

A background image of a starry night sky. The sky is dark blue and black, filled with numerous small, bright stars of various colors. In the center-right of the image, there is a prominent, bright yellow-white star with a soft, glowing halo. The overall scene is a deep space or night sky view.

Neutrino magnetic moment, sterile neutrinos and Big Bang Nucleosynthesis

Baha Balantekin
University of Wisconsin

INT
July 11, 2017

At lower energies, beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \dots$$

Majorana
neutrino
mass
(unique)

Includes
Majorana
neutrino
magnetic
moment



$$\mu_\nu \propto \frac{m_\nu}{\Lambda^2}$$

Introduce a magnetic moment operator, $\hat{\mu}$

Example: Neutrino-electron scattering via magnetic moment

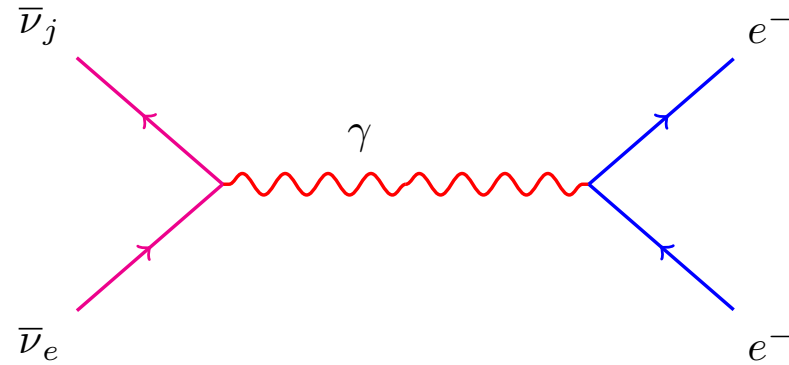
$$\sigma \propto \sum_i |\langle \nu_i | \hat{\mu} | \nu_e \rangle|^2 = \langle \nu_e | \hat{\mu}^\dagger \hat{\mu} | \nu_e \rangle$$

Dirac magnetic moment $\hat{\mu}^\dagger = \hat{\mu}$

Majorana magnetic moment $\hat{\mu}^T = -\hat{\mu}$

The matrix representation of this operator is best given in the mass basis

Neutrino-electron scattering at reactors



$$\frac{d\sigma_{ij}}{dt} = \frac{\alpha^2 \pi \mu_{ij}^2}{2m_e^2 \lambda} \left[\frac{1}{t} \left(2\lambda + 4m_e^2 m_i^2 + 2A \delta m^2 + 2m_e^2 \delta m^2 + [\delta m^2]^2 \right) + (2A + \delta m^2) + \frac{2m_e^2 (\delta m^2)^2}{t^2} \right]$$

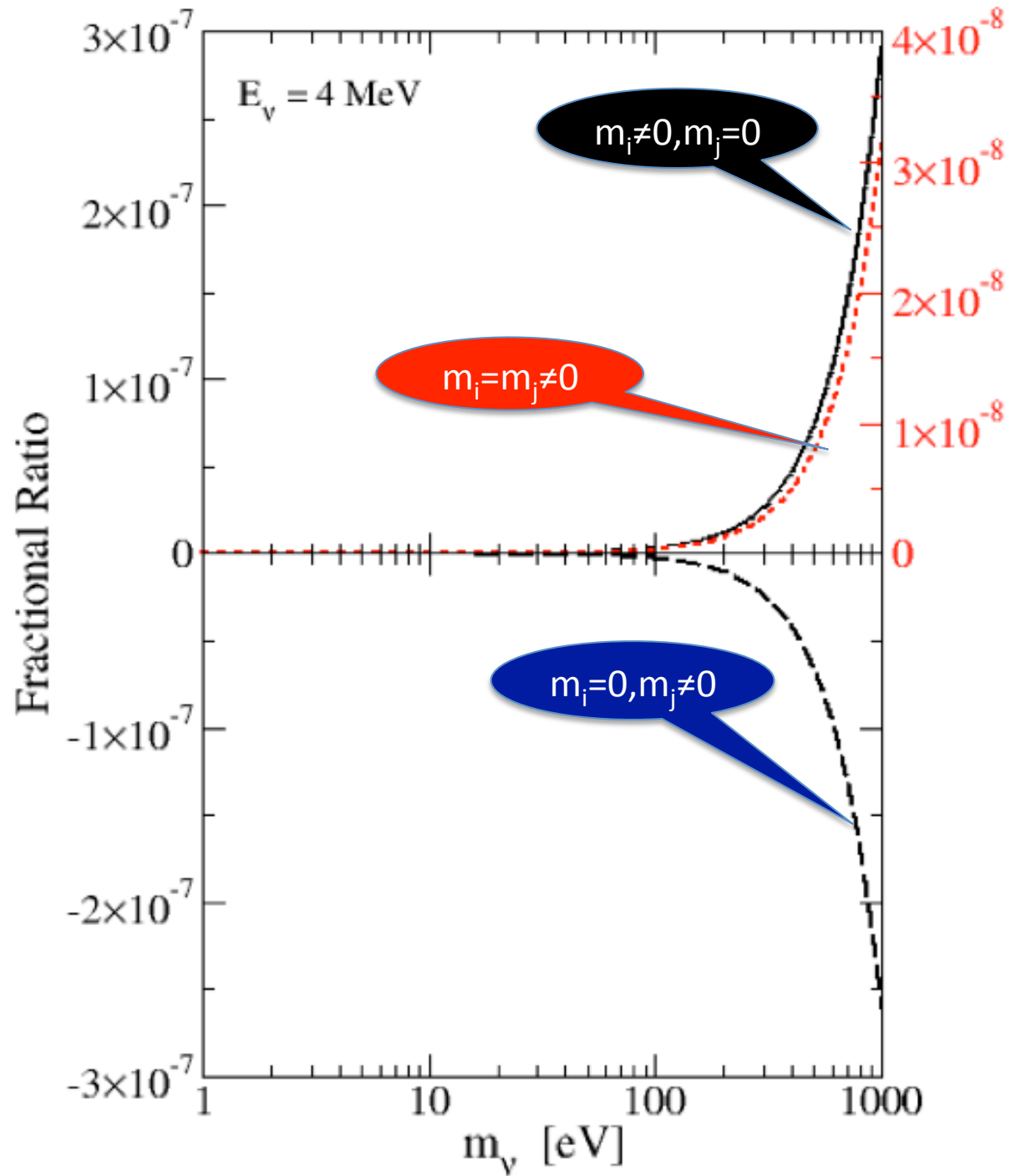
$$\delta m^2 = m_i^2 - m_j^2$$

$$A = s - m_e^2 - m_i^2$$

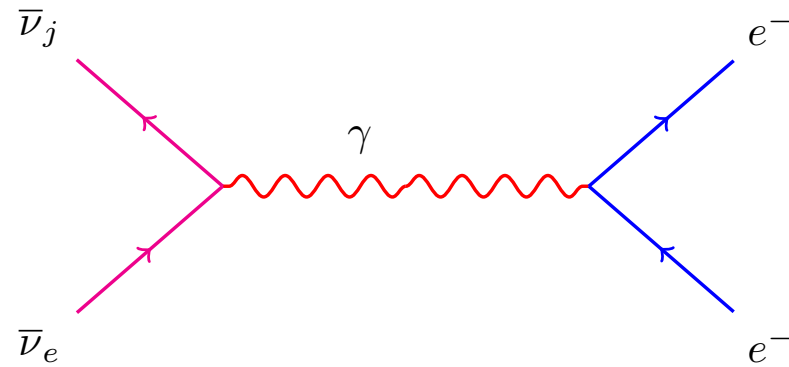
$$\lambda = A^2 - 4m_e^2 m_i^2$$

$$\frac{\left\langle \frac{d\sigma}{dt} \right\rangle_{m \neq 0} - \left\langle \frac{d\sigma}{dt} \right\rangle_{m=0}}{\left\langle \frac{d\sigma}{dt} \right\rangle_{m=0}}$$

