

Study of single- β and $2\nu\beta\beta$ decays within an effective theory for collective nuclei

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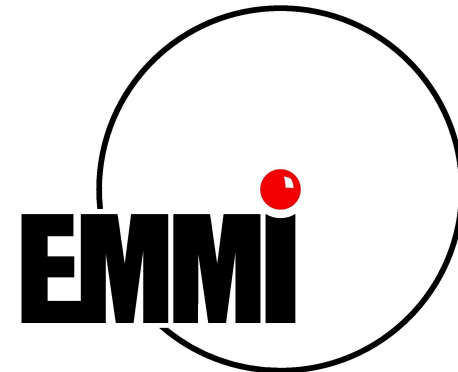


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Even-even nuclei

- Effective Hamiltonian and power counting
- E2 properties

Odd-mass nuclei

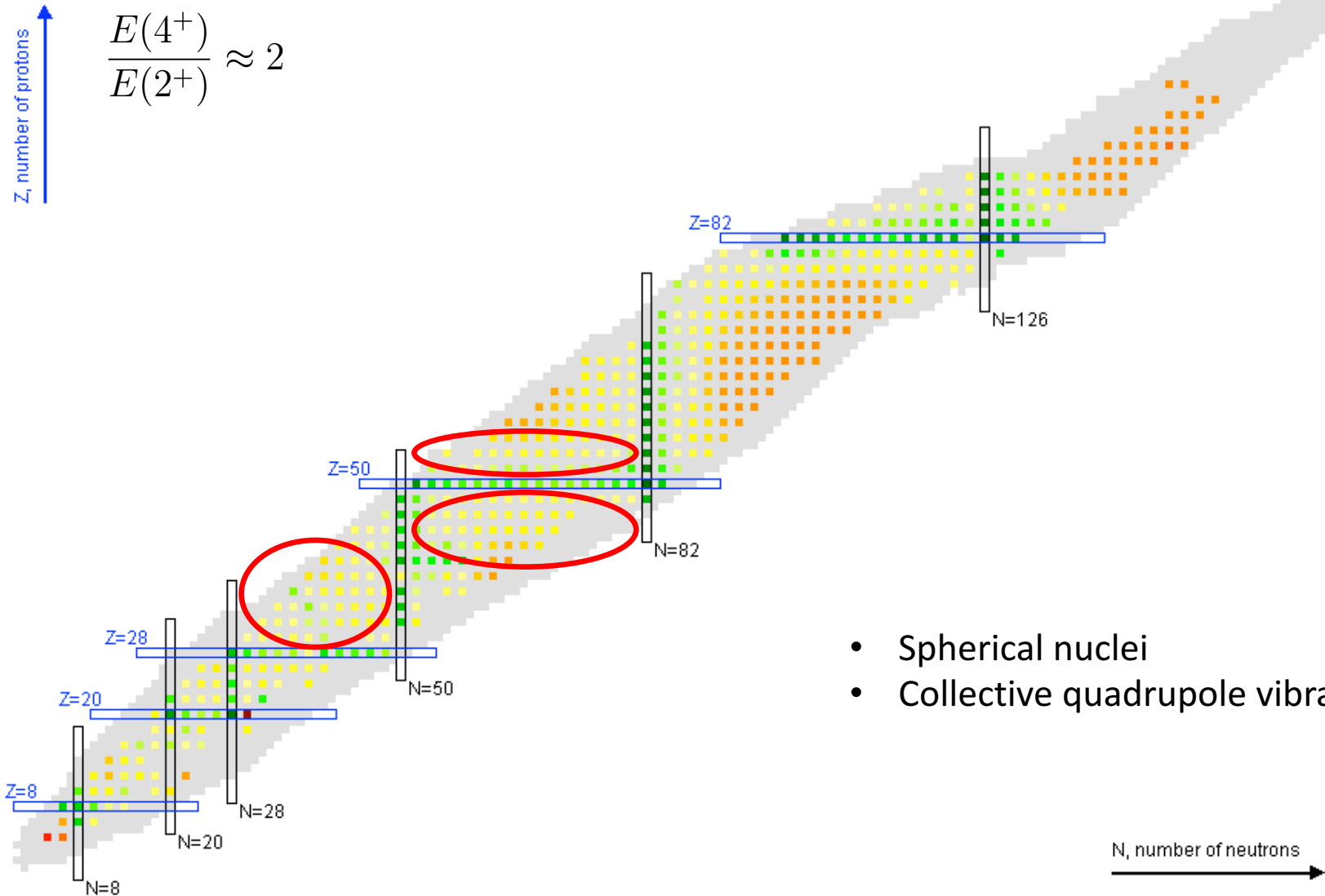
- Power counting for fermion operators
- Spectra at NNLO
- E2 and M1 properties

β decays from odd-odd nuclei

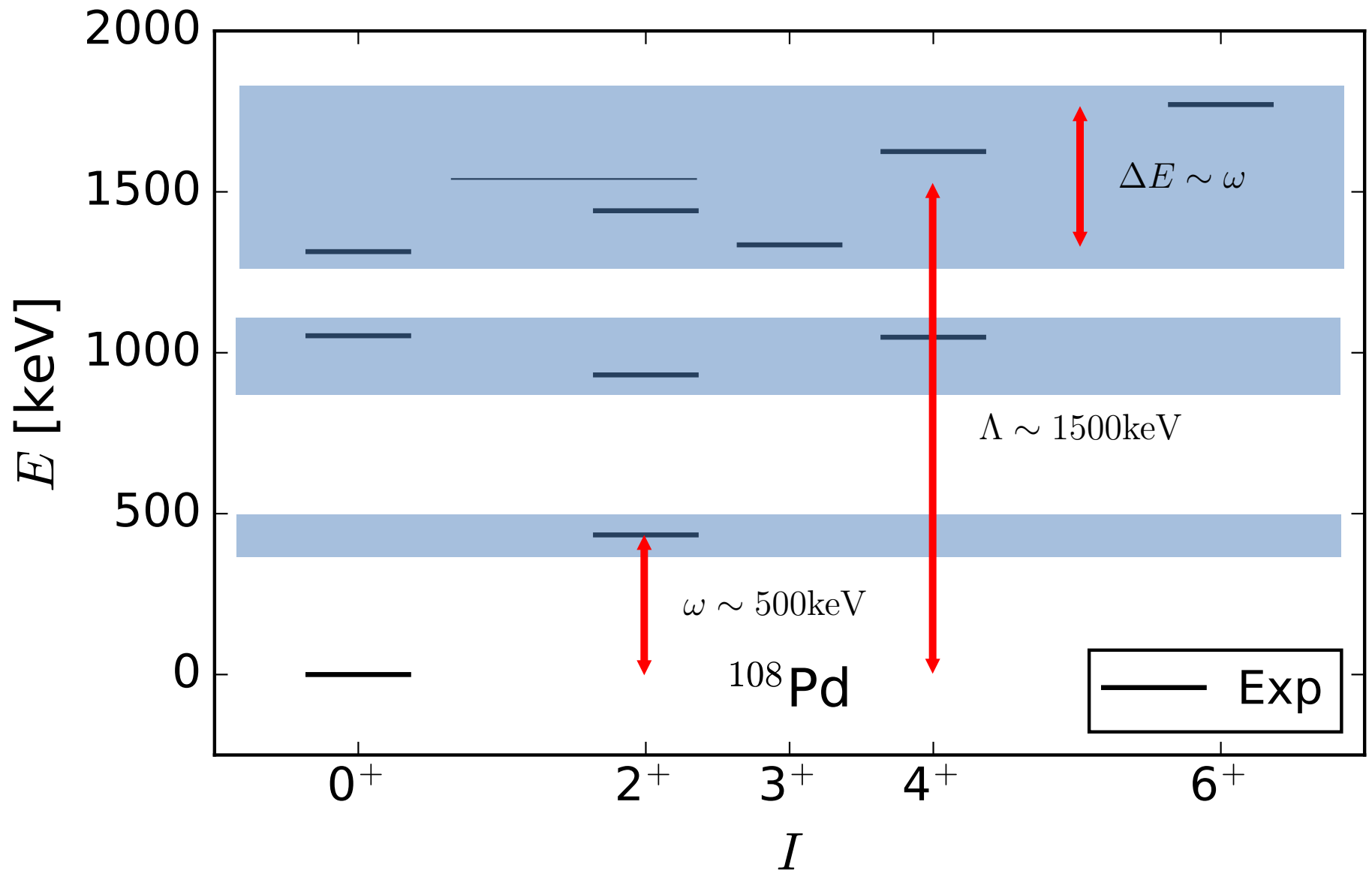
- Low-lying odd-odd states
- Effective Gamow-Teller operator
- Uncertainty estimates

$\beta\beta$ decays in the SSD approximation

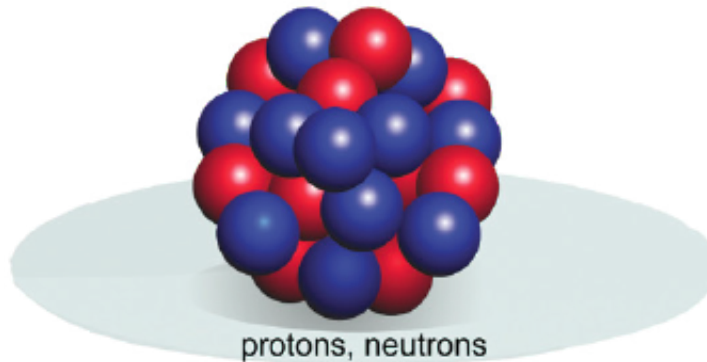
- Uncertainty estimate



- Spherical nuclei
- Collective quadrupole vibrations



Energy



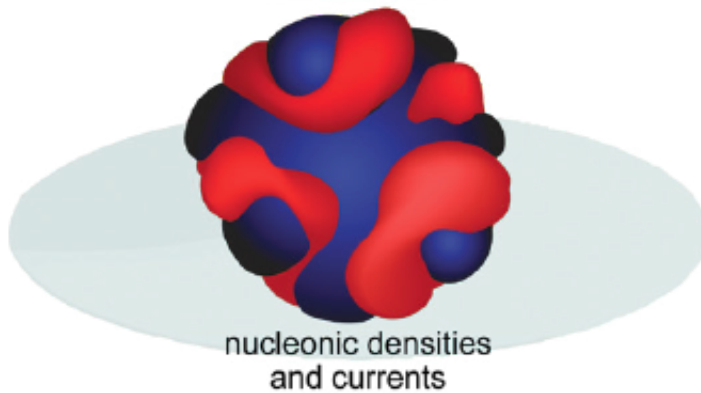
Chiral EFT

- Nucleon and pion fields

Pionless EFT

- Nucleon fields

BREAKDOWN SCALE $\Lambda \sim 1500\text{keV}$



Collective EFT

- Phonons
- Few fermions

$$\frac{\omega}{\Lambda} \ll 1$$

$$\omega \sim 500\text{keV}$$

Degrees of freedom: quadrupole-boson creation and annihilation operators

$$[d_\mu, d_\nu^\dagger] = \delta_{\mu\nu},$$

The states are constructed as phonon excitations of the reference state

$$(d^\dagger)^n_M |0\rangle$$

Rank-two tensors

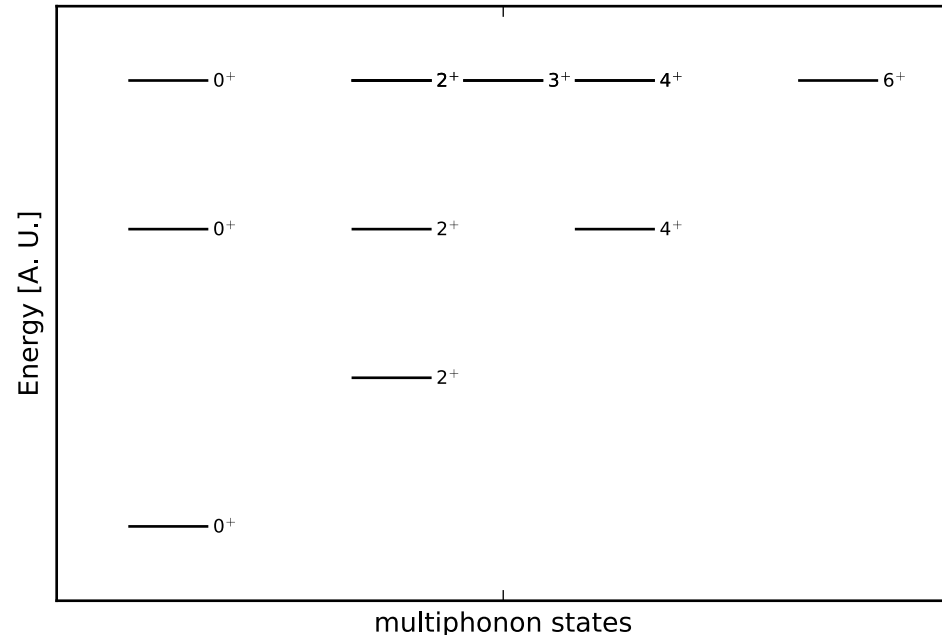
$$d_\mu^\dagger \quad \text{and} \quad \tilde{d}_\mu = (-1)^\mu d_{-\mu}$$

