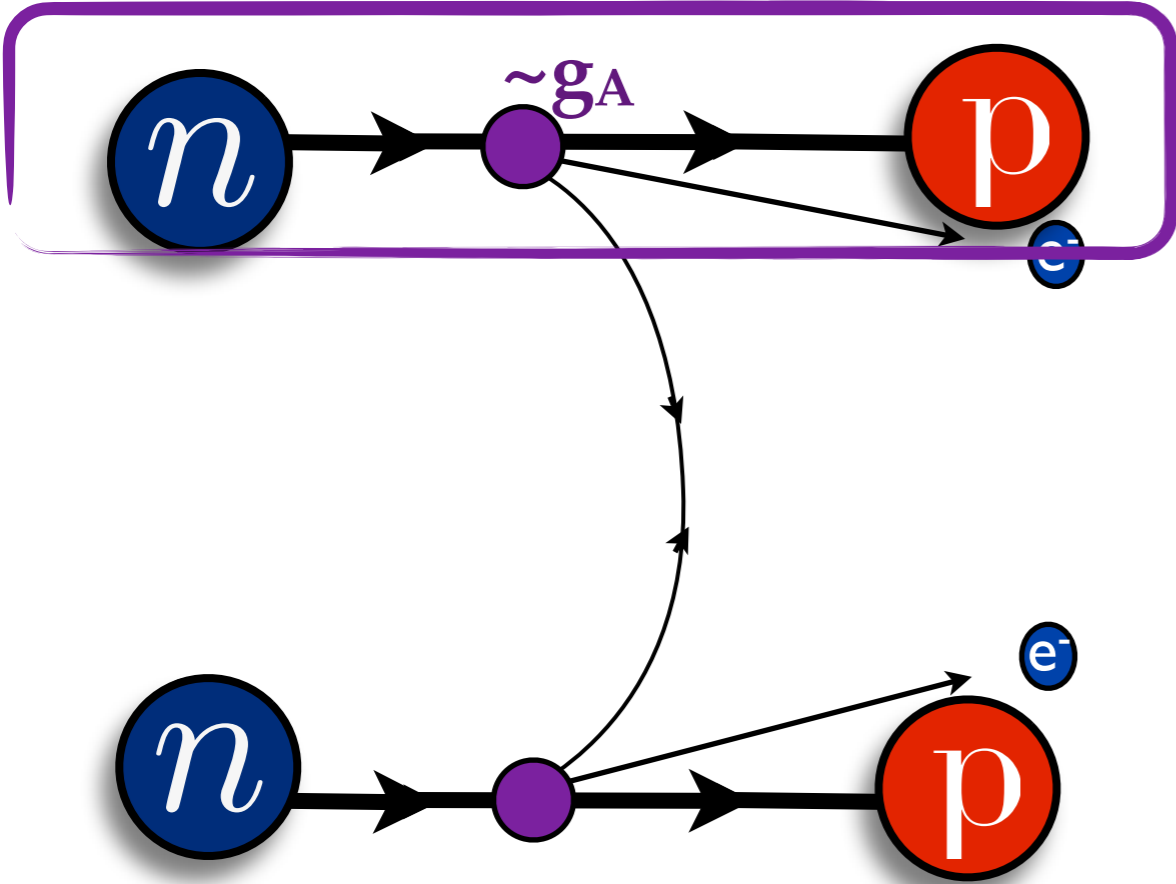


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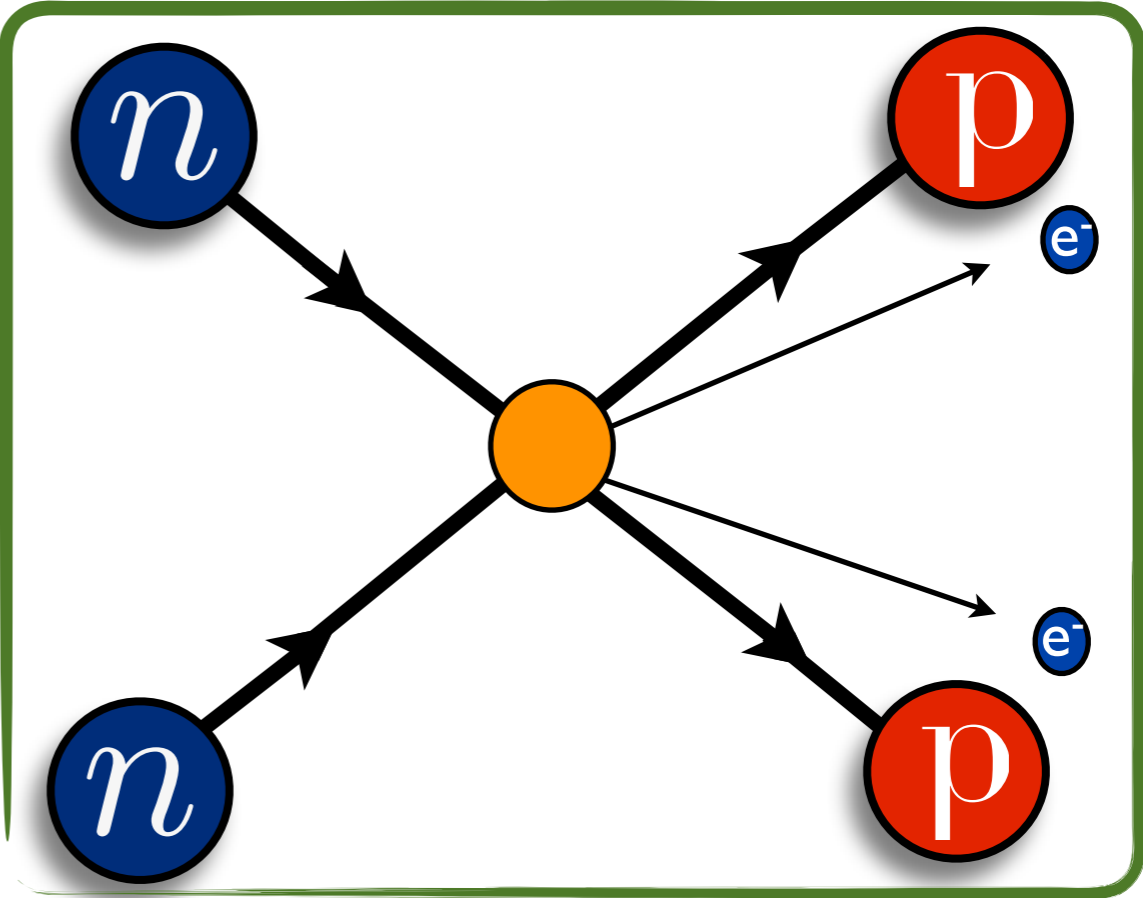
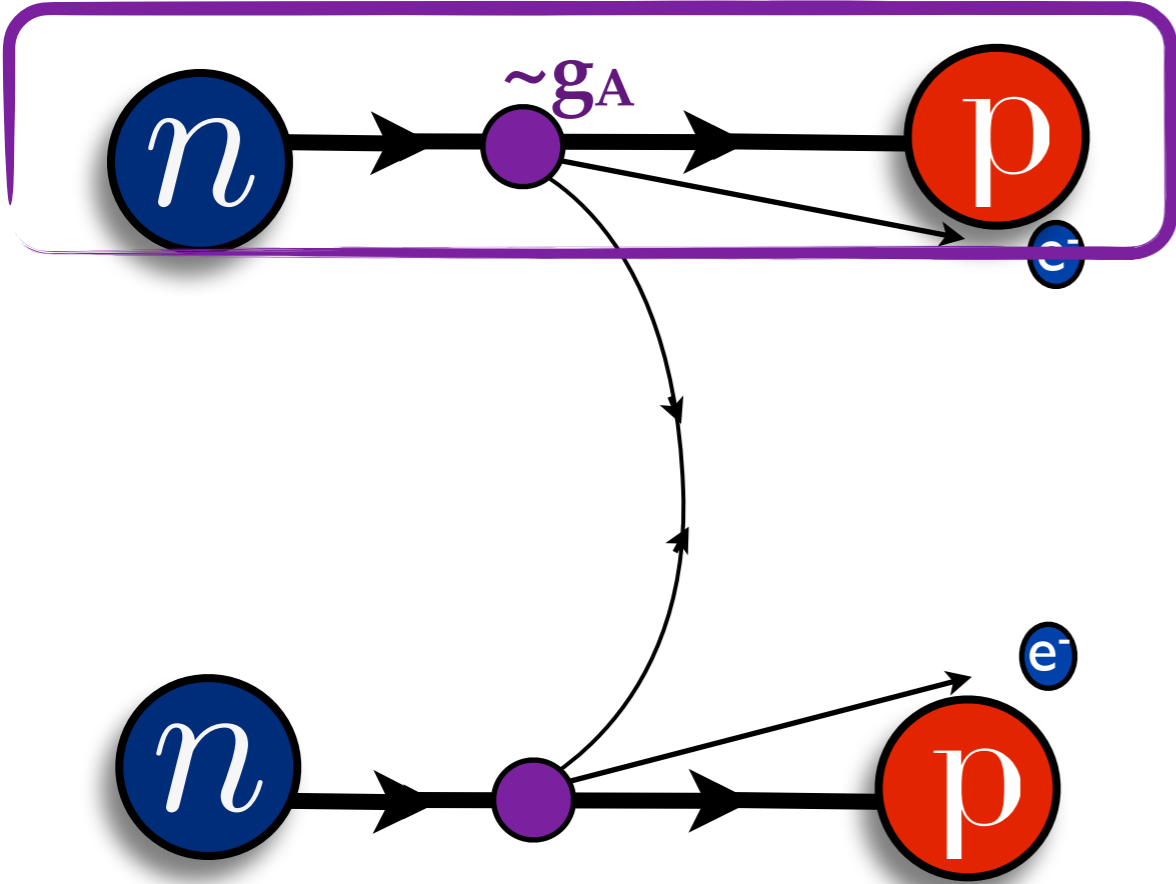
Prezeau, Ramsey-Musolf,  
Vogel (2003)



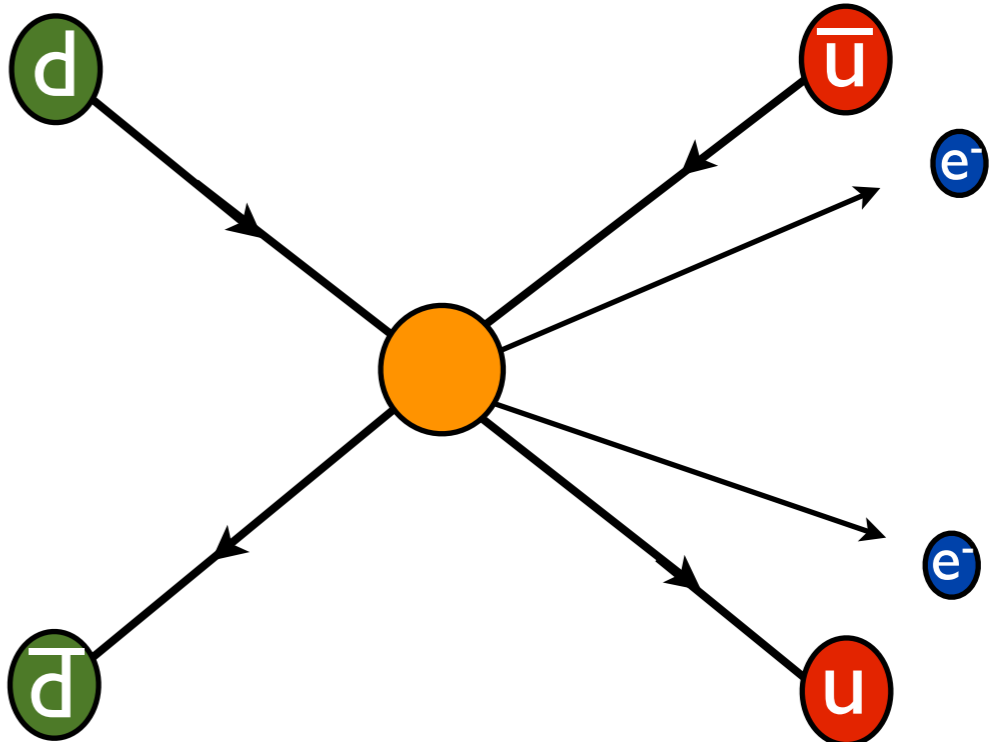


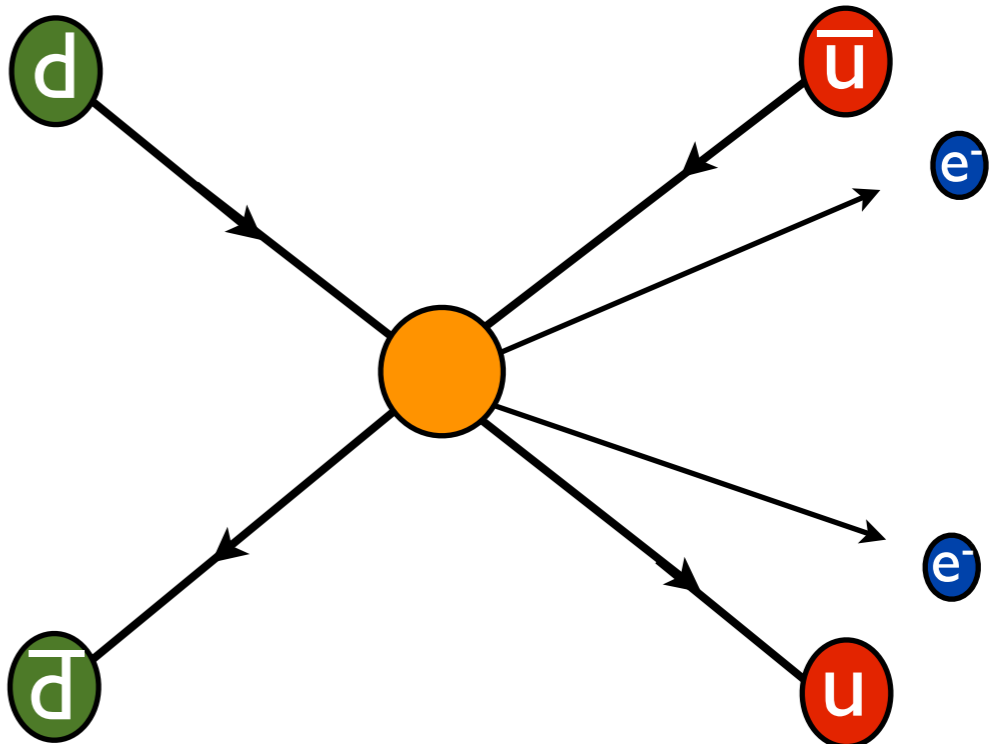
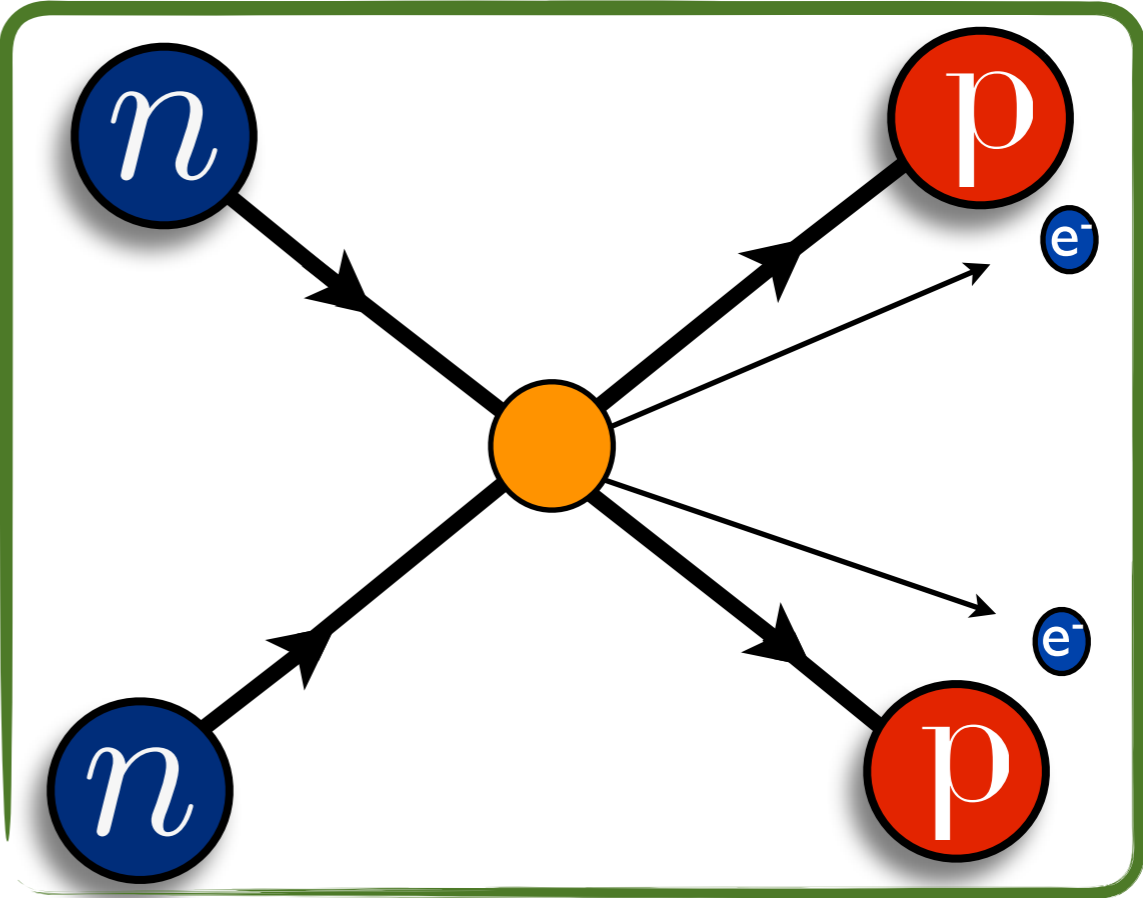
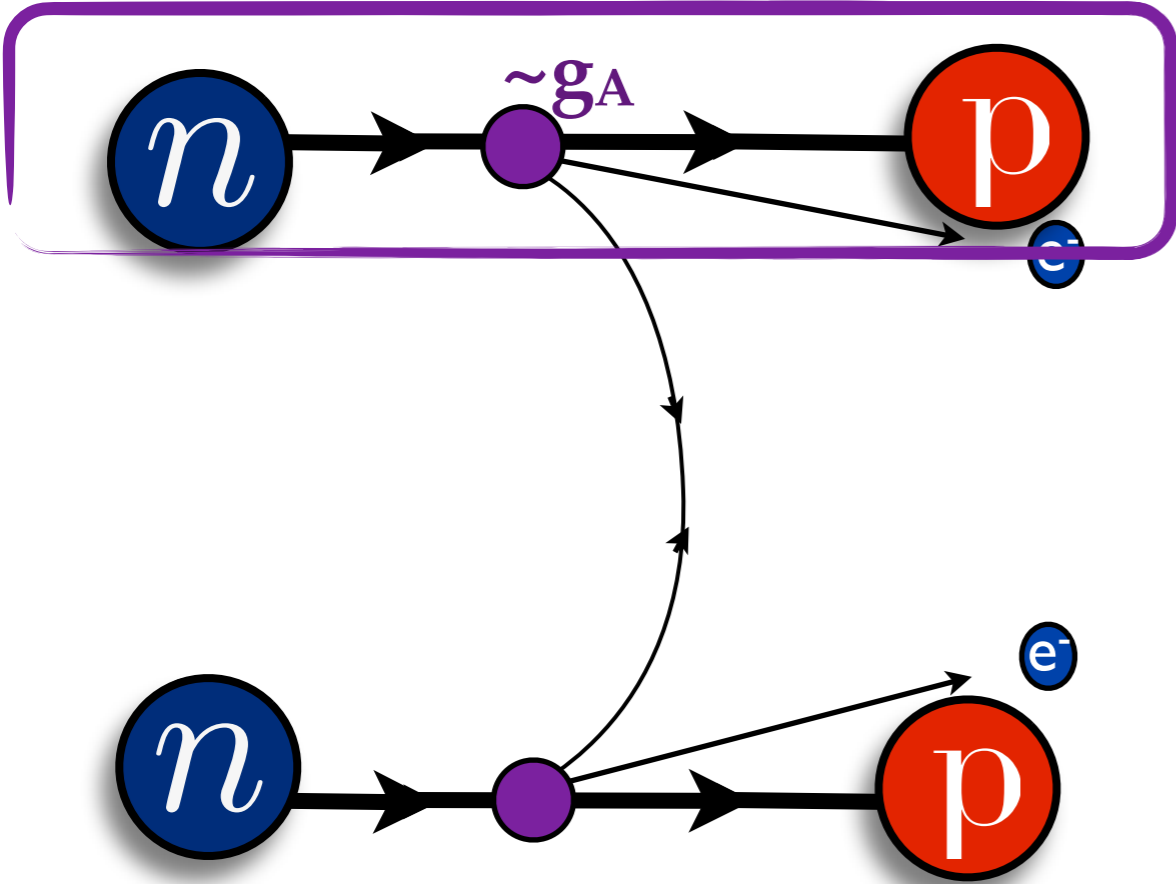


