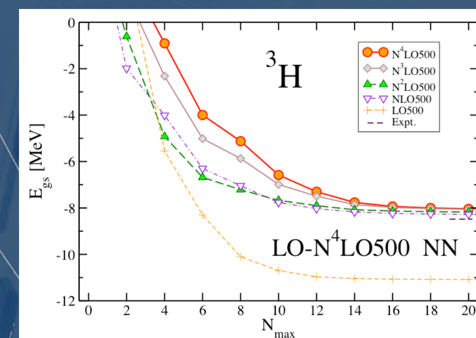
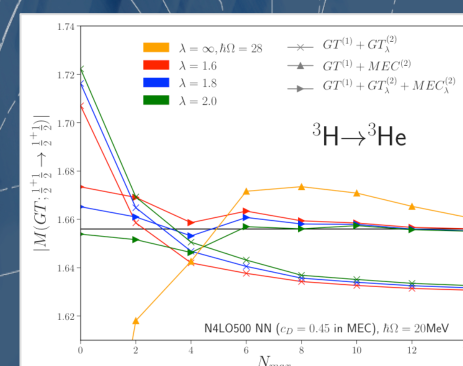
Canada's national laboratory
for particle and nuclear physics
and accelerator-based science

Ab initio β-decay calculations with SRG evolved chiral currents

INT Program INT-17-2a
Neutrinoless Double Beta Decay
June 20, 2017



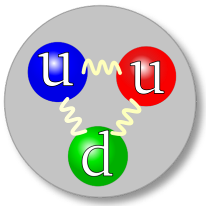
Petr Navratil | TRIUMF



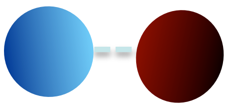
Collaborators:
Sofia Quaglioni, Kyle Wendt (LLNL)
Angelo Calci, **Peter Gysbers,** Jason Holt (TRIUMF)
Gaute Hagen, Micah Schuster (ORNL)
Mihai Horoi (CMU), Jon Engel (NCU), Doron Gazit (Hebrew U)

Outline

- New high precision chiral interactions
- Chiral currents
- SRG evolution of operators
- NCSM calculations of ^{3}H, ^{6}He, ^{14}C beta decay
- Initial double-beta decay applications

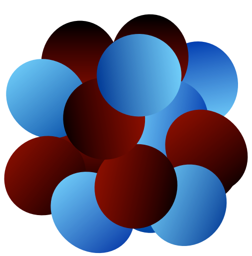


Low-energy QCD



NN+3N interactions
from chiral EFT

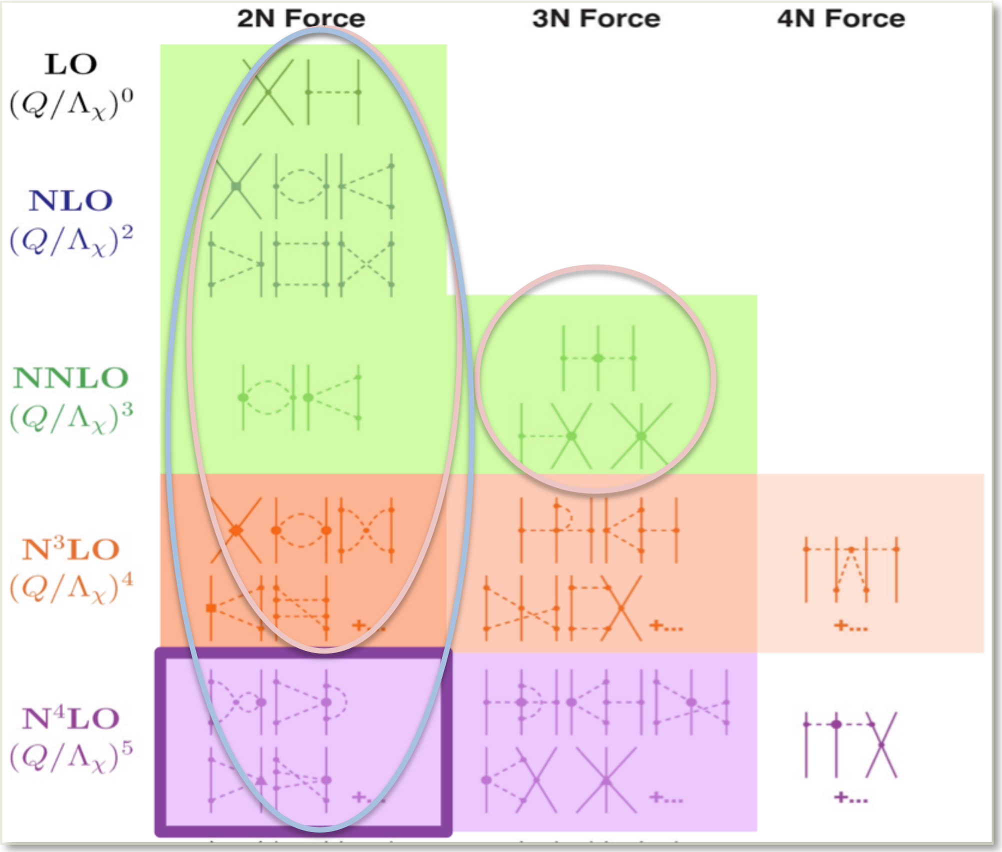
…or accurate
meson-exchange
potentials



Nuclear structure and reactions

Chiral Effective Field Theory

- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order (Q/Λ_χ)
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



Λ_χ~1 GeV :
Chiral symmetry breaking scale

N^{4}LO500 NN

N^{3}LO NN+N^{2}LO 3N
(NN+3N400, NN+3N500)

- # Meson-exchange current

PHYSICAL REVIEW C **67**, 055206 (2003)

Parameter-free effective field theory calculation for the solar proton-fusion and hep processes

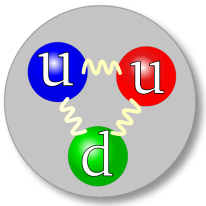
T.-S. Park,[1,2,3] L. E. Marcucci,[4,5] R. Schiavilla,[6,7] M. Viviani,[5,4] A. Kievsky,[5,4] S. Rosati,[5,4] K. Kubodera,[1,2]
D.-P. Min,[8] and M. Rho[1,9]

- weak axial current
  - one-body: LO - Gamow-Teller

$$\boldsymbol{A}_l = -g_A \tau_l^- e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_l}\left[\boldsymbol{\sigma}_l + \frac{2(\bar{\boldsymbol{p}}_l \boldsymbol{\sigma}_l\cdot\bar{\boldsymbol{p}}_l - \boldsymbol{\sigma}_l \bar{\boldsymbol{p}}_l^2) + i\boldsymbol{q}\times\bar{\boldsymbol{p}}_l}{4m_N^2}\right]$$

  - two-body: MEC

$$\boldsymbol{A}_{12} = \frac{g_A}{2m_N f_\pi^2}\frac{1}{m_\pi^2+\boldsymbol{k}^2}\left[-\frac{i}{2}\tau_\times^- \boldsymbol{p}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{k} + 4\hat{c}_3 \boldsymbol{k}\boldsymbol{k}\cdot(\tau_1^-\boldsymbol{\sigma}_1+\tau_2^-\boldsymbol{\sigma}_2) + \left(\hat{c}_4+\frac{1}{4}\right)\tau_\times^- \boldsymbol{k}\times[\boldsymbol{\sigma}_\times\times\boldsymbol{k}]\right] + \frac{g_A}{m_N f_\pi^2}[2\hat{d}_1(\tau_1^-\boldsymbol{\sigma}_1+\tau_2^-\boldsymbol{\sigma}_2)+\hat{d}_2\tau_\times^a\boldsymbol{\sigma}_\times], \qquad (19)$$



Low-energy QCD

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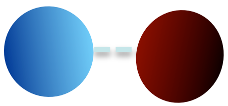


NN+3N interactions from chiral EFT

...or accurate meson-exchange potentials

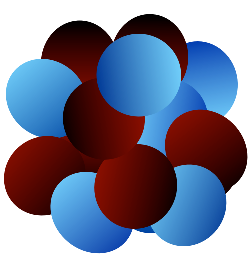
↓

$$H|\Psi\rangle = E|\Psi\rangle$$

Many-Body methods

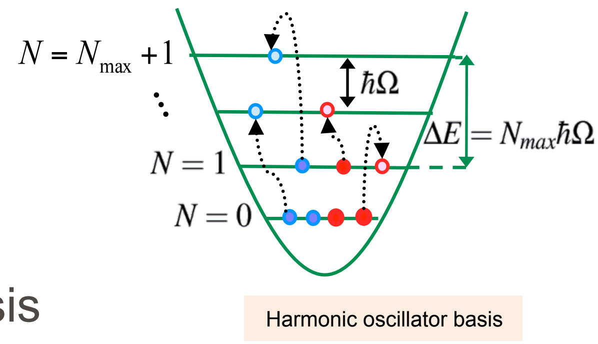
NCSM, NCSM/RGM, NCSMC, CCM, SCGF, GFMC, HH, Nuclear Lattice EFT...