A.B.B. & Vassh



Neutrino-electron scattering at reactors



$$\frac{d\sigma_{ij}}{dt} = \frac{\alpha^2 \pi \mu_{ij}^2}{2m_e^2 \lambda} \left[\frac{1}{t} \left(2\lambda + 4m_e^2 m_i^2 + 2A \delta m^2 + 2m_e^2 \delta m^2 + [\delta m^2]^2 \right) + (2A + \delta m^2) + \frac{2m_e^2 (\delta m^2)^2}{t^2} \right]$$

$$\delta m^2 = m_i^2 - m_j^2$$

$$A = s - m_e^2 - m_i^2$$

$$\lambda = A^2 - 4m_e^2 m_i^2$$

A reactor experiment measuring electron antineutrino magnetic moment is an inclusive one, i.e. it sums over all the neutrino final states

$$\frac{d\sigma}{dT_e} = \frac{\alpha^2 \pi}{m_e^2} \mu_{\text{eff}}^2 \left[\frac{1}{T_e} - \frac{1}{E_\nu} \right]$$

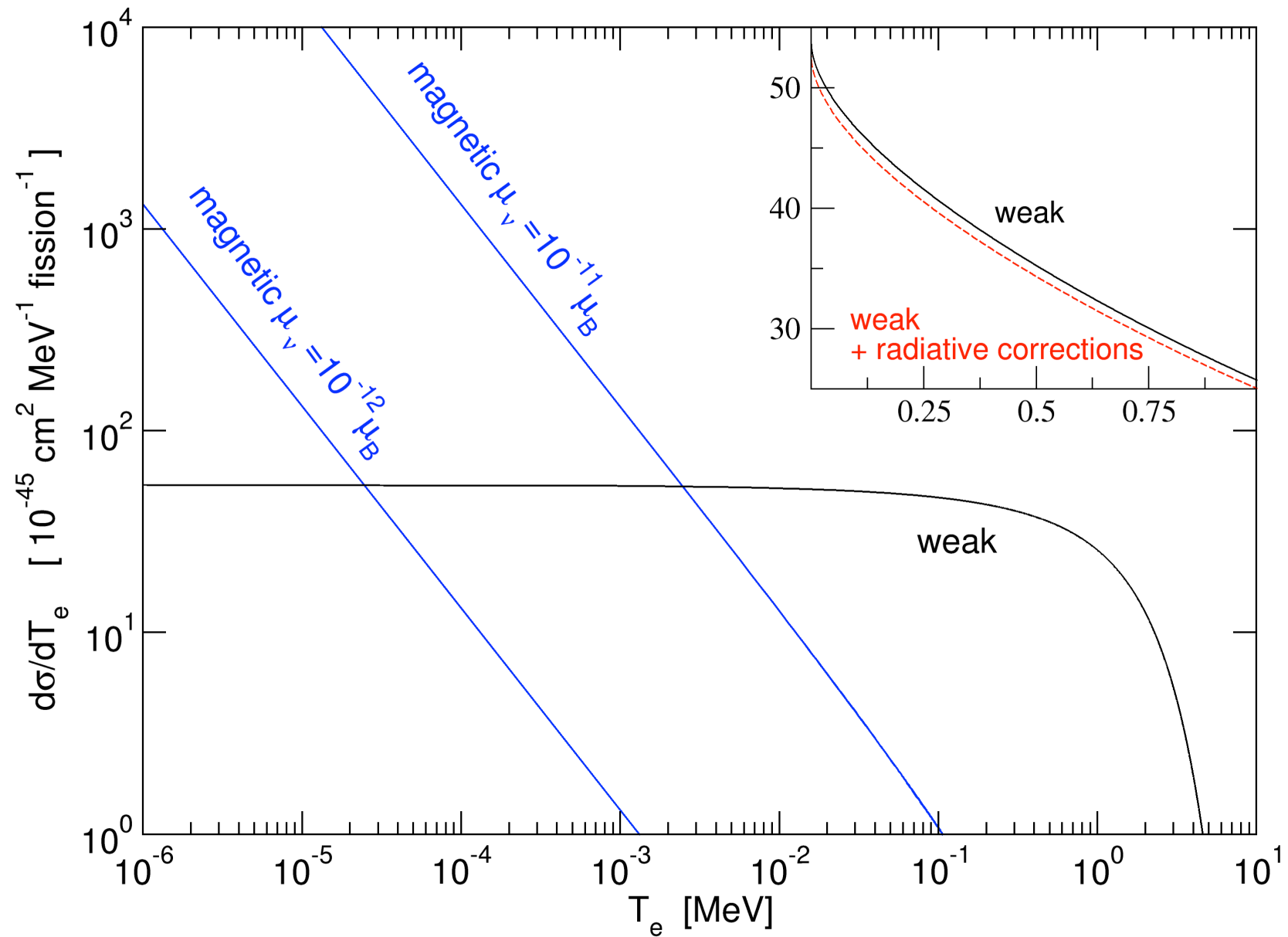
$$\mu_{\text{eff}}^2 = \sum_i \left| \sum_j U_{ej} e^{-iE_j L} \mu_{ji} \right|^2$$

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[(g_V + g_A)^2 + (g_V - g_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2) \frac{m_e T}{E_\nu^2} \right] \leftarrow \text{weak}$$

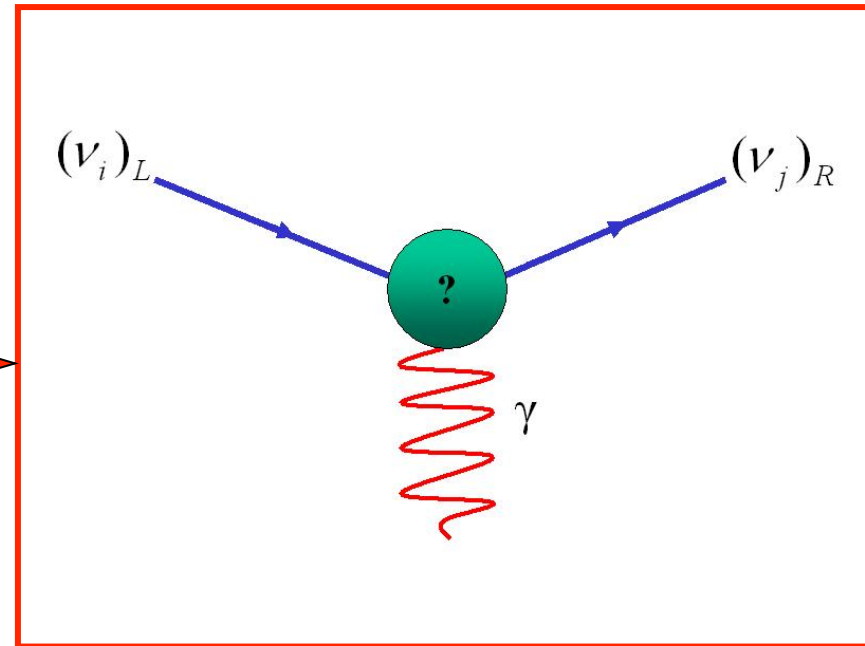
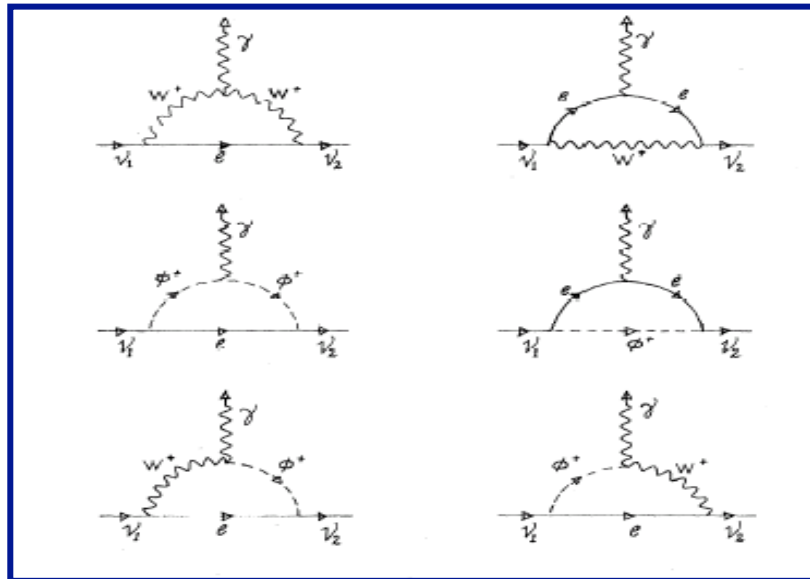
$$+ \frac{\pi \alpha^2 \mu^2}{m_e^2} \left(\frac{1}{T} - \frac{1}{E_\nu} \right) \leftarrow \text{magnetic}$$

