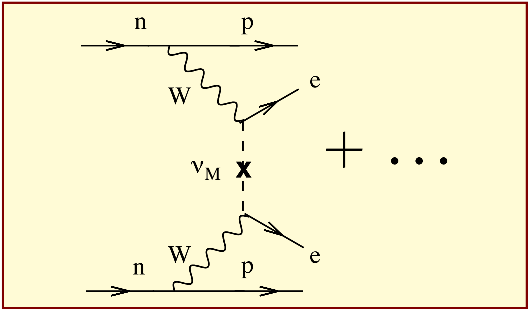
Computing $\beta\beta$ Nuclear Matrix Elements

J. Engel

June 13, 2017



Light-ν Exchange Tells Us Neutrino Mass

$$[T^{0\nu}_{1/2}]^{-1} = \sum_{\text{spins}} \int |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} - Q) \frac{d^3p_1}{2\pi^3} \frac{d^3p_2}{2\pi^3}$$

Amplitude $Z_{0\nu}$ contains lepton part

$$\sum_k \bar{e}(x)\gamma_\mu(1-\gamma_5)U_{ek}\underbrace{\nu_k(x)\,\overline{\nu^c_k}}(y)\gamma_\nu(1+\gamma_5)U_{ek}e^c(y)\ ,$$

where ν's are Majorana mass eigenstates.
Contraction gives neutrino propagator:

$$\sum_k \bar{e}(x)\gamma_\mu(1-\gamma_5)\frac{q^\rho\gamma_\rho + m_k}{q^2 - m_k^2}\gamma_\nu(1+\gamma_5)e^c(y)\ U^2_{ek}\ ,$$

The $q^\rho\gamma_\rho$ part vanishes in trace, leaving a factor

$$m_{\text{eff}} \equiv \sum_k m_k U^2_{ek}.$$

Nuclear Part For Light-ν Exchange

$$M_{0\nu} = M^{GT}_{0\nu} - \frac{g_V^2}{g_A^2} M^F_{0\nu} + \ldots$$

with

$$M^{GT}_{0\nu} = \langle F| \, | \sum_{i,j} H(r_{ij}) \, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \; \tau_i^+ \tau_j^+ \; |I\rangle + \ldots$$

$$M^{F}_{0\nu} = \langle F \,| \sum_{i,j} H(r_{ij}) \; \tau_i^+ \tau_j^+ \; |I\rangle + \ldots$$

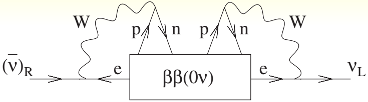
$$H(r) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2} \quad \text{roughly} \propto 1/r$$

Contribution to integral peaks at $q \approx 100$ MeV inside nucleus.

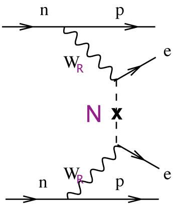
Corrections are from “forbidden” terms, weak nucleon form factors, many-body currents …

Totally New Physics Could Contribute

If neutrinoless decay occurs then ν's are Majorana, no matter what:



but light neutrinos may not drive the decay:



Exchange of heavy right-handed neutrino in left-right symmetric model.

Amplitude of "exotic" mechanism:

$$\frac{Z_{0\nu}^{\text{heavy}}}{Z_{0\nu}^{\text{light}}} \approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \left(\frac{\langle q^2 \rangle}{m_{\text{eff}}\, m_N}\right)$$

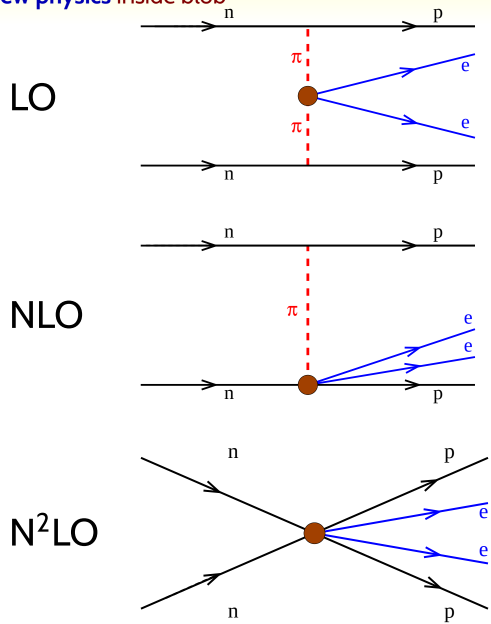
$\langle q^2 \rangle \approx 10^4\ \text{MeV}^2$

$$\approx \mathbf{1} \quad \text{if} \quad m_N \approx 1\,\text{TeV} \quad \text{and} \quad m_{\text{eff}} \approx \sqrt{\Delta m^2_{\text{atm}}}$$