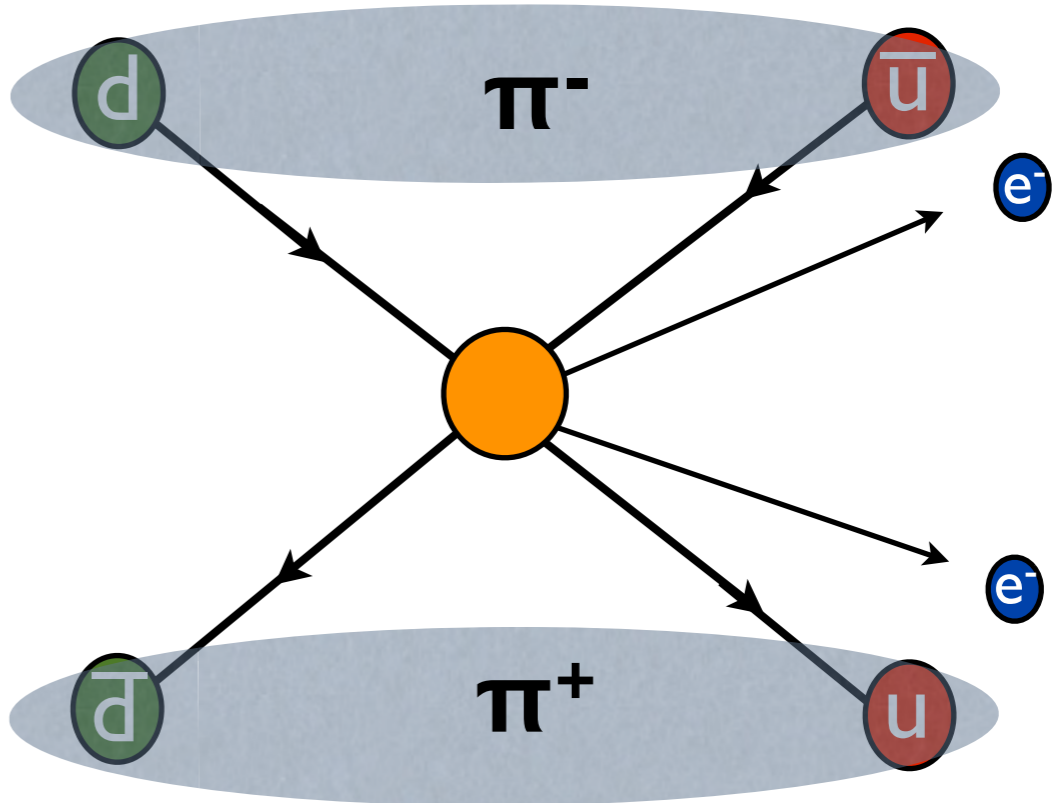
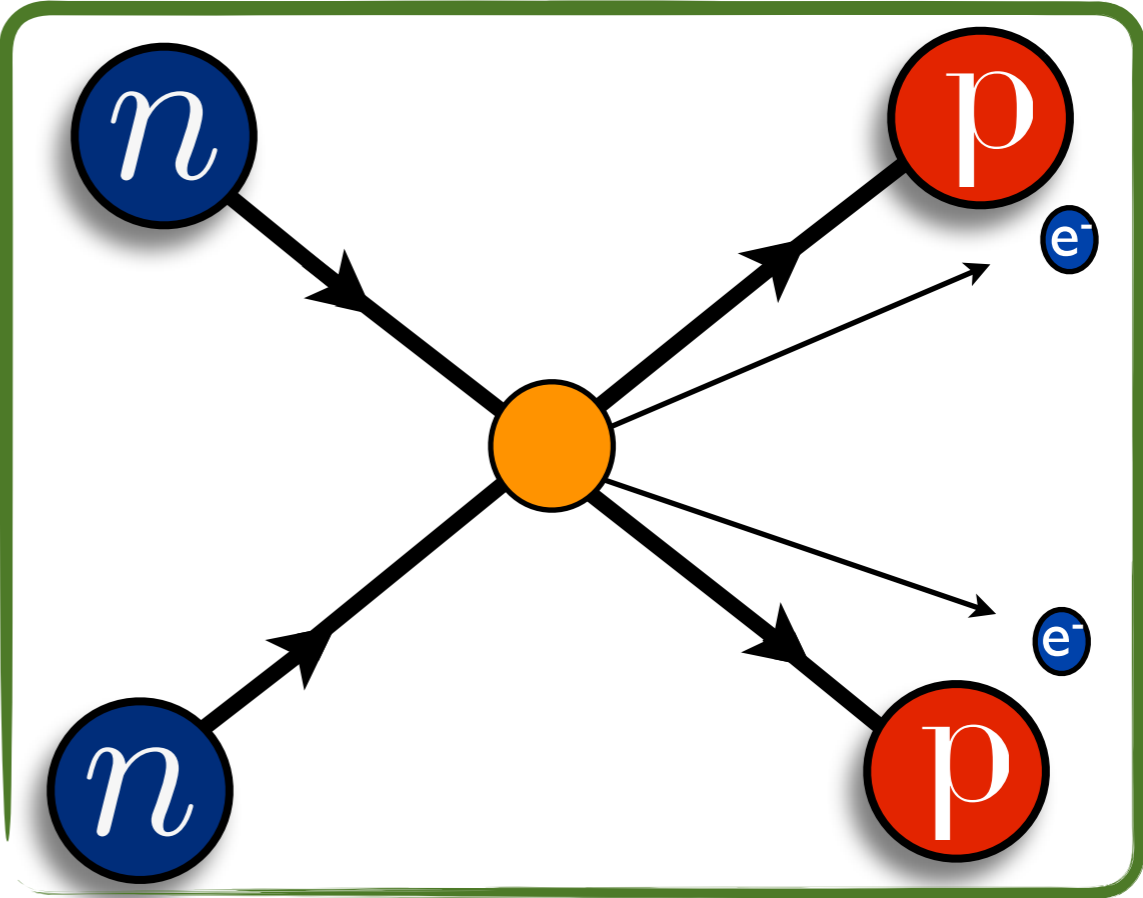
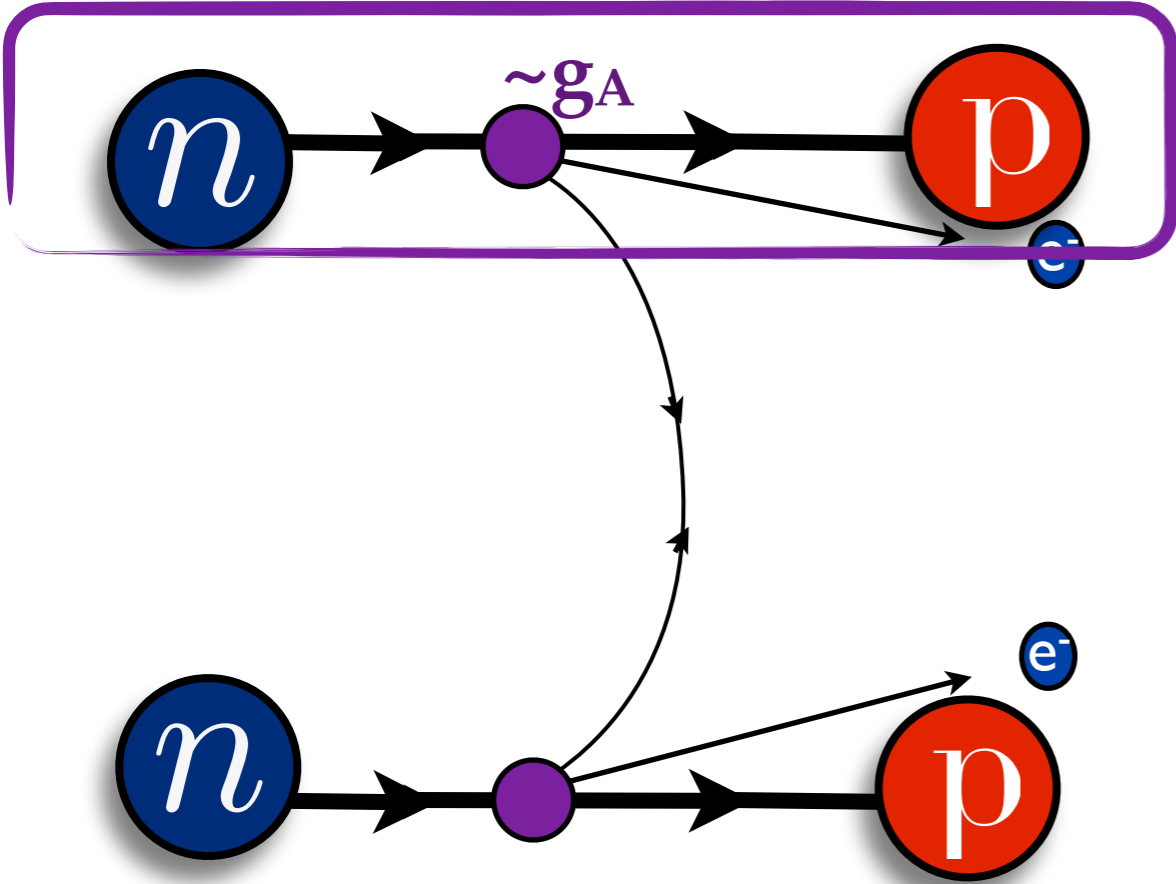
Prezeau, Ramsey-Musolf,  
Vogel (2003)



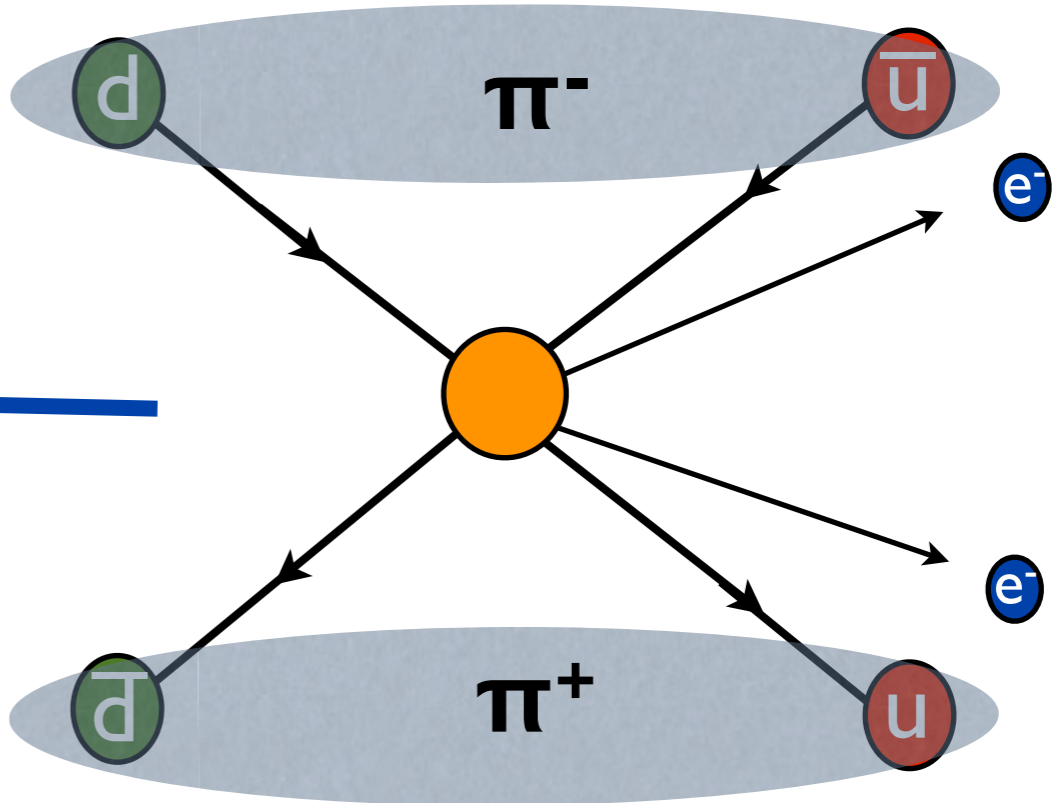
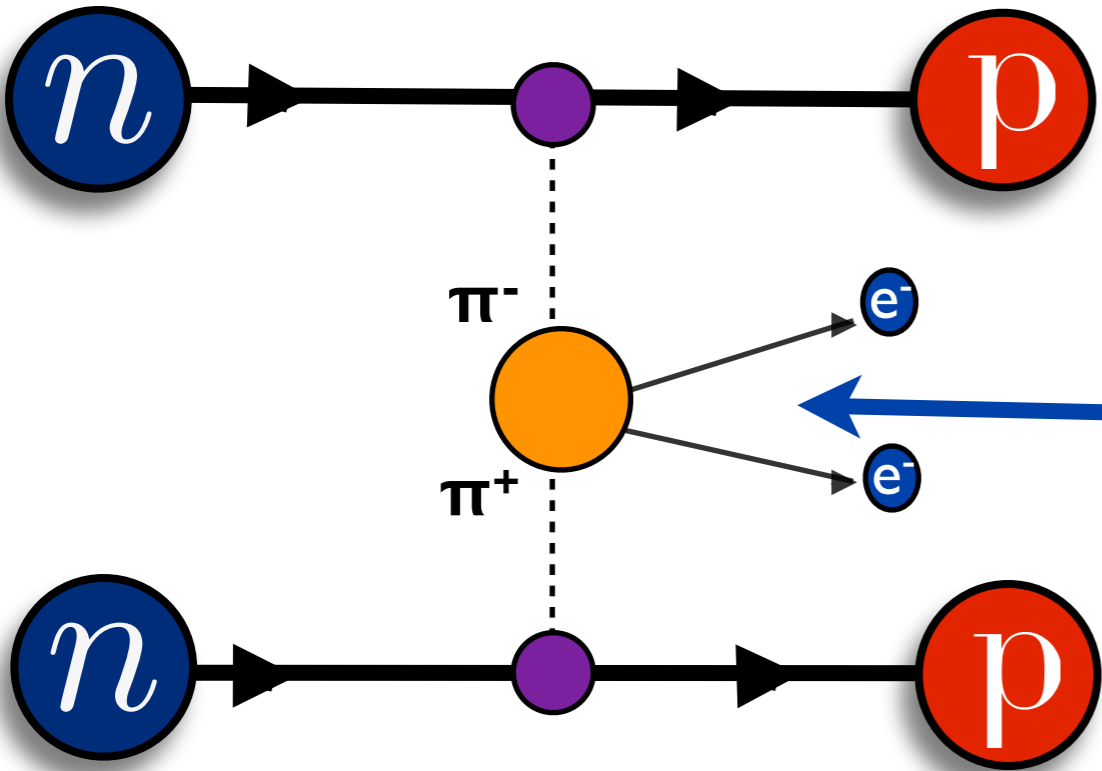
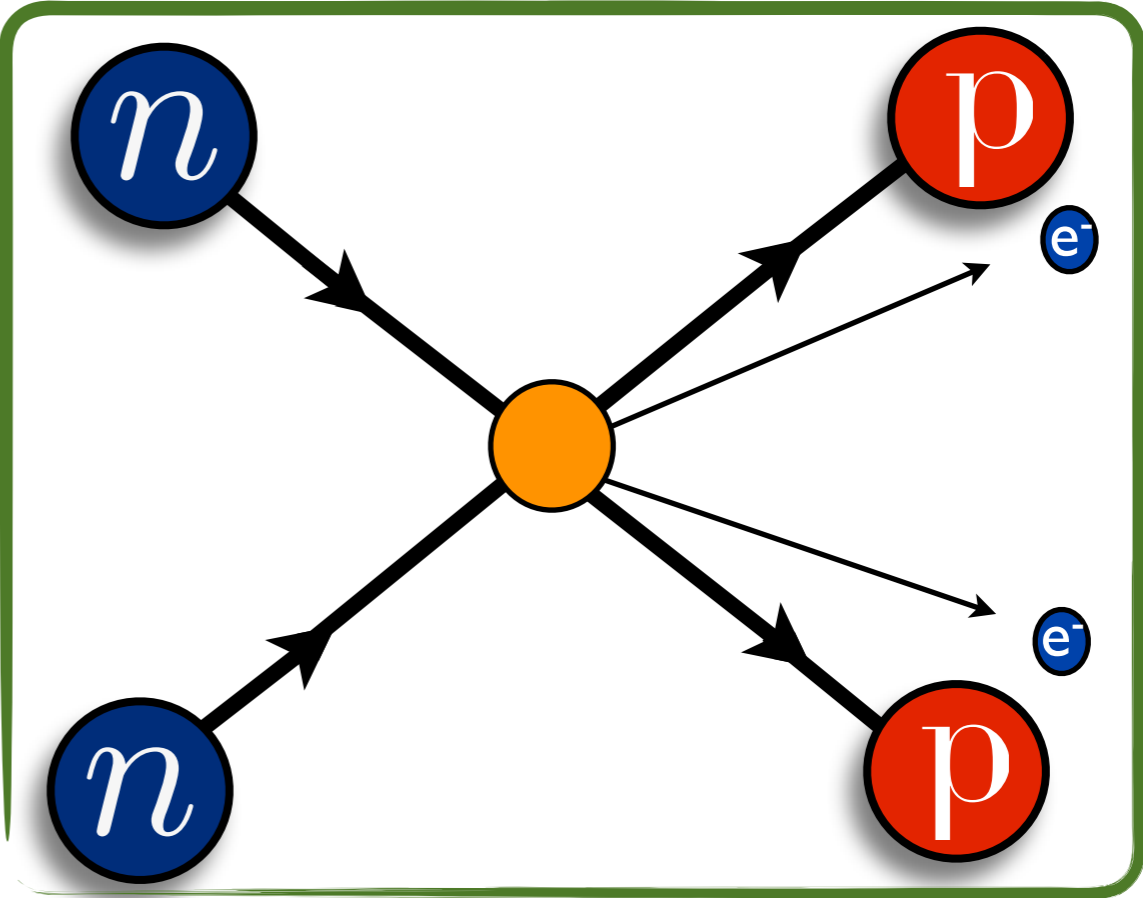
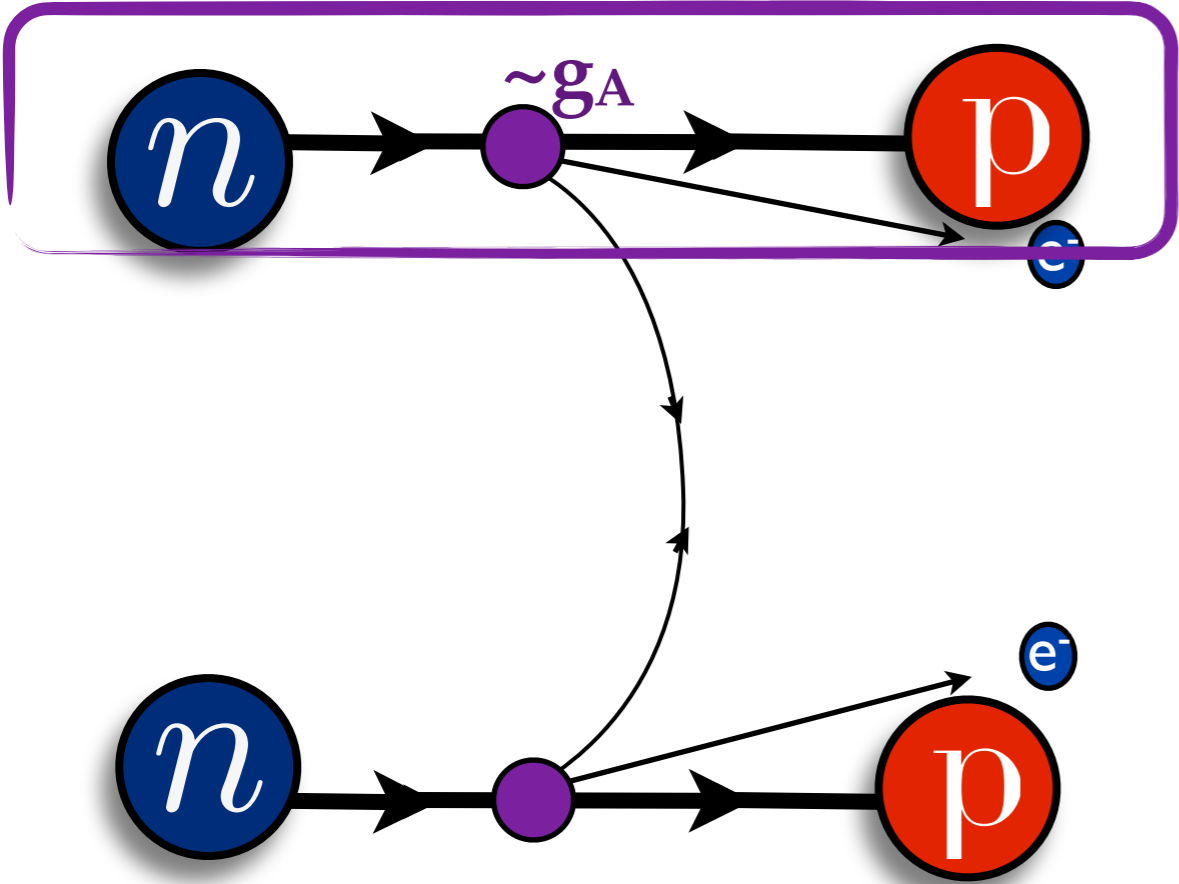
$\Lambda \ll \Lambda_{\text{QCD}}$



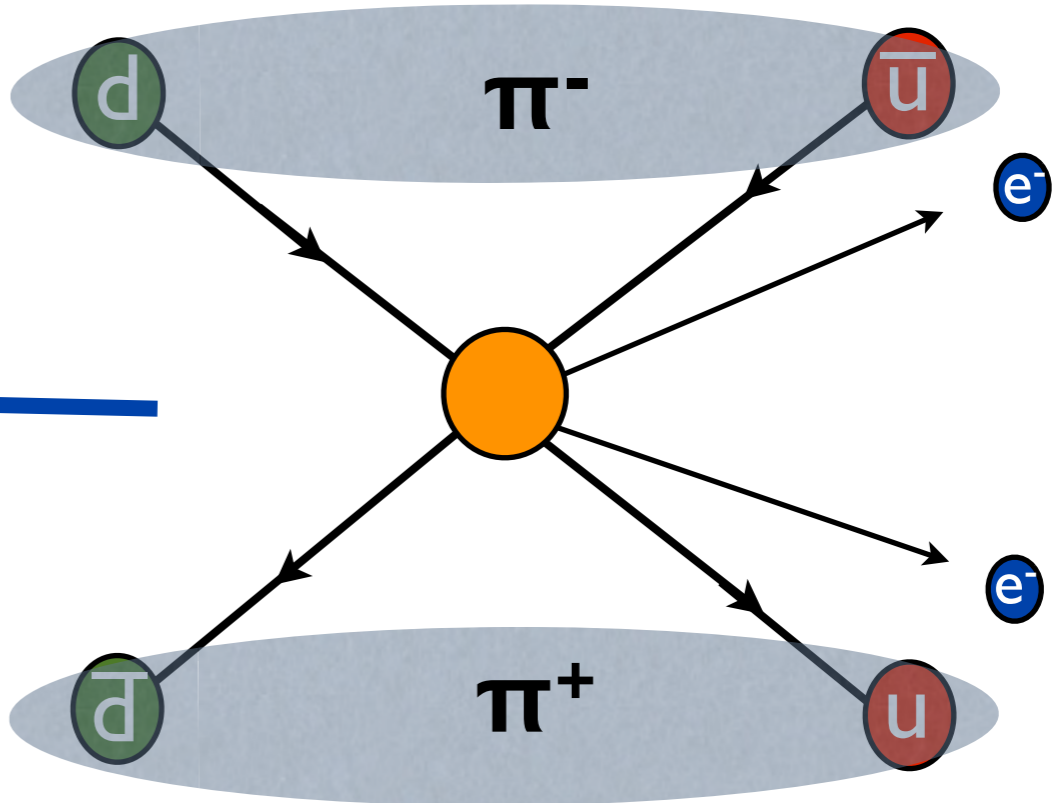
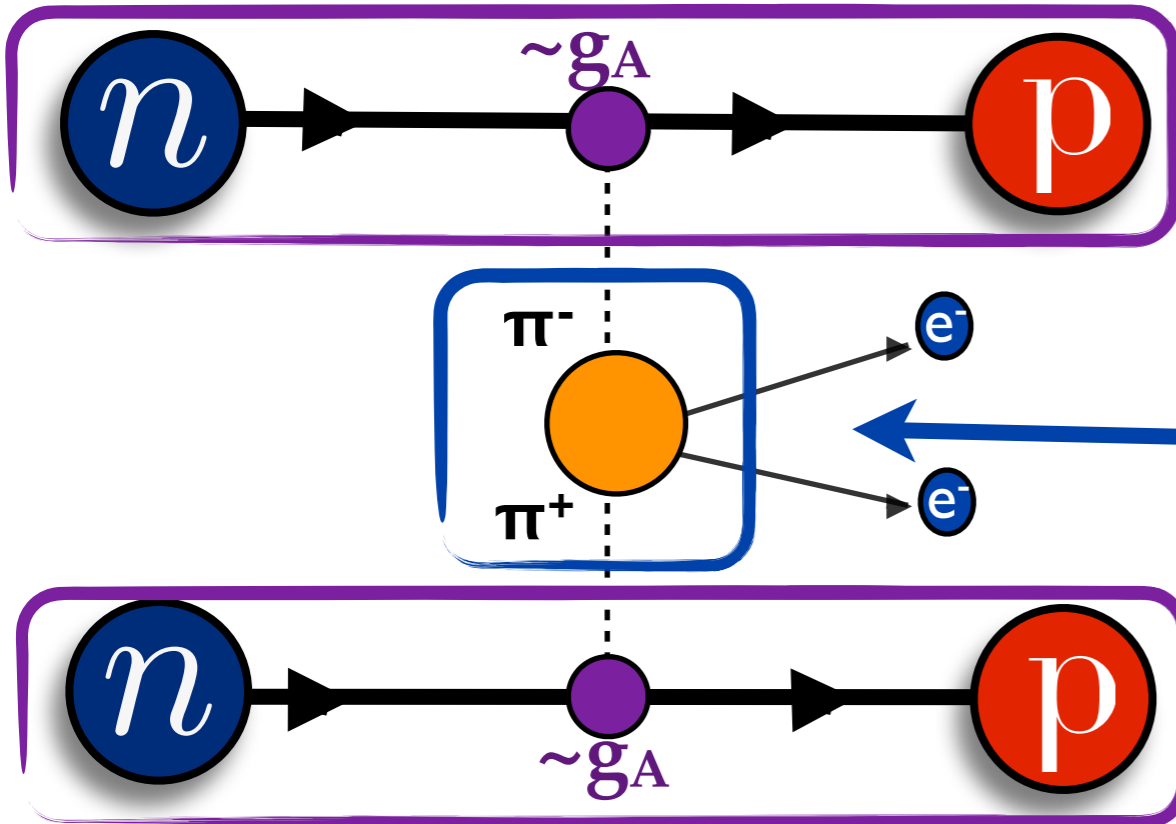
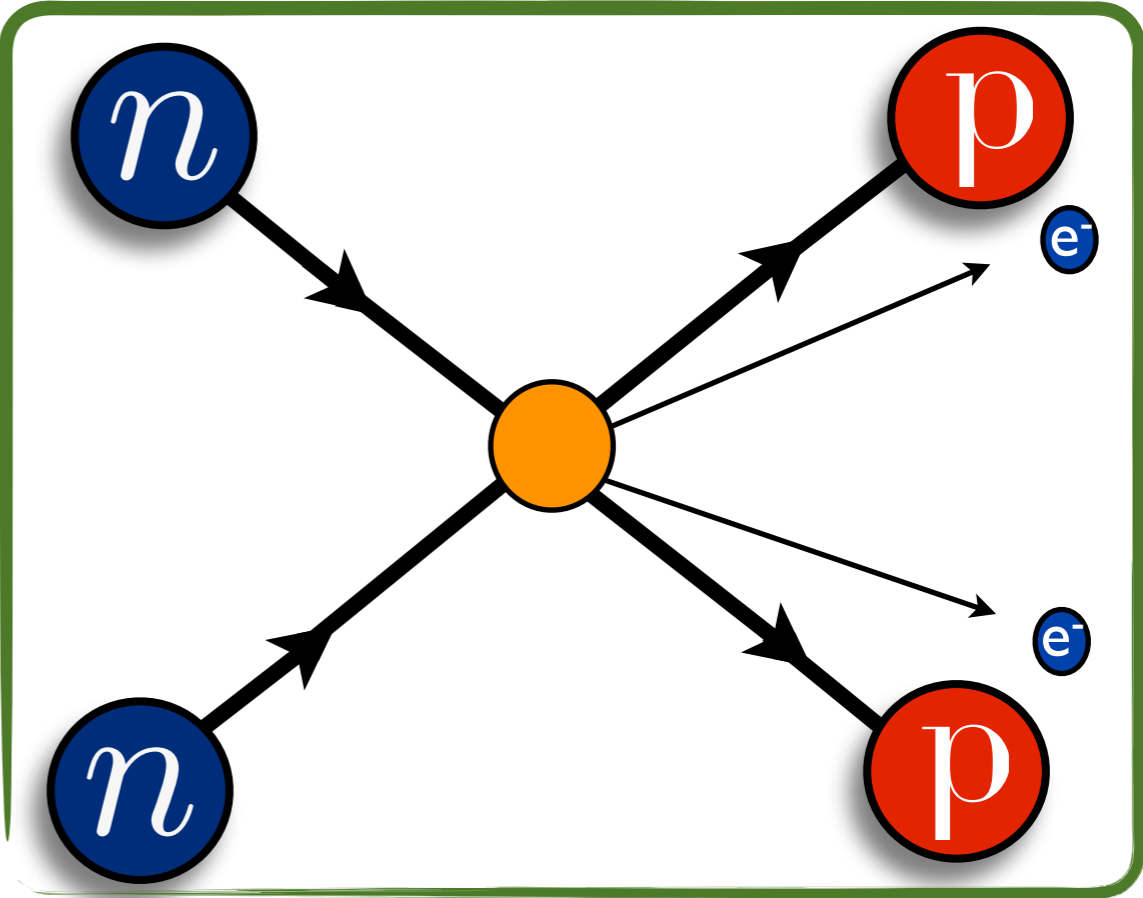
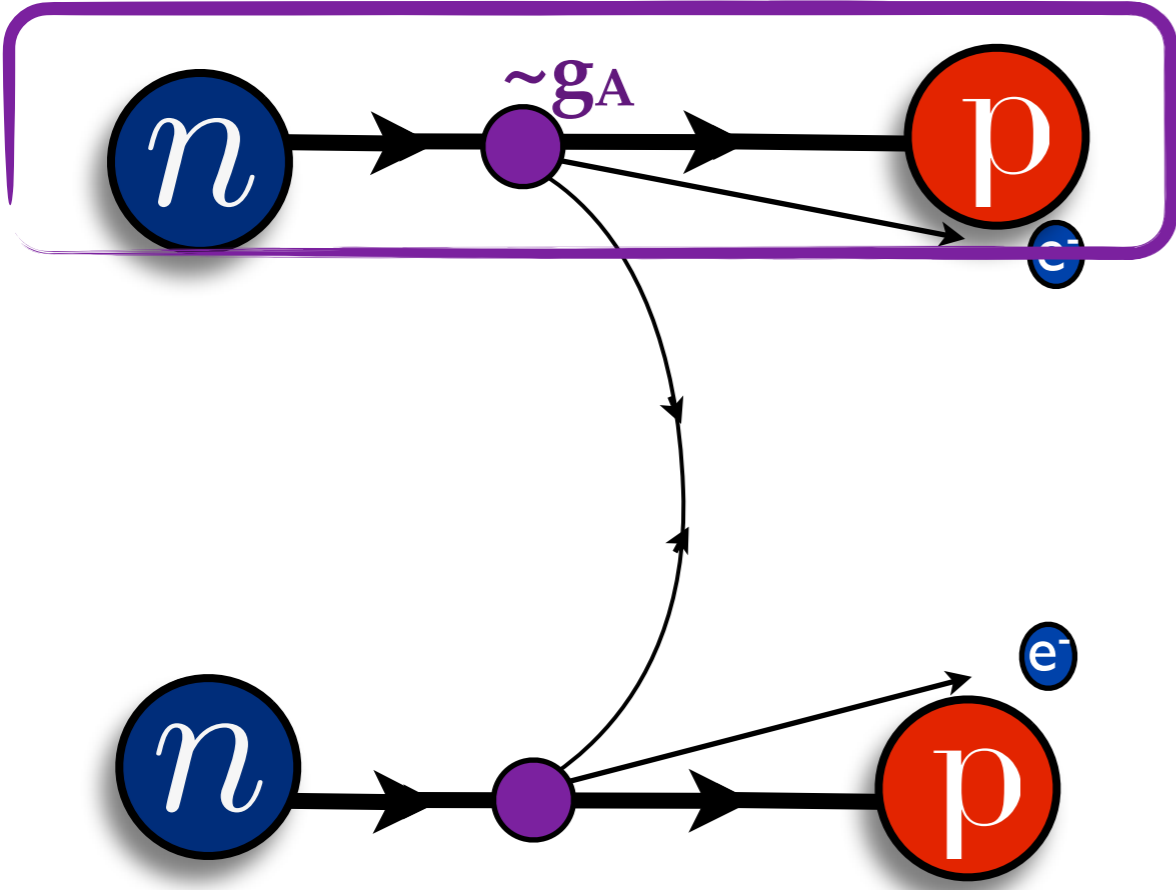