$$g_V = 2 \sin^2 \theta_W + 1/2$$

$$g_A = \begin{cases} +1/2 & \text{for electron neutrinos} \\ -1/2 & \text{for electron antineutrinos} \end{cases}$$



Neutrino Magnetic Moment in the Standard Model



Symmetry principles: $\mu_\nu \rightarrow 0$ as $m_\nu \rightarrow 0$

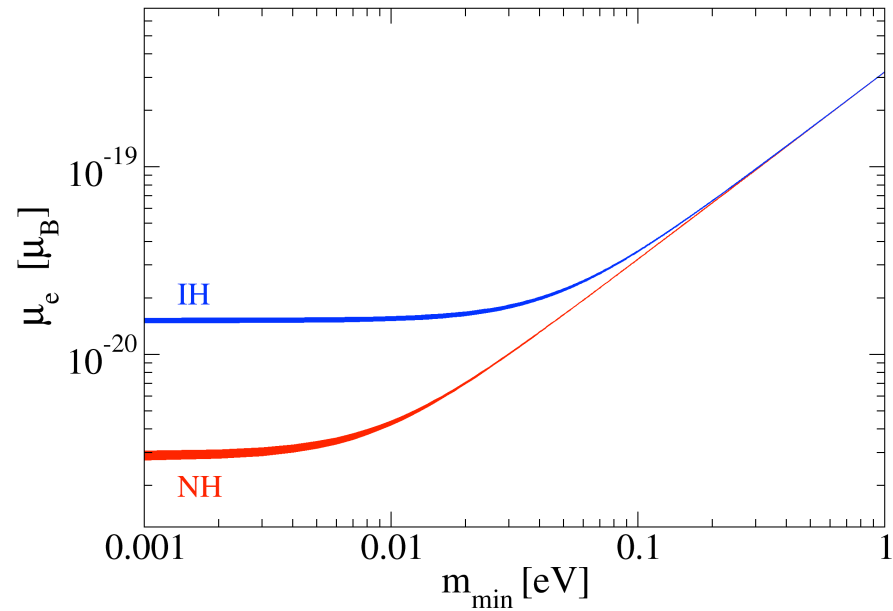
$$\mu_{ij} = -\frac{eG_F}{8\sqrt{2}\pi^2} (m_i + m_j) \sum_\ell U_{li} U_{lj}^* f(r_\ell)$$

$$f(r_\ell) \approx -\frac{3}{2} + \frac{3}{4} r_\ell + \dots, \quad r_\ell = \left(\frac{m_\ell}{M_W} \right)^2$$

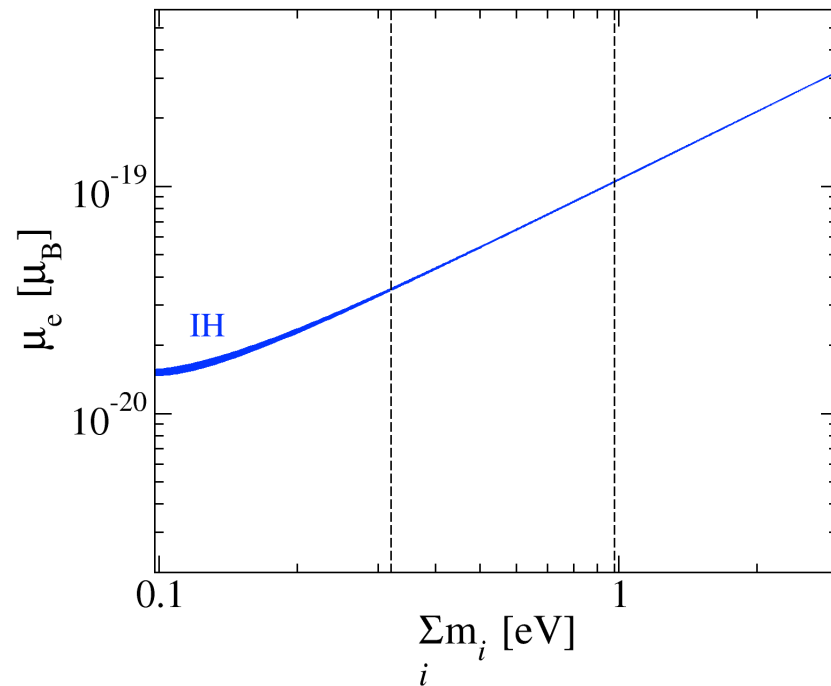
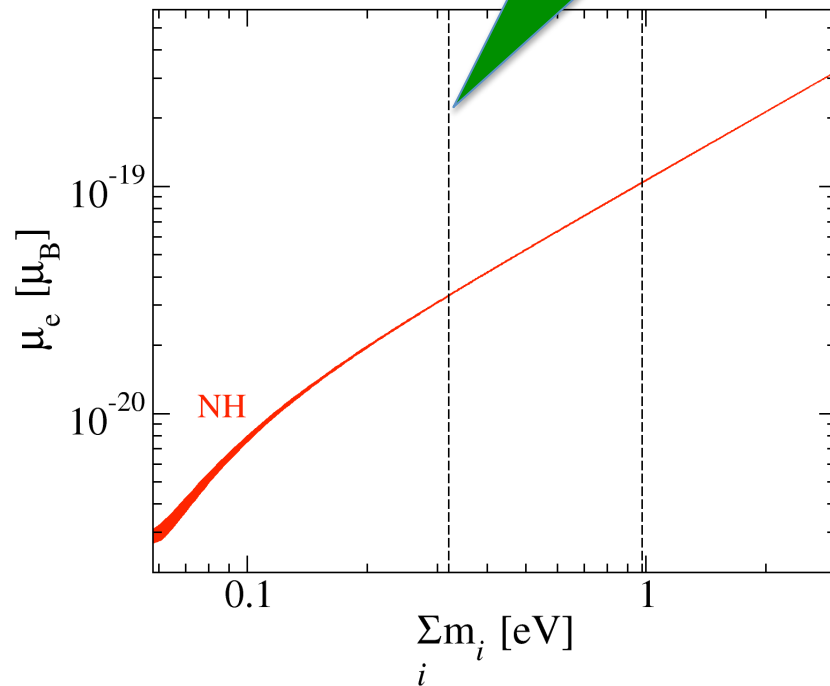
Standard Model (Dirac)

Standard Model (only)
 contribution to the
 Dirac neutrino
 magnetic moment
 measured at reactors

