Status and prospects of neutrino oscillations

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The award of the 2015 Nobel Prize to T. Kajita and A. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass" was a result of more than fifty years of efforts of many experimentalists and theoreticians

First idea of neutrino oscillations and mixing was pioneered in 1957-58 by B. Pontecorvo

First idea of flavor neutrino mixing was discussed by Maki, Nakagawa and Sakata in 1962

First model independent evidence in favor of disappearance of atmospheric ν_μ's was obtained in 1998 by the Super-Kamiokande collaboration

First model independent evidence of the disappearance of solar ν_e's was obtained by the SNO collaboration in 2001-2002

First model independent evidence of the disappearance of reactor $\bar{\nu}_e$'s was obtained by the KamLAND collaboration in 2002-2004

The discovery of neutrino oscillations was confirmed by many experiments: accelerator K2K, MINOS, T2K and NOvA, reactor DayaBay, RENO and Double Chooz, atmospheric IceCube

The study of neutrino oscillations is based on the following assumptions

I. Standard Model CC and NC interaction

$$\mathcal{L}_I^{CC}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{CC}(x) W^\alpha(x) + \mathrm{h.c.}$$

$$j_\alpha^{CC}(x) = 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha l_L(x)$$

$$\mathcal{L}_I^{NC}(x) = -\frac{g}{2\cos\theta_W} j_\alpha^{NC}(x) Z^\alpha(x)$$

$$j_\alpha^{NC}(x) = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha \nu_{lL}(x)$$

II. Neutrino mixing

$$\nu_{lL}(x) = \sum_{i=1}^{n} U_{li}\,\nu_{iL}(x), \quad l = e, \mu, \tau$$

$\nu_i(x)$ is the field of neutrino (Dirac or Majorana) with mass $m_i, \quad U^\dagger U = 1$

$n = 3$: oscillations between flavor neutrinos $\nu_l \rightleftarrows \nu_{l'}$

$n > 3$: oscillations between flavor neutrinos $\nu_l \rightleftarrows \nu_{l'}$ and flavor and sterile neutrinos $\nu_l \rightleftarrows \nu_{sL}$

I. A comment on the most plausible neutrino mass term
II. A comment on the phenomenology of neutrino oscillations
III. Some data, future

Neutrino masses, mixing and nature are determined by the neutrino mass term

For charged particle only one (Dirac) mass term is possible

For neutrinos ($Q = 0$) three different mass terms are possible

Dirac, Majorana, ($n = 3$) Dirac and Majorana $n = 3 + n_{\mathrm{sterile}}$

The most plausible neutrino mass term apparently is the Majorana mass term

After the discovery of the Higgs boson at LHC the Standard Model acquired the status of the theory of elementary particles in the electroweak range (up to $\sim$ 300 GeV)

What the SM teaches us?

In the framework of such general principles as local gauge symmetry, unification of the weak and electromagnetic interactions and Brout-Englert-Higgs spontaneous breaking of the electroweak symmetry nature chooses the simplest possibilities

Two-component neutrino fields ν_{lL} is the simplest possibility (2 dof), $SU(2)_L$ is the simplest nonabelian group, SM CC +NC+EM interaction is minimal gauge interaction, one Higgs doublet is the minimal possibility to generate masses of $W^{\pm}$ and Z^0 etc.

The simplest (and most natural) possibility: there are no right-handed neutrino fields ν_{lR} in SM

Means that neutrinos in the SM are massless

Neutrino masses are a signature of a beyond the SM physics

A neutrino mass term is a Lorenz-invariant product of left-handed and right-handed components. Can we build a neutrino mass term if we use only flavor left-handed fields ν_{lL}? The answer to this question was given by Gribov and Pontecorvo in 1969. Possible if we assume that the total lepton number L is not conserved. We must take into account that $(\nu_{lL})^c = C(\bar{\nu}_{lL})^T$ is right-handed component. Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\sum_{l',l}\bar{\nu}_{l'L}\,M_{l'l}(\nu_{lL})^c + \mathrm{h.c.} = -\frac{1}{2}\,\sum_{i=1}^{3} m_i\;\bar{\nu}_i\nu_i$$

Majorana mass term

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}$$

$\nu_i = C(\bar{\nu}_i)^T$ field of Majorana neutrino with mass m_i

The only possibility to build mass term if there are no right-handed fields. The most economical possibility

Neutrino mass therm in the Lagrangian is the only source of the violation of L. Neutrino masses and violation of L are tightly connected

But in this phenomenological approach neutrino masses are parameters. We have no any explanation of the smallness of neutrino masses

Modern effective Lagrangian approach (Weinberg) allows

- to obtain the Majorana mass term for neutrinos,
- to find an explanation of the smallness of neutrino masses,
- to predict existence of heavy Majorana fermions.

