$0\nu\beta\beta$ decay NMEs in large shell model space with the generator-coordinate method

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Generator Coordinate Method (GCM)

Generator Coordinate Method: an approach that treats large-amplitude fluctuations, which is essential for nuclei that cannot be approximated by a single mean field.

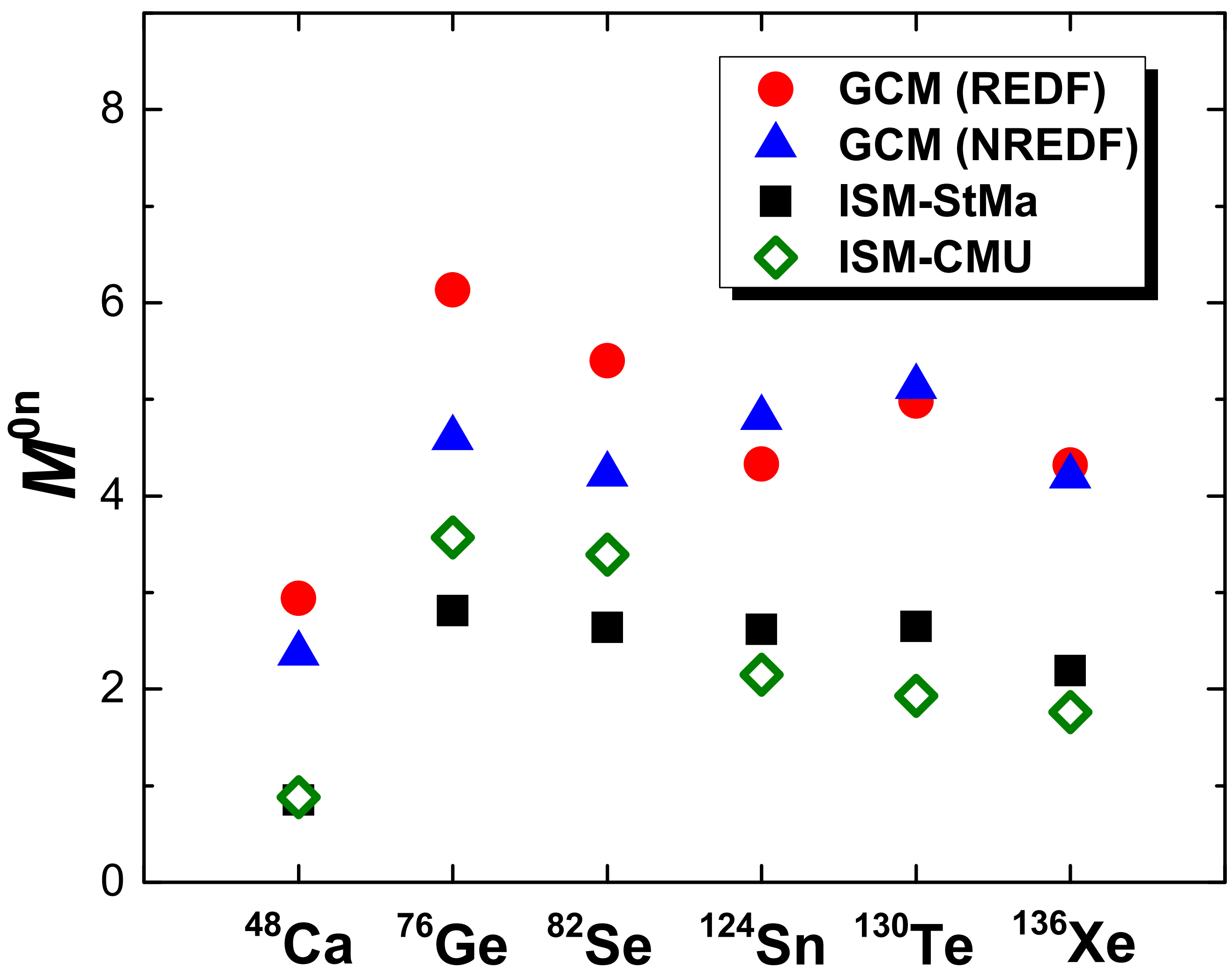
How it works:

Construct a set of mean-field states by constraining coordinates, e.g., quadrupole moment. Then diagonalize Hamiltonian in space of symmetry-restored nonorthogonal vacua with different amounts of quadrupole deformation.

GCM based on EDF has been applied to double-beta decay, however…

Comparison between GCM and SM

Current results with EDF-based GCM



Both the shell model and the EDF-based GCM could be missing important physics.

The discrepancy may be because:

- The GCM omits correlations.
- The shell model omits many single-particle levels

Our long-term goal is to combine the virtues of both frameworks through an EDF-based or *ab-initio* GCM that includes all the important shell model correlations and a large single-particle space.

To get closer to the ultimate goal:

We can use SM Hamiltonian in the GCM

Our short-term goal is more modest: a shell-model Hamiltonian-based GCM in one and two (and possibly more) shells.

Our Current Procedure

① Using a shell-model Hamiltonian

② HFB states $|\Phi(q)\rangle$ with multipole constraints *q*.

We are trying to include all possible collective correlations.

③ Angular momentum and particle number projection

$$|JMK;NZ;q\rangle = \hat{P}^J_{MK}\hat{P^N}\hat{P^Z}|\Phi(q)\rangle$$

④ Configuration mixing within GCM:

$$|\Psi^J_{NZ\sigma}\rangle = \sum_{K,q} f^{JK}_\sigma(q)|JMK;NZ;q\rangle$$

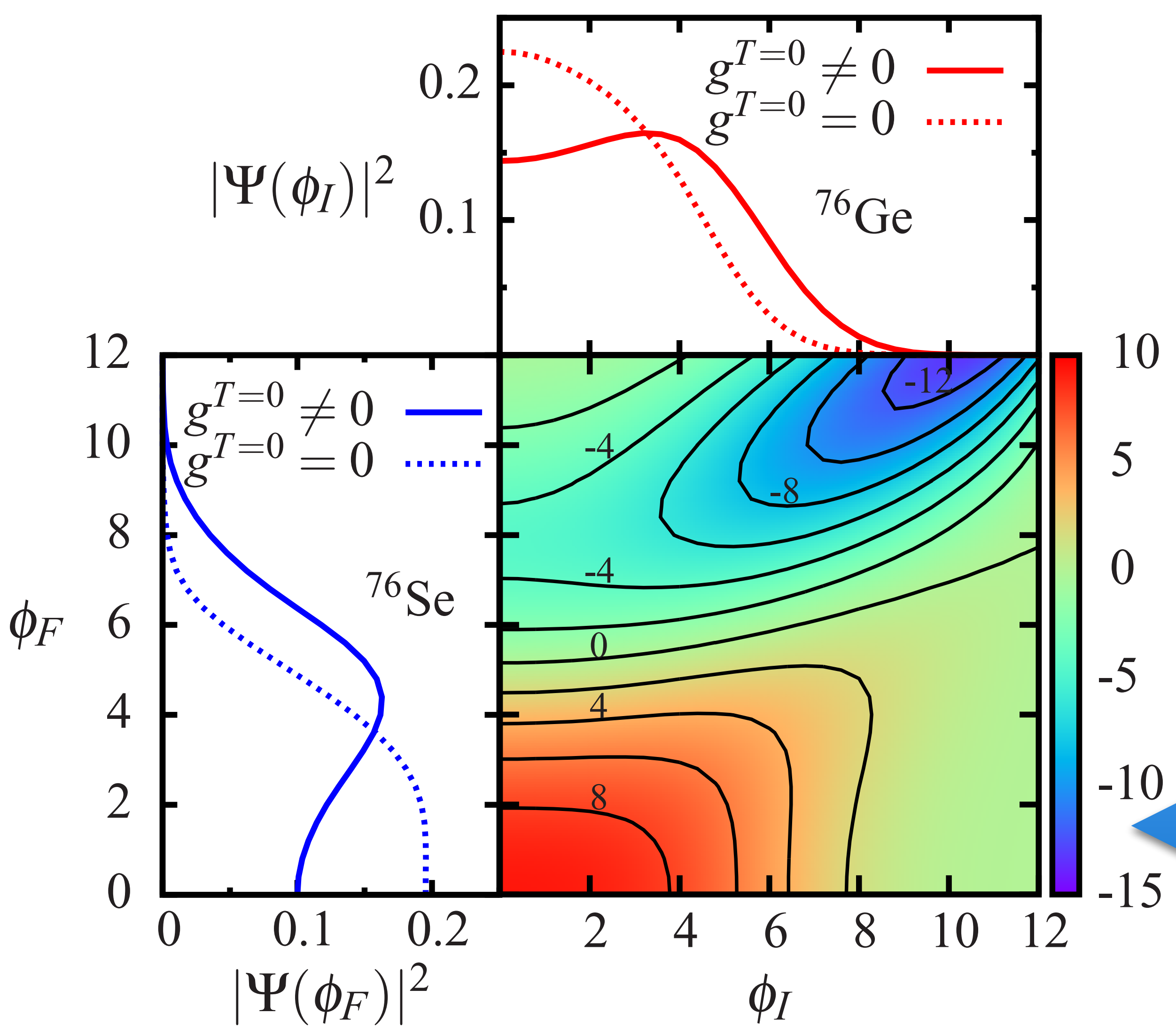
$$\sum_{K',q'}\{\mathcal{H}^J_{KK'}(q;q') - E^J_\sigma \mathcal{N}^J_{KK'}(q;q')\}f^{JK'}_\sigma(q') = 0 \quad \rightarrow \quad f^{JK}_\sigma(q)$$

$$M^{0\nu\beta\beta}_\xi = \langle\Psi^{J=0}_{N_fZ_f}|\hat{O}^{0\nu\beta\beta}_\xi|\Psi^{J=0}_{N_iZ_i}\rangle$$

Level 1 GCM: Axial shape and pn pairing fluctuation

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2}(P_0 + P_0^\dagger)$$

isoscalar *pn* pairing constrained



ϕ is the isoscalar pairing amplitude

$$\phi = \langle P_0 + P_0^\dagger \rangle / 2$$

$$P_0^\dagger = \frac{1}{\sqrt{2}} \sum_l \hat{l} [c_l^\dagger c_l^\dagger]^{L=0,S=1,T=0}_{M_S=0}$$