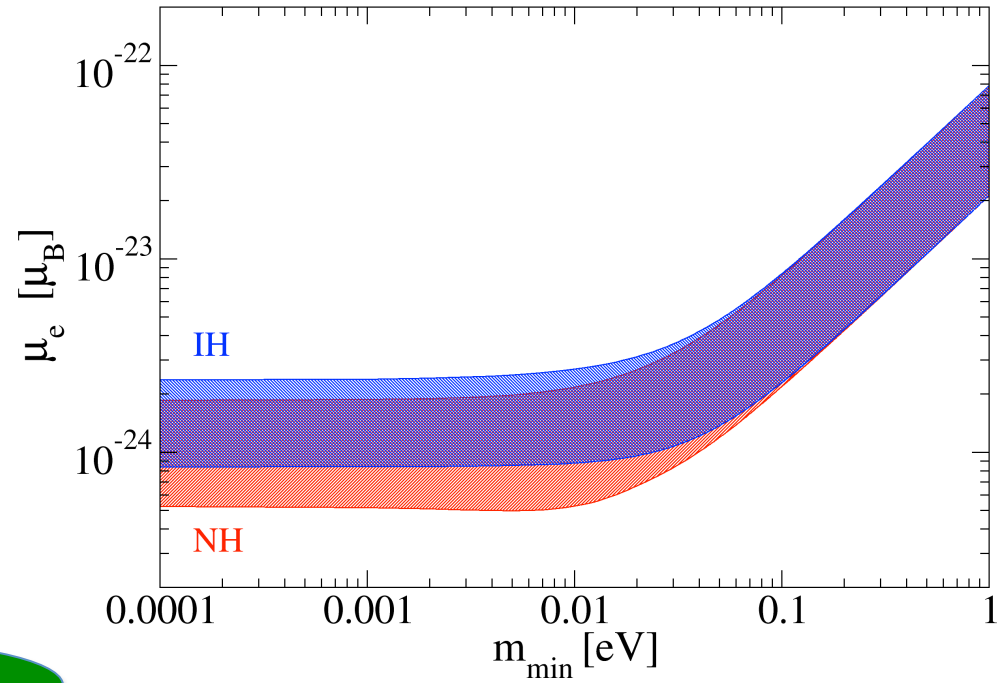
A.B.B., N. Vassh, PRD **89** (2014) 073013



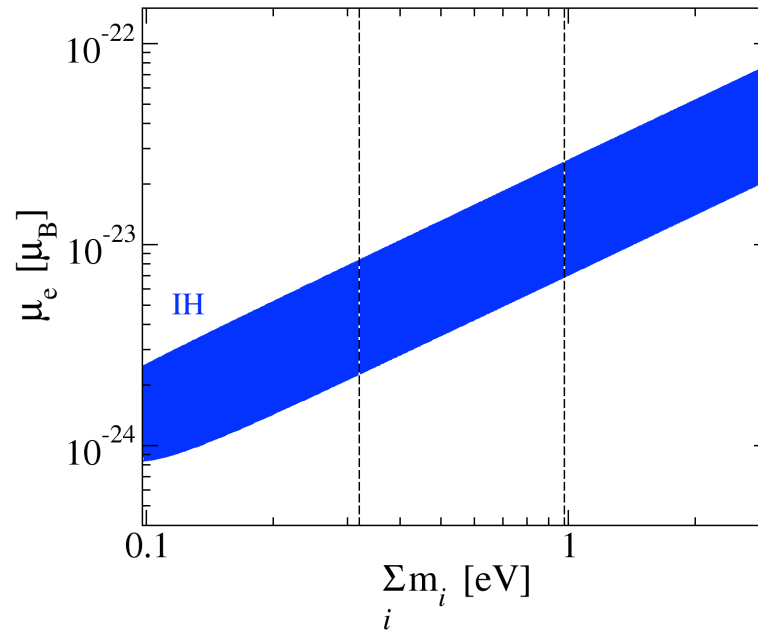
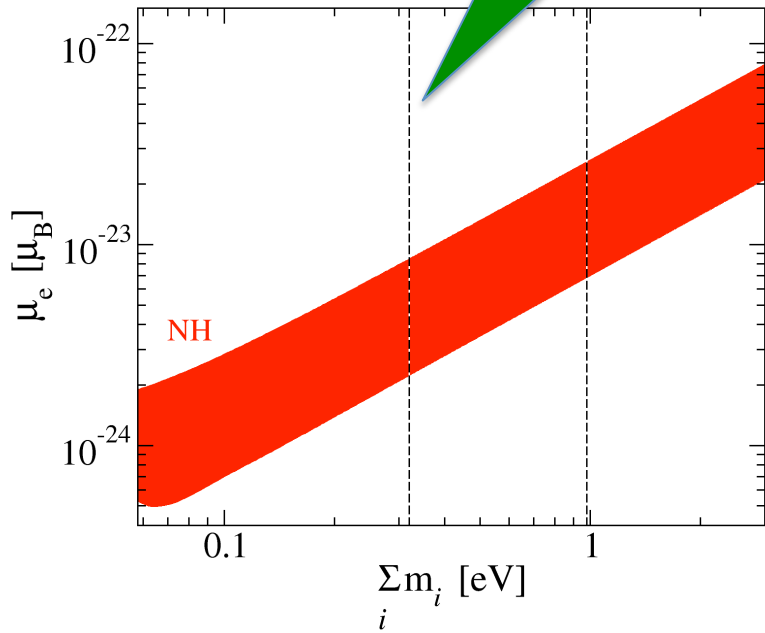
Cosmological limits

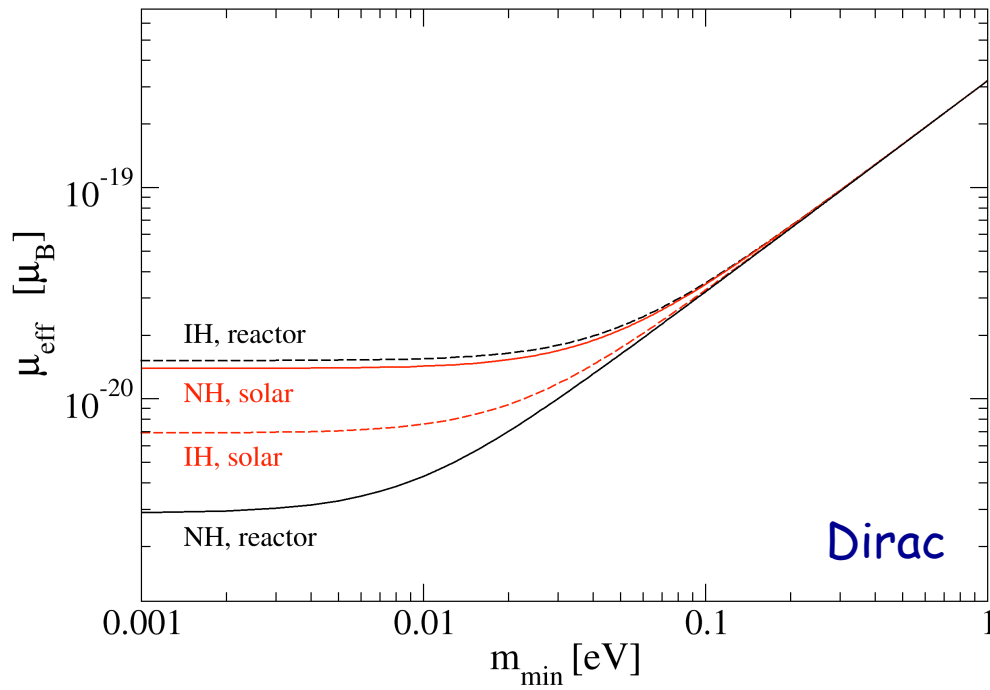


Standard Model (only)
contribution to the
Majorana neutrino
magnetic moment
measured at reactors



