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Neutrinoless Double-beta Decay
June 13 - July 14, 2017

NMEs for neutrinoless double-β decay in multi-reference and symmetry-restored CDFT

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Outline

- ❑ Introduction
- ❑ Nuclear matrix element in CDFT
- ❑ Numerical details
- ❑ Results and discussion
  - ^{150}Nd-Sm: Low-lying states; nuclear matrix elements
  - Implication on neutrino masses
- ❑ Summary and perspectives

Double-beta decay

A second-order weak process： two protons are simultaneously transformed into two neutrons, or vice versa, inside an atomic nucleus.

- Two-neutrino double-beta ($2\nu\beta\beta$) decay

Goeppert-Mayer 1935, Phys. Rev. 48, 512

$$(A, Z) \to (A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

- Neutrinoless double-beta ($0\nu\beta\beta$) decay

Majorana 1937, Nuovo Cim. 14, 171 Furry 1939, Phys. Rev. 56, 1184

$$(A, Z) \to (A, Z+2) + e^- + e^-$$

Lepton number violating process

Majorana's theory of neutrinos $\bar{\nu}_M = \nu_M$

The 2vββ mode is allowed in SM while 0vββ decay would go beyond SM. The $0\nu\beta\beta$ decay occurs only if neutrinos are Majorana particles and lepton numbers can be violated.

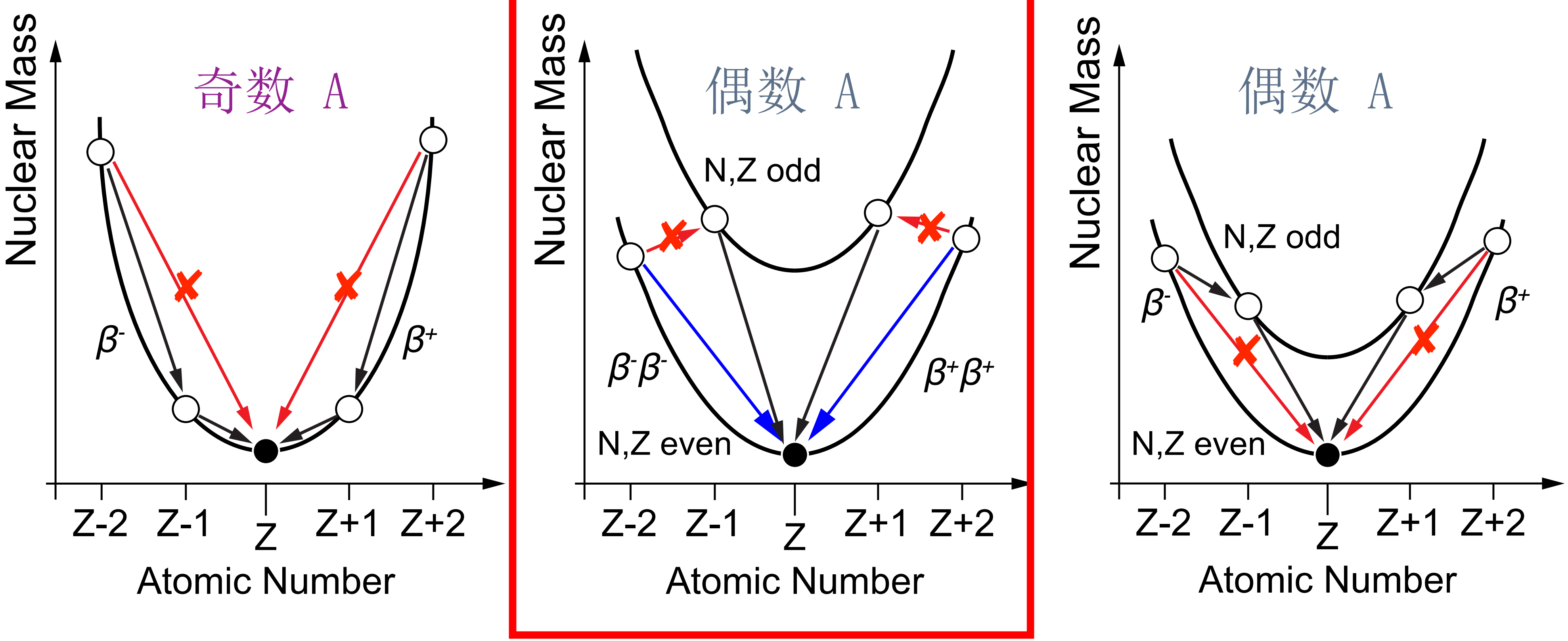
Experiments: searching for $\beta\beta$ decay

❖ Double-beta decay candidates ?

- Even-even nuclei
- Stable against beta decay (pairing correlation)

35 naturally occurred, $\beta^-\beta^-$ decay candidates

Haxton & Stephenson 1984, PPNP 12, 409 (Table 1)



Experiments: searching for $0\nu\beta\beta$ decay

Observed in 11 isotopes: $2\nu\beta\beta$ decay

$T_{1/2} = 10^{18}$-10^{24} yr

Isotope	$T^{2\nu}_{1/2}$ (yr)
^{48}Ca	$4.4^{+0.6}_{-0.5} \cdot 10^{19}$
^{76}Ge	$1.65^{+0.14}_{-0.12} \cdot 10^{21}$
^{82}Se	$(0.92 \pm 0.07) \cdot 10^{20}$
^{96}Zr	$(2.3 \pm 0.2) \cdot 10^{19}$
^{100}Mo	$(7.1 \pm 0.4) \cdot 10^{18}$
^{100}Mo-^{100}Ru(0^+_2)	$6.7^{+0.5}_{-0.4} \cdot 10^{20}$
^{116}Cd	$(2.87 \pm 0.13) \cdot 10^{19}$
^{128}Te	$(2.0 \pm 0.3) \cdot 10^{24}$
^{130}Te	$(6.9 \pm 1.3) \cdot 10^{20}$
^{136}Xe	$(2.19 \pm 0.06) \cdot 10^{21}$
^{150}Nd	$(8.2 \pm 0.9) \cdot 10^{18}$
^{150}Nd-^{150}Sm(0^+_2)	$1.2^{+0.3}_{-0.2} \cdot 10^{20}$
^{238}U	$(2.0 \pm 0.6) \cdot 10^{21}$
^{130}Ba, ECEC	$\sim 10^{21}$

Barabash 2015, Nucl. Phys. A 935, 52

$0\nu\beta\beta$ decay: NOT observed !

$T_{1/2} > 10^{19}$-10^{26} yr

new

Isotope	$T^{0\nu}_{1/2}$ (yr)	Collaboration	Year
^{48}Ca	$> 5.8 \cdot 10^{22}$	ELEGANT VI	2008
^{76}Ge	$> 5.3 \cdot 10^{25}$	GERDA	2017
^{82}Se	$> 3.6 \cdot 10^{23}$	NEMO-3	2011
^{96}Zr	$> 9.2 \cdot 10^{21}$	NEMO-3	2010
^{100}Mo	$> 1.1 \cdot 10^{24}$	NEMO-3	2014
^{116}Cd	$> 1.7 \cdot 10^{23}$	Solotvina	2003
^{124}Sn	$> 5.0 \cdot 10^{19}$	KIMS	2009
^{128}Te	$> 1.6 \cdot 10^{24}$	geochemistry	2011
^{130}Te	$> 2.8 \cdot 10^{24}$	CUORICINO	2011
^{136}Xe	$> 3.4 \cdot 10^{25}$	KamLAND-Zen	2013
^{150}Nd	$> 2.0 \cdot 10^{22}$	NEMO-3	2016

Schwingenheuer 2013, Ann. Phys. (Berlin) 525, 269

The calculation of the NME requires two main ingredients : One is **the decay operator**, which reflects the mechanism governing the decay process. The other is **the wave functions of the initial and final states**.

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Nuclear matrix elements

❖ The $0\nu\beta\beta$-decay rate

$$\Gamma^{0\nu} = G^{0\nu}(Q_{\beta\beta}, Z) \times |M^{0\nu}|^2 \times |\langle m_\nu \rangle|^2$$

Unknown

- Kinematic phase space factor $G^{0\nu}(Q_{\beta\beta}, Z)$ can be accurately determined.

Kotila & Iachello 2012, PRC 85, 034316

- Nuclear matrix element $M^{0\nu}$ depend on nuclear structure models.

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

Accurate nuclear matrix elements are crucial for extracting the effective neutrino mass.

Nuclear models for NME

Commonly used nuclear models:

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

- ❖ Configuration-interacting shell model (CISM) — Strasbourg-Madrid; Michigan; Tokyo
- ❖ Quasiparticle random phase approximation (QRPA) — Tübingen; Jyväskylä; UNC-Chapel Hill, Gilin
- ❖ Interacting boson model (IBM) — Yale
- ❖ Projected Hartree-Fock-Bogoliubov (PHFB) — Lucknow-UNAM
- ❖ Energy density functional (EDF) — GSI-Madrid

Non-relativistic models —
Non-relativistic approximation for decay operator

Call for comparative studies within a relativistic framework !

Covariant density functional theory (CDFT)

Covariant Density Functional Theory

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

- Relativistic Mean Field (RMF) theory

  Ring 1996, PPNP 37, 193; Vretenar, Afanasjev, Lalazissis, & Ring 2005, Phys. Rep. 409, 101
  Meng, Toki, Zhou, Zhang, Long, & Geng 2006, PPNP 57, 470
  Meng (editor) 2016, Int. Rev. Nucl. Phys. Vol. 10, World Scientific

- Beyond mean-field correlations
  - Angular momentum projection (AMP)
    Niksic, Vretenar, & Ring 2006, PRC 73, 034308
    Yao, Meng, Pena-Arteaga, & Ring 2008, CPL 25, 3609
  - Parity projection
    Yao, Zhou, & Li 2015, PRC 92, 041304(R)
  - Particle number projection (PNP)
    Niksic, Vretenar, & Ring 2006, PRC 74, 064309
    Yao, Hagino, Li, Meng, & Ring 2014, PRC 89, 054306
  - Generator coordinate method (GCM)

    Niksic, Vretenar, & Ring 2006, PRC 73, 034308
    Niksic, Vretenar, & Ring 2006, PRC 74, 064309
    Yao, Meng, Ring, & Vretenar 2010, PRC 81, 044311
    Yao, Mei, Chen, Meng, Ring, & Vretenar 2011, PRC 83, 014308

Relativistic description for the nuclear matrix element of $0\nu\beta\beta$ decay based on CDFT.

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Nuclear theory

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- *Ab inito*

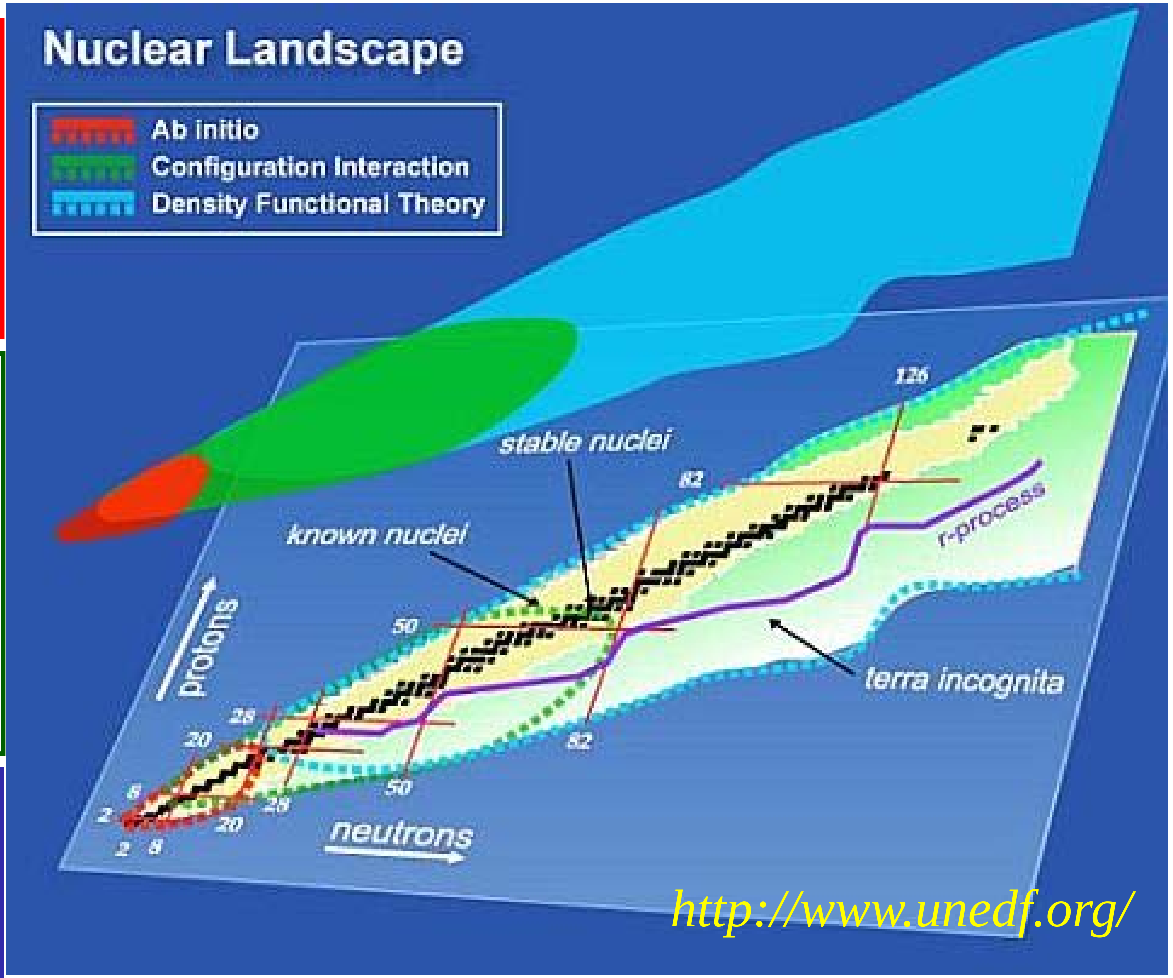
  Navratil, Vary, Barrett Phys. Rev. Lett. 84 (2000) 5728
  Bogner, Furnstahl, Schwenk
  Prog. Part. Nucl. Phys. 65 (2010) 94
  …

