Effective operators from wave function factorization

Scott Bogner
NSCL/FRIB Laboratory
& Department of Physics and Astronomy
Michigan State University

Some motivating questions

Can we understand the **general form** of effective operators independent of detailed implementation (SRG, OLS, Vlowk, UCOM,…)? Does it buy us anything?

What state/system independent aspects of NME in A-body systems that can be informed/extracted by few-body calculations?

Is there a way to identify/understand correlations between different observables?

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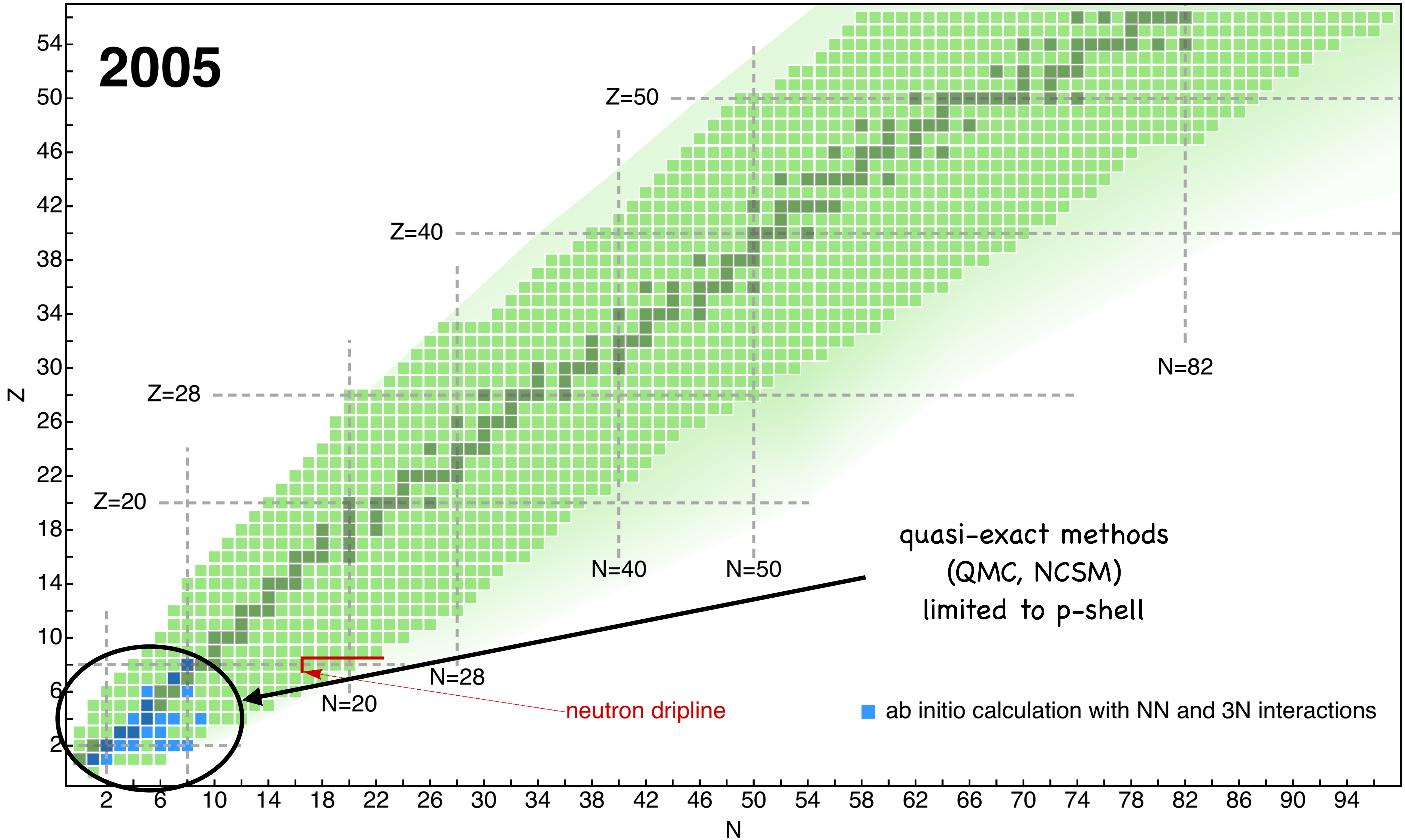
Disclaimer:

Anderson, SKB et al., PRC **82 (2010)**
SKB and Roscher, PRC **86 (2012)**

But see recent generalizations of Barnea, Bazak, Weiss, et al.

