Hadronic matrix elements for NDB decay from chiral SU(3)

INT
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based on:

MG, arXiv:1606.04549, submitted to JHEP
V. Cirigliano, W. Dekens, MG, E. Mereghetti, (PLB 2017, 1701.01443)

Neutrinoless double beta decay and TeV* scale physics

Motivation

Neutrinos have mass and search is on to discover the nature of their mass.

Ongoing or future experiments may detect a "neutrinoless double beta decay" signal.

Such a signal arises when neutrino masses violate lepton number (i.e., Majorana)

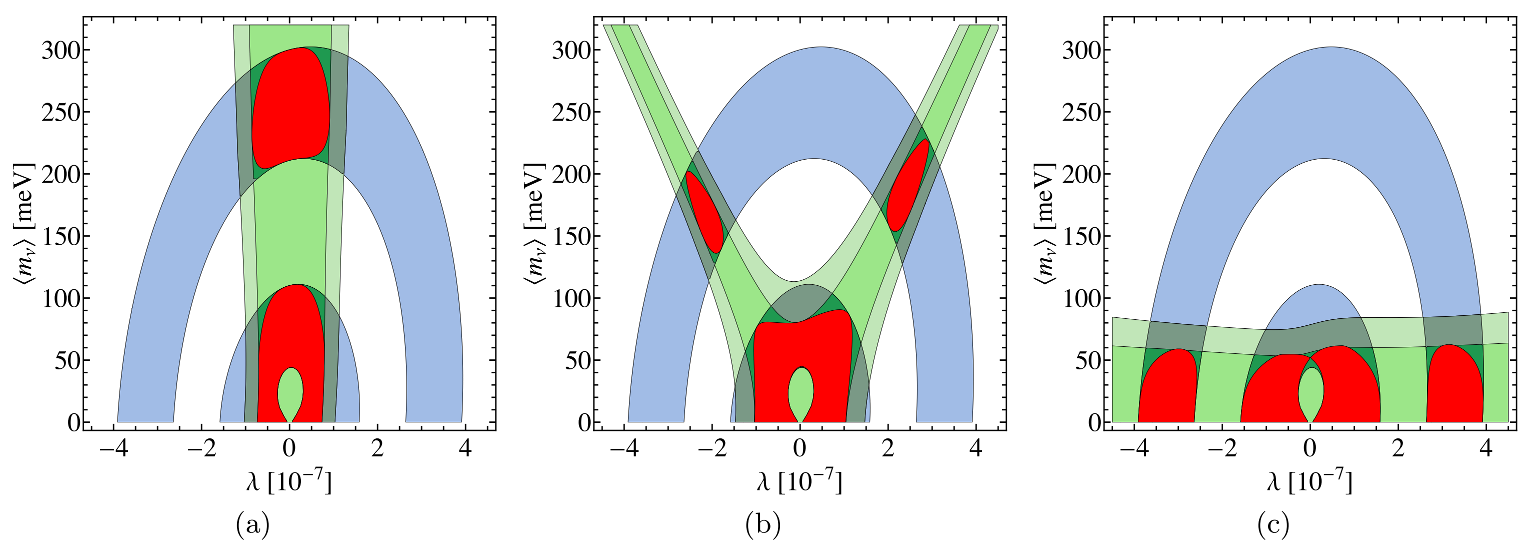
Question: is that the correct interpretation of such a signal?

Are there other (new physics scenario) interpretations?

New physics scenarios for neutrinoless double beta decay

Should a $\Delta L=2$ signal be detected, such exotic possibilities should be excluded before concluding that effect is due to Majorana neutrino exchange

Resolving competing explanations may need a next-generation detector reconstructing both electron kinematics (e.g. NEXT, SuperNEMO)

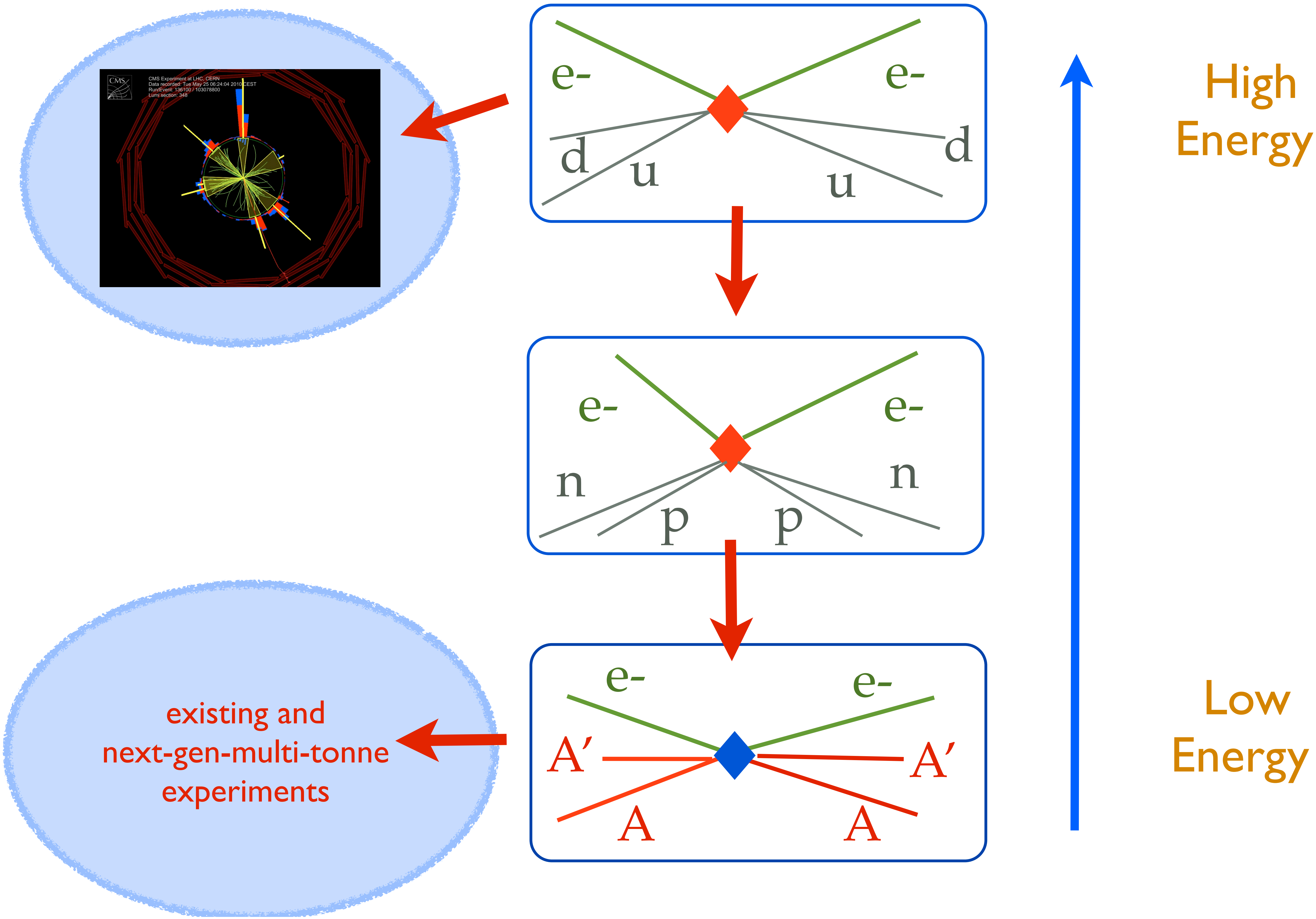


Comparison SuperNEMO sensitivity to various admixtures of W_R contribution (0%, 30%, 100%). Figure from Arnold et. al. (SuperNEMO, 2010)

- If hierarchy is "normal", then planned 0nubb have no chance of detecting Standard Model Majorana neutrinos (outside of the quasi-degenerate region)