- Shell model

  Caurier, Martínez-Pinedo, Nowacki, Poves, Zuker,
  Rev. Mod. Phys. 77 (2005) 427
  Otsuka, Honma, Mizusaki, Shimizu, Utsuno,
  Prog. Part. Nucl. Phys.47(2001)319
  Brown, Prog. Part. Nucl. Phys. 47 (2001) 517
  …

- **Density functional theory**

  Jones and Gunnarsson,
  Rev. Mod. Phys., 61 (1989) 689
  Bender, Heenen, Reinhard,
  Rev. Mod. Phys., 75 (2003) 121
  Ring, Prog. Part. Nucl. Phys.37(1996)193
  Meng, Toki, Zhou, Zhang, Long, Geng,
  Prog. Part. Nucl. Phys. 57 (2006) 470
  …

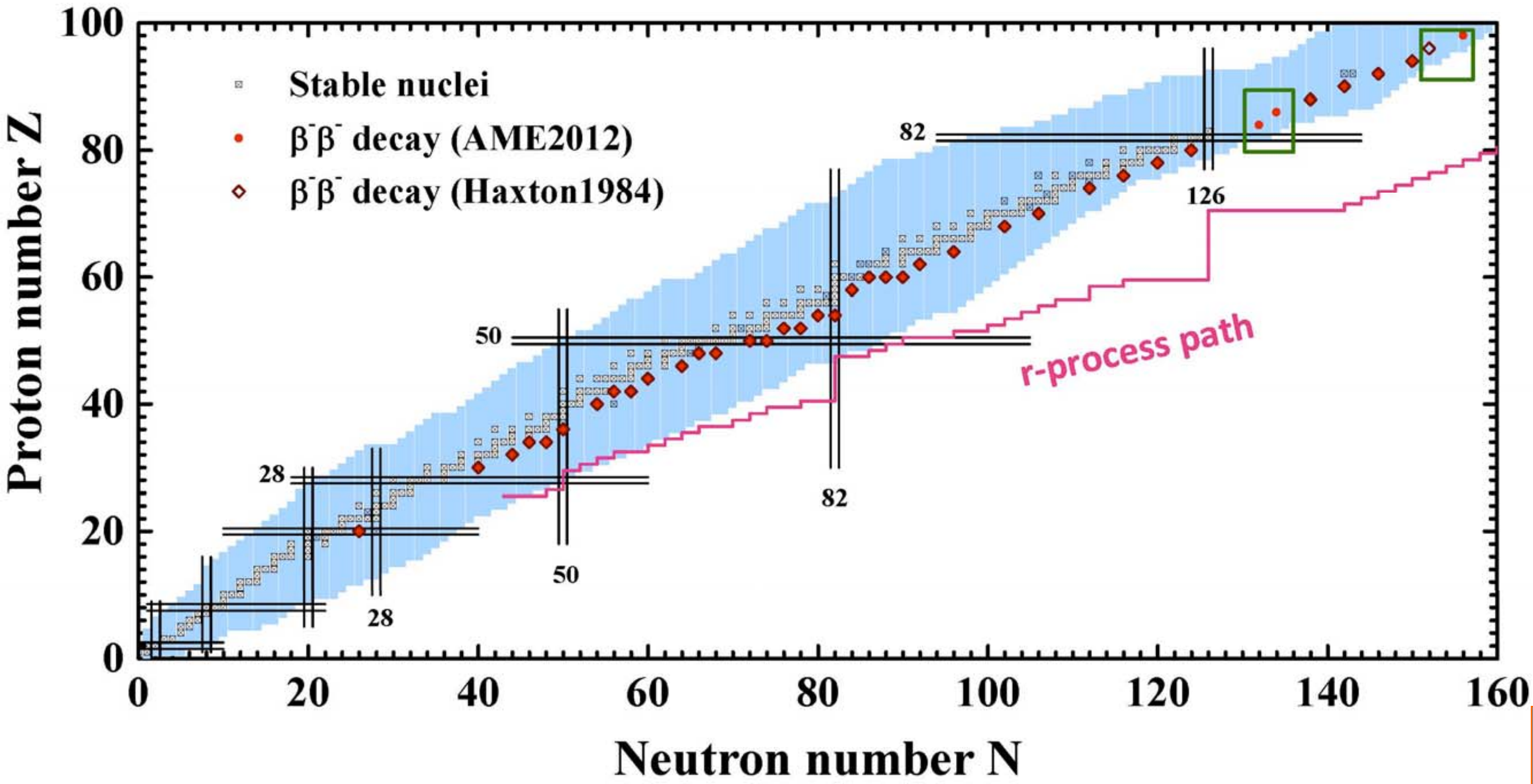


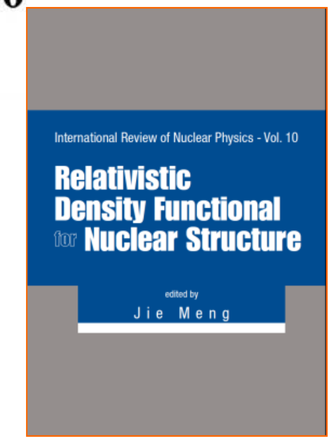
密度泛函理论有希望给出核素图上所有原子核性质的统一描述

Relativistic Density Functional for Nuclear Structure，International Review of Nuclear Physics Vol 10（World Scientific, 2016）

Nuclear neutrinoless double β-decay

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$





Relativistic Density Functional for Nuclear Structure， International Review of Nuclear Physics Vol 10（World Scientific, 2016）

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Hohenberg-Kohn theorem (1964)

The exact energy of a quantum mechanical many body system is a functional of the local density $\rho(\mathbf{r})$

$$E[\rho] = \langle \Psi | H | \Psi \rangle$$

This functional is universal. It does not depend on the system, only on the interaction.

One obtains the exact density $\rho(\mathbf{r})$ by a variation of the functional with respect to the density

note:

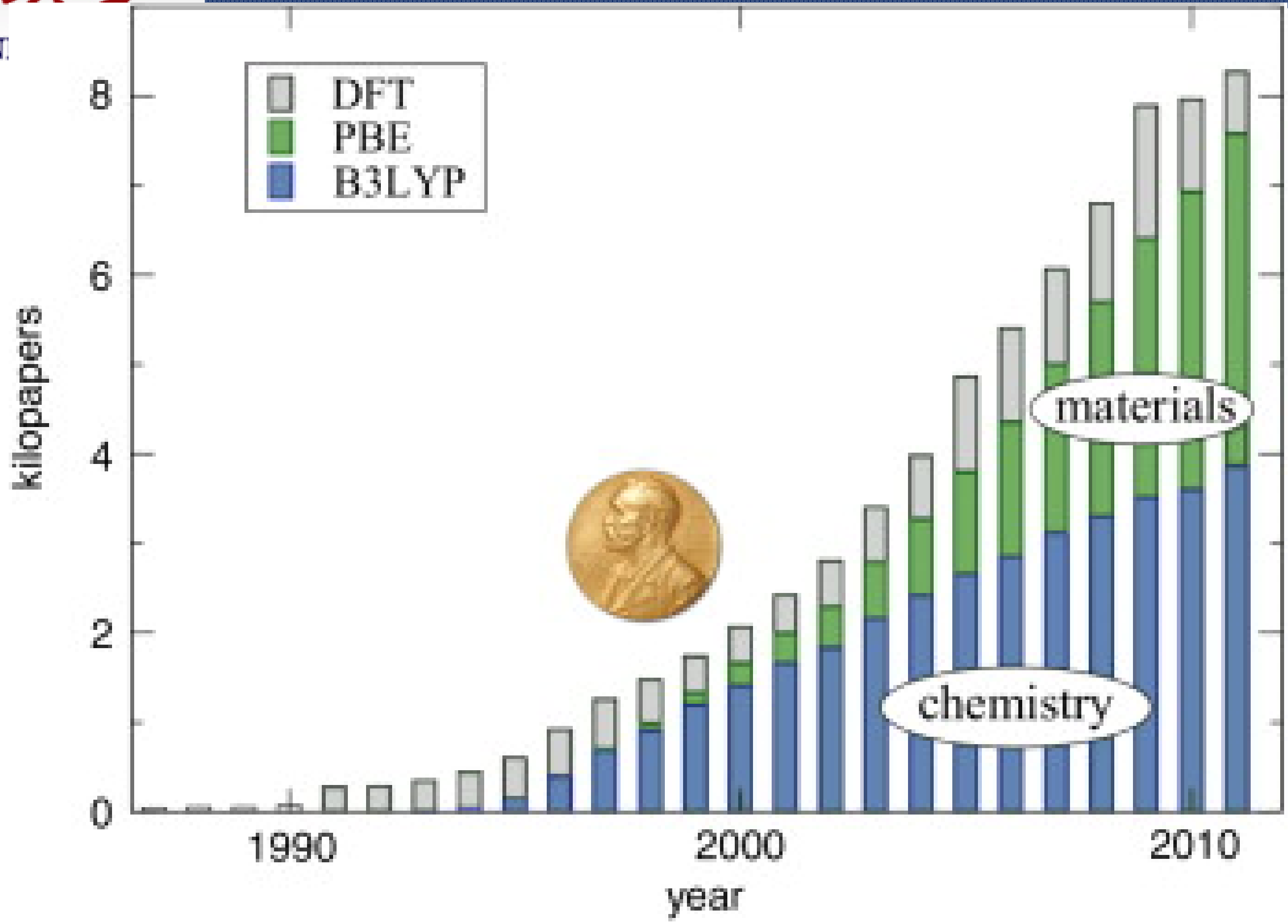
$\rho(\mathbf{r})$ is a function of 3 variables.

$\Psi(\mathbf{r}_1 \ldots \mathbf{r}_N)$ is a function of $3N$ variables.

Hohenberg

Kohn

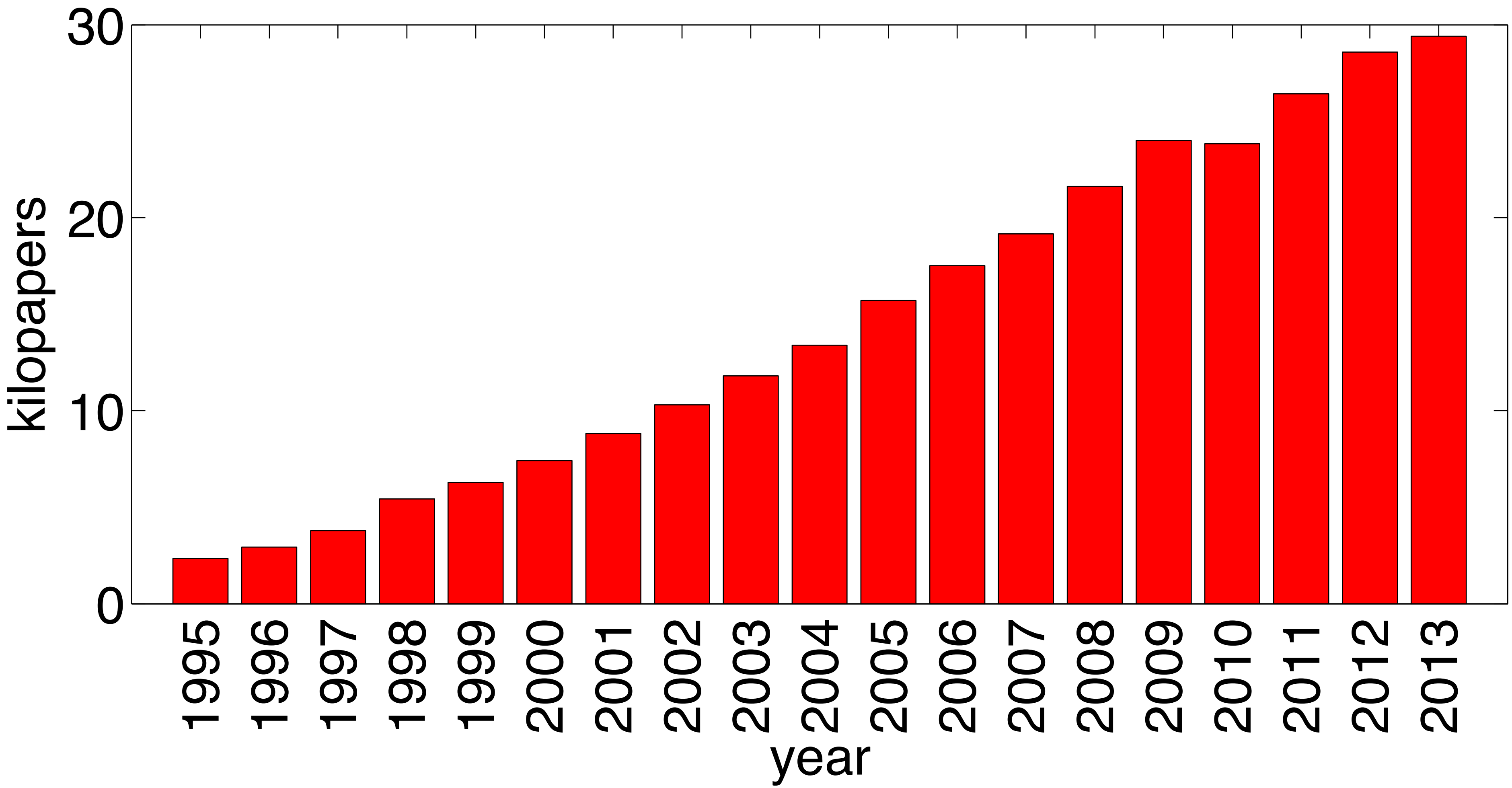
Nobel Prize in Chemistry 1998



The numbers of papers (in kilopapers) corresponding to the search of a topic “DFT” in Web of Knowledge (grey) for different and the most popular density functional potentials: B3LYP citations (blue), and PBE citations (green, on top of blue).

K. Burke, Perspective on density functional theory, J. Chem. Phys., 136 (2012) 150901 [1-9]

DFT papers



DFT: A Theory Full of Holes, Aurora Pribram-Jones, David A. Gross, Kieron Burke, Annual Review of Physical Chemistry (2014).

