

# Study of single- $\beta$  and  $2\nu\beta\beta$  decays within an effective theory for collective nuclei

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Even-even nuclei

- Effective Hamiltonian and power counting
- E2 properties

Odd-mass nuclei

- Power counting for fermion operators
- Spectra at NNLO
- E2 and M1 properties

 $\beta$  decays from odd-odd nuclei

- Low-lying odd-odd states
- Effective Gamow-Teller operator
- Uncertainty estimates

 $\beta\beta$  decays in the SSD approximation

Uncertainty estimate





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Chiral EFT

• Nucleon and pion fields

### Pionless EFT

• Nucleon fields



### Collective EFT

- Phonons ◆ Phonons<br>◆ Few fermions
	- $\omega \sim 500 \text{keV}$

Bertsch, Dean, Nazarewicz, SciDAC Rev. 6, 42 (2007) *<sup>v</sup>±*<sup>1</sup> ! *<sup>e</sup>±i*˜*v±*<sup>1</sup> (5)  $\overline{\phantom{a}}$ 

*<sup>µ</sup> <sup>d</sup><sup>µ</sup> <sup>µ</sup>* = 0*, <sup>±</sup>*1*, <sup>±</sup>*<sup>2</sup> ⇥

*<sup>µ</sup> <sup>N</sup>* (*n*)

r⇤

!



*n*ˆ ⌘ *a† · a.* ˜ (12)

Degrees of freedom: quadrupole-boson creation and annihilation operators  $\overline{C}$  the boson Hamiltonian ( $\overline{C}$ Degrees of freedom: quadrupole-boson creation and annihilation operators<br> **F** d *d* = *ex* 

$$
\left[ d_{\mu},d_{\nu}^{\dagger}\right] =\delta_{\mu\nu}\,,
$$

ond terms in the interaction Hamiltonian (15),

that of the one-phonon state in the even-even

*C<sup>k</sup>*

*<sup>µ</sup>* and *d<sup>µ</sup>* with *µ* = 2*,* 1*, ...,* 2 that

*mµn*⌫*M*(*m*)

Rank-two tensors Hamiltonian *<sup>M</sup>*(*I*) *· N* (*I*) <sup>=</sup> <sup>p</sup>2*<sup>I</sup>* + 1 ⇣ *I*⇡ = 2<sup>+</sup> *I*⇡ = 0<sup>+</sup>

*<sup>K</sup>*(*k*)

 $=$   $\frac{1}{2}$ 

erators *d†*

$$
d_\mu^\dagger \quad \text{and} \quad \tilde{d}_\mu = (-1)^\mu d_{-\mu}
$$

Most simple rotational-invariant Hamiltonian der (NLO) in the EFT for vibrational nuclei is



 $0^{+}$   $0^{-}$   $2^{+}$   $3^{+}$   $\leq$ *,* 4<sup>+</sup> **d**  $\frac{1}{2}$  d<sup>+</sup> multiphonon states  $\frac{1}{0^{+}}$ consists of a one-body term and a two-body term and a two-body term and a two-body term and a two-body term an  $0^+$   $2^+$   $3^+$   $4^+$   $6^+$ the unique two-body interaction for spin-1*/*<sup>2</sup>  $\pm$  shell.  $\overline{O^+}$  and  $\overline{O^+}$  and  $\overline{O^+}$  and  $\overline{O^+}$  and  $\overline{O^+}$  and  $\overline{O^+}$  and  $\overline{O^+}$ terms such as <sup>ˆ</sup>*j*<sup>2</sup> / *<sup>n</sup>*ˆ(2*n*ˆ) or ˆ*n*<sup>2</sup> because these are linear combinations of the terms already in- $\overline{\phantom{a}}$  2<sup>+</sup> The Hamiltonian (14) is not the Hamiltonian of free fermions but rather captures the  $\overline{\phantom{a}}$  interactions between fermions and the ground the ground the ground the ground the ground state ground the ground state state of the vibrating core. Let us discuss the ! *d d* ⇠ r⇤ d<sub>1</sub> *<sup>C</sup>*2*d*<sup>4</sup> ⇠ ✓*N*!  $-4^{+}$  $\frac{1}{2}$  B<br>  $\frac{1}{2}$  2 + *g*<sup>2</sup> *H*ˆNLO <sup>=</sup> *<sup>H</sup>*ˆNLO *<sup>H</sup>*ˆLO ⇠