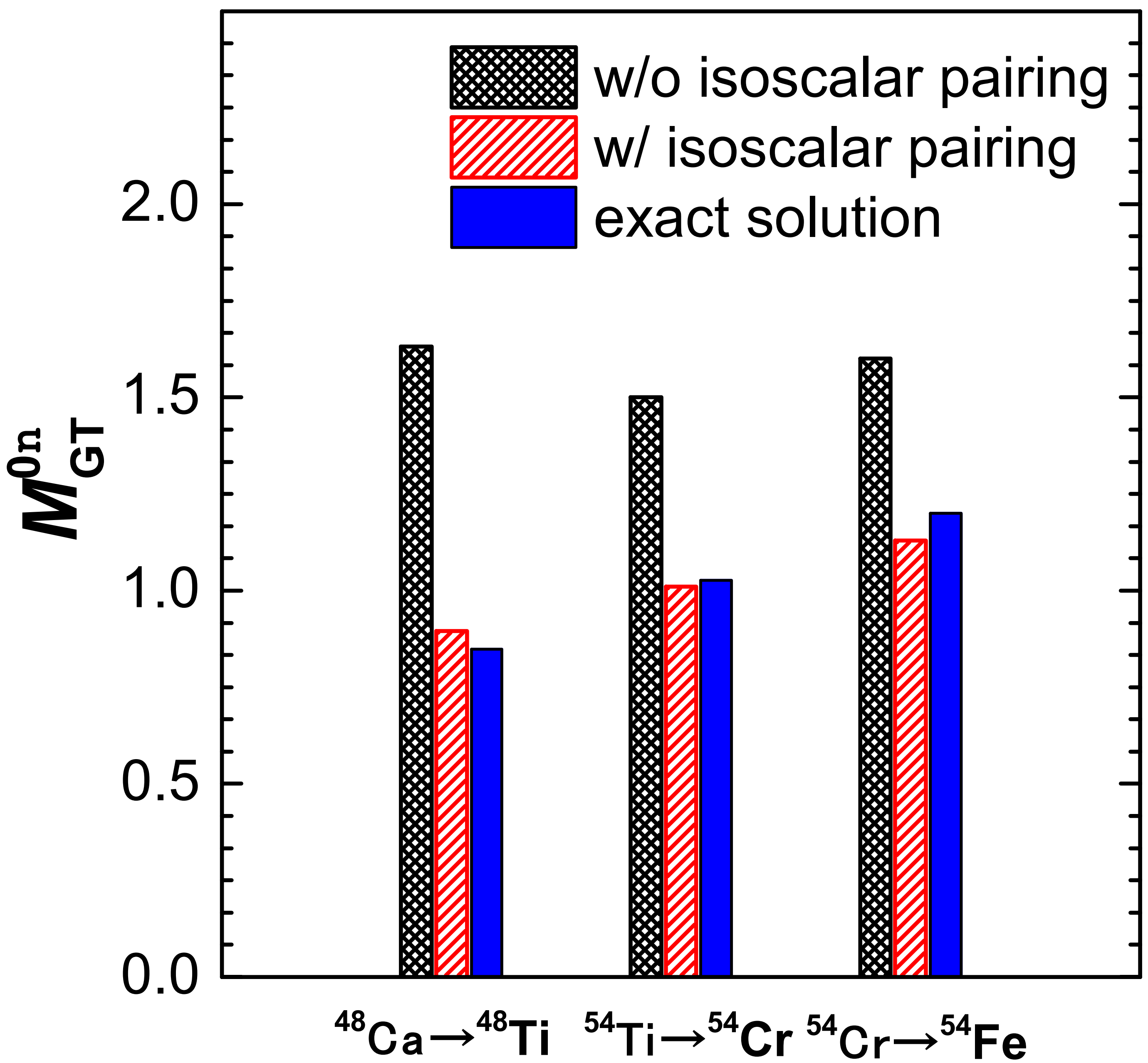
The wave functions are pushed into a region with large isoscalar pairing amplitude.

→ ***reduce the 0νββ NMEs.***

N. Hinohara and J. Engel, PRC 90, 031301(R) (2014)

Level 1 GCM: Axial shape and pn pairing fluctuation

To highlight the effects of isoscalar pairing:



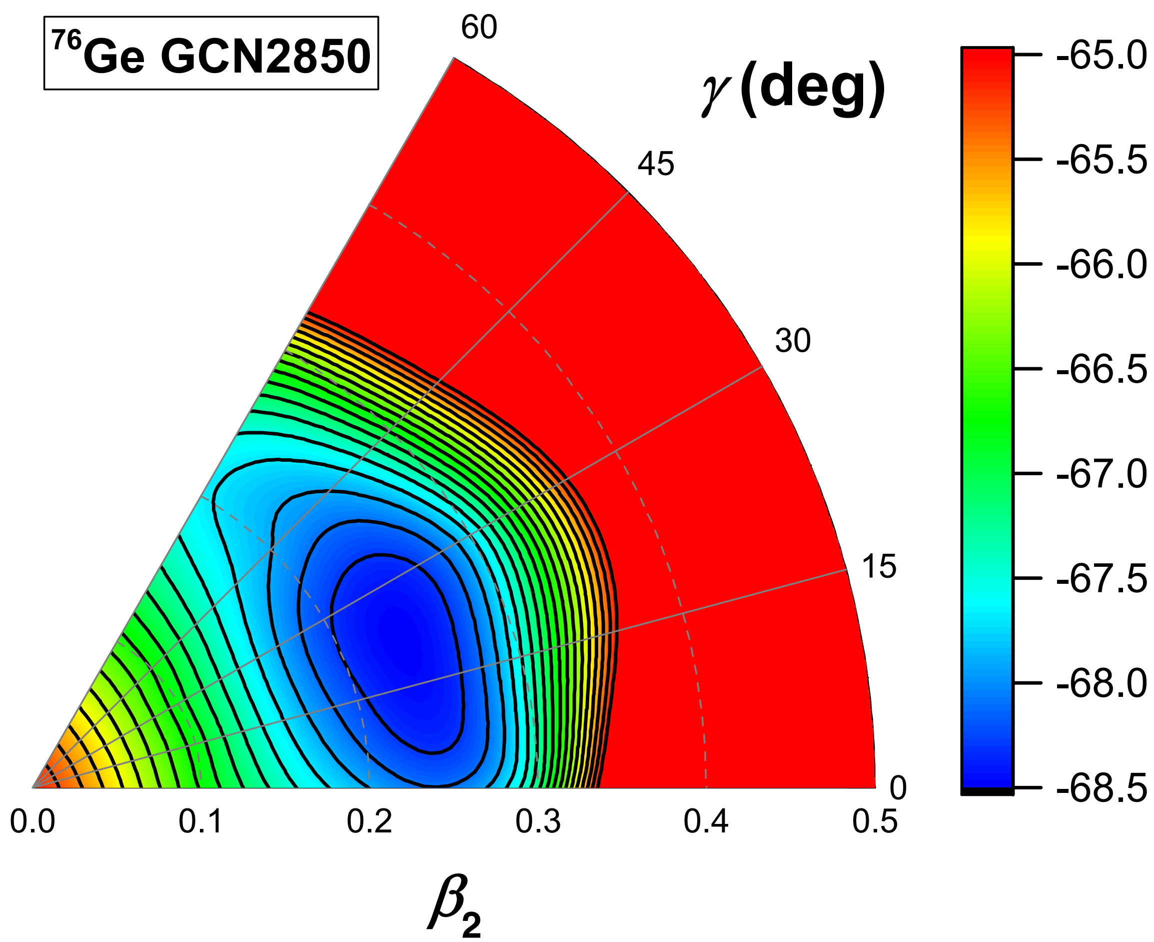
We use the KB3G interaction for two GCM calculations:

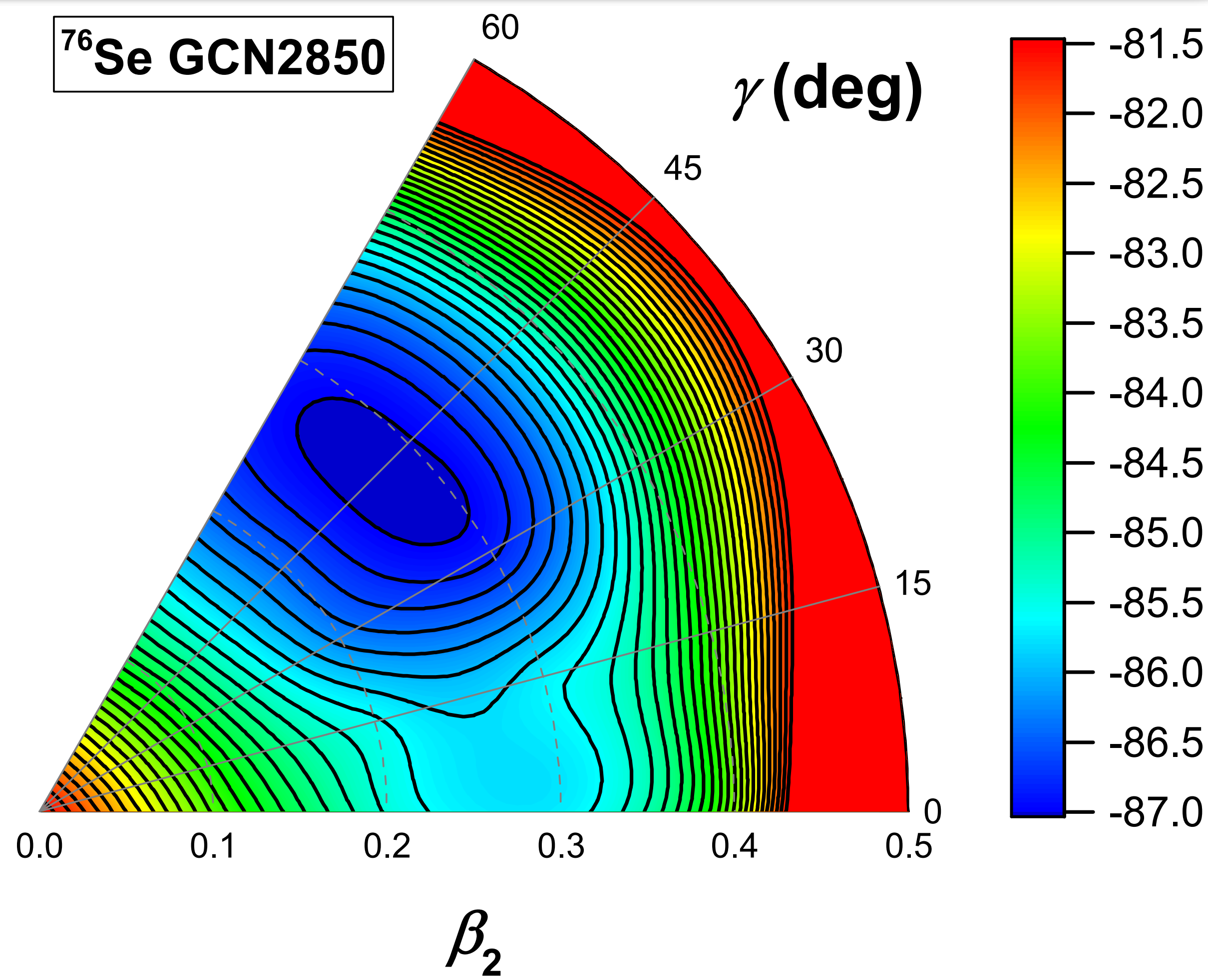
- **Black column**: we set all the two-body matrix elements of the Hamiltonian with $J = 1$ and $T = 0$ to zero, because those are the ones which isoscalar pairing acts through.
  M_{GT} is overestimated.
- **Red column:** we use the full KB3G Hamiltonian:
  M_{GT} is suppressed, close to SM.