Density functional theory in nuclei

Nuclear DFT has been introduced by **effective Hamiltonians**: by Vautherin and Brink (1972) using the Skyrme model as a vehicle

$$E = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | \hat{H}_{eff}(\hat{\rho}) | \Phi \rangle = E[\hat{\rho}]$$

Based on the philosophy of Bethe, Goldstone, and Brueckner one has a density dependent interaction in the nuclear interior G(ρ)

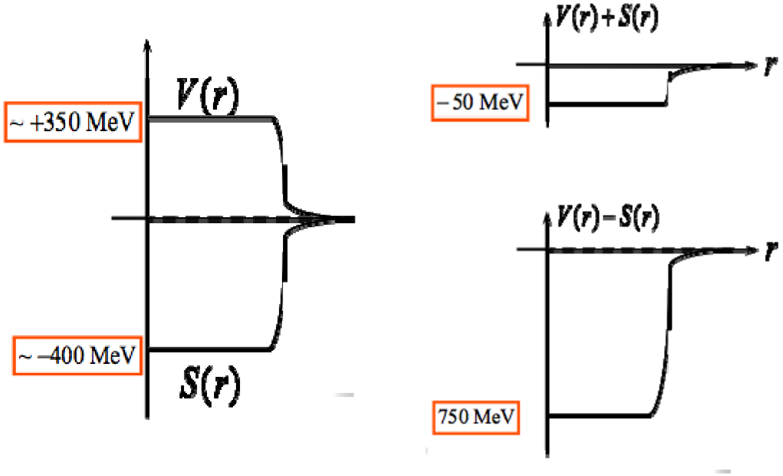
At present, the ansatz for E(ρ) is phenomenological:

- Skyrme: non-relativistic, zero range
- Gogny: non-relativistic, finite range (Gaussian)
- CDFT: Covariant density functional theory

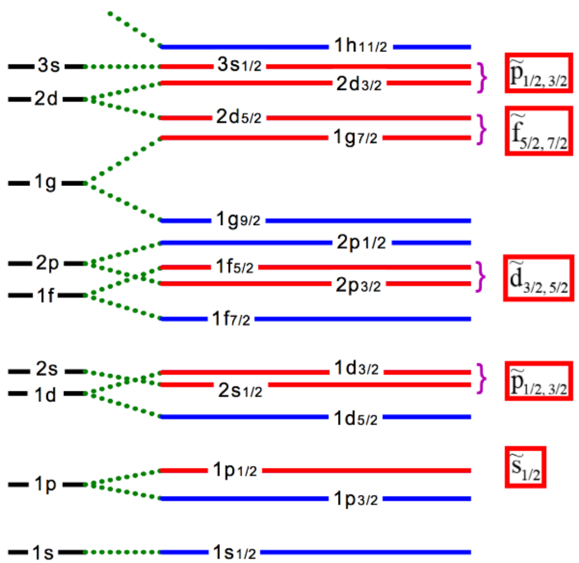
Why Covariant?

P. Ring Physica Scripta, T150, 014035 (2012)

- ✓ Spin-orbit automatically included
- ✓ Lorentz covariance restricts parameters
- ✓ Pseudo-spin Symmetry
- ✓ Connection to QCD: big V/S ~ ±400 MeV
- ✓ Consistent treatment of time-odd fields
- ✓ Relativistic saturation mechanism
- ✓ … **Liang, Meng, Zhou, Physics Reports 570 : 1-84 (2015).**



Pseudospin symmetry



$\tilde{p}_{1/2,3/2}$, $\tilde{f}_{5/2,7/2}$, $\tilde{d}_{3/2,5/2}$, $\tilde{p}_{1/2,3/2}$, $\tilde{s}_{1/2}$

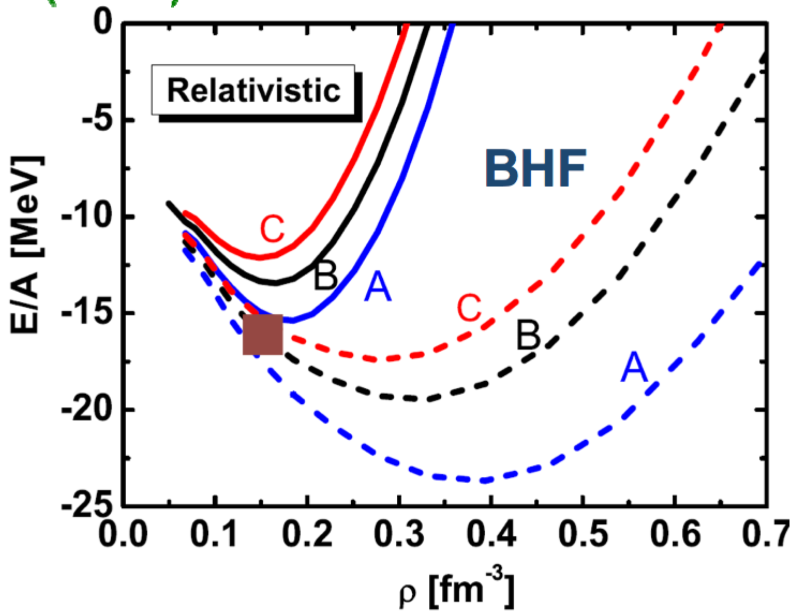
$$\begin{array}{lll} n-1, & l+2, & j=l+3/2 \\ n, & l, & j=l+1/2 \end{array}$$

$$\begin{array}{lcl} \text{pseudo-orbit} & : & \tilde{l}=l+1 \\ \text{pseudo-spin} & : & \tilde{s}=1/2 \end{array}$$

Hecht & Adler NPA137(1969)129

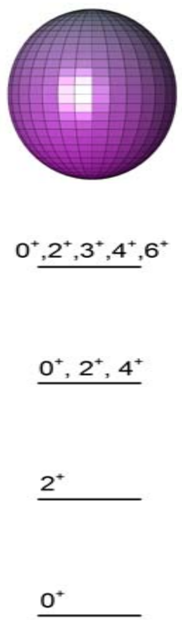
Arima, Harvey & Shimizu PLB 30(1969)517

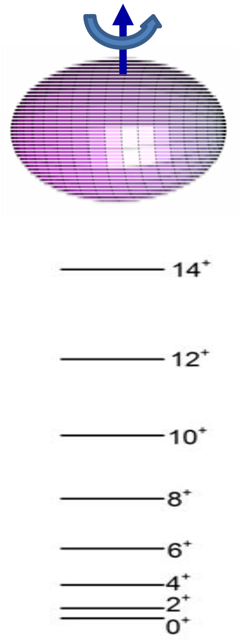
Ginocchio PRL 78, 436

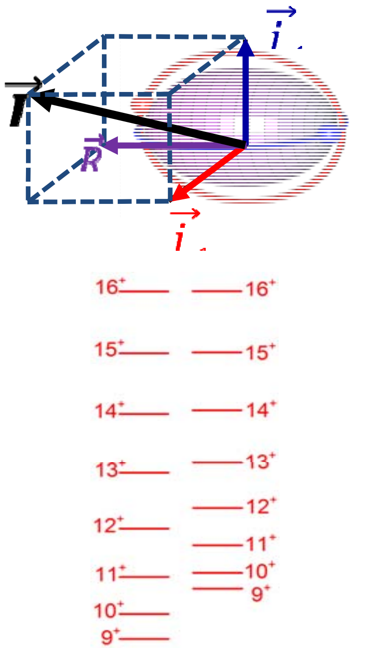


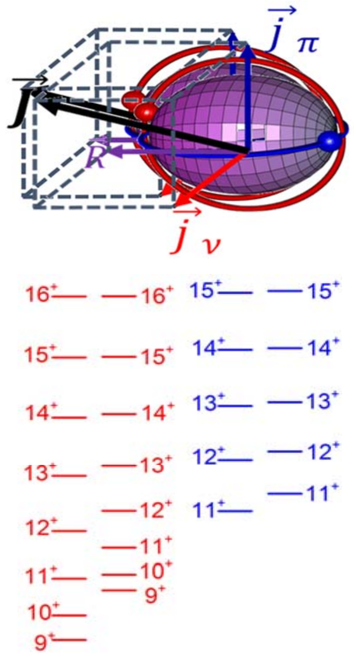
Brockmann & Machleidt, PRC42, 1965 (1990)

Novel excitation modes in atomic nucleus









$$H = \frac{1}{2}\sum_{\mu}\{B_2|\dot{\alpha}_{2\mu}|^2 + C_2|\alpha_{2\mu}|^2\}$$

$$H = \sum_{i=1}^{3}\frac{\hat{R}_i^2}{2\mathcal{J}_i}$$

$$H = \sum_{i=1}^{3}\frac{\hat{R}_i^2}{2\mathcal{J}_i} + \sum_{\tau\nu}\varepsilon_{\tau\nu}a_{\tau\nu}^{+}a_{\tau\nu}$$

$$H(q) = \sum_{i=1}^{3}\frac{\hat{R}_i^2}{2\mathcal{J}_i(q)} + \sum_{\tau\nu}\varepsilon_{\tau\nu}(q)a_{\tau\nu}^{+}(q)a_{\tau\nu}(q) + V(q)$$

vibration | **Rotation** | **Chiral Rotation** | **MχD**

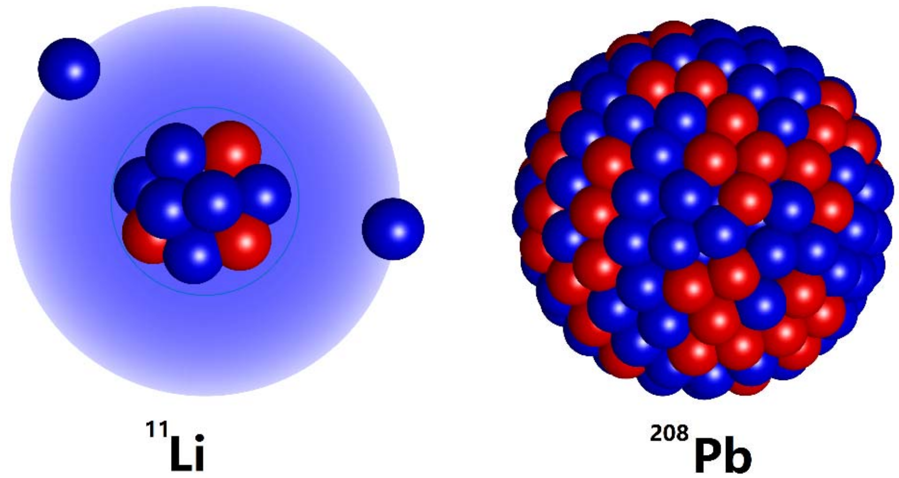
A. Bohr & B. Mottelson

Frauendorf & Meng, NPA 617 (1997) 131

Meng, Peng, Zhang, Zhou, PRC73 (2006) 037303

Self-consistent and microscopic description of halo

Meng, Toki, Zhou, Zhang, Long, Geng,
Progress in Particle and Nuclear Physics
57 (2006) 470-563



CDFT → Spin-orbit coupling

Bogoliubov equation → Pairing correlation

coordinate space → Continuum, discrete state

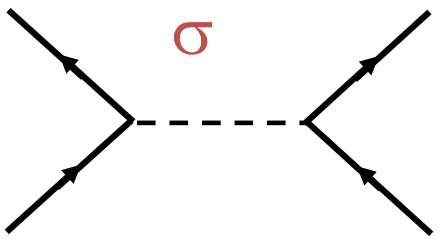
Pairing correlation + Continuum, discrete state → Coupling between bound state and continuum

Spin-orbit coupling + Coupling between bound state and continuum → Halo phenomena

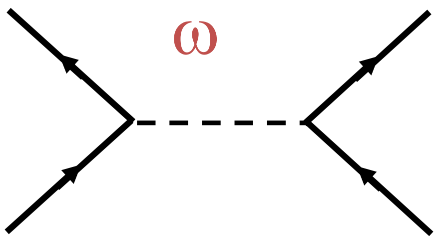
Brief introduction of CDFT

CDFT：Relativistic quantum many-body theory based on DFT and effective field theory for strong interaction

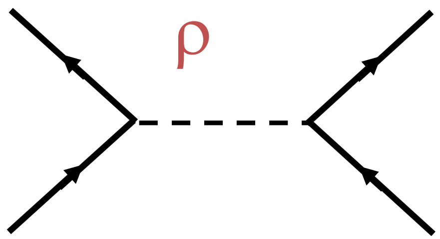
Strong force: Meson-exchange of the nuclear force



$(J^{\pi}\ T) = (0^{+}0)$



$(J^{\pi}\ T) = (1^{-}0)$



$(J^{\pi}\ T) = (1^{-}1)$

Sigma-meson:
attractive scalar field

Omega-meson:
Short-range repulsive

Rho-meson:
Isovector field

Electromagnetic force: The photon

Covariant Density Functional Theory

Elementary building blocks

$$(\bar{\psi}\mathcal{O}_\tau\Gamma\psi) \qquad \mathcal{O}_\tau \in \{1, \tau_i\} \qquad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$$

Densities and currents

Isoscalar-scalar $$\rho_S(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r})\psi_k(\mathbf{r})$$

Isoscalar-vector $$j_\mu(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r})\gamma_\mu\psi_k(\mathbf{r})$$

Isovector-scalar $$\vec{\rho}_S(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r})\vec{\tau}\psi_k(\mathbf{r})$$

Isovector-vector $$\vec{j}_\mu(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r})\vec{\tau}\gamma_\mu\psi_k(\mathbf{r})$$

Energy Density Functional

$$E_{kin} = \sum_k v_k^2 \int \bar{\psi}_k\left(-\gamma\nabla + m\right)\psi_k d\mathbf{r}$$

$$E_{2nd} = \frac{1}{2}\int (\alpha_S\rho_S^2 + \alpha_V\rho_V^2 + \alpha_{tV}\rho_{tV}^2)d\mathbf{r}$$

$$E_{hot} = \frac{1}{12}\int (4\beta_S\rho_S^3 + 3\gamma_S\rho_S^4 + 3\gamma_V\rho_V^4)d\mathbf{r}$$

$$E_{der} = \frac{1}{2}\int (\delta_S\rho_S\Delta\rho_S + \delta_V\rho_V\Delta\rho_V + \delta_{tV}\rho_{tV}\Delta\rho_{tV})d\mathbf{r}$$

$$E_{em} = \frac{e}{2}\int j_\mu^p A^\mu d\mathbf{r}$$

Equations of motion

For system with time invariance:

$$\left[\alpha\cdot\boldsymbol{p}+V(\boldsymbol{r})+\beta\left(M+S(\boldsymbol{r})\right)\right]\psi_i=\varepsilon_i\psi_i$$

$$\begin{cases} V(\boldsymbol{r})=\alpha_V\rho_V(\boldsymbol{r})+\gamma_V\rho_V^3(\boldsymbol{r})+\delta_V\Delta\rho_V(\boldsymbol{r})+\alpha_{TV}\rho_{TV}(\boldsymbol{r})+\delta_{TV}\Delta\rho_{TV}(\boldsymbol{r})+e\dfrac{1-\tau_3}{2}A(\boldsymbol{r}) \\ S(r)=\alpha_S\rho_S+\beta_S\rho_S^2+\gamma_S\rho_S^3+\delta_S\Delta\rho_S \end{cases}$$