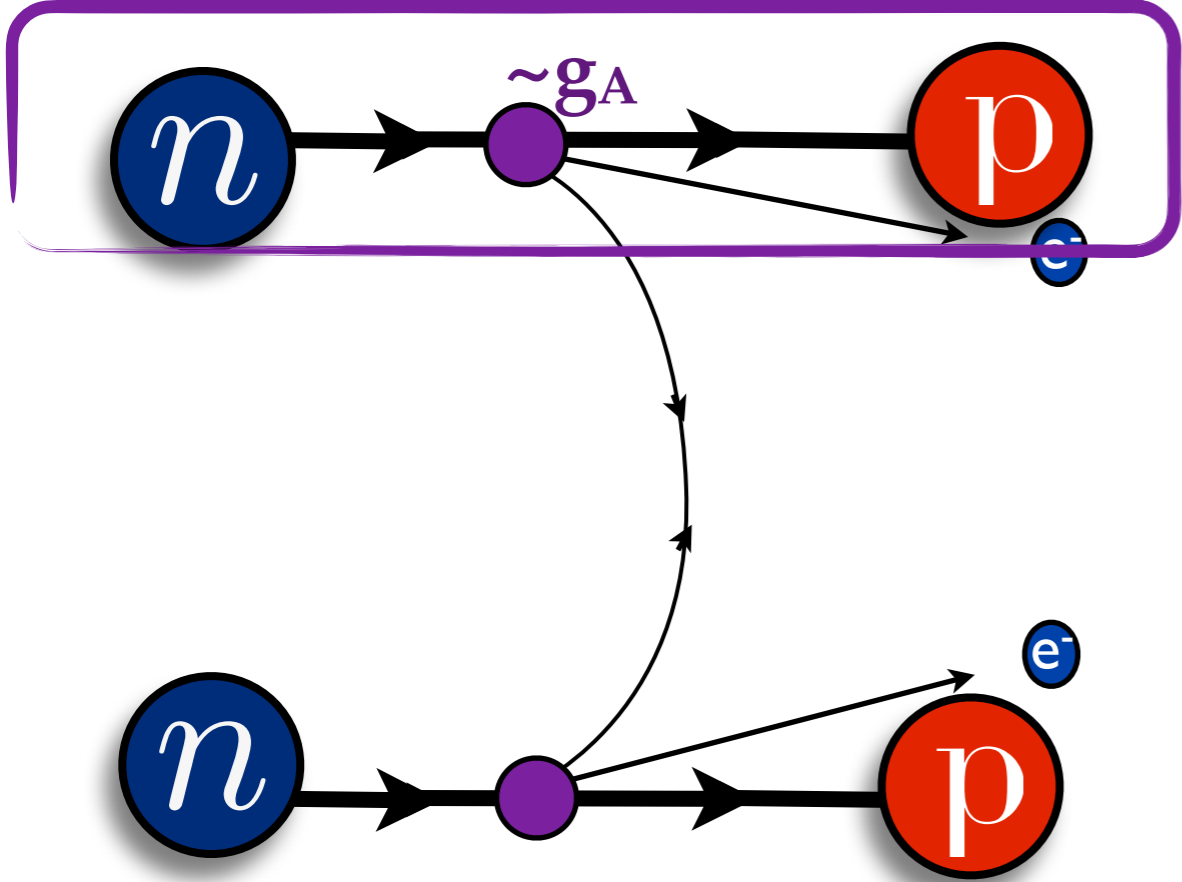


Prezeau, Ramsey-Musolf, Vogel (2003)

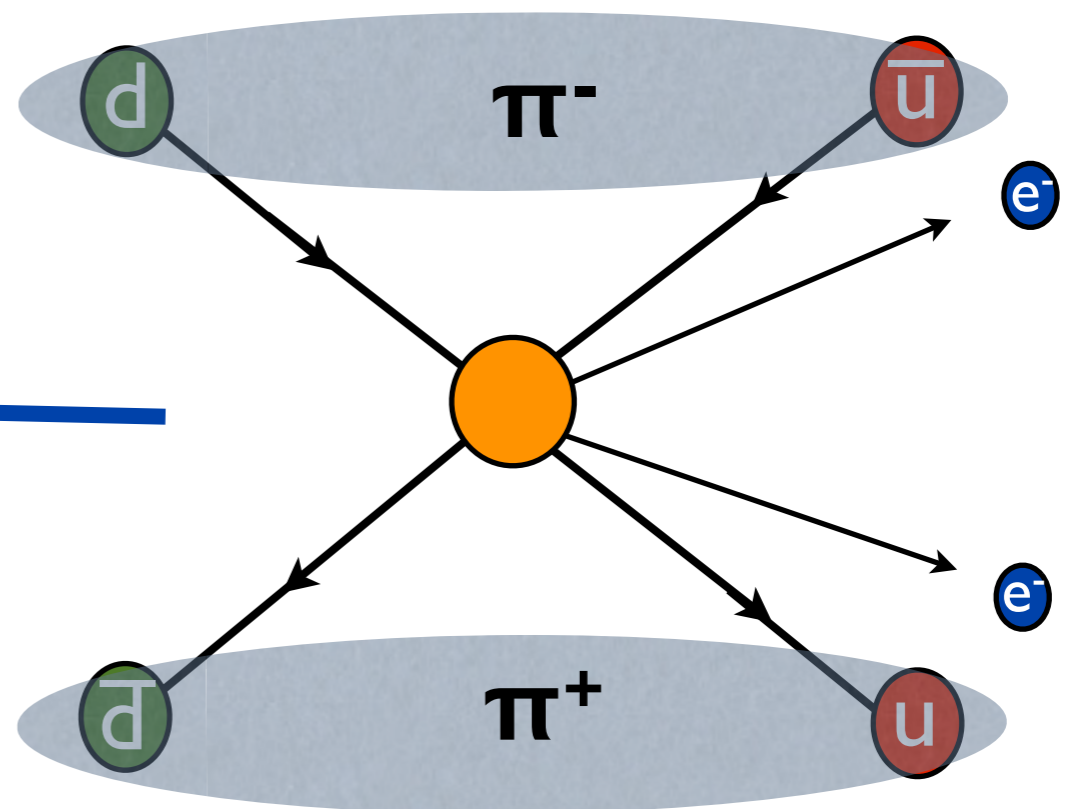


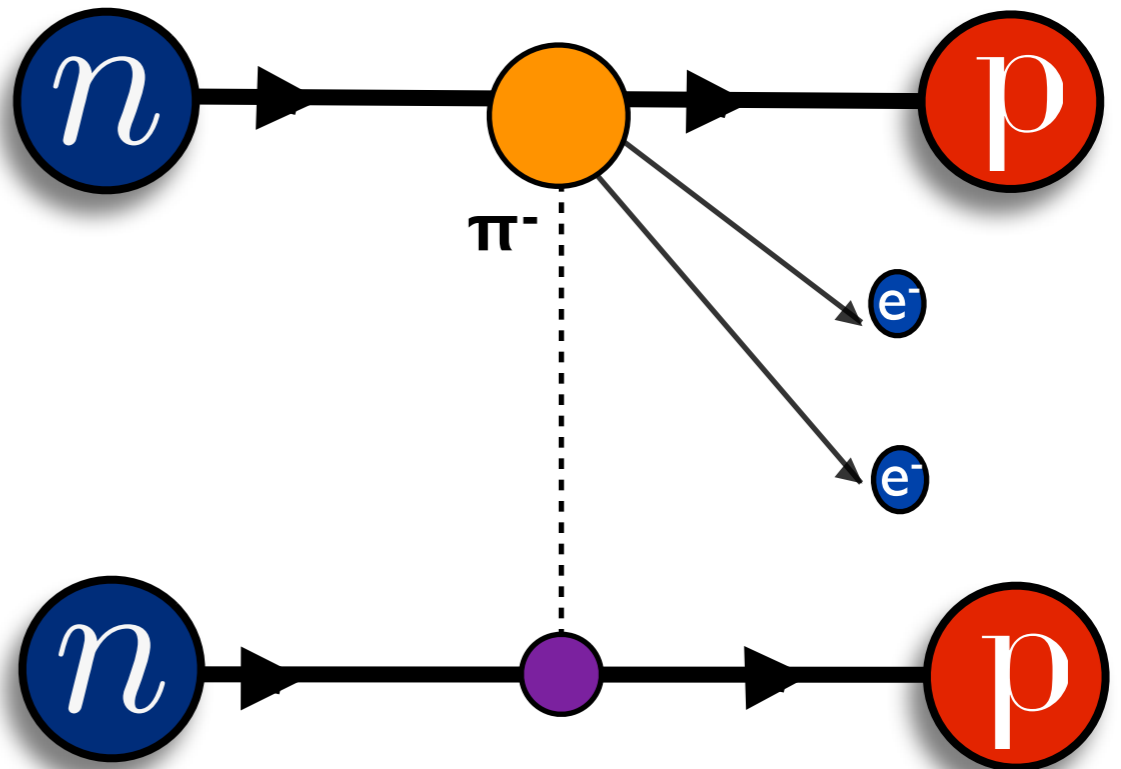
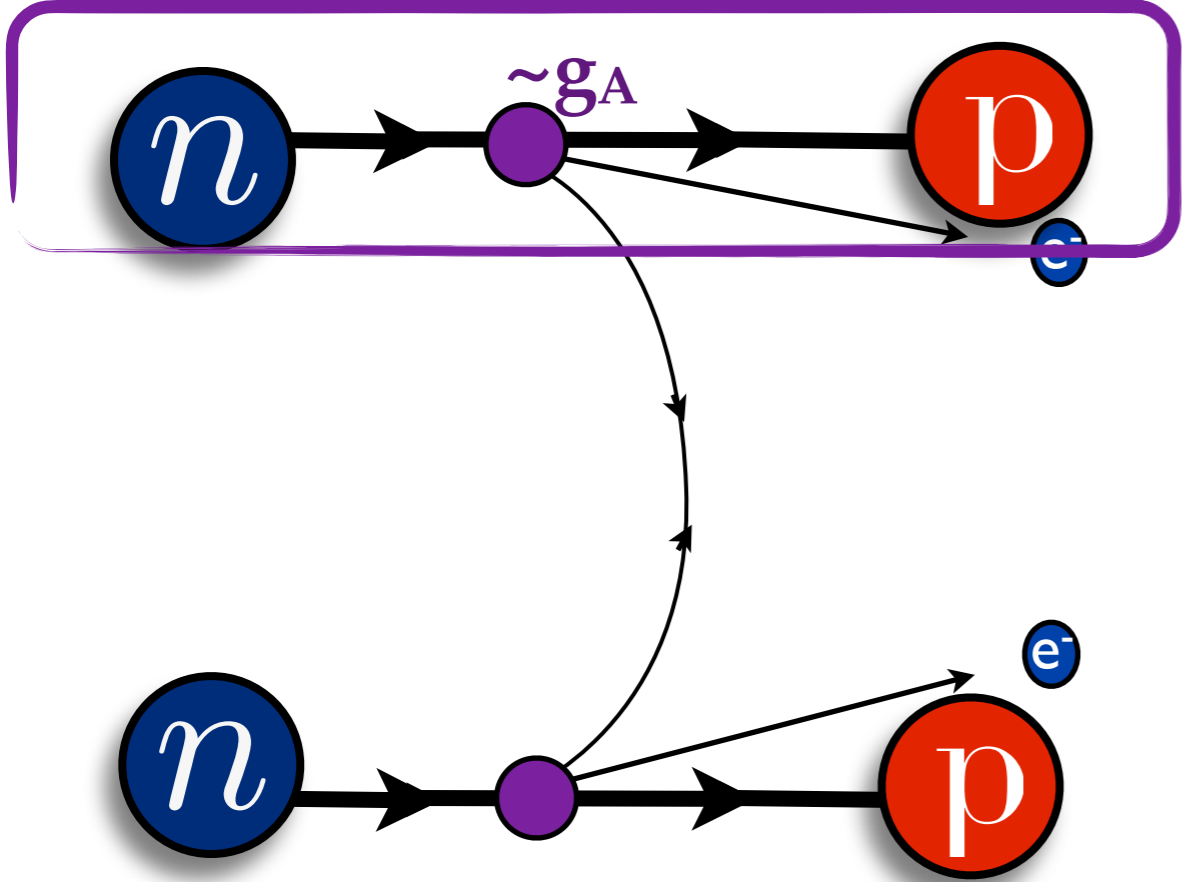
Prezeau, Ramsey-Musolf, Vogel (2003)



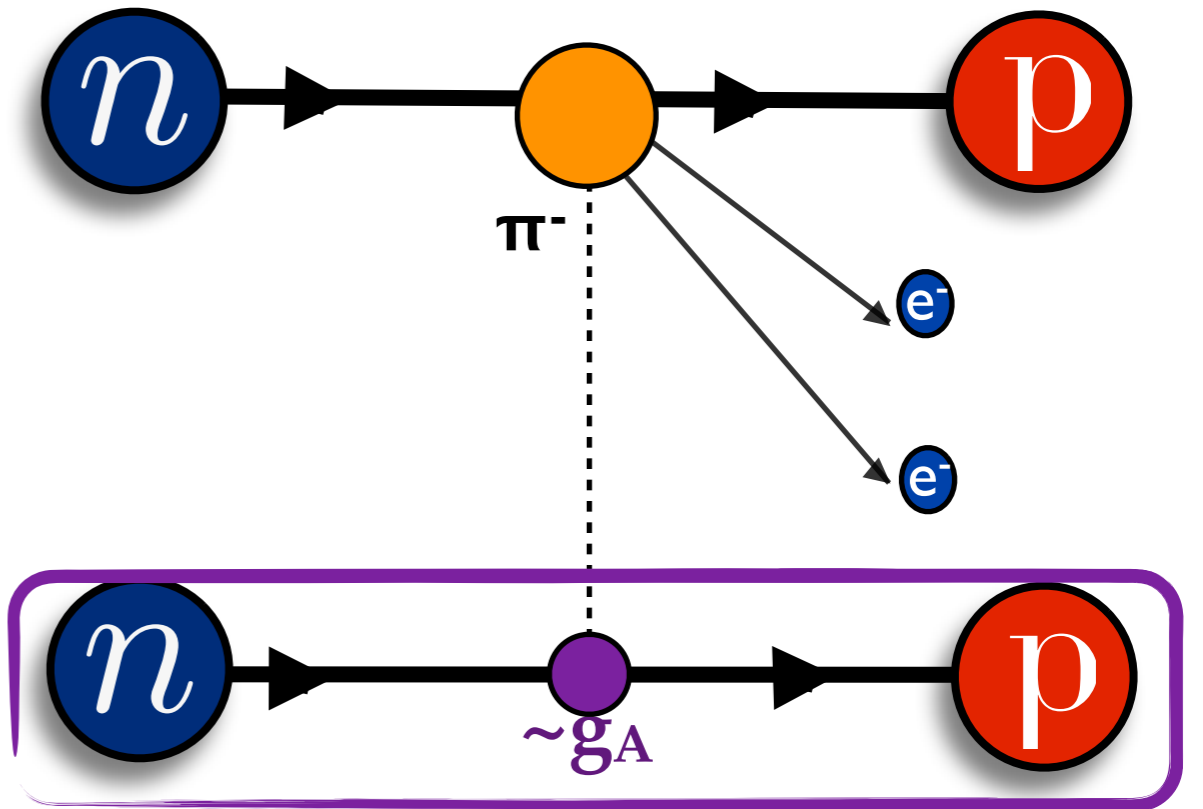
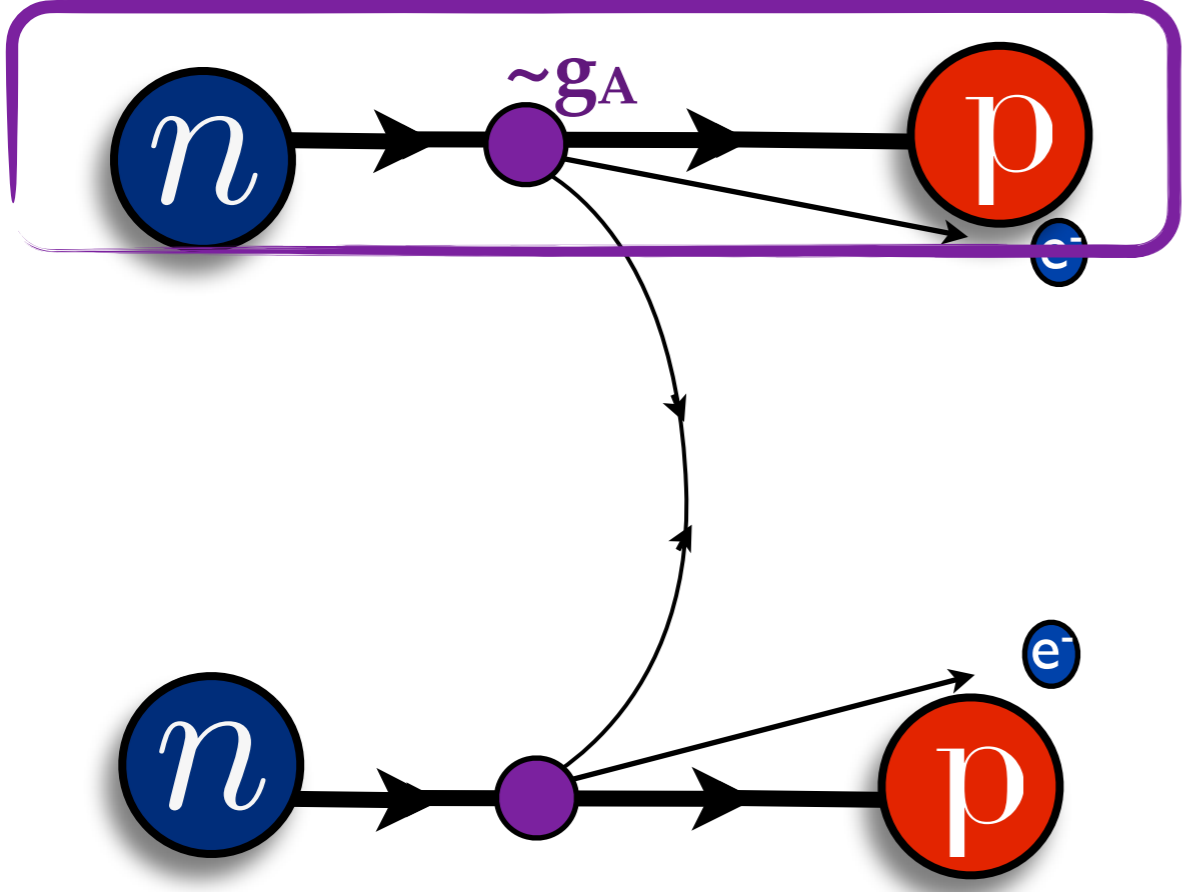


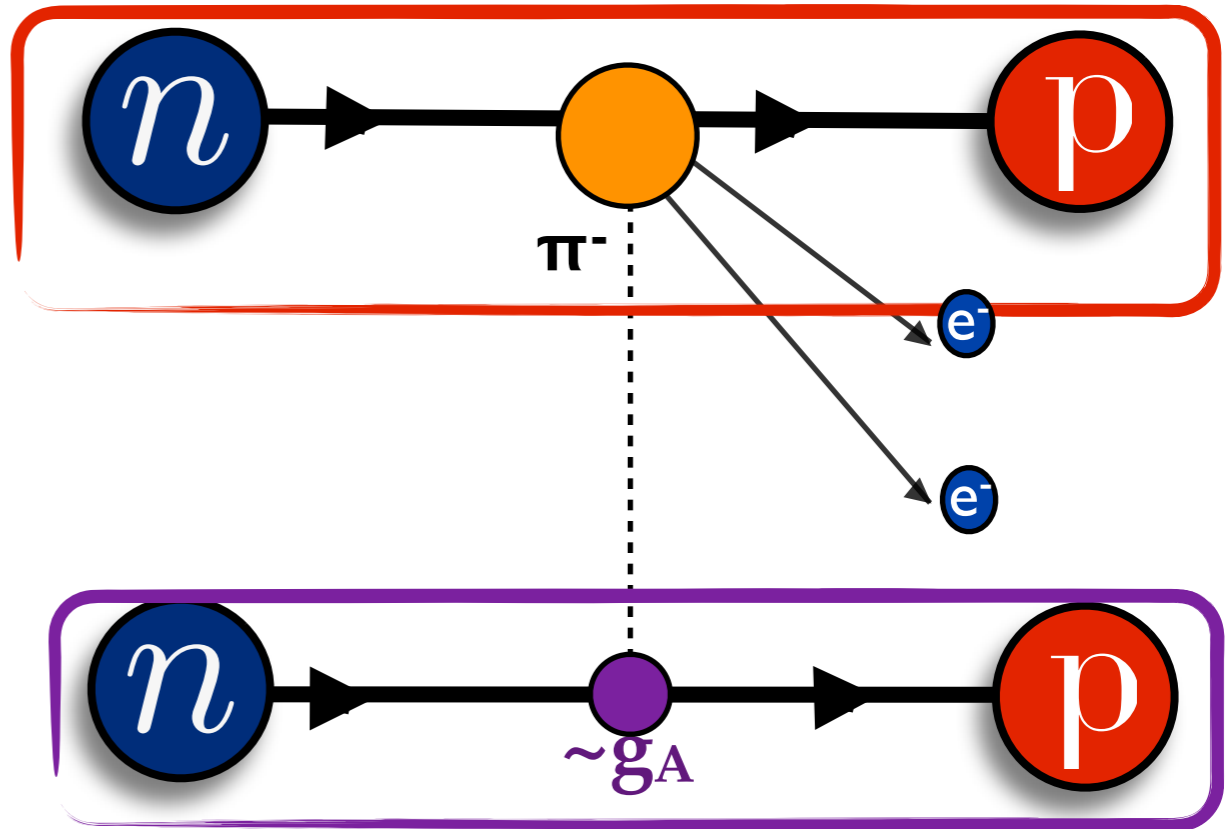
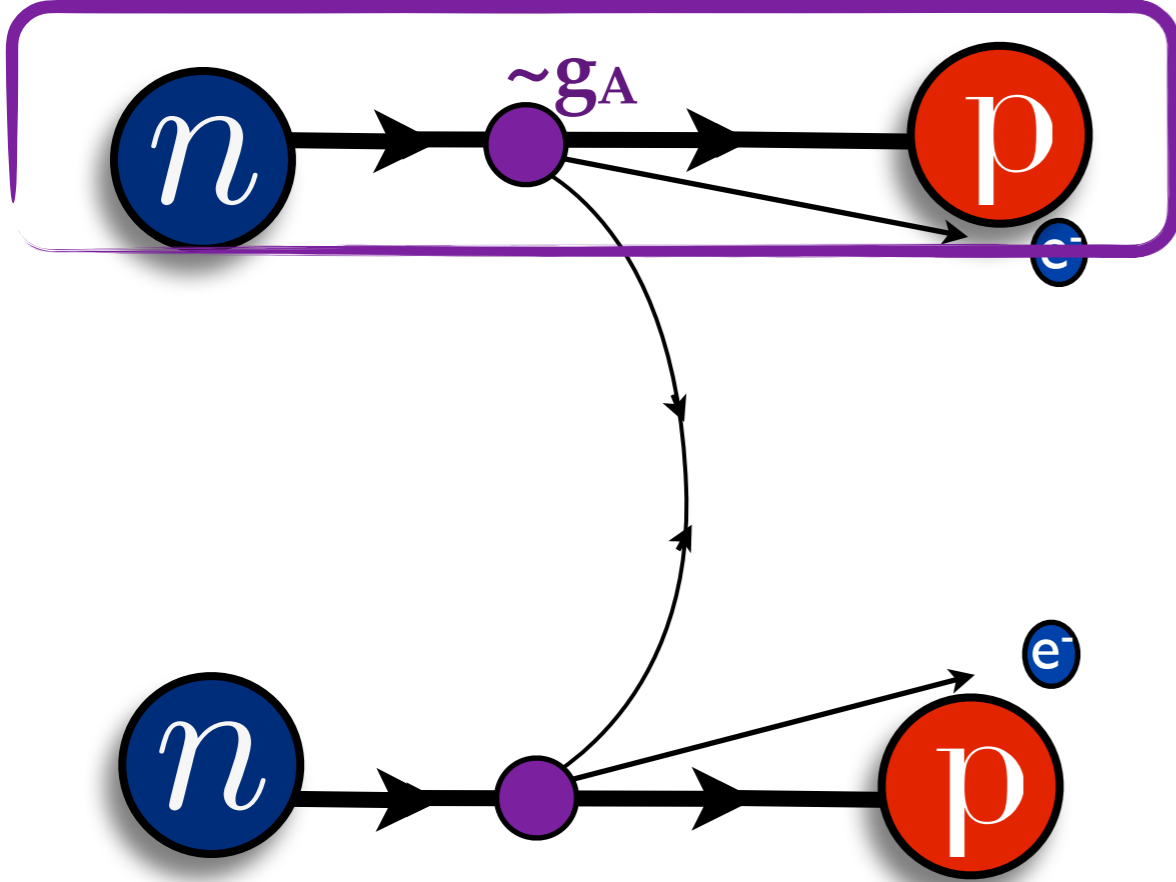
LEADING ORDER  
SHORT-RANGE