Level 2 GCM: Triaxial deformation

$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2}(P_0 + P_0^\dagger) - \lambda_2 Q_{22}$$

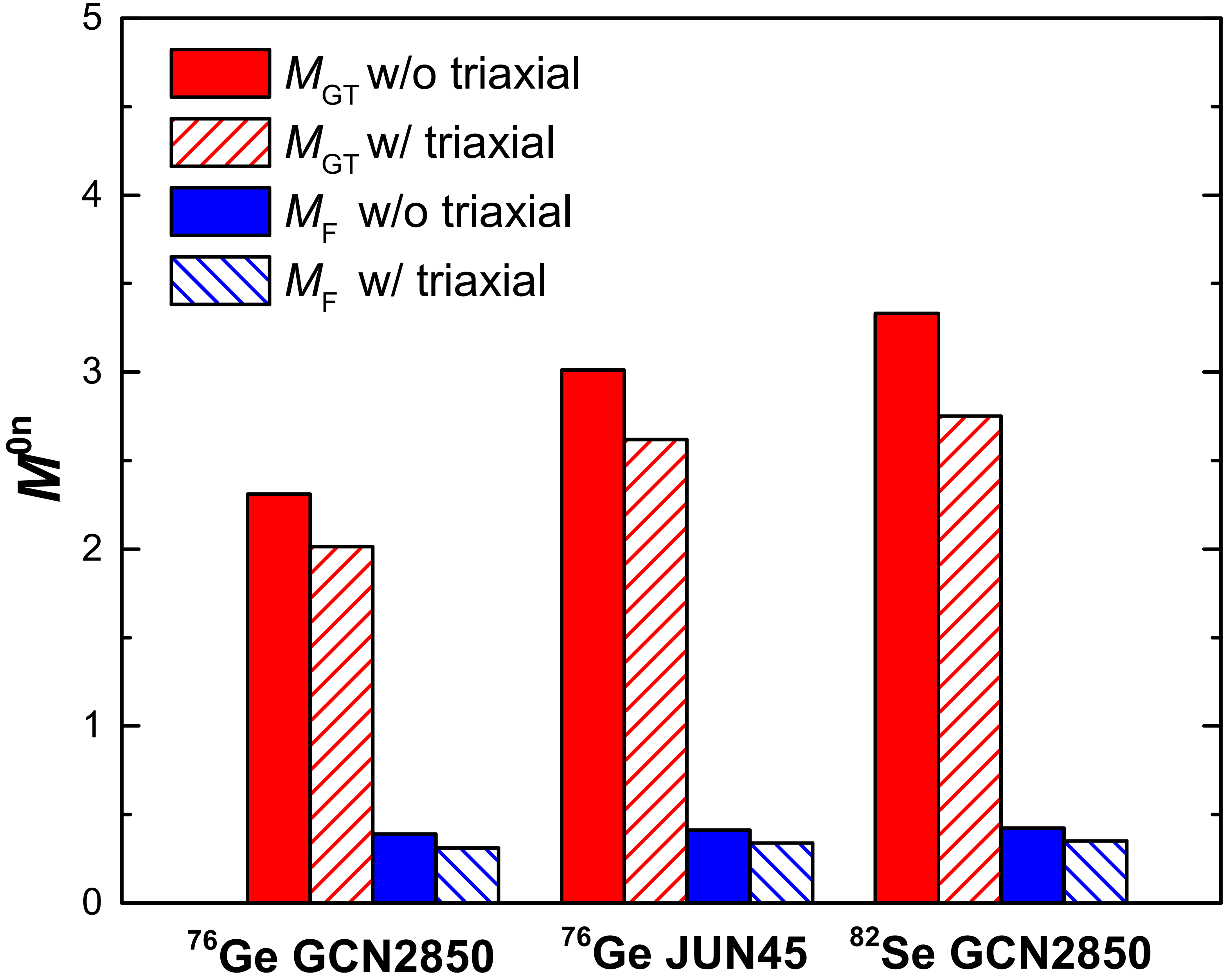
triaxial deformation constrained





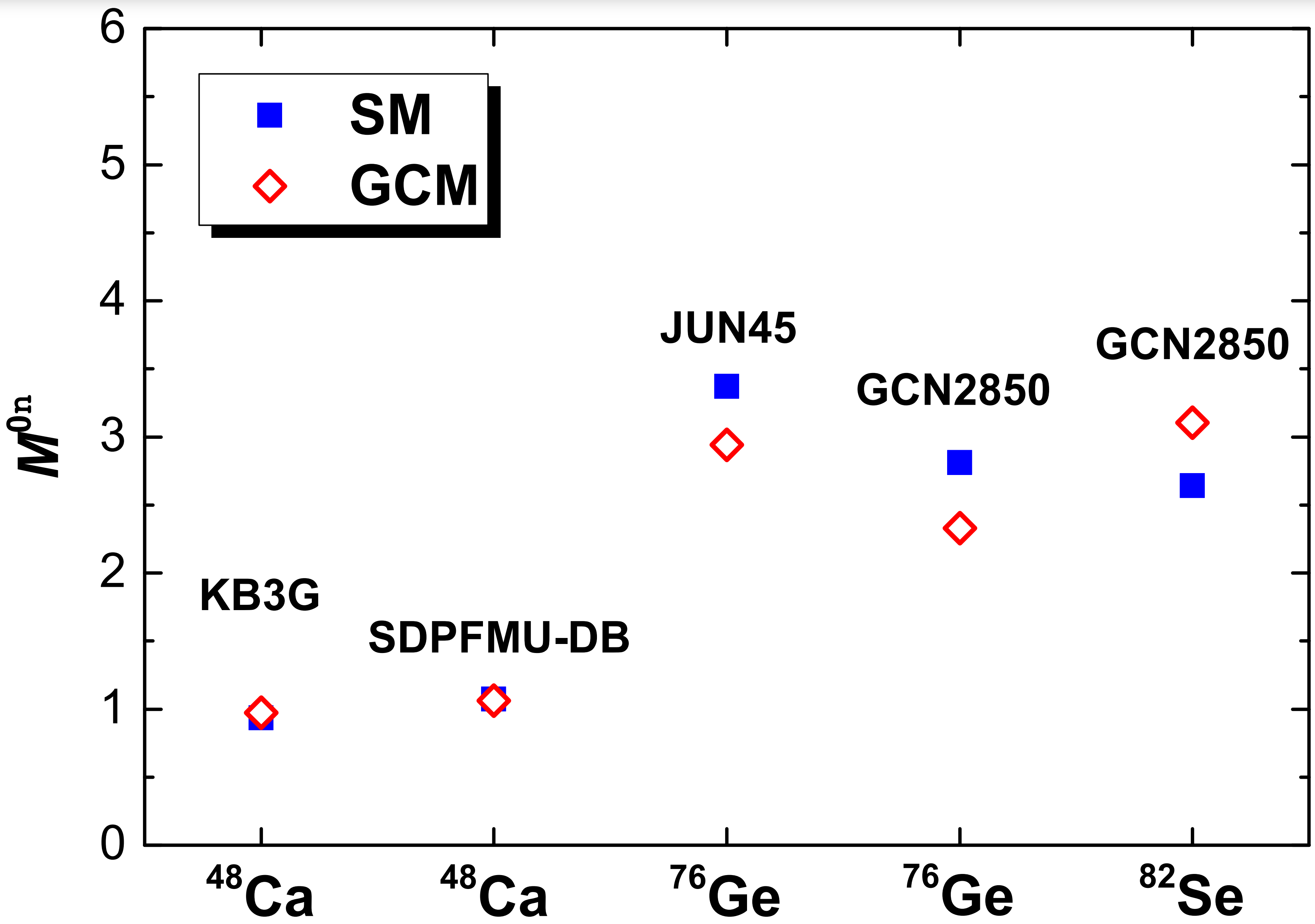
With GCN2850 or JUN45 interaction, projected potential energy surfaces for ^{76}Ge and ^{76}Se give minima with triaxial deformation.

Level 2 GCM: triaxial deformation



15%~20% reduction for both GT and Fermi part of NME if triaxial shape fluctuation is included.