Most simple rotational-invariant Hamiltonian

$$H_{\text{LO}} \equiv \omega_1 \hat{N}$$

$$\hat{N} \equiv d^\dagger \cdot \tilde{d}$$

$$\omega_1 \sim \omega$$



$$H_{\text{LO}} \equiv \omega_1 \hat{N}$$

$$\hat{N} \equiv d^\dagger \cdot \tilde{d}$$

$$\omega_1 \sim \omega$$

Scales

low energies

breakdown

$$H_{\text{LO}} \sim N\omega$$

$$H_{\text{LO}} \sim \Lambda$$

$$d \sim \sqrt{N}$$

$$d \sim \sqrt{\frac{\Lambda}{\omega}}$$

ASSUMPTION: Corrections shift the energies by ω at breakdown $\Delta E \sim \omega$

Example: Scale of a term with four boson operators at breakdown

$$C_2 d^4 \sim \omega \quad \text{or} \quad C_2 \sim \left(\frac{\omega}{\Lambda}\right)^2 \omega$$

At low energies

$$C_2 d^4 \sim \left(\frac{N\omega}{\Lambda}\right)^2$$

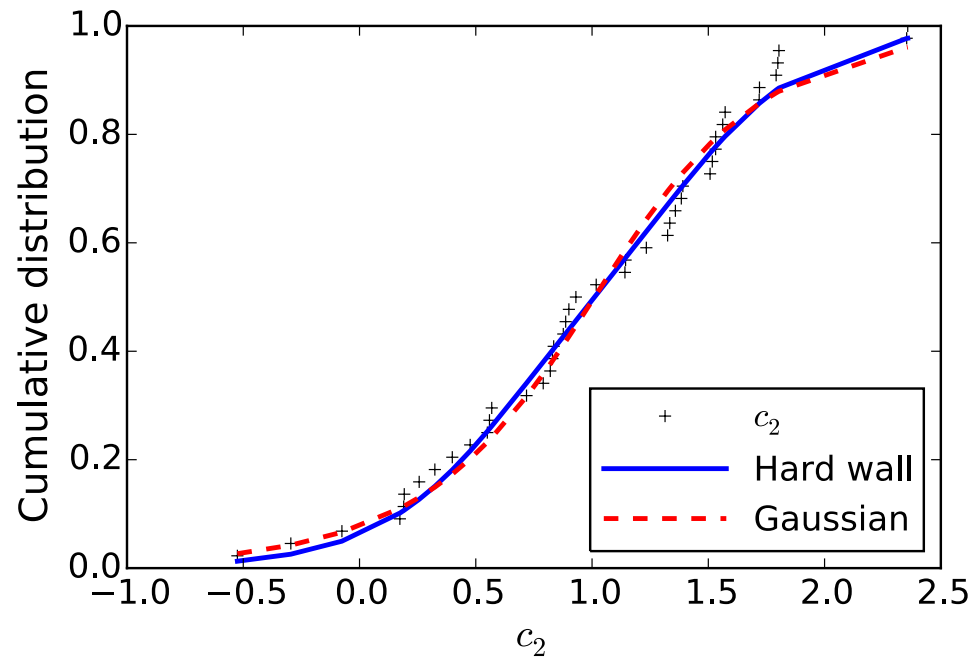
NNLO Hamiltonian

$$H_{\text{NNLO}} \equiv g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_J \hat{J}^2$$

With

$$\hat{\Lambda}^2 \equiv - (d^\dagger \cdot d^\dagger) (\tilde{d} \cdot \tilde{d}) + \hat{N}^2 - 3\hat{N},$$

$$\hat{J} = \sqrt{10} (d^\dagger \otimes \tilde{d})^{(1)}$$



Observables (energy as an example)

$$E = \omega \sum_n^{\infty} c_n \varepsilon^n,$$

$$\varepsilon \equiv \frac{N\omega}{\Lambda}$$

Expansion coefficients of order one
Assumption encoded into priors*

$$\text{pr}^{(G)}(\tilde{c}_i | c) = \frac{1}{\sqrt{2\pi s c}} e^{-\frac{\tilde{c}_i^2}{2s^2 c^2}}$$

$$\text{pr}(c) = \frac{1}{\sqrt{2\pi\sigma c}} e^{-\frac{\log^2 c}{2\sigma^2}}$$

*Cacciari, Houdeau; Nucl. J. High Energy Phys. **09** (2011) 039

*Furnstahl, et al.; J. Phys. G **42**, 034028 (2015)

Most general positive-parity rank-two tensor

$$\hat{Q} = Q_0 (d^\dagger + \tilde{d}) + Q_1 (d^\dagger \otimes \tilde{d})^{(2)}$$

ASSUMPTION: All terms scale similarly at breakdown

$$Q_1 \sim \sqrt{\frac{\omega}{\Lambda}} Q_0$$

Natural scaling

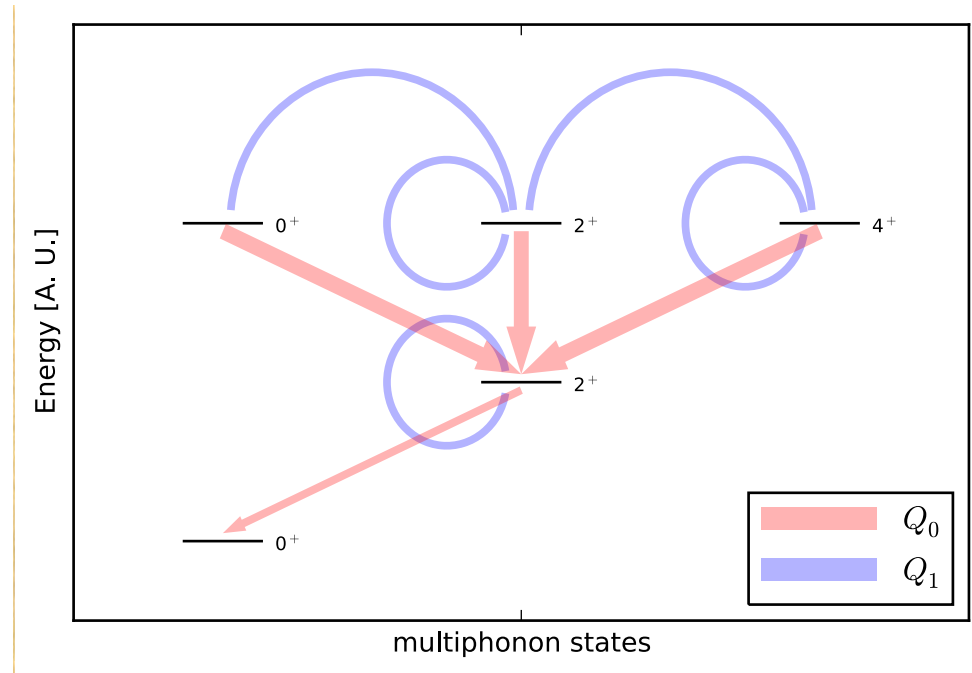
$$B \sim A \Rightarrow B \in \left[A \sqrt{\frac{\omega}{\Lambda}}, A \sqrt{\frac{\Lambda}{\omega}} \right]$$