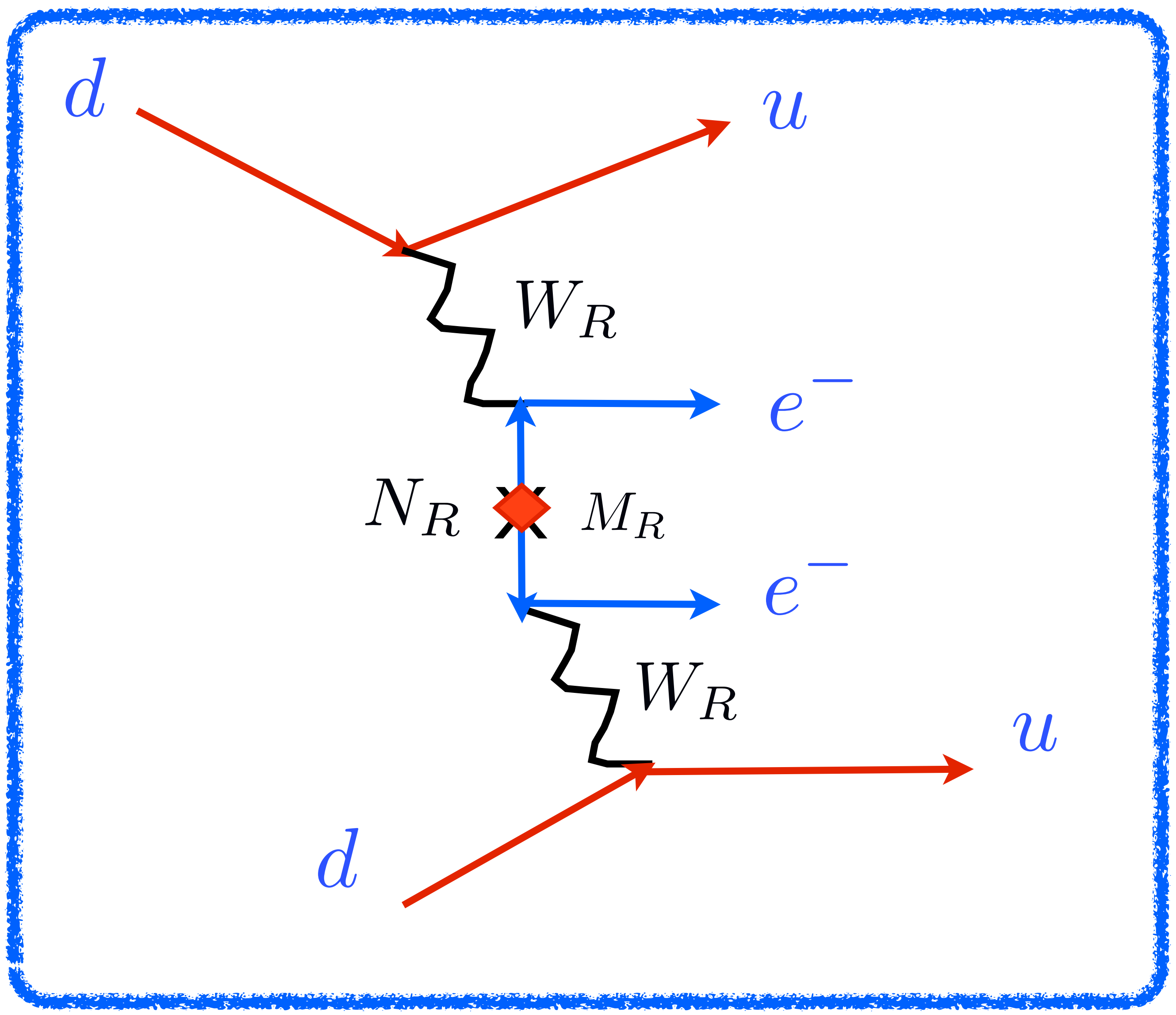
- In such a circumstance, only hope is for exotic scenarios

BSM contributions to neutrinoless beta decay



BSM contributions to neutrinoless beta decay:

Left-Right symmetric model



- new electroweak gauge bosons couple to right-handed currents

- new right-handed or "sterile" neutrinos, electroweak partners of Standard Model right-handed electron

- possibility for type-II see-saw at TeV scale

$$\mathcal{L}_Y = \frac{1}{2}\ell_L \frac{M_{\nu_L}}{\langle\Delta_L\rangle}\Delta_L \ell_L + \frac{1}{2}\ell_R \frac{M_{\nu_R}}{\langle\Delta_R\rangle}\Delta_R \ell_R + \text{h.c.}$$

- Assuming a type-II see-saw, C invariance leads

$$M_{\nu_R}/\langle\Delta_R\rangle = M^*_{\nu_L}/\langle\Delta_L\rangle^* \quad \text{or} \quad m_N \propto m_\nu$$

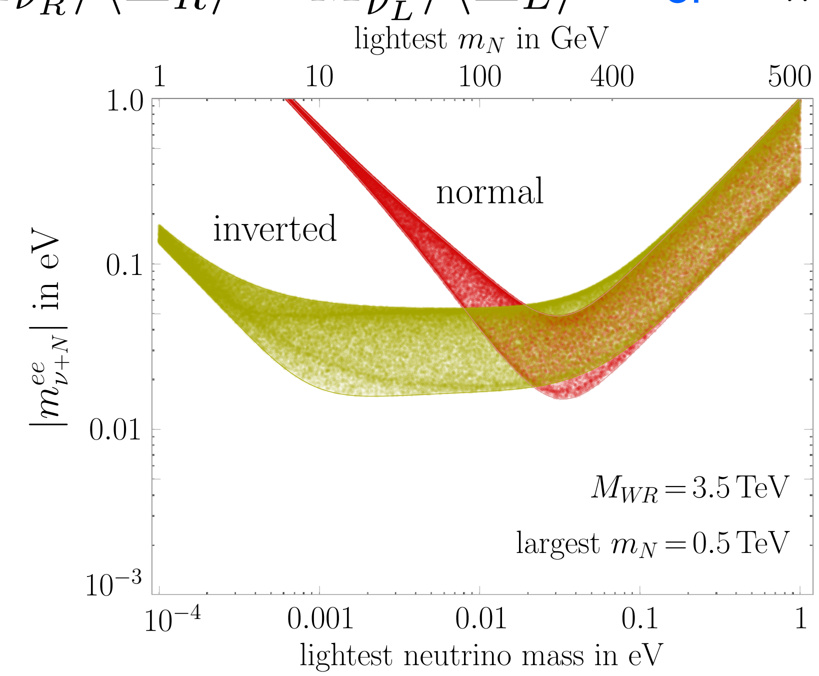
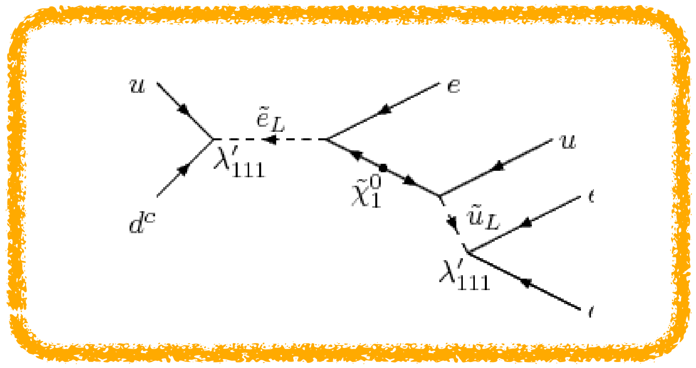