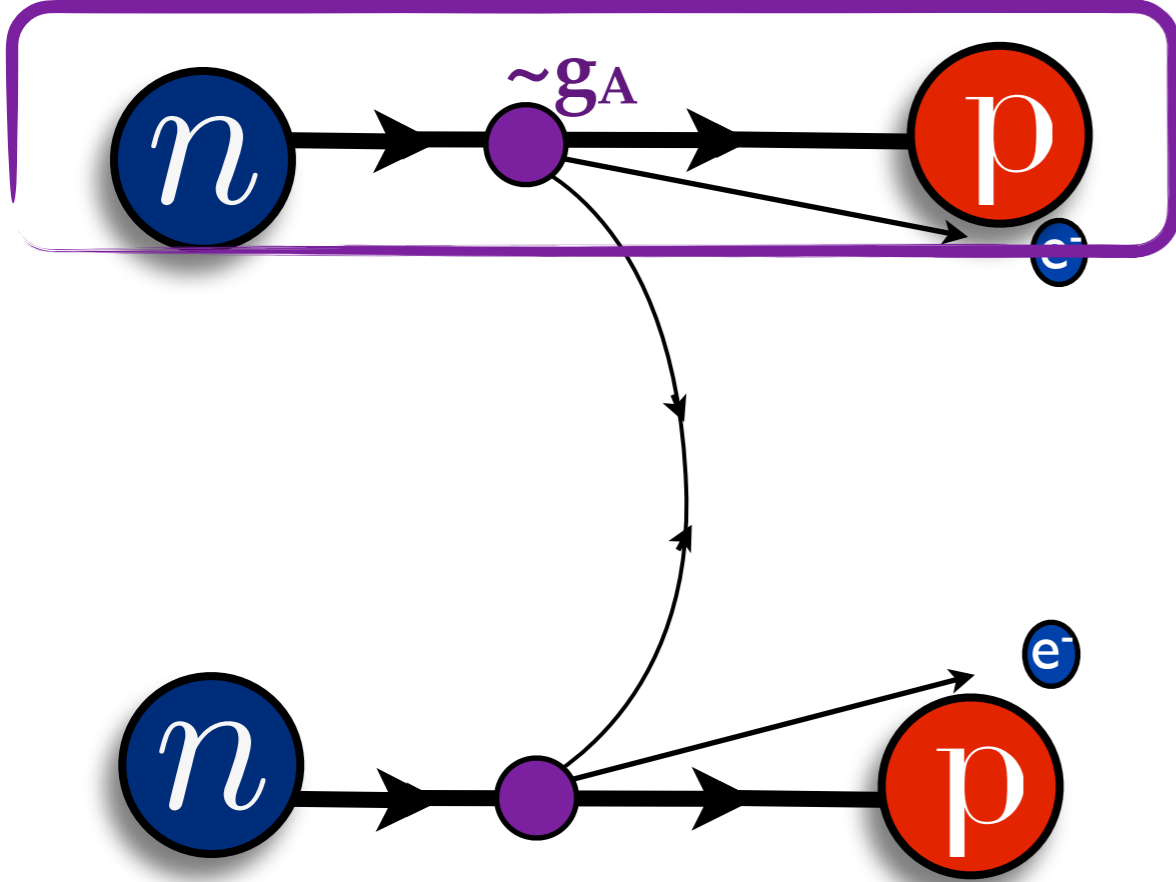


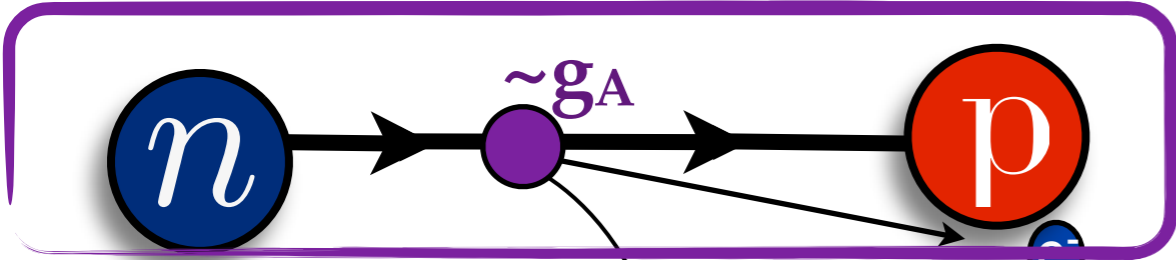




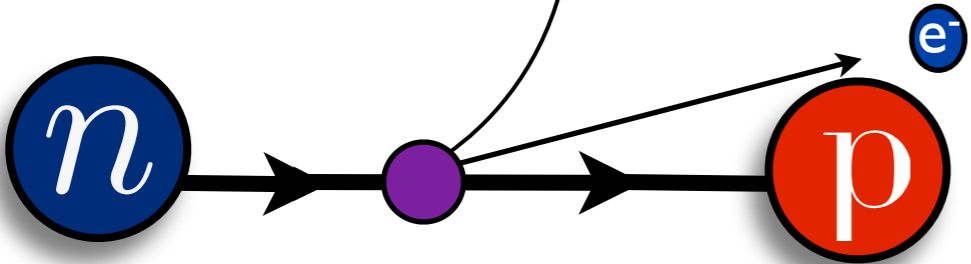




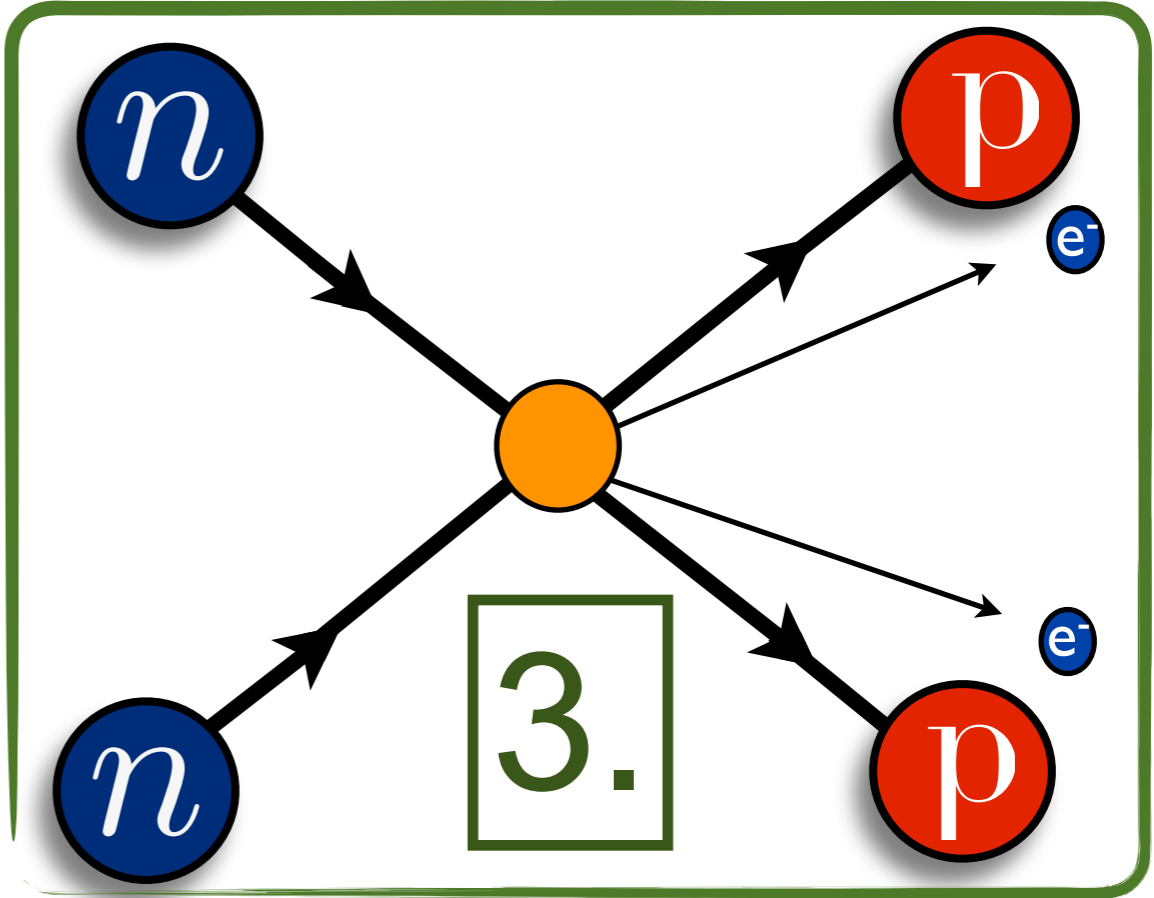
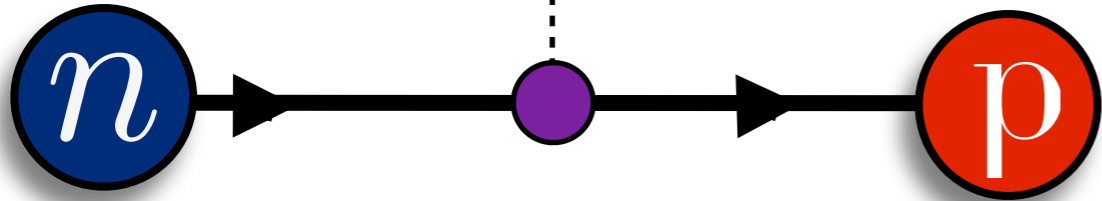
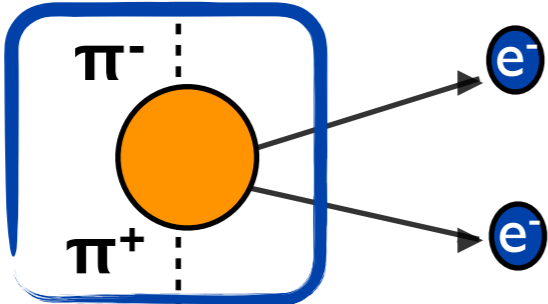




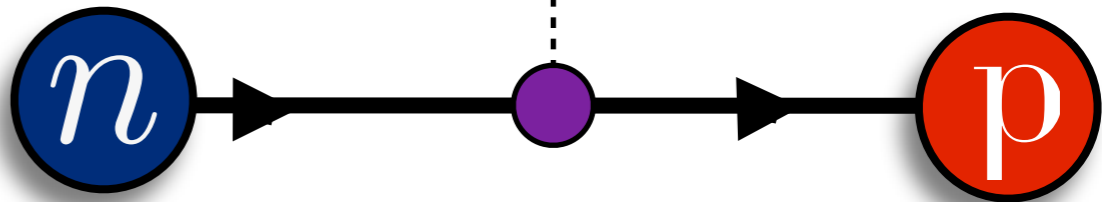
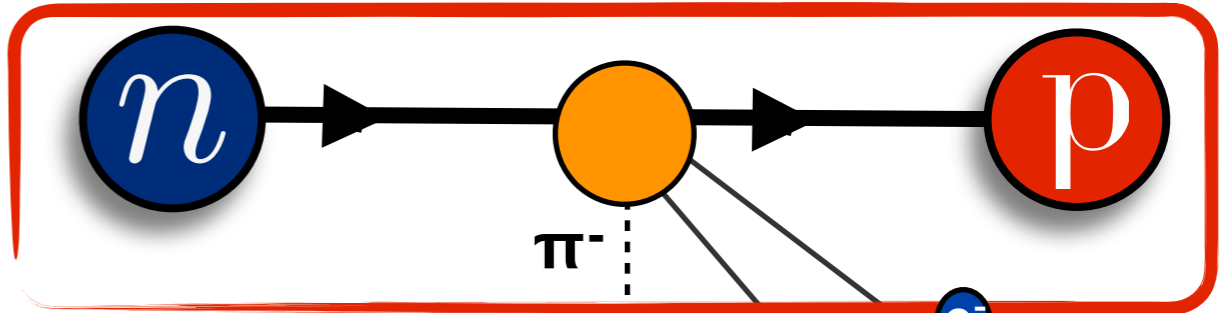
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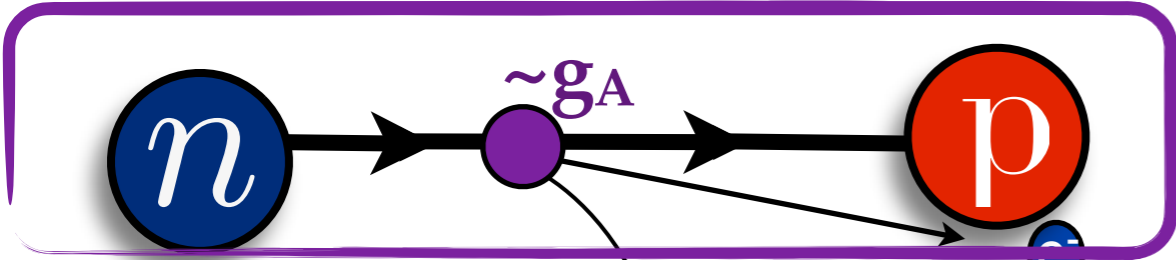


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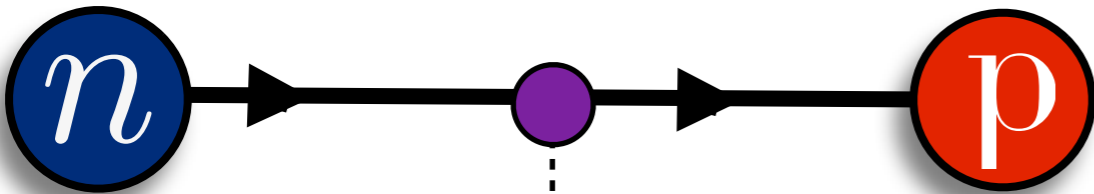
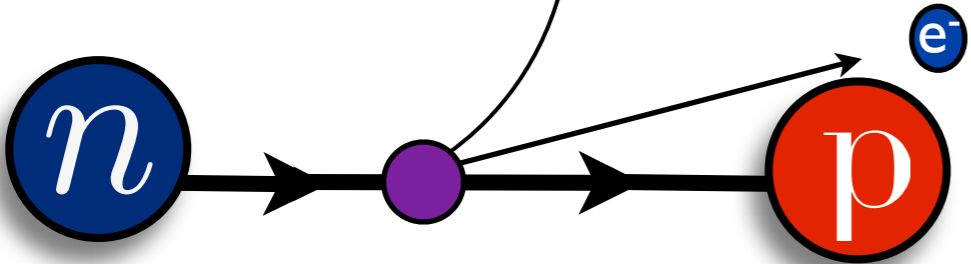


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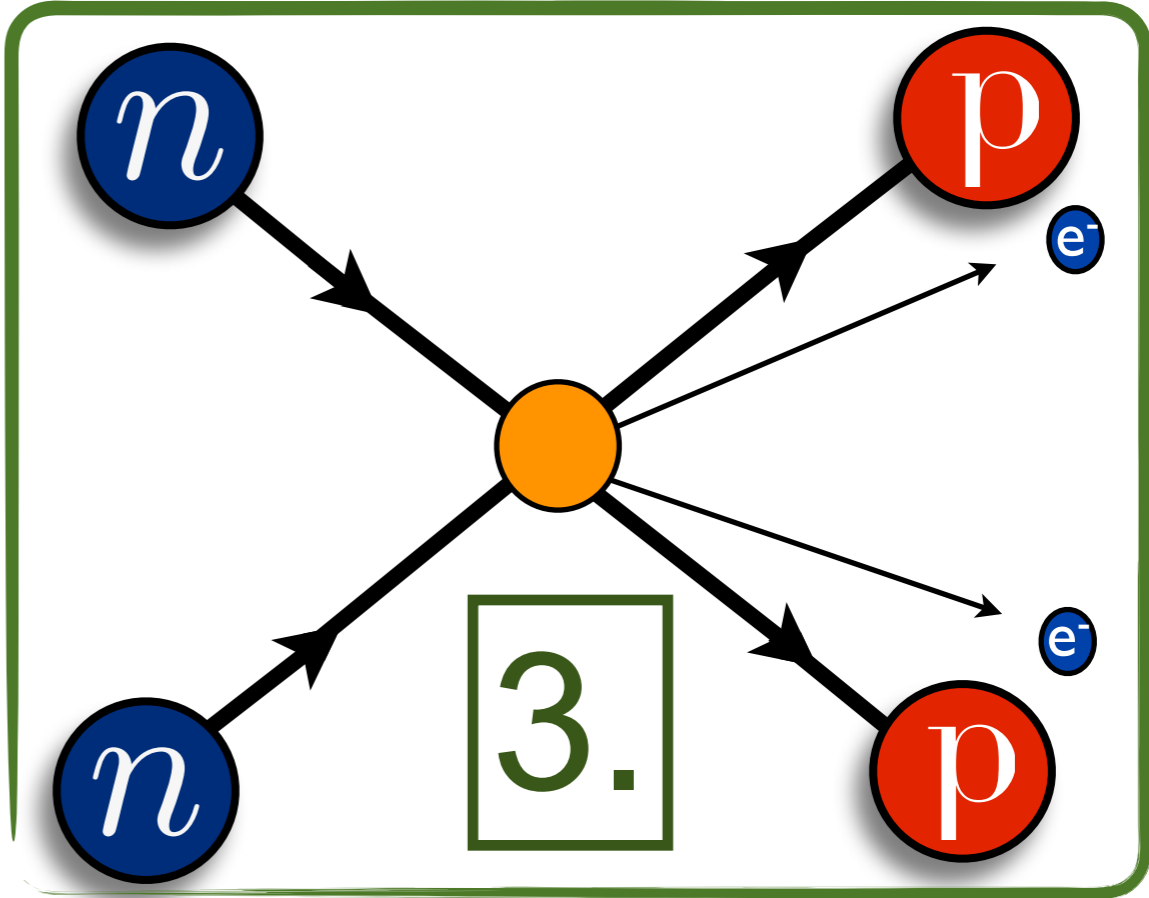
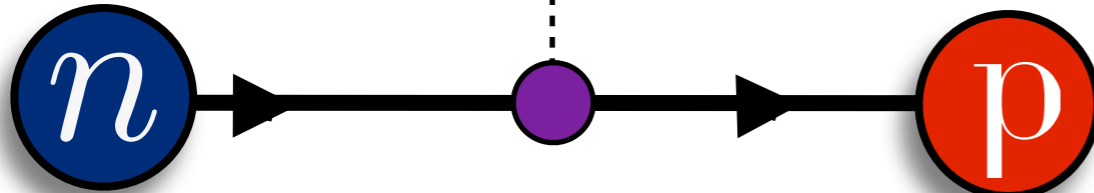
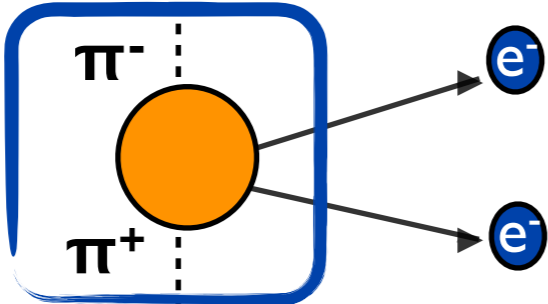




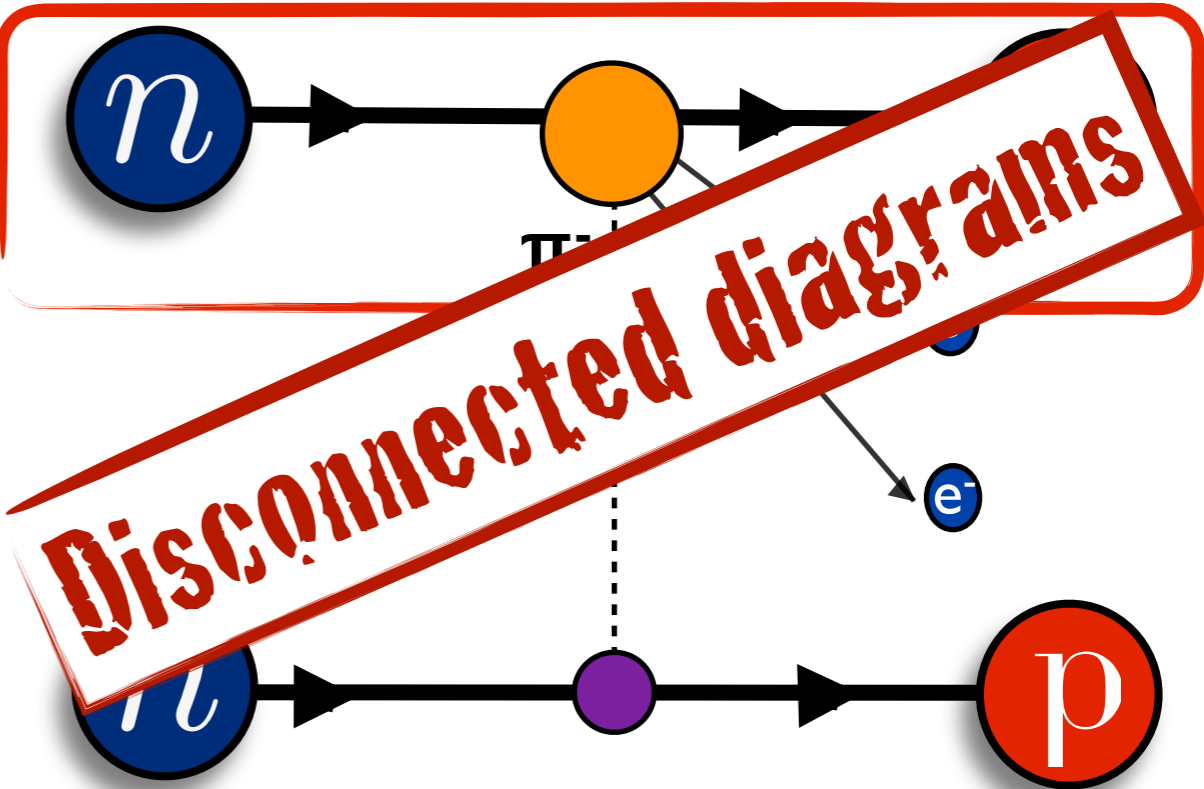
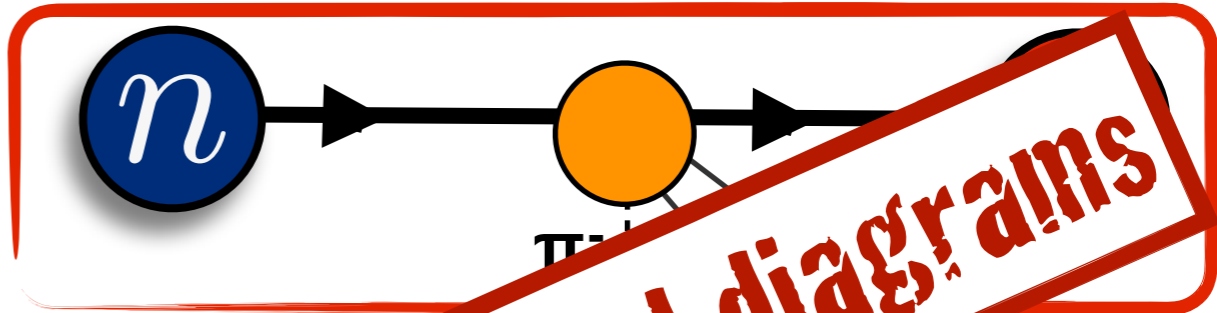
1.