*H*ˆNLO = *H*ˆLO + *g*!*N*ˆ + *g<sup>N</sup> N*ˆ <sup>2</sup> + *gv*⇤ˆ <sup>2</sup> + *g<sup>I</sup>* ˆ*I*<sup>2</sup> ⇤ order Casimir operator, respectively. For more Coello Pérez, Papenbrock; Phys. Rev. C **92**, 064309 (2015) *,* <sup>6</sup><sup>+</sup> ⇤ ⇠ <sup>3</sup>! (20) *H*(2015) *H* 

The states are constructed as phonon excitations of the reference state *<sup>d</sup><sup>µ</sup>* = (1)*<sup>µ</sup>d<sup>µ</sup>* !<sup>1</sup> ⇠ !

<sup>ˆ</sup><sup>j</sup> <sup>=</sup> <sup>1</sup>

⇤

p2

j.

$$
\left(d^{\dagger^n}\right)^{(I)}_M|0\rangle
$$

*,* 4<sup>+</sup>



$$
H_{\text{LO}} \equiv \omega_1 \hat{N}
$$
  

$$
\hat{N} \equiv d^{\dagger} \cdot \tilde{d}
$$

 $\omega_1 \sim \omega$ 

*,* 2<sup>+</sup>

 $C$ <sub>*d*</sub><sub>4</sub> **∴**  $C$ <sup>2</sup> <sub>∴</sub>  $C$ <sup>2</sup> ∴  $C$ <sup>2</sup>

*,* 3<sup>+</sup>

*,* 4<sup>+</sup> *I*⇡ = 0<sup>+</sup>

Example: Scale of a term with four boson operators at breakdown of the same rank *I* is [50] berators at breakdown

*Ck*

$$
\hat{V} \equiv d^{\dagger} \cdot \tilde{d}
$$
\n
$$
C_2 d^4 \sim \omega \text{ or } C_2 \sim \left(\frac{\omega}{\Lambda}\right)^2 \omega
$$
\n
$$
\omega_1 \sim \omega
$$

 $\frac{1}{2}$  and  $\frac{1}{2}$  are  $\frac{1}{2}$  and  $\frac{1}{2}$  are Hamiltonian (14) is not the Hamiltonian (14) is not th  $T_{\rm t}$  and  $T_{\rm t}$  at next-to-leading or- $\omega_1 \sim \omega$  and  $\omega_2$  and  $\omega_3$   $\sim$  *M*<sup>+</sup>  $\omega$ *N*  $\sim$  2  $\omega$ now energies

*<sup>K</sup>*(*k*)

 $\sim$   $\sim$ 

*I*⇡ = 2<sup>+</sup> *I*⇡ = 0<sup>+</sup>

*I*⇡ = 2<sup>+</sup> *I*⇡ = 0<sup>+</sup>

$$
C_2 d^4 \sim \left(\frac{N\omega}{\Lambda}\right)^2
$$

low energies breakdown  $H_{\text{LO}} \sim N \omega$   $H_{\text{LO}} \sim \Lambda$  $H_{\text{LO}} \sim \Lambda$ ⌘*n/*<sup>2</sup>  $\sqrt{\Lambda}$ *d* = !*N*ˆ ˜ *d*<sub>*i*</sub>  $\frac{d}{dx}$  = (10)*d*<sub>*n*</sub> (2) = (1)<sup>*n*</sup>  $\sqrt{1}$ *low energi* breakdown