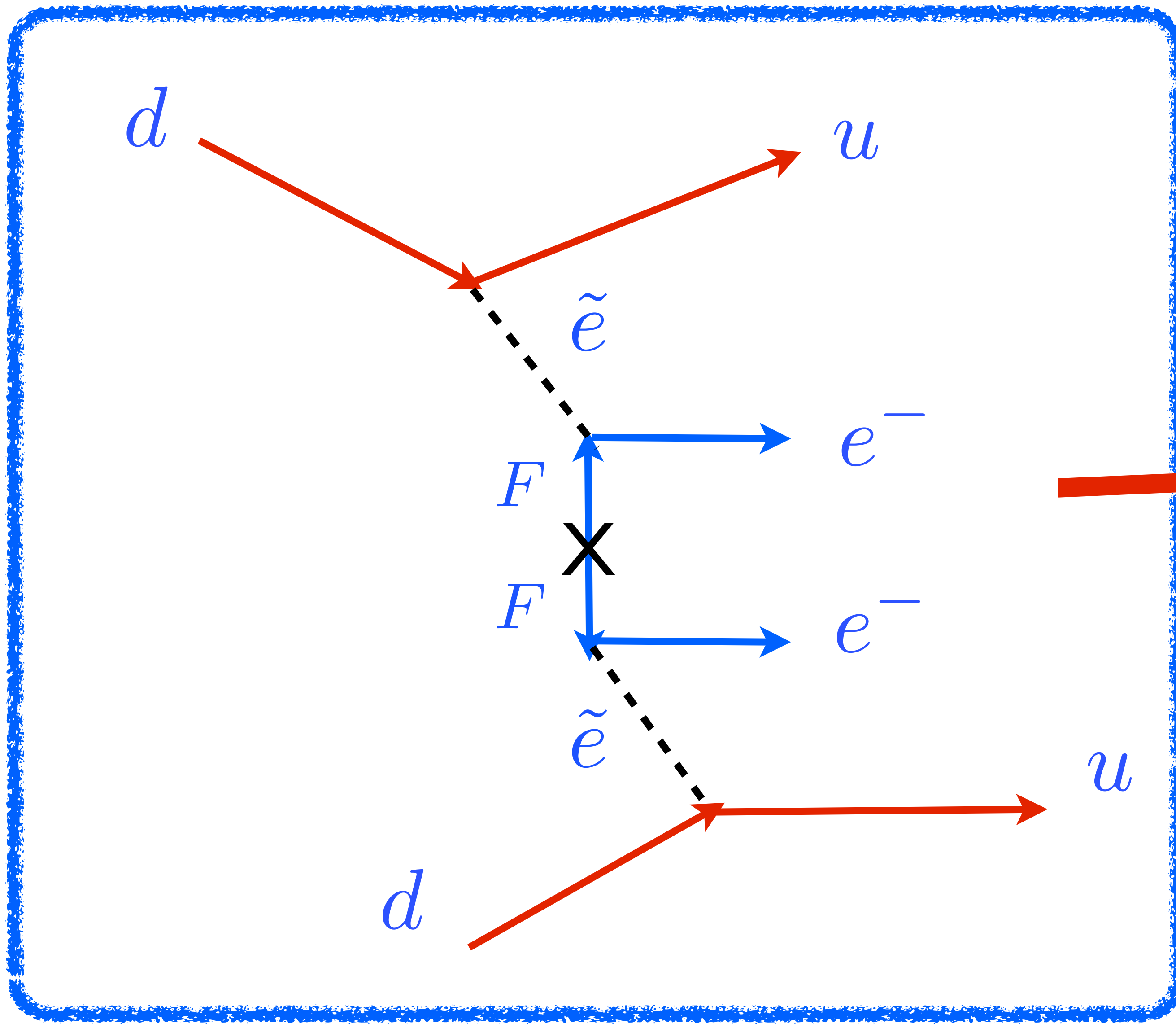
Figure from Tello, Nemevsek, Nesti, Senjanovic and Vissani, 2011

BSM contributions to neutrinoless beta decay:

R-parity violation inspired



- see also e.g. Deppisch, Hirsch, Pas, 2012



- new charged scalar leptons ("sleptons")
- new electroweak partners of the electron
- generate different contact operator at low energies

$$\mathcal{L}^{\rm eff}_{\rm LNV} = \frac{C_1}{\Lambda^5}\mathcal{O}_1 + {\rm h.c.} \quad , \quad \mathcal{O}_1 = \bar{Q}\tau^+ d\bar{Q}\tau^+ d\bar{L}L^C$$

see e.g. M. Ramsey-Musolf, T. Peng and P. Winslow, 2015 for thorough LHC collider phenomenology analysis (and see M. Ramsey Musolf's talk)

- R-M P W include leading 2 pion interactions and RGE analysis, backgrounds, detector sim.
- and determine signal acceptances - very model-dependent (see e.g., A. Friedland, MG, I. Shoemaker, L. Vecchi, 2012, in context of non-Standard Neutrino Interactions at the LHC)

Effective field theory analysis of BSM contributions to neutrinoless double beta decay

- new particles generating ΔL=2 processes have masses in multi-TeV scale.
- 0nubb process generated at very short distances.
- Leading effects of such TeV scale physics can be described by series of ΔL=2 violating operators involving only quarks and leptons

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \mathcal{L}_{\nu,M} + \sum_{i,d>4} \frac{c_i^d}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

e.g., $dd \to u u e^- e^-$

(collider signal:
Keung, Senjanovic, PRL, 1983)

At "low energy" - ie QCD scale - there are a number of "short distance" operators that contribute to neutrinoless double beta decay (Prezeau, Ramsey-Musolf and Vogel (PRD, 68, 2003))

$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^5_{\rm LNV}} \left[\sum_{i=\rm scalar} \left(c_{i,S}\, \bar{e} e^c + c'_{i,S}\, \bar{e}\gamma_5 e^c \right) O_i \;+\; \bar{e}\gamma_\mu\gamma_5 e^c \sum_{i=\rm vector} c_{i,V}\, O_i^\mu \right]$$

What is a minimal basis (MG, arXiv:1606.04549) ?

- leading ΔL=2 operator with two charged leptons has a minimum of 4 quarks, in other words, dimension 9
- For ΔL=2 phenomenology (e.g., 0nubb decay rates) need to know a minimal basis of operators, the set of relevant operators that cannot be reduced by Fierz operators
- Electromagnetic invariance: 24 (compared to 14 in prior literature): 8 scalar and 8 vector 4-quark operators
- Electroweak invariance: If scale Λ of ΔL=2 violating physics is much larger than the electroweak scale, effect of ΔL=2 physics appears as a series of higher dimension operators invariant under the full Standard Model gauge symmetry
- If color + electroweak invariance is imposed, then 11 operators at LO in v/Λ: 7 scalar and 4 vector
- At hadron colliders, if E << Λ, then collider only probing (color + electroweak invariant) ΔL=2 contact operators. In this "contact limit" can classify their experimental signatures.

Electroweak invariant dimension 9 operators:

collider signatures

operator	content	hadron collider signatures: same-sign dilepton	hadron collider signatures: e+MET	hadron collider signatures: dijet+ MET	Low Energy
			dimension 9		
LM1	$i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu Q_c)(\overline{u}_R\gamma_\mu d_R)(\overline{\ell}_b\ell^C_c)$	√	√	√	$\mathcal{O}_{1LR}\otimes(LL)$
LM2	$i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu\lambda^A Q_c)(\overline{u}_R\gamma_\mu\lambda^A d_R)(\overline{\ell}_b\ell^C_c)$	√	√	√	$\mathcal{O}^\lambda_{1LR}\otimes(LL)$
LM3	$(\overline{u}_R Q_a)(\overline{u}_R Q_b)(\overline{\ell}_a\ell^C_b)$	√	√	√	$\mathcal{O}_{2RL}\otimes(LL)$
LM4	$(\overline{u}_R\lambda^A Q_a)(\overline{u}_R\lambda^A Q_b)(\overline{\ell}_a\ell^C_b)$	√	√	√	$\mathcal{O}^\lambda_{2RL}\otimes(LL)$
LM5	$i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a d_R)(\overline{Q}_c d_R)(\overline{\ell}_b\ell^C_d)$	√	√	√	$\mathcal{O}_{2LR}\otimes(LL)$
LM6	$i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a\lambda^A d_R)(\overline{Q}_c\lambda^A d_R)(\overline{\ell}_b\ell^C_d)$	√	√	√	$\mathcal{O}^\lambda_{2LR}\otimes(LL)$
LM7	$(\overline{u}_R\gamma^\mu d_R)(\overline{u}_R\gamma_\mu d_R)(\overline{e}_R e^C_R)$	√	☹	☹	$\mathcal{O}_{3R}\otimes(RR)$
LM8	$(\overline{u}_R\gamma^\mu d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a d_R)(\overline{\ell}_b\gamma_\mu e^C_R)$	√	√	☹	$\mathcal{O}^\mu_{RRLR}\otimes(LR)$
LM9	$(\overline{u}_R\gamma^\mu\lambda^A d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a\lambda^A d_R)(\overline{\ell}_b\gamma_\mu e^C_R)$	√	√	☹	$\mathcal{O}^{\lambda\mu}_{RRLR}\otimes(LR)$
LM10	$(\overline{u}_R\gamma^\mu d_R)(\overline{u}_R Q_a)(\overline{\ell}_a\gamma_\mu e^C_R)$	√	√	☹	$\mathcal{O}^\mu_{RRRL}\otimes(LR)$
LM11	$(\overline{u}_R\gamma^\mu\lambda^A d_R)(\overline{u}_R\lambda^A Q_a)(\overline{\ell}_a\gamma_\mu e^C_R)$	√	√	☹	$\mathcal{O}^{\lambda\mu}_{RRRL}\otimes(LR)$

scalar 4-quark operators (7) (LM1–LM7)

vector 4-quark operators (4) (LM8–LM11)