Benchmarking: 0νββ NMEs given by GCM and SM



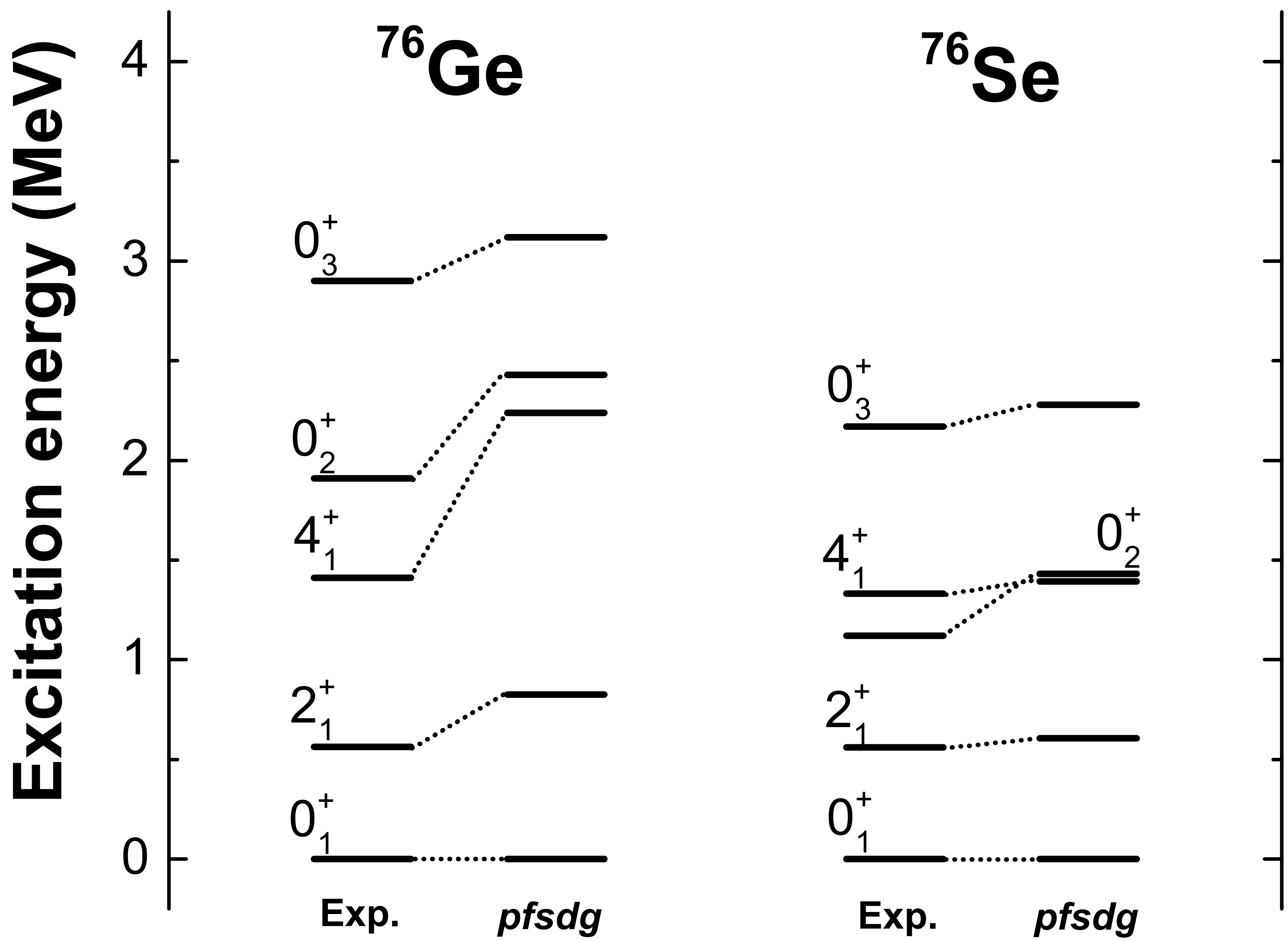
The NMEs given by SM and GCM are in good agreement, indicating that **the GCM captures most important valence-shell correlations.**

Multi-shell GCM

- In principle, effective *pfsdg*-shell interaction based on chiral EFT can be calculated by many-body perturbation theory (MBPT), similarity renormalization group (SRG) or couple cluster (CC).
- We employ an effective *pfsdg*-shell interaction calculated by Extended Kuo-Krenciglowa perturbative method, which are provided by J. D. Holt.
- The monopole part of the resulting Hamiltonian is sensitive to the three-body part of the initial interaction, which one generally reduces to an effective two-body interaction by summing the third particle over a set of occupied states.

pfsdg: 3N forces normal ordered with respect to ^{56}Ni
Not the ideal core, but we work with it nonetheless.

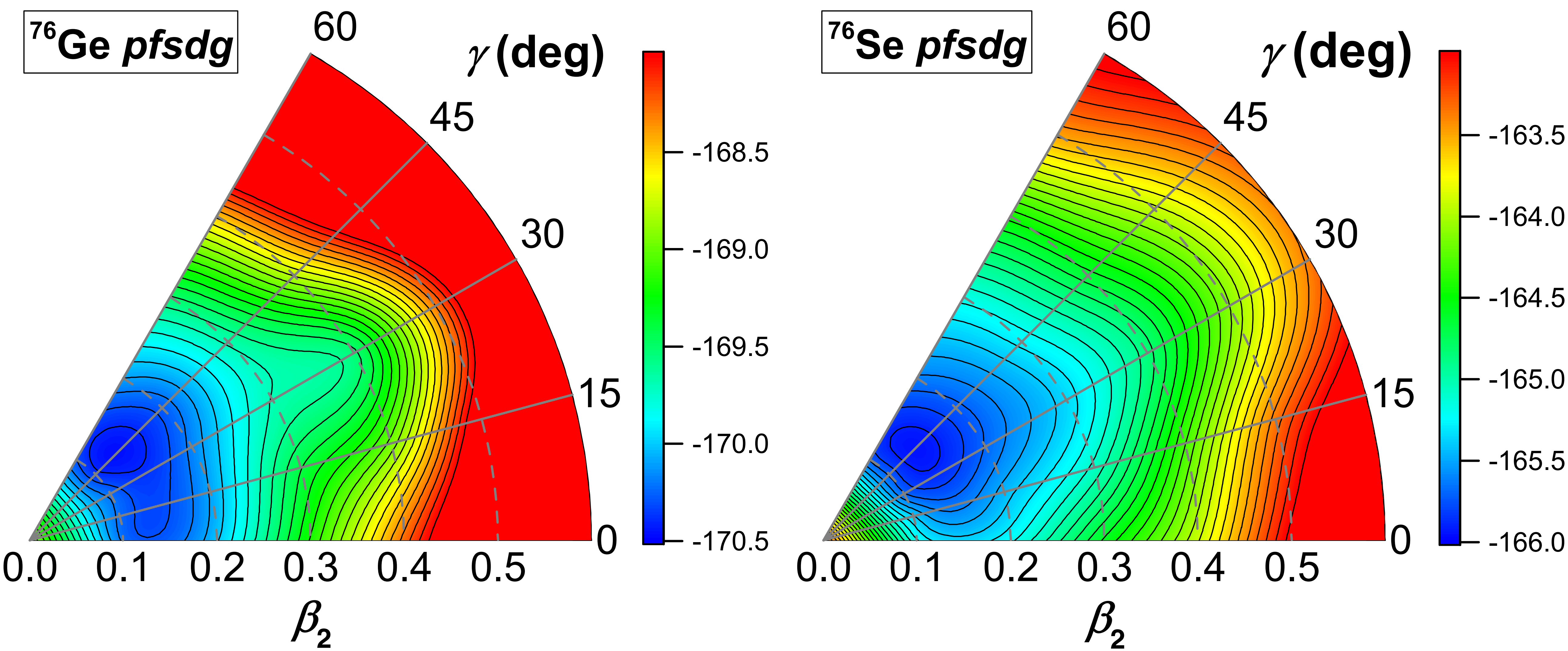
Multi-shell GCM: low-lying spectra



We optimize the single-particle energies for *pfsdg*-shell interactions by fitting the measured occupancies of valence neutron and proton orbits.

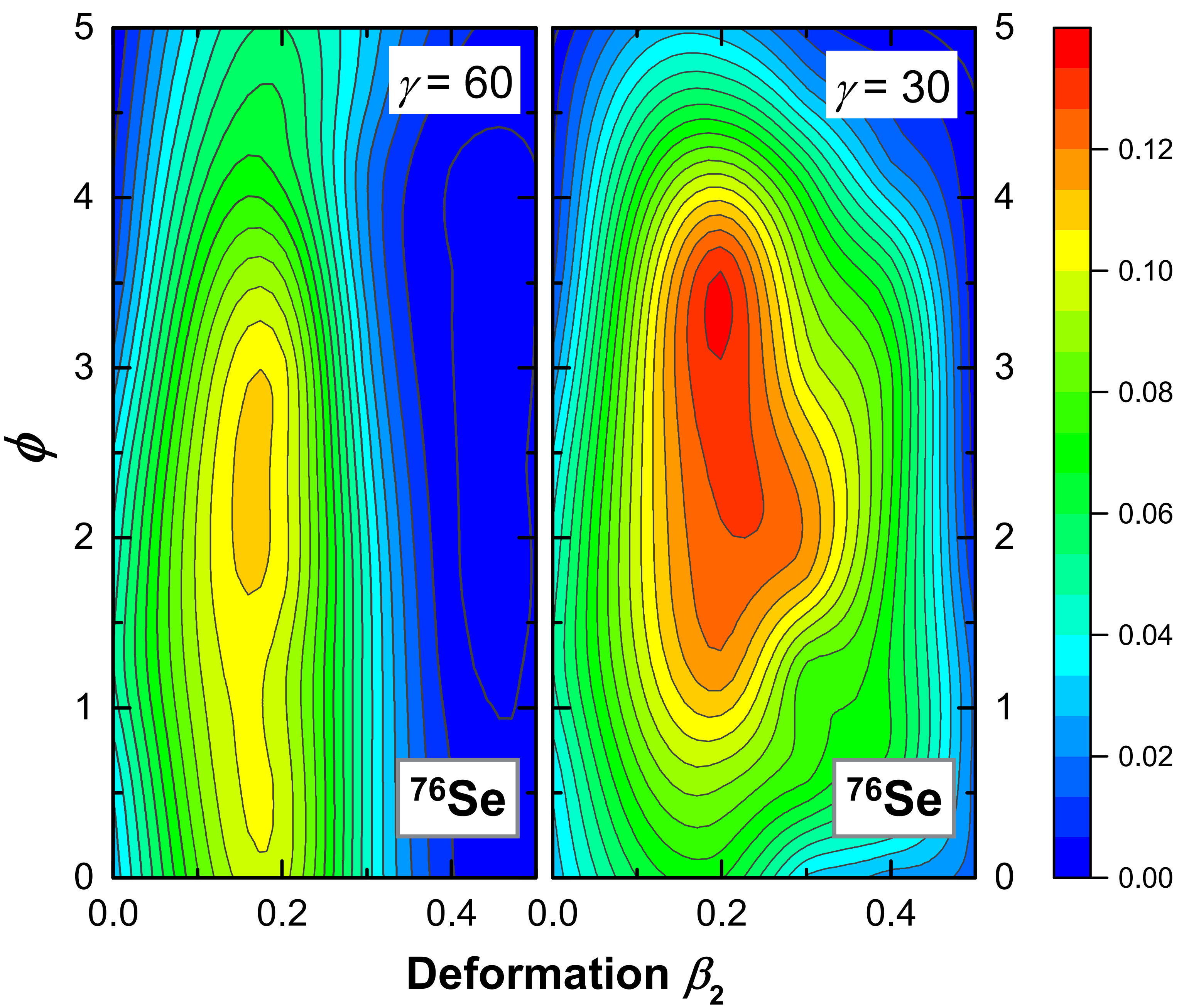
Low-lying level spectra agree well with the experimental data:
give us confidence on two-shell GCM based on *pfsdg* Hamiltonian.