PRL **114** (2015) arXiv:1612.00923
PRC **92** (2015) arXiv:1705.02592

Progress in *Ab Initio* Calculations



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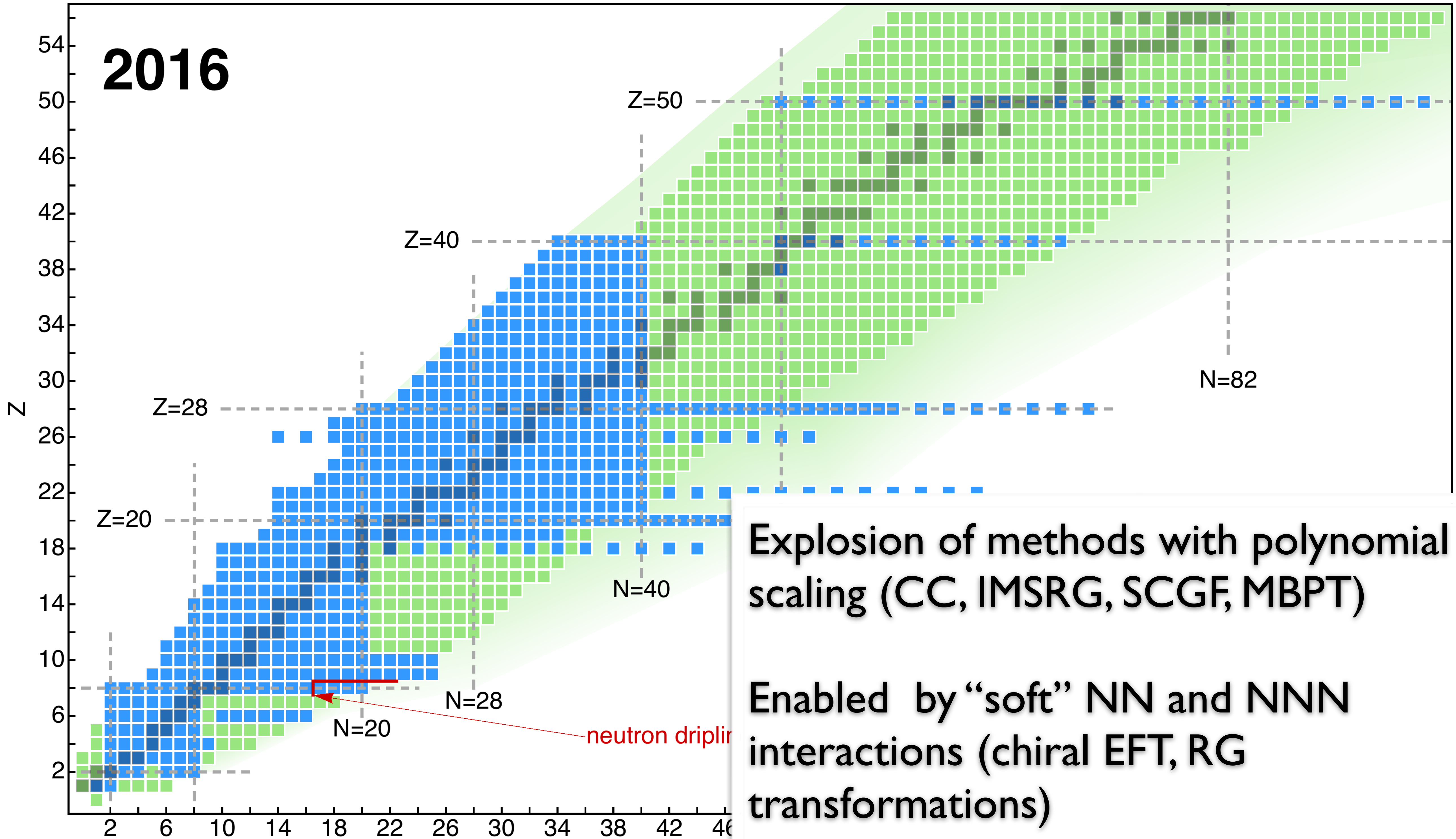
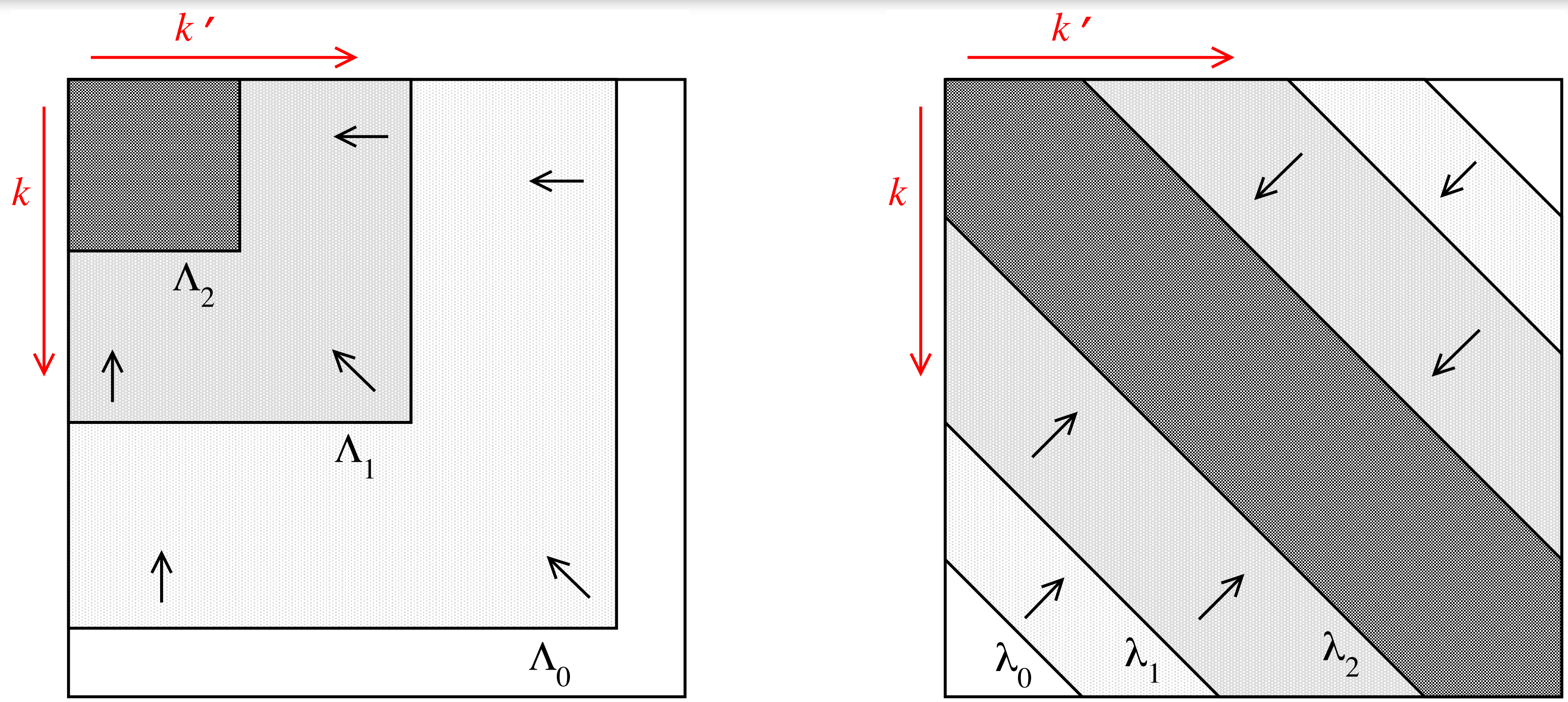


Explosion of methods with polynomial scaling (CC, IMSRG, SCGF, MBPT)

Enabled by "soft" NN and NNN interactions (chiral EFT, RG transformations)

Renormalization Group Methods



Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65** (2010)

Folklore => **@ low Λ** simple Ψ(Λ) ==> complicated O(Λ) ?

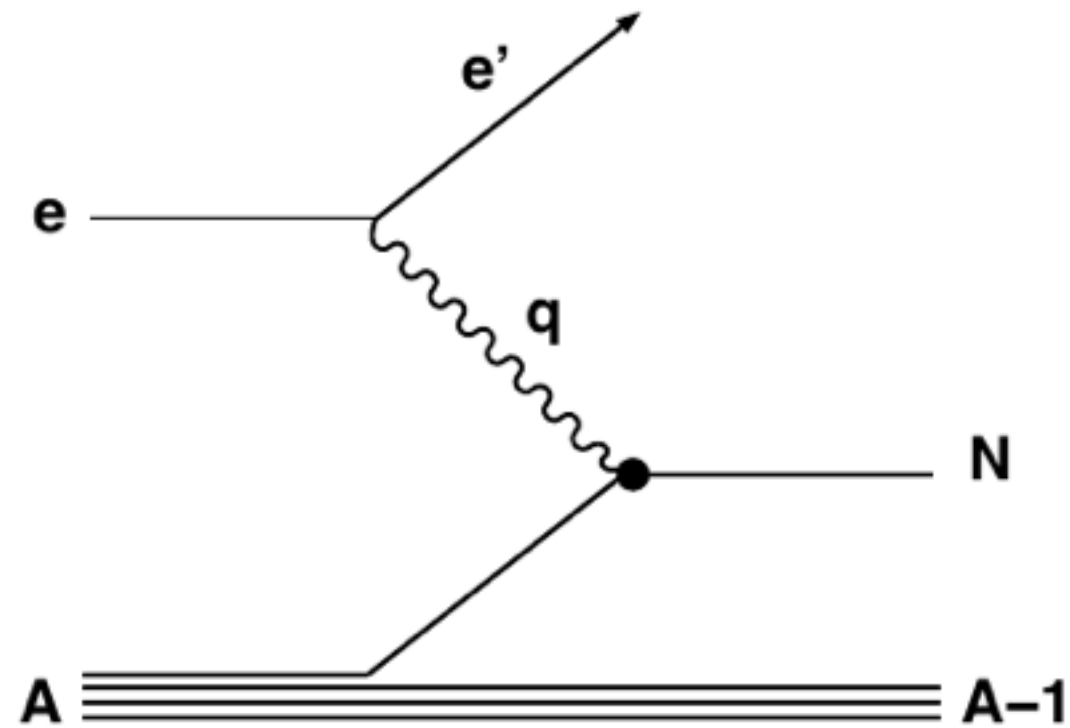
What about large q >> Λ operators?