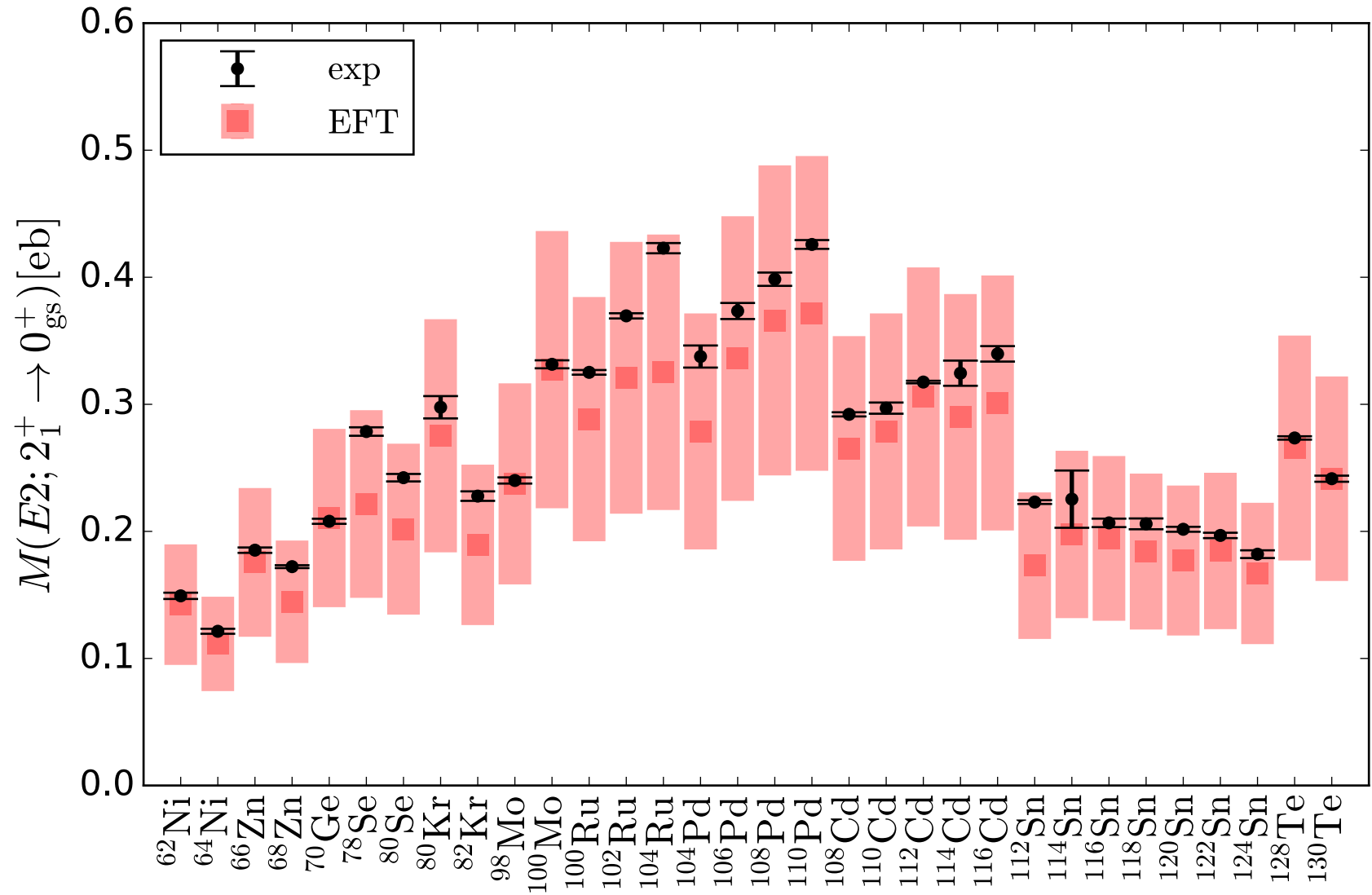
LO

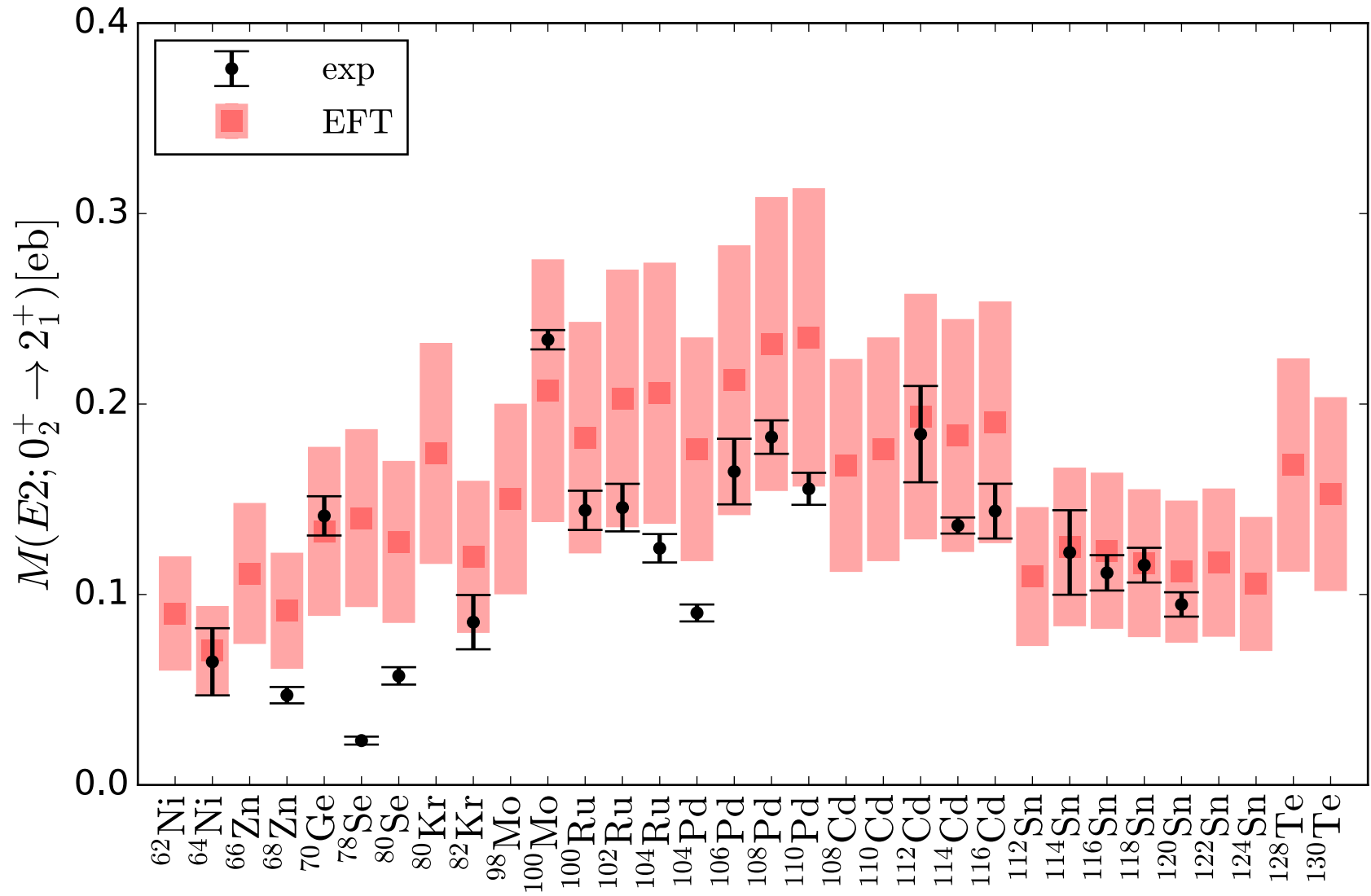
- Phonon-annihilating transitions

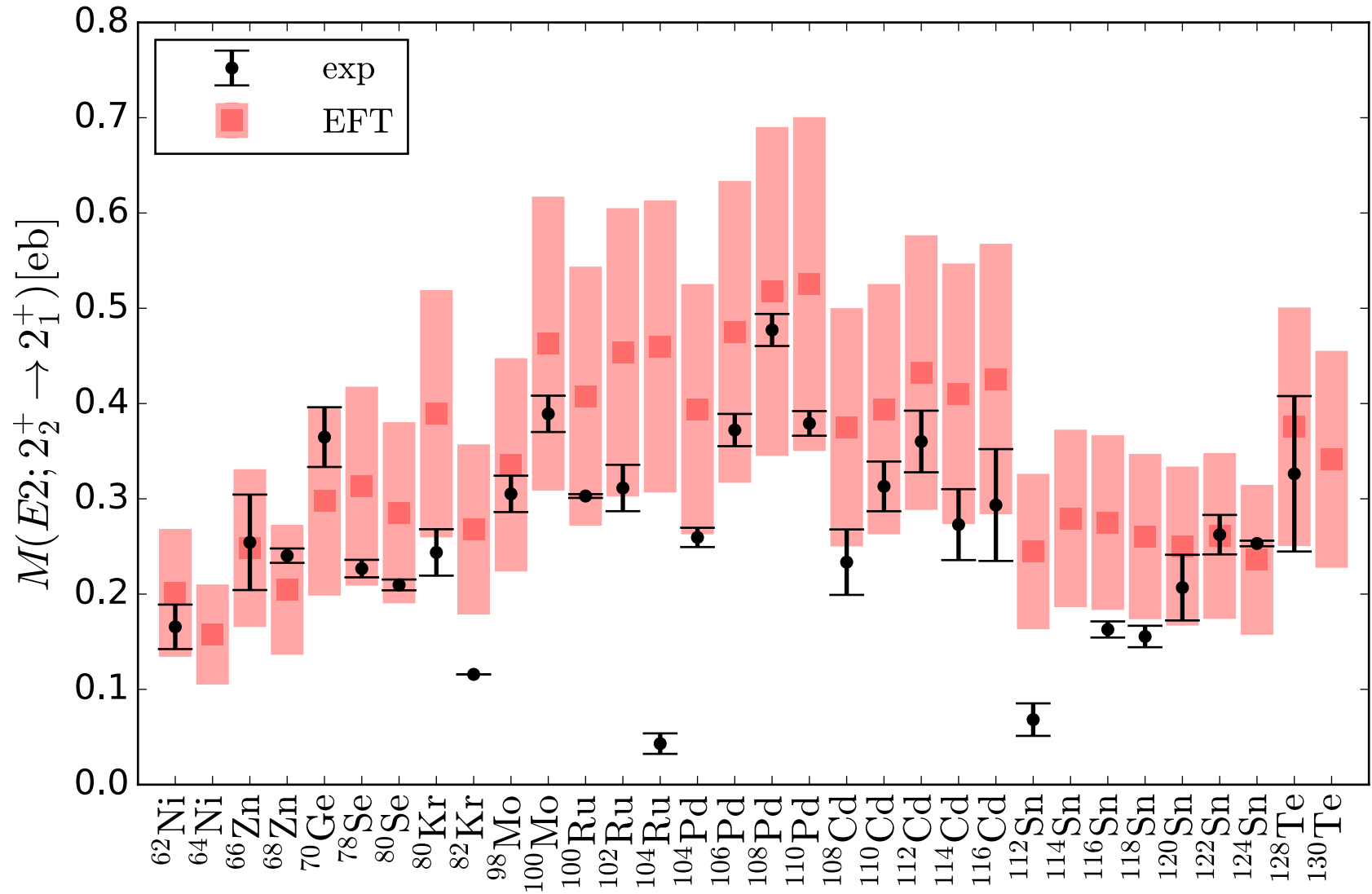
NLO

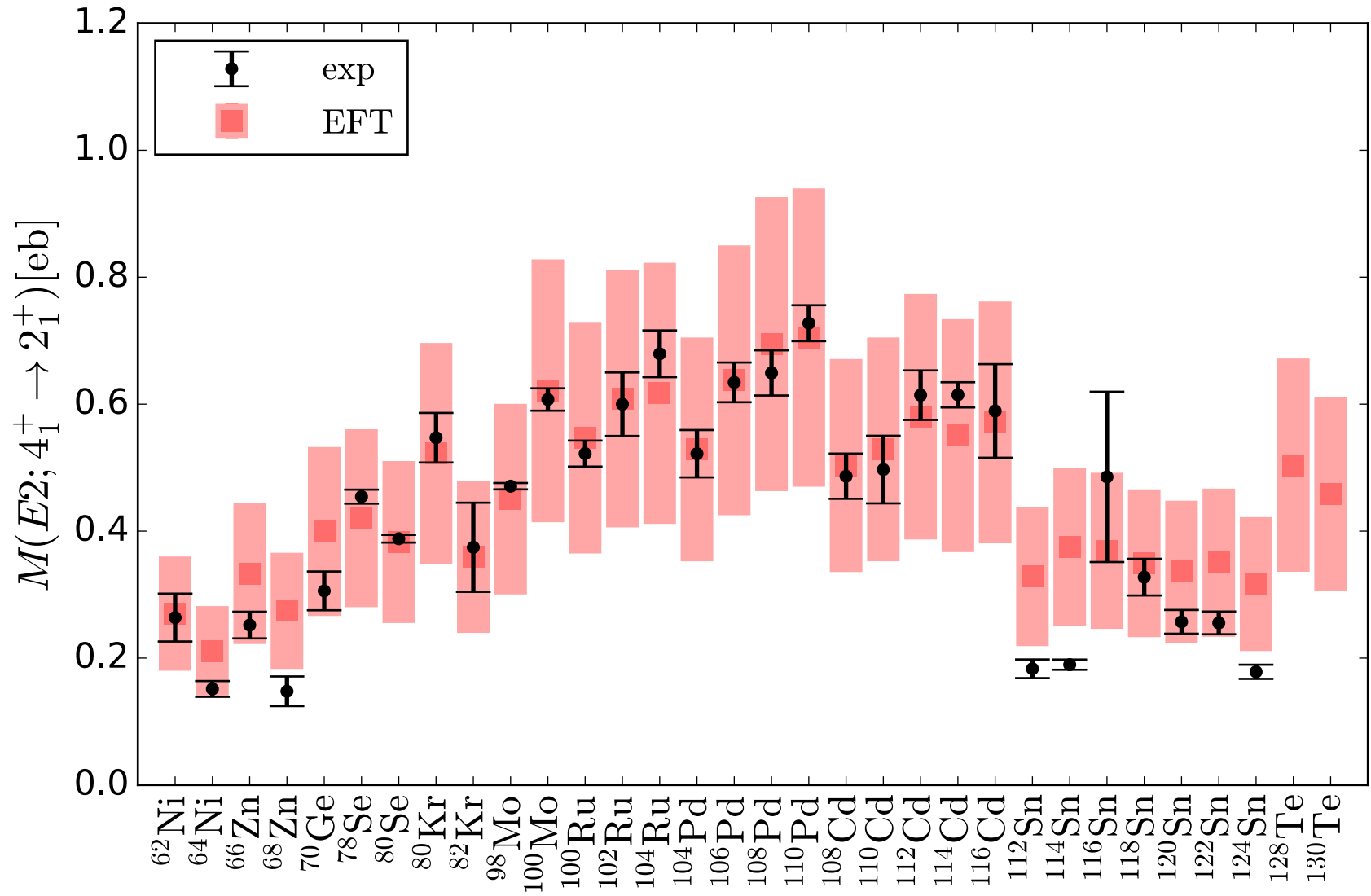
- Phonon-conserving transitions
- Static E2 moments