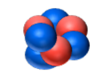
↓



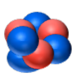
Nuclear structure and reactions

- *Ab initio* no-core shell model
  - Short- and medium range correlations
  - Bound-states, narrow resonances
  - Equivalent description in relative-coordinate and Slater determinant basis





NCSM

(A) 

$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

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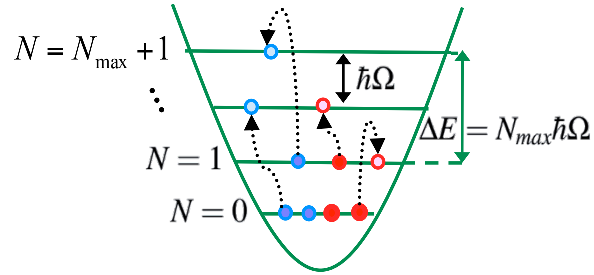
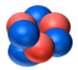


Harmonic oscillator basis

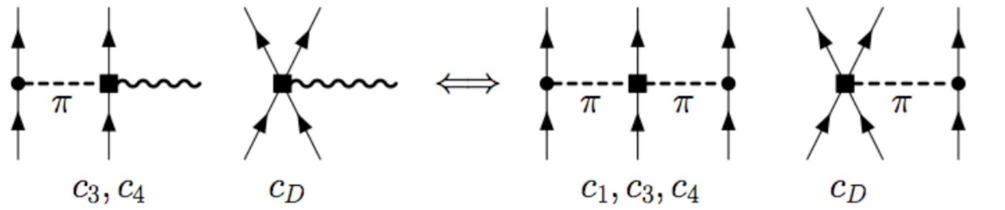


$$(A)\quad \Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

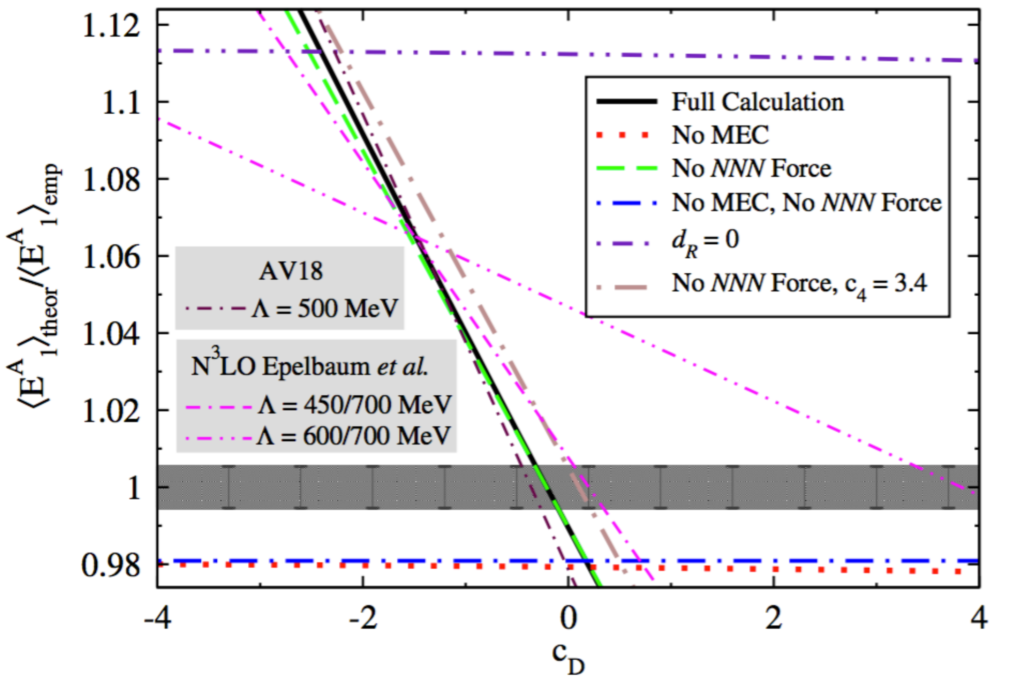
$$(A)\quad \Psi_{\mathrm{SD}}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{\mathrm{SD}} \Phi_{\mathrm{SD}\,Nj}^{HO}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

Axial vector currents & 3N forces in chiral EFT

Park et al., Gardestig & Phillips, ...



- Nuclear currents are obtained consistently
  - LO: standard single-nucleon terms
  - N^{2}LO: first appearance of two-body currents
  - Two-body axial vector currents predicted by NN and 3N couplings



- ^{3}H binding energy and β-decay half-life uncorrelated
  - Used to fully constrain N^{2}LO 3N force (c_E, c_D) in A=3

PRL **103,** 102502 (2009) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2009

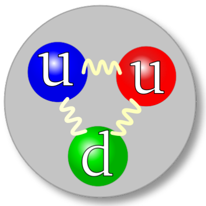
Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory

Doron Gazit

Institute for Nuclear Theory, University of Washington, Box 351550, Seattle, Washington 98195, USA

Sofia Quaglioni and Petr Navrátil

Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA



Low-energy QCD

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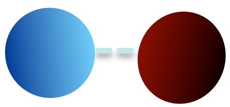
NN+3N interactions from chiral EFT

…or accurate meson-exchange potentials



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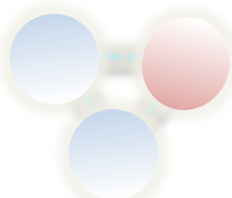
Unitary/similarity transformations

Identity or SRG or OLS or UCOM … Softens NN, induces 3N



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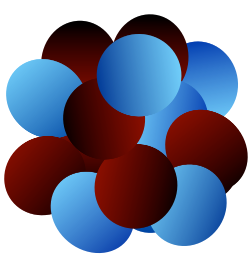
Many-Body methods

NCSM, NCSM/RGM, CCM, SCGF, GFMC, HH, Nuclear Lattice EFT…

$$H|\Psi\rangle = E|\Psi\rangle$$

↓

Nuclear structure and reactions



Similarity Renormalization Group (SRG) evolution

- Continuous transformation driving Hamiltonian to band-diagonal form with respect to a chosen basis
- Unitary transformation $H_\alpha = U_\alpha H U_\alpha^+$ $\qquad U_\alpha U_\alpha^+ = U_\alpha^+ U_\alpha = 1$

$$\frac{dH_\alpha}{d\alpha} = \frac{dU_\alpha}{d\alpha} H U_\alpha^+ + U_\alpha H \frac{dU_\alpha^+}{d\alpha} = \frac{dU_\alpha}{d\alpha} U_\alpha^+ U_\alpha H U_\alpha^+ + U_\alpha H U_\alpha^+ U_\alpha \frac{dU_\alpha^+}{d\alpha}$$

$$= \frac{dU_\alpha}{d\alpha} U_\alpha^+ H_\alpha + H_\alpha U_\alpha \frac{dU_\alpha^+}{d\alpha} = [\eta_\alpha, H_\alpha]$$

$$\eta_\alpha \equiv \frac{dU_\alpha}{d\alpha} U_\alpha^+ = -\eta_\alpha^+$$

anti-Hermitian generator

- Setting $\eta_\alpha = [G_\alpha, H_\alpha]$ with Hermitian G_α

$$\frac{dH_\alpha}{d\alpha} = [[G_\alpha, H_\alpha], H_\alpha]$$

- Customary choice in nuclear physics $G_\alpha = T$ …kinetic energy operator
  - band-diagonal in momentum space plane-wave basis
- Initial condition $H_{\alpha=0} = H_{\lambda=\infty} = H$ $\qquad \lambda^2 = 1/\sqrt{\alpha}$

The SRG transformation maintains the same eigenvalues for the Hamiltonian

$$\hat{H}\left|\psi_k\right\rangle = E_k\left|\psi_k\right\rangle \rightarrow \hat{H}_\alpha\left|\psi_{k,\alpha}\right\rangle = E_k\left|\psi_{k,\alpha}\right\rangle$$

But to extract additional observables from the wavefunction while taking advantage of the SRG tranformation, the corresponding operators must be transformed

$$\left\langle\psi_i\right|\hat{O}\left|\psi_f\right\rangle = \left\langle\psi_{i,\alpha}\right|\hat{O}_\alpha\left|\psi_{f,\alpha}\right\rangle \text{ where } \hat{O}_\alpha = U_\alpha\hat{O}U_\alpha^\dagger$$

The transformation matrix can be extracted from the eigenfunctions of the Hamiltonian

$$U_\alpha = \sum_k \left|\psi_{k,\alpha}\right\rangle\left\langle\psi_k\right|$$

H_α, O_α:
2-body part determined in A=2 system,
3-body part determined in A=3 system,
...