The method of the effective Lagrangian is a general method which allows to describe effects of a beyond the Standard Model physics

The effective Lagrangian is a nonrenormalizable dimension five or more operator invariant under the $SU(2)_L \times U(1)_Y$ transformations and built from the Standard Model fields

The only effective Lagrangian which generate neutrino mass term (Weinberg)

$$\mathcal{L}_I^{\mathrm{eff}} = -\frac{1}{\Lambda}\ \sum_{l',l} (\bar{\psi}^{lep}_{l'L}\tilde{\phi})\ X'_{l'l}\ C\ (\bar{\psi}^{lep}_{lL}\tilde{\phi})^T + \mathrm{h.c.}$$

$\mathcal{L}_I^{\mathrm{eff}}$ does not conserve the total lepton number L

The constant Λ characterizes a scale of a beyond the SM physics

After the spontaneous symmetry breaking we come to the Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\frac{v^2}{\Lambda} \sum_{l',l} \bar{\nu}_{l'L}\, X_{l'l} (\nu_{lL})^c + \mathrm{h.c.} = -\frac{1}{2}\ \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i$$

in which $m_i = \frac{v^2}{\Lambda}\ x_i = \frac{v}{\Lambda}\ (x_i v)$

x_i is the eigenvalue of the matrix X, $(x_i v)$ is a "typical SM mass",

$v \simeq 246$ GeV (vev)

$$\frac{v}{\Lambda} = \frac{\mathrm{scale\ \ of\ \ SM}}{\mathrm{scale\ \ of\ \ a\ \ new\ \ physics}}$$

Smallness of neutrino masses can be ensured if we assume that a scale Λ of a new lepton number violating physics is much larger than the electroweak scale v

$\mathcal{L}_I^{\mathrm{eff}}$ is the only Lagrangian which has dimension five. Neutrinos are the most sensitive probe of a new physics

Can we estimate Λ?

We do not know m_i and Yukawa constants x_i

However, if we assume hierarchy of neutrino masses

$(m_1 \ll m_2 \ll m_3)$ $m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV.

We have $\Lambda \simeq 1.2 \cdot 10^{15}\ x_3\ \mathrm{GeV}$

If $\Lambda \simeq \mathrm{TeV}$ in this case $x_3 \simeq 10^{-12}$ (too small, fine tuning). If $x_3 \simeq 1$ in this case $\Lambda \simeq 10^{15}$ GeV (GUT scale).

$\mathcal{L}_I^{\mathrm{eff}}$ can be a result of an exchange of virtual heavy Majorana leptons between lepton-Higgs pairs

Main implications

1. Neutrinos with definite masses ν_i are Majorana particles. Investigation of $0\nu\beta\beta$-decay is the first priority problem.
2. The number of neutrinos with definite masses must be equal to the number of the flavor neutrinos (three). No transitions of flavor neutrinos into sterile states are allowed.
3. Heavy Majorana leptons with masses much larger than v must exist. Existence of such leptons could explain the barion asymmetry of the Universe

Neutrino oscillations

$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\, \nu_{iL}(x)$, U is a unitary PMNS matrix, is the relation between fields

The notion of the flavor neutrinos ν_l $(l = e, \mu, \tau)$is determined by the CC Lagrangian: ν_μ is a particle which is produced together with μ^+ in the decay $\pi^+ \to \mu^+ + \nu_\mu$ etc. The phenomenology of neutrino oscillations is based on the assumption that the state of ν_l with momentum $\vec{p}$ is given by a coherent superposition of the states of neutrinos with definite masses

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^{*}\ |\nu_i\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i, momentum $\vec{p}$ and energy $E_i \simeq E + \frac{m_i^2}{2E}$

This relation means that we can not resolve production of ultrarelativistic neutrinos with different masses in weak decays and neutrino reactions. It is a consequence of the Heisenberg uncertainty relation

Small neutrino mass-squared differences can be resolved in special experiments with a large distance between neutrino source and neutrino detector. A possibility to resolve neutrino mass-squared differences is based on the time-energy uncertainty relation

$$\frac{|\Delta m^2_{ki}|}{2E}\ L \geq 1, \quad \Delta m^2_{ki} = m^2_i - m^2_k$$