How do interpretations change with Λ?

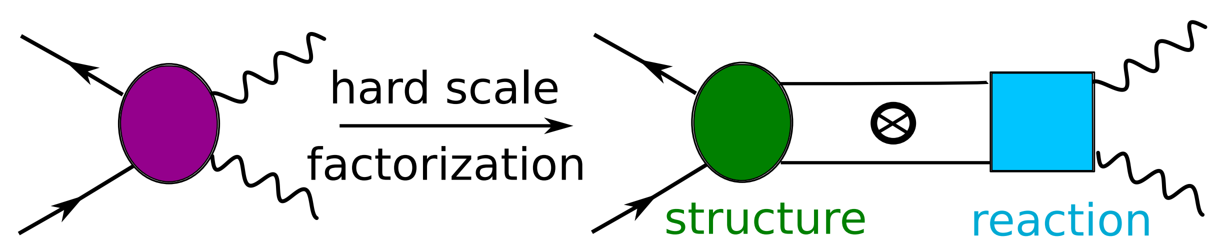
Basic Problem

- Goal: Extract nuclear properties from experiments and predict them from theory
- $$\frac{d\sigma}{d\Omega} \propto \left|\langle\psi_f|\widehat{O}(q)|\psi_i\rangle\right|^2$$
- $$\underbrace{\langle\psi_f|}_{\text{structure}}\;\overbrace{\widehat{O}(q)}^{\text{reaction}}\;\underbrace{|\psi_i\rangle}_{\text{structure}}$$

e.g., nucleon knockout reaction

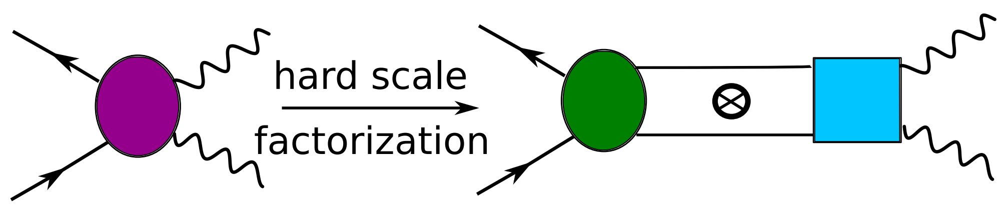


- Factorization to isolate components and extract process-independent properties

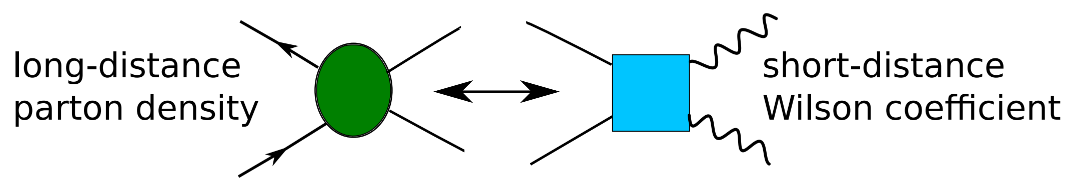


Analogy with DIS in QCD

High-E QCD



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation not unique, depends on the scale μ_f
- Form factor F_2 independent of μ_f but pieces not
- $f_a(x, \mu_f)$ runs with $\mu_f^2 = Q^2$, but is process independent

Low-E Nuclear

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

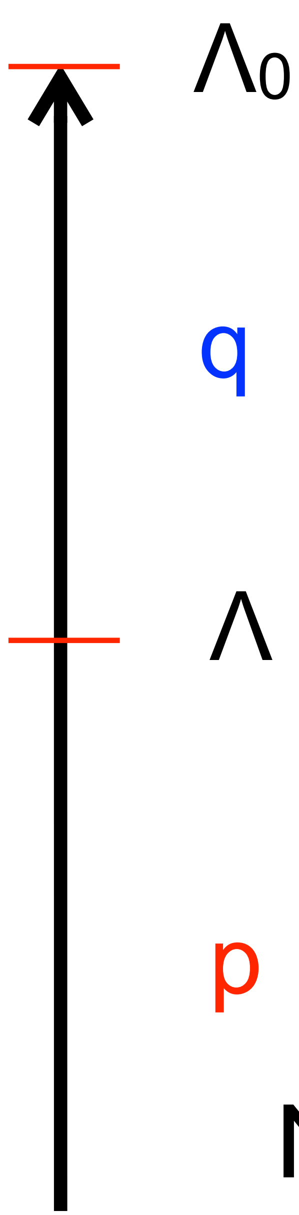
$$\sigma^{if} = \sum_{|J_i - J_f| \leq j \leq J_i + J_f} S_j^{if} \, \sigma_{sp}$$

Open Questions

- When does factorization hold?
- What is the scale/scheme dependence of extracted props?
- Can we extract at one scale and evolve to another?
- Scale/scheme dependence of interpretations?
- Structure of evolved operators?

Ground rules

- Want to understand form of effective operators without getting bogged down in a particular scheme (OLS, SRG, etc.)



I only assume for **low-energy** states

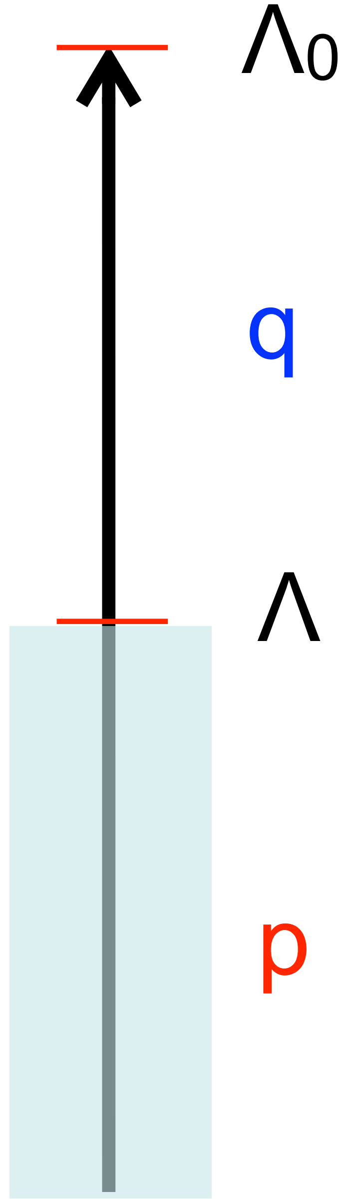
I) $P|\psi_n^{\Lambda_0}\rangle \approx Z(\Lambda)|\psi_n^{\Lambda}\rangle$