Implementation up to two-body terms:

Peter Gysbers (McMaster/TRIUMF)

The matrix U is calculated blockwise, for relative coordinate two-nucleon eigenstates:

$$|(A=2)kJ^{\pi}TT_z\rangle = \sum_{n,\ell} c^{k}_{n\ell s}\,|n\ell s J^{\pi}TT_z\rangle$$

The corresponding submatrix of $\hat{H}$ is evolved then diagonalized to produce a matrix $U^{J^{\pi}TT_z}_{\alpha}$

Compute the matrix elements of the bare operator: $\langle k' J'^{\pi'} T' T'_z ||\, \hat{O}^{(K)}\, || k J^{\pi} T T_z\rangle$

Matrix elements of the evolved operator are:

$$\langle k' J'^{\pi'} T' T'_z, \alpha ||\, U^{J'^{\pi'}T'T'_z}_{\alpha} \hat{O}^{(K)} U^{\dagger J^{\pi}TT_z}_{\alpha}\, || k J^{\pi} T T_z, \alpha\rangle$$

Implementation up to two-body terms:

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The matrix U is calculated blockwise, for relative coordinate two-nucleon eigenstates:

$$|(A=2)kJ^\pi TT_z\rangle = \sum_{n,\ell} c^k_{n\ell s} |n\ell s J^\pi TT_z\rangle$$

The corresponding submatrix of $\hat{H}$ is evolved then diagonalized to produce a matrix $U_\alpha^{J^\pi TT_z}$

Compute the matrix elements of the bare operator: $\langle k' J'^{\pi'} T' T'_z || \hat{O}^{(K)} || k J^\pi TT_z\rangle$

Matrix elements of the evolved operator are:

$$\langle k' J'^{\pi'} T' T'_z, \alpha || U_\alpha^{J'^{\pi'} T' T'_z} \hat{O}^{(K)} U_\alpha^{\dagger J^\pi TT_z} || k J^\pi TT_z, \alpha\rangle$$

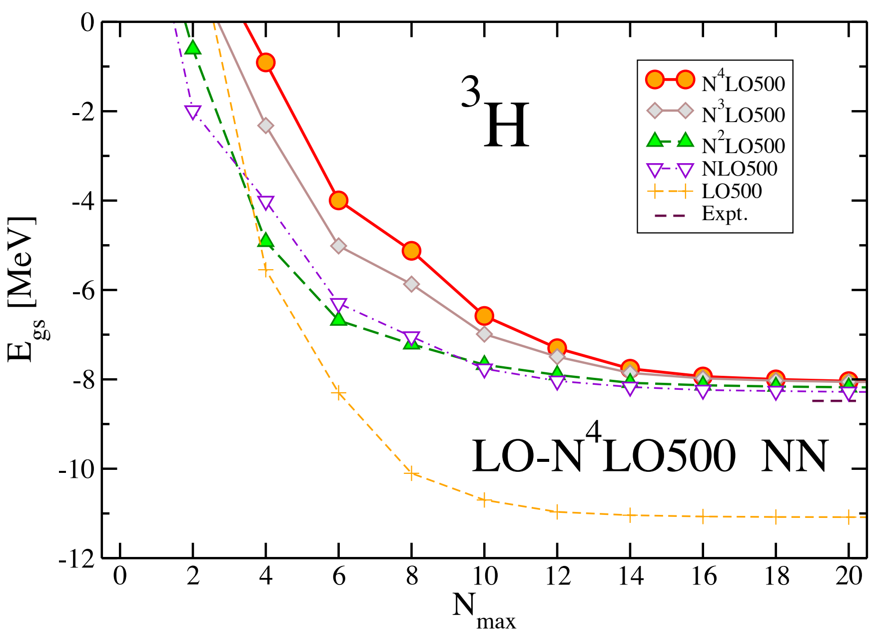
Converting from the two-nucleon Jacobi basis to the single particle basis:

$$\langle a'b' J'^{\pi'} T' T'_z || \hat{O}^{(K)}_\alpha || ab J^\pi TT_z\rangle \qquad a \equiv \{n_a, \ell_a, j_a\}$$

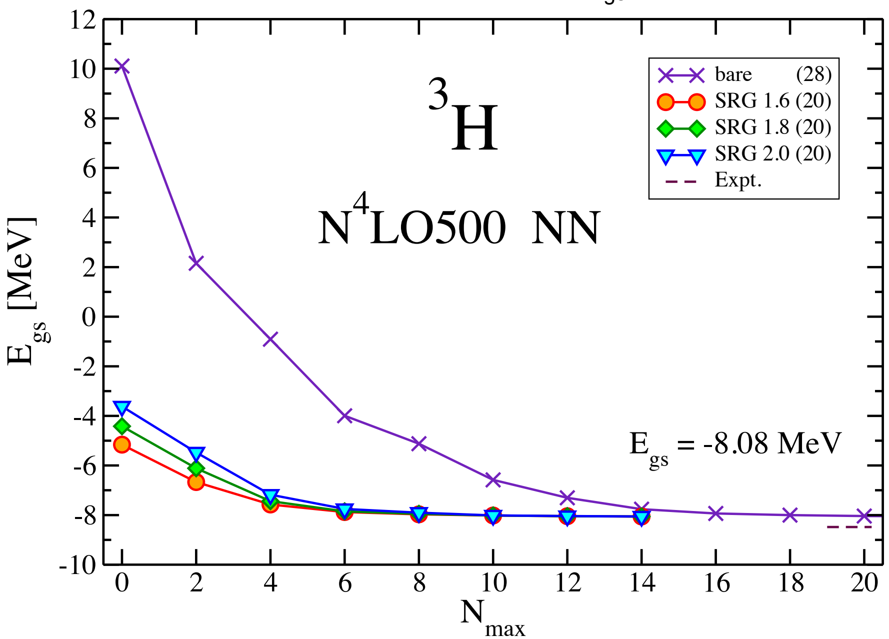
$$= \sum C^{*a'b'}_{n'\ell's'} C^{ab}_{n\ell s} \langle n'\ell's' J'^{\pi'} T' T'_z || \hat{O}^{(K)}_\alpha || n\ell s J^\pi TT_z\rangle$$

Code NCSMV2B

- Systematic from LO to N⁴LO
- High precision – χ^2/datum = 1.15
  - D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91, 014002 (2015).
  - D. R. Entem, R. Machleidt, and Y. Nosyk, arXiv:1703.05454.



Original EM 2003 N³LO NN E_{gs}=-7.85 MeV



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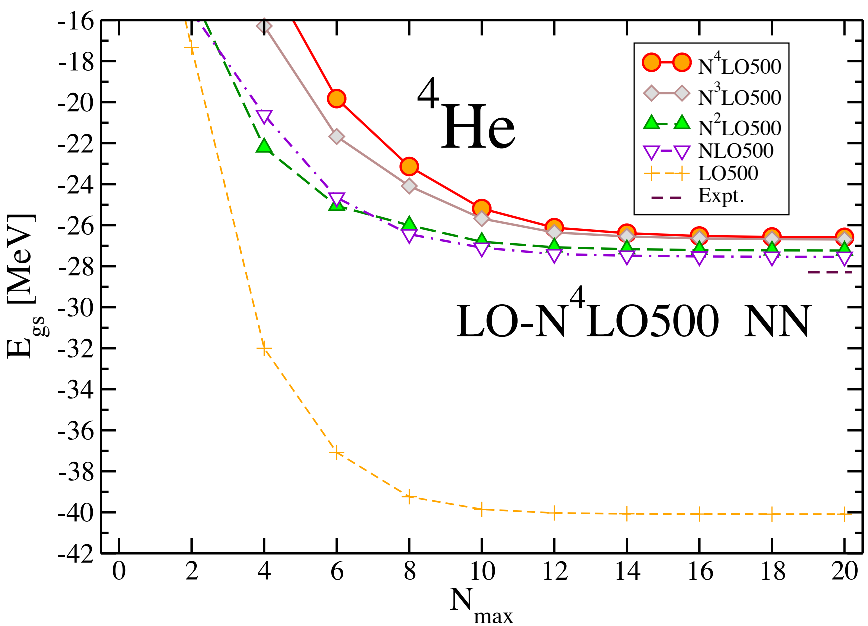
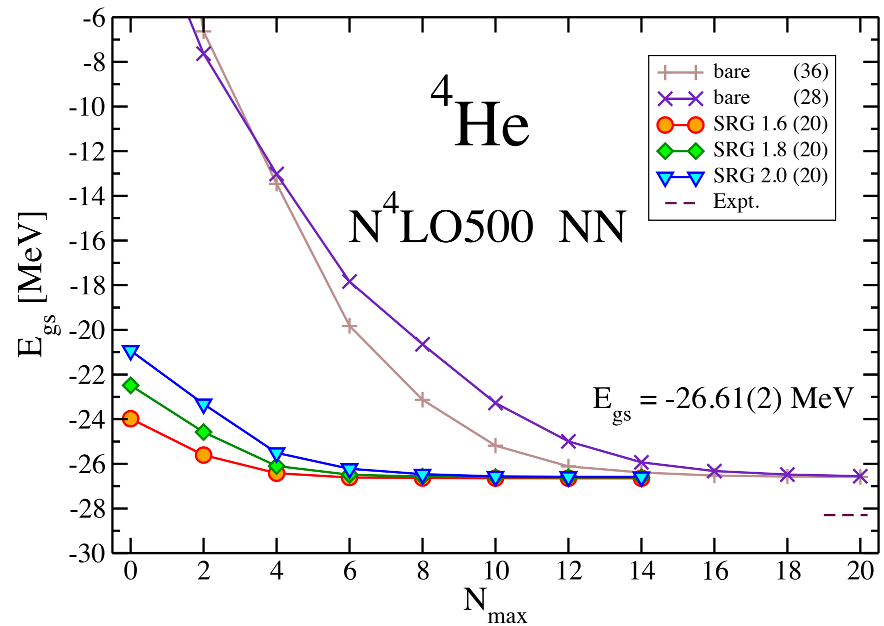