 $\frac{d}{d} \sim$ 

*d*  $\sim$ 

 $d \sim \sqrt{N}$  the  $d \sim \sqrt{\frac{\Lambda}{\omega}}$ 

 $d \sim \sqrt{\frac{1}{\omega}}$ 

 $d \sim \sqrt{N}$ 

 $d \sim \sqrt{N}$ 

 $\overline{a}$ 

Scales

*<sup>d</sup><sup>µ</sup>* = (1)*<sup>µ</sup>d<sup>µ</sup>* (19)

 $\overline{a}$ 

*,* 2<sup>+</sup>

*I*⇡ = 2<sup>+</sup> *I*⇡ = 0<sup>+</sup>

*n d*<sup>*x*</sup> *d z*<sub>2</sub>  $\frac{1}{2}$   $\frac{1$ 

*I*⇡ = 2<sup>+</sup> *I*⇡ = 0<sup>+</sup>

ASSUMPTION: Corrections shift the energies by  $\omega$  at breakdown *n*<br>
at preakud  $\mathsf{DN}\text{:}\ \mathsf{Corrections}\ \mathsf{shift}\ \mathsf{the}\ \mathsf{energies}\ \mathsf{akdown\ \ }\Delta E\sim\omega$ ⌘*n/*<sup>2</sup>  $\omega$  at breakdown  $\Delta P$  $\mathsf{H}$  assume from: Correction<br>by  $\omega$  at breakdown  $\Delta E$ ✓*N*! ◆<sup>2</sup> !!<br>|  $\frac{1}{2}$ ⇤ .<br>ماد  ${\sf down}\;\, \Delta E \sim \omega$ 

**NNLO Hamiltonian** and We note that *d† <sup>µ</sup>* and **H**<sup>*N*</sup> + *M*<sup>2</sup> + *M*<sup>2</sup> + *M*<sup>2</sup> + *M*<sup>2</sup> + *g*<sup>*I*</sup> *d*<sub>*I*</sub><sup>*n*</sup> + *d*<sub>*I</sub>* 

$$
H_{\rm NNLO} \equiv g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_J \hat{J}^2
$$

With which the term  $\sim$  in Eq. (17) sets the overall set  $\mathsf{With}$ 

*,* <sup>6</sup><sup>+</sup> ⇤ ⇠ <sup>3</sup>! (20)

$$
\hat{\Lambda}^2 \equiv -\left(d^{\dagger} \cdot d^{\dagger}\right) \left(\tilde{d} \cdot \tilde{d}\right) + \hat{N}^2 - 3\hat{N},
$$
  
energies  

$$
\hat{\mathbf{J}} = \sqrt{10} \left(d^{\dagger} \otimes \tilde{d}\right)^{(1)}
$$

✓*N*!

nuclei, respectively. The centroids of the *I* = *J ± j*

◆<sup>2</sup>



 $1.0$ odd-mass states are shown as blue crosses.  $\mathcal{U}^{\ddagger}$  $\mathbb{P}$  provide us with the opportunity to  $\mathbb{P}$  $\frac{1}{2}$  theoretical uncertainties. While theoretical uncertainties is  $c_2$  $\mathsf{p}$  0.2  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  and wall Gaussian  $\begin{array}{c|cccc}\n0.0 & \overline{z} & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 3.5 \\
\hline\n\end{array}$ about the distribution of  $\frac{c_0}{c_0}$ form of priors. Employing Bayesian statistics *c X*% (25) *X* = *X*<sup>0</sup>  $\begin{array}{ccc} 1.0 \ \textcolor{red}{\overline{ \text{ }} } \end{array}$  $\overline{a}$  theoretical uncertainties. While theoretical uncertainties. While the theoretical uncertainties.  $\overline{p}$  and  $t = \frac{1}{\sqrt{2\pi}}$  $\frac{1}{20}$  0.6 assumptions on  $\frac{1}{20}$ about the distribution of the LECs  $\downarrow$  $\geq 0.4$  $\overline{a}$  over unknown parameters  $\overline{a}$  over unknown parameters  $\overline{a}$  $\begin{array}{ccc} \text{E} & 0.2 \end{array}$  Hard w  $\overrightarrow{O}$  |  $\overrightarrow{I}$   $\over$  $-1.0$   $-0.5$  0.0 0.0 0.1 0.1  $\sigma$ 

Observables (energy as an example)  $\mathbf C$ *E*NNLO(*I*⇡)  $\frac{1}{2}$  and  $\frac{1}{2}$  an *<sup>c</sup>n*"*<sup>n</sup> <sup>c</sup>*<sup>2</sup> <sup>=</sup> *H*ˆNLO gy a **an Chain** Observables (energy as an example)

*<sup>c</sup>*2(*I*⇡) ⌘

✓*N*!