↔ RPV-inspired theory (LM6)

↔ LR symmetric theory (LM7)

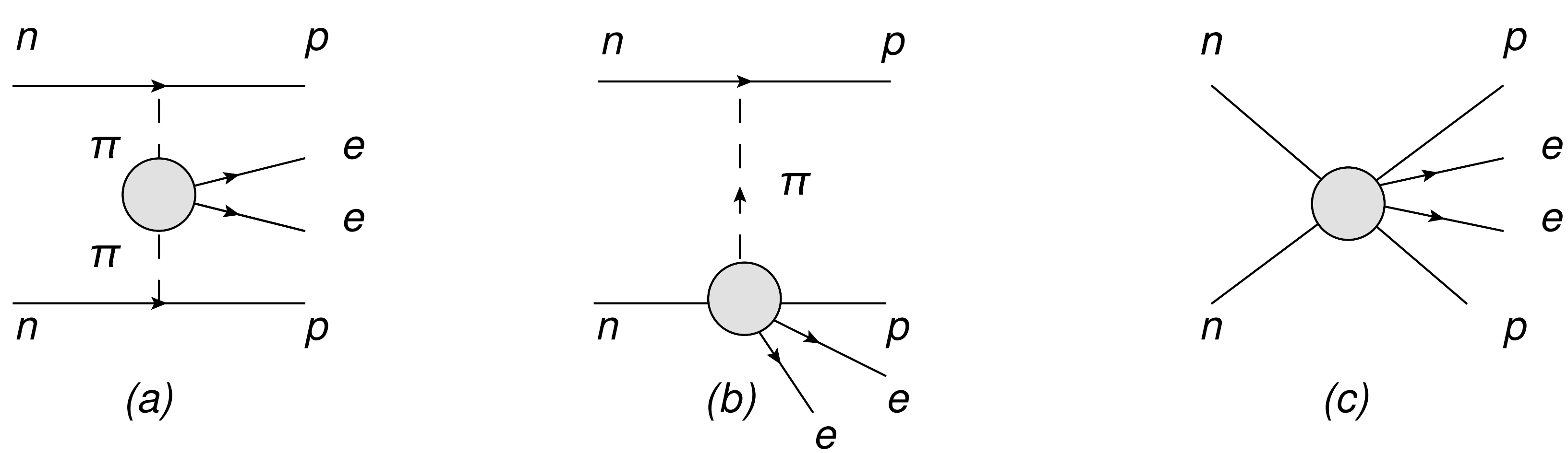
Table from MG, arXiv:1606.04549

- Set up systematic formalism for χPT operators in low-energy effective field theory
- Applied general formalism to identify which operators contribute at LO to eeππ interactions (i.e., which ops. in χPT dominate ΔL=2 amplitude over effects of eeπNN and eeNNNN interactions)

Effective field theory analysis of BSM contributions to neutrinoless double beta decay: Weinberg power counting

- Quarks couple to everything, so expect 4 quark operator to generate many multi-hadron interactions

- Two pion interaction important (Faessler, S. Kovalenko, F. Simkovic, and J. Schwieger, 1996; Prezeau, Ramsey-Musolf and Vogel (PRD, 68, 2003)) but not consistently implemented in other literature

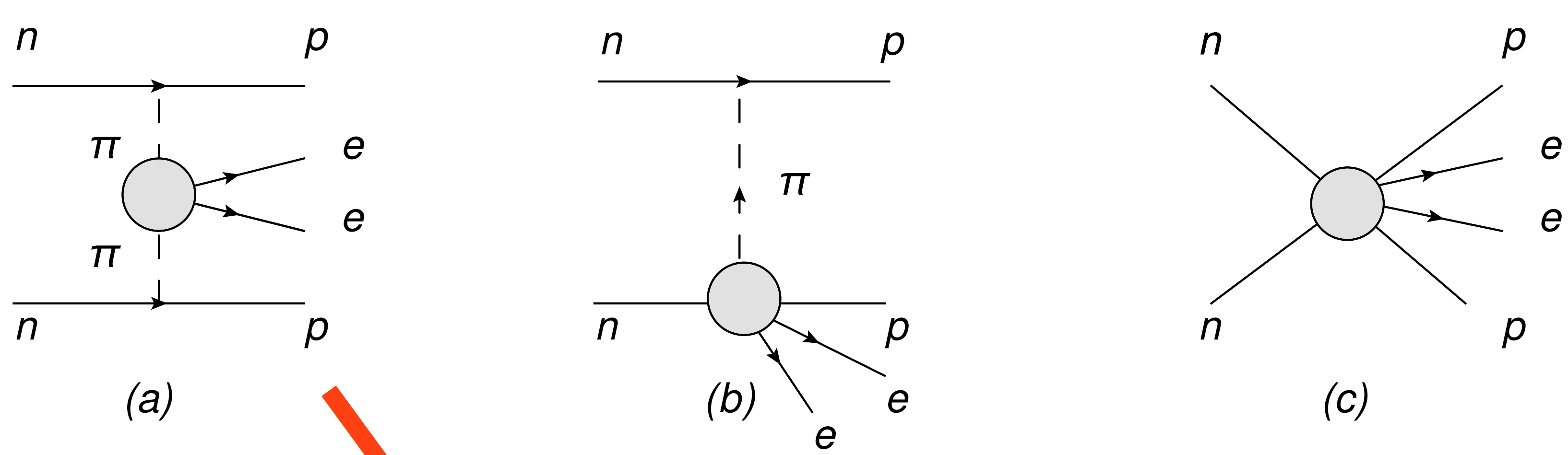


$O(q^{-2+\Delta_{\mathcal{O}}(\pi\pi)})$ $O(q^{-1+\Delta_{\mathcal{O}}(\pi NN)})$ $O(q^{0+\Delta_{\mathcal{O}}(NNNN)})$

- A number of analyses comparing LHC projections and 0nubb limits only include 4-nucleon interactions, "conservatively" suppressing limits from 0nubb experiments (unfairly promotes the competitiveness of the LHC)

- Here power counting is for free field theory only - need to insert inside a nucleus and test power-counting (see S. Pastore's talk)

Effective field theory analysis of BSM contributions to neutrinoless double beta decay: Estimate of long-distance pion exchange



$$O(q^{-2+\Delta_{\mathcal{O}}(\pi\pi)})$$

$$\mathcal{A}_{\pi\pi} \simeq \frac{1}{\Lambda^5_{\rm LNV}} \frac{M_{\langle \pi^+|O_i|\pi^-\rangle}}{f_\pi^2 q^2} \sim 10^2 \frac{1}{\Lambda^5_{\rm LNV}} \frac{M_{\langle O_i\rangle}}{10^{-2}} \frac{(100\ {\rm MeV})^4}{f_\pi^2 q^2}$$

$$\mathcal{A}_{\rm SM} \simeq G_F^2 \frac{m_{\beta\beta}}{q^2}$$

chiral PT estimate: $M_{\langle O_i\rangle} \sim 10^{-2}\ (O_{2,3,4,5}, O'_{2,3})$

Effective field theory analysis of BSM contributions to neutrinoless double beta decay (MG, arXiv:1606.04549)

General $\Delta L=2$ 4-quark scalar operator (following Savage 1999)

$$\mathcal{O} = T^{ab}_{cd}(\overline{q}^c\Gamma q_a)(\overline{q}^d\Gamma' q_b),\ T^{ab}_{cd} = (\tau^+)_c{}^a(\tau^+)_d{}^b$$

Transform T such that O is formally chirally invariant

$$\begin{aligned} q_L &\to Lq,\ q_R \to Rq_R, \\ T &\to T\otimes X_1\otimes X_2\otimes X_3\otimes X_4,\ X_i\in\{L, R, L^\dagger, R^\dagger\} \end{aligned}$$

Construct pion and nucleon operators in chiral theory such that they are formally chirally invariant

$$T^{ab}_{cd}\tilde{\mathcal{O}}^{cd}_{ab}(\pi, N)$$

Effective field theory analysis of BSM contributions to neutrinoless double beta decay: Weinberg power counting