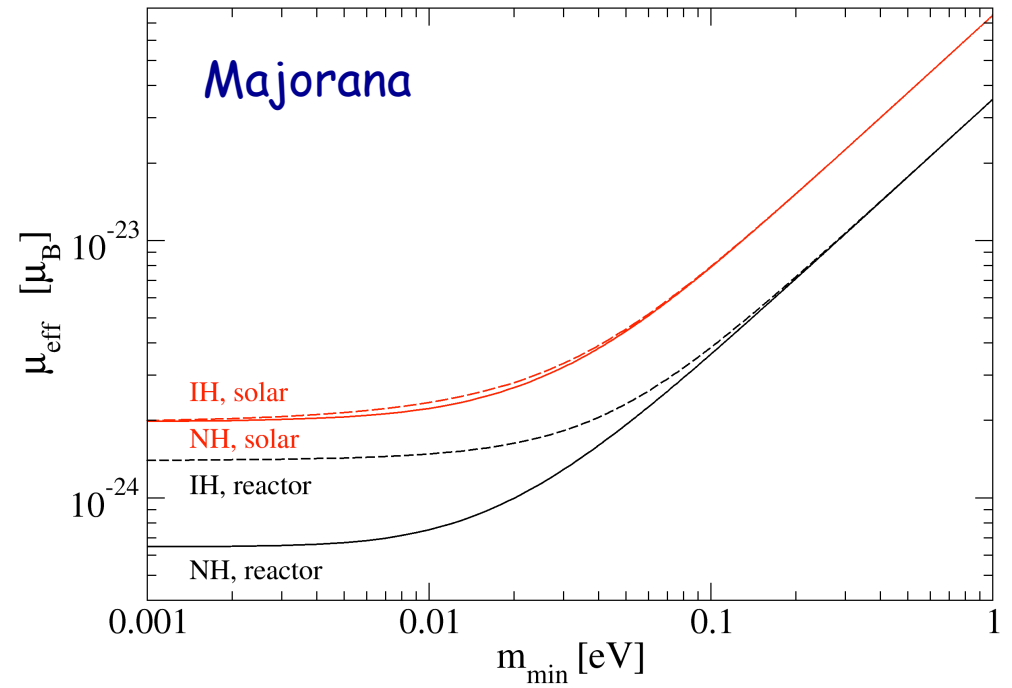
Cosmological limits

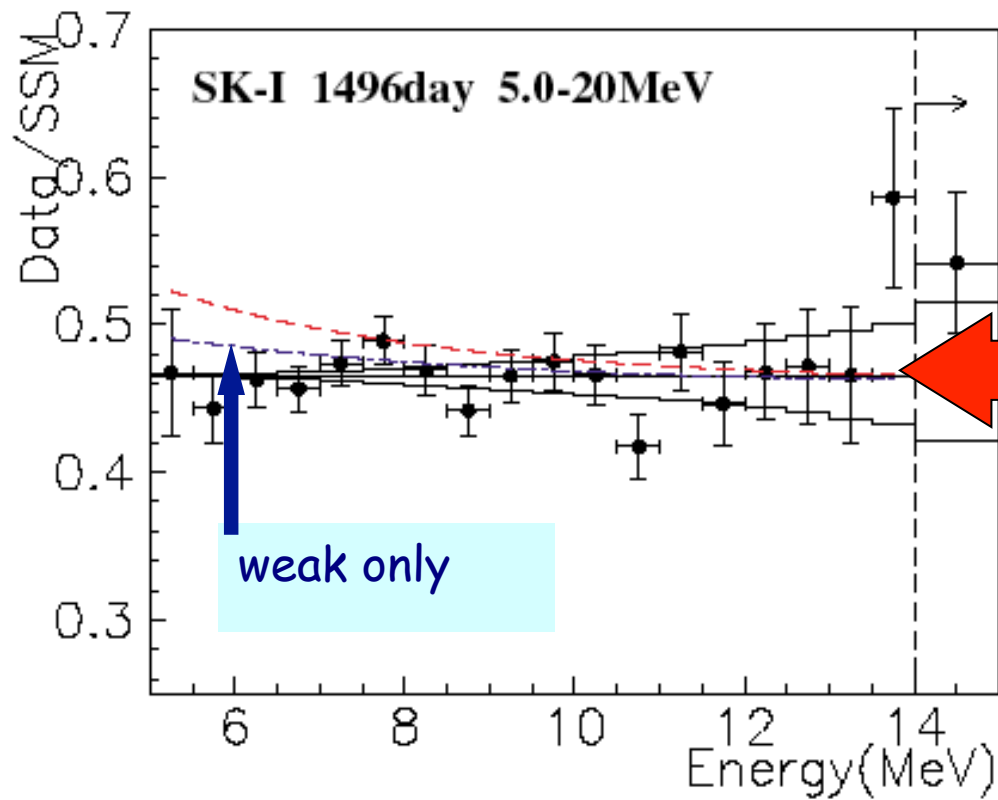




Reactors vs. solar
Cerenkov detectors

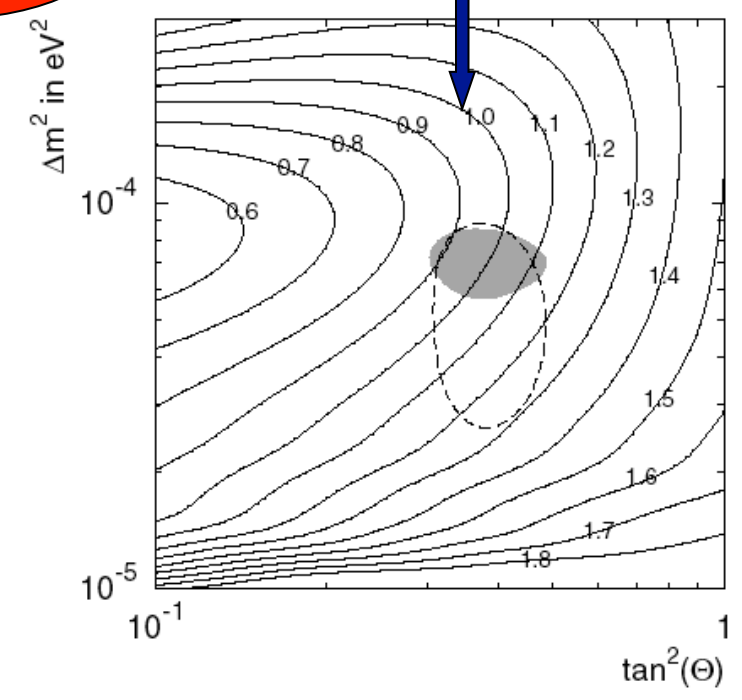
A.B.B. & N. Vassh
AIP Conf.Proc. 1604 (2014) 150
arXiv:1404.1393





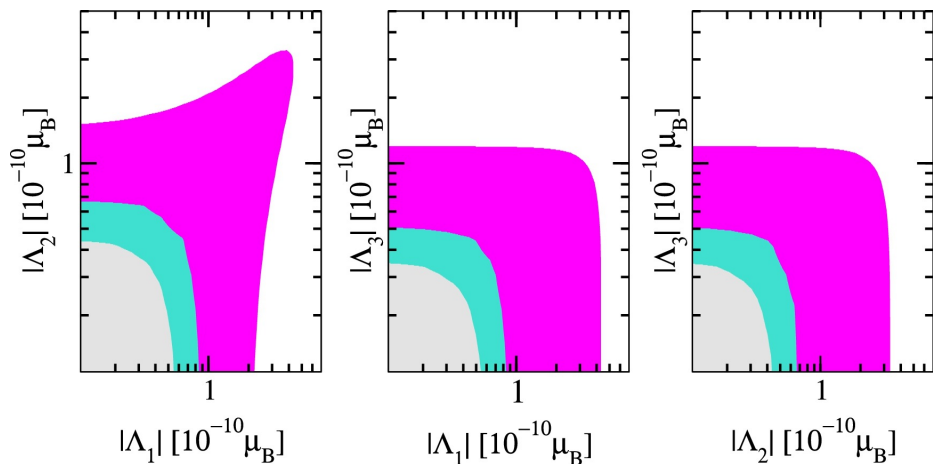
additional $\mu_\nu < 10^{-10} \mu_B$

$\mu_\nu = 10^{-10} \mu_B$



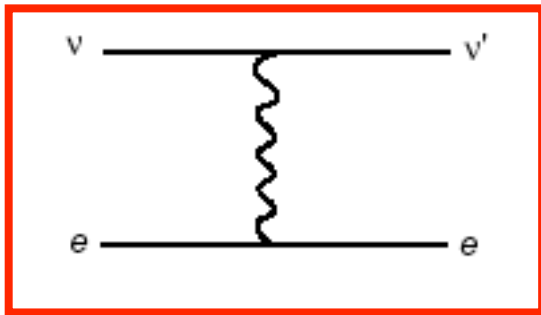
SuperK: $\mu_\nu \leq (3.6 \times 10^{-10}) \mu_B$ at 90% C.L.