So exotic stuff can occur with roughly the same rate as light-ν exchange. Upcoming talks by W. Rodejohann, M. Ramsey-Musolf

Generic Chiral Ordering of Diagrams (Prezau, Ramsey-Msuolf, Vogel)

New physics inside blob



These lead to two-nucleon operators with different space dependence from that of the “standard” operator.

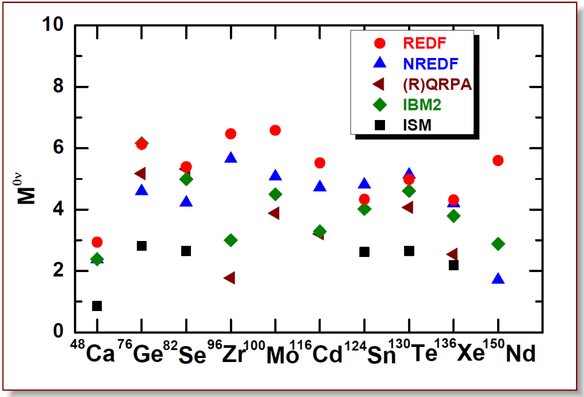
Upcoming talks by A. Nicholson, M. Graesser, M. Savage

Nuclear Matrix Elements: The Situation at Present

Light-ν Echange

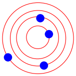
Significant spread. And all the models could be missing important physics.

Uncertainty hard to quantify.



Nuclear Structure: Contrasting the Approaches

Starting point is always mean field(s)



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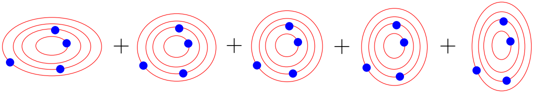
$\cdots +$  $+ \cdots$

"Energy-Density Functional Theory" employs Generator-Coordinate Method (GCM), which mixes many such states with different collective properties.

Upcoming talk by T. Rodriguez

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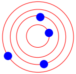
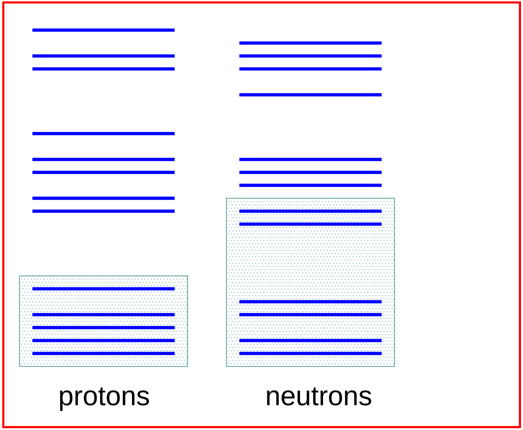


Other methods build on single independent-particle state.



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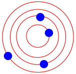

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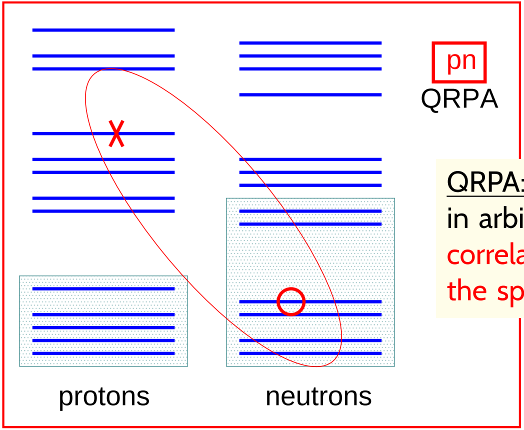

QRPA: Large single-particle spaces in arbitrary single mean field; simple correlations and excitations within the space.