With $\xi = \mathrm{Exp}[\pi \cdot \tau / 2F_\pi], \; \xi \to L\xi U^\dagger = U\xi R^\dagger, \; N \to UN$

Construct "proto-O" out of products of ξ's such that

$$(proto - \tilde{\mathcal{O}}) \quad \to \quad (proto - \tilde{\mathcal{O}}) \otimes Y_1 \otimes Y_2 \otimes Y_3 \otimes Y_4, \; Y_i \in \{U, U^\dagger\}$$

To construct invariants

- only pions: takes all possible traces
- pions and two nucleons: multiply by two N fields in all possible ways, take all possible traces
- Four nucleons: multiply in by 4 nucleon fields in all possible ways
- can also generate new operators involving higher chiral order using chiral transformation properties of quark mass and covariant derivative

Example: Operators from W_R exchange (Left-right-symmetric model)

$$\mathcal{O}_{3R} \equiv (\bar{q}_R \gamma^\mu \tau^+ q_R)(\bar{q}_R \gamma_\mu \tau^+ q_R)$$
$$T^{ab}_{cd} \to T^{\alpha\beta}_{\rho\sigma} R^\rho_c R^\sigma_d R^{\dagger a}_\alpha R^{\dagger b}_\beta$$
$$proto - \tilde{\mathcal{O}}_{3R} = T^{ab}_{cd} \xi^{\dagger i}_a \xi^{\dagger j}_b \xi^c_k \xi^d_l$$

To construct invariants

- only pions: takes all possible traces -> all vanish (in this example)
- Four nucleons: multiply in by 4 nucleon fields in all possible ways -> non-vanishing operator involving 4 nucleons
- can also generate new operators involving higher chiral order using chiral transformation properties of quark mass and covariant derivative -> Find a number of single and double trace operators, e.g.

$$\mathrm{tr}(\mathcal{D}^\mu \xi \tau^+ \mathcal{D}_\mu \xi^\dagger \xi \tau^+ \xi^\dagger)$$

For this operator, expect first non-vanishing two-pion matrix element at NLO -- which we confirmed using chiral SU(3) -- and first non-vanishing 4 nucleon matrix element at LO

Electroweak invariant dimension 9 operators:

two-pion couplings

operator	content	hadron collider signatures: same-sign dilepton	hadron collider signatures: e+MET	hadron collider signatures: dijet+ MET	Low Energy	χPT $(\pi\pi)$
dimension 9						
LM1	$i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu Q_c)(\overline{u}_R\gamma_\mu d_R)(\overline{\ell}_b\ell^C_c)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}_{1LR}\otimes(LL)$	LO
LM2	$i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu\lambda^A Q_c)(\overline{u}_R\gamma_\mu\lambda^A d_R)(\overline{\ell}_b\ell^C_c)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}^\lambda_{1LR}\otimes(LL)$	LO
LM3	$(\overline{u}_R Q_a)(\overline{u}_R Q_b)(\overline{\ell}_a\ell^C_b)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}_{2RL}\otimes(LL)$	LO
LM4	$(\overline{u}_R\lambda^A Q_a)(\overline{u}_R\lambda^A Q_b)(\overline{\ell}_a\ell^C_b)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}^\lambda_{2RL}\otimes(LL)$	LO
LM5	$i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a d_R)(\overline{Q}_c d_R)(\overline{\ell}_b\ell^C_d)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}_{2LR}\otimes(LL)$	LO
LM6	$i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a\lambda^A d_R)(\overline{Q}_c\lambda^A d_R)(\overline{\ell}_b\ell^C_d)$	$\surd$	$\surd$	$\surd$	$\mathcal{O}^\lambda_{2LR}\otimes(LL)$	LO
LM7	$(\overline{u}_R\gamma^\mu d_R)(\overline{u}_R\gamma_\mu d_R)(\overline{e}_R e^C_R)$	$\surd$	$\ddot{\frown}$	$\ddot{\frown}$	$\mathcal{O}_{3R}\otimes(RR)$	NNLO
LM8	$(\overline{u}_R\gamma^\mu d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a d_R)(\overline{\ell}_b\gamma_\mu e^C_R)$	$\surd$	$\surd$	$\ddot{\frown}$	$\mathcal{O}^\mu_{RRLR}\otimes(LR)$	-
LM9	$(\overline{u}_R\gamma^\mu\lambda^A d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a\lambda^A d_R)(\overline{\ell}_b\gamma_\mu e^C_R)$	$\surd$	$\surd$	$\ddot{\frown}$	$\mathcal{O}^{\lambda\mu}_{RRLR}\otimes(LR)$	-
LM10	$(\overline{u}_R\gamma^\mu d_R)(\overline{u}_R Q_a)(\overline{\ell}_a\gamma_\mu e^C_R)$	$\surd$	$\surd$	$\ddot{\frown}$	$\mathcal{O}^\mu_{RRRL}\otimes(LR)$	-
LM11	$(\overline{u}_R\gamma^\mu\lambda^A d_R)(\overline{u}_R\lambda^A Q_a)(\overline{\ell}_a\gamma_\mu e^C_R)$	$\surd$	$\surd$	$\ddot{\frown}$	$\mathcal{O}^{\lambda\mu}_{RRRL}\otimes(LR)$	-

7 "scalar" quark operators (LM1–LM7)

4 "vector" quark operators (LM8–LM11)

LM6 ↔ RPV-inspired theory

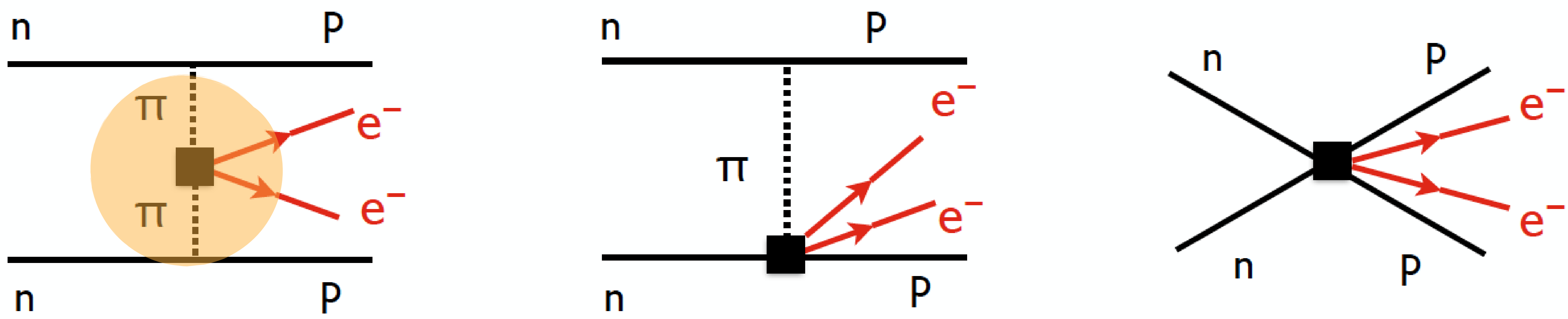
LM7 ↔ LR symmetric theory

Table from MG, arXiv:1606.04549

- Only one pair of scalar operators suppressed in chiPT counting (O_1,O'_1)
- Confirm two-pion interactions from vector operators suppressed by electron mass through NNLO (Prezeau, Ramsey-Musolf, Vogel)

Effective field theory analysis of BSM contributions to neutrinoless double beta decay: two pion matrix elements

V. Cirigliano, W. Dekens, MG, E. Mereghetti, 1701.01443, PLB 2017



From the minimal basis, 8 scalar quark operators:

$$
\begin{aligned}
O_1 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \; \bar{q}_L^\beta \gamma^\mu \tau^+ q_L^\beta \\
O_2 &= \bar{q}_R^\alpha \tau^+ q_L^\alpha \; \bar{q}_R^\beta \tau^+ q_L^\beta \\
O_3 &= \bar{q}_R^\alpha \tau^+ q_L^\beta \; \bar{q}_R^\beta \tau^+ q_L^\alpha \\
O_4 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \; \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta \\
O_5 &= \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta \; \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\alpha
\end{aligned}
$$

$$+ \, O'_{1,2,3} \text{ from } L \leftrightarrow R \text{ on } O_{1,2,3}$$

For 0nubb phenomenology, need matrix elements

$$\langle \pi^+ | O_i | \pi^- \rangle$$

Two pion matrix elements

- O_1, O_1' two pion matrix element determined by M. Savage (1999) using chiral SU(3) symmetry to relate $\pi\pi$ amplitude to $\Delta I=3/2$ K-> $\pi\pi$ decay
- we were able to extend Savage's analysis to all such operators, by relating two pion matrix elements to those involving $\Delta S=1, 2$ matrix elements which are now accurately computed on the lattice
- preliminary lattice computations exist for two pion matrix elements
- (Nicholson et. al., 2015, see A. Nicholson's talk)