Multi-shell GCM: collective wave function



- Larger model space: triaxially deformed as predicted.
- How does triaxial shape influence NMEs?

Multi-shell GCM: triaxial deformation

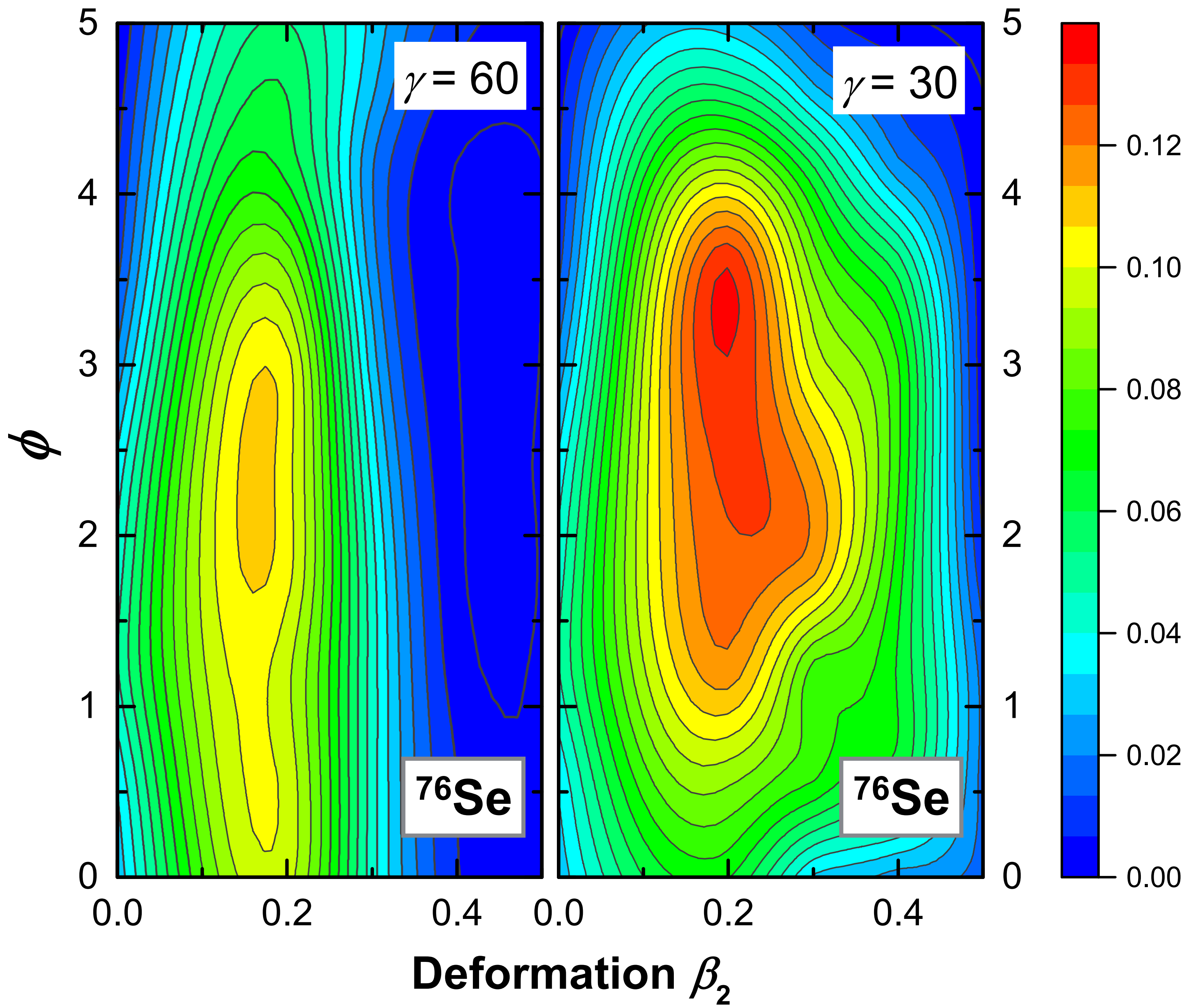


With triaxially deformed configurations, the wave functions:

① are pushed to the region with larger isoscalar *pn* pairing.

② spread widely to the region with larger deformation

Multi-shell GCM: triaxial deformation



	$M^{0\nu}_{\rm GT}$	$-(\frac{g_v}{g_A})^2 M^{0\nu}_{\rm F}$	$M^{0\nu}_{\rm T}$	$M^{0\nu}$
$pfsdg$	2.01	0.35	−0.02	2.34

With triaxially deformed configurations, the wave functions:
① are pushed to the region with larger isoscalar *pn* pairing.
② spread widely to the region with larger deformation

$M_{\rm GT}$ is reduced from 3.25 without triaxial deformation to 2.01 with triaxial deformation.

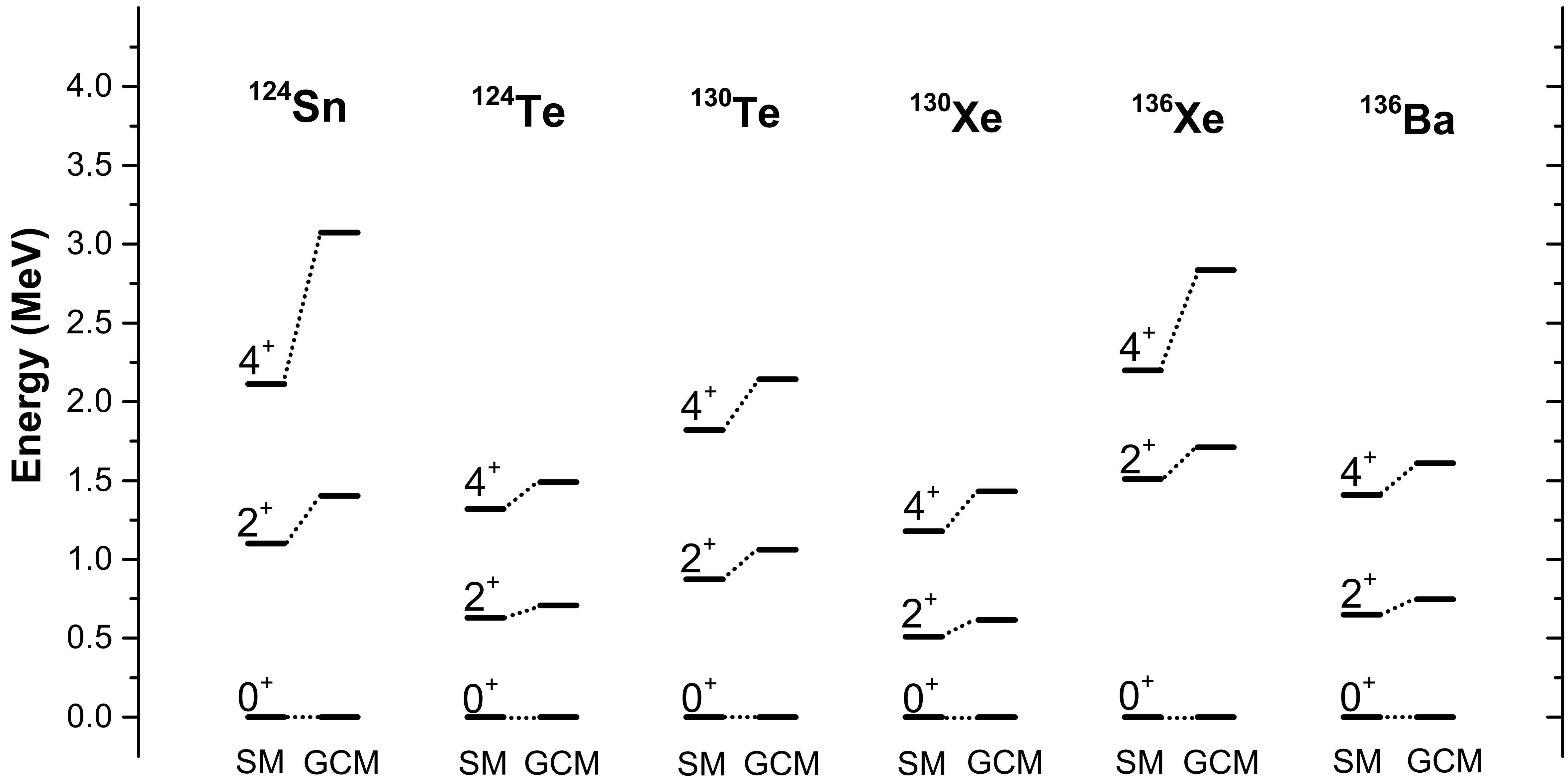
GCM with jj55 space

We want to extend the Hamiltonian-based GCM to larger model space and heavier $0\nu\beta\beta$-decay candidates (e.g., ^{150}Nd), for which no effective shell-model interaction exists.