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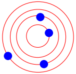

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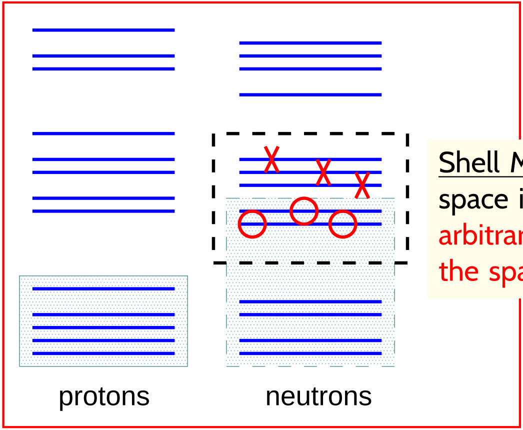

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

Nuclear Structure: Contrasting the Approaches

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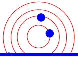
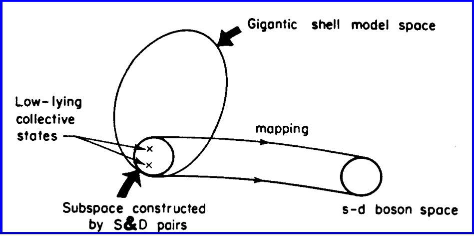
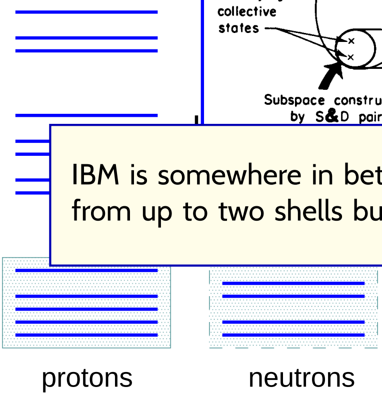


Other me … ticle state.





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… field;
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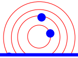
IBM is somewhere in between, mapping matrix elements from up to two shells but truncating to collective pairs.

Nuclear Structure: Contrasting the Approaches

Starting point is always mean field(s)



Can we improve and combine these methods?

Can we avoid fitting parameters to data directly in heavy nuclei? That's not a bad thing, but makes it hard to estimate accuracy when calculating something different from anything ever measured.

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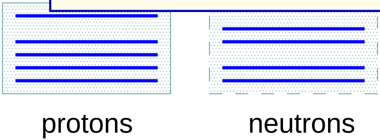
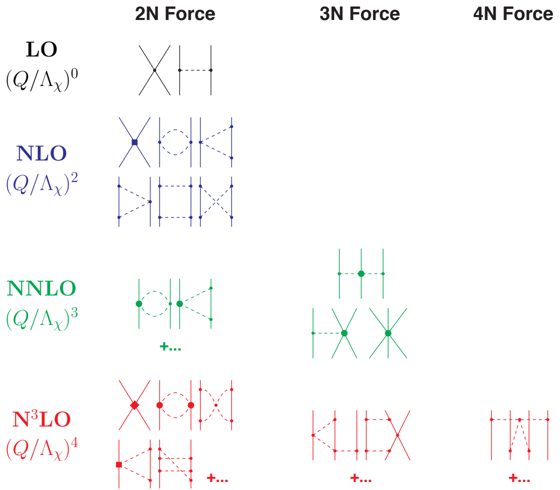


The Way Forward: Ab Initio Nuclear Structure?

Often starts with chiral effective field theory.

Nucleons, pions sufficient below chiral-symmetry breaking scale.