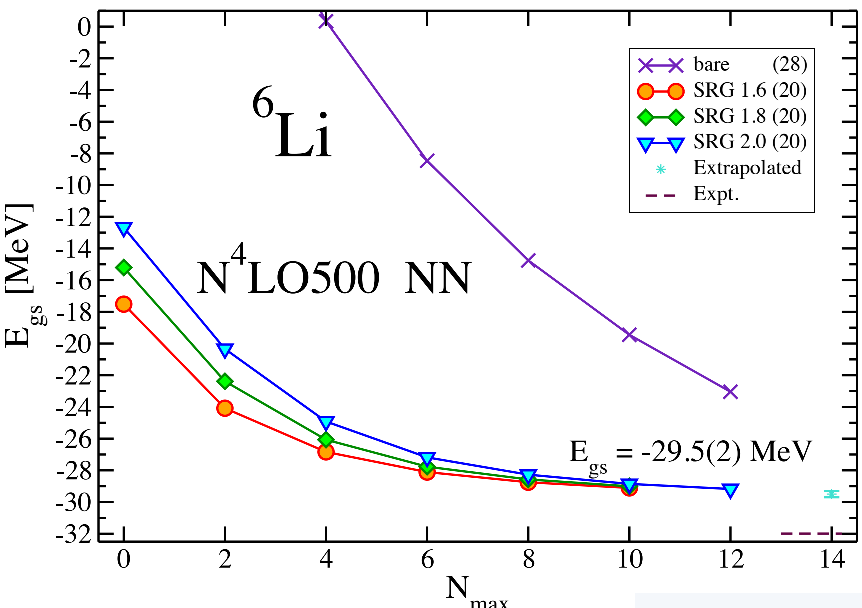
Original EM 2003 N^{3}LO NN E_{gs}=-25.39 MeV

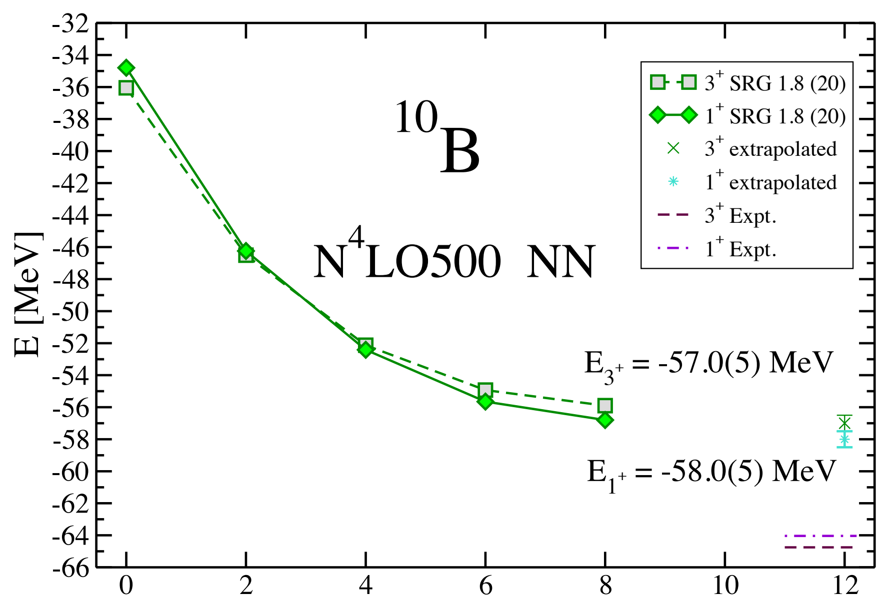


Chiral EFT interactions up to N^4LO

- Systematic from LO to N^4LO
- High precision – χ^2/datum = 1.15
  - D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91, 014002 (2015).
  - D. R. Entem, R. Machleidt, and Y. Nosyk, arXiv:1703.05454.

Original EM 2003 N^3LO NN E_{gs}=-28.0(5) MeV





Chiral NN with SRG 2- and 3-body induced

$$\hat{O} = GT^{(1)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT^{(2)}_\alpha + \dots$$

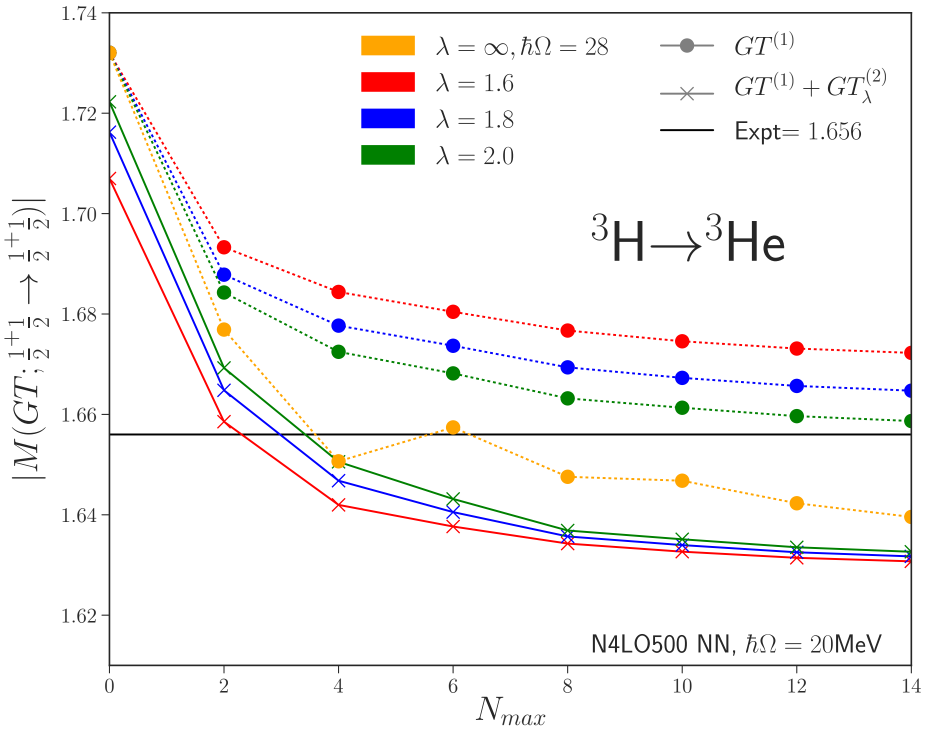
Operator:

Gamow-Teller (1-body)

$$\langle GT^{(2)}_\alpha \rangle_{A=2} = \langle (GT^{(1)})_\alpha \rangle_{A=2} - \langle GT^{(1)}_\alpha \rangle_{A=2}$$

Potential: "N⁴LO NN"

- chiral NN @ N^4LO, Machleidt PRC91 (2015), 500MeV cutoff



Hamiltonian:
chiral NN+3N with SRG 2- and 3-body induced
(except orange line: bare chiral NN+3N)