STEP1: We move forwards to ^{124}Sn, ^{130}Te, and ^{136}Xe to check how GCM with shell-model Hamiltonian works for them.

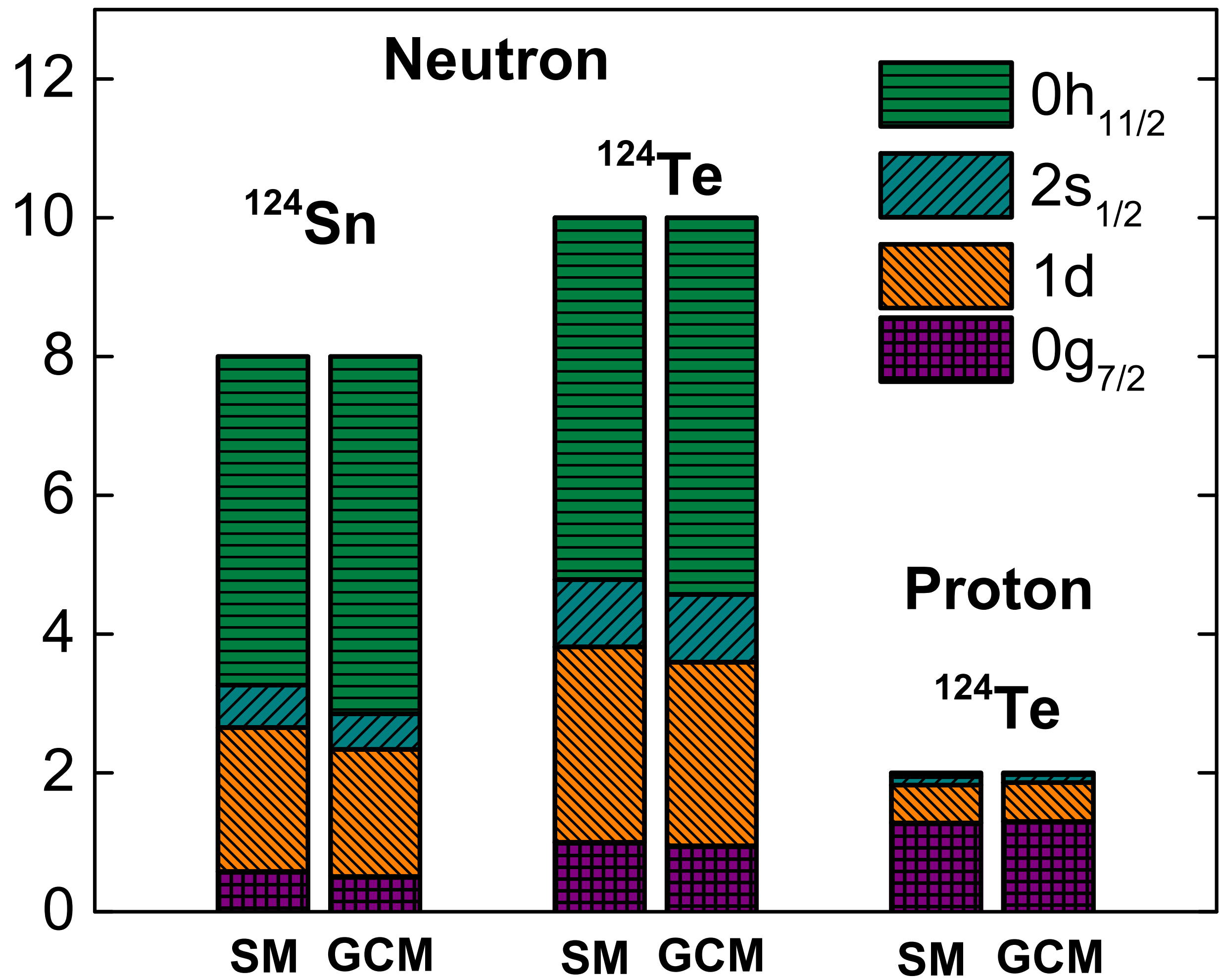
- We use the SVD effective Hamiltonian within $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$. $2s_{1/2}$, $0h_{11/2}$ orbits (called *jj*55 model space here). Prof. Horoi's group has done a lot of shell-model calculation with this interaction, providing a great testing ground.
- Because these nuclei are considered to be nearly spherical or slightly deformed, only axial deformation, isoscalar pairing, and isovector pn pairing are treated as coordinates (but separately for latter two).

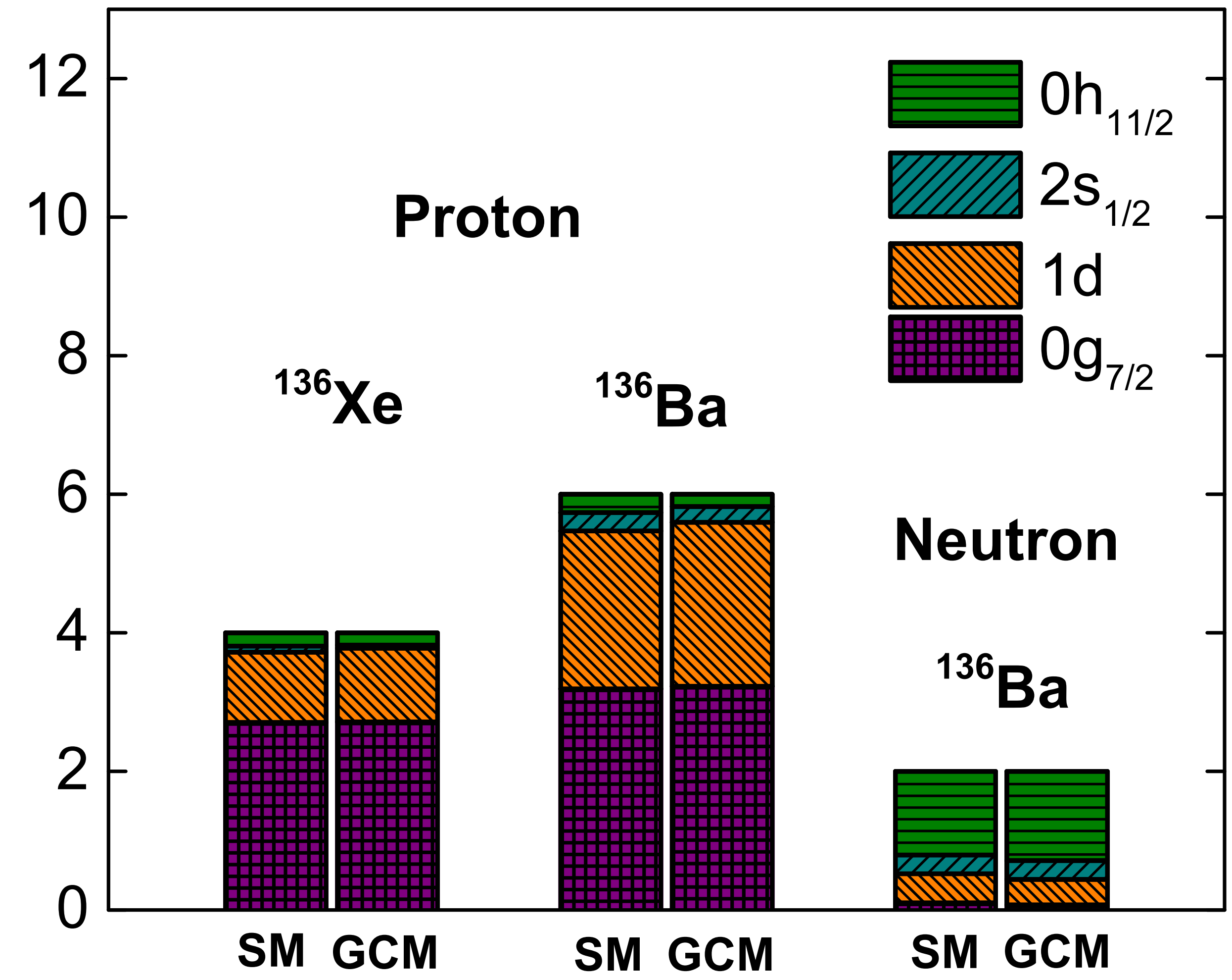
GCM with jj55 space



GCM shows a reasonable agreement with SM in low-lying states, though they are more overestimated for spherical nuclei ^{124}Sn and ^{136}Xe.

GCM with jj55 space





The theoretical neutron shell vacancies and proton shell occupancies given by GCM are very close to the exact diagonalization from SM.