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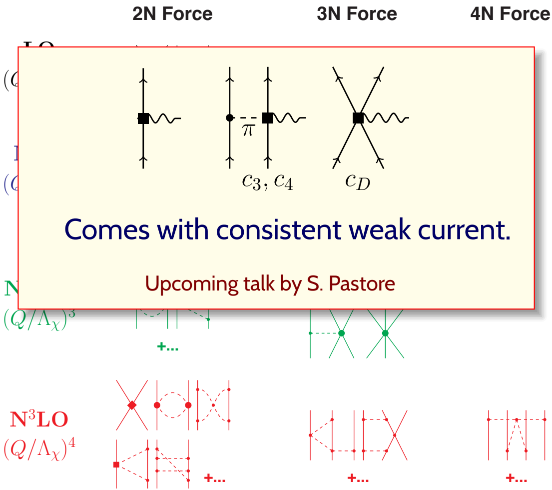
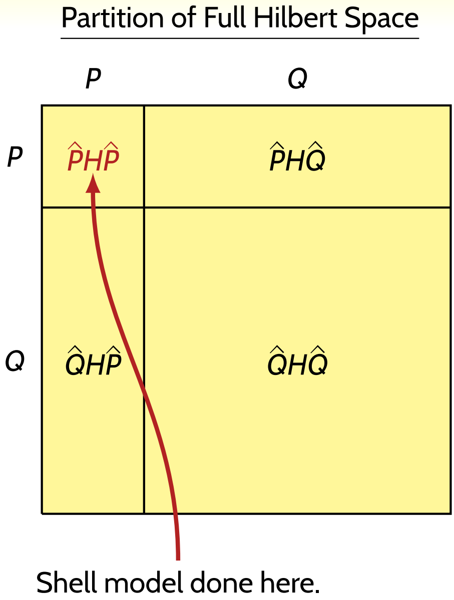


Comes with consistent weak current.

Upcoming talk by S. Pastore

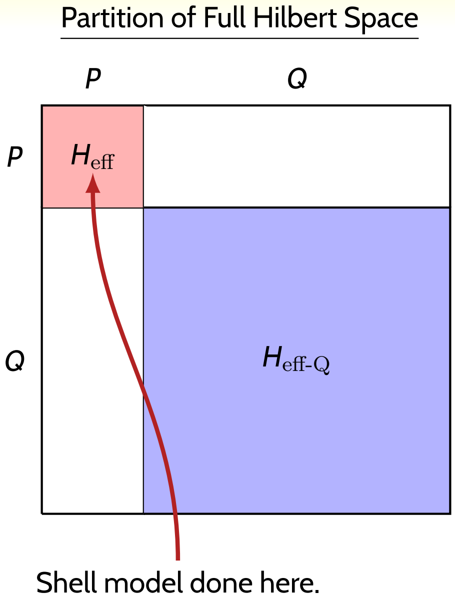
Ab Initio Shell Model



P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing d most important eigenvalues.

Ab Initio Shell Model



P = valence space
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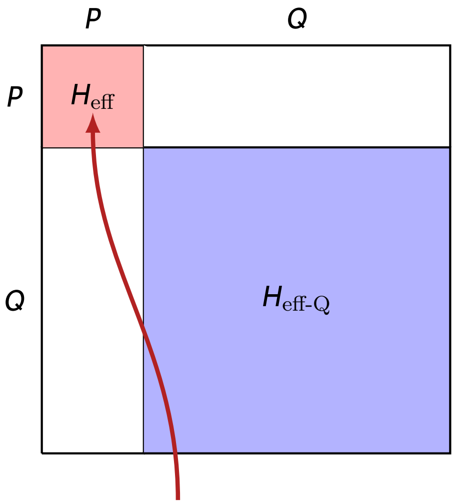
Task: Find unitary transformation to make *H* block-diagonal in *P* and *Q*, with H_{eff} in *P* reproducing *d* most important eigenvalues.

Ab Initio Shell Model

Partition of Full Hilbert Space



Shell model done here.

P = valence space
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Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing d most important eigenvalues.

For transition operator $\hat{M}$, must apply same transformation to get $\hat{M}_{\rm eff}$.

Upcoming talks by W.C. Haxton, L. Coraggio, H. Hergert

Ab Initio Shell Model

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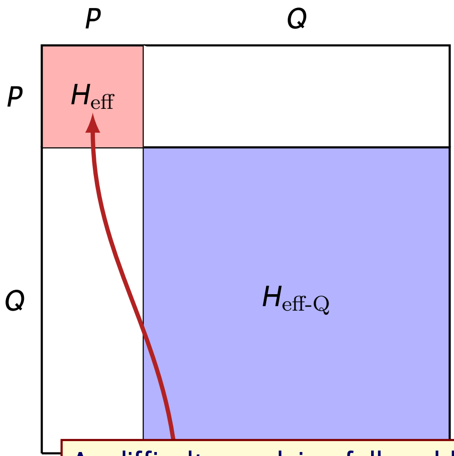

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Upcoming talks by W.C. Haxton, L. Coraggio, H. Hergert

As difficult as solving full problem. But idea is that N-body effective operators may not be important for N >2 or 3.