$^3H \rightarrow {}^3He$ β decay

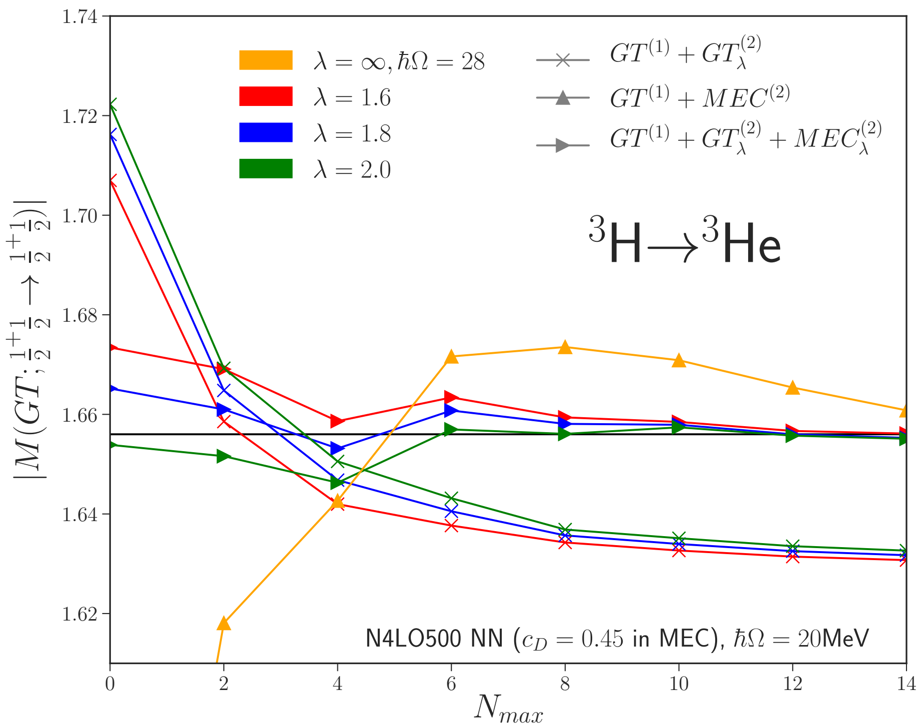
$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT^{(2)}_\alpha + MEC^{(2)}_\alpha + \dots$$

Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body) Park (2003)

Potential: "N^{4}LO NN"

- chiral NN @ N^{4}LO, Machleidt PRC91 (2015), 500MeV cutoff
- LEC $c_D = 0.45$ determined



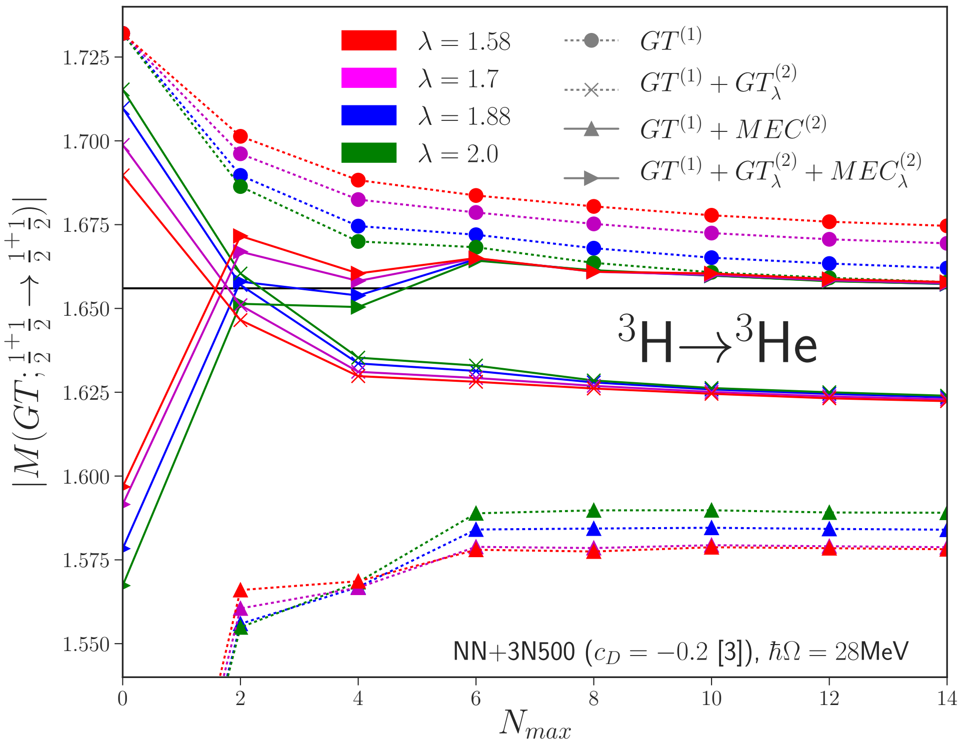
Original EM 2003 N^{3}LO NN c_D=-0.2 (3N attractive)

Determination of the c_D parameter relevant to chiral 3N force c_D=0.45 (3N repulsive)

$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT^{(2)}_\alpha + MEC^{(2)}_\alpha + \ldots$$

Potential: "NN+3N500"

- chiral NN @ N^{3}LO, Entem & Machleidt PRC68 (2003), 500MeV cutoff
- chiral 3N @ N^{2}LO, Navrátil Few-Body Sys. 41 (2007), 500MeV cutoff
- LEC $c_D = -0.2$ determined by Gazit PRL103 (2009)



Hamiltonian:
chiral NN+3N with SRG 2- and 3-body induced

Precision measurement of ^{6}He beta decay

PHYSICAL REVIEW C **86**, 035506 (2012)

Precision measurement of the ^{6}He half-life and the weak axial current in nuclei

A. Knecht,[1,*] R. Hong,[1] D. W. Zumwalt,[1] B. G. Delbridge,[1] A. García,[1] P. Müller,[2] H. E. Swanson,[1] I. S. Towner,[3] S. Utsuno,[1] W. Williams,[2,†] and C. Wrede[1,‡]

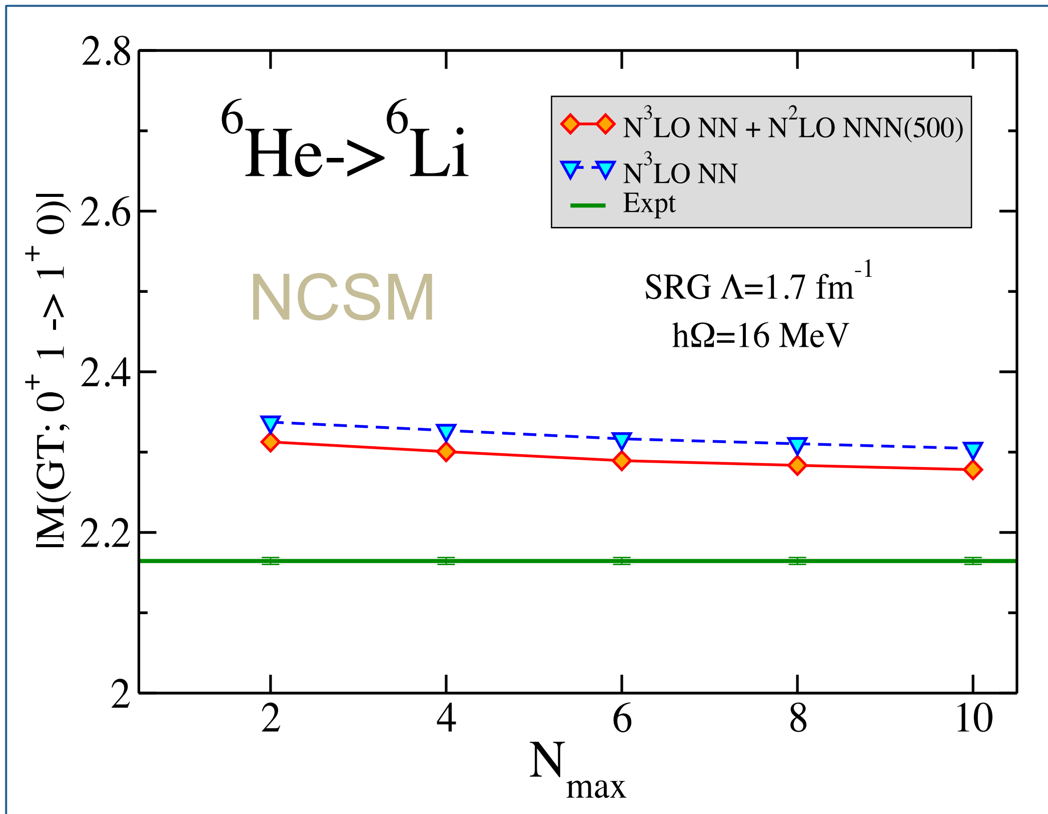
… challenge and test
of *ab initio* calculations,
nuclear forces
and currents

Improvement with the NNN interaction

MEC must be included

Also:

Operator renormalization & continuum



$^6He \rightarrow ^6Li$ β decay

Precision measurement of ^6He beta decay

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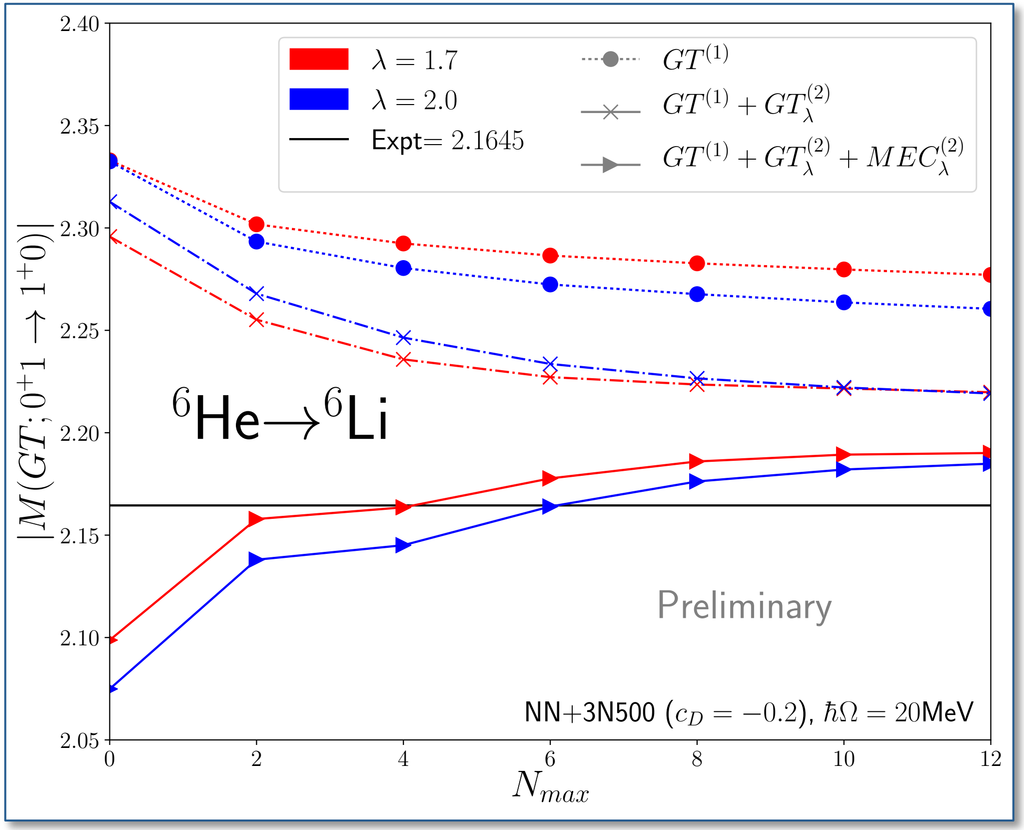
… challenge and test of *ab initio* calculations, nuclear forces and currents

Improvement with the NNN interaction
Improvement with MEC
& operator renormalization

Still to be done: continuum



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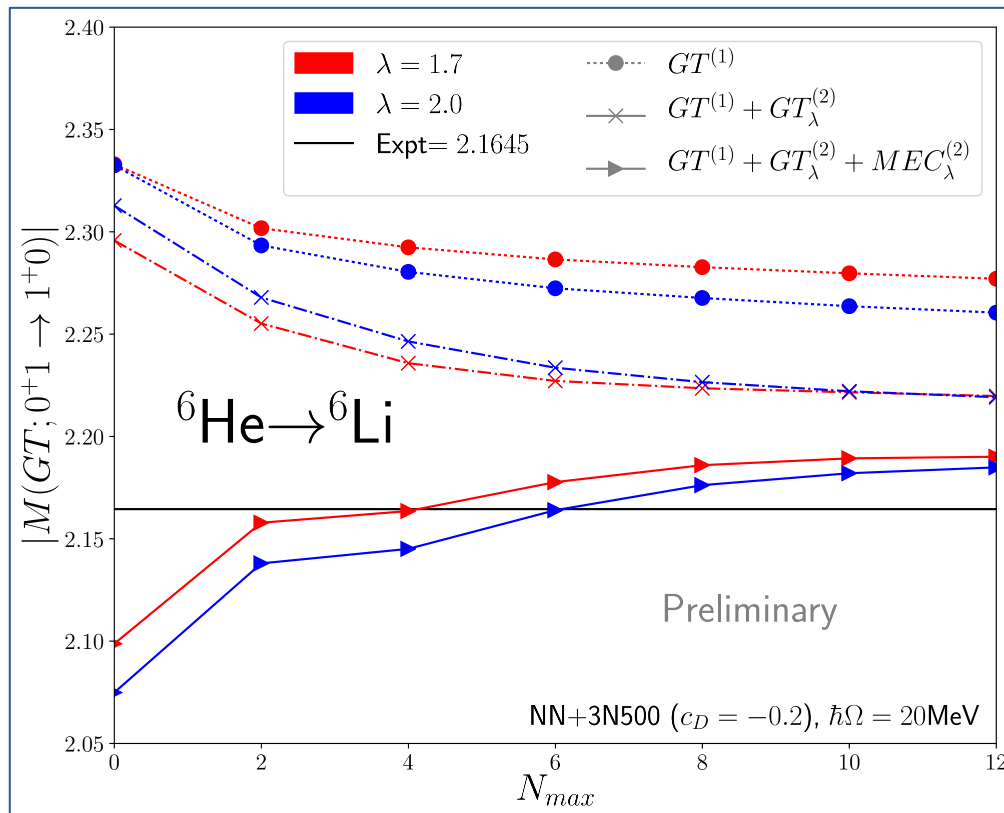
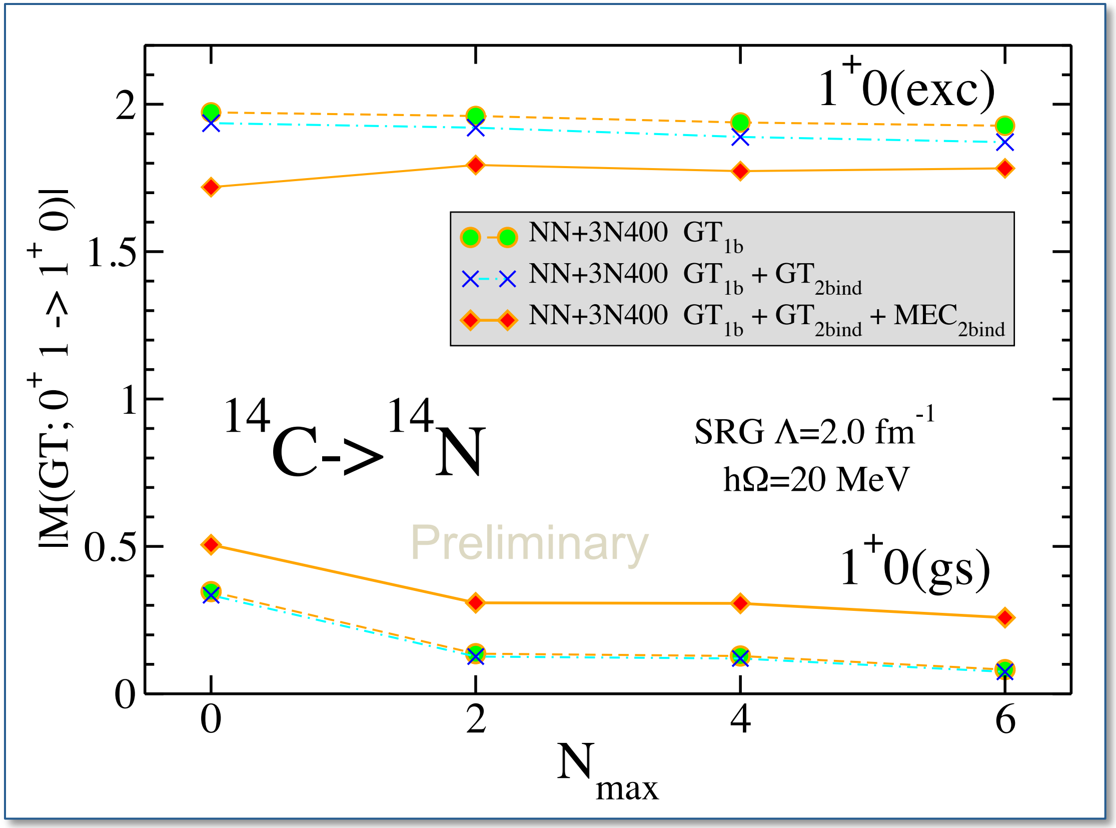