Without Klein-Gordon equation

$$\begin{cases} \rho_s(\boldsymbol{r})=\sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_v(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_3(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r}) \\ \rho_c(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\dfrac{1-\tau_3}{2}\psi_i(\boldsymbol{r}) \end{cases}$$

Mean-field

❖ Variation of energy under constraints

$$\boxed{\langle\Phi(q)|\hat{H}|\Phi(q)\rangle = E_{\rm CDF}}$$

$$\delta\langle\Phi(q)|\hat{H} - \sum_{\tau=n,p}\lambda_\tau\hat{N}_\tau - \sum_{\lambda=1,2,3}C_\lambda(\hat{Q}_{\lambda 0}-q_\lambda)^2 - C_{22}(\hat{Q}_{22}-q_{22})^2\ldots|\Phi(q)\rangle = 0$$

① Intrinsic wave functions

$$|\Phi(q)\rangle,\quad q = (\beta_{20},\beta_{30},\beta_{22},\ldots)$$

deformation parameters

$$\beta_{\lambda\mu} \equiv \frac{4\pi}{3AR^\lambda}q_{\lambda\mu},\qquad R = 1.2A^{1/3}$$

- ✘ good particle numbers
- ✘ good angular momentum
- ✘ good parity
- ✘ shape mixing

Beyond mean-field

$$M^{0\nu} = \langle \Psi_F | \hat{\mathcal{O}}^{0\nu} | \Psi_I \rangle$$

❖ Restoration of broken symmetries

② Projected wave functions

$$|JMK; NZ; \pi; q\rangle = \hat{P}^J_{MK} \hat{P}^N \hat{P}^Z \hat{P}^\pi |\Phi(q)\rangle$$

$$\hat{P}^J_{MK} = \frac{2J+1}{8\pi^2} \int d\Omega D^{J*}_{MK}(\Omega) \hat{R}(\Omega)$$

$$\hat{P}^{N_\tau} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi_\tau e^{i\varphi_\tau(\hat{N}_\tau - N_\tau)}$$

$$\hat{P}^{\pi=\pm} = \frac{1}{2}(1+\pi\hat{\mathcal{P}})$$

❖ Mixing of configurations

$$\frac{\delta}{\delta f_\alpha^{JK_a\pi}(q_a)} \frac{\langle \Psi_\alpha^{JM\pi} | \hat{H} | \Psi_\alpha^{JM\pi} \rangle}{\langle \Psi_\alpha^{JM\pi} | \Psi_\alpha^{JM\pi} \rangle} = 0$$

or Hill-Wheeler-Griffin Equation

③ GCM wave functions

$$\left|\Psi_\alpha^{JM\pi}(N,Z)\right\rangle = \sum_{\kappa \in \{q,K\}} f_\alpha^{JK\pi}(q)\, \hat{P}^J_{MK} \hat{P}^N \hat{P}^Z \hat{P}^\pi \left|\Phi(q)\right\rangle, \quad \alpha = 1, 2, \ldots$$

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

For the decay operator, the starting point is the semileptonic charged-current weak Hamiltonian.

By using the long-wave approximation for the outgoing electrons and neglecting the small energy transfer between nucleons, the NME $M^{0\nu}$ of the $0\nu\beta\beta$ decay can be obtained.

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Scattering matrix

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

❖ Hamiltonian of weak interaction:

$$\boxed{\mathcal{H}_\beta(x) = \frac{G_F \cos\theta_C}{\sqrt{2}} j_L^\mu \mathcal{J}_\mu^\dagger + \text{h.c.}}$$

where G_F is Fermi constant, θ_C is Cabbibo angle;

$j_L^\mu = 2(\bar{e}_L \gamma^\mu \nu_{eL})$ Is leptonic current; $\mathcal{J}_\mu^\dagger$ is nucleonic current.

❖ Scattering matrix:

$$\boxed{\langle f | \mathrm{i}T | i \rangle = \langle p_1, p_2; \Psi_F \left| \frac{(-\mathrm{i})^2}{2!} \int d^4x_1 d^4x_2 \hat{T}(\mathcal{H}_\beta(x_1)\mathcal{H}_\beta(x_2)) \right| \Psi_I \rangle}$$

spinor of electron neutrino propagator

$$\langle f | \mathrm{i}T | i \rangle = 4 \left(\frac{G_F \cos\theta_C}{\sqrt{2}} \right)^2 \frac{(-\mathrm{i})^2}{2!} N_{p_1} N_{p_2} \int \boxed{\bar{u}(p_1) e^{\mathrm{i}p_1 x_1}} \gamma^\mu \boxed{\langle 0 | \hat{T}\left(\nu_{eL}(x_1) \nu_{eL}^T(x_2)\right) | 0 \rangle}$$

$$\times \gamma^{\nu T} \boxed{\bar{u}^T(p_2) e^{\mathrm{i}p_2 x_2}} \boxed{\langle \Psi_F | \hat{T}\left(\mathcal{J}_\mu^\dagger(x_1) \mathcal{J}_\nu^\dagger(x_2)\right) | \Psi_I \rangle} d^4x_1 d^4x_2 - (p_1 \leftrightarrow p_2)$$

spinor of electron strong interaction part

$$N_{p_{1,2}} = \frac{1}{(2\pi)^{3/2} \sqrt{2p_{1,2}^0}}$$

Bilenky 2010, In Lecture Notes in Physics, vol. 817. Springer-Verlag

Nucleonic current

$$\langle \Psi_F | \hat{T}\left(\mathcal{J}_\mu^\dagger(x_1)\mathcal{J}_\nu^\dagger(x_2)\right)|\Psi_I\rangle$$

- Vector (V), weak magnetic (M), axial-vector (A), pseudo scalar (P) currents:

Simkovic, Pantis, Vergados, & Faessler 1999, PRC 60, 055502

$$\mathcal{J}_\mu^\dagger(x) = \bar{\psi}(x)\left[\boxed{g_V(q^2)\gamma_\mu} + \boxed{ig_M(q^2)\frac{\sigma_{\mu\nu}}{2m_p}q^\nu} - \boxed{g_A(q^2)\gamma_\mu\gamma_5} - \boxed{g_P(q^2)q_\mu\gamma_5}\right]\tau_-\psi(x)$$

m_p is nucleon mass, q_μ is momentum transfer, $\psi(x)$ is nucleon field;

$\tau_- \equiv (\tau_1 - i\tau_2)/2$ is isospin lowering operator.

Coupling coefficients with form factors (dipole approx.):

$$g_V(q^2) = \frac{g_V}{(1+q^2/M_V^2)^2} \qquad g_A(q^2) = \frac{g_A}{(1+q^2/M_A^2)^2}$$

$$g_M(q^2) = (\mu_p - \mu_n)g_V(q^2) \qquad g_P(q^2) = 2m_p\frac{g_A(q^2)}{q^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)$$

Coupling constants

$g_V = 1$

$g_A = 1.254$

Cutoffs

$M_V = 842$ MeV

$M_A = 1090$ MeV

Magnetic moment

$(\mu_p - \mu_n) = 3.70$

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Numerical details: mean-field

- $0\nu\beta\beta$ nuclei: ^{48}Ca-Ti, ^{76}Ge-Se, ^{82}Se-Kr, ^{96}Zr-Mo, ^{100}Mo-Ru, ^{116}Cd-Sn, ^{124}Sn-Te, ^{130}Te-Xe, ^{136}Xe-Ba, ^{150}Nd-Sm

- Axial deformation

- Spherical Harmonic Oscillator shells:

  10 major shells for A <= 100; 12 major shells for A > 100.

- Relativistic energy density functional: PC-PK1

Zhao, Li, Yao & Meng 2010, PRC 82, 054319

- Pairing correlation: zero-range δ force with a smooth cutoff

$$\boxed{V_\tau^\delta(\boldsymbol{r}_1, \boldsymbol{r}_2) = V_\tau \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)}$$

Vn = −314.55 MeV fm^3, Vp = −346.50 MeV fm^3 , fitted to the average pairing gaps in ^{150}Nd given by the separable force.

Numerical details: beyond mean-field

❖ Angular momentum projection for axial states:

$$\hat{P}^J_{00} = \frac{2J+1}{2}\int_0^\pi \sin\theta d\theta \; d^J_{00}(\theta) e^{i\theta \hat{J}_y}$$

In Gaussian-Legendre integration: $N_\theta = 7$, $\theta \in [\,0, \pi/2\,]$

❖ Particle number projection:

$$\hat{P}^N = \frac{1}{2\pi}\int_0^{2\pi} d\varphi e^{i\varphi(\hat{N}-N)}$$

In Gaussian-Legendre integration: $N_\varphi = 7$, $\varphi \in [\,0, \pi\,]$

❖ Generator coordinate method:

In shape mixing: $Nq = 11$, $\beta_2 \in [\,-0.4, 0.6\,]$, $\Delta\beta_2 = 0.1$

Numerical details: NME part

- Computation

CPU Time around 10^4 hours per nucleus

$$M^{0\nu}(q_I,q_F)=\sum_{abcd}\langle ab|\hat{O}|cd\rangle\times\langle q_F|\left[c_a^{(\pi)\dagger}c_b^{(\pi)\dagger}c_d^{(\nu)}c_c^{(\nu)}\right]\hat{P}^{J=0}\hat{P}^{N}\hat{P}^{Z}\hat{P}^{\pi=+}|q_I\rangle$$

$$M^{0\nu}=\sum_{q_I,q_F}f^{*}_{0_F^+}(q_F)f_{0_I^+}(q_I)M^{0\nu}(q_I,q_F)$$

Number of mesh points to consider:

$N_\theta = 7$

$N_\varphi = 7$

$N_q = 11$

$$N_{tot.} = N_\theta \times N_\varphi^2 \times N_q^2 \sim 10^5$$

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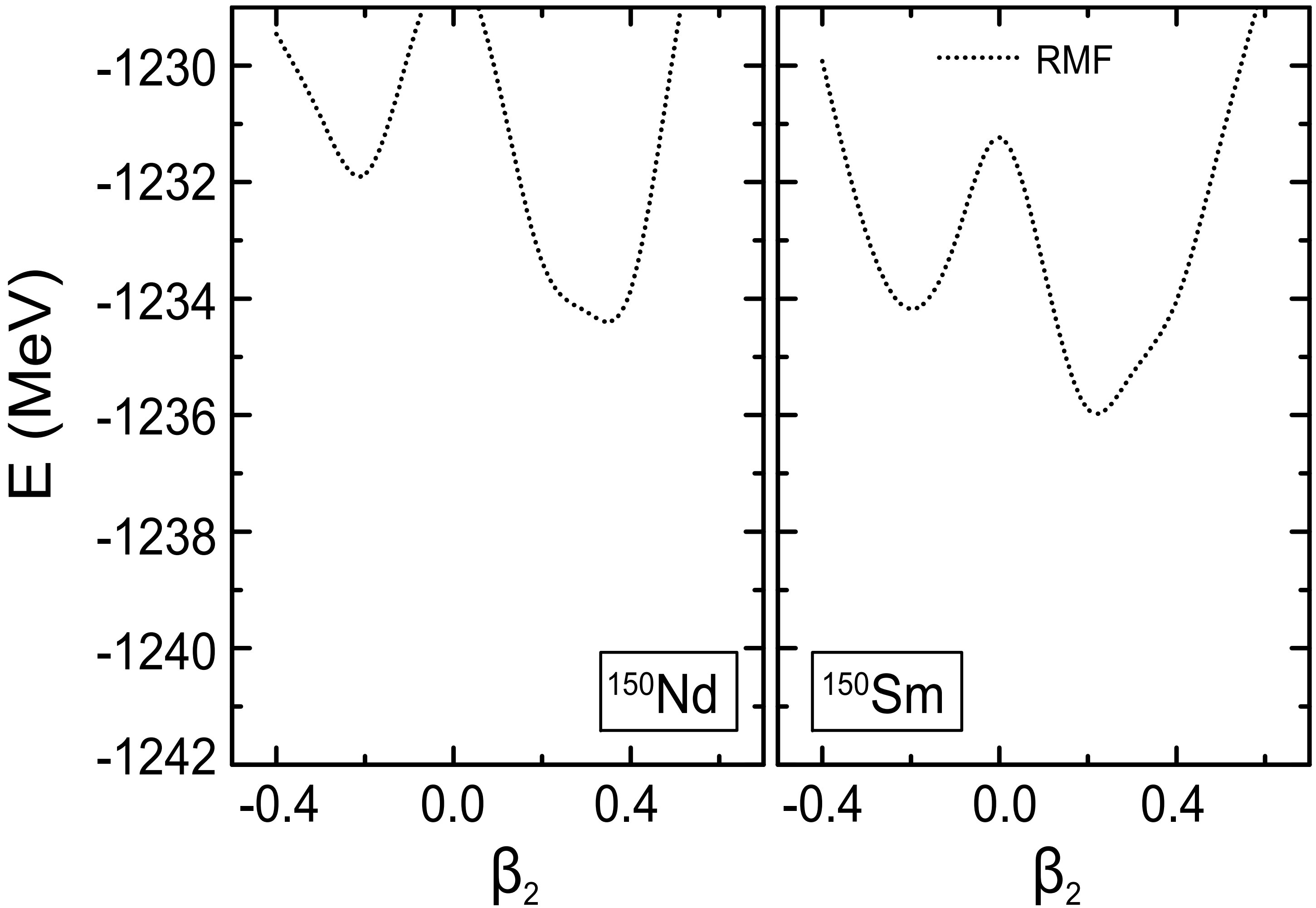
Potential energy curves