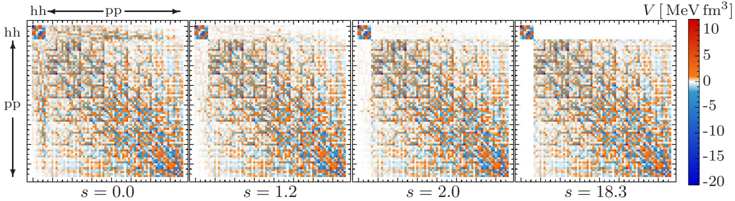
Shell model done here.

In-Medium Similarity Renormalization Group

One way to determine the transfomation

Flow equation for effective Hamiltonian. Asymptotically decouples shell-model space.

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \quad H(\infty) = H_{\text{eff}}$$

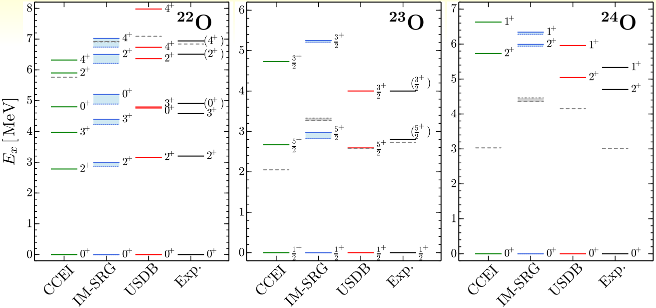


Hergert et al.

Trick is to keep all 1- and 2-body terms in *H* at each step *after normal ordering*.

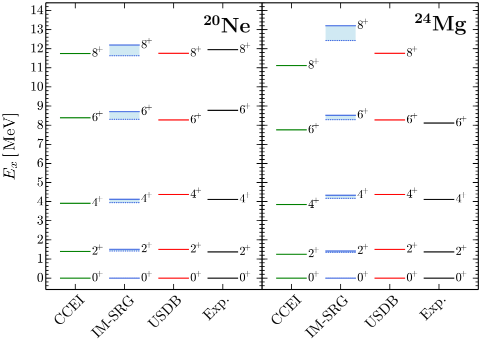
If shell-model space contains just a single state, approach yields ground-state energy. If it is a typical valence space, result is effective interaction and operators.

Ab Initio Calculations of Spectra



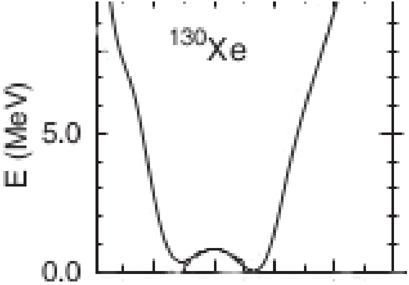
Neutron-rich oxygen isotopes

Deformed nuclei



Complementary Way Forward: Improved GCM?

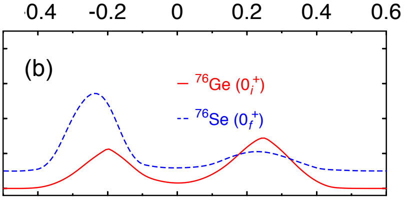
Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle$. Then diagonalize (usually non-ab-initio) H in space of symmetry-restored quasiparticle vacua with different $\langle Q_0 \rangle$.



β_2 = deformation

Robledo et al.: Minima at $\beta_2 \approx \pm.15$

Collective wave functions



β_2

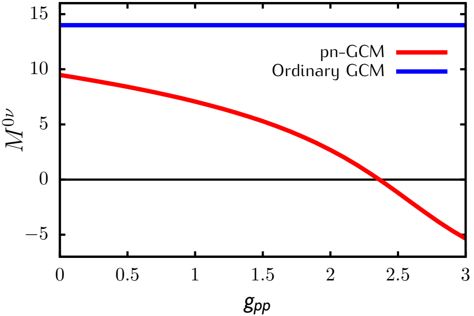
Rodriguez and Martinez-Pinedo: Wave functions peaked at $\beta_2 \approx \pm.2$

Can be improved by including crucial **neutron-proton pairing amplitude** as collective coordinate...