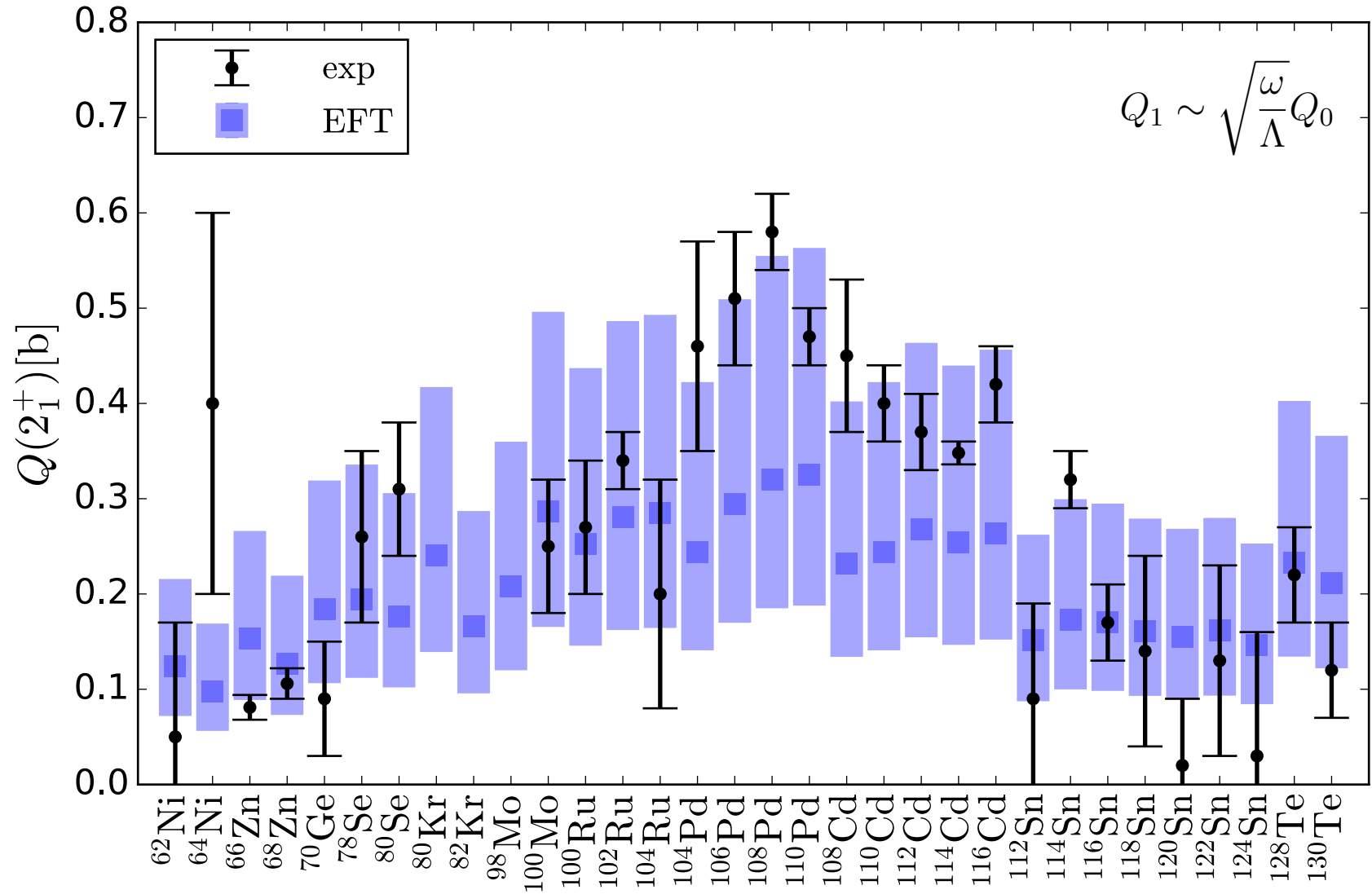


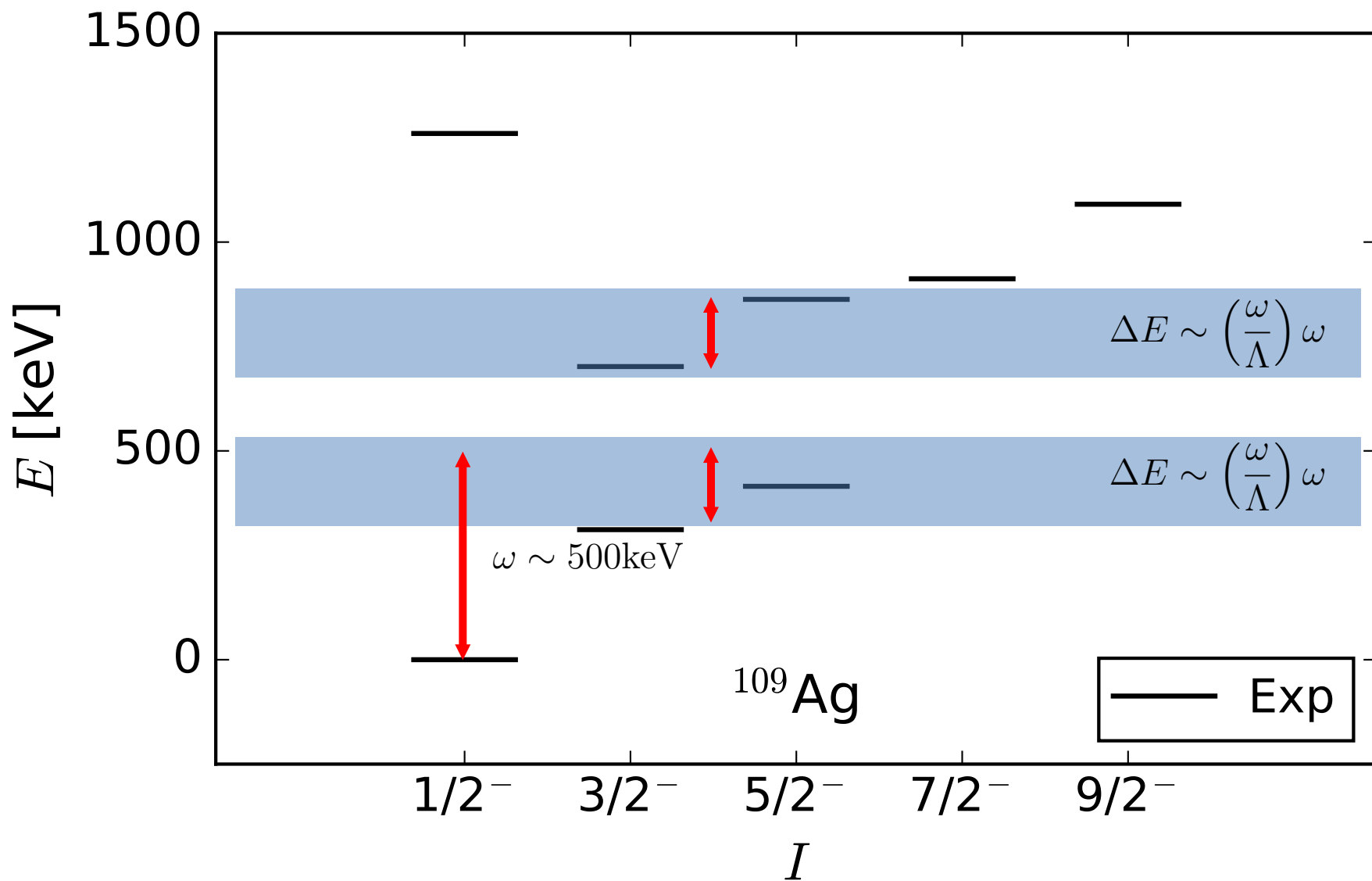












Degrees of freedom: fermion creation and annihilation operators for a fermion in a $j^\pi = 1/2^-$ orbital

$$\{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu}$$

PROPOSAL: The matrix element of an operator with an n -fermion factor scales as

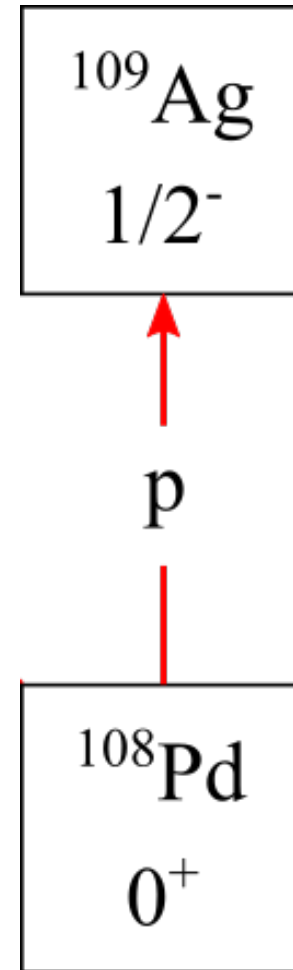
$$\langle \hat{O}_n \rangle \sim \langle \hat{O}_{n-1} \rangle \frac{\omega}{\Lambda}$$

NLO Hamiltonian

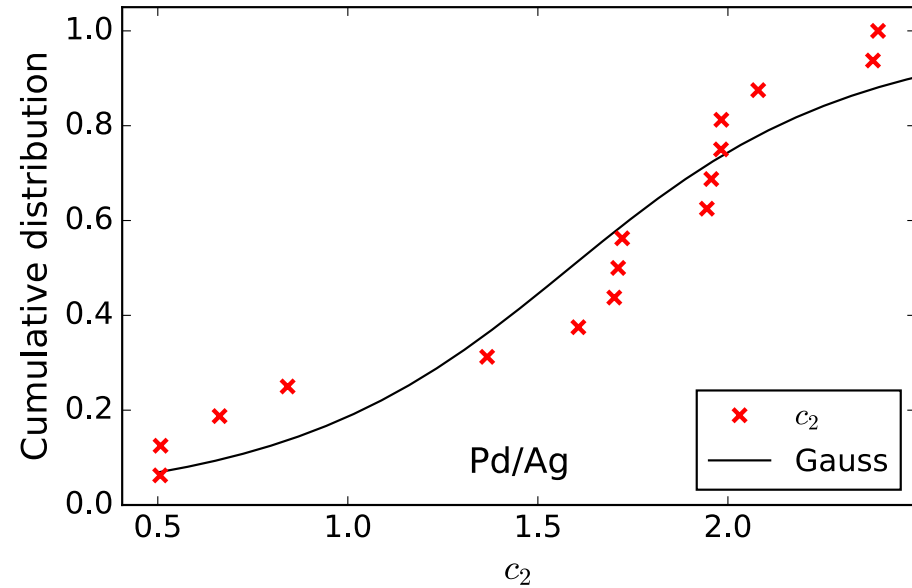
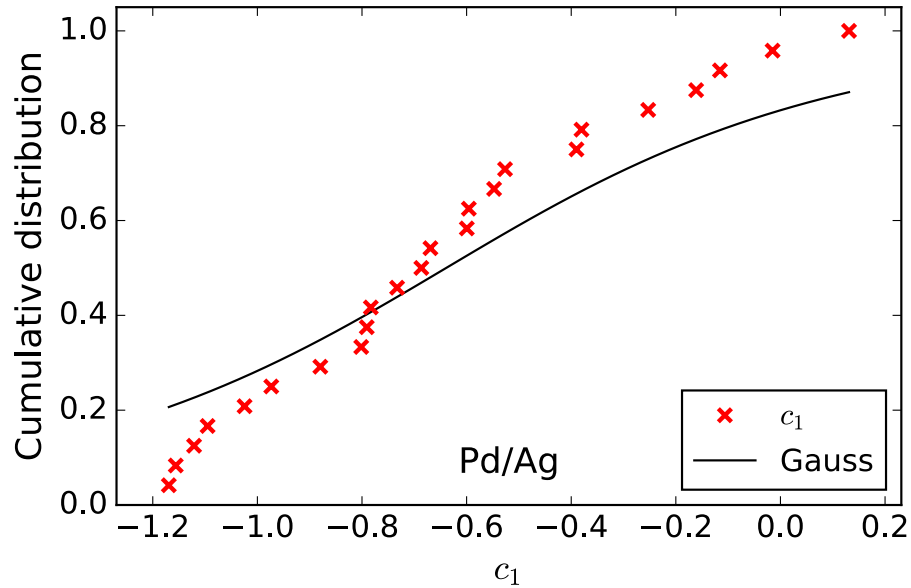
$$H_{\text{NLO}} \equiv g_{Jj} \hat{\mathbf{J}} \cdot \hat{\mathbf{j}} + \omega_2 \hat{N} \hat{n}$$

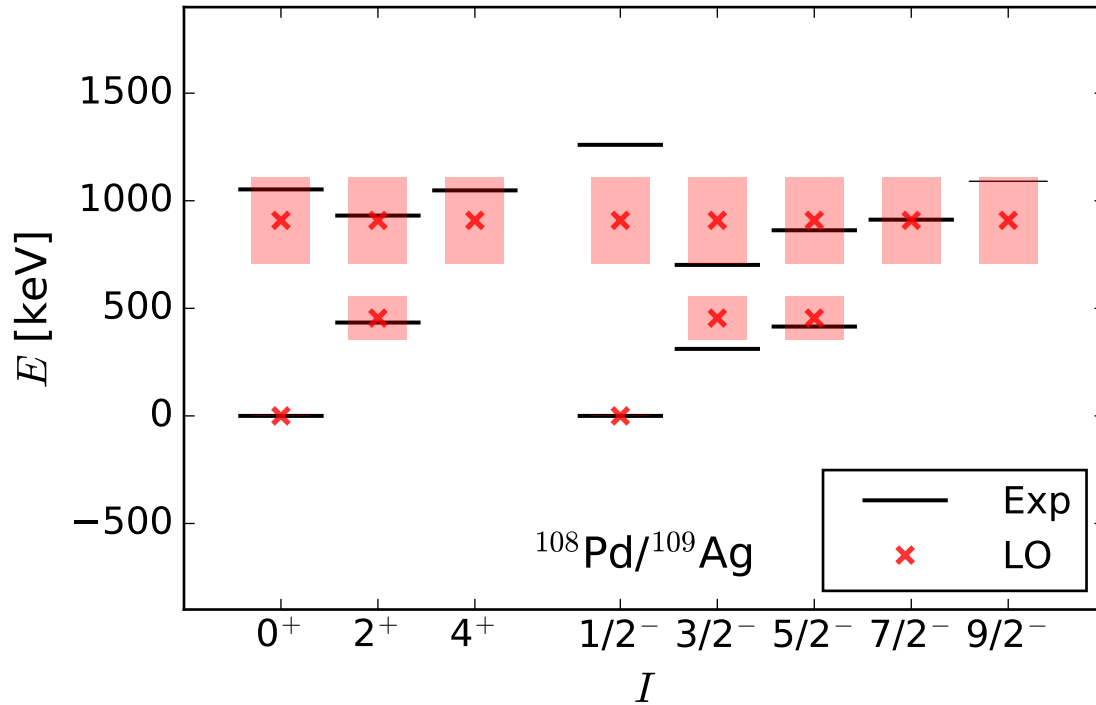
with

$$\hat{\mathbf{j}} = \frac{1}{\sqrt{2}} (a^\dagger \otimes \tilde{a})^{(1)} \quad \text{and} \quad \hat{n} \equiv a^\dagger \cdot \tilde{a}$$



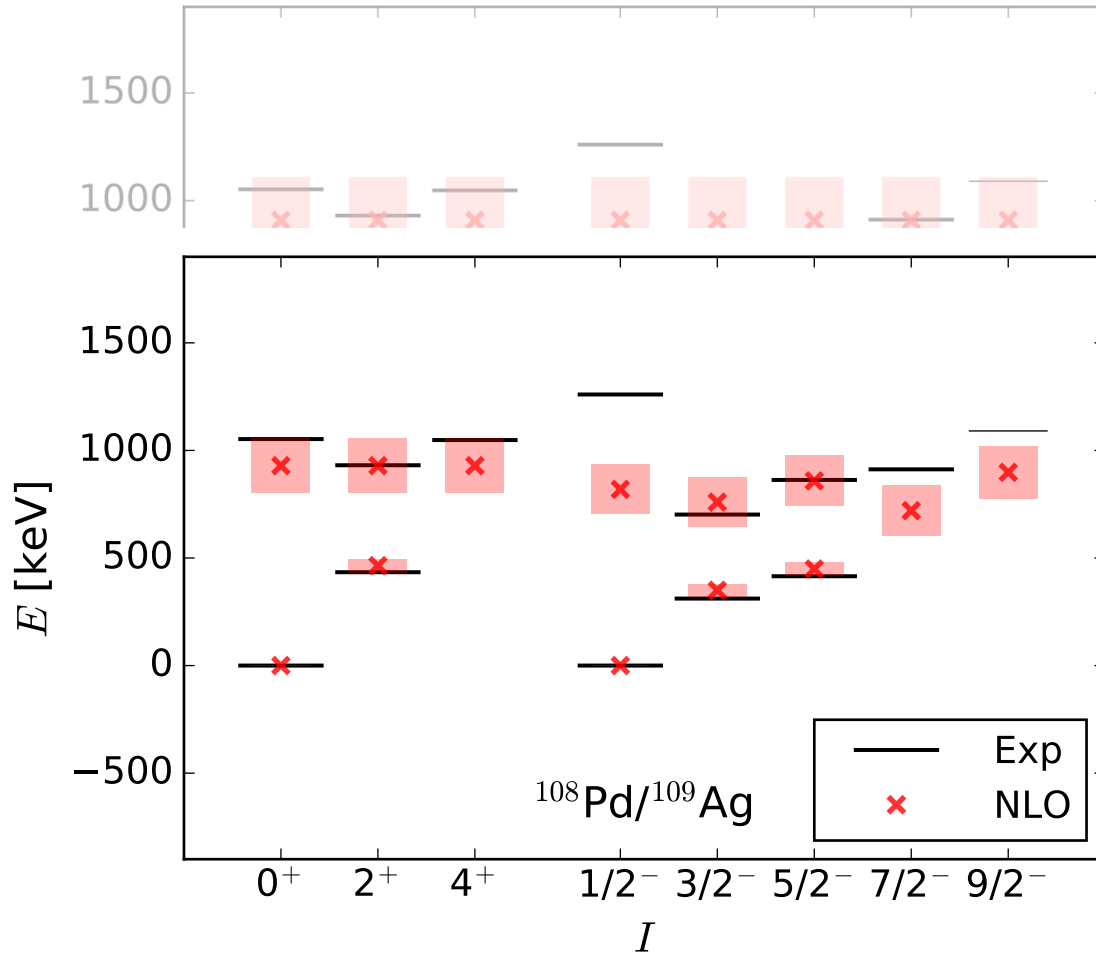
Observables $E = \omega \sum_n^{\infty} c_n \varepsilon^n$, $\varepsilon \equiv \frac{N\omega}{\Lambda}$