II) $Q|\psi_n^{\Lambda}\rangle \approx 0$

Met by all softening transformations I know of…

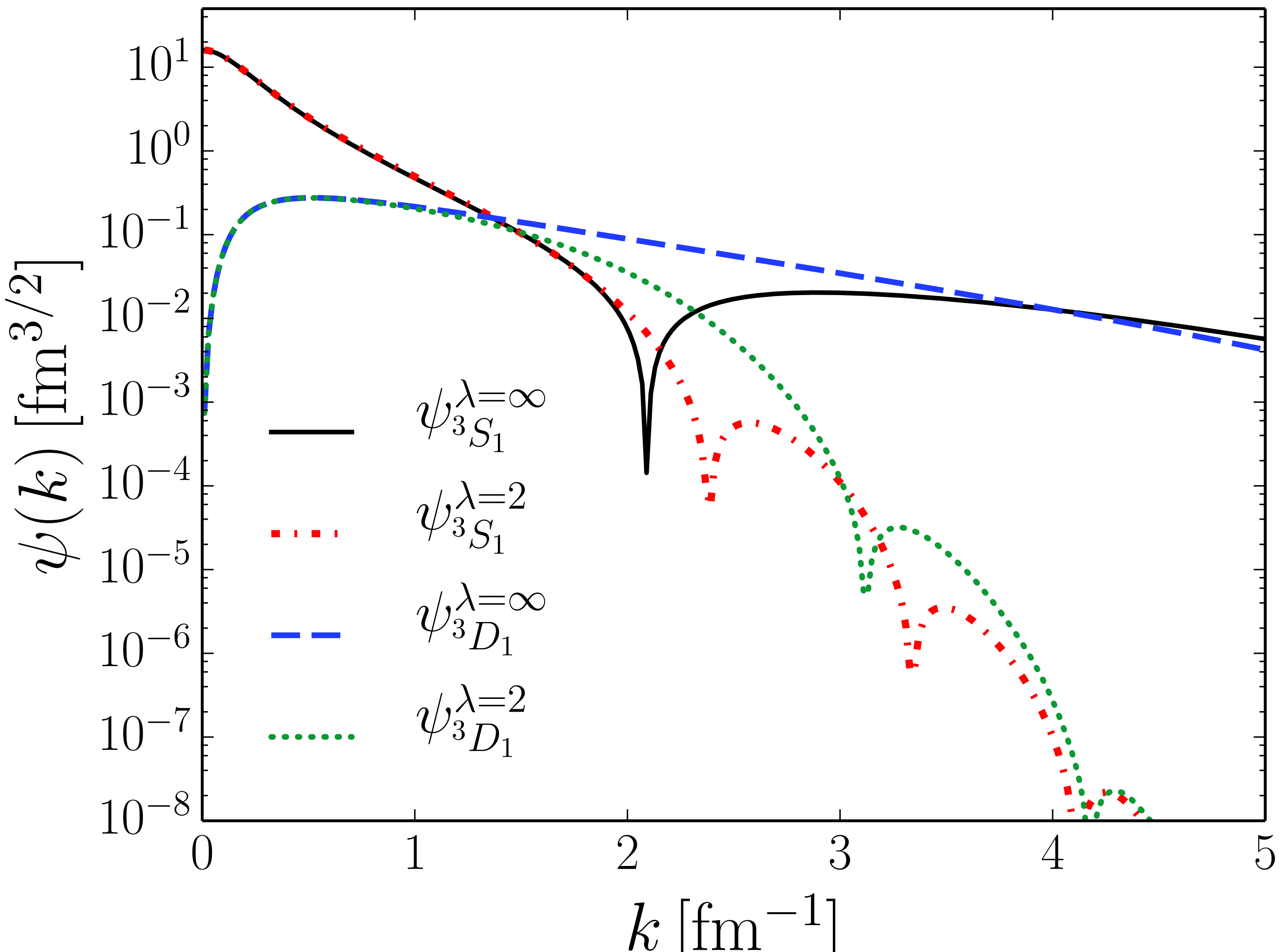
Wave function factorization

Consider low-k components of low-E wf's for A=2.



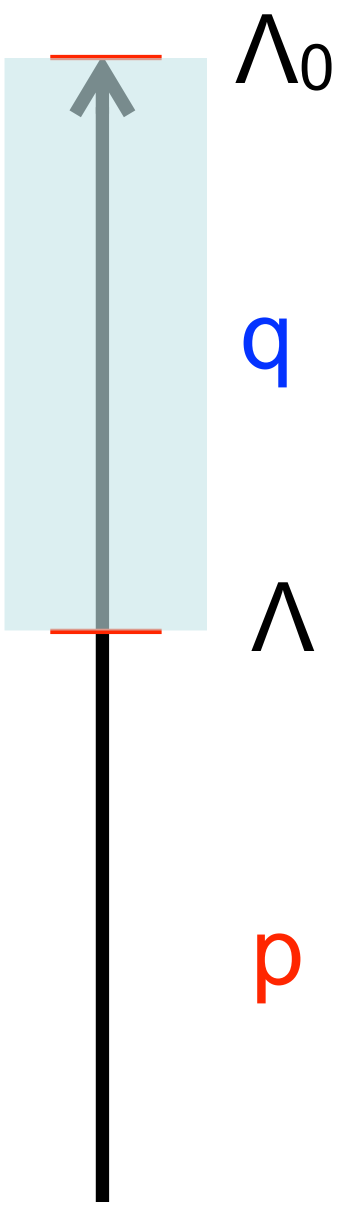
RG doesn't change long distance/IR structure

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p})$$



Wave function factorization

Consider high-k components of low-E wf's for A=2.



Scale separation ($E_\alpha << \Lambda^2 << q^2$)

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda)\int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \eta(\mathbf{q};\Lambda)\int_0^\Lambda d^3p\, \mathbf{p}^2 Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) \cdots$$

Anderson et al., PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)

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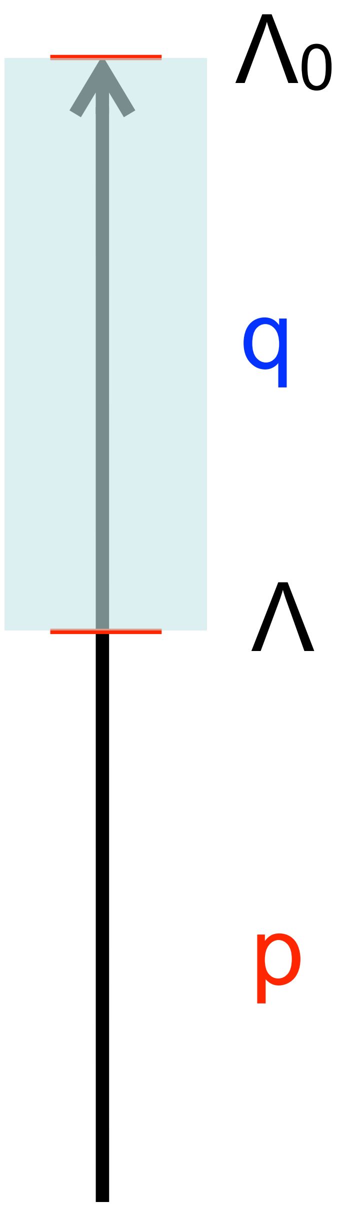

Scale separation ($E_\alpha << \Lambda^2 << q^2$)

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3p\, Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p}) + \eta(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3p\, \mathbf{p}^2 Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p}) \cdots$$

Operator Product Expansion
of wave function a-la Lepage

$$\gamma(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \langle \mathbf{q} | \frac{1}{QH^{\Lambda_0}Q} | \mathbf{q}' \rangle V^{\Lambda_0}(\mathbf{q}', 0)$$

$$\beta(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \langle \mathbf{q} | \frac{1}{QH^{\Lambda_0}Q} | \mathbf{q}' \rangle \frac{\partial^2}{\partial p^2} V^{\Lambda_0}(\mathbf{q}', \mathbf{p}) \Big|_{\mathbf{p}=0}$$

State-independent
Wilson Coefficients

Anderson et al., PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)