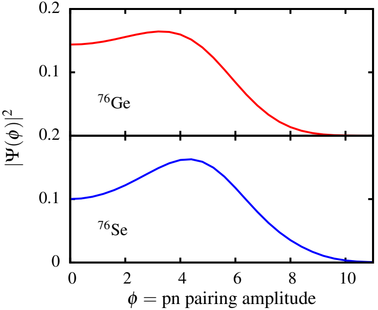
GCM Example: Proton-Neutron (pn) Pairing

Can build possibility of pn correlations into mean field. They are frozen out in mean-field minimum, but included in GCM.

0νββ matrix element



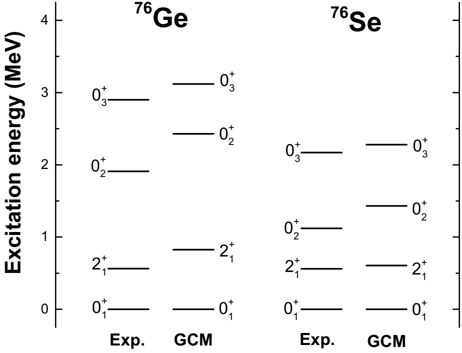
Collective pn-pairing wave functions



Proton-neutron pairing significantly reduces matrix element.

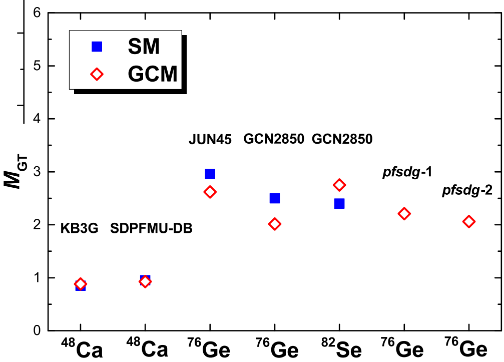
Upcoming talk by N. Hinohara

GCM in Shell-Model Spaces



GCM Spectrum in 2 Shells

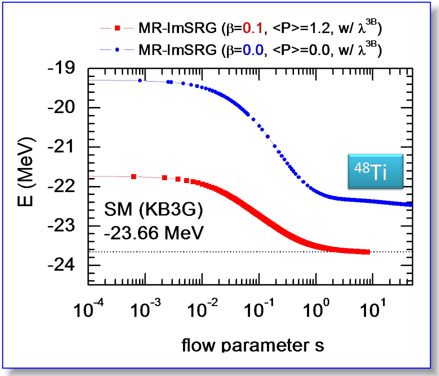
ββ Matrix Elements in 1 and 2 Shells



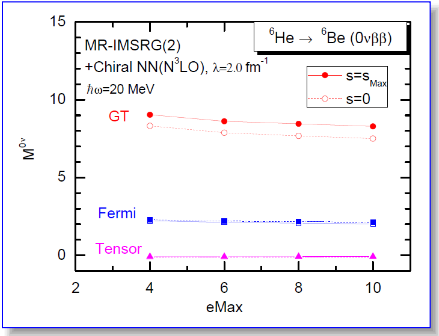
Combining GCM and Ab Initio Methods

GCM incorporates some correlations that are hard to capture automatically (e.g. shape coexistence). So use it to construct initial "reference" state, let IMSRG, do the rest.

Energy evolution in single shell



Ab initio 0νββ matrix element in mirror pair



From J. Yao

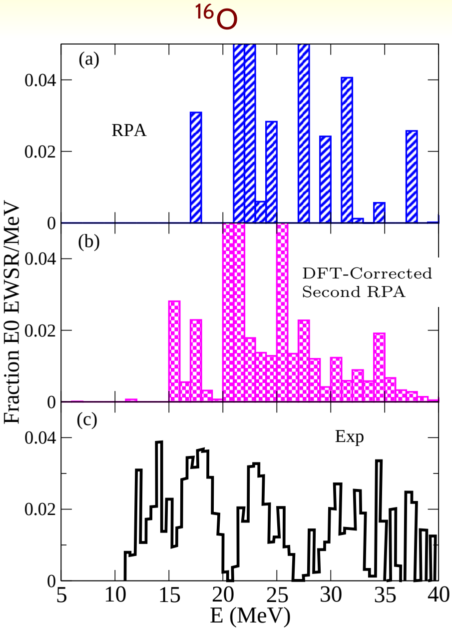
In progress: Matrix elements for detector nuclei.