GCM with jj55 space

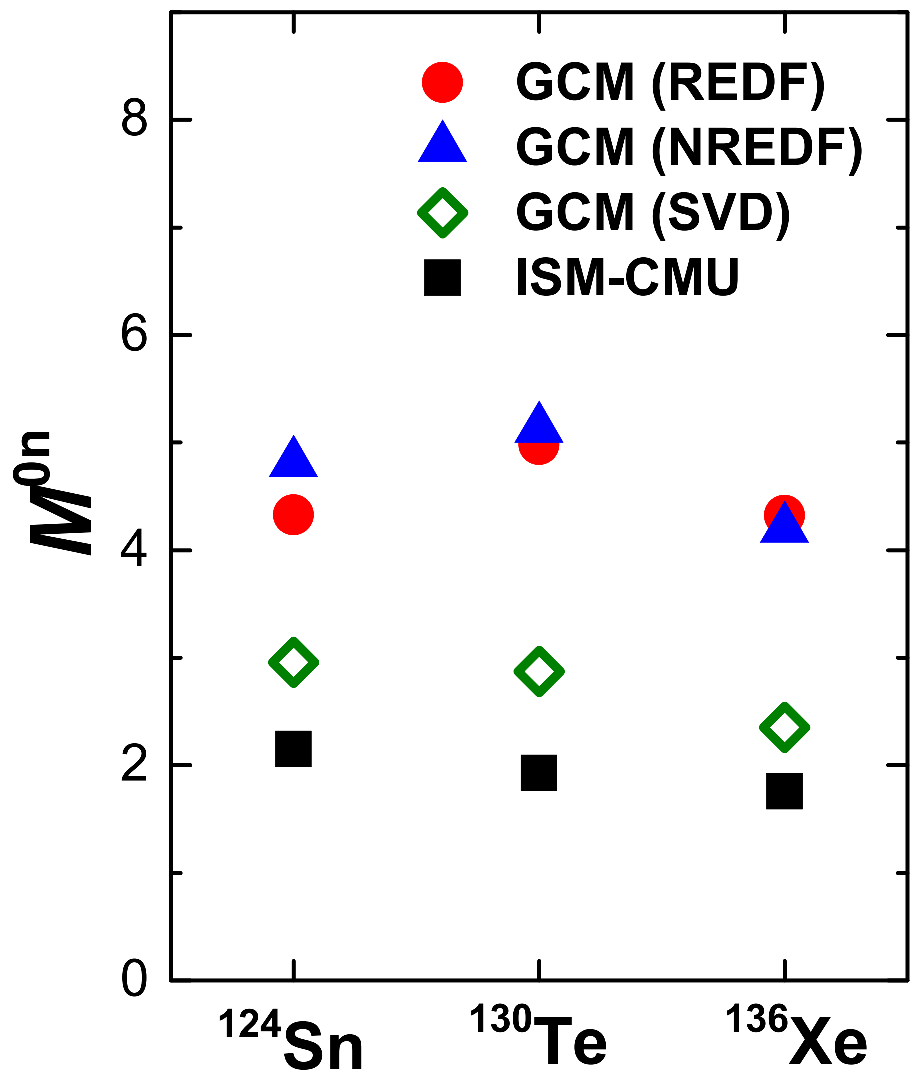
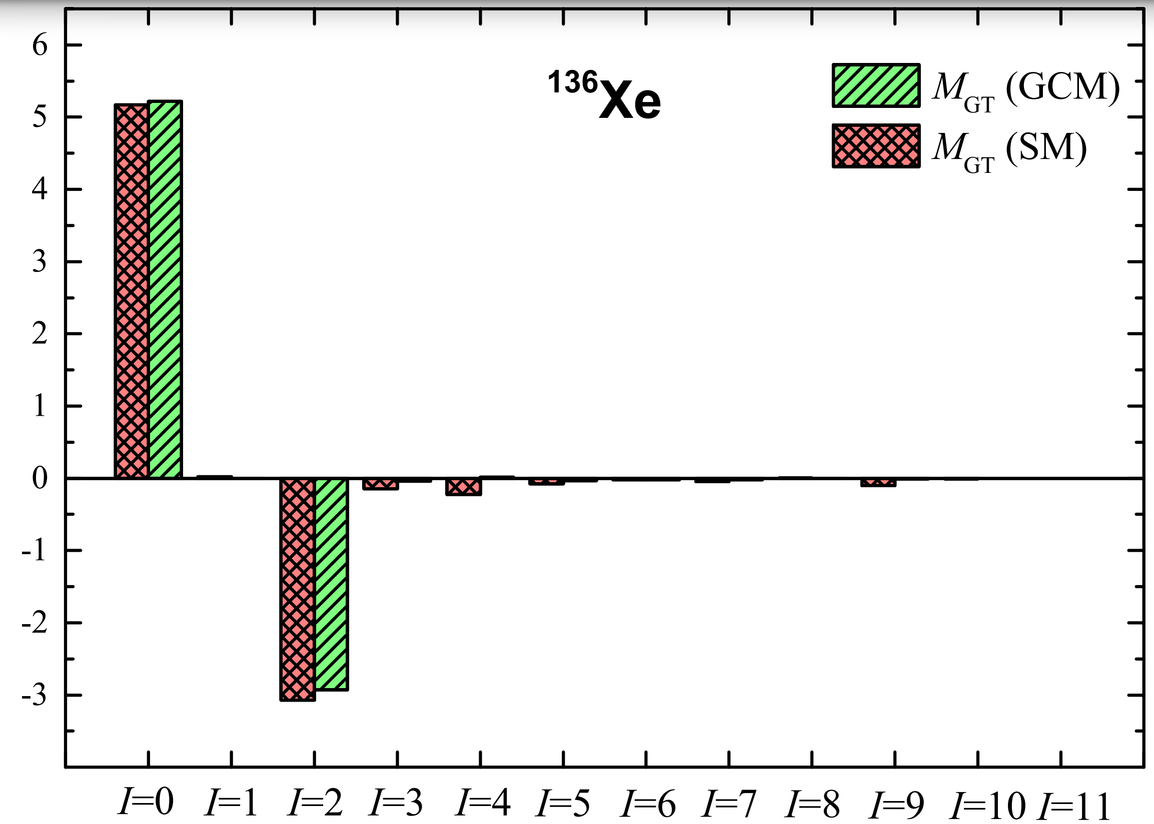


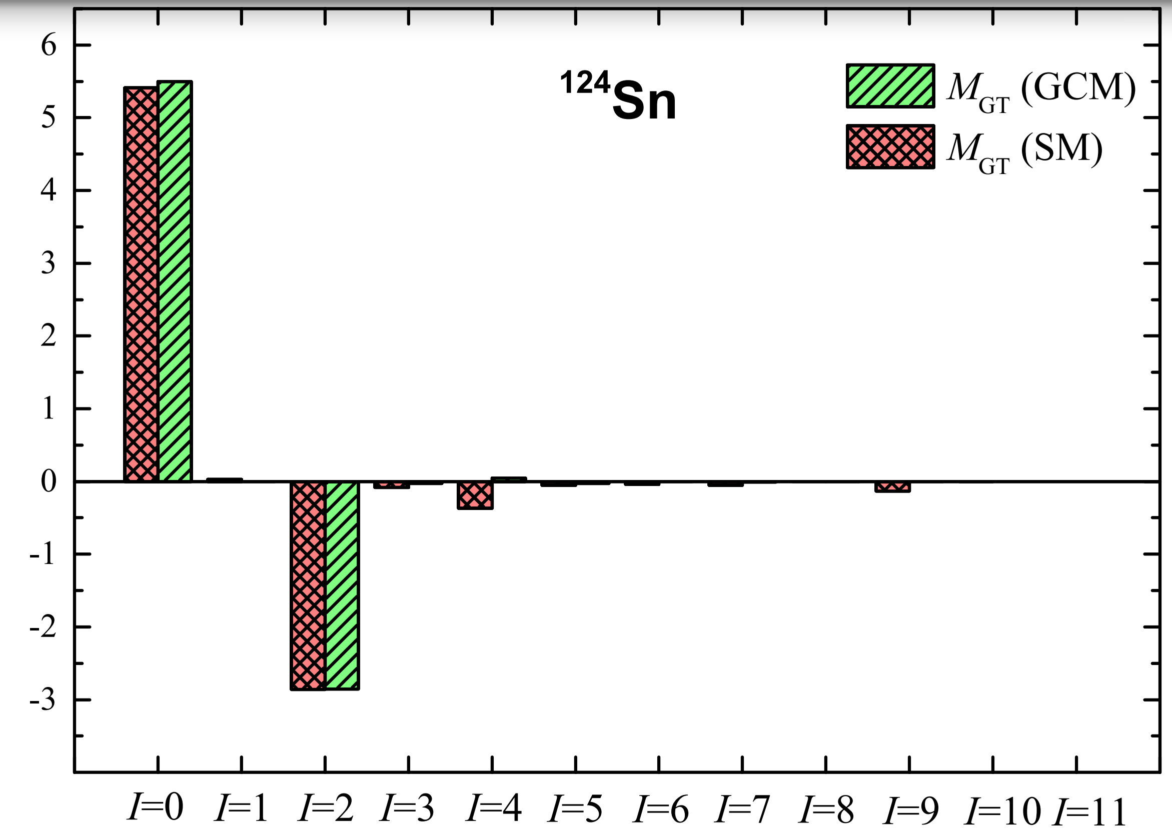
TABLE I: The g.s. energies and NMEs obtained with SVD interaction by using GCM and SM for ^{124}Sn, ^{130}Te, and ^{136}Xe. CD-Bonn SRC parametrization was used.

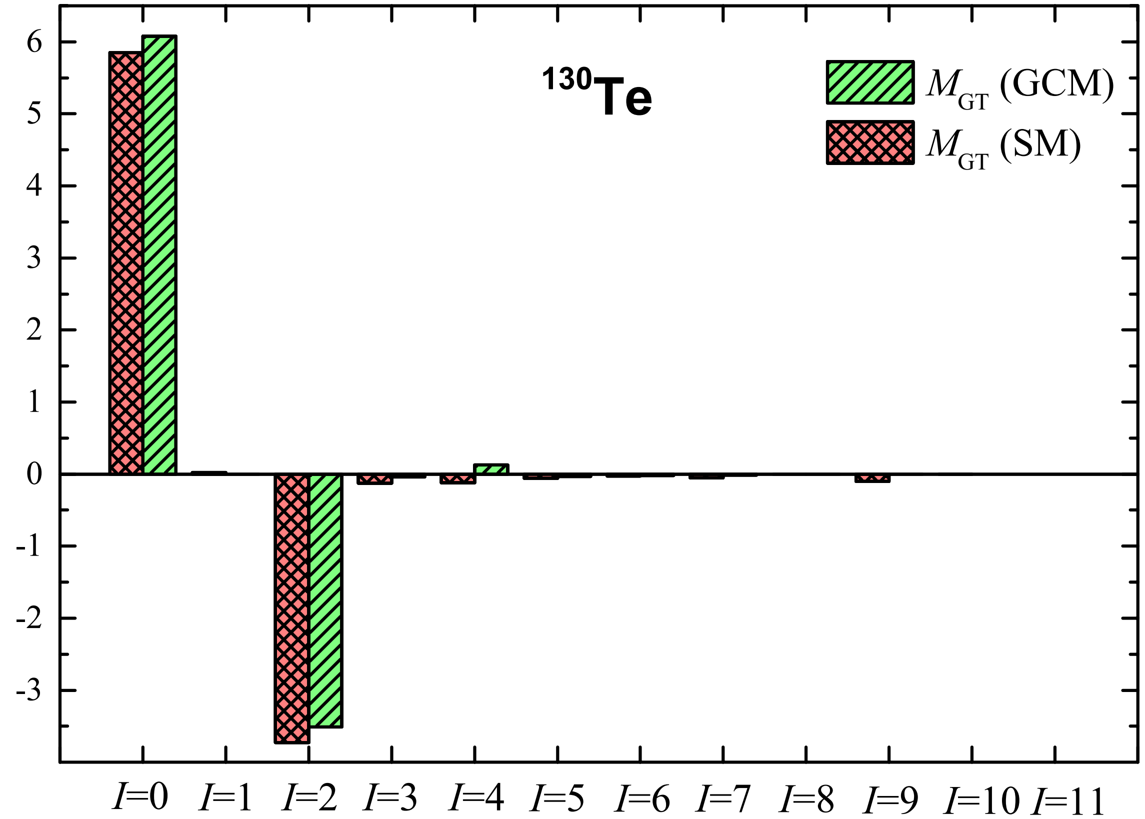
	g.s Energy (MeV)		NMEs			
	^{124}Sn	^{124}Te	$M^{0\nu}_{\rm GT}$	$M^{0\nu}_{\rm F}$	$M^{0\nu}_{\rm T}$	$M^{0\nu}$
GCM	−15.659	−23.056	2.62	−0.58	−0.03	2.96
SM	−16.052	−24.446	1.85	−0.47	0.01	2.15
	^{130}Te	^{130}Xe				
GCM	−25.646	−32.510	2.57	−0.51	−0.02	2.87
SM	−26.039	−33.313	1.66	−0.44	−0.01	1.94
	^{136}Xe	^{136}Ba				
GCM	−34.896	−40.282	2.19	−0.32	−0.02	2.37
SM	−34.971	−40.745	1.50	−0.40	−0.01	1.76