2.



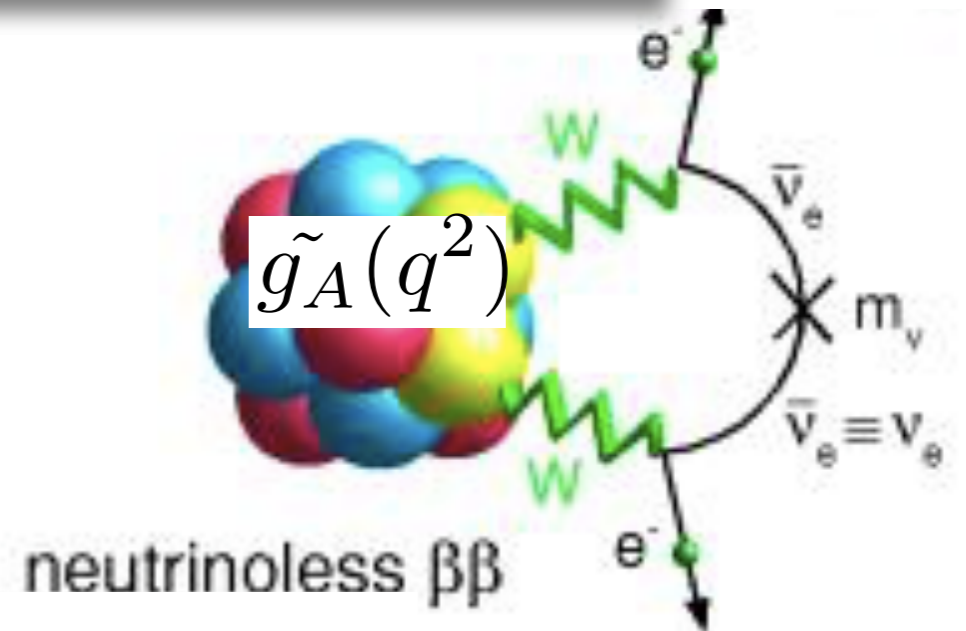
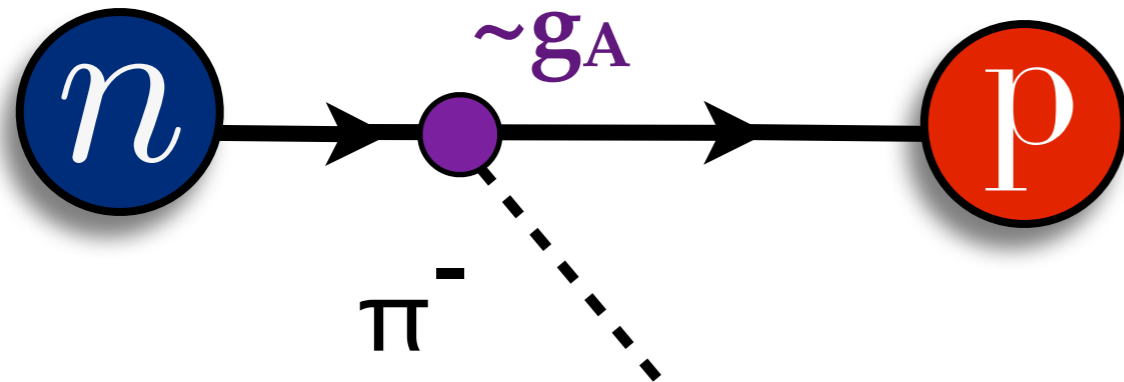
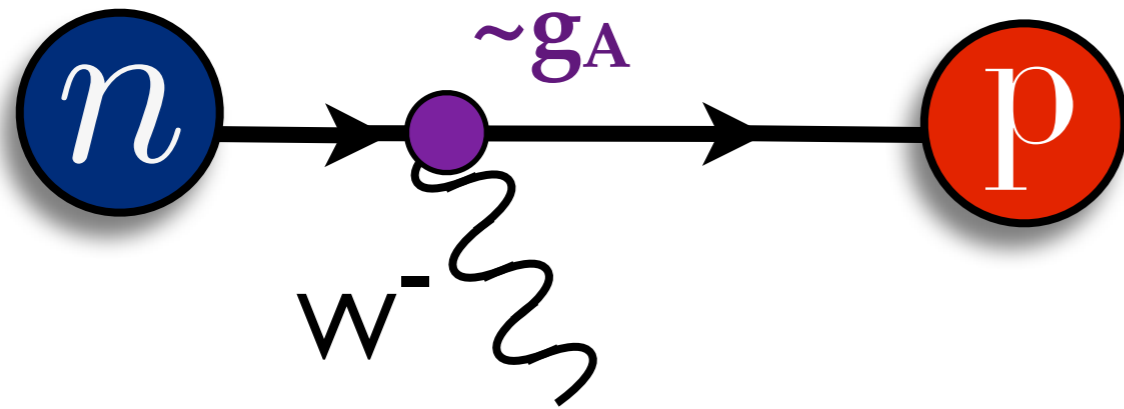
3.



Disconnected diagrams

1.

# Nucleon axial charge, $g_A$



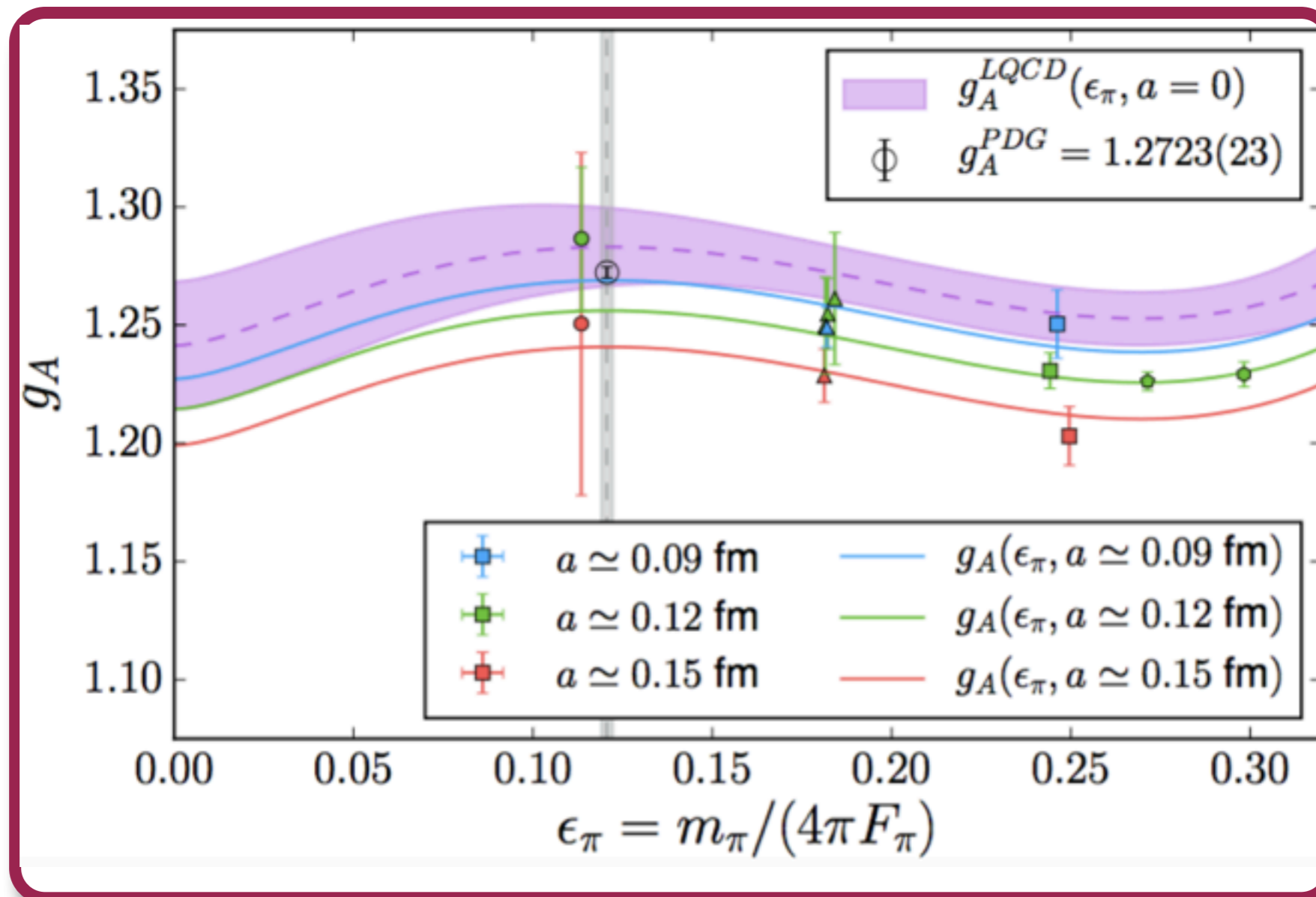
- $g_A^{\text{exp}} = 1.2723(23)$
- Less well known: in-medium modifications to  $g_A$  at relevant momentum scale NPLQCD (2017)
- Notoriously difficult “benchmark” for nuclear physics from LQCD!

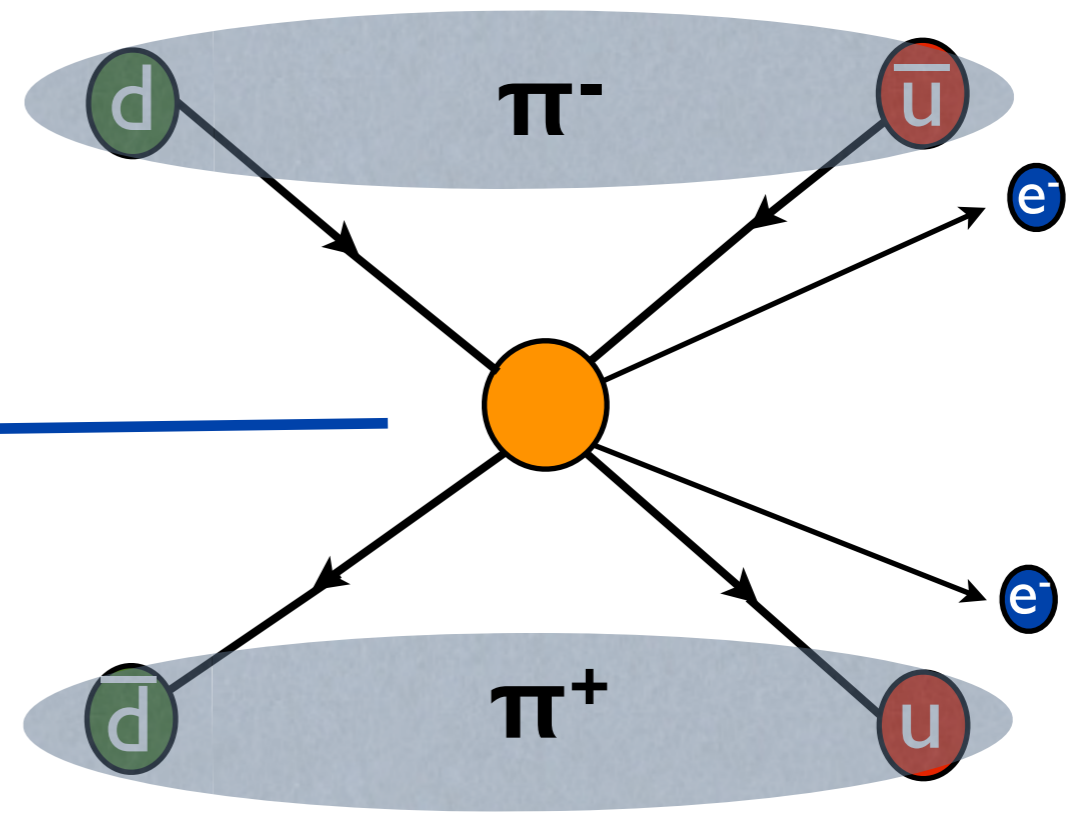
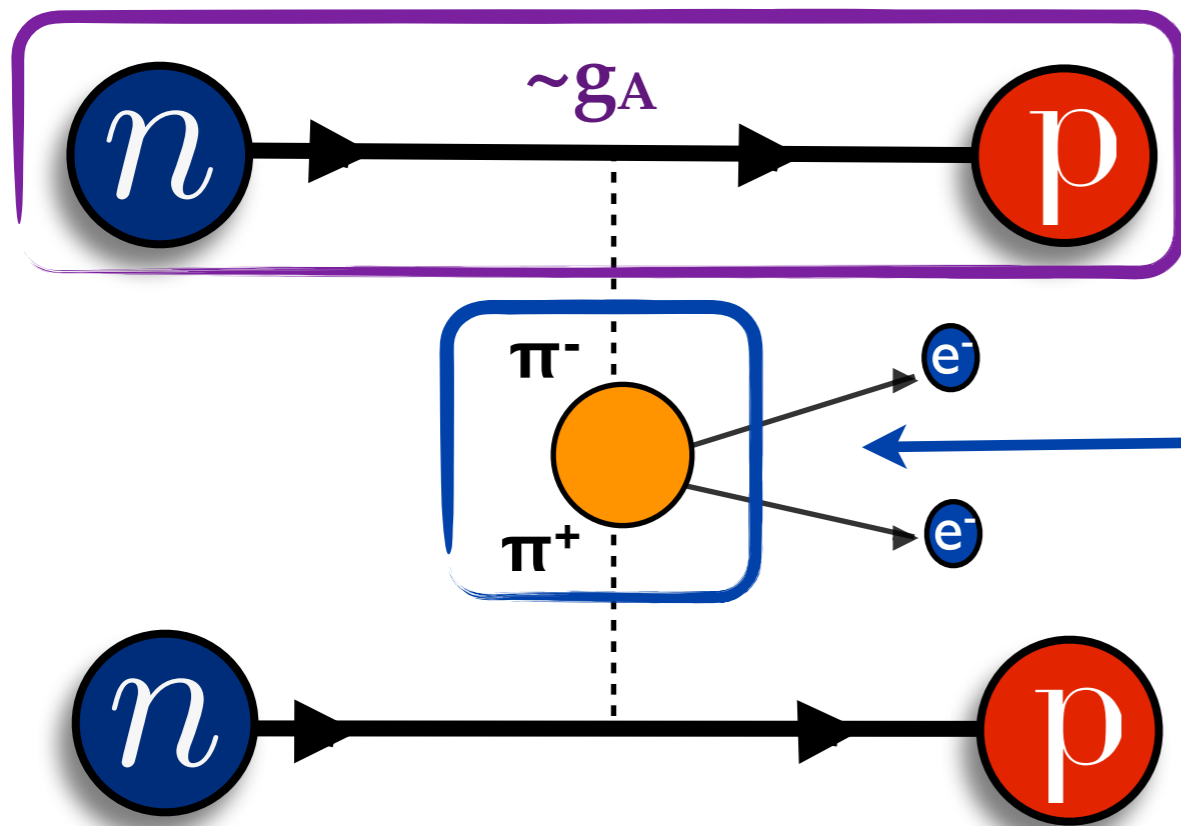
# Nucleon axial charge, $g_A$

arXiv:1704.01114  
 $g_A^{\text{LQCD}} = 1.283 \pm 0.017$



+ C. Bouchard, C. Monahan,  
K. Orginos





2.

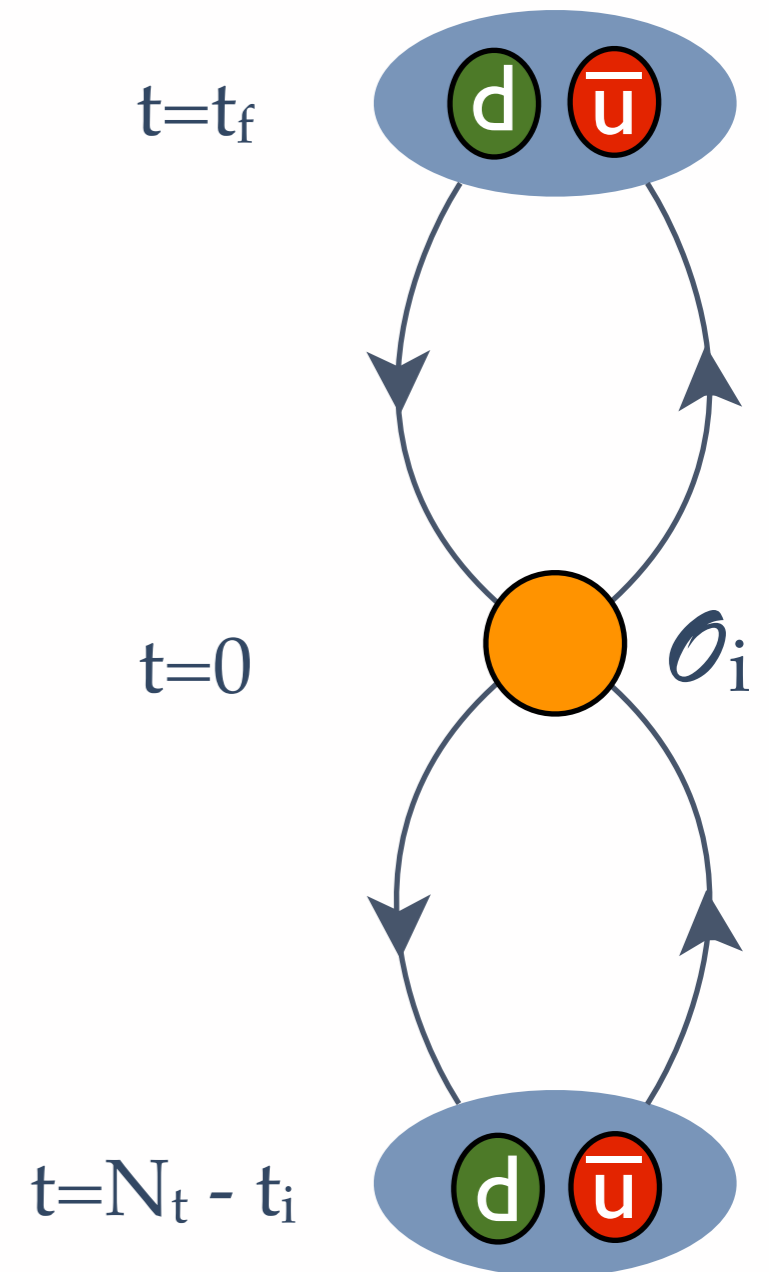
$\pi^- \rightarrow \pi^+$  Transition:  
no direct experimental input



# Long-range pion calculation

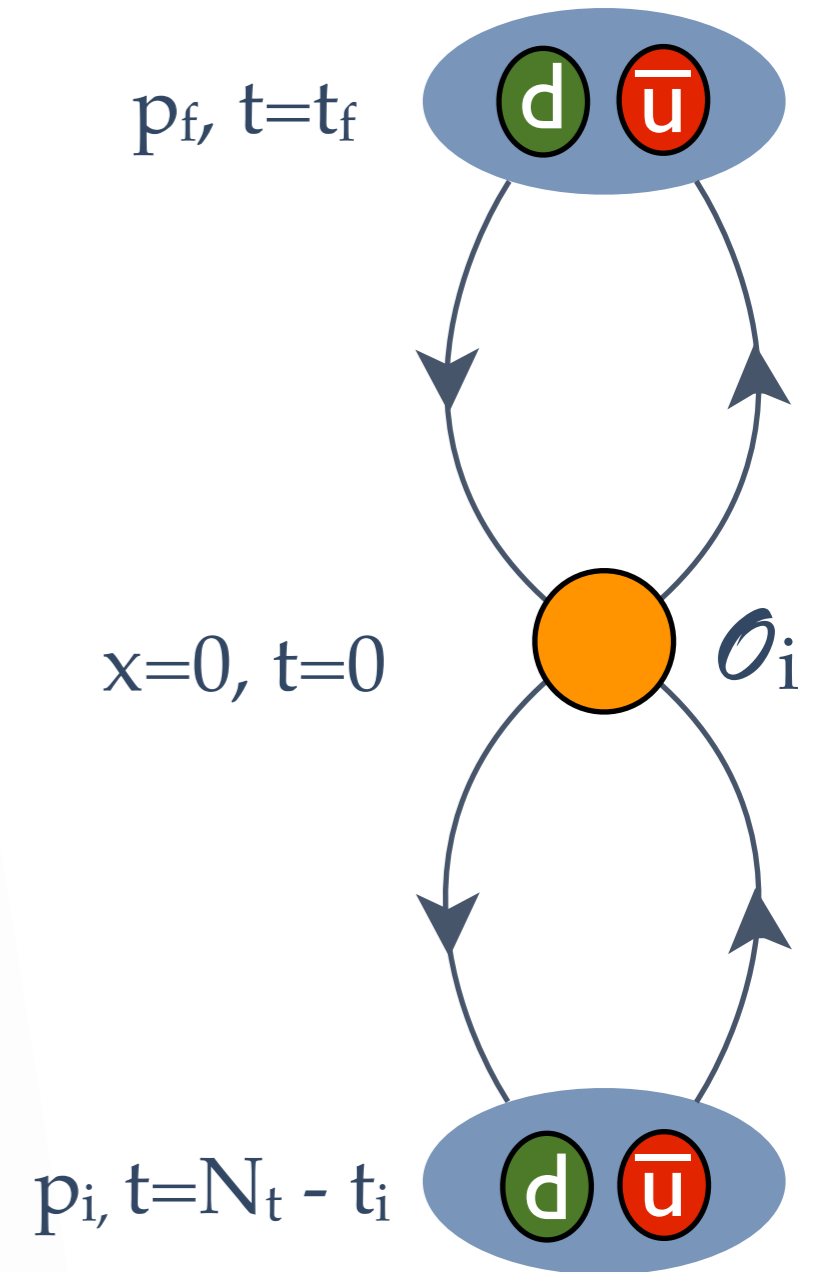
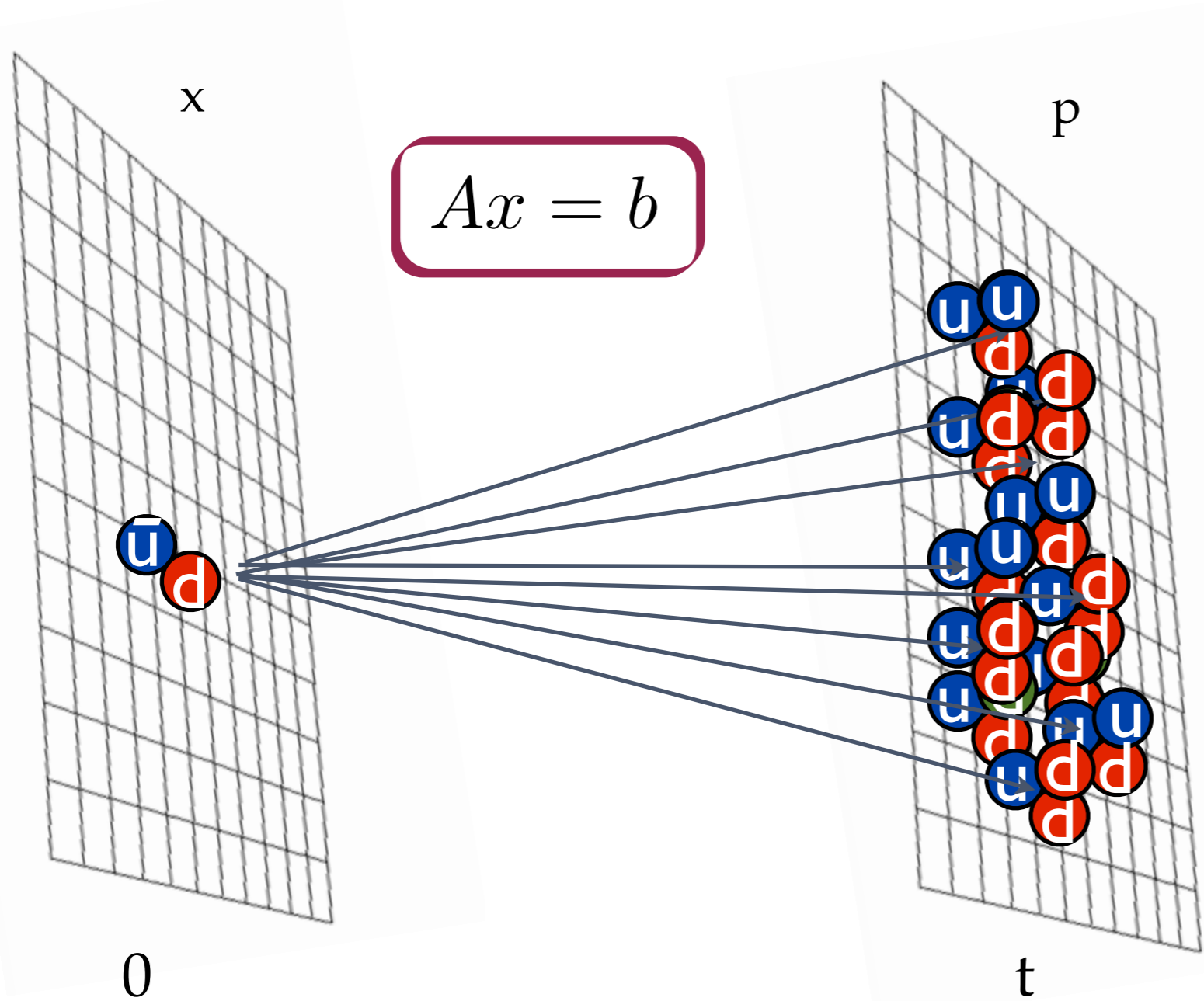
- Evolution in Euclidean time leads to exponential damping of excited states
- Easy to compute pion physics on the lattice!
  - Clean signals
  - Single particle

$$\begin{aligned}\langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle &= \langle \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) \rangle \\ &= \sum_n |\langle 0 | \mathcal{O} | n \rangle|^2 e^{-E_n t} \xrightarrow{t \rightarrow \infty} \langle 0 | \mathcal{O} | 0 \rangle e^{-E_0 t}\end{aligned}$$