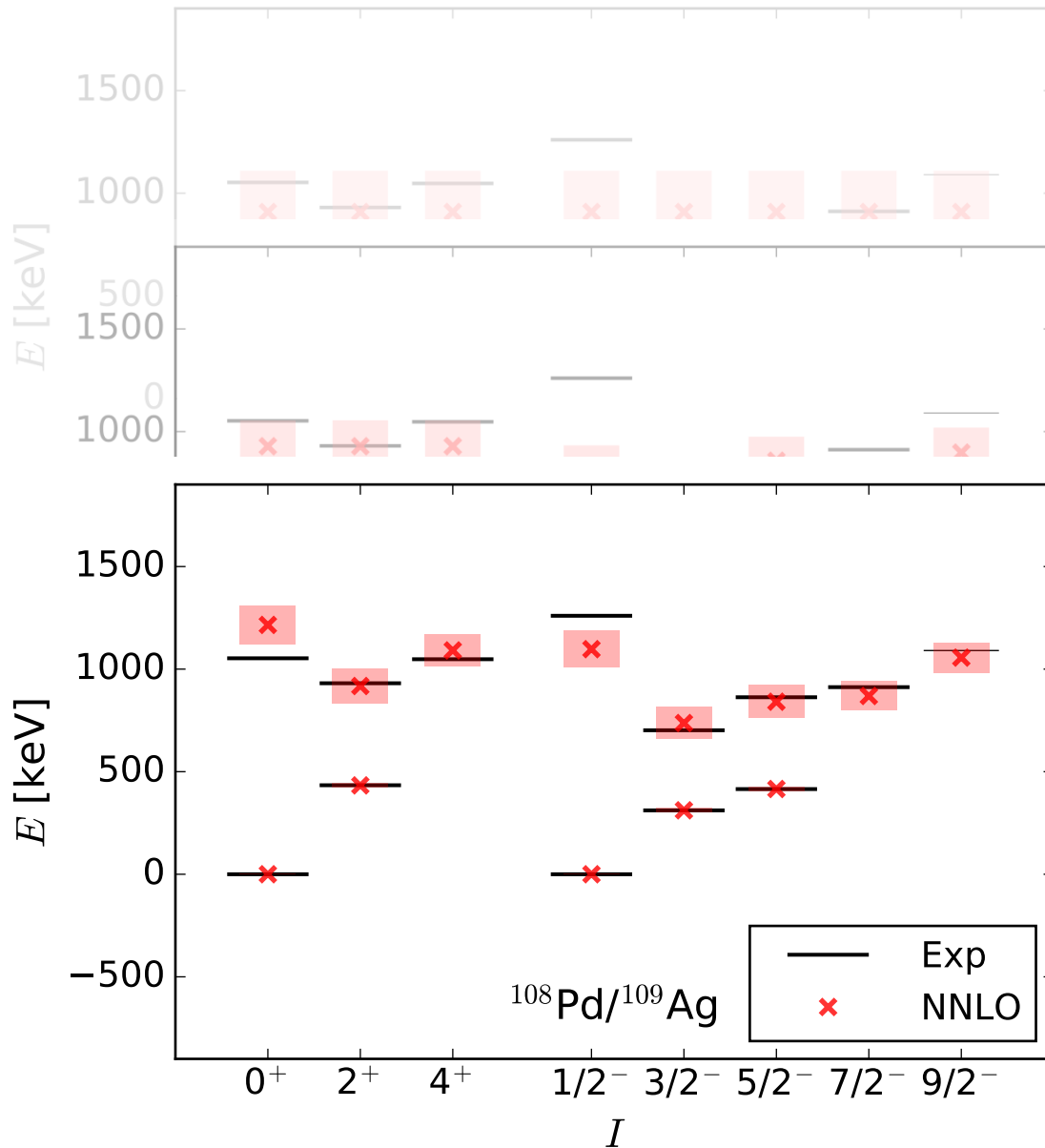




LO:

- One LEC
- Harmonic behavior





LO:

- One LEC
- Harmonic behavior

NLO:

- Two additional LECs
- Particle-core interactions

NNLO:

- Three additional LECs
- Anharmonic corrections

Accuracy and precision show an order-by-order increase of precision at the expense of reduced predictive power

$$\hat{Q} = Q_0 (d^\dagger + \tilde{d}) + Q_1 (d^\dagger \otimes \tilde{d})^{(2)}$$

Nucleus	$I_i^\pi \rightarrow I_f^\pi$	$M(E2)_{\text{exp}}$	$M(E2)_{\text{EFT}}$
^{102}Ru	$2_1^+ \rightarrow 0_1^+$	0.370(2)	0.306(102)
	$0_2^+ \rightarrow 2_1^+$	0.146(12)	0.194(65)
	$2_2^+ \rightarrow 2_1^+$	0.311(24)	0.433(144)
	$4_1^+ \rightarrow 2_1^+$	0.600(50)	0.581(194)
^{103}Rh	$\frac{3}{2}_1^- \rightarrow \frac{1}{2}_1^-$	0.297(16)	0.274(91)
	$\frac{5}{2}_1^- \rightarrow \frac{1}{2}_1^-$	0.402(14)	0.336(112)
	$\frac{1}{2}_2^- \rightarrow \frac{3}{2}_1^-$		0.173(58)
	$\frac{1}{2}_2^- \rightarrow \frac{5}{2}_1^-$	0.244(23)	0.212(71)
	$\frac{3}{2}_2^- \rightarrow \frac{3}{2}_1^-$		0.324(108)
	$\frac{3}{2}_2^- \rightarrow \frac{5}{2}_1^-$		0.212(71)
	$\frac{5}{2}_2^- \rightarrow \frac{3}{2}_1^-$	0.100(7)	0.212(71)
	$\frac{5}{2}_2^- \rightarrow \frac{5}{2}_1^-$	0.121(9)	0.424(141)
	$\frac{7}{2}_1^- \rightarrow \frac{3}{2}_1^-$	0.408(66)	0.520(173)
	$\frac{7}{2}_1^- \rightarrow \frac{5}{2}_1^-$		0.173(58)
	$\frac{9}{2}_1^- \rightarrow \frac{5}{2}_1^-$	0.531(40)	0.613(204)

Nucleus	$I_i^\pi \rightarrow I_f^\pi$	$M(E2)_{\text{exp}}$	$M(E2)_{\text{EFT}}$
^{108}Pd	$2_1^+ \rightarrow 0_{\text{gs}}^+$	0.398(5)	0.341(114)
	$0_2^+ \rightarrow 2_1^+$	0.183(9)	0.216(72)
	$2_2^+ \rightarrow 2_1^+$	0.477(17)	0.482(161)
	$4_1^+ \rightarrow 2_1^+$	0.649(36)	0.647(216)
^{109}Ag	$\frac{3}{2}_1^- \rightarrow \frac{1}{2}_{\text{gs}}^-$	0.322(161)	0.305(102)
	$\frac{5}{2}_1^- \rightarrow \frac{1}{2}_1^-$	0.397(8)	0.373(124)
	$\frac{1}{2}_2^- \rightarrow \frac{3}{2}_1^-$		0.193(64)
	$\frac{1}{2}_2^- \rightarrow \frac{5}{2}_1^-$		0.236(79)
	$\frac{3}{2}_2^- \rightarrow \frac{3}{2}_1^-$	0.356(87)	0.361(120)
	$\frac{3}{2}_2^- \rightarrow \frac{5}{2}_1^-$		0.236(79)
	$\frac{5}{2}_2^- \rightarrow \frac{3}{2}_1^-$	0.176(44)	0.236(79)
	$\frac{5}{2}_2^- \rightarrow \frac{5}{2}_1^-$	0.197(69)	0.472(157)
	$\frac{7}{2}_1^- \rightarrow \frac{3}{2}_1^-$		0.579(193)
	$\frac{7}{2}_1^- \rightarrow \frac{5}{2}_1^-$		0.193(64)
	$\frac{9}{2}_1^- \rightarrow \frac{5}{2}_1^-$	0.664(54)	0.682(227)

Most general operator of rank one

$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$

LO term:

- Two LECs
- Phonon-conserving transition
- Static M1 moments

NLO term:

- Two LECs
- Phonon-annihilating transition

Nucleus	I_i^π	μ_{exp}	μ_{EFT}
^{102}Ru	2_1^+	0.85(3)	1.02(34)
	2_2^+		1.02(34)
	4_1^+		2.04(68)
^{103}Rh	1_1^-	-0.09	-0.08(3)
	3_1^-	0.77(7)	0.97(32)
	5_1^-	1.08(4)	0.93(31)
	7_1^-	2.00(60)	2.04(68)
	9_1^-	2.80(50)	1.95(65)
	2_1^-		
^{106}Pd	2_1^+	0.79(2)	0.91(30)
	2_2^+	0.71(10)	0.91(30)
	4_1^+	1.80(40)	1.81(60)
^{107}Ag	1_1^-	-0.11	-0.11(4)
	3_1^-	0.98(9)	0.88(29)
	5_1^-	1.02(9)	0.80(27)
	7_1^-		1.85(62)
	9_1^-		1.71(57)
	2_1^-		
^{108}Pd	2_1^+	0.71(2)	0.93(31)
	2_2^+		0.93(31)
	4_1^+		1.86(62)
^{109}Ag	1_1^-	-0.13	-0.13(4)
	3_1^-	1.10(10)	0.91(30)
	5_1^-	0.85(8)	0.80(26)
	7_1^-		1.91(64)
	9_1^-		1.72(57)
	2_1^-		

$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$

LECs fitted to static M1 moments

Nucleus	$I_i^\pi \rightarrow I_f^\pi$	$M(M1)_{\text{exp}}$	$M(M1)_{\text{EFT}}$
^{103}Rh	$5_1^- \rightarrow 3_1^-$		0.512(171)
	$5_1^- \rightarrow 3_2^-$		0.512(171)
	$9_1^- \rightarrow 7_1^-$		0.697(232)
	$2_1^- \rightarrow 2_1^-$		
^{107}Ag	$5_1^- \rightarrow 3_1^-$	0.595(36)	0.506(169)
	$5_1^- \rightarrow 3_2^-$		0.506(169)
	$9_1^- \rightarrow 7_1^-$		0.689(230)
	$2_1^- \rightarrow 2_1^-$		
^{109}Ag	$5_1^- \rightarrow 3_1^-$	0.680(55)	0.551(184)
	$5_1^- \rightarrow 3_2^-$		0.551(184)
	$9_1^- \rightarrow 7_1^-$		0.750(250)
	$2_1^- \rightarrow 2_1^-$		

$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$

Nucleus	$I_i^\pi \rightarrow I_f^\pi$	$M(M1)_{\text{exp}}$	$M(M1)_{\text{EFT}}$	
^{103}Rh	$\frac{3}{2}^- \rightarrow \frac{1}{2}^-$	0.568(24)	0.566(189)	
	$\frac{1}{2}^- \rightarrow \frac{3}{2}^-$		0.358(119)	
	$\frac{3}{2}^- \rightarrow \frac{3}{2}^-$		0.624(208)	
	$\frac{3}{2}^- \rightarrow \frac{5}{2}^-$		0.291(97)	
	$\frac{5}{2}^- \rightarrow \frac{3}{2}^-$	0.250(18)	0.291(97)	
	$\frac{5}{2}^- \rightarrow \frac{5}{2}^-$	0.299(22)	0.289(96)	
	$\frac{7}{2}^- \rightarrow \frac{5}{2}^-$		1.074(358)	
	^{109}Ag	$\frac{3}{2}^- \rightarrow \frac{1}{2}^-$	0.560(36)	0.612(204)
		$\frac{1}{2}^- \rightarrow \frac{3}{2}^-$		0.387(129)
$\frac{3}{2}^- \rightarrow \frac{3}{2}^-$		0.655(143)	0.506(169)	
$\frac{3}{2}^- \rightarrow \frac{5}{2}^-$			0.371(124)	
$\frac{5}{2}^- \rightarrow \frac{3}{2}^-$		0.401(89)	0.371(124)	
$\frac{5}{2}^- \rightarrow \frac{5}{2}^-$		0.669(134)	0.523(174)	
$\frac{7}{2}^- \rightarrow \frac{5}{2}^-$			1.162(387)	