◆<sup>3</sup>

$$
E = \omega \sum_{n=1}^{\infty} c_n \varepsilon^n,
$$
  

$$
\varepsilon \equiv \frac{N\omega}{\Lambda}
$$

Expansion coefficients of order one Assumption encoded into priors<sup>\*</sup> nuclei. These distributions, with means *µ*<sup>1</sup> and *µ*2, <u>respectively, can be a building by the approximation  $\mathbf{r}$ </u>  $f(x)$  =  $f(x)$   $\geq f(x)$   $\geq f(x)$   $\geq f(x)$   $\geq f(x)$ eter *c*, associated with the width of the distribu-

$$
pr^{(G)}(\tilde{c}_i|c) = \frac{1}{\sqrt{2\pi}sc}e^{-\frac{\tilde{c}_i^2}{2s^2c^2}}
$$
\n
$$
pr(c) = \frac{1}{\sqrt{2\pi}}e^{-\frac{\log^2 c}{2\sigma^2}}
$$

 $\star$  c chose log-normal priors for the LECs' distribution of the LECs' distribution  $\gamma$  . Let the assumption that *c* is log-normal distributed "Cacciari, Houdeau; Nucl. J. High Energy Phys. **09** (2011) 039<br>\*Furnstahl, et al.; J. Phys. G **42**, 034028 (2015) \*Cacciari, Houdeau; Nucl. J. High Energy Phys. **09** (2011) 039

 $\sqrt{2\pi}\sigma c$ 

 $\text{pr}(c) = \frac{c}{\sqrt{2\pi}\sigma c}e^{-2\sigma^2}$  $\sqrt{2\pi\sigma}$ 



Most general positive-parity rank-two tensor  $\mathbb{R}^2$ <sup>1</sup> 0.79 (2) 0.79 (24) Most general positive-parity rank-two tens

$$
\hat{Q}=Q_0\left(d^\dagger+\tilde{d}\right)+Q_1\left(d^\dagger\otimes\tilde{d}\right)^{(2)}
$$

ASSUMPTION: All terms scale similarly at *A***B** = *Q*<sup>2</sup>*C***<sub>***L***</sub>** *C***<sub>***C***</sub>***C***<sub>***C***</sub>** *C***<sub>***C***</sub>** r! rms <mark>!</mark> *Q*<sup>1</sup> *Q*<sup>0</sup> i<mark>larly at</mark> ⇡ <sup>0</sup>*.*<sup>47</sup> *<sup>Q</sup>*<sup>1</sup> ⇡ <sup>0</sup>*.*<sup>41</sup> *<sup>Q</sup>*<sup>1</sup> terms scale similarly at

$$
Q_1\sim \sqrt{\frac{\omega}{\Lambda}} Q_0
$$

**Natural scaling**  $\blacksquare$  Natural scaling **Natu** 

<sup>1</sup> 0.79 (2) 0.79 (24) <sup>2</sup> 0.71 (10) 0.79 (49) <sup>1</sup> 1.8 (4) 1.93 (49)

vibrational mode. For details we refer the reader to Ref [1].