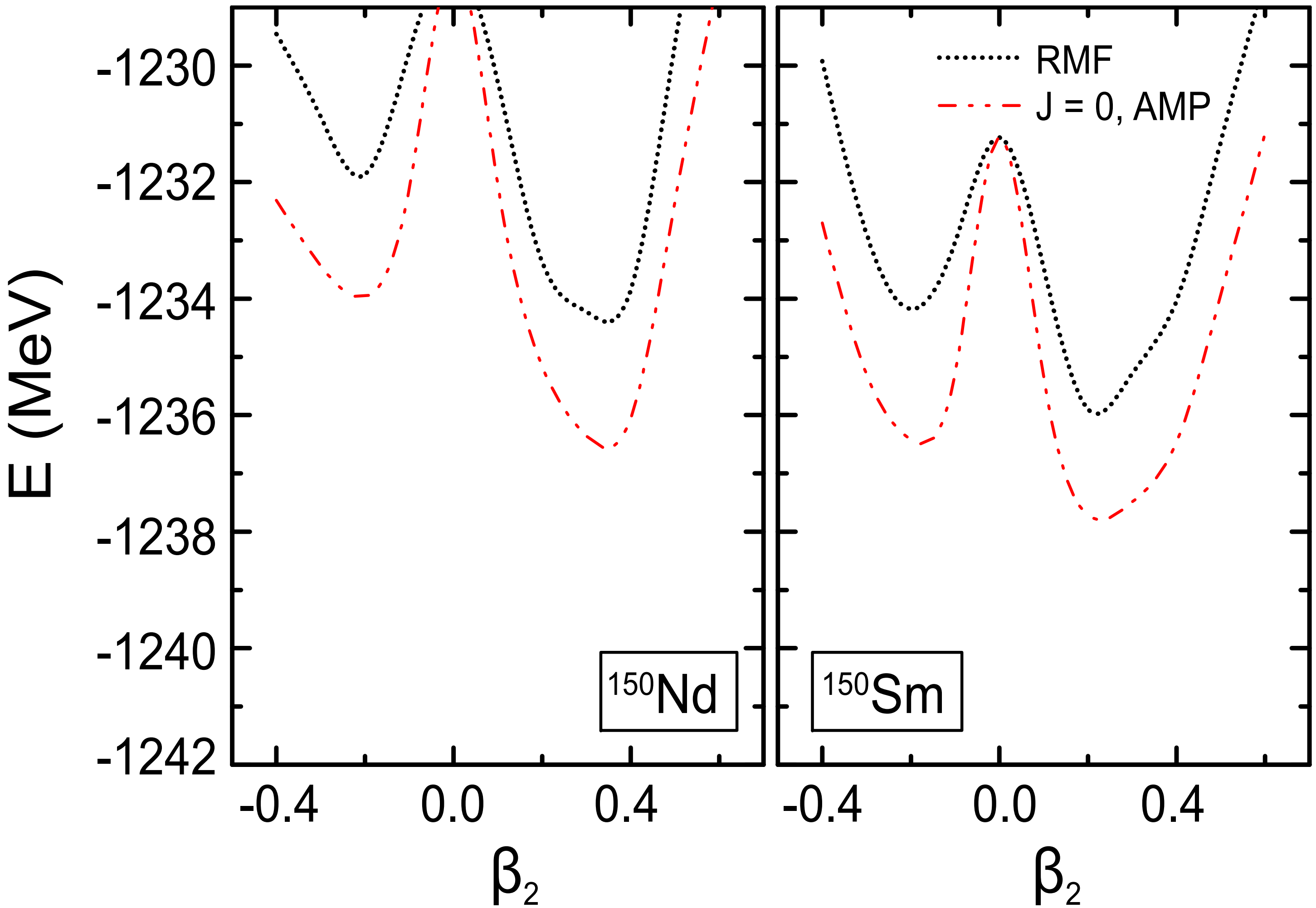
❖ Mean-field



Song, Yao, Ring, & Meng 2014, PRC 90, 054309

Potential energy curves

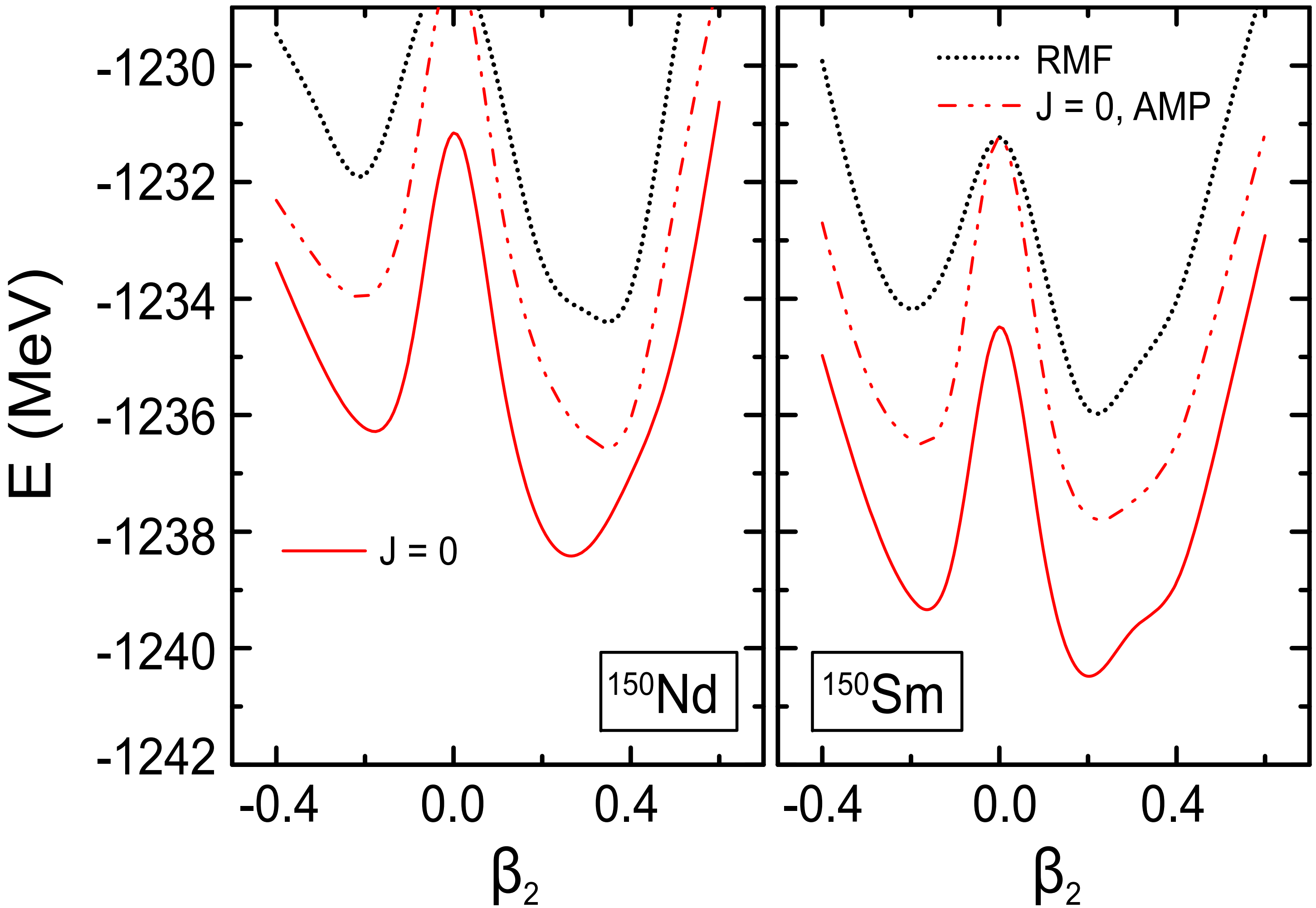
❖ Mean-field ⇨ AMP J=0



Song, Yao, Ring, & Meng 2014, PRC 90, 054309

Potential energy curves

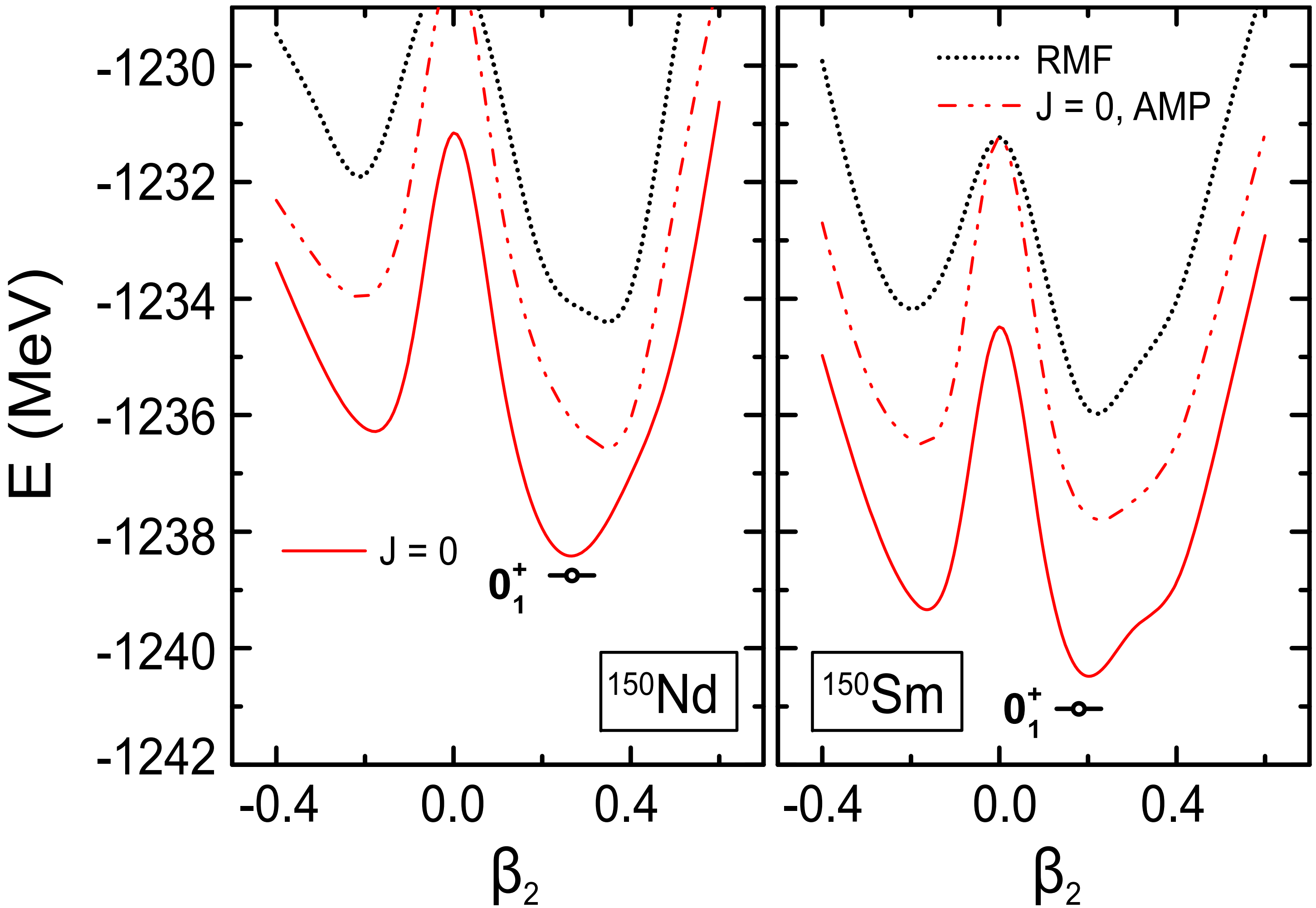
- Mean-field ⇨ AMP J=0 ⇨ PNAMP J=0



Song, Yao, Ring, & Meng 2014, PRC 90, 054309

Potential energy curves

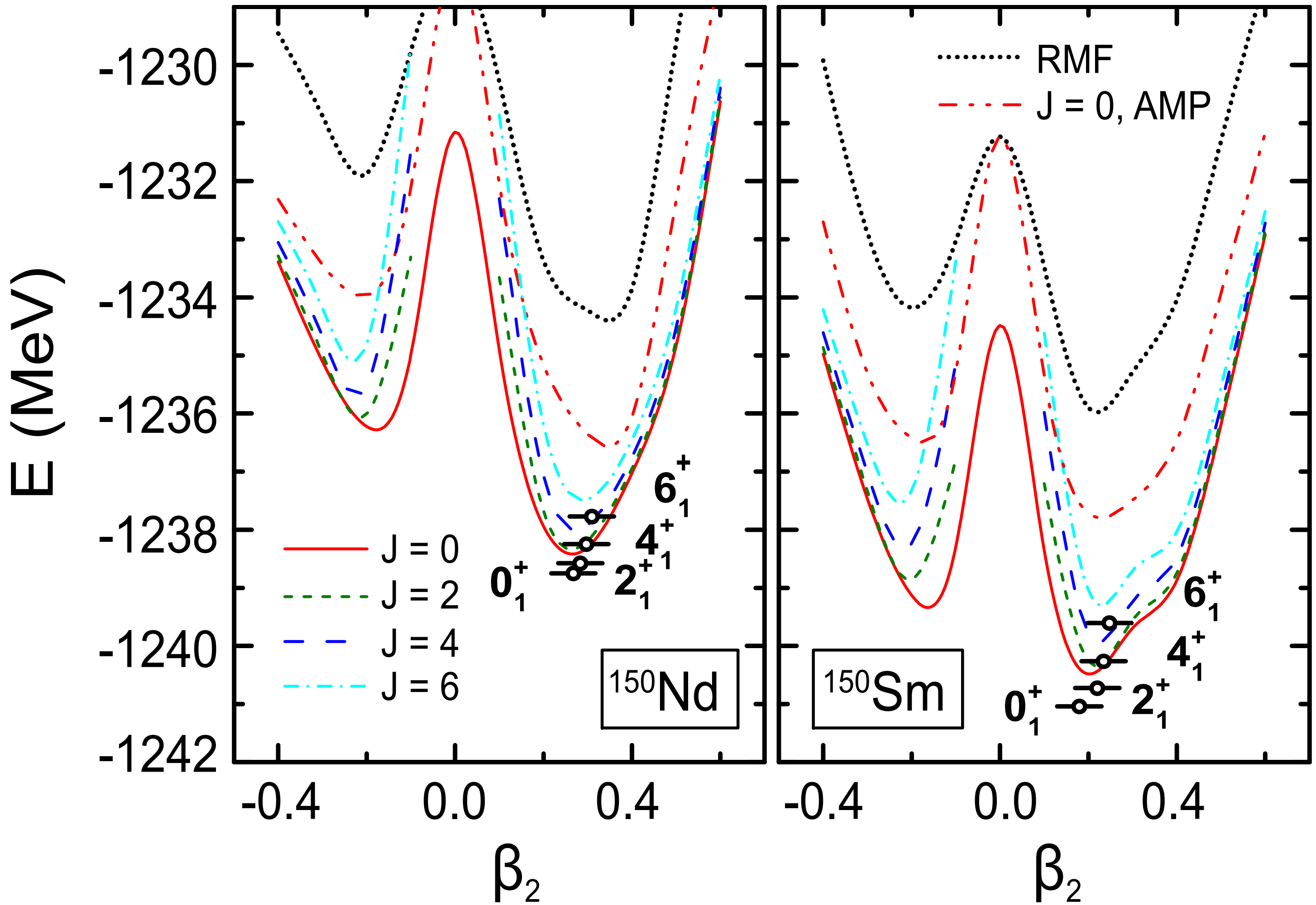
❖ Mean-field ⇨ AMP J=0 ⇨ PNAMP J=0 ⇨ GCM ground state