Low-lying positive-parity odd-odd states are constructed as

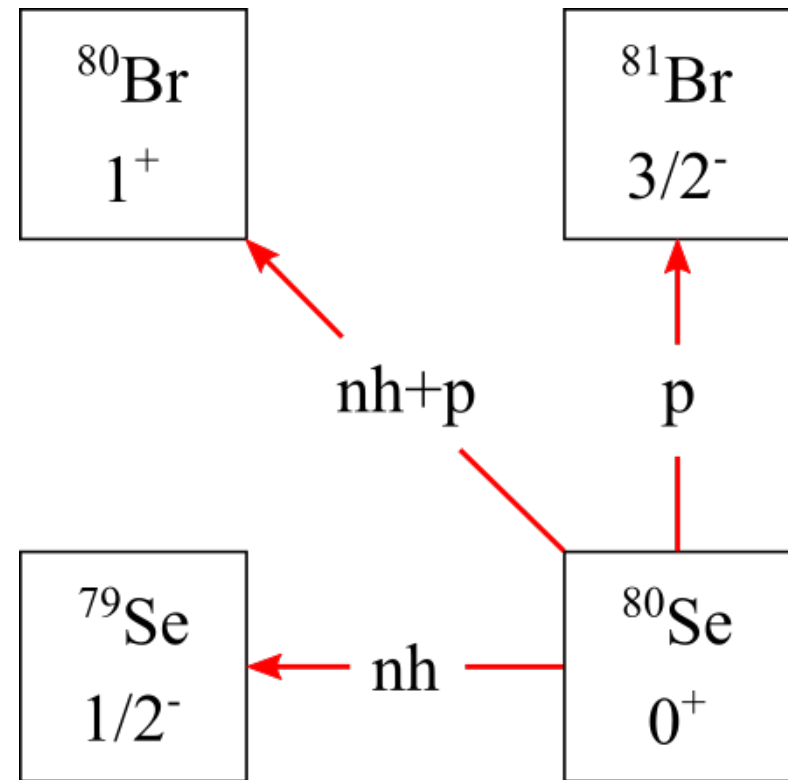
$$|IM; j_p; j_n\rangle = \sum_{\mu\nu} C_{j_n\mu j_p\nu}^{IM} n_{\mu}^{\dagger} p_{\nu}^{\dagger} |0\rangle$$

where

$$|j_n - j_p| \leq I \leq j_n + j_p$$

and

$$\pi_n \pi_p = 1$$



Most general rank-one operator coupling
odd-odd and even-even states

From the power counting

$$\frac{C_{\beta\ell}}{C_\beta} \stackrel{\text{EFT}}{\sim} 0.58 \binom{+42}{-25} \quad \text{and} \quad \frac{C_{\beta L\ell}}{C_\beta} \stackrel{\text{EFT}}{\sim} 0.33 \binom{+25}{-14}$$

$$\begin{aligned} \hat{O}_\beta = & C_\beta (\tilde{p} \otimes \tilde{n})^{(1)} \\ & + \sum_\ell C_{\beta\ell} \left[(d^\dagger + \tilde{d}) \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} \\ & + \sum_{L\ell} C_{\beta L\ell} \left[(d^\dagger \otimes d^\dagger + \tilde{d} \otimes \tilde{d})^{(L)} \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} \end{aligned}$$

LO term:

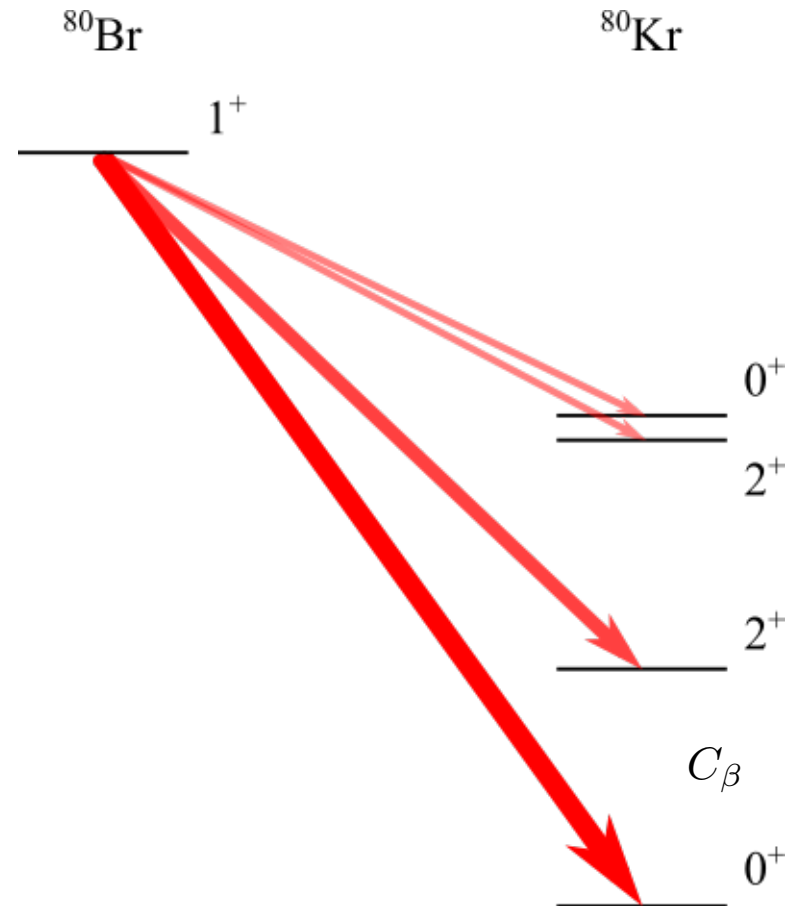
- Couples states with $\Delta\mathcal{N} = 0$

NLO term:

- Couples states with $\Delta\mathcal{N} = 1$

NNLO term:

- Couples states with $\Delta\mathcal{N} = 2$



Corrections to odd-odd Hamiltonian

$$\sqrt{\omega/\Lambda}$$

Corrections to the effective GT operator

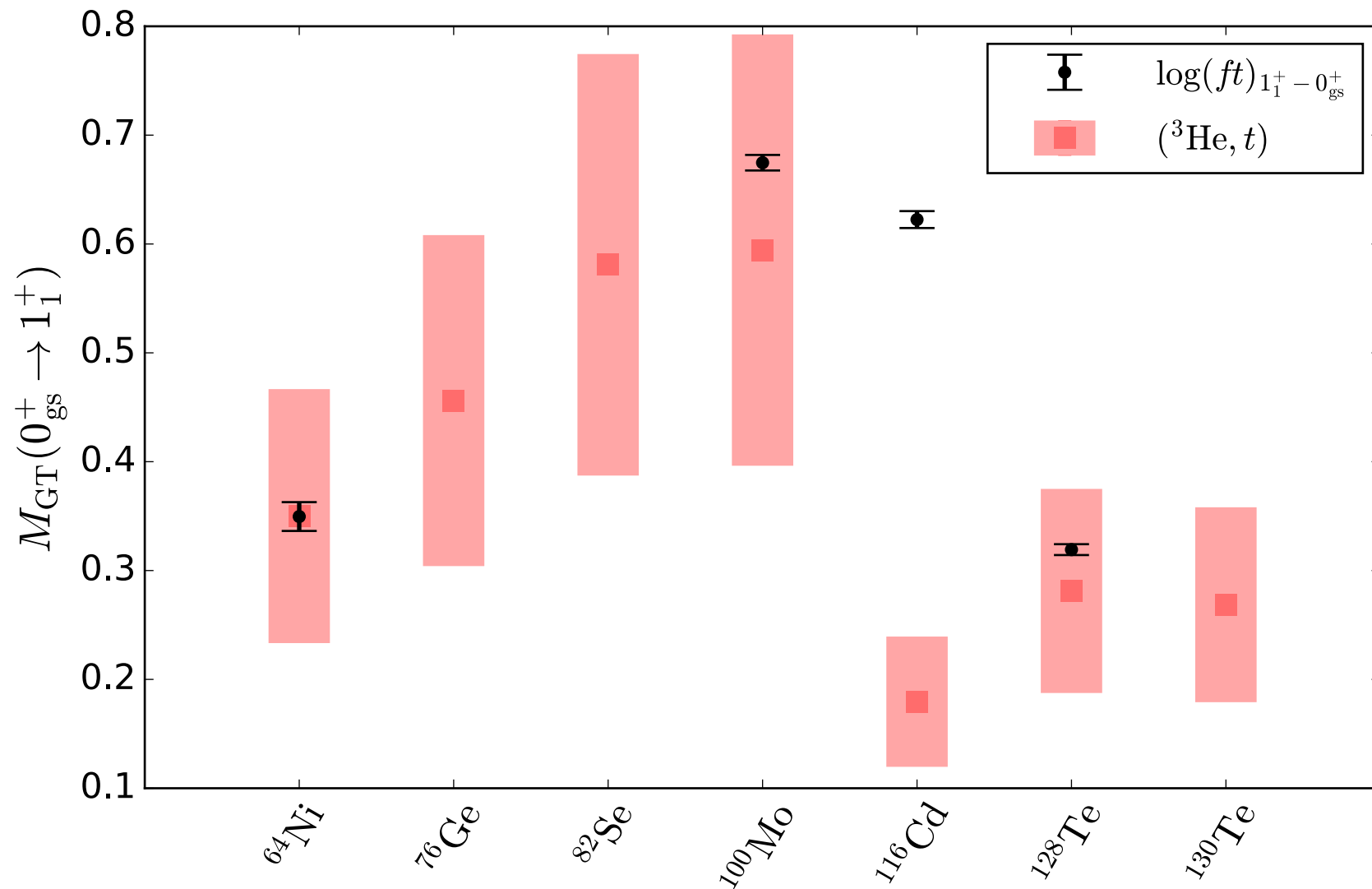
$$\omega/\Lambda$$

Uncertainty estimate

$$\Delta \langle 0 || \hat{O}_\beta || I; j_p; j_n \rangle \stackrel{\text{EFT}}{\sim} \langle 0 || \hat{O}_\beta || I; j_p; j_n \rangle \frac{\omega}{\Lambda}$$

or

$$\Delta \log(ft)_{if} \stackrel{\text{EFT}}{\sim} \frac{2}{\ln 10} \frac{\omega}{\Lambda}$$



Popescu et al., Phys. Rev. C **79**, 064312 (2009)

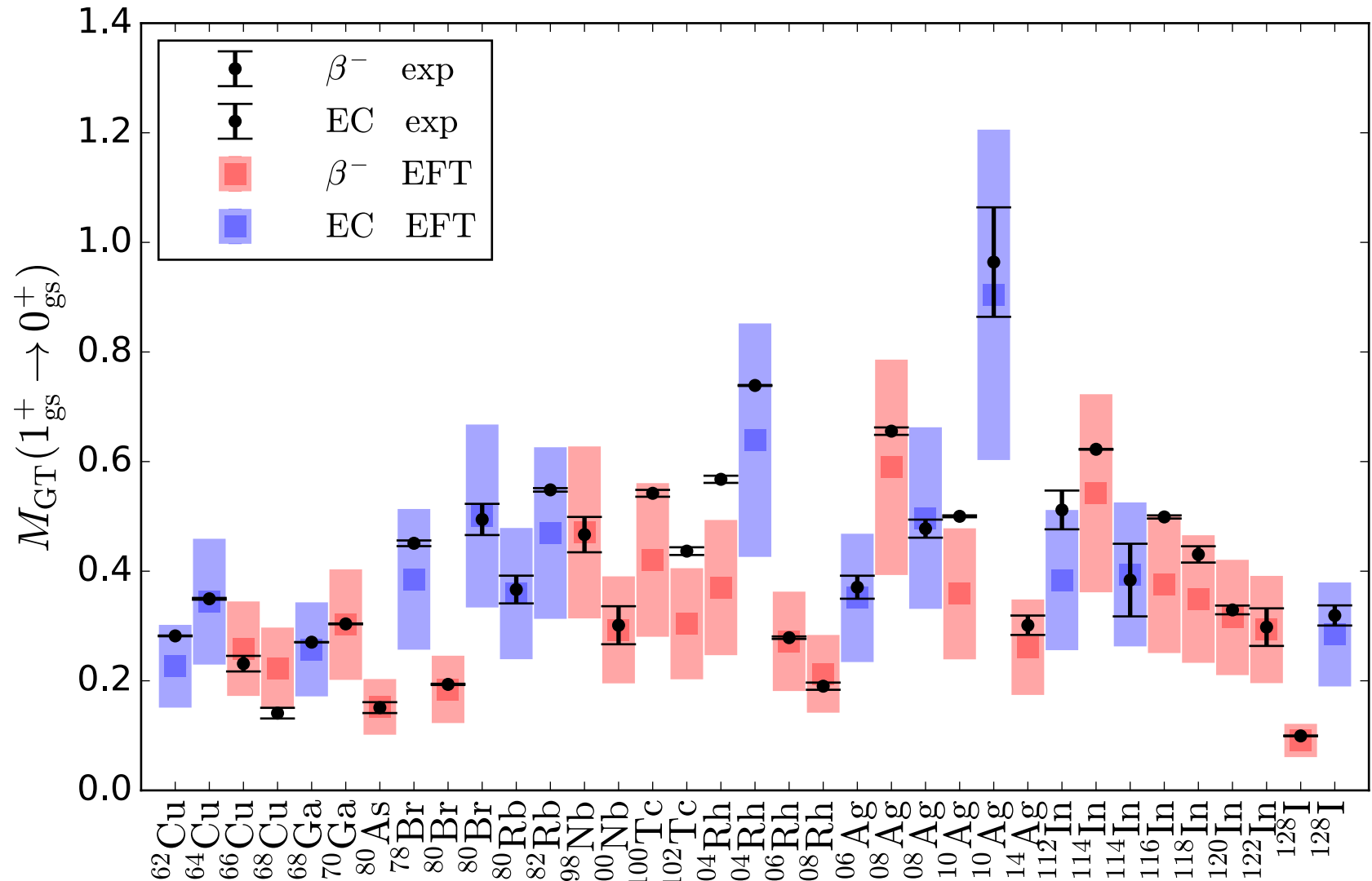
Thies et al., Phys. Rev. C **86**, 014304 (2012)