The NMEs given by our SVD-based GCM are closer to the exact result, ~40% larger than SM results, most of them come from GT part.

GCM with jj55 space

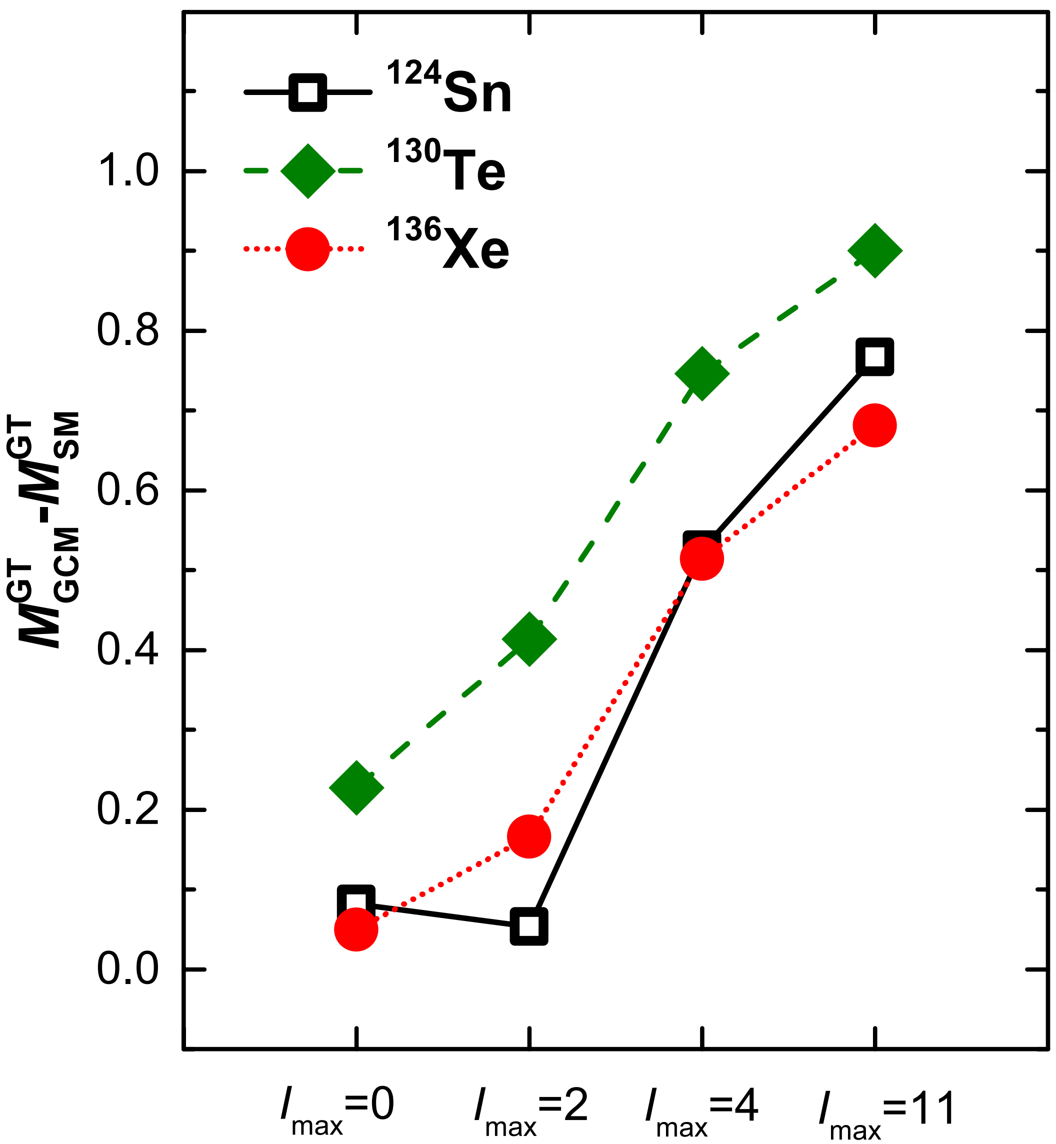






- The dramatic cancellation between I = 0 and I = 2 is well described in GCM.
- The largest discrepancy for I=0 and I=2 occurs in ^{130}Te.
- GCM results barely have I>3 contributions.

GCM with jj55 space

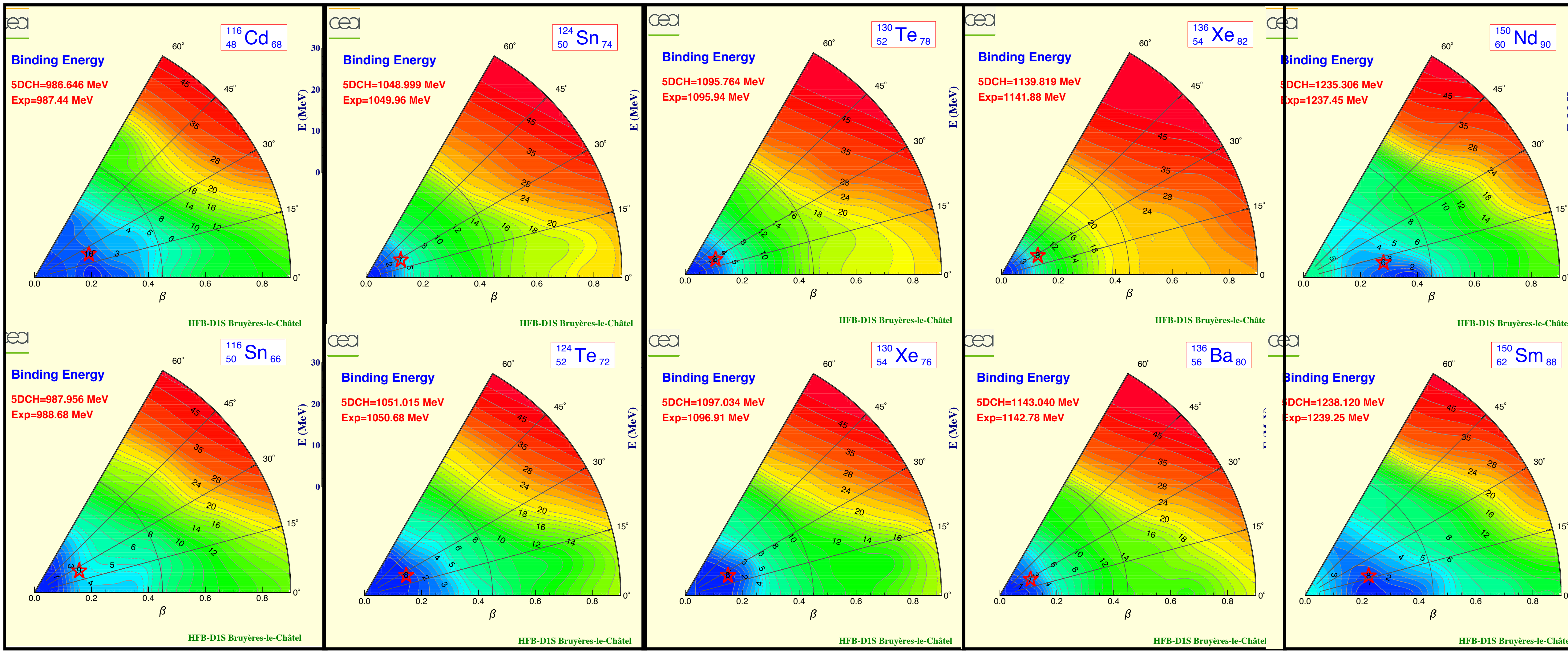


more than 50% overestimation is from *l*>3 contributions: High-seniority correlations?

Some potential improvement:

- treat deformation, isovector pairing, isoscalar pairing as coordinates at the same time.
- High-seniority correlations should be considered. (e.g., quasiparticle excitation?)
- Or we should include triaxially deformed configurations

HFB-PES



CEA-Bruyeres-le-Chatel data base

Page from Thomas R. Rodriguez’s talk at TRIUMF

Summary

- We are trying to combine the virtues of the shell model and EDF calculations by including all collective correlations in the GCM.

- Tests against exact solutions in one shell indicate that we indeed have captured important valence-space correlations.

- Calculation has been extended to two major shell (e.g., *pfsdg* shell) model space, which is out of scope of the conventional SM. Including triaxially deformed configurations significantly affect the calculated NMEs.

- Extending to *jj*55 model space indicates that high-seniority correlations may be required.

Summary

Collaborators:

- Mihai Horoi, CMU
- Andrei Neascu, CMU
- Jonathan Engel, UNC
- Jiangming Yao, UNC
- Longjun Wang, UNC
- Jason Holt, TRIUMF
- Nobuo Hinohara, University of Tsukuba
- Javier Menendez, University of Tokyo

Thank you for your attention!