Song, Yao, Ring, & Meng 2014, PRC 90, 054309

Potential energy curves

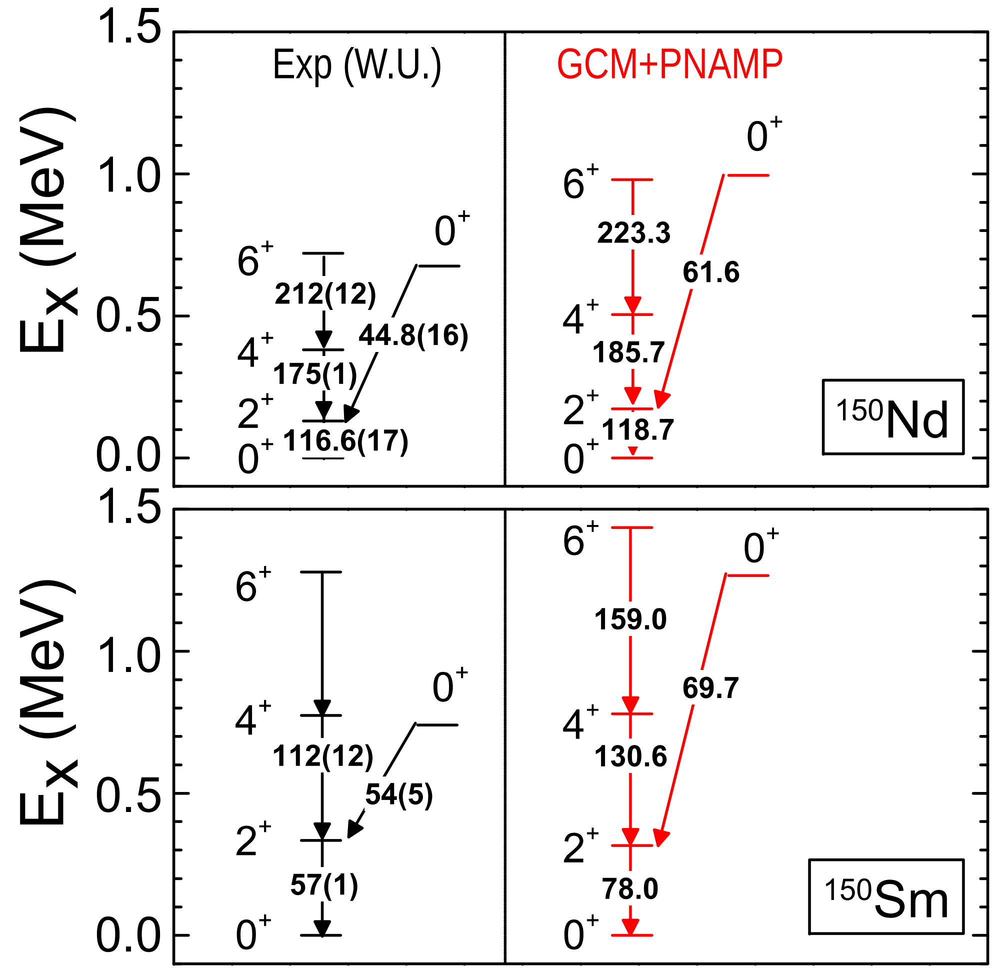
❖ Mean-field ⇨ AMP J=0 ⇨ PNAMP J=0 ⇨ GCM ground state



Song, Yao, Ring, & Meng 2014, PRC 90, 054309

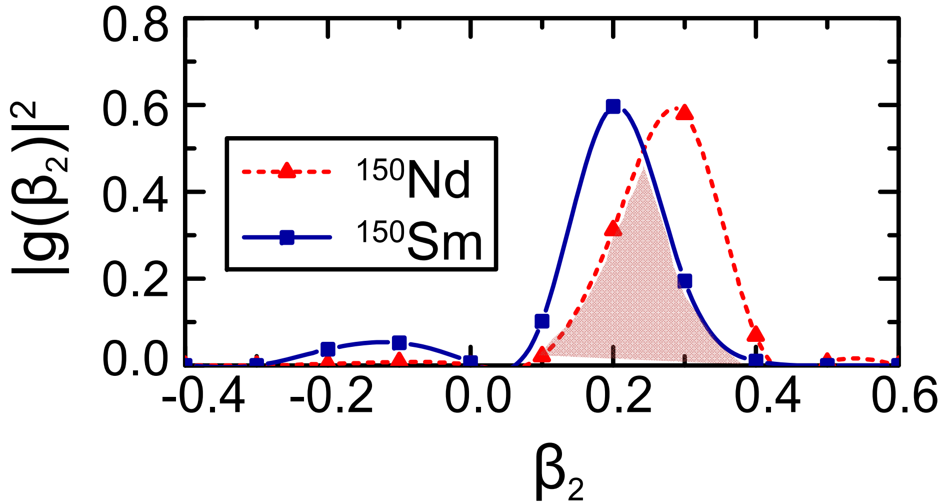
Low-lying excitation states

❖ Energy spectra and E2 transition



1 e² fm⁴ = 0.0211 W.u.

❖ Collective wave functions of g.s.



- Deformation:

  ^{150}Nd: $\beta_2 = 0.3$; ^{150}Sm: $\beta_2 = 0.2$.
- Shape mixing is significant.
- Large overlap between initial and final wave functions.

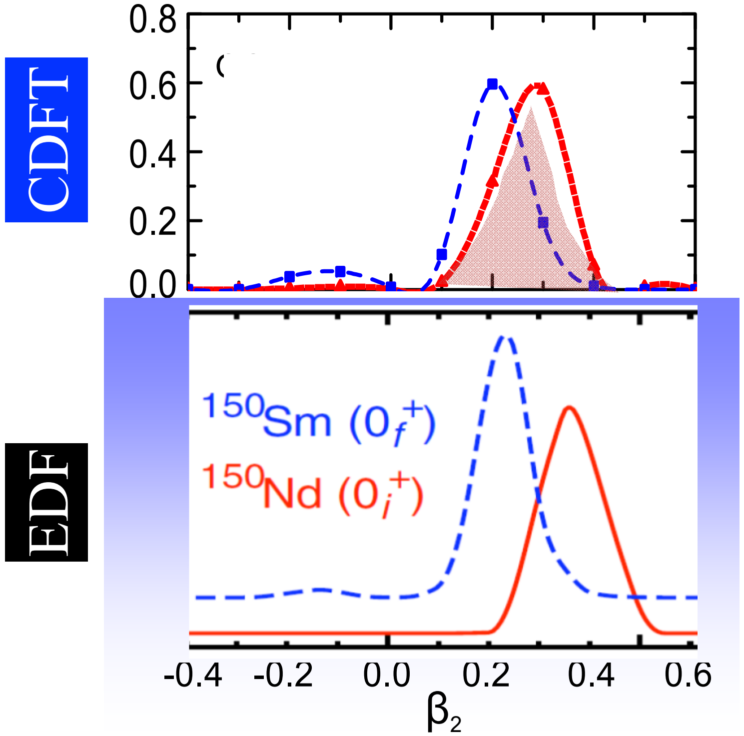
Song, Yao, Ring, & Meng 2014, PRC 90, 054309

NME of ^{150}Nd -> ^{150}Sm

❖ Results in different models:

Song, Yao, Ring, & Meng 2017, PRC 95, 024305

	CDFT	EDF	PHFB	QRPA-Tü	QRPA-NC	IBM
light-ν NME	**5.46**	1.71 / 2.19	2.49 – 3.31	3.37	3.14 / 2.71	2.67
heavy-ν NME	**218.2**	–	77.3 – 97.8	–	–	116.0



Discrepancy with **EDF** :

- **CDFT** gives larger overlap between WFs.

CDFT better than **EDF** in reproducing :

- E2 transition in ^{150}Nd and ^{150}Sm.
- Quantum phase transition in ^{150}Nd.

Nikšić, Vretenar, Lalazissis, & Ring 2007, PRL 99, 092502
Li, Nikšić, Vretenar, Meng, Lalazissis, & Ring 2009, PRC 79, 054301

Results of non-relativistic Gogny EDF:

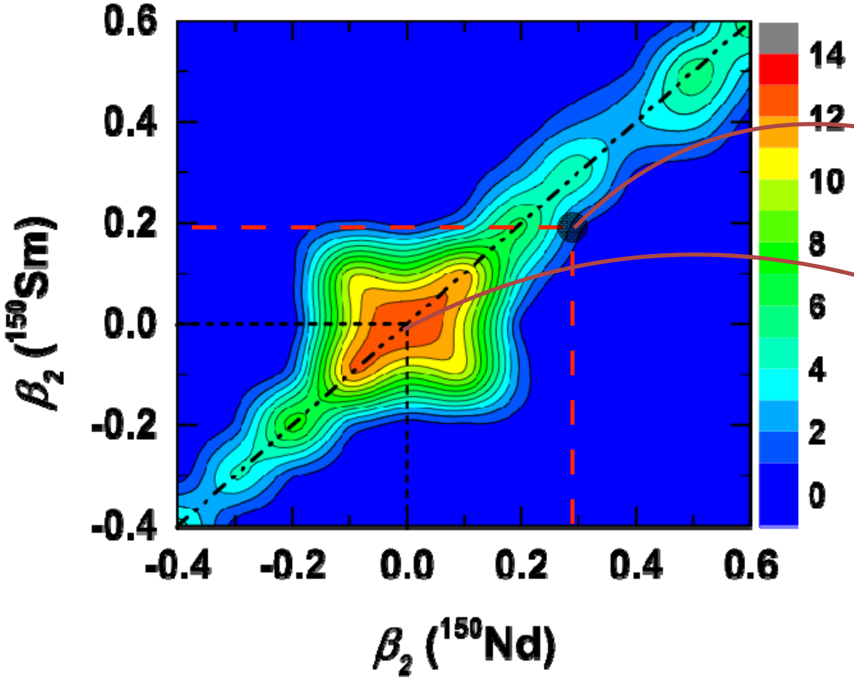
Rodriguez & Martinez-Pinedo 2010, PRL 105, 252503

Deformation and shape mixing

❖ NME at fixed deformations: $M^{0\nu}(\beta_2^F, \beta_2^I)$

GCM

$\boxed{M^{0\nu} = 5.46}$



$M^{0\nu}(\beta_2^F = 0.2, \beta_2^I = 0.3) = 3.27$

$M^{0\nu}(\beta_2^F = \beta_2^I = 0) = 13.53$

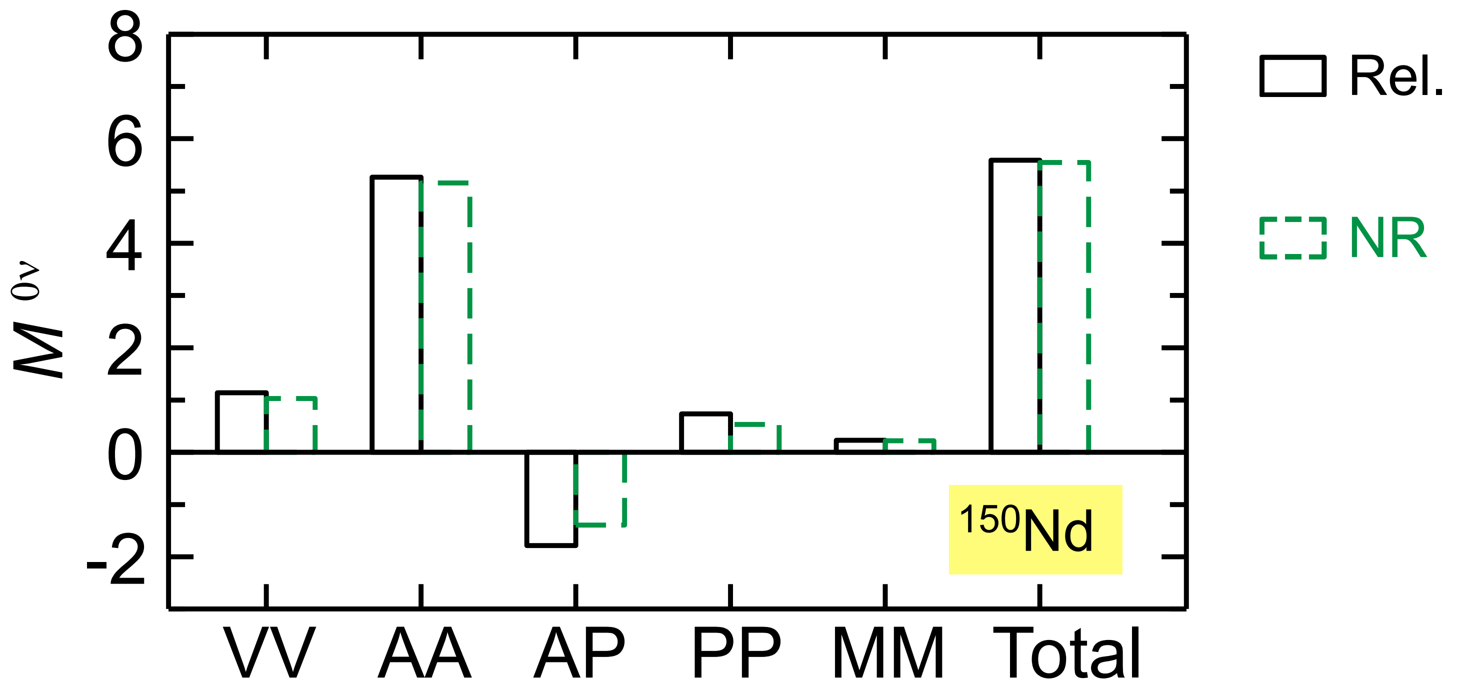
- $0\nu\beta\beta$ decay is suppressed by the difference in deformations of initial and final states: Shape mixing is important.
- $0\nu\beta\beta$ decay is favored if both nuclei are spherical.
- The same applies for heavy-neutrino NMEs.