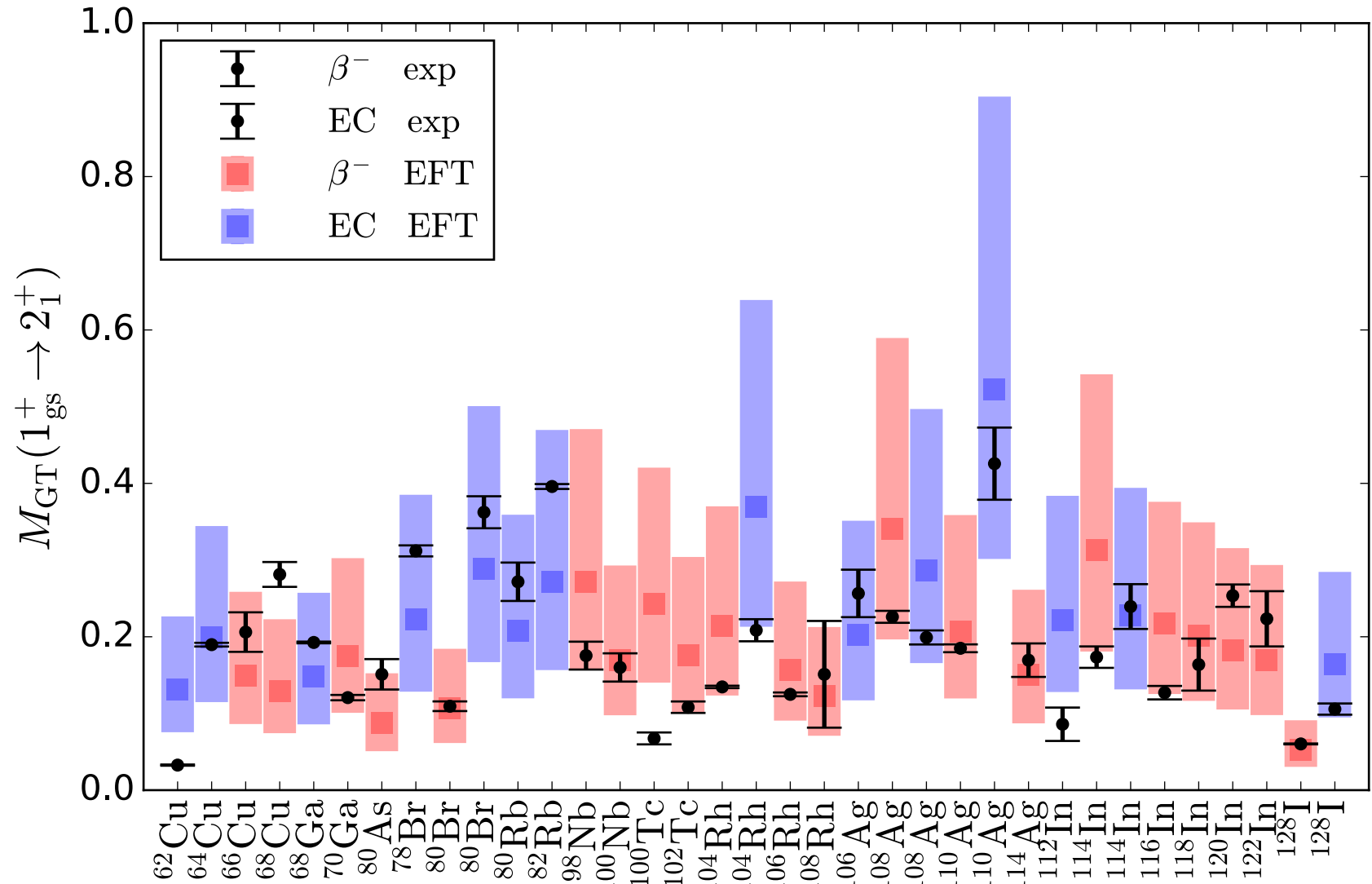
Frekers et al., Phys. Rev. C **94**, 014614 (2016)

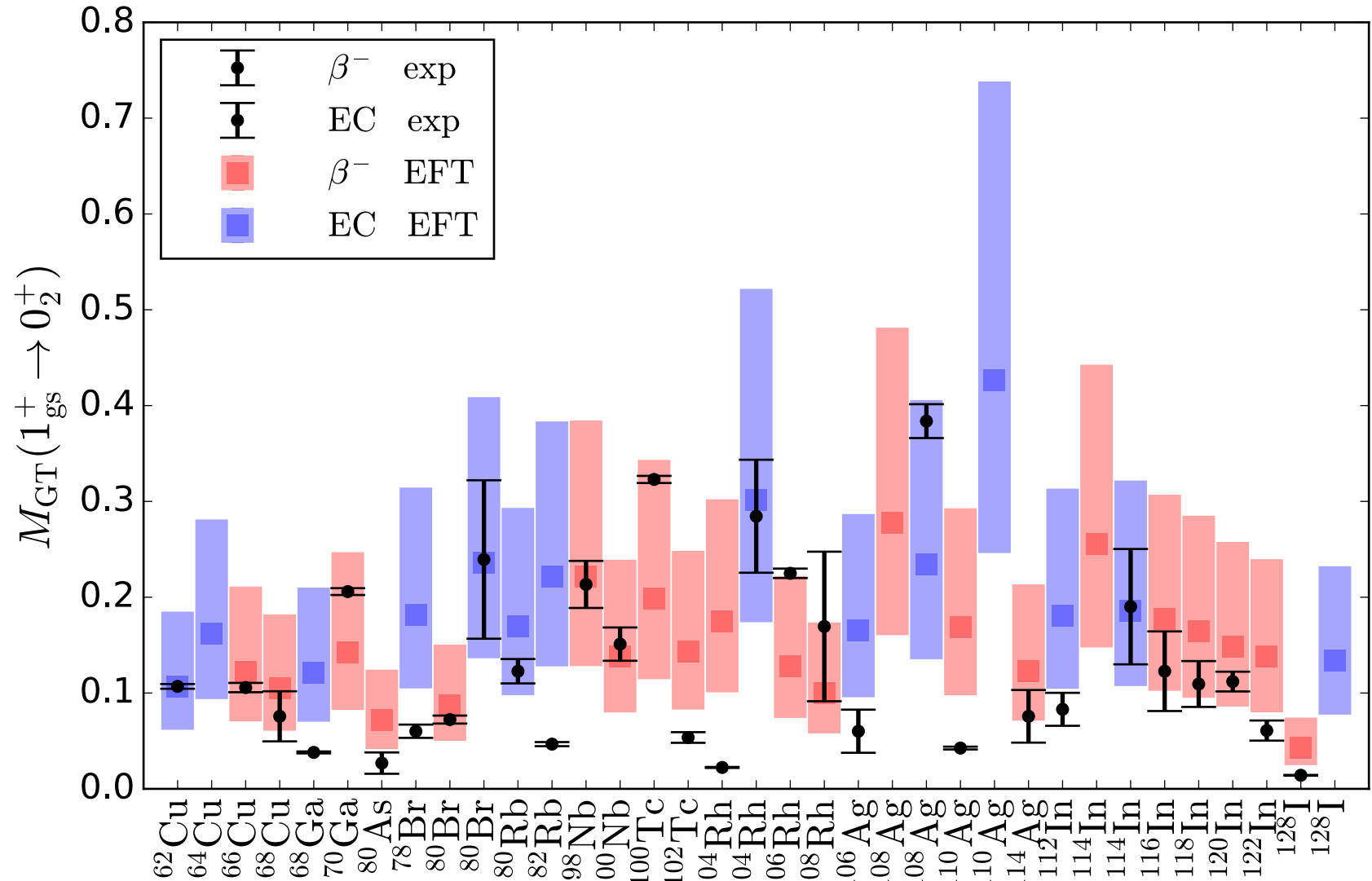
Thies et al., Phys. Rev. C **86**, 044309 (2012)

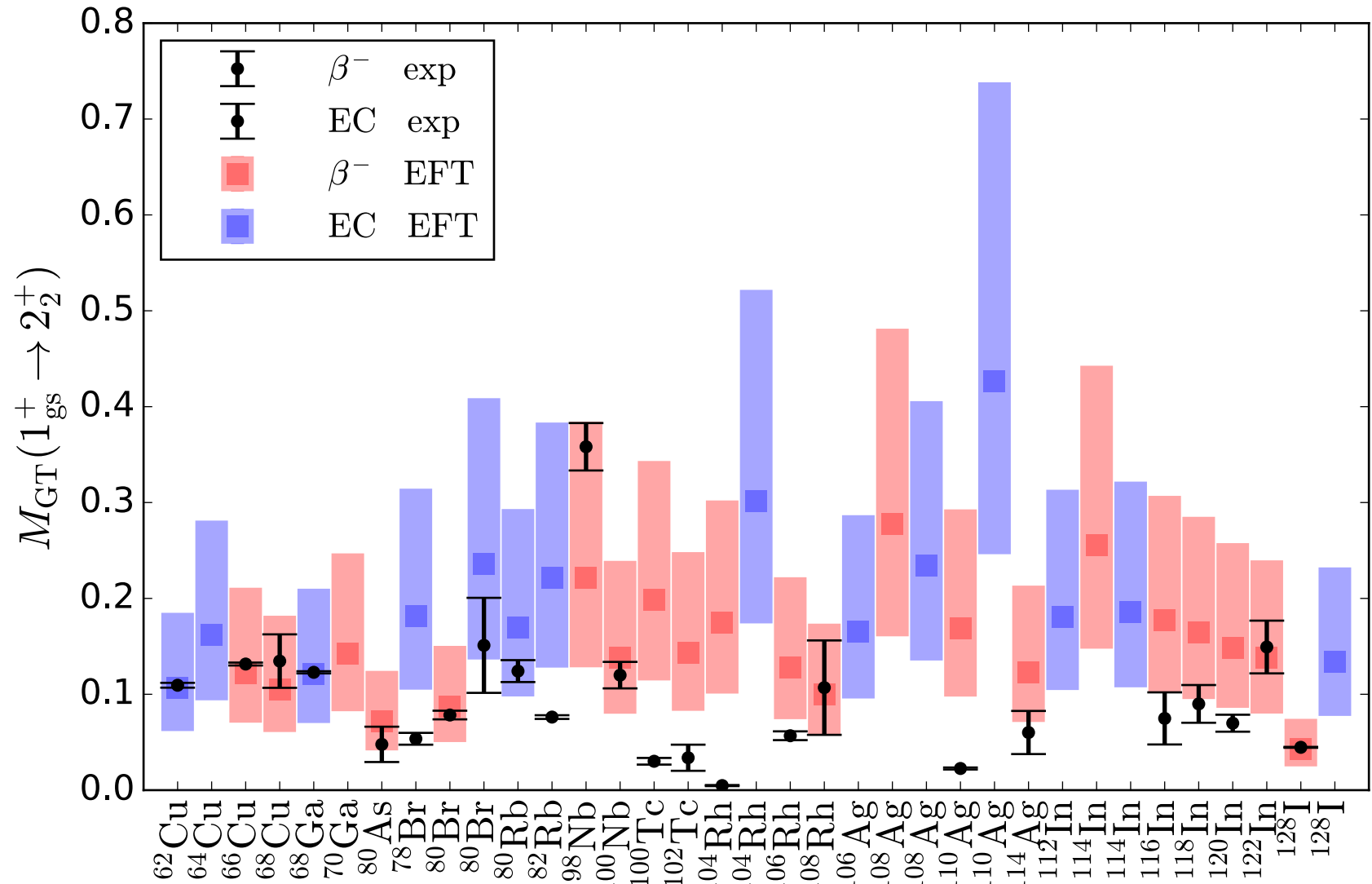
Akimune et al., Phys. Lett. B **394**, 23 (1997)

Puppe et al., Phys. Rev. C **86**, 044603 (2012)