For the di↵erent terms of the operator (3) we have

State *µ* EFT

$$
B \sim A \quad \Rightarrow \quad B \in \left[ A \sqrt{\frac{\omega}{\Lambda}}, A \sqrt{\frac{\Lambda}{\omega}} \right]
$$

LO *<sup>i</sup>* ! *<sup>I</sup>*⇡

• Phonon-annihilating transitions *II*10p*I*(*<sup>I</sup>* + 1) *<sup>f</sup>* ) = **•** Phonon-annihilating tran *<sup>Q</sup>*(*I*⇡) = <sup>h</sup>*I*⇡*||Q*ˆ*||I*⇡<sup>i</sup>



### NLO

- Phonon-conserving transitions *µ*(4<sup>+</sup>)=2*µ*(2<sup>+</sup>) n-conserving transitio<br>E2 moments
- Static E2 moments • Static E2 moment  $\ddot{\phantom{0}}$ *I<br><i>I***I** *II*























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p

*<sup>µ</sup>*(*I*) = <sup>r</sup>4⇡



Degrees of freedom: fermion creation and annihilation operators for a fermion in a  $j^{\pi} = \frac{1}{2}^{-}$  orbital The spin <sup>1</sup>*/*<sup>2</sup> fermion is described in terms of Degrees of freedom: fermion creation and estimate. It is important to realize that the  $\dot{s}^{\pi} = 1$  $J =$ annihilation operators for a fermion in a<br>We note that the three-body term is a set of the three-body term is a set of the three- $\alpha$ counting of the EFT is in powers of the small  $\alpha$ parameter !*/*⇤. For details, we refer the reader

rections introduce anharmonicities. The power

comparison to the three-body term involving

ond terms in the interaction Hamiltonian (15),

respectively.

neighbor, given by the matrix element of the matrix element of the matrix element of the matrix element of the

$$
\left\{ a_{\mu },a_{\nu }^{\dagger }\right\} =\delta _{\mu \nu }
$$

PROPOSAL: The matrix element of an PROPOSAL: The matrix element of an<br>operator with an *n*-fermion factor scales as  $\frac{1}{2}$  with an *H* Termon factor searcs as operator with an  $n$ -fermion factor sca The spin <sup>1</sup>*/*<sup>2</sup> fermion is described in terms of *.* (3) operato fermion creation and annihilation operators *a†* The matrix element of an freedom. For an operator *O*ˆ*<sup>n</sup>* consisting of 2*n*  $\frac{1}{2}$  filation and creation operators, we can consider the creation of  $\frac{1}{2}$ 

$$
\langle \hat{O}_n\rangle \sim \langle \hat{O}_{n-1}\rangle \frac{\omega}{\Lambda}
$$

NLO Hamiltonian *<sup>H</sup>*LO ⌘ !1*N,* <sup>ˆ</sup> (18) NIO Hamiltonian In most of this paper, ⌫ = <sup>1</sup>*/*2*,* sponding angular momentum operator is a sensible parameter operator is  $\mathcal{L}$ ergy diameterisant between the two distributions of two distributions of the two distributions of two distributions of the two distributions of the two distributions of the two distributions of the two distributions of the

freedom are

$$
H_{\rm NLO} \equiv g_{Jj} \hat{\bf J} \cdot \hat{\bf j} + \omega_2 \hat{N} \hat{n}
$$

with Here, we used the spherical rank-<sup>1</sup>*/*<sup>2</sup> tensor ˜*a* with components of the c with  $\frac{1}{2}$  momentum operator is a set of  $\frac{1}{2}$  $\frac{1}{2}$  $\mathsf{with} \quad \mathsf{with} \quad \mathsf{$ 

*µ<sup>a</sup>* ⇠ *µp.* (55)

Static *M*1 moments for the ground state in

$$
\hat{\mathbf{j}} = \frac{1}{\sqrt{2}} \left( a^\dagger \otimes \tilde{a} \right)^{(1)} \quad \text{and} \ \ \hat{n} \equiv a^\dagger \cdot \tilde{a}
$$

Coolle Dérez, Depenbroak: Dhug Dou munisti<br>angular momenta Coello Pérez, Papenbrock; Phys. Rev. C 94, 054316 (2016) neighbor  $\epsilon$ , and the matrix element of the matrix element  $\epsilon$ Coello Pérez, Papenbrock; Phys. Rev. C **94**, 054316 (2016)