Song, Yao, Ring, & Meng 2014, PRC 90, 054309

Relativity and SRC (light neutrinos)

❖ NMEs w/o SRC

Song, Yao, Ring, & Meng 2017, PRC 95, 024305



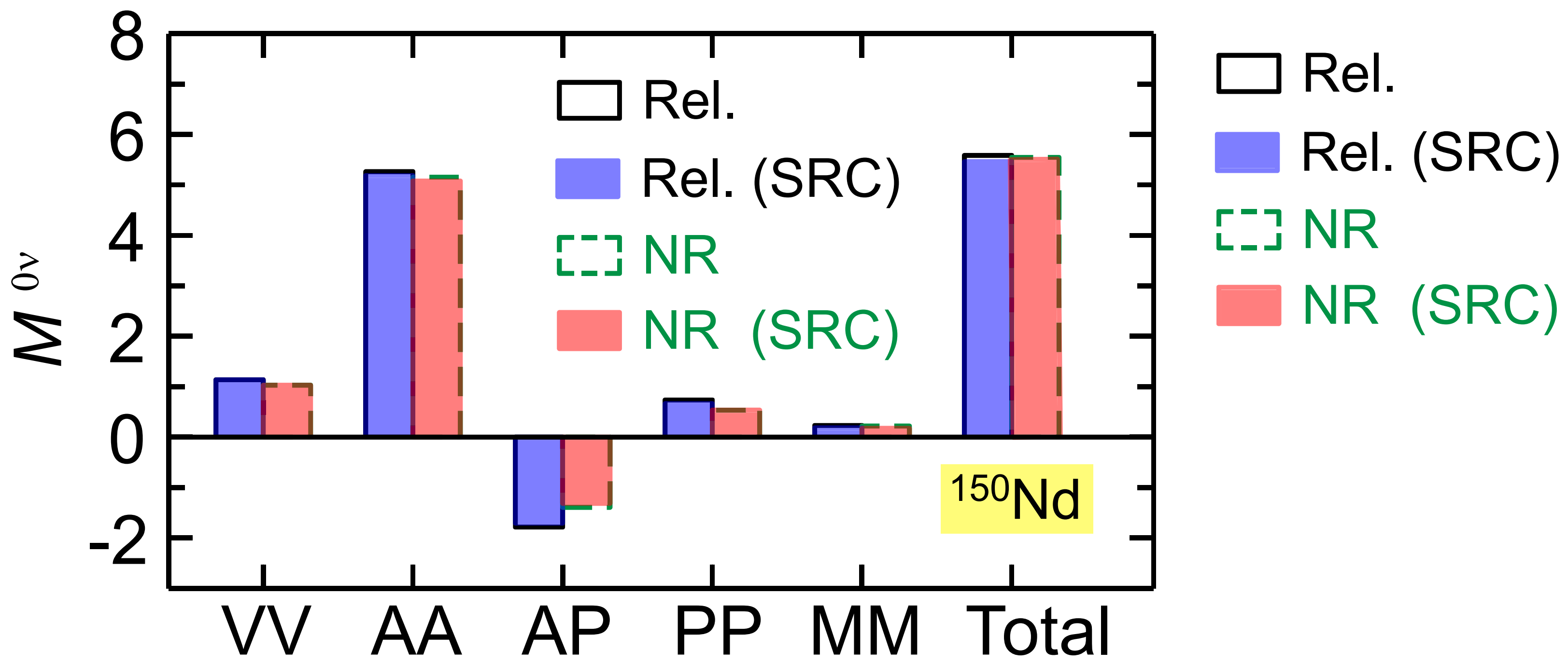
In the NMEs with light neutrinos:

➢ Relativistic effects are negligible.

Relativity and SRC (light neutrinos)

❖ NMEs w/ SRC

Song, Yao, Ring, & Meng 2017, PRC 95, 024305



In the NMEs with light neutrinos:

- ➢ Relativistic effects are negligible.
- ➢ SRC effects are negligible.

Relativity and SRC (light neutrinos)

❖ Test with other nuclei (spherical symmetry)

Effects of relativity

$$\Delta_{\mathrm{Rel.}} \equiv (M^{0\nu}_{\mathrm{Rel.}} - M^{0\nu}_{\mathrm{NR}})/M^{0\nu}_{\mathrm{Rel.}}$$

	w/o SRC	w/ SRC
^{48}Ca	−2%	−1%
^{76}Ge	−1%	−3%
^{82}Se	−1%	−3%
^{96}Zr	1%	−1%
^{100}Mo	1%	−1%
^{116}Cd	1%	−1%
^{124}Sn	−1%	−2%
^{130}Te	−1%	−2%
^{136}Xe	−1%	−3%
^{150}Nd	1%	−0%

Effects of SRC

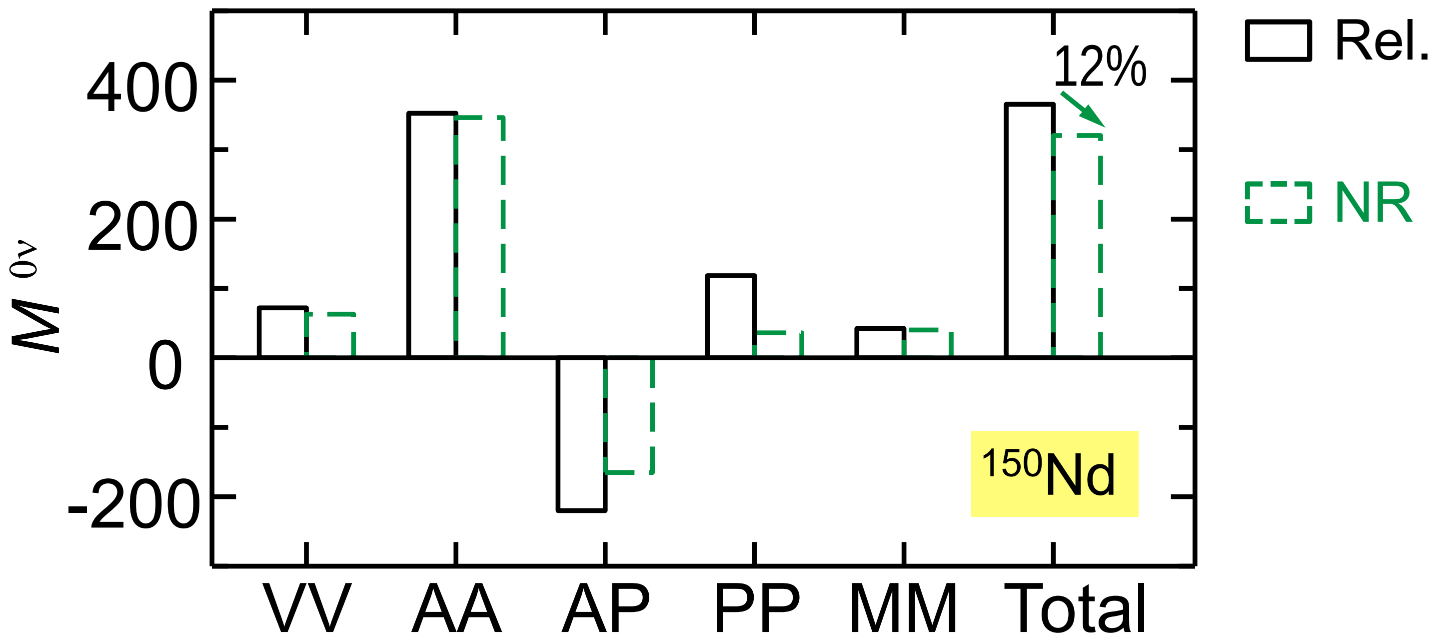
$$\Delta_{\mathrm{src}} \equiv (M^{0\nu}_{\mathrm{bare}} - M^{0\nu}_{\mathrm{src}})/M^{0\nu}_{\mathrm{bare}}$$

	Δ_{src}
^{48}Ca	1%
^{76}Ge	2%
^{82}Se	2%
^{96}Zr	2%
^{100}Mo	2%
^{116}Cd	2%
^{124}Sn	2%
^{130}Te	2%
^{136}Xe	2%
^{150}Nd	2%

Relativity and SRC (heavy neutrinos)

❖ NMEs w/o SRC

Song, Yao, Ring, & Meng 2017, PRC 95, 024305



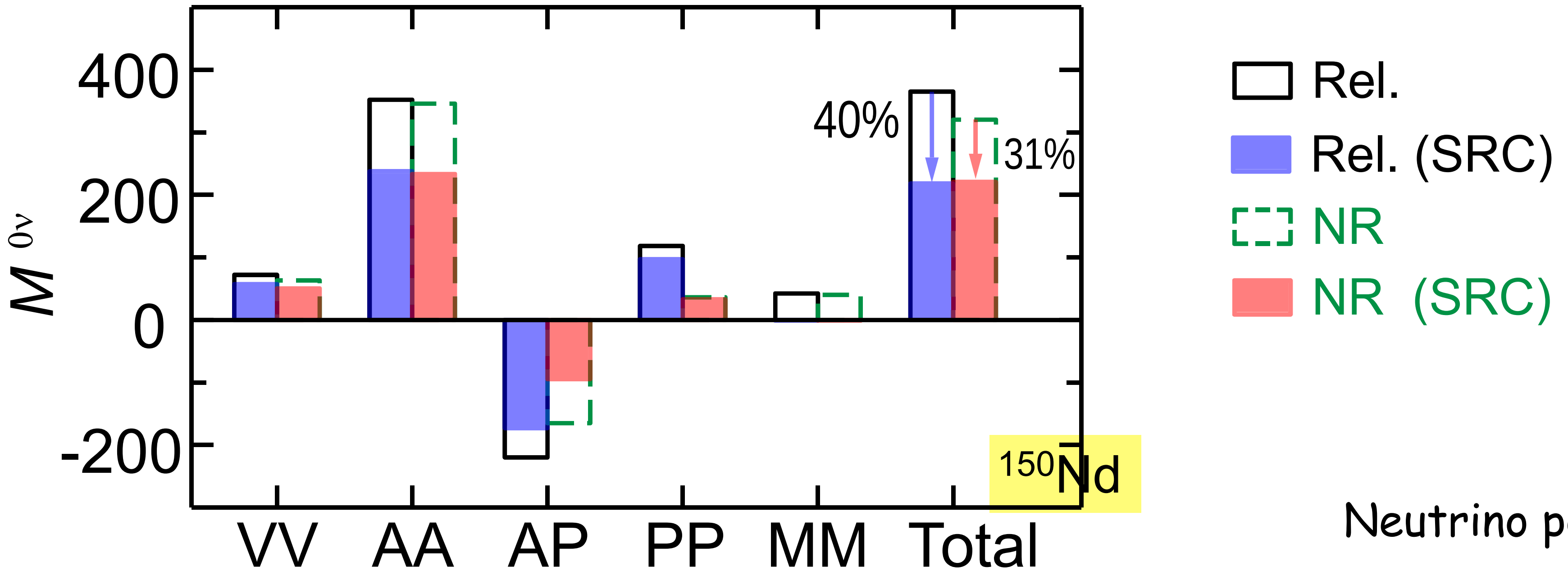
In the NMEs with heavy neutrinos:

- Relativistic effects are important: 12% of the total NME w/o SRC.

Relativity and SRC (heavy neutrinos)

❖ NMEs w/ SRC

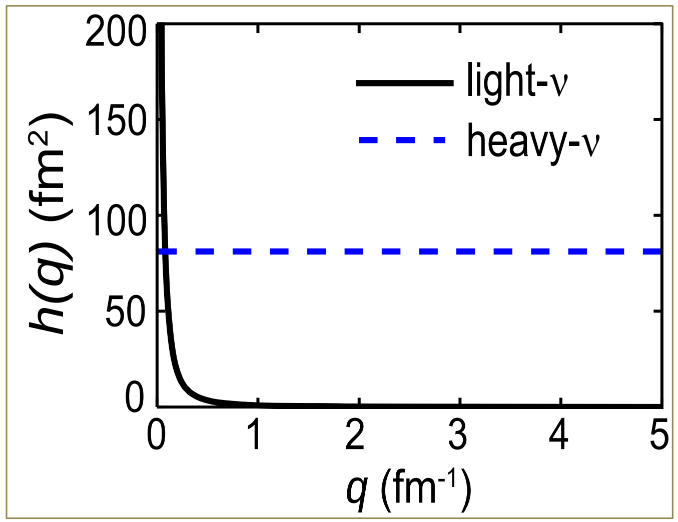
Song, Yao, Ring, & Meng 2017, PRC 95, 024305



In the NMEs with heavy neutrinos:

- Relativistic effects are important: 12% of the total NME w/o SRC.
- Relativistic effects are eliminated by taking into account the SRC.
- SRCs are important: 40% of the total NME.