L is the distance between neutrino source and detector

If at $t = 0$ flavor neutrino ν_l is produced at time t

$$|\nu_l\rangle_t = e^{-iHt}\ |\nu_l\rangle = \sum_{l'} |\nu_{l'}\rangle\ (\sum_i U_{l'i}\ e^{-iE_i t}\ U^*_{li})$$

The probability of the $\nu_l \to \nu_{l'}$ transition

$$P(\nu_l \to \nu_{l'}) = |\delta_{l'l} - 2i \sum_{i \neq p} U_{l'i}\ e^{-i\Delta_{pi}} \sin \Delta_{pi}\ U^*_{li}|^2$$

p is an arbitrary fixed index and $\Delta_{pi} = \frac{\Delta m^2_{pi} L}{4E}$

$\nu_l \to \nu_{l'}$ ($\bar{\nu}_l \to \bar{\nu}_{l'}$) transition probability

$$\mathrm{P}(\nu_l \to \nu_{l'})(\mathrm{P}(\bar{\nu}_l \to \bar{\nu}_{l'})) = \delta_{l'l} - 4 \sum_i |U_{li}|^2(\delta_{l'l} - |U_{l'i}|^2) \ \sin^2 \Delta_{pi}$$

$$+8 \sum_{i>k}[\mathrm{Re}\ (U_{l'i}U_{li}^* U_{l'k}^* U_{lk}) \ \cos(\Delta_{pi} - \Delta_{pk})$$

$$\pm \mathrm{Im}\ (U_{l'i}U_{li}^* U_{l'k}^* U_{lk}) \ \sin(\Delta_{pi} - \Delta_{pk})] \ \sin \Delta_{pi} \sin \Delta_{pk}$$

$i \neq p$, $k \neq p$ only one interference term (usually three)

Usually $m_2 > m_1$, $\Delta m^2_{12} = \Delta m^2_S > 0$

Δm^2_S is the solar mass-squared difference. Atmospheric neutrino mass-squared difference Δm^2_A is about 30 times larger. Two possibilities for the third mass m_3 and, correspondingly, for the neutrino mass spectrum

1. Normal ordering (NO) $m_3 > m_2 > m_1$, $\Delta m^2_{23} = \Delta m^2_A$.
2. Inverted ordering (IO) $m_2 > m_1 > m_3$, $|\Delta m^2_{13}| = \Delta m^2_A$.

Definition of Δm^2_A does not depend on the mass ordering (a lot of confusion in literature)

Neutrino oscillation parameters obtained from the global analysis of the oscillation data

Parameter	Normal Ordering	Inverted Ordering
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
δ (in $^\circ$)	(261^{+51}_{-59})	(277^{+40}_{-46})
Δm^2_S	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$
Δm^2_A	$(2.524^{+0.039}_{-0.040})\cdot 10^{-3}\ \mathrm{eV}^2$	$(2.514^{+0.038}_{-0.041})\cdot 10^{-3}\ \mathrm{eV}^2$

Neutrino oscillations parameters are known with accuracies (3 -10)% The major aim of future neutrino oscillation experiments

- to establish the neutrino mass ordering,
- to measure the *CP* phase δ,
- to determine neutrino oscillation parameters with 1% accuracy

Some latest results

T2K: 30 GeV proton beam (J-PARC facility), ND (280 m) and FD (Super-Kamiokande, 295 km), FD is 2.5° off-axes, narrow energy spectrum peaks at 0.6 GeV

First search for CP violation. For the normal mass ordering

$-3.13 \leq \delta \leq -0.39$ at 90% CL

$\delta = 0, \pi$ (CP conservation) is excluded at 90% CL

NOvA NuMI beam at Fermilab, ND (1 km), FD (810 km) FD is 14.6 mrad of-axes, narrow energy spectrum peaks at 2 GeV

For the normal mass ordering $\Delta m_A^2 = (2.67 \pm 0.11) \cdot 10^{-3}\ \mathrm{eV}^2$

$\sin^2\theta_{23} = 0.404^{+0.030}_{-0.022}\ \mathrm{or}\ 0.624^{+0.022}_{-0.030}$

Maximum mixing ($\theta_{23} = \frac{\pi}{2}$) is disfavored at 2.6 σ

33 ν_e candidates due to $\nu_\mu \to \nu_e$ were observed. Using Daya Bay value of $\sin^2\theta_{13}$ NOvA excludes inverted mass ordering at 93% CL (assuming $\theta_{23} < \frac{\pi}{4}$, for all values of δ)

Two remarks

I. In scenarios we discussed there are no sterile neutrinos

Sterile neutrinos have no standard weak interaction and can not be detected directly. Two ways to reveal their existence

- Detect flavor neutrinos and prove that transition (survival) probability depends on additional large mass-squared difference(s)
- Detect neutrinos via NC processes.