⇤

! *E*NLO ⇠

⇤

*E*LO ⇠



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$$
\text{Observables}\ \ E=\omega\sum_{n}^{\infty}c_{n}\varepsilon^{n}\,,\ \ \varepsilon\equiv\frac{N\omega}{\Lambda}
$$

⇤

*<sup>c</sup>n*"*<sup>n</sup> <sup>c</sup>*<sup>2</sup> <sup>=</sup> *H*ˆNLO

+*M*



#### Order-by-order improvement



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#### Order-by-order improvement



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- LO:
- •One LEC
- •Harmonic behavior

#### NLO:

- •Two additional LECs
- •Particle-core interactions

#### Order-by-order improvement



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$$
\hat{Q}=Q_0\left(d^\dagger+\tilde{d}\right)+Q_1\left(d^\dagger\otimes\tilde{d}\right)^{(2)}
$$





Data: Nuclear Data Sheets



Most general operator of rank one

$$
\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[ \left( d^\dagger + \tilde{d} \right) \otimes \left( \mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}
$$

r<br>Hara

*II*10p*I*(*<sup>I</sup>* + 1)

LO term:

- **Two LECs**
- Phonon-conserving transition
- Static M1 moments

NLO term:

- Two LECs
- Phonon-annihilating transition





 $\mathcal{L}_{\mathcal{D}}$  does been from 68% DOB intervals.

$$
\hat{\mu}=\hspace{-0.5mm}\mu_d\hat{\bf J}+\mu_a\hat{\bf j}+\left[\left(d^\dagger+\tilde{d}\right)\otimes\left(\mu_{d1}\hat{\bf J}+\mu_{a1}\hat{\bf j}\right)\right]^{(1)}
$$

#### LECs fitted to static M1 moments TABLE XII. Reduced transition probabilities for the state of the state of the state of the state of the state o<br>Table XII. Reduced to the state of the state ECs fitted to static M1 moments **in Weiss**



Data: Nuclear Data Sheets



$$
\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[ \left( d^\dagger + \tilde{d} \right) \otimes \left( \mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}
$$

TABLE XIV. Reduced transition probabilities for  $\mathcal{R}$ 

*µ*(4<sup>+</sup>)=2*µ*(2<sup>+</sup>)



<sup>2</sup> 0*.*551(184)

<sup>1</sup> 0*.*750(250)

⇡*<sup>p</sup>*

*<sup>n</sup>* orbital, and the *p* operators create and annihilate a proton or proton hole in a *j*



Low-lying positive-parity odd-odd states are constructed as Low-lying positive-parity odd-odd states are  $\sqrt{80}$  $E_{\text{L}}$  constructed as  $E_{\text{L}}$ respectively. The spin and parity of the orbital in which the odd fermion lies in a particular odd-mass nucleus are Edw thing positive parity odd bad state.<br>Constructed as  $f_{\text{SUSL}}$  assumed by the low-enters that at LO only the description of the low-energy lying of the low-energy lying orbital enters the low-energy lying of the low-energy lying orbital enters the low-energy lying of the

*<sup>µ</sup>,* (1)

$$
|IM;j_p;j_n\rangle = \sum_{\mu\nu} C^{IM}_{j_n\mu j_p\nu} n^\dagger_\mu p^\dagger_\nu |0\rangle
$$

where

inferred from the spin and parity of the later's ground state. While additional orbitals may be accessible to the odd

 $\overline{\phantom{a}}$ 

terms of which  $\mathbb{E}_{\mathcal{F}}$  is written as

$$
|j_n - j_p| \le I \le j_n + j_p
$$





⇡*<sup>p</sup>*



Most general rank-one operator coupling odd-odd and even-even states  $\frac{C_{\beta\ell}}{C_{\beta}} \stackrel{\rm EFT}{\sim} 0.58(^{+42}_{-25})$  and  $\frac{C_{\beta L\ell}}{C_{\beta}}$ Most general rank-one operator coupling  $C_{\beta\ell \text{ EFT}}$   $_{\alpha \text{ FO}$  $(+42)}$  and  $C_{\beta L}$  $C_\beta$ 