$^{14}C \rightarrow {}^{14}N$ β decay

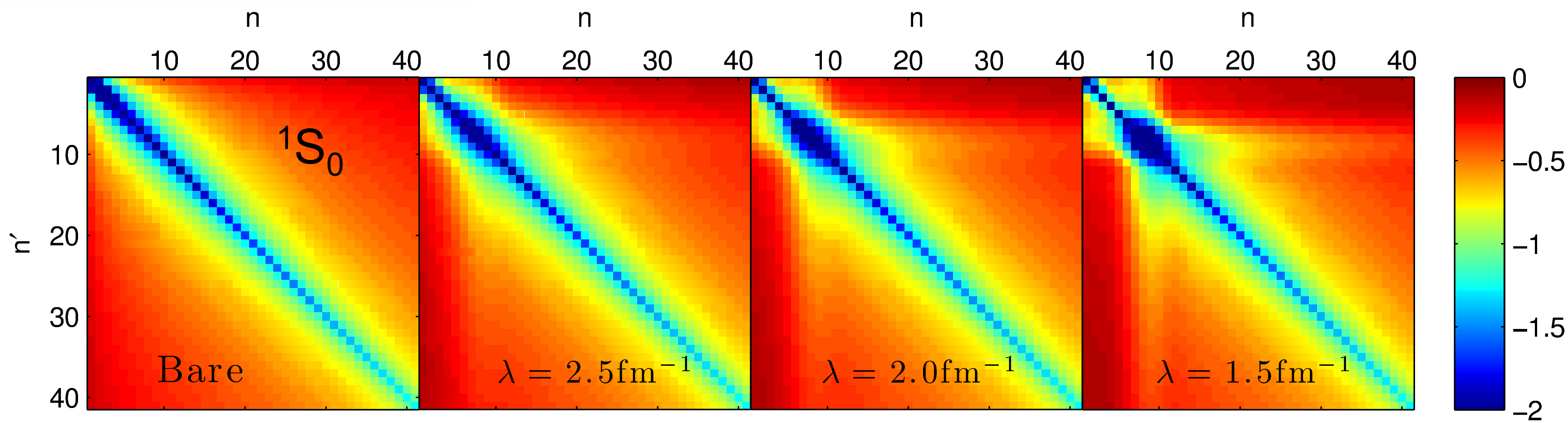


Carbon dating:
Super-allowed transition to the ground state very weak
NNN interaction suppresses it
MEC appears to enhance it

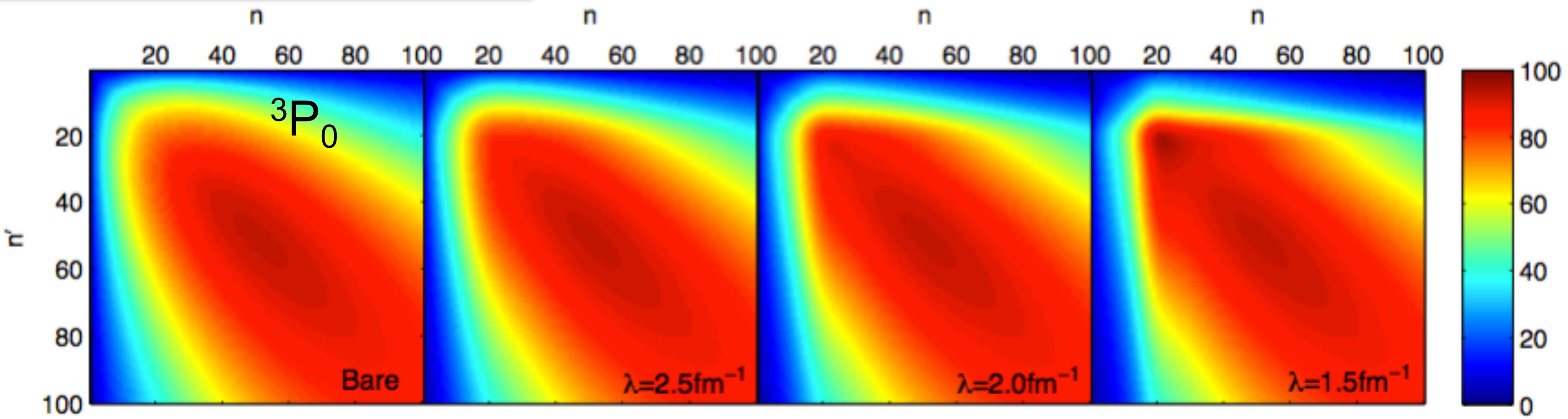
SRG 2-body evolution of the 0νββ operator

In collaboration with Quaglioni, Schuster, Horoi, Engel, Holt

Light-Neutrino 0νββ



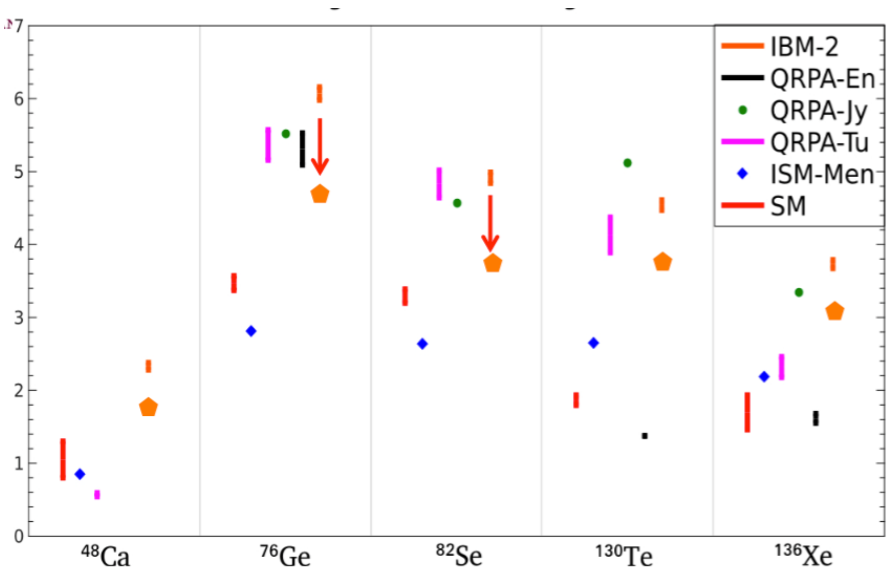
Heavy-Neutrino 0νββ

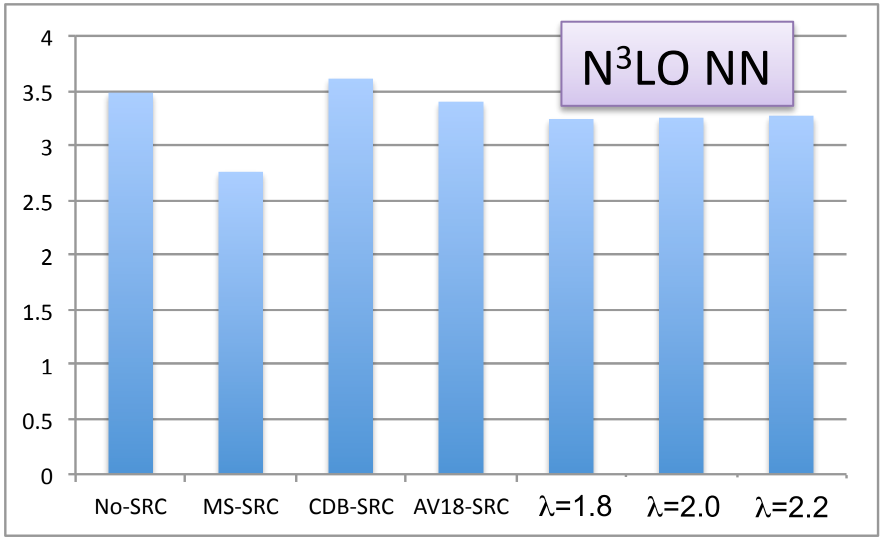


Application to ^{76}Ge $0\nu\beta\beta$ matrix elements

In collaboration with Quaglioni, Schuster, Horoi, Engel, Holt

- Matrix elements for light-neutrino exchange mechanism





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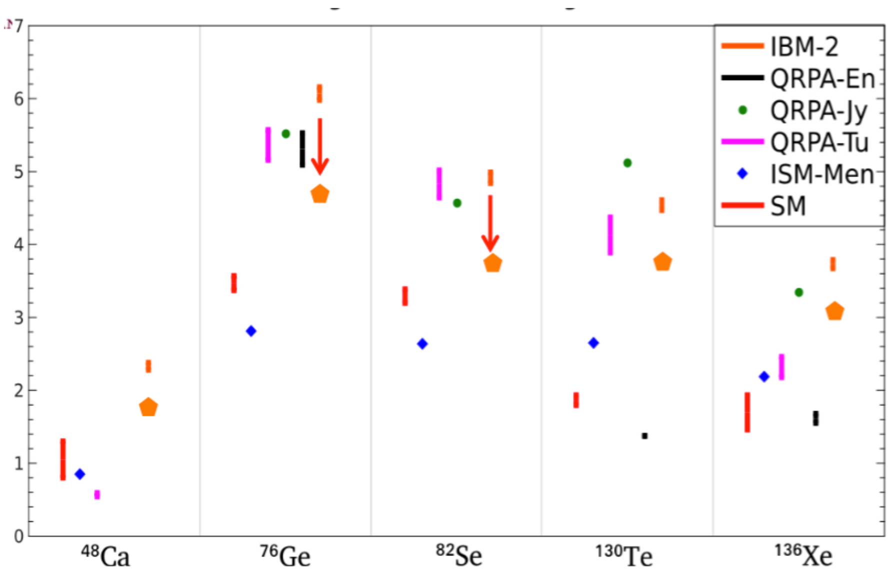