SuperK + KamLAND: $\mu_\nu \leq (1.1 \times 10^{-10}) \mu_B$ at 90% C.L.

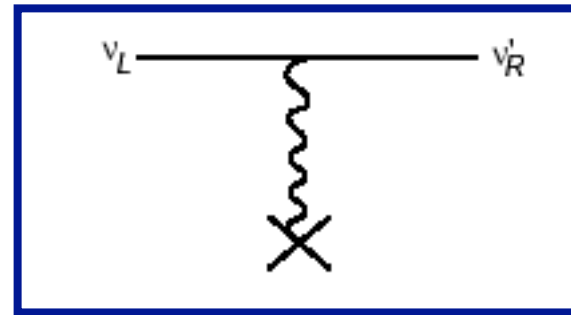


Canas, et al analysis of the Borexino Data

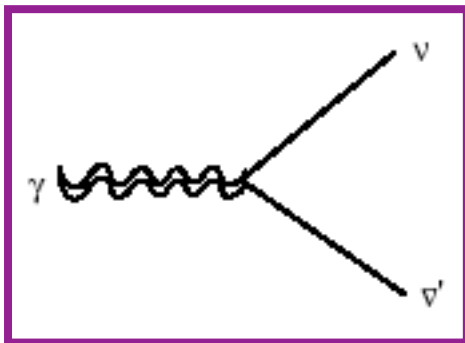
Physical Processes with a Neutrino Magnetic Moment



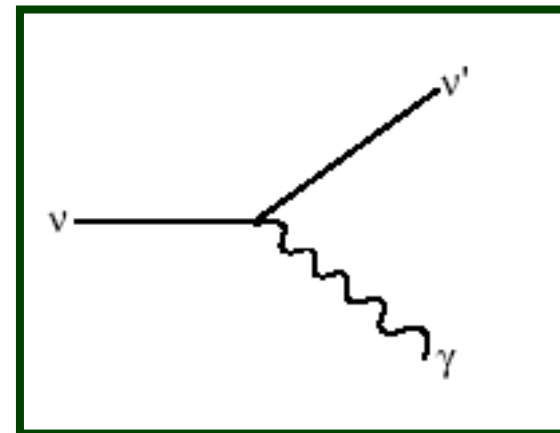
ν - e scattering



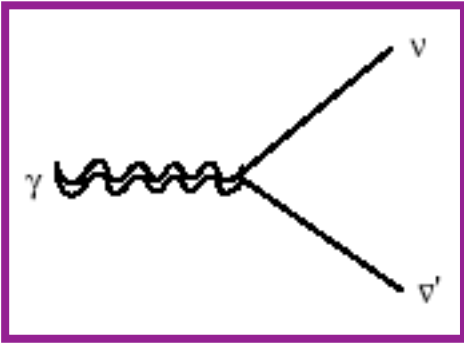
Spin-flavor precession



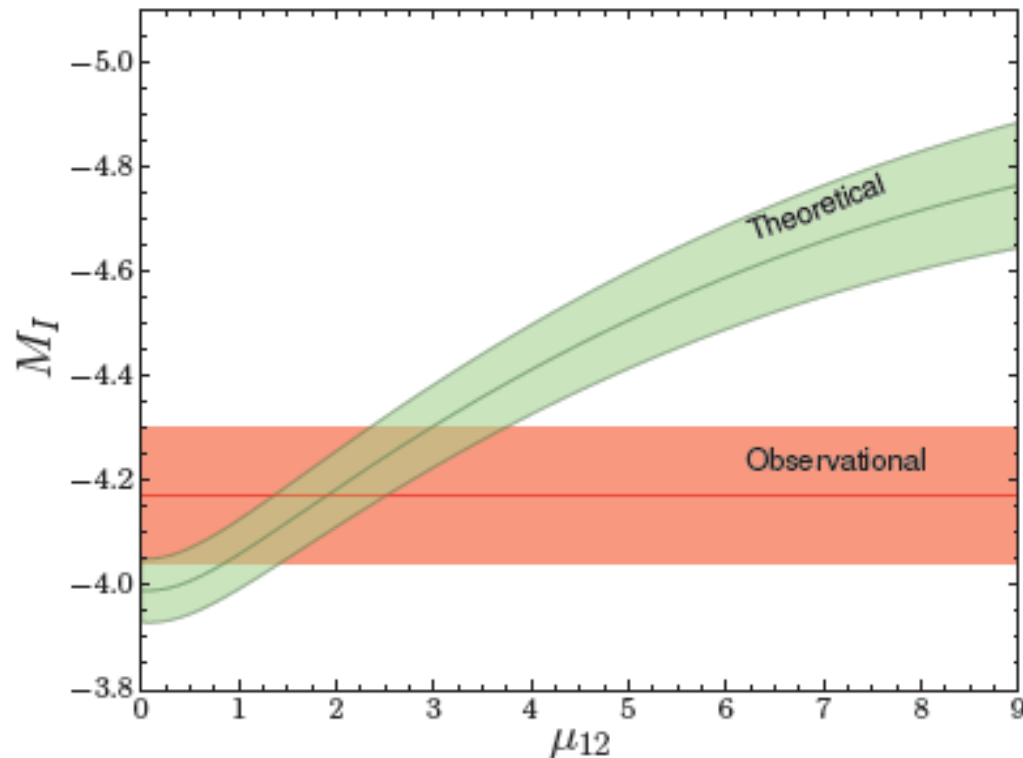
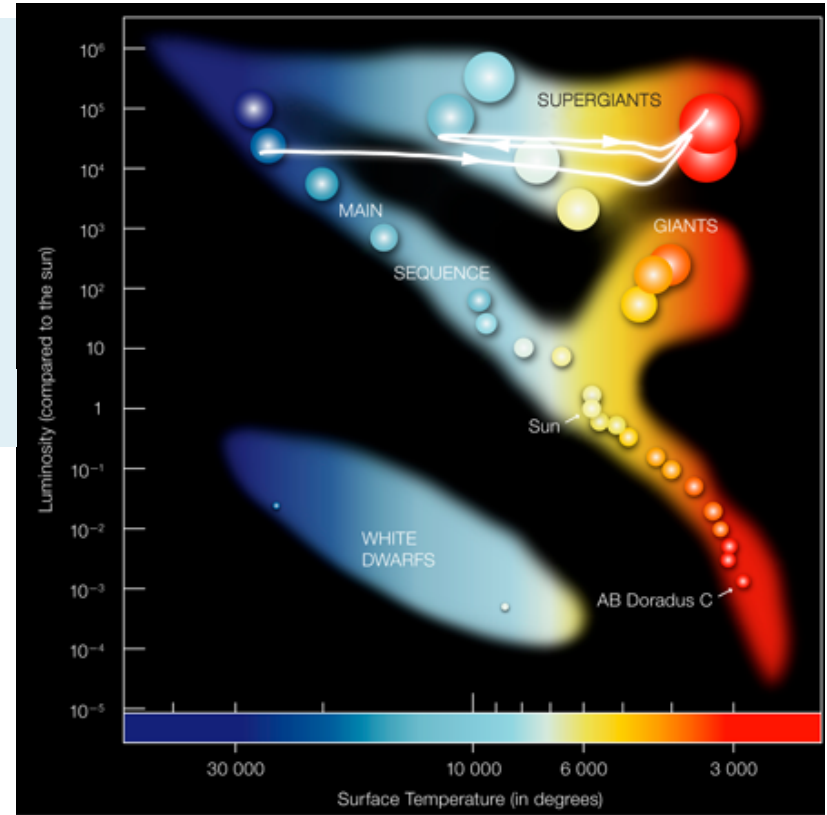
Plasmon decay