Wave function factorization

LO: $\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$ → state-independent ratio for well-separated scales

$$\frac{\psi_\alpha^{\Lambda_0}(\mathbf{q})}{\psi_\alpha^\Lambda(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$

$$|E_\alpha| \lesssim \Lambda^2 \quad |\mathbf{q}| \gtrsim \Lambda$$

Wave function factorization

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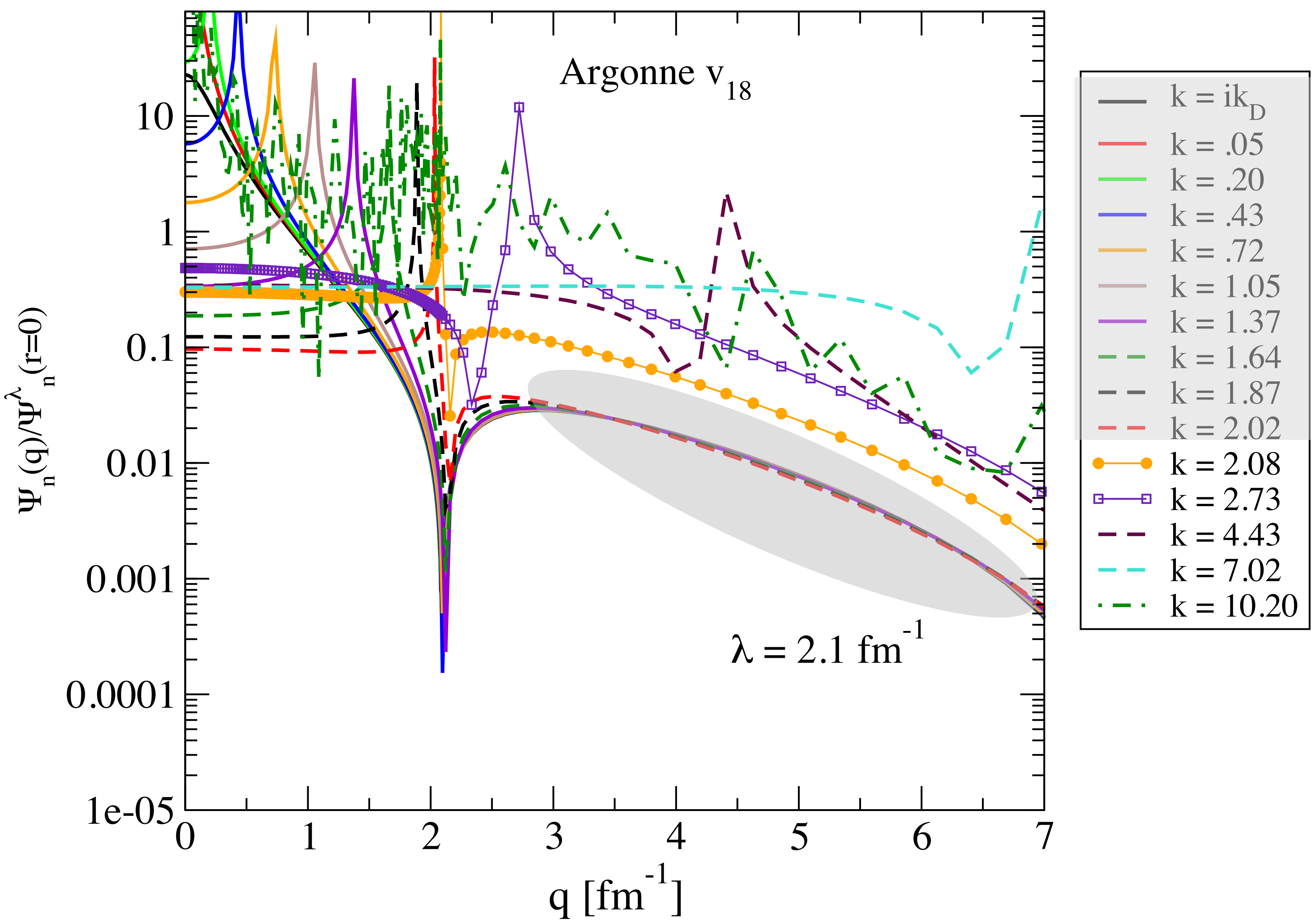

$$\frac{\psi_\alpha^{\Lambda_0}(\mathbf{q})}{\psi_\alpha^\Lambda(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$

$$|E_\alpha| \lesssim \Lambda^2 \quad |\mathbf{q}| \gtrsim \Lambda$$

Effective operators from w.f. factorization

$$
\begin{aligned}
\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \;=\;& \int_0^{\Lambda} dp \int_0^{\Lambda} dp' \, \psi_\alpha^{\Lambda_0 *}(p) O(p,p') \psi_\alpha^{\Lambda_0}(p') + \int_0^{\Lambda} dp \int_{\Lambda}^{\Lambda_0} dq \, \psi_\alpha^{\Lambda_0 *}(p) O(p,q) \psi_\alpha^{\Lambda_0}(q) \\
&+ \int_{\Lambda}^{\Lambda_0} dq \int_0^{\Lambda} dp \, \psi_\alpha^{\Lambda_0 *}(q) O(q,p) \psi_\alpha^{\Lambda_0}(p) + \int_{\Lambda}^{\Lambda_0} dq \int_{\Lambda}^{\Lambda_0} dq' \, \psi_\alpha^{\Lambda_0 *}(q) O(q,q') \psi_\alpha^{\Lambda_0}(q')
\end{aligned}
$$

Effective operators from w.f. factorization

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle = \int_0^\Lambda dp \int_0^\Lambda dp' \, \psi_\alpha^{\Lambda_0 *}(p) O(p,p') \psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp \int_\Lambda^{\Lambda_0} dq \, \psi_\alpha^{\Lambda_0 *}(p) O(p,q) \psi_\alpha^{\Lambda_0}(q)$$

$$+ \int_\Lambda^{\Lambda_0} dq \int_0^\Lambda dp \, \psi_\alpha^{\Lambda_0 *}(q) O(q,p) \psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq \int_\Lambda^{\Lambda_0} dq' \, \psi_\alpha^{\Lambda_0 *}(q) O(q,q') \psi_\alpha^{\Lambda_0}(q')$$

Now use:

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p \, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \cdots$$ OPE for w.f.'s

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$ IR structure unaltered

$$O(q,p) \approx O(q,0) + \cdots$$ Scale separation

Effective operators from w.f. factorization

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^{\Lambda} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda} \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle + \cdots$$

state-independent
high-q physics
depends on operator

state dependent
soft m.e. (low-k)
same for all high-q operators

E.g.,

$$g^{(0)}(\Lambda) \equiv 2Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q}\, O(0,q)\gamma(q;\Lambda) + Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q} \int_\Lambda^{\Lambda_0} d\tilde{q}' \, \gamma^*(q;\Lambda) O(q,q') \gamma(q';\Lambda)$$

Generically:

$$\widehat{O}_\Lambda = Z_\Lambda^2 \, \widehat{O}_{\Lambda_0} + g^{(0)}(\Lambda)\, \delta(\mathbf{r}) + g^{(2)}(\Lambda)\, \nabla^2 \delta(\mathbf{r}) + \cdots$$

Scaling of high momentum operators

How does an operator that probes high-momentum w.f. components look in a low-momentum effective theory?

$$\langle\psi_\alpha^{\Lambda_0}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda_0}\rangle \approx Z_\Lambda^2\langle\psi_\alpha^{\Lambda}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda}\rangle + g^{(0)}(\Lambda)\,\langle\psi_\alpha^{\Lambda}|\delta^{(3)}(\mathbf{r})|\psi_\alpha^{\Lambda}\rangle + \cdots$$

= 0 since $P_\Lambda O_{\Lambda_0} P_\Lambda = 0$

E.g., momentum distribution for q >> Λ

$$\langle\psi_\alpha^{\Lambda_0}|a_{\mathbf{q}}^\dagger a_{\mathbf{q}}|\psi_\alpha^{\Lambda_0}\rangle \approx \gamma^2(\mathbf{q};\Lambda)\,Z_\Lambda^2|\langle\psi_\alpha^{\Lambda}|\delta(\mathbf{r})|\psi_\alpha^{\Lambda}\rangle|^2$$

low-E states have the same large-q tails

Generalize to arbitrary **A-body** states?