*µ*⌫

$$
\hat{O}_{\beta} = C_{\beta} (\tilde{p} \otimes \tilde{n})^{(1)} + \sum_{\ell} C_{\beta \ell} \left[ \left( d^{\dagger} + \tilde{d} \right) \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} + \sum_{\ell} C_{\beta \ell \ell} \left[ \left( d^{\dagger} \otimes d^{\dagger} + \tilde{d} \otimes \tilde{d} \right)^{(L)} \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} \qquad \qquad \text{as } \ell \ge 0
$$

LO term:

vibrational mode. For details we refer the reader to  $\mathbb{R}^n$ . For details we refer the reader to Ref  $\mathbb{R}^n$ .

• Couples states with  $\Delta N = 0$ from 1<sup>+</sup>, 2<sup>+</sup> and 3<sup>+</sup> odd-odd ground states to the 0<sup>+</sup> Couples states with  $\Delta \mathcal{N} = 0$ 

NLO term:

NNLO term:  $T_{\rm t}$ 

• Couples states with  $\Delta \mathcal{N} = 2$ 

#### From the power counting of the odd-mass nuclei adjacent to both the even-even and odd-odd nuclei involved in the decay of interest. It is **H**<br>Manusu panud rom the power counting 2 om tł ne power counting

⇠ *C*

2

a<br>Tan

5 a<br>Tan

2 <sup>1</sup> ! <sup>3</sup> 2  <sup>1</sup> ! <sup>3</sup> 2

upling 
$$
\frac{C_{\beta\ell}}{C_{\beta}} \stackrel{\text{EFT}}{\sim} 0.58(^{+42}_{-25}) \text{ and } \frac{C_{\beta L\ell}}{C_{\beta}} \stackrel{\text{EFT}}{\sim} 0.33(^{+25}_{-14})
$$

⇤ or

2



 $\sqrt{\frac{1}{2}}$ 

Corrections to odd-odd Hamiltonian

This would require the control of the cont

Corrections to the effective GT operator expected to scale as <sup>p</sup>!*/*⇤*|IM*; *<sup>N</sup>* = 0; *<sup>j</sup>p*; *<sup>j</sup>n*i. It is naively expected for the contribution to the matrix element (5) These two kinds of corrections, expected to scale as

 $\omega/\Lambda$  $\omega/\Lambda$  $t_{\rm eff}$  denote the Hamiltonian mixing state with phonon-number di $\alpha$  $\omega / \Lambda$  $\omega/\Lambda$ 

 $\sqrt{\omega/\Lambda}$ 

 $\mathbf{v}$  to the Hamiltonian mixing state with phonon-number di $\mathbf{v}$ 

Uncertainty estimate is the one we associate to the one we associate to the matrix element ( $\alpha$ ). Due to the identification in the id *ID*<br>*Incertainty estimate* officertainty commate  $\alpha$  estimate the EFT presented here is a presented here is a presented here is a promis-

$$
\Delta \langle 0 || \hat{O}_\beta || I; j_p; j_n \rangle \stackrel{\rm EFT}{\sim} \langle 0 || \hat{O}_\beta || I; j_p; j_n \rangle \frac{\omega}{\Lambda}
$$

or Taylor expansion of the logarithm of Eq. (8) with <sup>h</sup>*f||O*ˆ*||i*i⇡h0*||O*ˆ*||I*; *<sup>j</sup>p*; *<sup>j</sup>n*i(1 *<sup>±</sup>* !*/*⇤), that is,  $\alpha$ r  $\mathcal{L}_{\mathcal{A}}$ , this natural term also be associated to the PPQ matrix element. Thus, the EFT have  $\mathcal{L}_{\mathcal{A}}$ 

$$
\Delta \log (ft)_{if} \stackrel{\rm EFT}{\sim} \frac{2}{\ln 10} \frac{\omega}{\Lambda}
$$

mental data on 000 decay yet, the fitting of the fitting of the LEC END of the Hamiltonian order corrections to **needs** to a be described to be described to a strongly correct to the data strongly correct to a strongly correct of the data strongly correct to a strongly correct to a strongly correct to a strongly correct to a strongl  $\equiv$   $\sqrt[3]{\mathbb{Z}}$  dating the E LO approximation for the low-lying odd-odd states.