Two pion matrix elements

- 4 quark operators belong to irreducible representations of $SU(3)_L \times SU(3)_R$

$$q_{L,R} \quad \sim \quad \mathbf{3_{L,R}}$$

$$\begin{aligned}
O_1 &= \bar{q}^\alpha_L \gamma_\mu \tau^+ q^\alpha_L \ \bar{q}^\beta_L \gamma^\mu \tau^+ q^\beta_L \\
O_2 &= \bar{q}^\alpha_R \tau^+ q^\alpha_L \ \bar{q}^\beta_R \tau^+ q^\beta_L \\
O_3 &= \bar{q}^\alpha_R \tau^+ q^\beta_L \ \bar{q}^\beta_R \tau^+ q^\alpha_L \\
O_4 &= \bar{q}^\alpha_L \gamma_\mu \tau^+ q^\alpha_L \ \bar{q}^\beta_R \gamma^\mu \tau^+ q^\beta_R \\
O_5 &= \bar{q}^\alpha_L \gamma_\mu \tau^+ q^\beta_L \ \bar{q}^\beta_R \gamma^\mu \tau^+ q^\alpha_R
\end{aligned}$$

$\xrightarrow{SU(3)_L \times SU(3)_R}$

$$O_1 \sim \mathbf{27_L} \otimes \mathbf{1_R}$$

$$O_{2,3} \sim \mathbf{6_L} \otimes \overline{\mathbf{6}}\mathbf{_R}$$

$$O_{4,5} \sim \mathbf{8_L} \otimes \mathbf{8_R}$$

- + O'$_{1,2,3}$ by L <--> R from O$_{1,2,3}$; by parity same QCD matrix element

- $O_1 \sim (\overline{u}_L d_L)(\overline{u}_L d_L) \quad \rightarrow \quad I = (2_L, 0_R)$

$$\mathbf{8} \otimes \mathbf{8} \quad = \quad \mathbf{27} + \mathbf{10} + \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}$$

and only 27 contains I=2 → $27_L \otimes 1_R$

- $O_{2,3} \sim (\overline{u}_R d_L)(\overline{u}_R d_L) \quad \rightarrow \quad I = (1_L, 1_R)$

and contains “symmetric component” → $6_L \otimes \overline{6}_R$

Chiral perturbation theory

First consider $O_{2,3,4,5}$+ $O'_{2,3,4,5}$, then return to O_1, O_1'

$$U = \exp\left(\frac{\sqrt{2}i\pi}{F_0}\right), \qquad \pi = \begin{pmatrix} \frac{\pi_3}{\sqrt{2}} + \frac{\pi_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_3}{\sqrt{2}} + \frac{\pi_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\pi_8 \end{pmatrix} \qquad U \rightarrow LUR^\dagger$$

$$\begin{aligned} O_{4,5} &= \overline{q}_L T^a \gamma^\mu q_L \ \overline{q}_R T^b \gamma_\mu q_R \\ T^a &\rightarrow L T^a L^\dagger \\ T^b &\rightarrow R T^b R^\dagger \end{aligned}$$

Only $\mathrm{Tr}\ T^a U T^b U^\dagger$ is formally invariant

Chiral perturbation theory

$$U = \exp\left(\frac{\sqrt{2} i\pi}{F_0}\right), \qquad \pi = \begin{pmatrix} \frac{\pi_3}{\sqrt{2}} + \frac{\pi_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_3}{\sqrt{2}} + \frac{\pi_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\pi_8 \end{pmatrix} \qquad U \rightarrow LUR^\dagger$$

$$\begin{aligned} O_{2,3} &= \bar{q}_R T^a q_L \ \ \bar{q}_R T^b q_L \\ T^{a,b} &\rightarrow R T^{a,b} L^\dagger \end{aligned}$$

Here there are two formal invariants

$\mathrm{Tr}\ T^a U T^b U$ and $\mathrm{Tr}\ T^a U$

Specific linear combination keeps the 6 and projects out the 3*

Matching quark operators onto chiral operators

$$O_{2,3} \quad : \quad O^{a,b}_{6\times\bar{6}} = \bar{q}_R T^a q_L \ \ \bar{q}_R T^b q_L \Big|_{6\times\bar{6}} \quad \rightarrow \quad g_{6\times\bar{6}} \, \frac{F_0^4}{8} \left[\mathrm{Tr}\left(T^a U T^b U\right) + \mathrm{Tr}\left(T^a U\right) \mathrm{Tr}\left(T^b U\right)\right]$$

$$O_{4,5} \quad : \quad O^{a,b}_{8\times 8} = \bar{q}_L T^a \gamma_\mu q_L \ \ \bar{q}_R T^b \gamma^\mu q_R \quad \rightarrow \quad g_{8\times 8} \, \frac{F_0^4}{4} \, \mathrm{Tr}\left(T^a U T^b U^\dagger\right) \ ,$$

- Non-perturbative dynamics encoded in each low-energy constant

$$g_{6\otimes\bar{6}}, \ \ g_{8\otimes 8}$$

  - for each chiral rep, each color contraction has its own LEC g
  - ΔL=2 operators $\quad T^a \quad \rightarrow \quad T^1 + iT^2$
  - K–Kbar mixing ΔS=2 operators

$$T^a \quad \rightarrow \quad T^6 - iT^7$$

Matching quark operators onto chiral operators

$O_{2,3}$: $O^{a,b}_{6\times\bar{6}} = \bar{q}_R T^a q_L \;\; \bar{q}_R T^b q_L\Big|_{6\times\bar{6}} \;\; \to \;\; g_{6\times\bar{6}} \, \frac{F_0^4}{8} \left[\mathrm{Tr}\left(T^a U T^b U\right) + \mathrm{Tr}\left(T^a U\right)\mathrm{Tr}\left(T^b U\right)\right]$

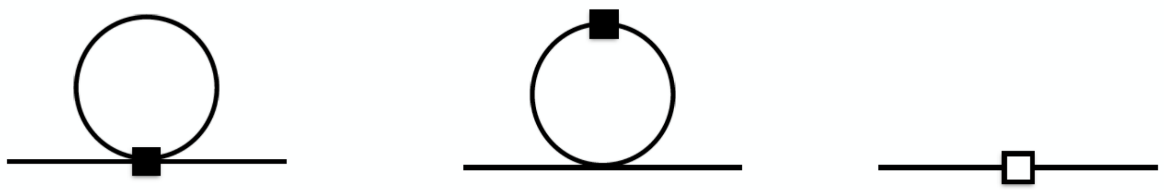
$O_{4,5}$: $O^{a,b}_{8\times 8} = \bar{q}_L T^a \gamma_\mu q_L \;\; \bar{q}_R T^b \gamma^\mu q_R \;\; \to \;\; g_{8\times 8} \, \frac{F_0^4}{4} \, \mathrm{Tr}\left(T^a U T^b U^\dagger\right) \; ,$

$$\mathcal{M}^{\pi\pi}_{6\times\bar{6}} \equiv \langle \pi^+ | O^{1+i2,1+i2}_{6\times\bar{6}} | \pi^- \rangle \;\; = \;\; \langle \bar{K}^0 | O^{6-i7,6-i7}_{6\times\bar{6}} | K^0 \rangle \equiv \mathcal{M}^{K\bar{K}}_{6\times\bar{6}}$$

$$\mathcal{M}^{\pi\pi}_{8\times 8} \equiv \langle \pi^+ | O^{1+i2,1+i2}_{8\times 8} | \pi^- \rangle \;\; = \;\; \langle \bar{K}^0 | O^{6-i7,6-i7}_{8\times 8} | K^0 \rangle \equiv \mathcal{M}^{K\bar{K}}_{8\times 8}$$