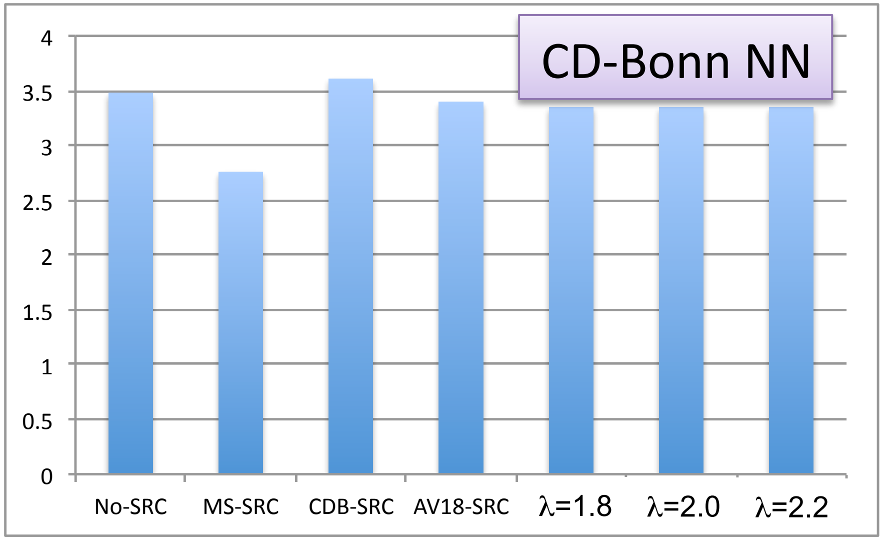

Application to ^{76}Ge $0\nu\beta\beta$ matrix elements
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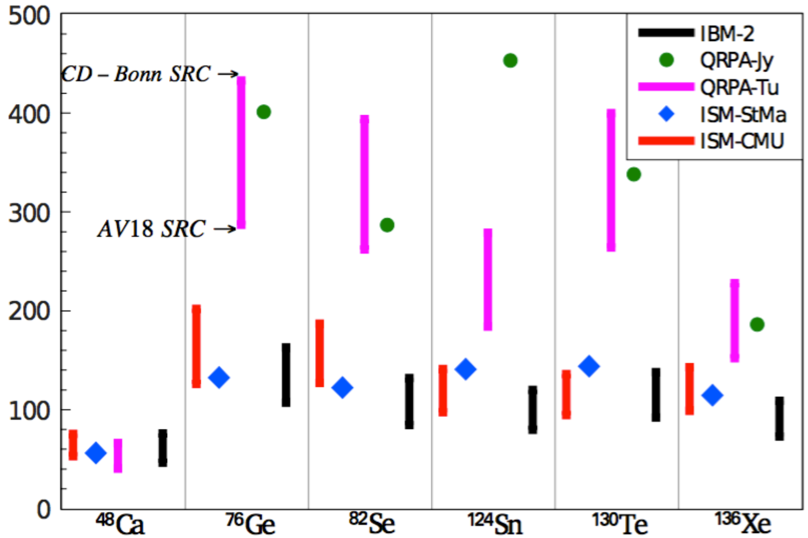
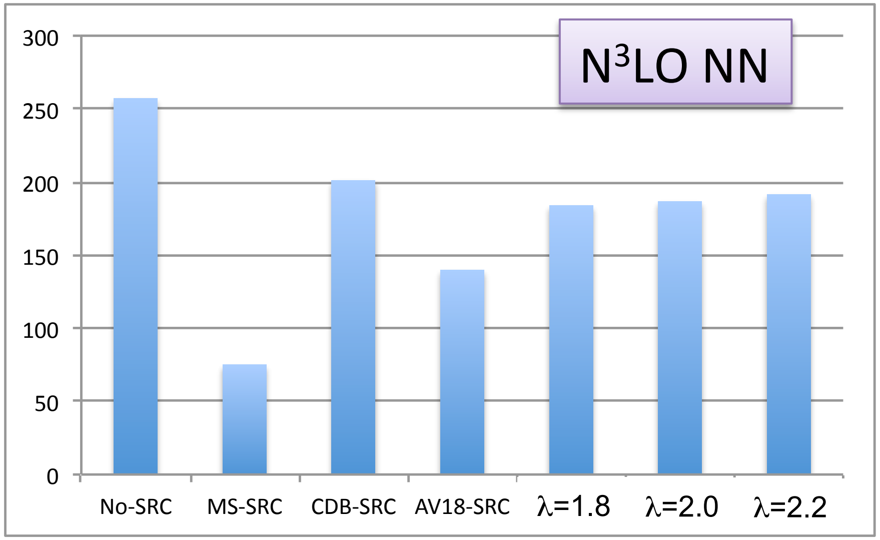
- Matrix elements for heavy-neutrino exchange mechanism





Application to ^{76}Ge $0\nu\beta\beta$ matrix elements

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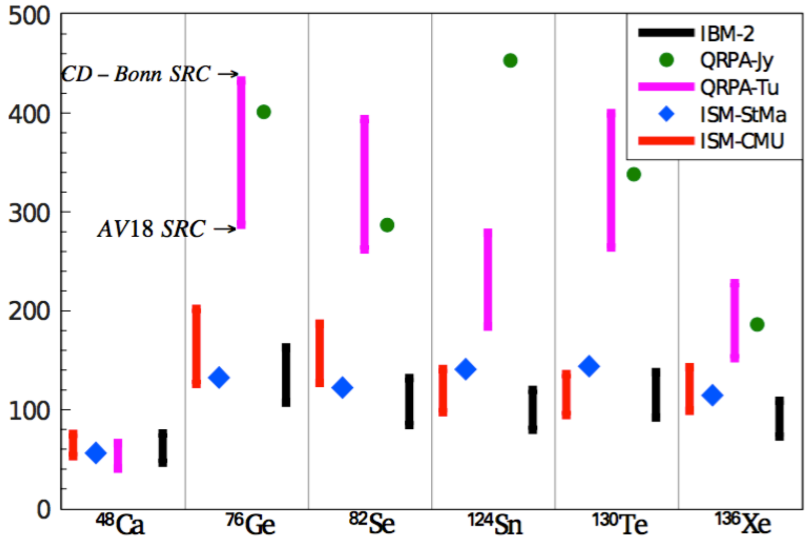

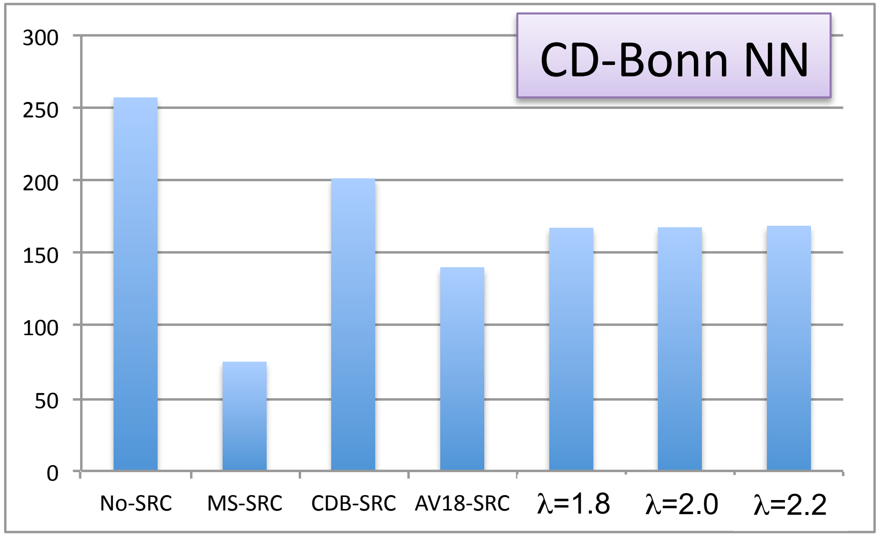

Conclusions and Outlook

- SRG evolution **important** for β decay operators
  - both GT and MEC
  - as well as neutrinoless double-beta decay (especially with heavy neutrino)

- Implemented on two-body level

- Generalization to three-body terms straightforward
  - although technically challenging
  - Codes: NCSMV2b -> MANYEFF
    - Beware of the transformation from relative to single-particle basis