Scaling of high momentum tails

SKB and Roscher, PRC **86** (2012)

Creation/annihilation operators under RG evolution:

$$a^{(\Lambda)\dagger}_{\mathbf{q}} = a^{\dagger}_{\mathbf{q}} + \sum_{\mathbf{k_1},\mathbf{k_2}} C^{\Lambda}_{\mathbf{q}}(\mathbf{k_1},\mathbf{k_2}) a^{\dagger}_{\mathbf{k_1}} a^{\dagger}_{\mathbf{k_2}} a_{\mathbf{k_1}+\mathbf{k_2}-\mathbf{q}} + \cdots \equiv a^{\dagger}_{\mathbf{q}} + \delta a^{(\Lambda)\dagger}_{\mathbf{q}}$$

fixed from RGE in A=2 system

Scaling of high momentum tails

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Scale separation ($\Lambda << q < \Lambda_0$):

$$\begin{aligned} \langle \psi_{\alpha,A}^{\Lambda_0} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda_0} \rangle &= \langle \psi_{\alpha,A}^{\Lambda} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \delta a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} + \delta a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda} \rangle \\ &\approx \langle \psi_{\alpha,A}^{\Lambda} | \delta a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda} \rangle \end{aligned}$$

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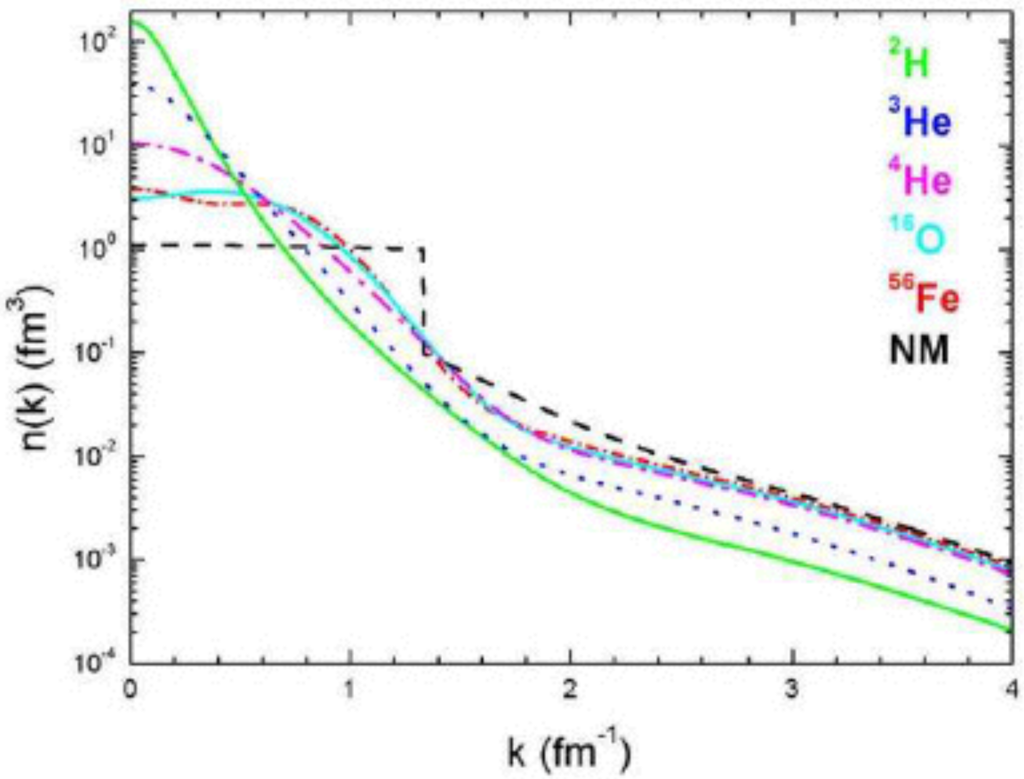
&\approx \langle \psi_{\alpha,A}^{\Lambda} | \delta a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda} \rangle \\
&\approx \gamma^2(\mathbf{q};\Lambda) \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}^{\Lambda} Z_{\Lambda}^2 \langle \psi_{\alpha,A}^{\Lambda} | a_{\frac{\mathbf{K}}{2}+\mathbf{k}}^{\dagger} a_{\frac{\mathbf{K}}{2}-\mathbf{k}}^{\dagger} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi_{\alpha,A}^{\Lambda} \rangle
\end{aligned}$$