Neutrino decay



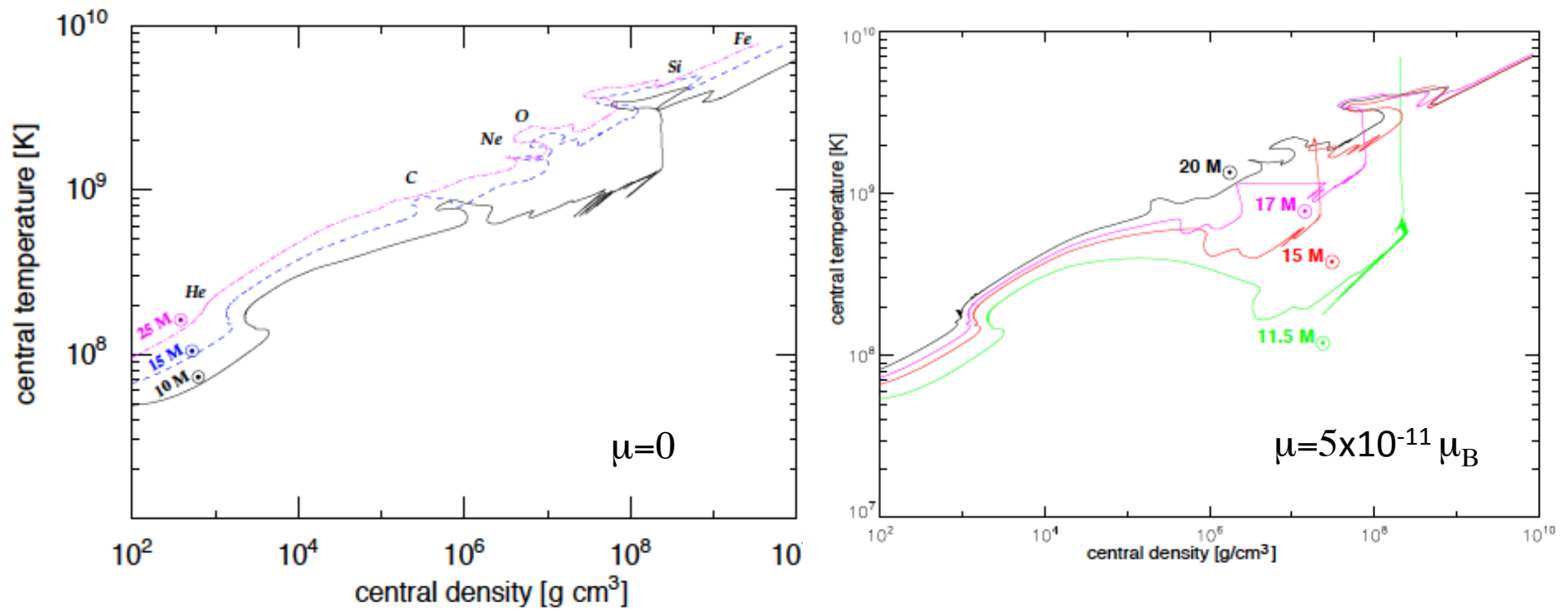
A large enough neutrino magnetic moment implies enhanced plasmon decay rate: $\gamma \rightarrow \nu\bar{\nu}$. Since the neutrinos freely escape the star, this in turn cools a red giant star faster delaying helium ignition.



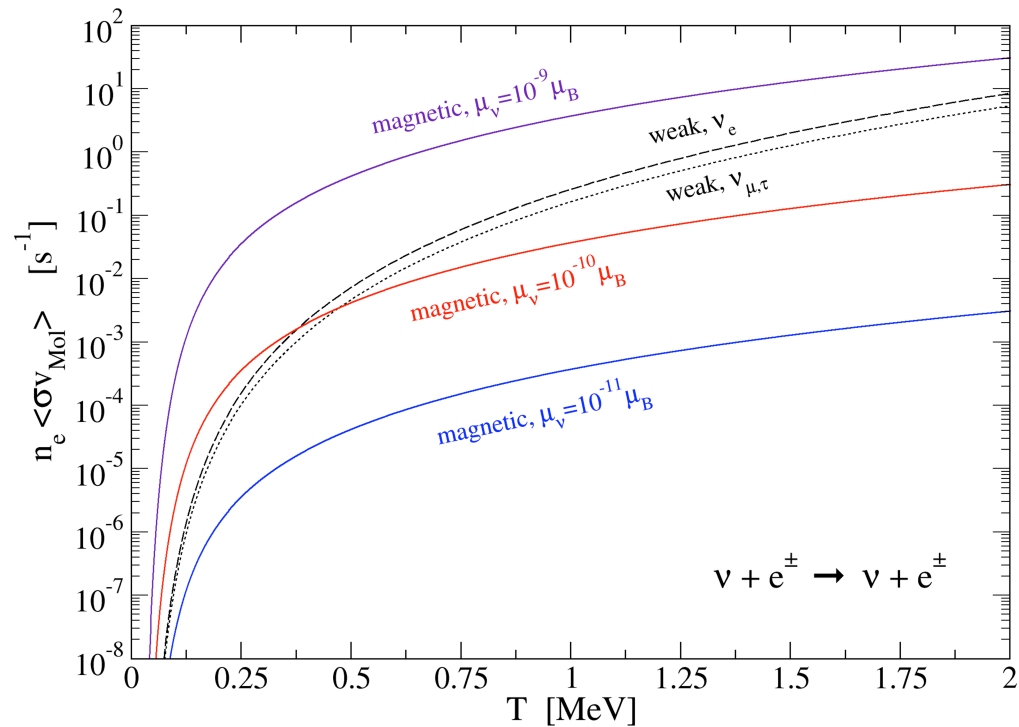
Globular cluster M5
 $\rightarrow \mu_\nu < 4.5 \times 10^{-12} \mu_B$
 (95% C.L.)

arXiv:1308.4627

Neutrino magnetic moment may impact stellar evolution

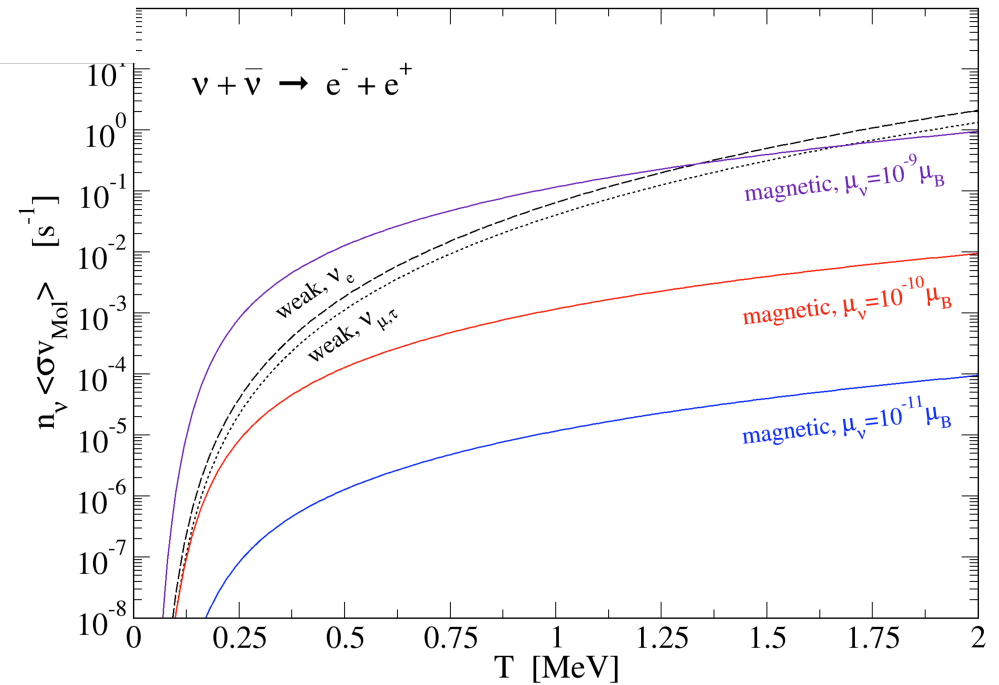


Heger, Friedland, Giannotti and Cirigliano, *Astrophys.J.* **696**, 608 (2009)