Could be considerably harder.

Improving RPA/QRPA

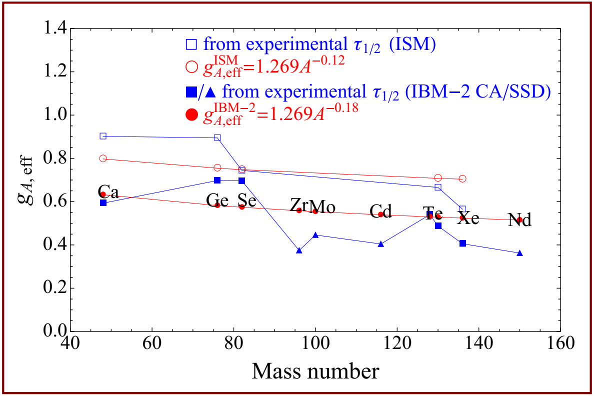
Upcoming talks by
J. Terasaki, C. Robin

RPA produces states in intermediate nucleus, but form is restricted to 1p-1h excitations of ground state. Second RPA adds 2p-2h states.



Issue Facing All Models: "g_A"

<u>40-Year-Old Problem:</u> Effective g_A needed for single-beta and two-neutrino double-beta decay in shell model and QRPA.



from F. Iachello

If 0ν matrix elements quenched by same amount as 2ν matrix elements, experiments will be much less sensitive; rates go like fourth power of g_A.

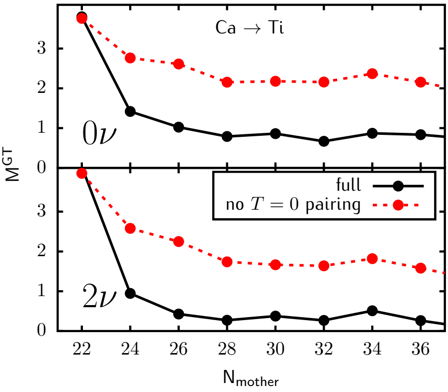
Upcoming talks by S. Pastore, L. Coraggio?

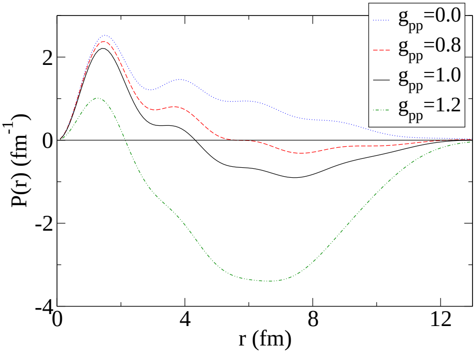
Arguments Suggesting Strong Quenching of 0ν

- Both β and $2\nu\beta\beta$ rates are strongly quenched, by consistent factors.
- Forbidden (2^-) decay among low-lying states appears to exhibit similar quenching.
- Quenching due to correlations shows weak momentum dependence in low-order perturbation theory.

Arguments Suggesting Weak Quenching of 0ν

- Many-body currents seem to suppress 2ν more than 0ν.
- Enlarging shell model space to include some effects of high-*j* spin-orbit partners reduces 2ν more than 0ν.
- Neutron-proton pairing, related to spin-orbit partners and investigated pretty carefully, suppresses 2ν more than 0ν.





Large *r* contributes more to 2ν.

We Hope to Resolve the Issue Soon

Problem must be due to some combination of:

1. Truncation of model space.

   Should be fixable in ab-initio shell model, which compensates effects of truncation via effective operators.

2. Many-body weak currents.

   Size still not clear, particularly for $0\nu\beta\beta$ decay, where current is needed at finite momentum transfer q.

   Leading terms in chiral EFT for finite q only recently worked out. Careful fits and use in decay computations will happen in next year or two.

Finally…

- Also coming…Talks on neutrino oscillations by S. Bilenky and double charge exchange by N. Auerbach.

- Topical collaboration will speed progress in next few years.
  Or else …I don't want to think about it.

That's all.

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