- hard (high q) physics
- Universal (state-indep)
- fixed from A=2

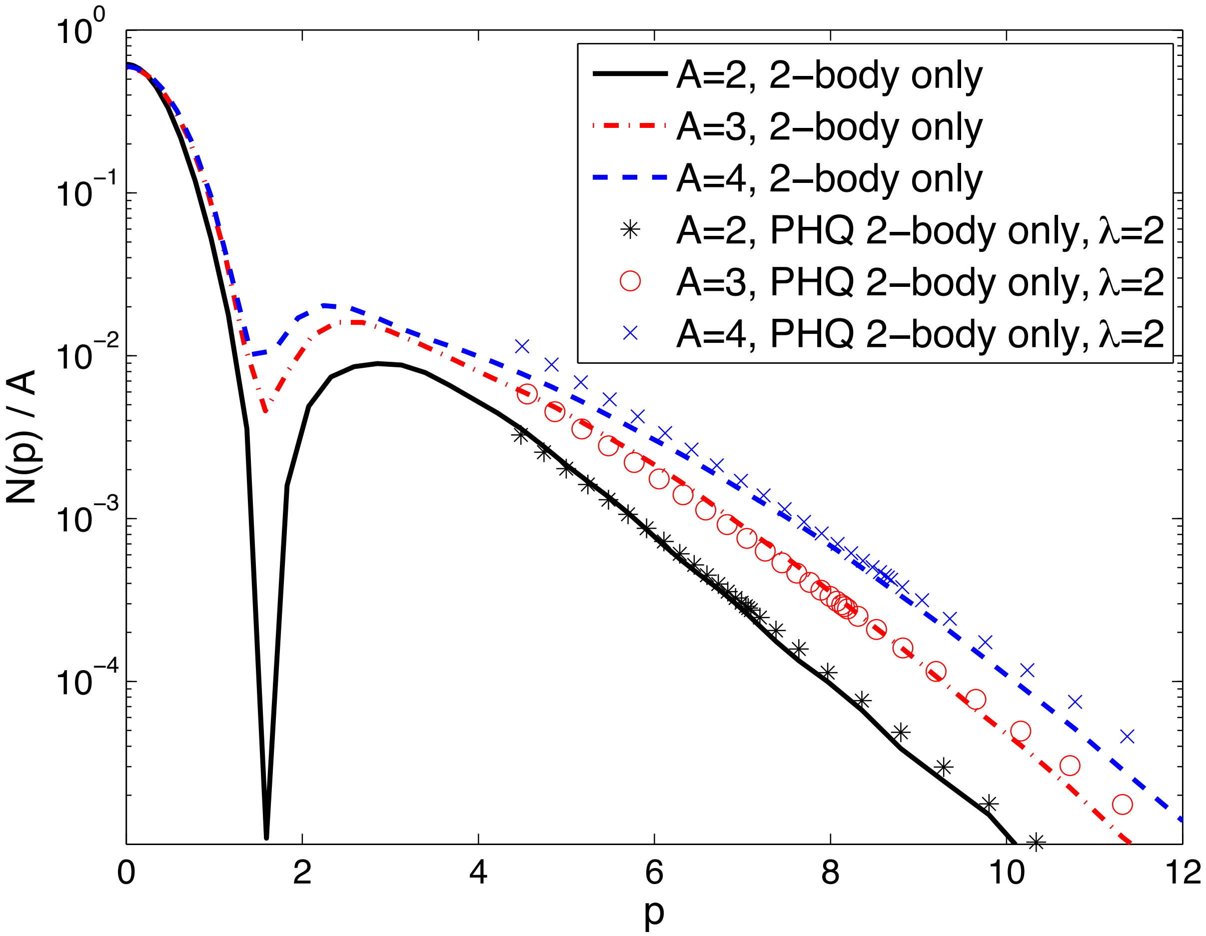
X

- soft (low-k) m.e.
- same for all high-q probes
- A-dependent scale factor

Scaling of high momentum tails



[From C. Ciofi degli Atti and S. Simula]



natural explanation why high-q tails scale

$$C(A,2) \equiv \frac{n_A(\mathbf{q})}{n_D(\mathbf{q})} \sim \frac{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,A} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle}{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,D} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,D} \rangle}$$

Scaling of high momentum tails

E.g., static structure functions $\widehat{S}(\mathbf{q}) = \widehat{\rho}^{\dagger}(\mathbf{q})\widehat{\rho}(\mathbf{q})$

$$\langle\psi^{\Lambda_0}_{\alpha,A}|\widehat{S}(\mathbf{q})|\psi^{\Lambda_0}_{\alpha,A}\rangle \approx \left\{2\gamma(\mathbf{q};\Lambda) + \sum_{\mathbf{P}}\gamma(\mathbf{P}+\mathbf{q};\Lambda)\gamma(\mathbf{P};\Lambda)\right\} \times \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\Lambda} Z^2_{\Lambda}\,\langle\psi^{\Lambda}_{\alpha,A}|a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle$$

Universal (state-indep) q-dependence => connects few-body and A-body

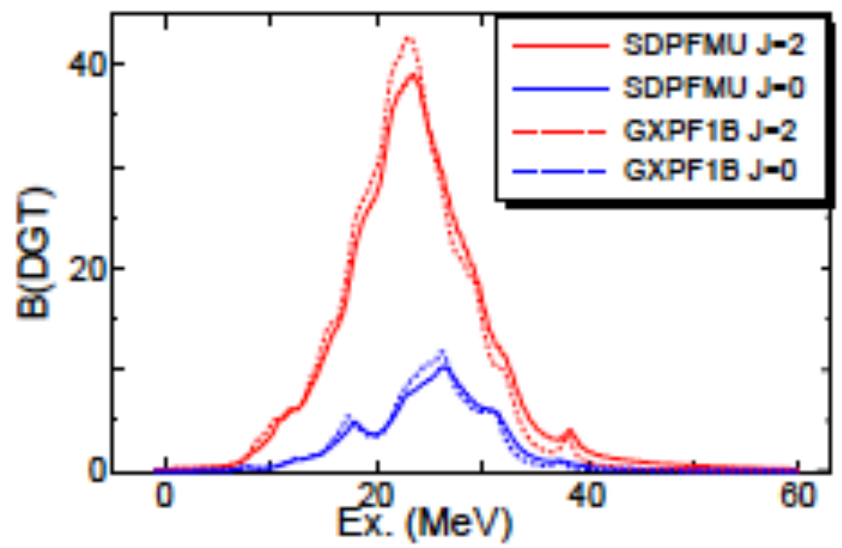
State dependence encoded in low-k m.e. =>

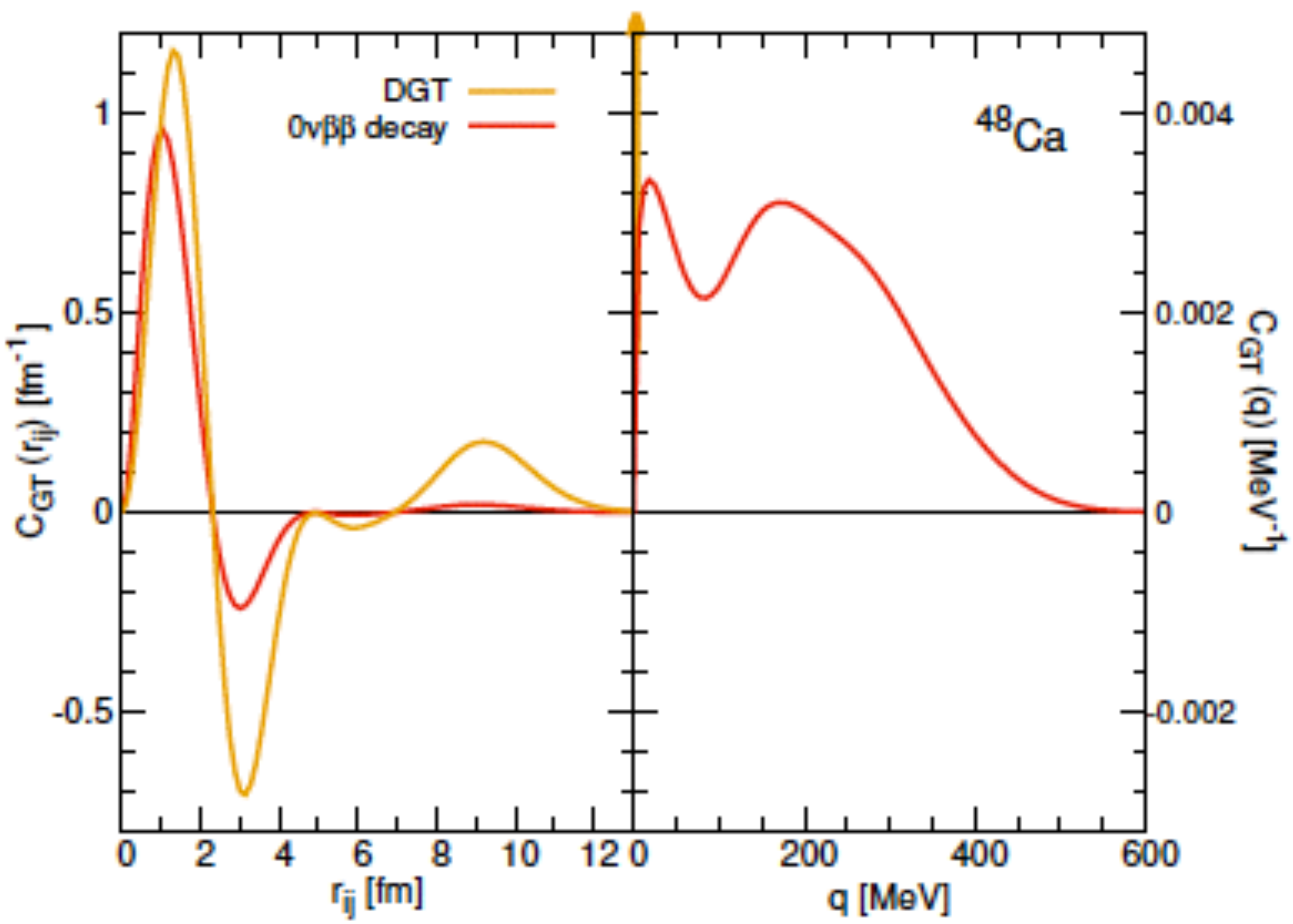
linear correlations between observables with same leading OPE? (Javier's talk)