 $\sqrt{\omega/\Lambda}$ 

#### LEC from charge-exchange reaction experiments





Popescu et al., Phys. Rev. C **79**, 064312 (2009) Thies et al., Phsy. Rev. C **86**, 014304 (2012) Frekers et al., Phys. Rev. C **94**, 014614 (2016)

Thies et al., Phys. Rev. C **86**, 044309 (2012) Akimune et al., Phys. Lett. B **394**, 23 (1997) Puppe et al., Phys. Rev. C **86**, 044603 (2012)

















is almost completely exhausted by the isobaric analogue

*t*<sup>2</sup>⌫ *if*







*D*<sup>22</sup>

ASSUMPTION: Energies and matrix elements of 1<sup>+</sup> states scale as This yields the following matrix element  $\overline{r}$  states scale as <sup>1</sup> ) + *n*! *,* (29)

tainty with the EFT assuming that the  $\Gamma$  assuming that the low-lying  $\Gamma$ 

$$
E(1_{n+1}^+) \stackrel{\text{EFT}}{\sim} E(1_1^+) + n\omega \quad \text{and} \quad M_{\text{GT}}\left(0_{\text{gs}}^+ \to 1_{n+1}^+\right) \stackrel{\text{EFT}}{\sim} \left(\frac{\omega}{\Lambda}\right)^{n/2} M_{\text{GT}}\left(0_{\text{gs}}^+ \to 1_1^+\right)
$$

**Omitted contribution** م<br>Omitted contribution

$$
\Delta M_{\rm GT}^{2\nu} (0_{\rm gs}^+ \to 0_{\rm gs}^+)
$$
  
\n
$$
\sum_{n=1}^{\rm EFT} \sum_{n=1}^{\infty} \left(\frac{\omega}{\Lambda}\right)^n \frac{M_{\rm GT}(1_1^+ \to 0_{\rm gs}^+) M_{\rm GT}(0_{\rm gs}^+ \to 1_1^+)}{(D_{11} + n\omega)/m_e c^2}
$$
  
\n
$$
= \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11} + \omega}{\omega}\right) M_{\rm GT}^{2\nu} (0_{\rm gs}^+ \to 0_{\rm gs}^+),
$$

where

$$
\Phi(z, s, a) \equiv \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s}
$$

The percentual uncertainty depends on the energy scales involved transition )<br>זע ⇤*,* <sup>1</sup>*,* es involve a transiti is the Lerch transcendent. The relative uncertainty dependent to the contract to uncertainty of the uncertainty  $\sigma$ ,  $\frac{1}{2}$  ergy scales involved transi

$$
\delta(\text{gs} \to \text{gs}) = \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11} + \omega}{\omega}\right)
$$





Coello Pérez, Menéndez, Schwenk; in preparation Data: Barabash, Nucl. Phys. A 935, 52 (2015)



We studied single- $\beta$  and  $2\nu\beta\beta$  decays within a collective EFT

- Matrix elements for single- $\beta$  decays to the ground and excited states of spherical nuclei can be consistently described within an EFT that describes the later systems as even-even collective cores coupled to an additional neutron and/or proton
- Matrix elements for  $2\nu\beta\beta$  decays calculated within the SSD approximation consistently describe observed decays when the theoretical uncertainty estimate is taken into account
- NLO corrections to the Hamiltonian for the odd-odd system and the effective GT operator are expected to decrease the uncertainty by a factor of  $1/3$ . These corrections are feasible

**FUTURE GOAL** 

Calculate matrix elements for  $0\nu\beta\beta$  decays with uncertainty estimates. This would require us to write the corresponding operators in terms of the effective degrees of freedom. Since there is no available experimental data the LECs to other nuclear structure calculations, or highly correlated data







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# Thanks