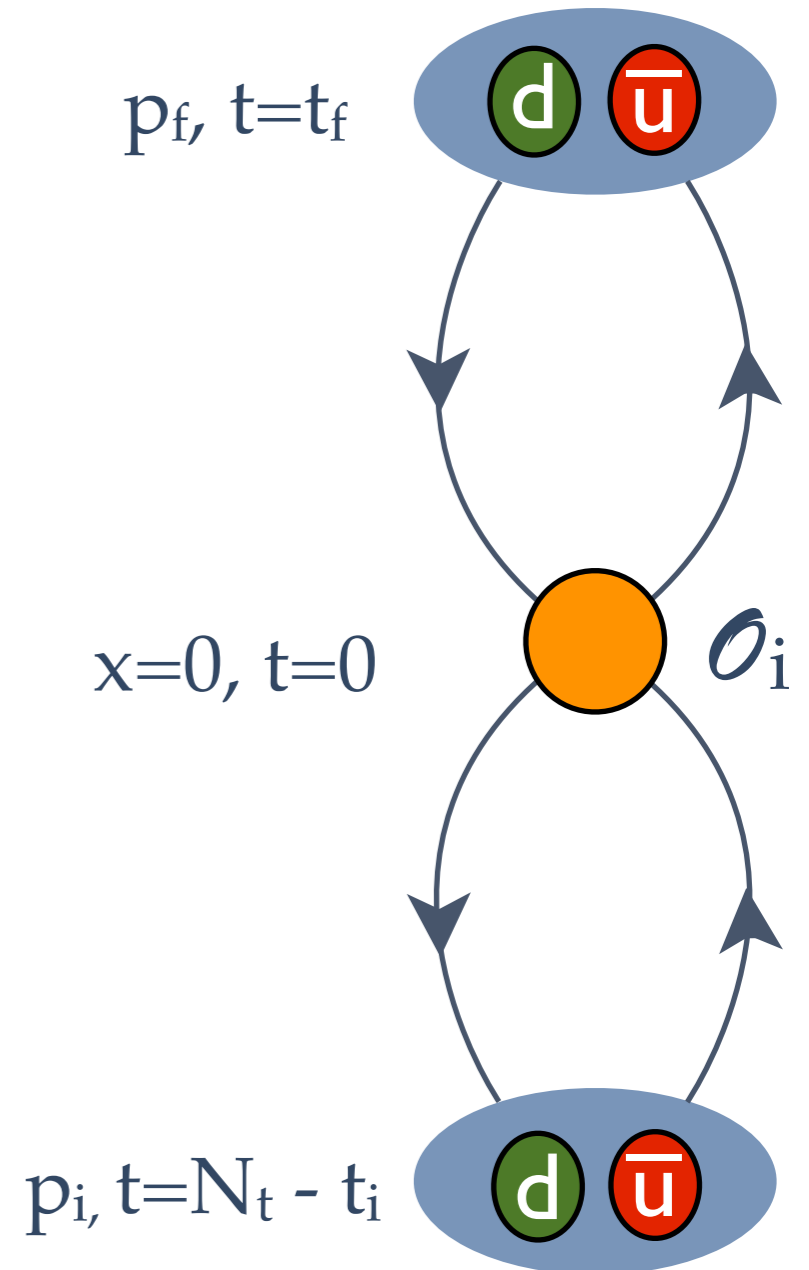
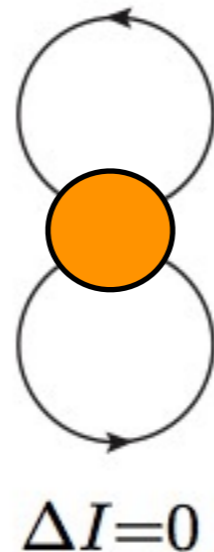
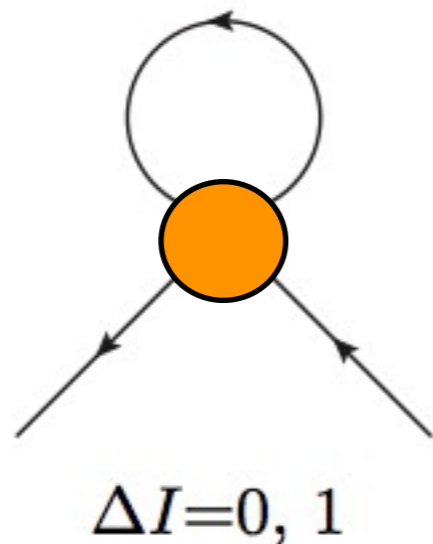
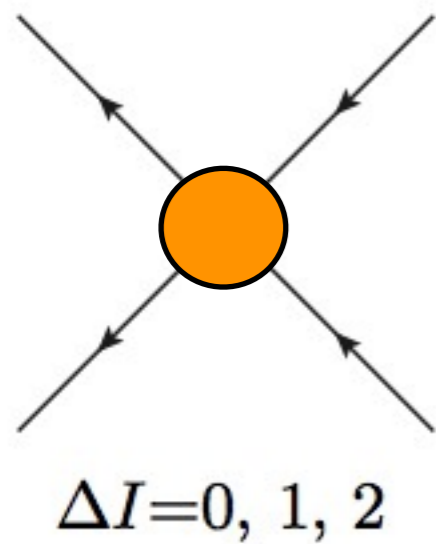
# Long-range pion calculation

- Can perform exact momentum projection at source and sink



# Long-range pion calculation

- Can perform exact momentum projection at source and sink
- $\Delta I = 2$  no disconnected pieces from operators



XPT:

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNNNe e$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

$$\mathcal{O}_{1+}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

$$\mathcal{O}_{2\pm}^{ab} = (\bar{q}_R \tau^a q_L)(\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R)(\bar{q}_L \tau^b q_R),$$

$$\mathcal{O}_{3\pm}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

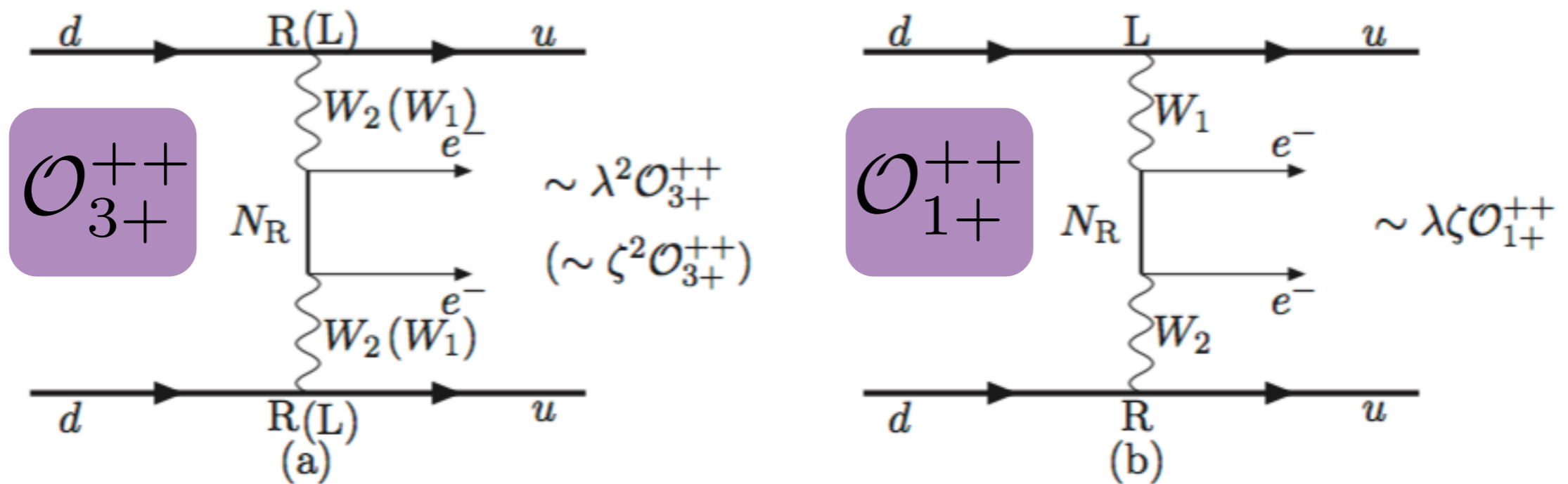
$$\mathcal{O}_{4\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L),$$

$$\mathcal{O}_{5\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L).$$

XPT:

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNNNee$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

### Left-right symmetric models



# Contractions

- QCD interactions can mix colors below the electroweak scale
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

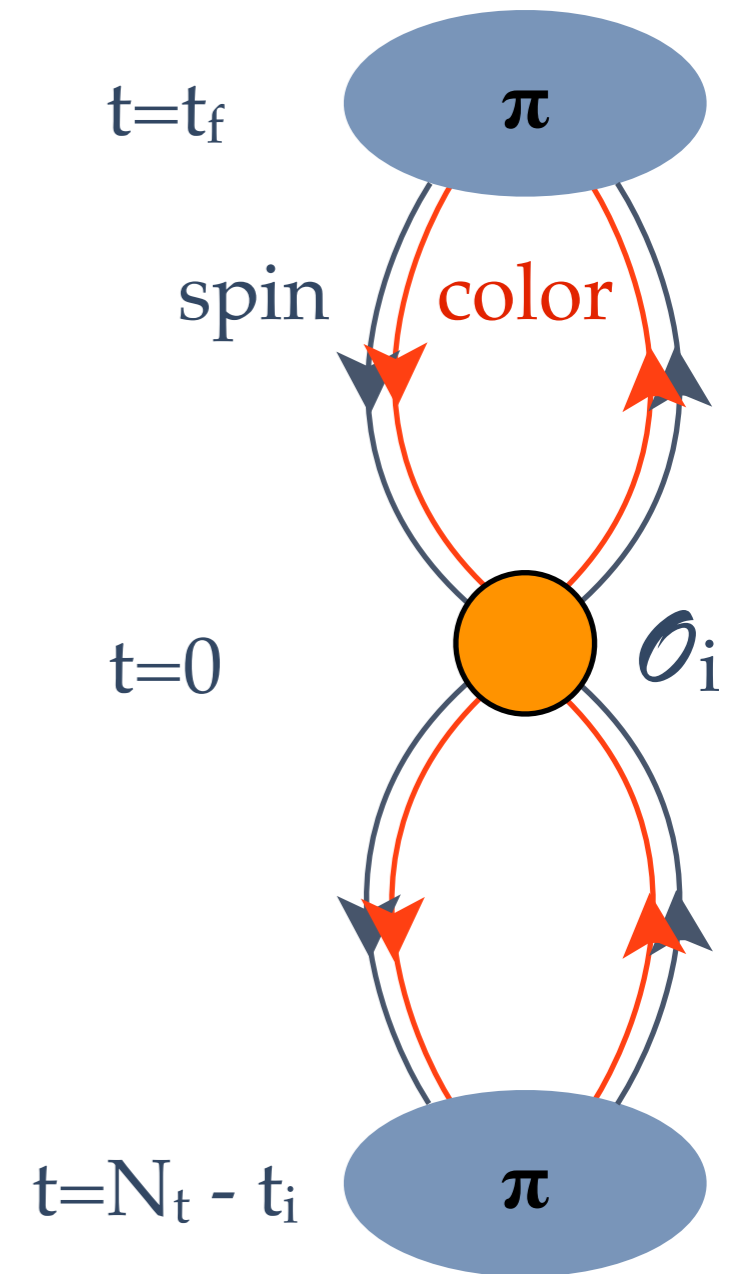
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}'_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}'_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



# Contractions

- QCD interactions can mix colors below the electroweak scale
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

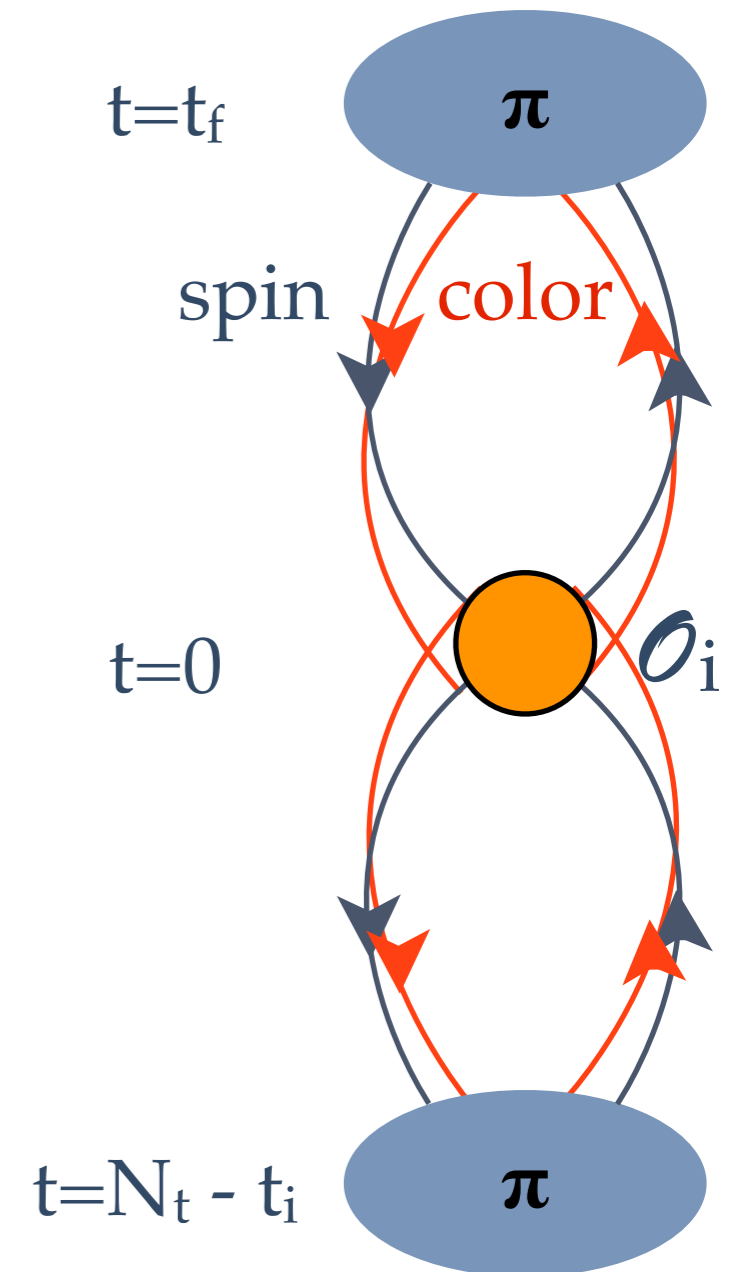
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}'_{1+}{}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}'_{2+}{}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



# Lattice Ensembles

## HISQ ensembles

$a[fm] : m_\pi[MeV]$	310	220	135
0.15	$16^3 \times 48, m_\pi L \sim 3.78$	$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12		$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$	$32^3 \times 64, m_\pi L \sim 4.29$	$48^3 \times 64, m_\pi L \sim 3.91$
0.12		$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$	$48^3 \times 96, m_\pi L \sim 4.73$	

- Möbius DWF on HISQ
- Gradient flow method for smearing configs
  - $m_{\text{res}} < 0.1 m_\ell$  for moderate  $L_5$

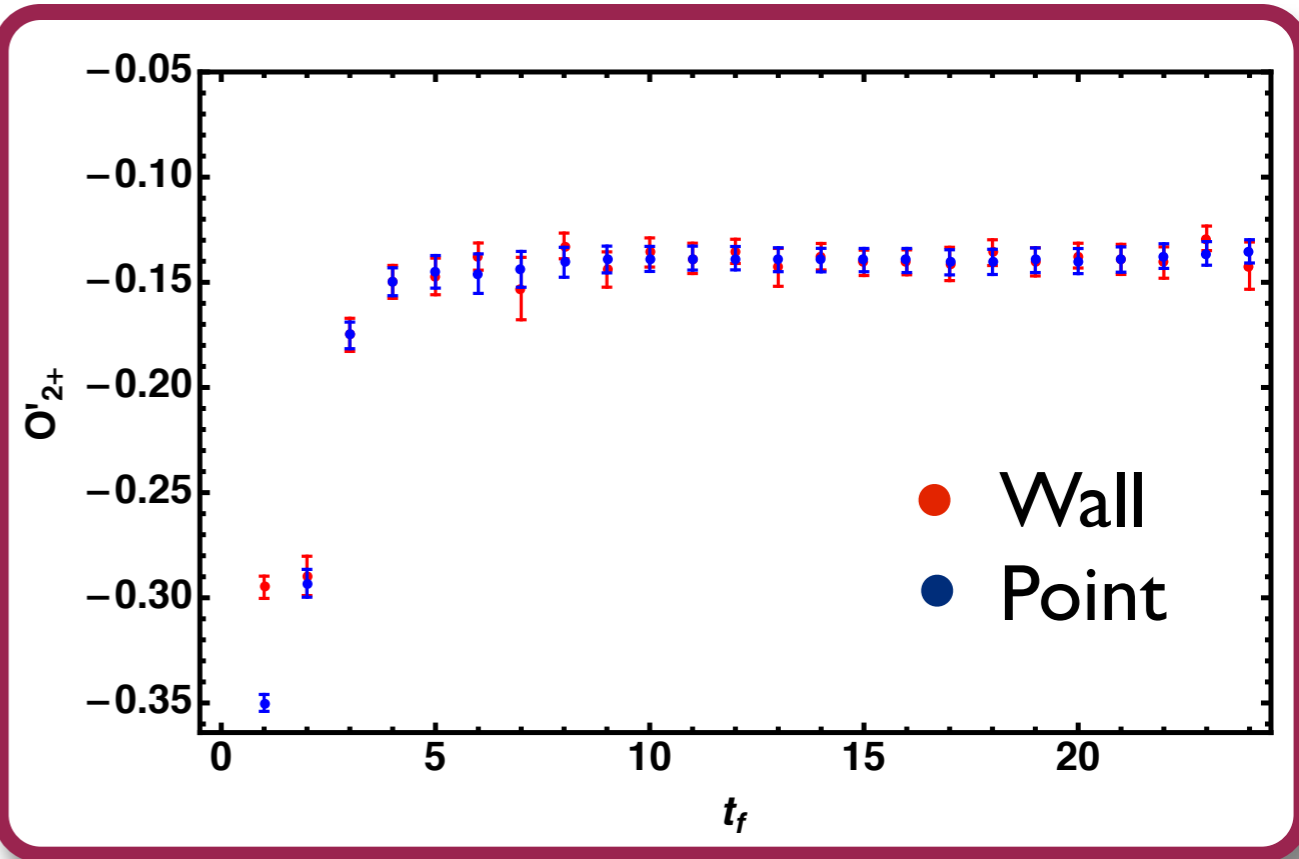
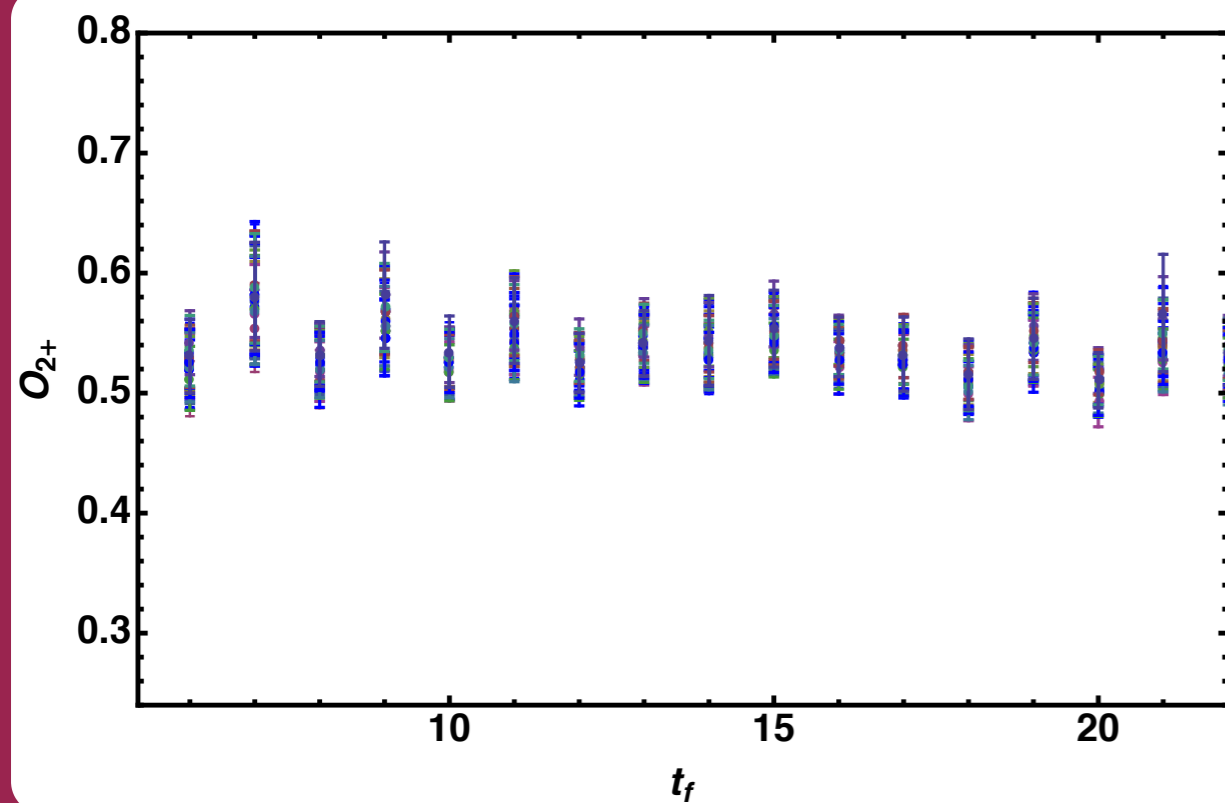
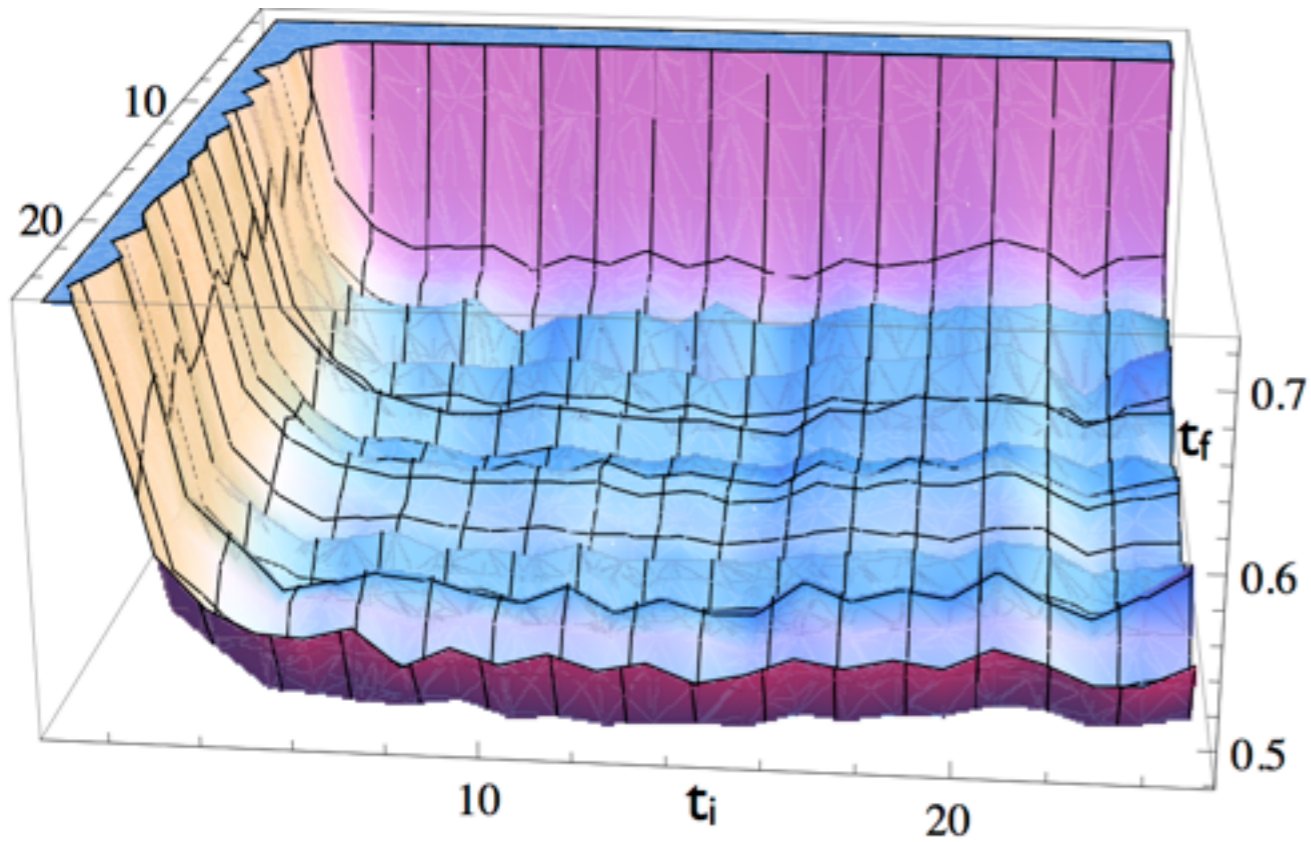
MILC Collaboration Phys. Rev. D87 (2013) 054505