First indication in favor of existence of sterile neutrinos were obtained in the short baseline LSND experiment. Appearance of $\bar{\nu}_e$'s in the transition $\bar{\nu}_\mu \to \bar{\nu}_e$ was observed ($L \simeq 30$ m, $20 \leq E \leq 60$ MeV). From analysis of the data

$$(\sin^2 2\theta_{e\mu}, \ \ \Delta m^2_{14})_{\rm best\ fit} = 3 \cdot 10^{-3}, \ 1.2 \ \ {\rm eV}^2$$

In the MiniBooNE experiment ($L \simeq 540$ m, $200 \leq E \leq 1250$ MeV) $\bar{\nu}_e$'s events, which can be explained by $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations, were observed. From data analysis $(\sin^2 2\theta_{e\mu}, \ \Delta m^2_{14})_{\rm best\ fit} = (1 \cdot 10^{-2}, \ 0.5 \ {\rm eV}^2)$ (compatible with the LSND data)

Reactor neutrino anomaly. Data of old reactor short baseline experiments were reanalyzed with a new antineutrino flux. The old average ratio r of observed and predicted reactor antineutrino events was equal to $r = 0.976 \pm 0.024$. The new ratio $r = 0.938 \pm 0.023$ is not equal to one at 98.6 % C.L

Gallium anomaly. In the short baseline calibration experiments performed by GALLEX and SAGE collaborations the observed numbers of $\nu_e +^{71}\mathrm{Ga} \to e^- +^{71}\mathrm{Ge}$ events was smaller than the expected numbers: 0.86 ± 0.05

From global analysis of data of all short baseline experiments a disagreement (tension) between $\bar\nu_\mu \to \bar\nu_e$ and $\nu_e \to \nu_e$ indications and non observation of the disappearance $\nu_\mu \to \nu_\mu$ was found

In recent IceCube, Daya Bay, MINOS experiments no indications in favor of transitions of flavor neutrinos into sterile states were found

Many new short baseline neutrino experiments on the search for sterile neutrinos with masses ~ 1 eV are going on or in preparation

Source experiments: SOX (BOREXINO), CeLAND (KamLAND), BEST (SAGE), ...

Short baseline reactor neutrino experiments: Nucifer (France), NEOS (Korea), DANSS (Russia), Stereo (France)...

Short baseline accelerator neutrino experiments: SBN three detectors short baseline experiment (Fermilab), OscSNS (Oak Ridge), IsoDAR (KamLAND),...

The problem of sterile neutrinos will be definitely solved in a near future

II. Matrix element of the $0\nu\beta\beta$-decay is proportional to the effective Majorana mass

$$|m_{\beta\beta}| = |\sum_i U^2_{ei}\ m_i —$$

Strongly depends on Majorana neutrino mass hierarchy

Inverted hierarchy: $m_3 \ll m_1 \ll m_2$

$$|m_{\beta\beta}| \simeq \cos^2\theta_{13}\ \sqrt{\Delta m^2_A}\ (1 - \sin^2 2\,\theta_{12}\ \sin^2\alpha)^{\frac{1}{2}}$$

α is (unknown) Majorana phase difference

From neutrino oscillation data

$$2 \times 10^{-2} \lesssim |m_{\beta\beta}| \lesssim 5 \times 10^{-2}\,\mathrm{eV}$$

In the next generation of experiments on the search for $0\nu\beta\beta$-decay this region is planned to be reached

Normal hierarchy: $m_1 \ll m_2 \ll m_3$

$$|m_{\beta\beta}| \simeq \left| \cos^2\theta_{13} \sin^2\theta_{12} \sqrt{\Delta m_S^2} + e^{2i\,\alpha} \sin^2\theta_{13} \sqrt{\Delta m_A^2} \right|$$

$$2 \times 10^{-3}\ \mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 4 \times 10^{-3}\,\mathrm{eV}$$

Good news. The most plausible possibility -neutrinos with definite masses ν_i are Majorana particles.

$0\nu\beta\beta$-decay is allowed process

Bad news. Normal mass hierarchy is a possibility (indication from data)

Next after next $0\nu\beta\beta$ experiments probably will be needed