Double GT to ground state and $0\nu\beta\beta$ decay

Double GT transition to ground state of final nucleus closest similarity to $0\nu\beta\beta$ decay

Both matrix elements tiny sum rule fraction



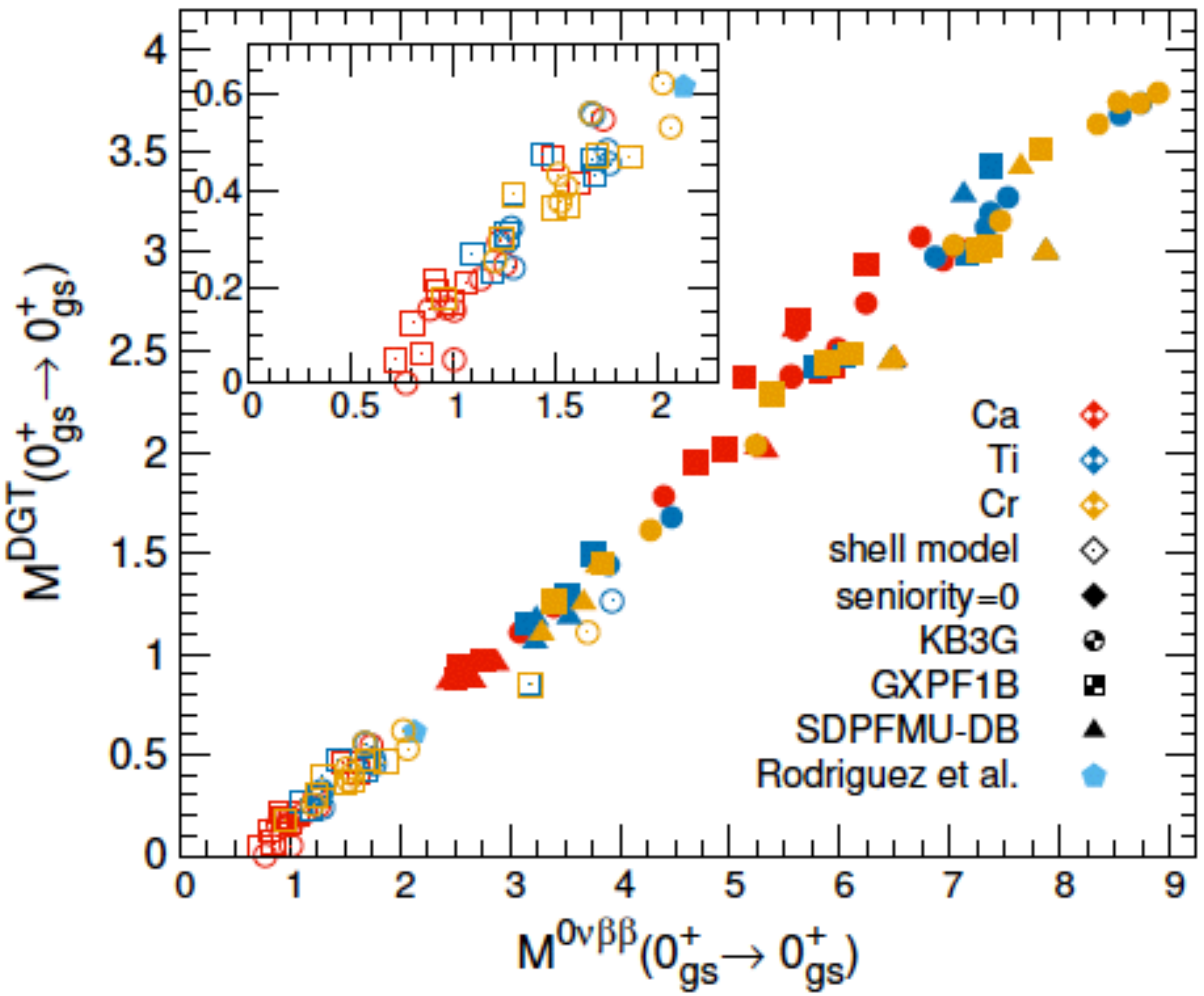


$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part also dominant in double GT matrix element cancellation of long range parts

Shimizu, JM, Yako, in preparation

Double GT to ground state and $0\nu\beta\beta$ decay



Linear correlation
Double GT, $0\nu\beta\beta$ decay matrix elements

$$M^{\rm DGT} = \sqrt{B(DGT_-;0;0^+_{gs} \rightarrow 0^+_{gs})}$$

Correlation similar across Ca, Ti, Cr nuclei

with and without seniority correlations

$0.5 \lesssim M \lesssim 9$

for different shell model interactions

in the *pf* shell
sdpf configuration space

Scaling of high momentum tails

Factorization of generic high-q operators (schematic)

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle \quad = \quad \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle$$

$$\hat{O}_{\mathbf{q}}^{\Lambda_0} = a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \, , \sum_{\mathbf{p},\mathbf{p}'} a_{\mathbf{p}+\mathbf{q}}^{\dagger} a_{\mathbf{p}} a_{\mathbf{p}'}^{\dagger} a_{\mathbf{p}'+\mathbf{q}} \cdots$$

Expand evolved operator as polynomial in creation/annihilation operators at $\boldsymbol{\Lambda}_0$

$$\hat{O}_{\mathbf{q}}^{\Lambda} = \sum_{\alpha} g_{\mathbf{q}}^{\alpha}(\Lambda)\, \hat{A}_{\alpha}^{\Lambda_0}$$

c-number running couplings

string of creation/annihilation operators

$\alpha = \mathbf{p}_1 \dots \mathbf{p}_\beta$

Scaling of high momentum tails

Factorization of generic high-q operators (schematic)

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle = \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle \qquad \Lambda \ll \mathbf{q} \ll \Lambda_0$$

$$= \sum_\alpha g_{\mathbf{q}}^{\alpha}(\Lambda) \, \langle \Psi_n^{\Lambda} | \hat{A}_{\alpha}^{\Lambda_0} | \Psi_n^{\Lambda} \rangle$$

1) Decoupling => only modes $p < \Lambda$ in α contribute

2) Taylor expand c-# coefficients about $p = 0$

=> q-dependence **factorizes**
=> state-dependence from soft matrix elements A_α

Scaling if leading term dominates

Conclusions

- Simple decoupling + scale separation arguments generically give the form of effective operators softened by OLS, SRG, Vlowk,…
- Can we use scaling of A-body tails w.r.t. few-body systems to constrain the form of short-distance contributions to NME?
- Can we use factorization/OPE-like arguments to identify quantities that correlate w/0vBB NME?
- How do interpretations change as Λ varied by RG transformations (See Sushong More's talk)