Narayanan, Neuberger (2006), Luscher (2010)

Callat arXiv:1701.07559

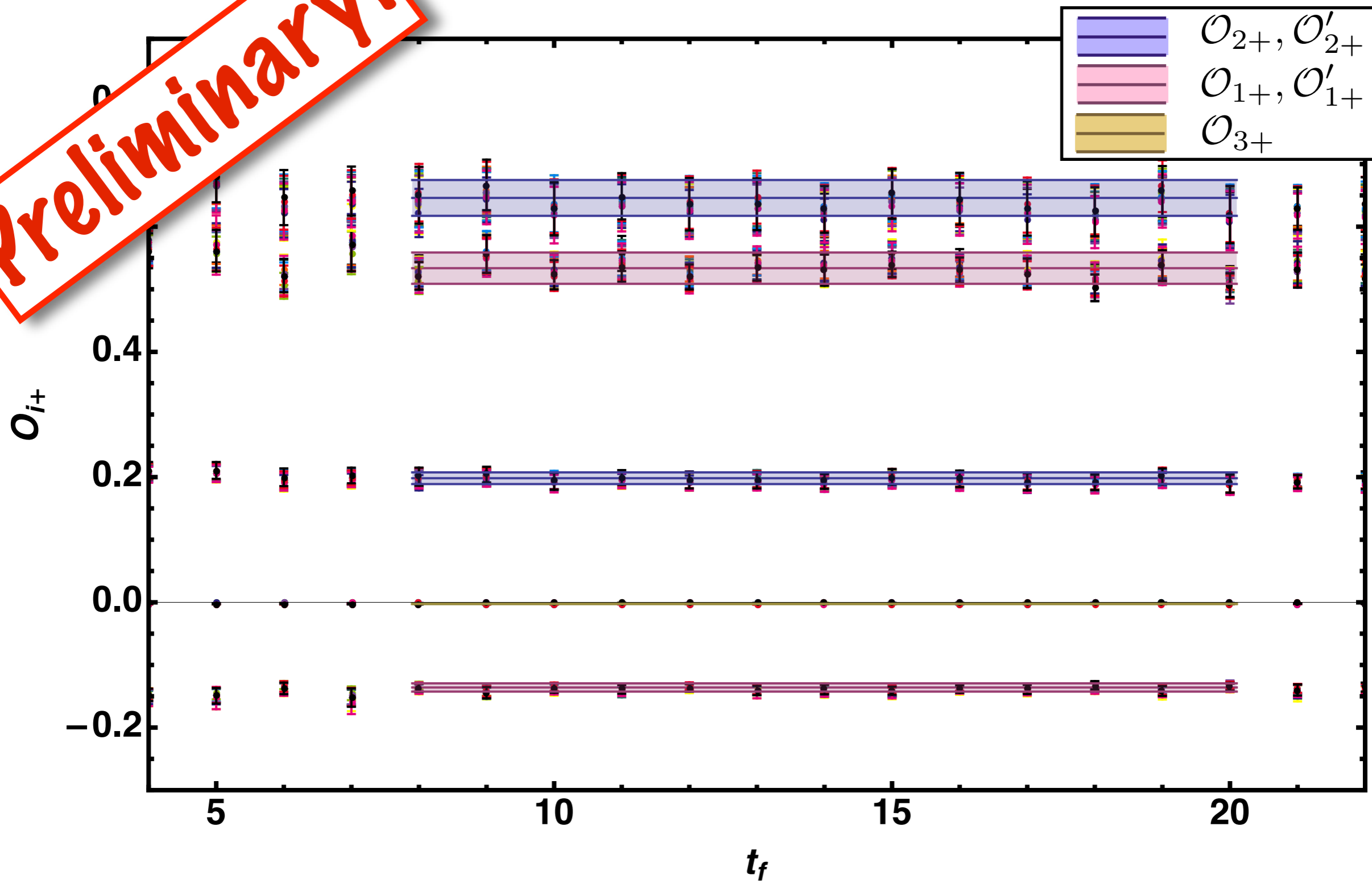


# Signals

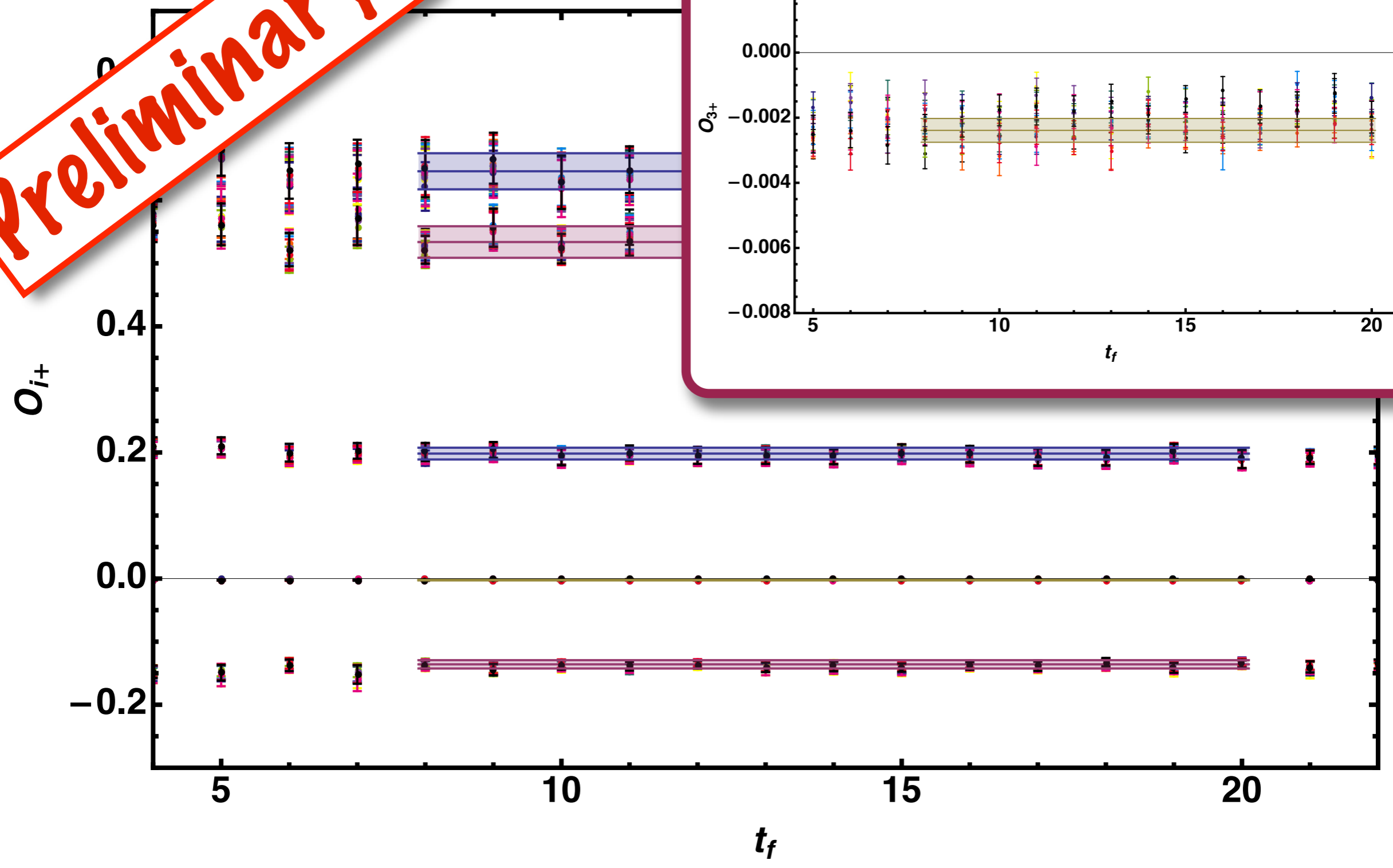


- $m_\pi \sim 135$  MeV
- $L = 5.76$  fm
- $a = 0.12$  fm

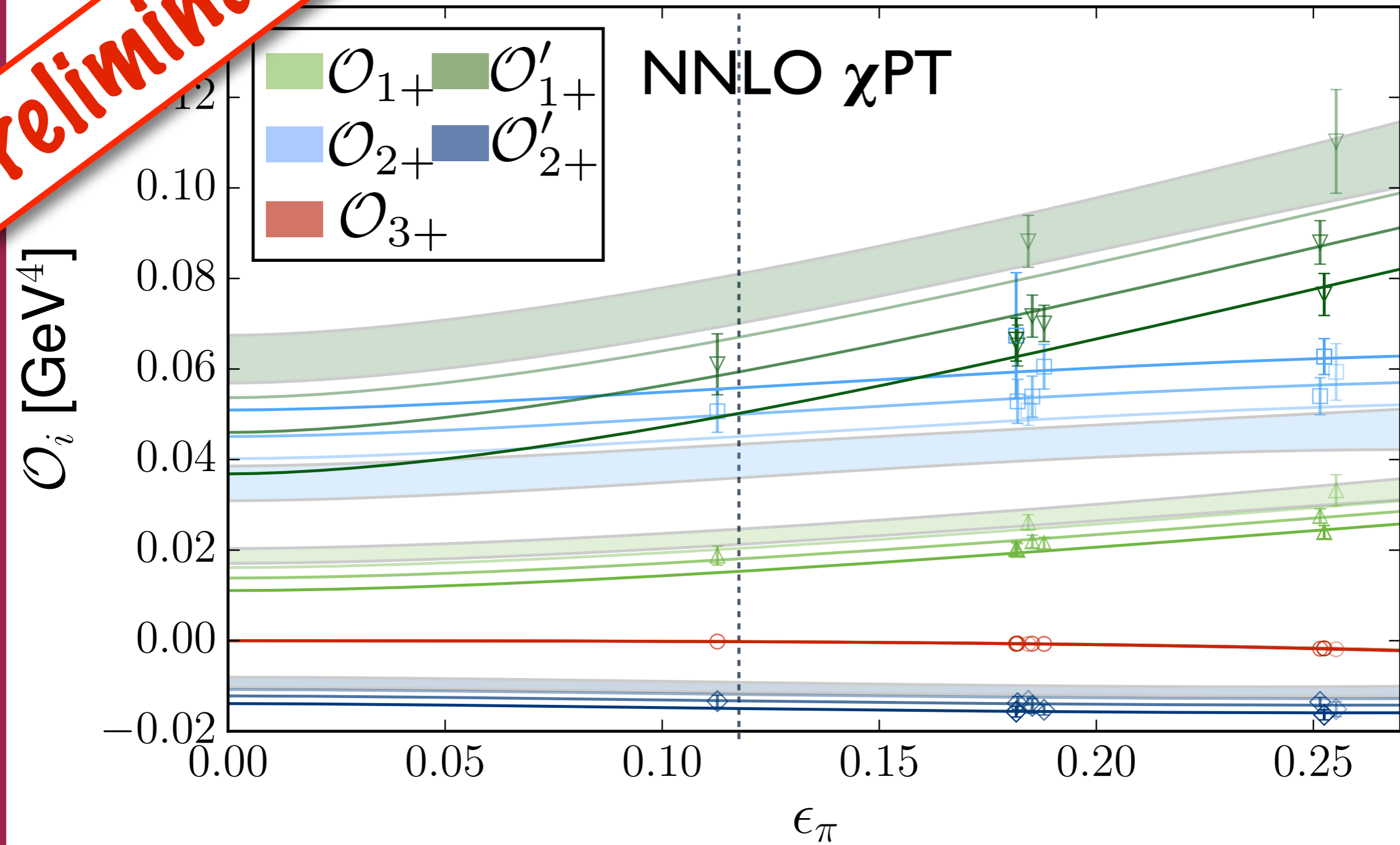
**Preliminary!**



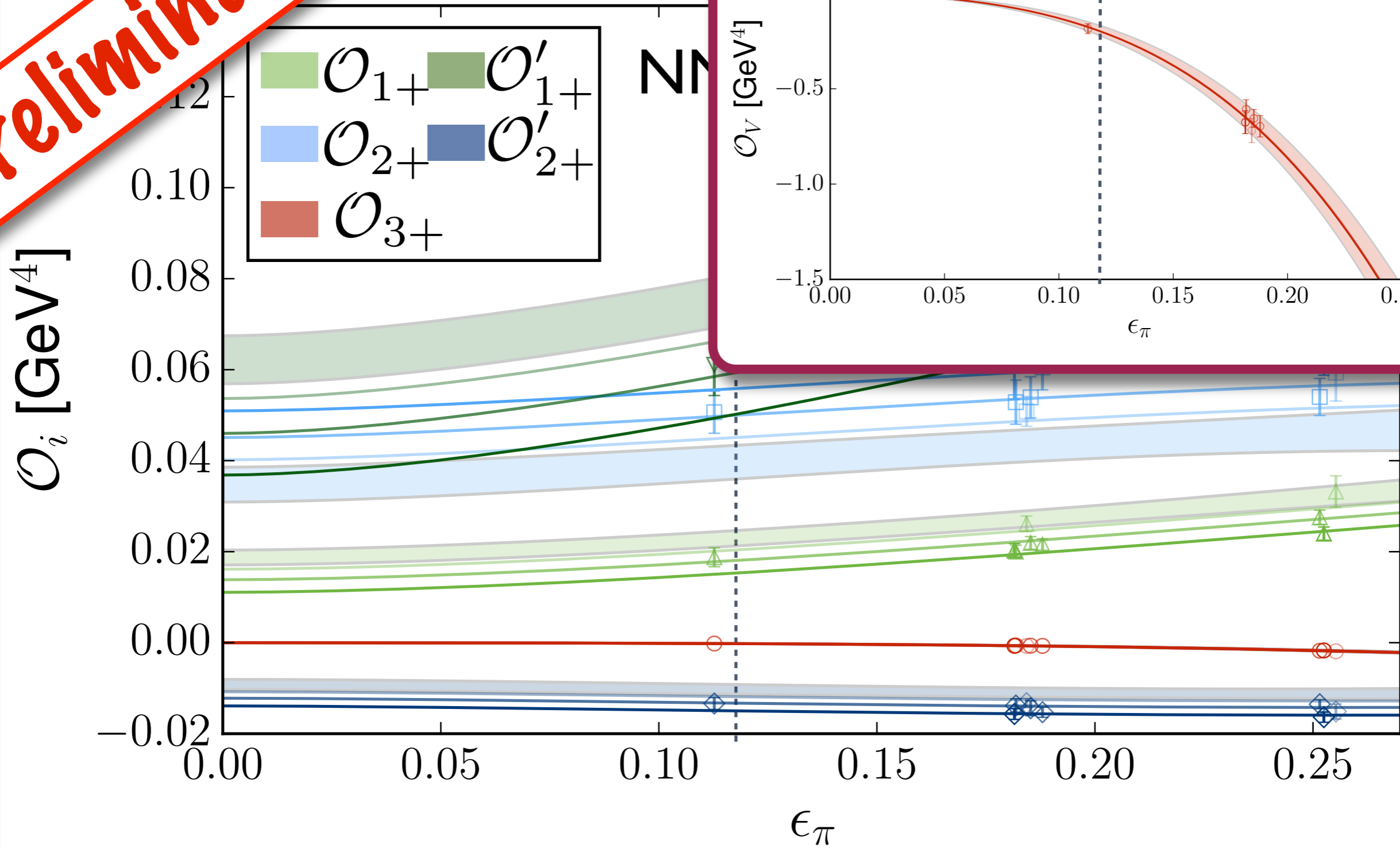
**Preliminary!**



Preliminary!

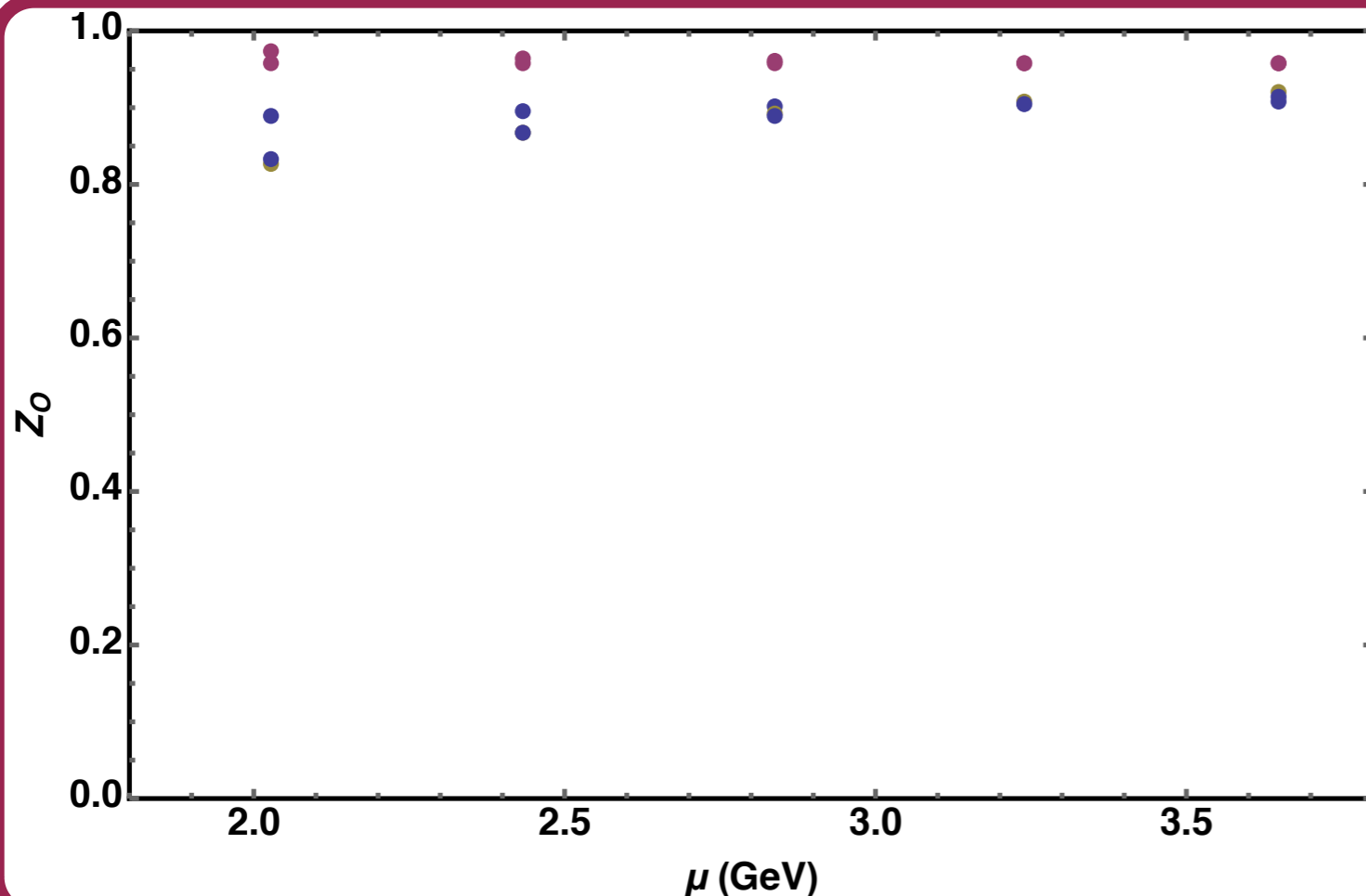


**Preliminary!**

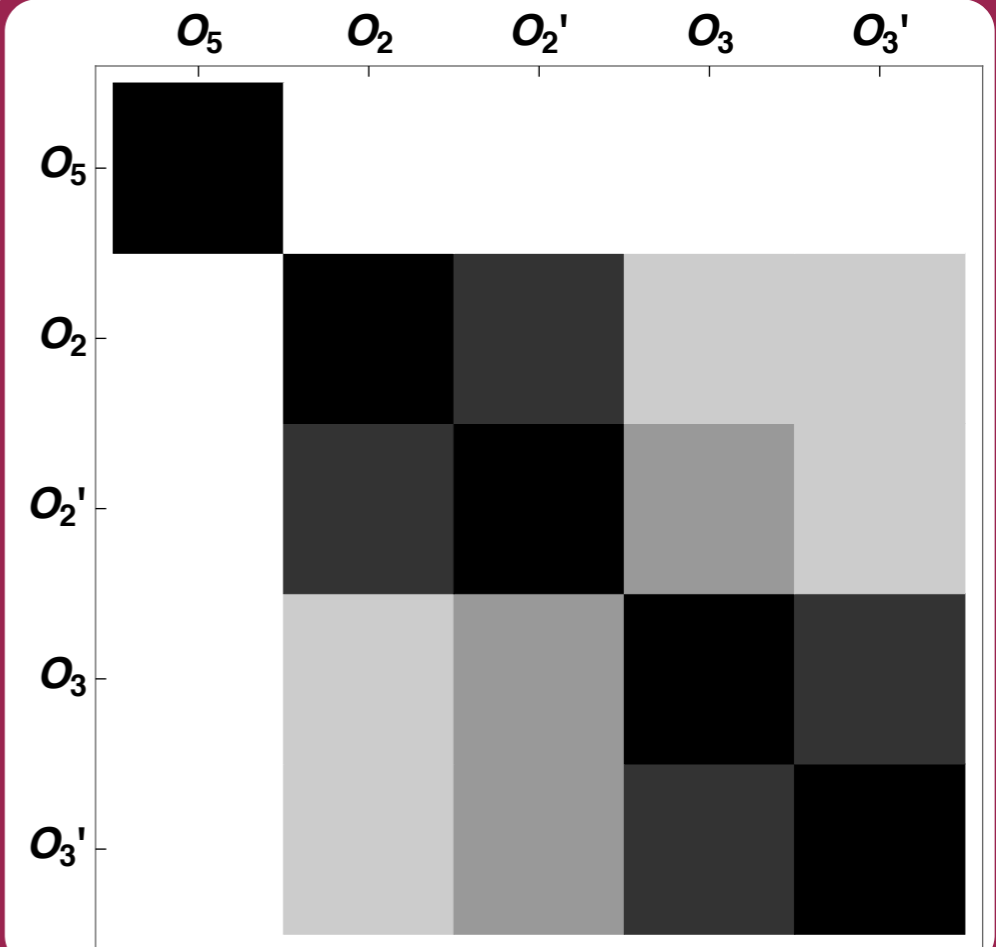


# Renormalization

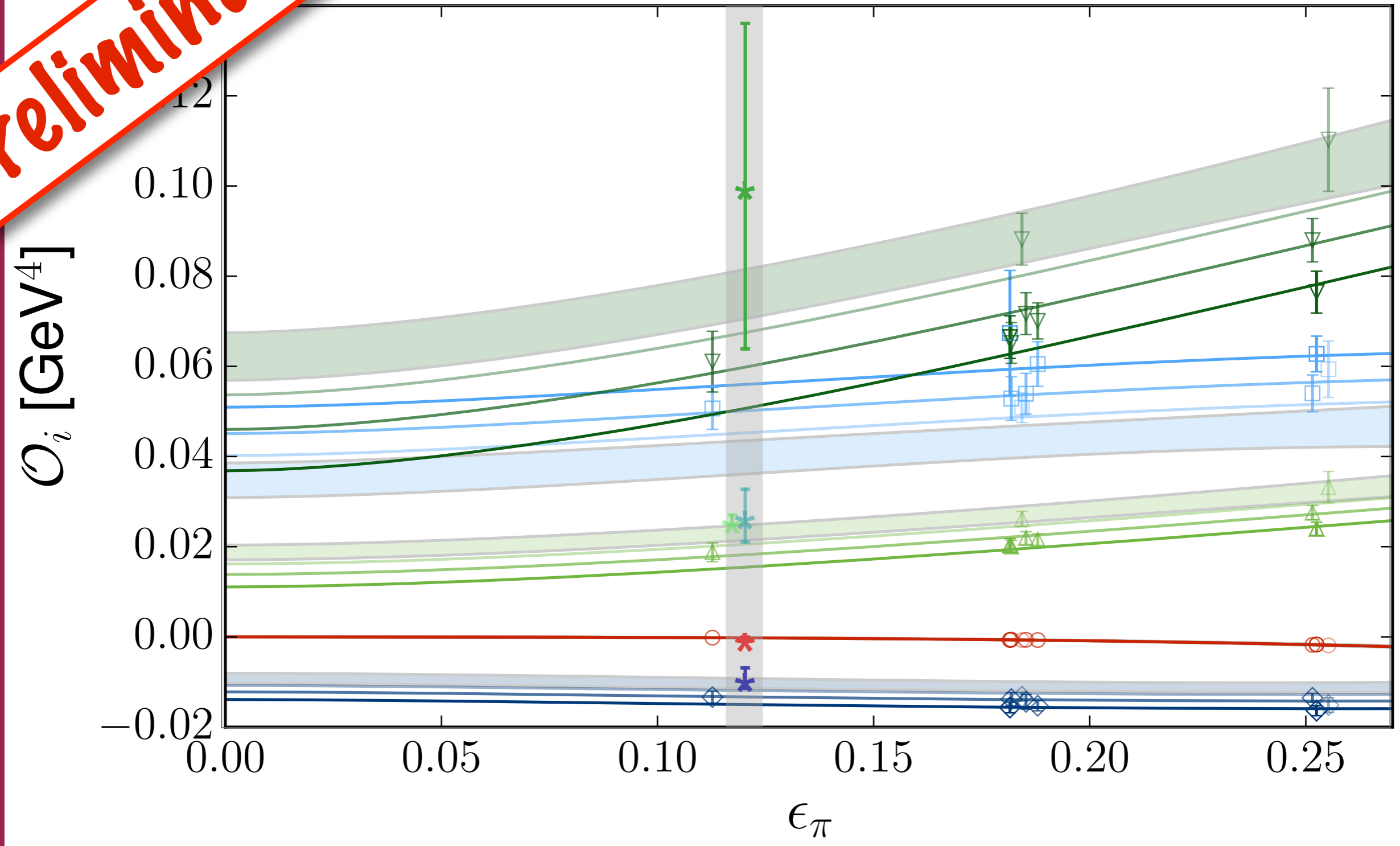
- Lattice perturbation theory is difficult and poorly convergent
- Nonperturbative running (RI-SMOM) to match onto  $\overline{\text{MS}}$