GT matrix elements for $2\nu\beta\beta$ decay

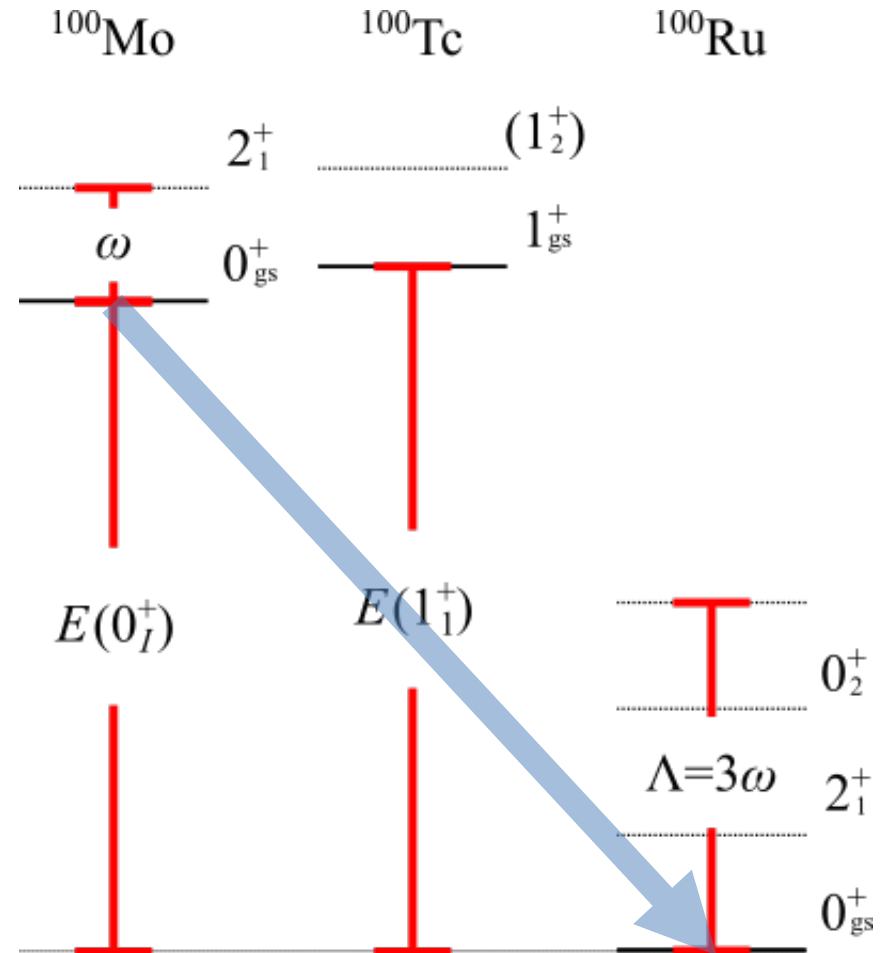
$$M_{\text{GT}}^{2\nu} = \sum_n \frac{\langle f || \sum_a \sigma_a \tau_a^+ || 1_n^+ \rangle \langle 1_n^+ || \sum_b \sigma_b \tau_b^+ || i \rangle}{D_{nf}/m_e}$$

with

$$D_{nf} = E_n - \frac{E_i - E_f}{2}$$

SSD approximation

$$M_{\text{GT}}^{2\nu}(i \rightarrow f) \approx \frac{M_{\text{GT}}(1_1^+ \rightarrow 0_f^+) M_{\text{GT}}(0_i^+ \rightarrow 1_1^+)}{D_{1f}/m_e c^2}$$



ASSUMPTION: Energies and matrix elements of 1^+ states scale as

$$E(1_{n+1}^+) \stackrel{\text{EFT}}{\sim} E(1_1^+) + n\omega \quad \text{and} \quad M_{\text{GT}}(0_{\text{gs}}^+ \rightarrow 1_{n+1}^+) \stackrel{\text{EFT}}{\sim} \left(\frac{\omega}{\Lambda}\right)^{n/2} M_{\text{GT}}(0_{\text{gs}}^+ \rightarrow 1_1^+)$$

Omitted contribution

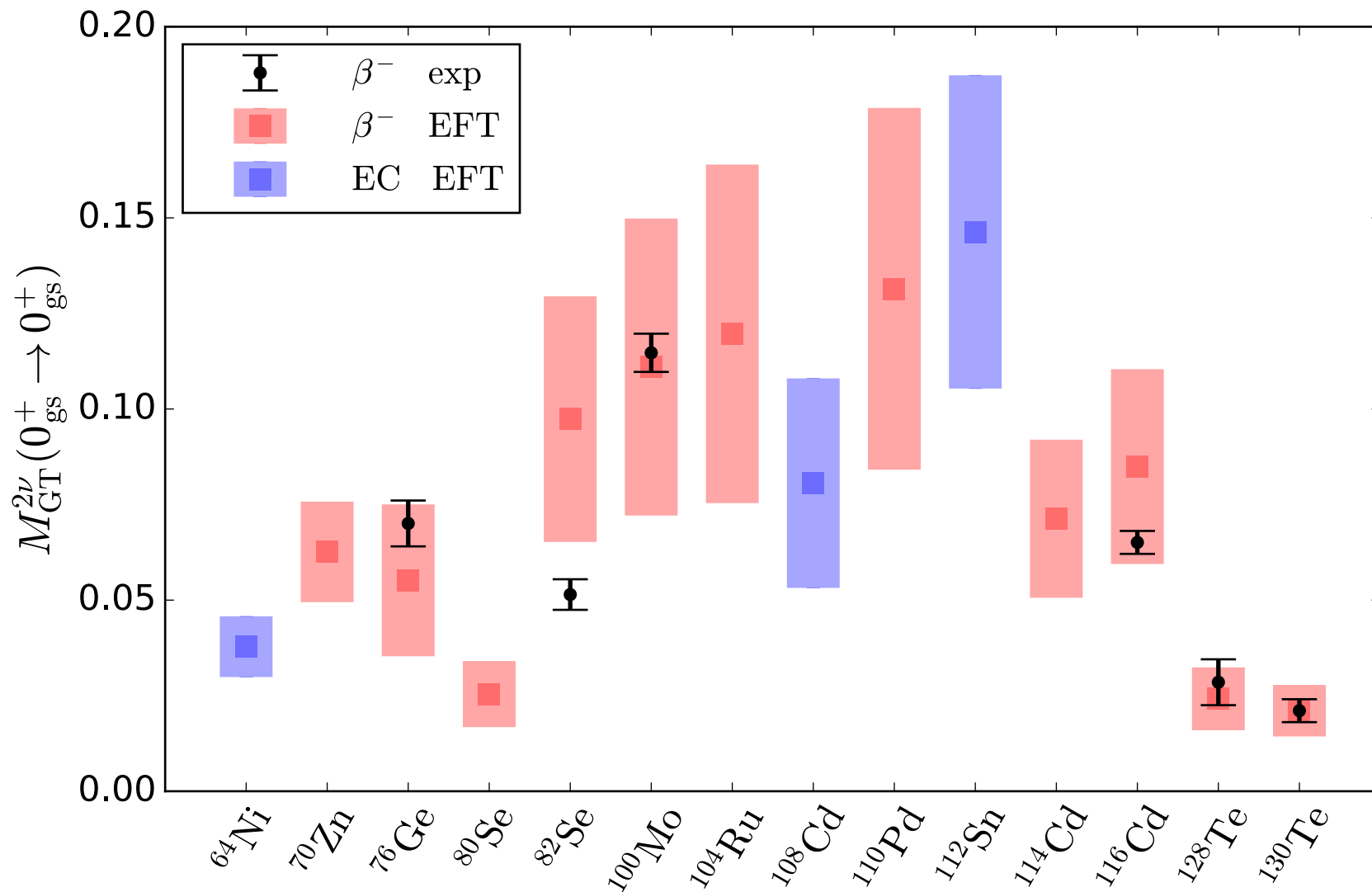
$$\begin{aligned} \Delta M_{\text{GT}}^{2\nu}(0_{\text{gs}}^+ \rightarrow 0_{\text{gs}}^+) & \stackrel{\text{EFT}}{\sim} \sum_{n=1} \left(\frac{\omega}{\Lambda}\right)^n \frac{M_{\text{GT}}(1_1^+ \rightarrow 0_{\text{gs}}^+) M_{\text{GT}}(0_{\text{gs}}^+ \rightarrow 1_1^+)}{(D_{11} + n\omega)/m_e c^2} \\ & = \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11} + \omega}{\omega}\right) M_{\text{GT}}^{2\nu}(0_{\text{gs}}^+ \rightarrow 0_{\text{gs}}^+), \end{aligned}$$

where

$$\Phi(z, s, a) \equiv \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s}$$

The percentual uncertainty depends on the energy scales involved transition

$$\delta(\text{gs} \rightarrow \text{gs}) = \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11} + \omega}{\omega}\right)$$

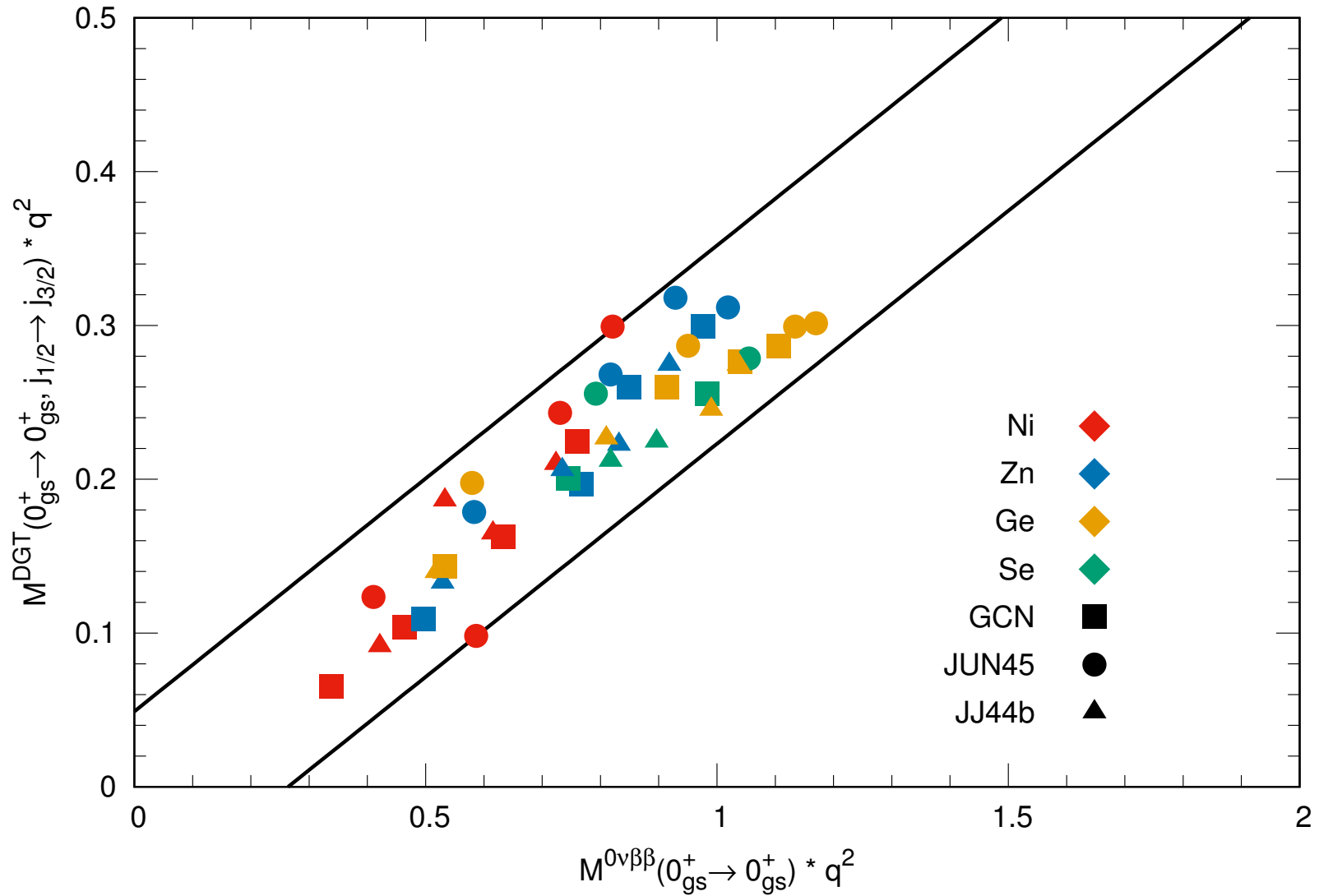


We studied single- β and $2\nu\beta\beta$ decays within a collective EFT

- Matrix elements for single- β decays to the ground and excited states of spherical nuclei can be consistently described within an EFT that describes the later systems as even-even collective cores coupled to an additional neutron and/or proton
- Matrix elements for $2\nu\beta\beta$ decays calculated within the SSD approximation consistently describe observed decays when the theoretical uncertainty estimate is taken into account
- NLO corrections to the Hamiltonian for the odd-odd system and the effective GT operator are expected to decrease the uncertainty by a factor of 1/3. These corrections are feasible

FUTURE GOAL

- Calculate matrix elements for $0\nu\beta\beta$ decays with uncertainty estimates. This would require us to write the corresponding operators in terms of the effective degrees of freedom. Since there is no available experimental data the LECs to other nuclear structure calculations, or highly correlated data





Thanks