(LO in chiPT)

Quark masses (pion masses) break chiral symmetry. So previous relations modified at NLO. We did a loop computation to estimate the size of that splitting.



$$\mathcal{M}^{\pi\pi}_{8\times 8} = \mathcal{M}^{K\bar{K}}_{8\times 8} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{8\times 8}) = \mathcal{M}^{K\bar{K}}_{8\times 8} \times R_{8\times 8}$$

$$\mathcal{M}^{\pi\pi}_{6\times \bar{6}} = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{6\times \bar{6}}) = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times R_{6\times \bar{6}} \; ,$$

$$\Delta_{8\times 8} = \frac{1}{(4\pi F_0)^2}\left[\frac{m_\pi^2}{4}(-4+5L_\pi) - m_K^2(-1+2L_K) + \frac{3}{4}m_\eta^2 L_\eta - a_{8\times 8}\,(m_K^2 - m_\pi^2)\right]$$

$$\Delta_{6\times \bar{6}} = \frac{1}{(4\pi F_0)^2}\left[-\frac{m_\pi^2}{4}(4-3L_\pi) - m_K^2(-1+2L_K) + \frac{5}{4}m_\eta^2 L_\eta - a_{6\times \bar{6}}\,(m_K^2 - m_\pi^2)\right]$$

$$\left(L_{\pi,K,\eta} \equiv \log \mu_\chi^2 / m^2_{\pi,K,\eta}\right)$$

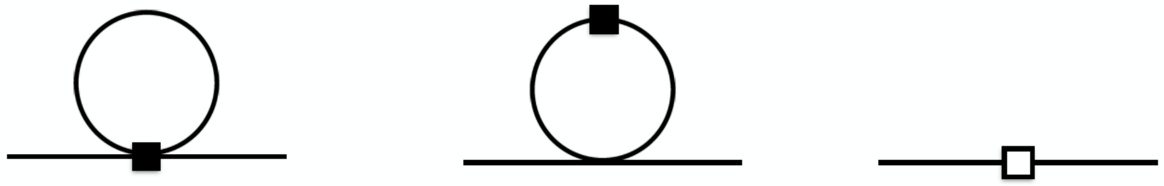
- We agree with loop corrections to K-Kbar (Becirevic, Villadoro, 2004)
- Counter-terms from NLO local operators have the form (V. Cirigliano, E. Golowich, 2000)

$$\delta^{K\bar{K}}_{8\times 8} = a_{8\times 8}\, m_K^2 + b_{8\times 8}\left(m_K^2 + \frac{1}{2}m_\pi^2\right)$$

$$\delta^{\pi\pi}_{8\times 8} = a_{8\times 8}\, m_\pi^2 + b_{8\times 8}\left(m_K^2 + \frac{1}{2}m_\pi^2\right)$$

- Low-energy coefficients {a} could be extracted (in principle) from K-Kbar mixing computed using lattice QCD at different values for the quark masses

Estimating central value and uncertainty



$$\mathcal{M}^{\pi\pi}_{8\times 8} = \mathcal{M}^{K\bar{K}}_{8\times 8} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{8\times 8}) = \mathcal{M}^{K\bar{K}}_{8\times 8} \times R_{8\times 8}$$
$$\mathcal{M}^{\pi\pi}_{6\times \bar{6}} = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{6\times \bar{6}}) = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times R_{6\times \bar{6}} \, ,$$

- For central value for Δ's, set renormalization scale to rho mass and counter-terms =0
- Adopted two prescriptions for estimating the error due to unknown $\delta^{K\bar{K}}_{8\times 8}, \delta^{\pi\pi}_{8\times 8}$
  - Naive-dimensional analysis : $|a_{8\times 8, 6\times \bar{6}}| \sim O(1)$

    gives $\Delta_{8\times 8} = 0.02(20), \; \Delta_{6\times \bar{6}} = 0.07(20)$
  - O(1) change in (log) renormalization scale (Manohar '96): $\Delta_n^{(\mathrm{ct})} = \pm |d\Delta_n^{(\mathrm{loops})}/d(\log \mu_\chi)|$

    gives $\Delta_{8\times 8} = 0.02(36), \; \Delta_{6\times \bar{6}} = 0.07(16)$
- For final analysis, chose $\Delta_{8\times 8} = 0.02(30) \; , \; \Delta_{6\times \bar{6}} = 0.07(20)$
  - This choice gives
$$R_{8\times 8} = 0.72(21) \; (\sim 30\% \text{ uncertainty})$$
$$R_{6\times \bar{6}} = 0.76(14) \; (\sim 20\% \text{ uncertainty})$$

Relate our operators to those defined by FLAG (Aoki et.al, 1607.00299)

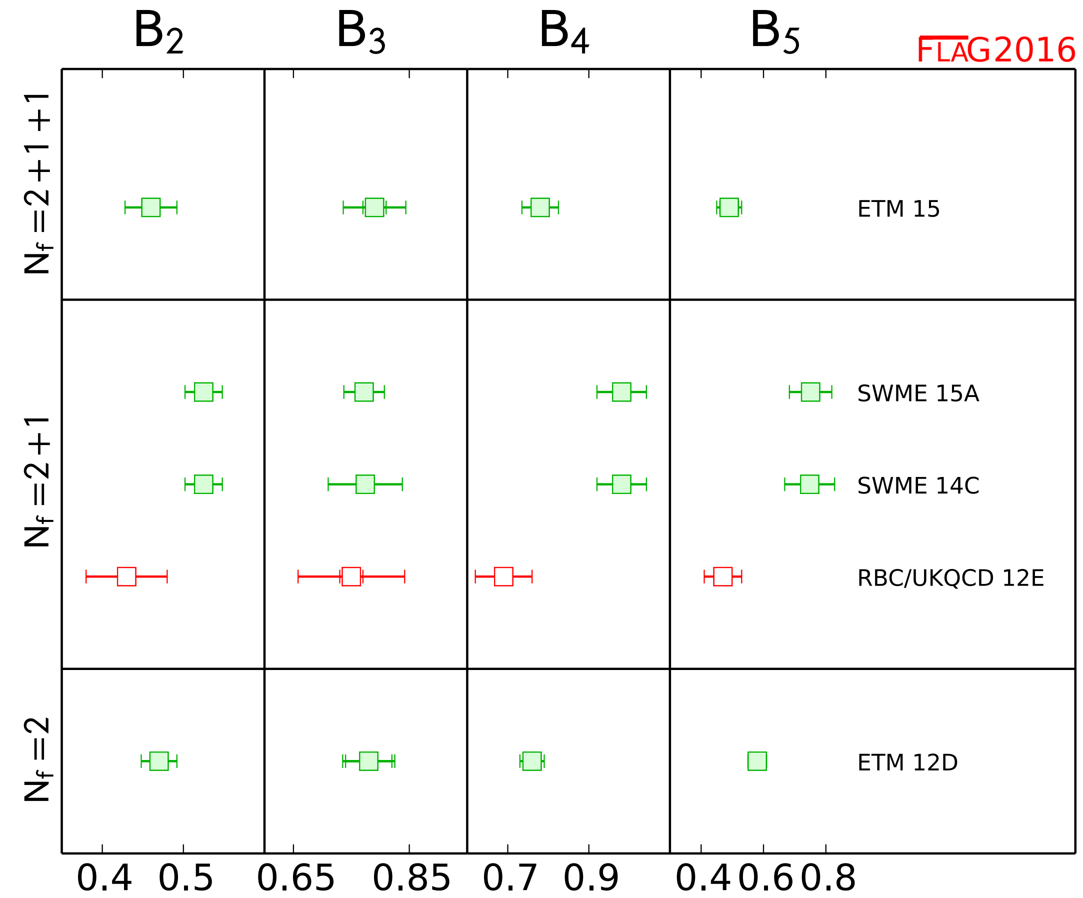
average central values for Nf=2+1 and Nf=2+1+1

$$\langle\pi^+|O_2|\pi^-\rangle = -\frac{5}{12} B_2\, K \times R_{6\times\bar{6}}, \qquad K = \frac{2\, F_K^2\, m_K^4}{(m_d+m_s)^2}$$

$$\langle\pi^+|O_3|\pi^-\rangle = \frac{1}{12} B_3\, K \times R_{6\times\bar{6}}$$

$$\langle\pi^+|O_4|\pi^-\rangle = -\frac{1}{3} B_5\, K \times R_{8\times 8}$$

$$\langle\pi^+|O_5|\pi^-\rangle = -B_4\, K \times R_{8\times 8}$$



LQCD input: B_2, B_3: O(10%) error

B_4, B_5: O(20%) error

$$\langle\pi^+|O_1|\pi^-\rangle = (1.0\pm0.1\pm0.2)\times10^{-4}\ \mathrm{GeV}^4$$

$$\langle\pi^+|O_2|\pi^-\rangle = -(2.7\pm0.3\pm0.5)\times10^{-2}\ \mathrm{GeV}^4$$

$$\langle\pi^+|O_3|\pi^-\rangle = (0.9\pm0.1\pm0.2)\times10^{-2}\ \mathrm{GeV}^4$$

$$\langle\pi^+|O_4|\pi^-\rangle = -(2.6\pm0.8\pm0.8)\times10^{-2}\ \mathrm{GeV}^4$$

$$\langle\pi^+|O_5|\pi^-\rangle = -(11\pm2\pm3)\times10^{-2}\ \mathrm{GeV}^4$$

Fractional error:

O_2, O_3: O(20%) error

O_5 : O(40%) error

O_4 : O(35%) error

Updating M. Savage's (1999) determination of $\langle \pi^+ | O_1 | \pi^- \rangle$

Observation is that

$$O_1,\ O_{\Delta S=2},\ Q_2^{(27\otimes 1)} \in \mathbf{27}$$

$$\updownarrow$$

$$K^+ \quad \rightarrow \quad \pi^+ \pi^0$$

$$\begin{aligned} Q_2^{(27\times 1)} &\rightarrow g_{27\times 1}\ F_0^4\ \left(L_{\mu 32} L^{\mu}_{11} + \frac{2}{3} L_{\mu 31} L^{\mu}_{12} \right) \\ O_{\Delta S=2} &\rightarrow \frac{5}{3}\, g_{27\times 1}\ F_0^4\ L_{\mu 32} L^{\mu}_{32} \\ 4\, O_1 &\rightarrow \frac{5}{3}\, g_{27\times 1}\ F_0^4\ L_{\mu 12} L^{\mu}_{12} \end{aligned}$$

$$L^{\mu}_{ij} = i (U^{\dagger} \partial^{\mu} U)_{ij}$$

- Chiral loops and counter terms again give:

$$\langle \pi^+|O_1|\pi^-\rangle = \frac{5}{3}\, g_{27\times 1}\, m_\pi^2 F_\pi^2 \left\{1 + \frac{m_\pi^2}{(4\pi F_0)^2}(-1+3L_\pi) + \delta_{27\times 1}^{\pi\pi}\right\}$$

$$\langle \pi^+\pi^0|iQ_2|K^+\rangle = \frac{5}{3}\, g_{27\times 1}\, F_\pi\, \left(m_K^2 - m_\pi^2\right) \left\{1 + \Delta_{27}^{K^+\pi^+\pi^0}\right\}$$

- for ΔS=1 part, loops are small, and counter terms found to also be small at large Nc because of factorization of Q_2 into product of currents (Cirigliano, Ecker, Neufeld, Pich, 2004)
- lattice QCD computation of K-> pi pi O(10%) error (Blum et. al. 2015)

--> $g_{27} = 0.34(3)_{LQCD}(2)_{chiPT}$

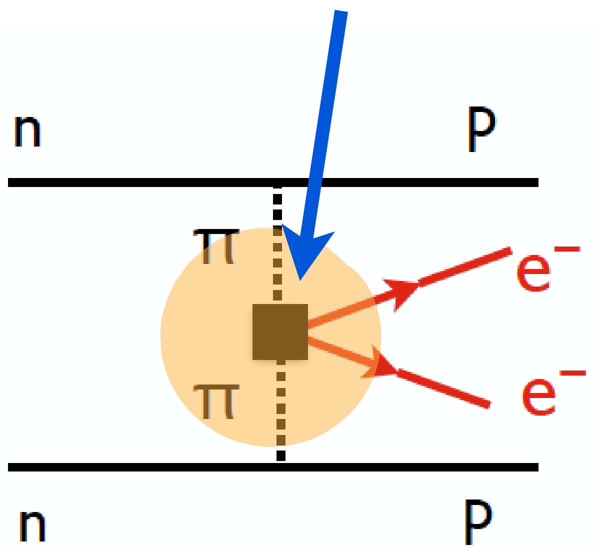
- with 20% error in $\delta_{27\times 1}^{\pi\pi}$ gives our estimate for O_1 :

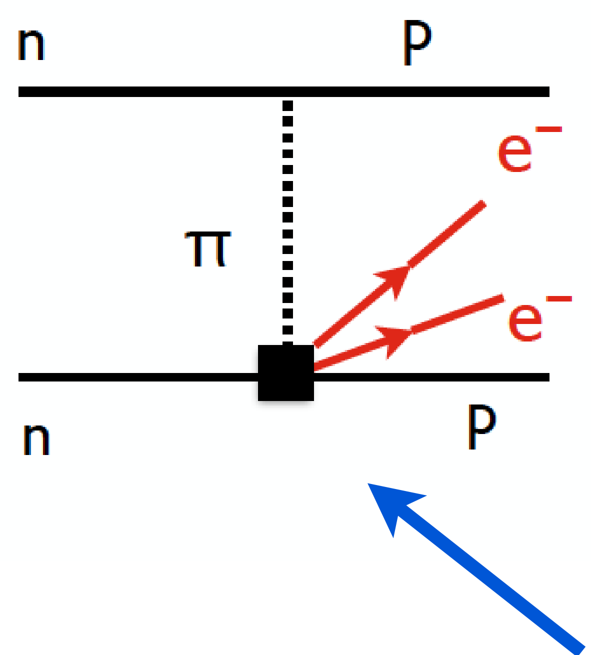
$$\langle \pi^+|O_1|\pi^-\rangle = (1.0 \pm 0.1 \pm 0.2) \times 10^{-4}\ \mathrm{GeV}^4$$

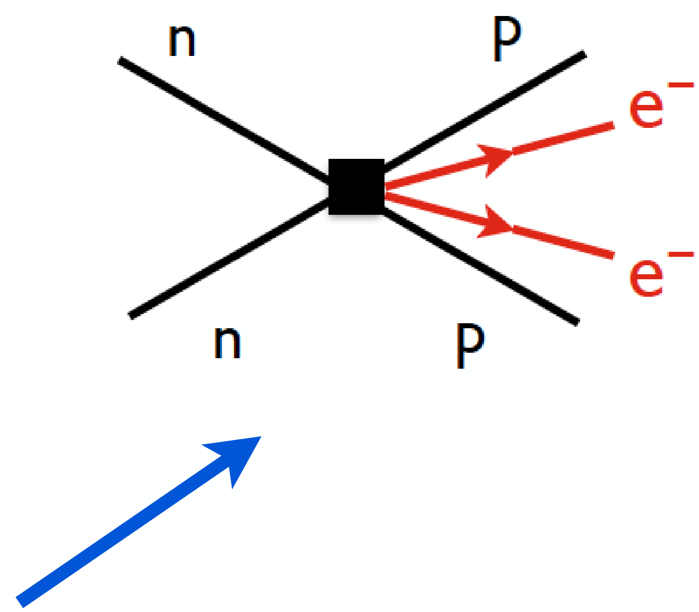
- As expected from general considerations, this matrix element is suppressed compared to other ΔL=2 two pion matrix elements

Summary

progress on these interactions from
LQCD and chiral PT







progress on these interactions from
LQCD just beginning

Summary

- New sources of $\Delta L=2$ LNV could dominate "standard non-standard" contribution (i.e., long-distance Majorana neutrino mass contribution)

- If neutrino hierarchy is "normal"*, such non-conventional sources for $\Delta L=2$ LNV and 0nubb only physics case for discovery

- Discussed possibilities, from both model-dependent and effective field theory descriptions. In contact limit reduced set of electroweak invariant operators.

- first chiral estimates of *all* two pion matrix elements arising from scalar quark operators, necessary ingredient for leading 0nubb matrix elements arising from such non-conventional sources

- expect error to be improved only through direct LQCD computations

- big inverse problem if $\Delta L=2$ LNV discovered, but that is a good situation to be in

*and outside of the quasi-degenerate region