Neutrino potentials:



Relativity and SRC (heavy neutrinos)

❖ Test with other nuclei (spherical symmetry)

Effects of relativity

$$\Delta_{\rm Rel.} \equiv (M^{0\nu}_{\rm Rel.} - M^{0\nu}_{\rm NR})/M^{0\nu}_{\rm Rel.}$$

	w/o SRC	w/ SRC
^{48}Ca	15%	−2%
^{76}Ge	10%	−6%
^{82}Se	11%	−5%
^{96}Zr	11%	−2%
^{100}Mo	11%	−2%
^{116}Cd	12%	−3%
^{124}Sn	10%	−3%
^{130}Te	10%	−3%
^{136}Xe	10%	−3%
^{150}Nd	13%	−0%

Effects of SRC

$$\Delta_{\rm src} \equiv (M^{0\nu}_{\rm bare} - M^{0\nu}_{\rm src})/M^{0\nu}_{\rm bare}$$

	$\Delta_{\rm src}$
^{48}Ca	43%
^{76}Ge	43%
^{82}Se	42%
^{96}Zr	42%
^{100}Mo	42%
^{116}Cd	41%
^{124}Sn	41%
^{130}Te	41%
^{136}Xe	41%
^{150}Nd	40%

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NME and neutrino masses

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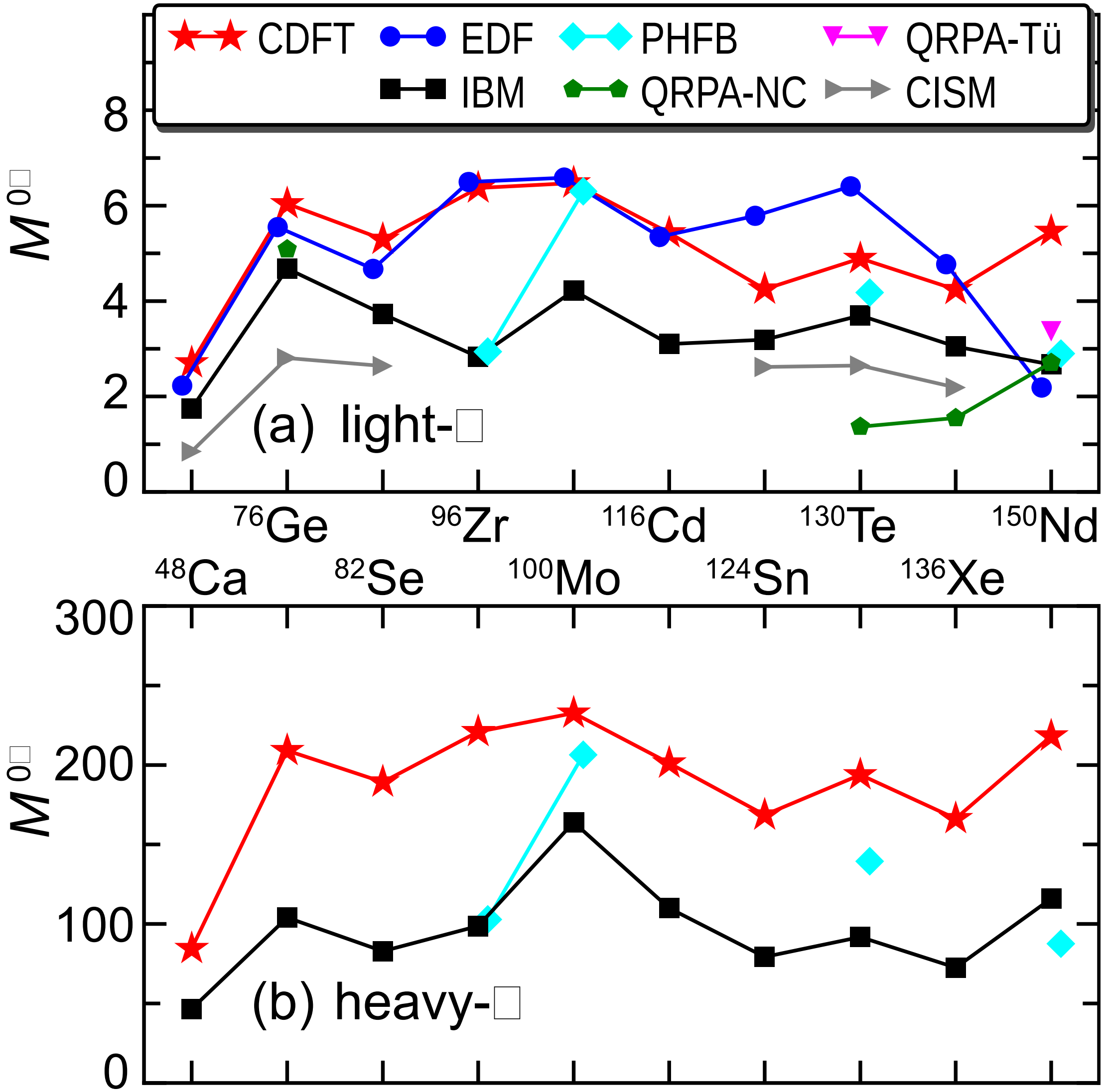
Song, Yao, Ring, & Meng 2017, PRC 95, 024305

	半衰期下限	相空间因子	中微子有效质量			
	$T_{1/2}^{0\nu}$	$G_{0\nu}$	light-ν		heavy-ν	
	(10^{22} yr)	(10^{-15} yr $^{-1}$)	$M^{0\nu}$	$\lvert\langle m_\nu\rangle\rvert$	$M^{0\nu}$	$\lvert\langle m_{\nu_h}^{-1}\rangle\rvert^{-1}$
^{48}Ca	5.8	24.81	2.71	< 3.2	84.5	> 4.7
^{76}Ge	5300	2.363	6.04	< 0.15	209.1	> 109.2
^{82}Se	36	10.16	5.30	< 1.0	189.3	> 16.9
^{96}Zr	0.92	20.58	6.37	< 3.7	220.9	> 4.5
^{100}Mo	110	15.92	6.48	< 0.38	232.6	> 45.4
^{116}Cd	17	16.70	5.43	< 1.1	201.1	> 15.8
^{124}Sn	0.005	9.04	4.25	< 114	168.5	> 0.17
^{130}Te	280	14.22	4.89	< 0.33	193.8	> 57.1
^{136}Xe	10700	14.58	4.24	$\boxed{< 0.06}$	166.3	$\boxed{> 306.5}$
^{150}Nd	2.0	63.03	5.46	< 1.7	218.2	> 11.4

* The unit of $\lvert\langle m_\nu\rangle\rvert$ is eV; The unit of $\lvert\langle m_{\nu_h}^{-1}\rangle\rvert^{-1}$ is 10^6 GeV.

Comparison with other models

Song, Yao, Ring, & Meng 2017, PRC 95, 024305

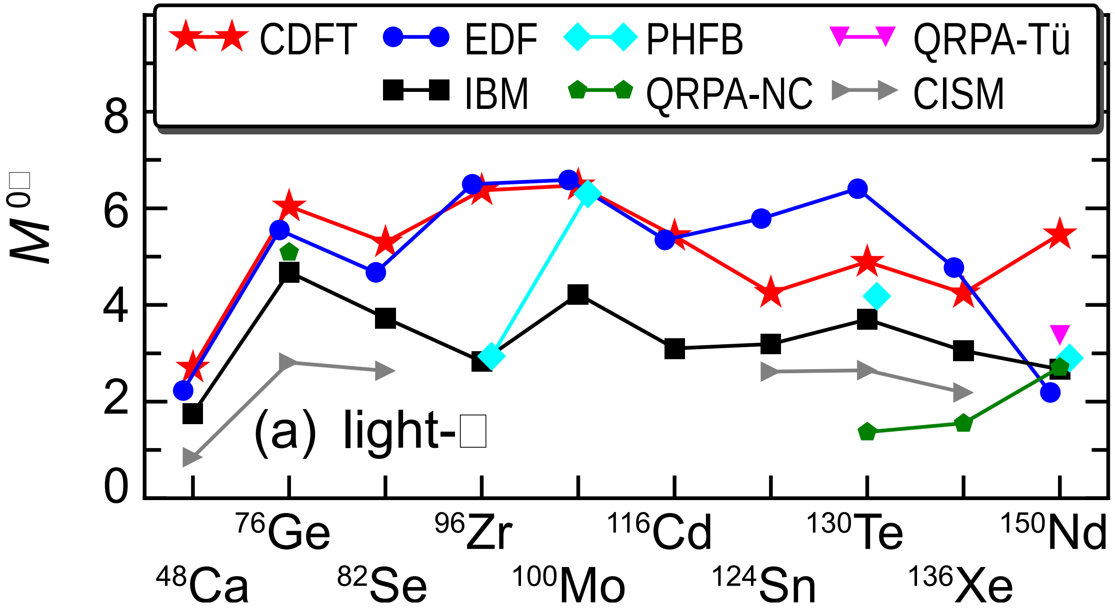


Selective results in comparison

- Deformations are considered.
- Use unquenched $g_A \sim 1.26$;

  radius $R = 1.2A^{1/3}$ fm.
- Uncertainties from SRCs ~ 5%
  - AV18, CCM/Jastrow (CDFT, IBM, PHFB);
  - UCOM (EDF, CISM);
  - CD Bonn, G matrix (QRPA-Tu);
  - neglected (QRPA-NC).

Comparison with other models



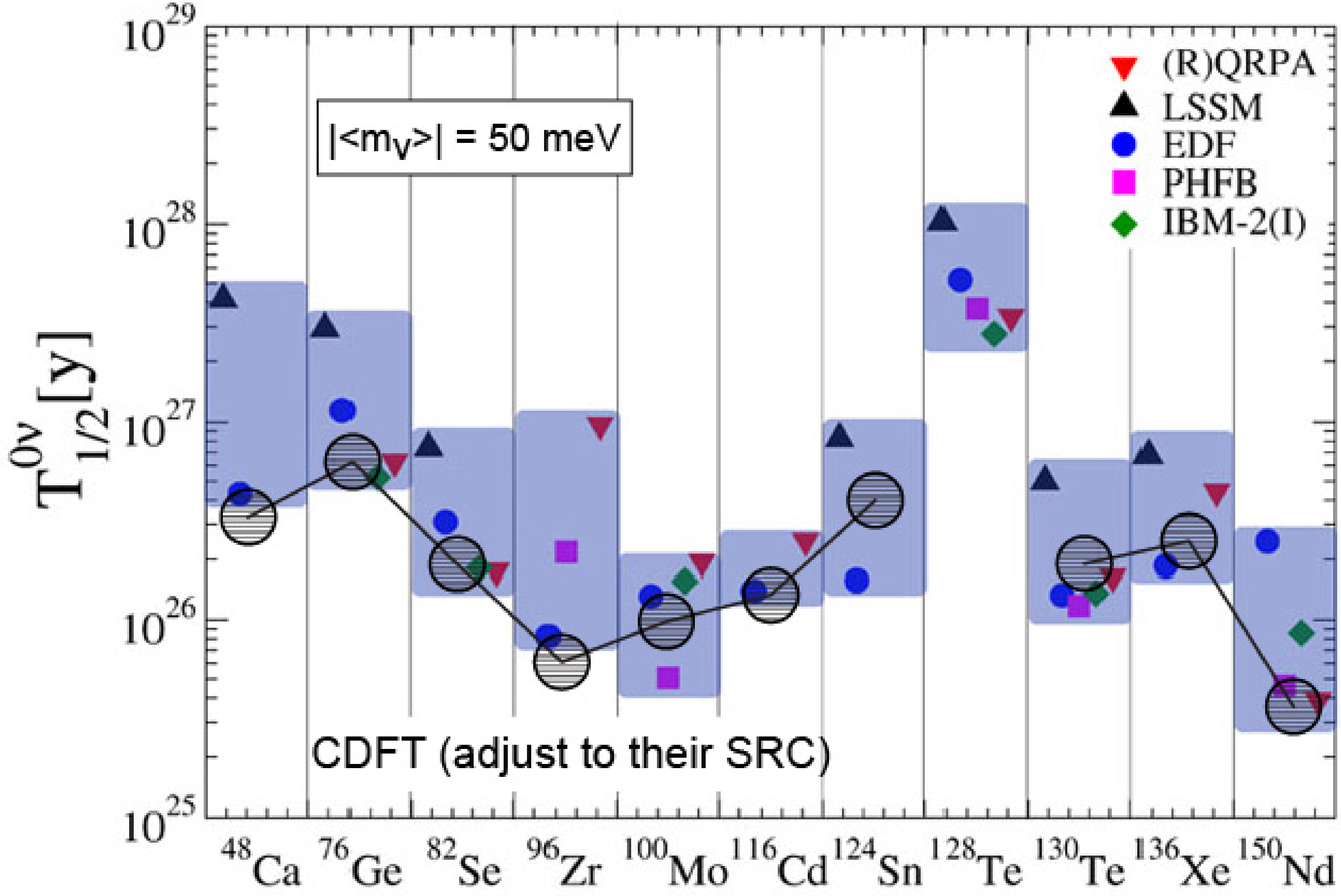
- CDFT highly in consistent with EDF, except ^{124}Sn, ^{130}Te, and ^{150}Nd.
- CDFT results give the upper boundary of all NMEs, except in ^{124}Sn and ^{130}Te.

- Uncertainties in NME will translate into the effective neutrino masses.
- To reduce the uncertainties:
  - Nuclear models:

    Shell-model like: To enlarge model space

    Mean-field based: To include more correlations
  - Details in the NME calculation: treatment of the intermediate states, short-range correlations, form factors … Many open questions

DOUBLE-BETA DECAY HALF-LIFE OF SEVERAL NUCLEI FROM DIFFERENT NUCLEAR MANY-BODY CALCULATIONS IN THE LIGHT NEUTRINO EXCHANGE SCENARIO



Vergados, Ejiri and Simkovic, Rep. Prog. Phys. 75 106301 (2012)

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Summary and perspectives

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Summary

First relativistic description for the nuclear matrix elements of $0\nu\beta\beta$ decay based on covariant density functional theory.

- ✓ Deformation, relativity, and short-range correlation
- ✓ Effective neutrino masses
- ✓ Comparison with other nuclear models

Perspectives

- Short-range correlations from relativistic ab initio calculations
- Effects of triaxiality: ^{76}Ge
- Proton-neutron pairing and restoration of isospin symmetry
- New generator coordinates: particle number (pairing) fluctuation

Collaborators：K. Hagino, P. Ring, **L. S. Song**, J. M. Yao

Thank you for your attention！