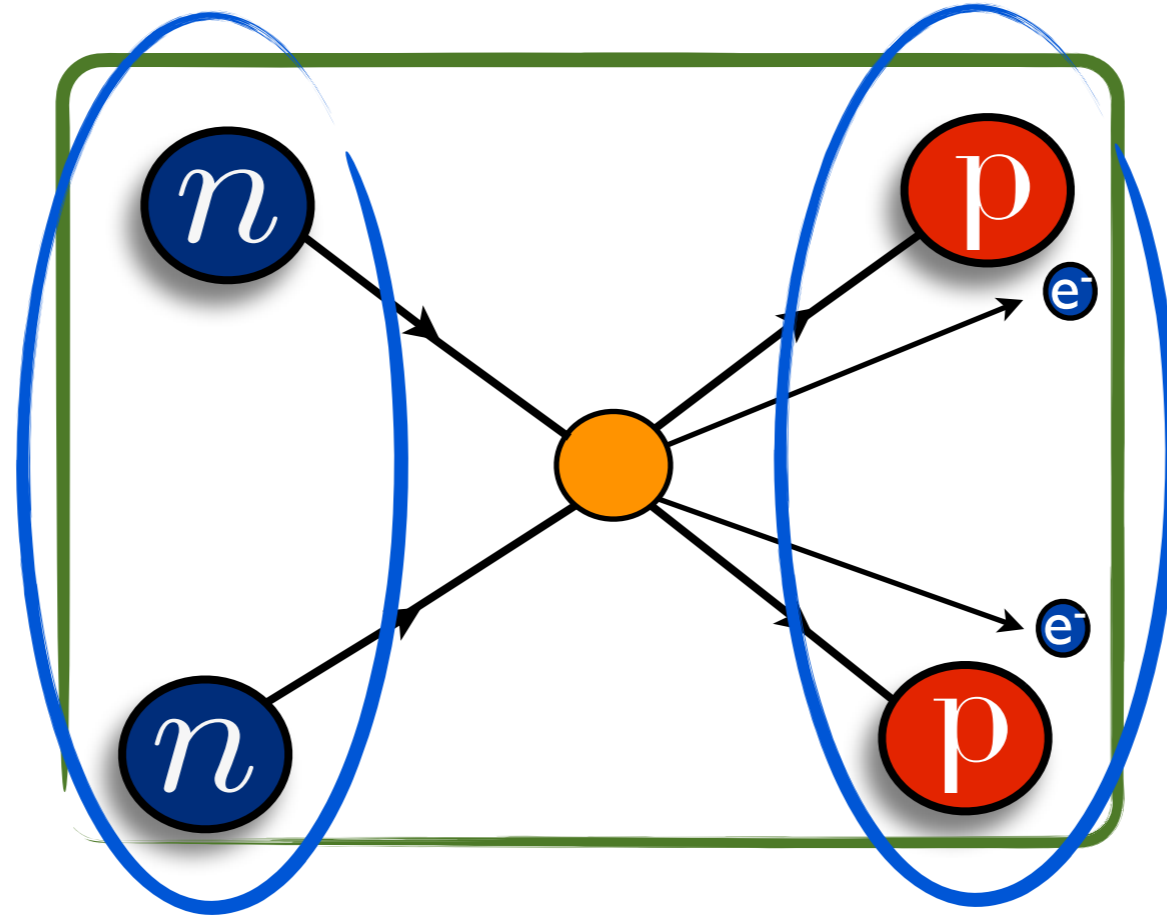


Mixing matrix



**Preliminary!**





3.

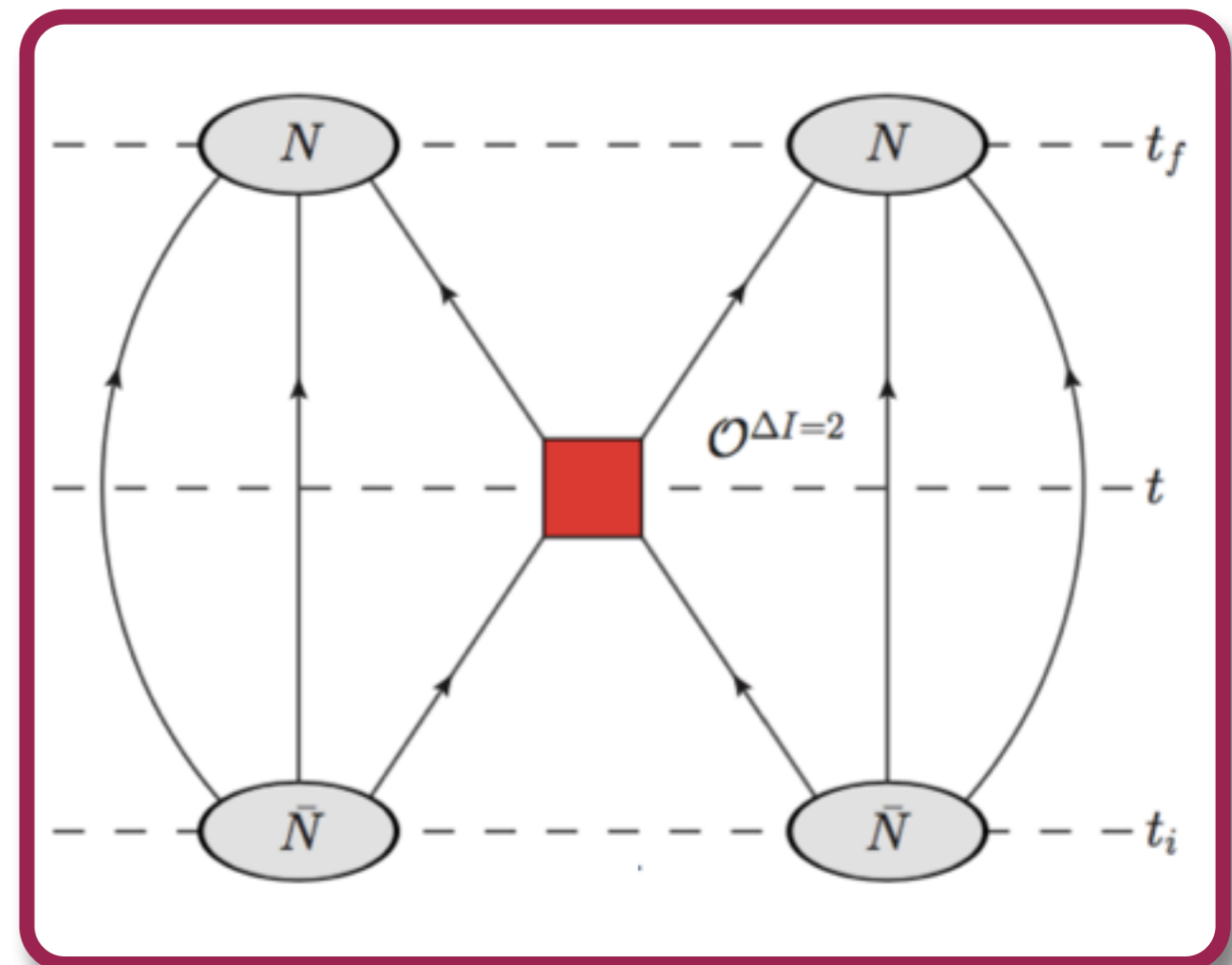
Two-nucleon  
contact



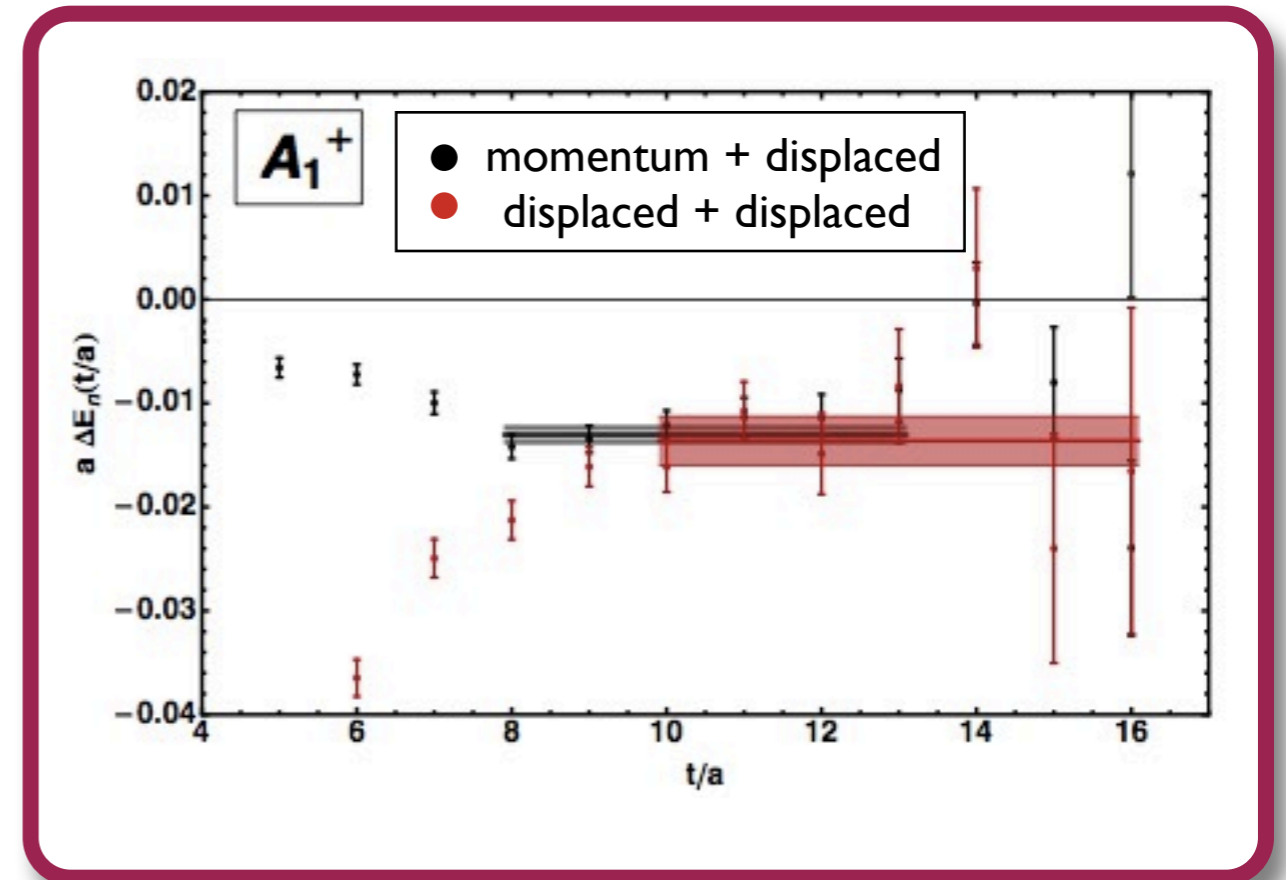
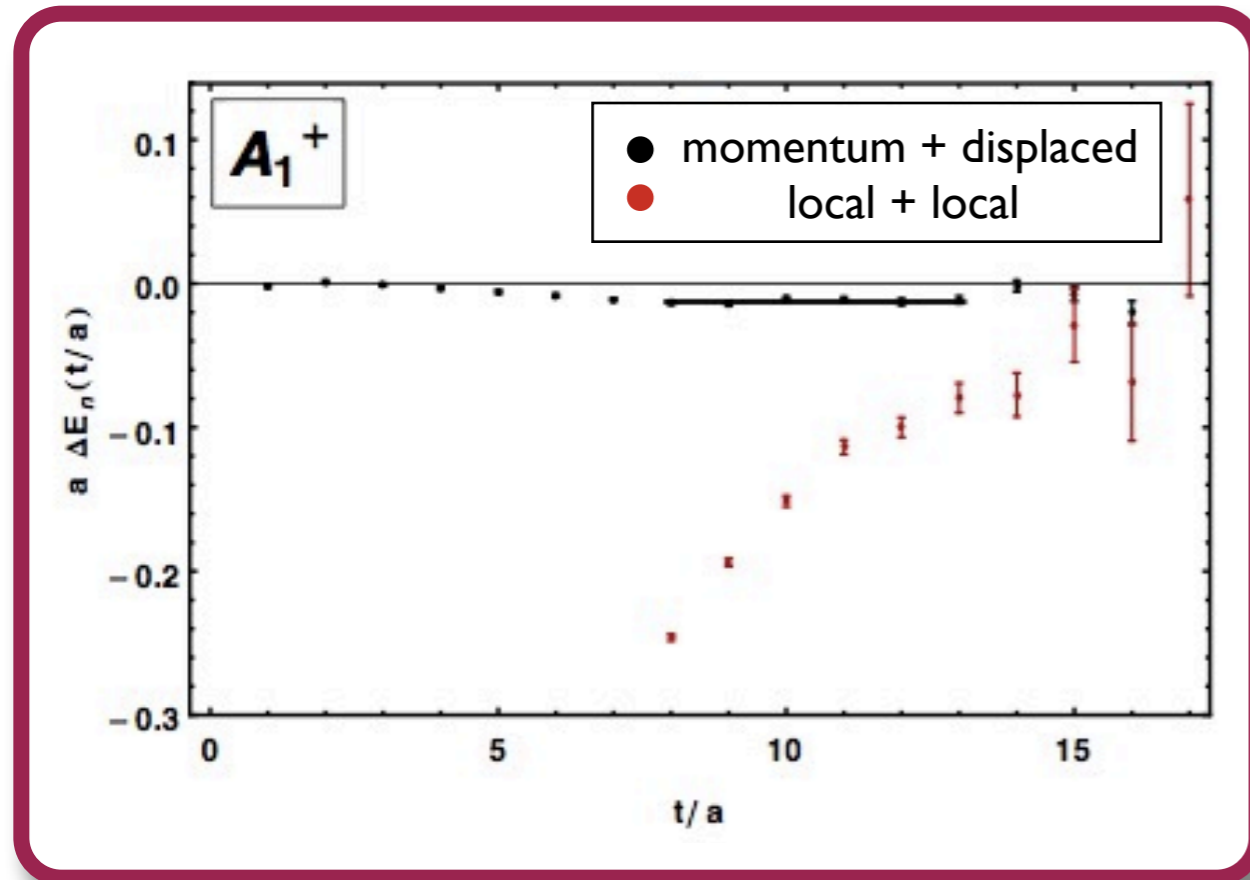
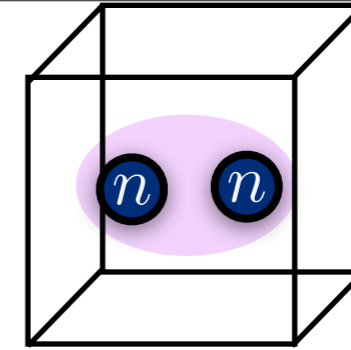
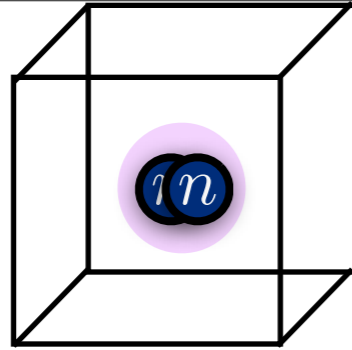


# Two-nucleon contact

- Nucleons and multi-particle states are much more difficult!
  - exponentially poor signal-to-noise problem, small excited state energy splittings, ....
- Isospin limit: 576 contractions\*
- Must deal with multi-particle states in a finite volume
- Ops must be in position space
  - otherwise all-to-all propagators connect to quark operator



\*Doi & Endres, Originos et. al., Günther et. al.

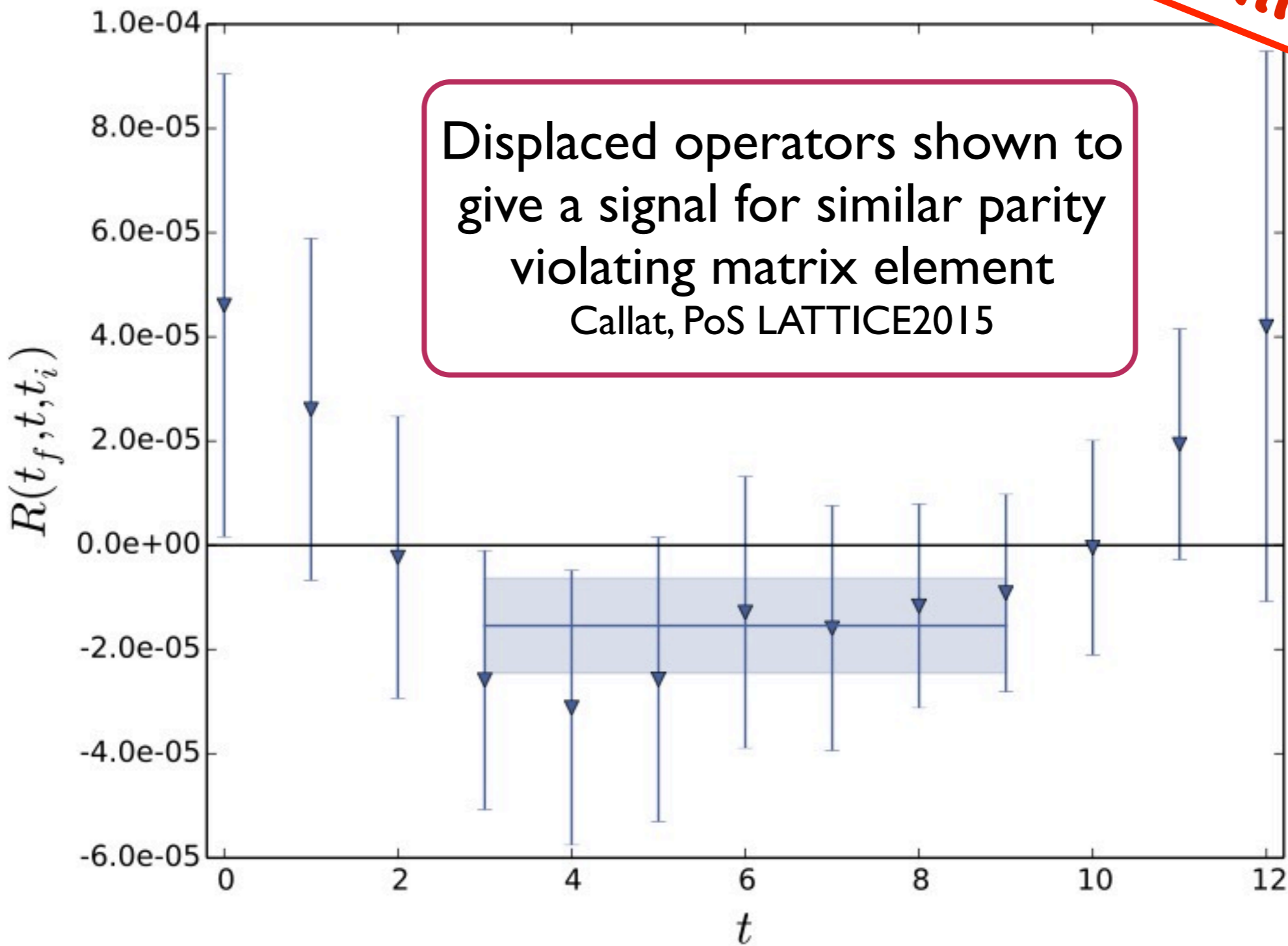


Need displaced operators

Callat arXiv:1508.00886 (2015)

Iso-clover cfs,  $m_\pi \sim 800$  MeV (W. Detmold, R. Edwards, D. Richards, K. Orginos)

Preliminary

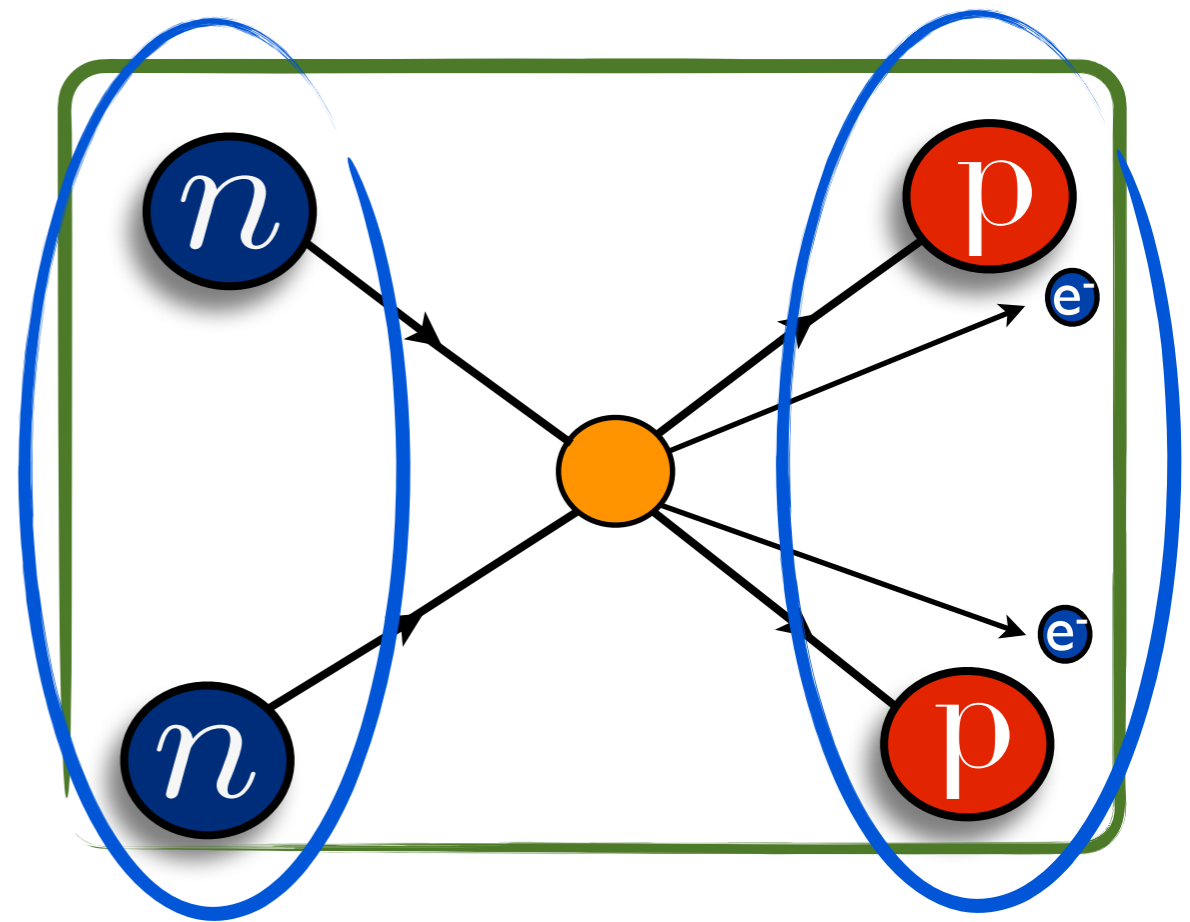


Callat arXiv:1508.00886 (2015)

Iso-clover cfs,  $m_\pi \sim 800$  MeV (W. Detmold, R. Edwards, D. Richards, K. Orginos)

3.

## Two-nucleon contact



- Some new developments:
  - Exponentially improved NN operators
    - will allow us to lower the pion mass
  - HOBET in a periodic box
    - more direct path from finite volume lattice results to nuclear many-body techniques (talk by W. Haxton)



# Summary

- LQCD can be used as a step toward connecting experimental signals to BSM models
- Nucleon axial charge
  - Finally achieved accuracy with LQCD!
- $\pi^- \rightarrow \pi^+$  matrix element
  - Leading short-range contribution
  - To do: complete renormalization
- Two-nucleon contact
  - Testing new method for two nucleon operators
  - Machinery in place for calculating 3-point function

• LBL/UCB: C.C. Chang, AN, A. Walker-Loud

• LLNL: P. Vranas

• NERSC: T. Kurth

• Jülich: E. Berkowitz

• BNL: E. Rinaldi

• nVidia: M.A. Clark

• JLab: B. Joo

• Plymouth: N. Garron

• WM/LBL: D. Brantley,  
H. Monge-Comacho