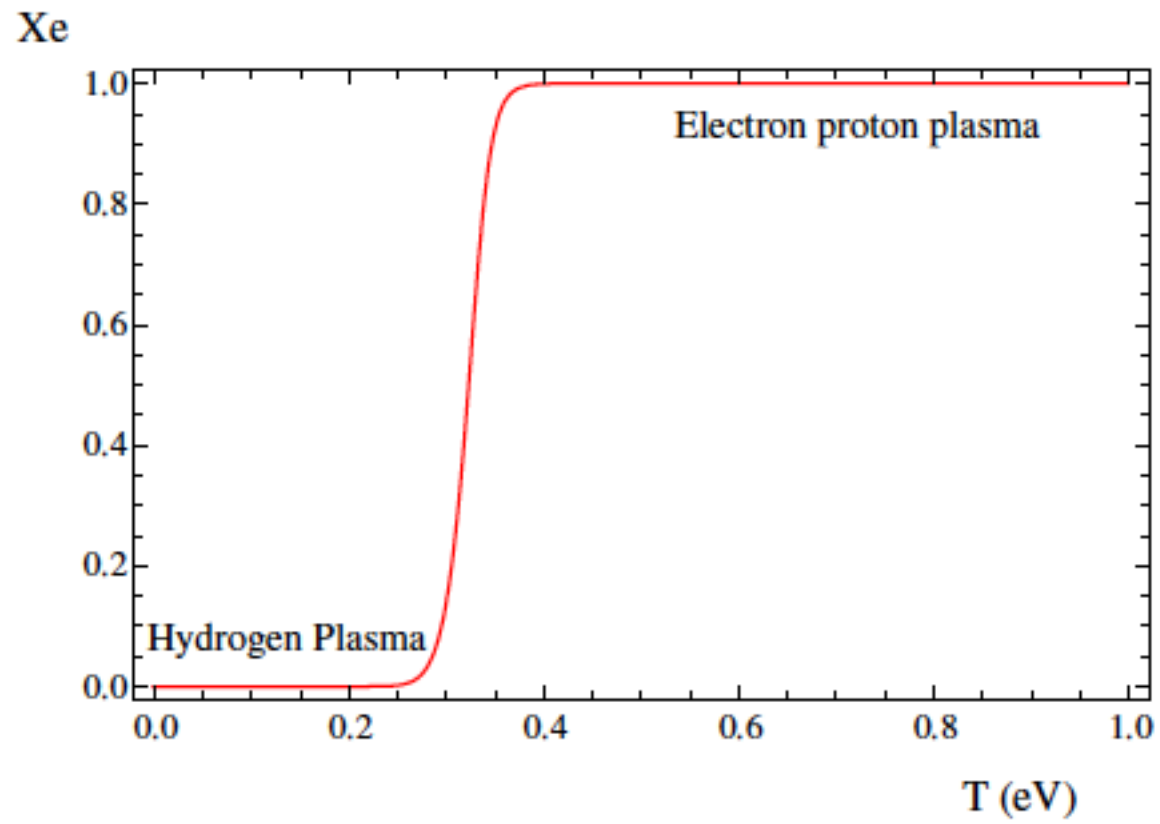


The effect of the neutrino magnetic moment on neutrino decoupling in the BBN epoch

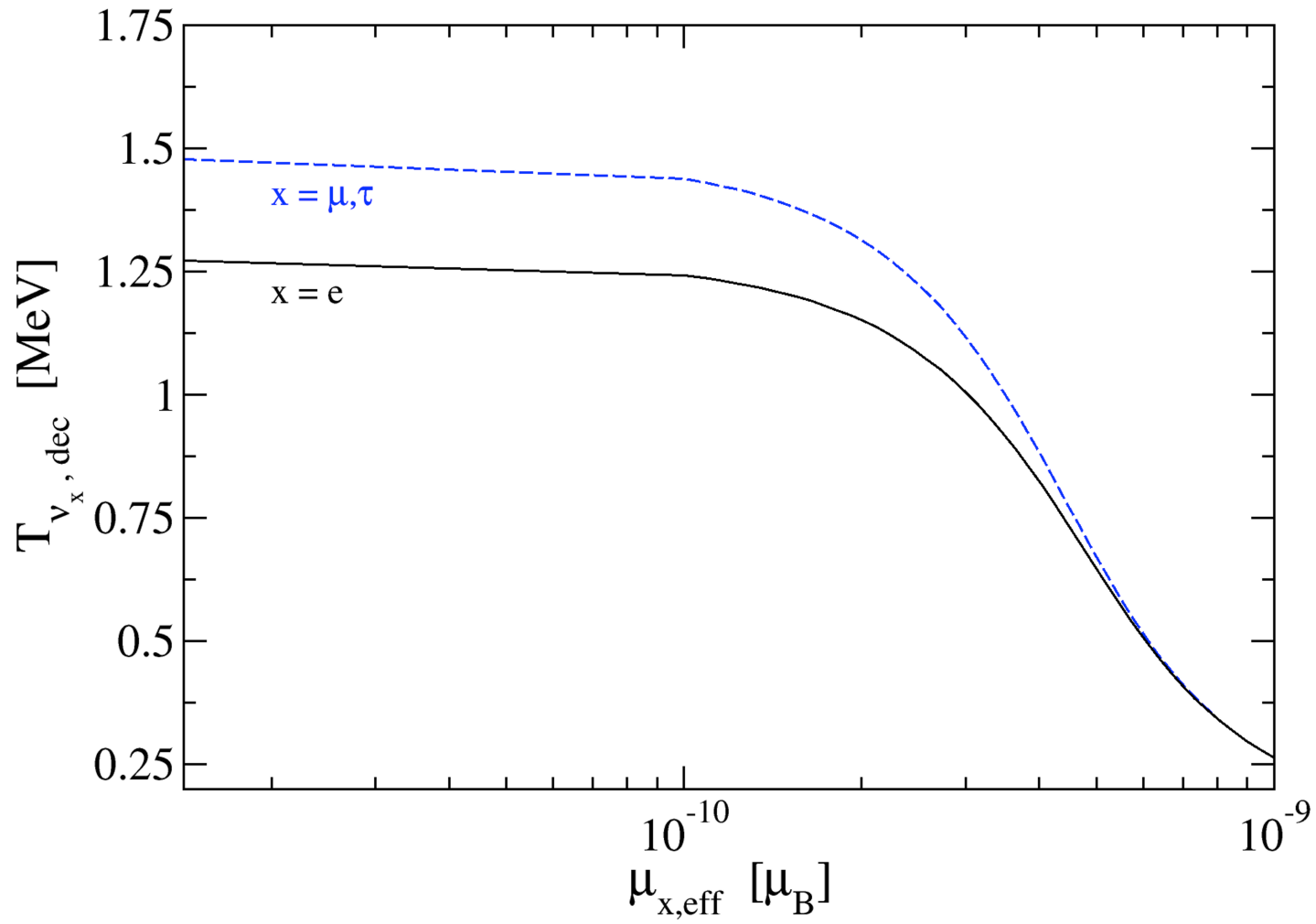
Vassh, Grohs, Balantekin, Fuller,
Phys. Rev. D **92**, 125020 (2015)



Ionization rate

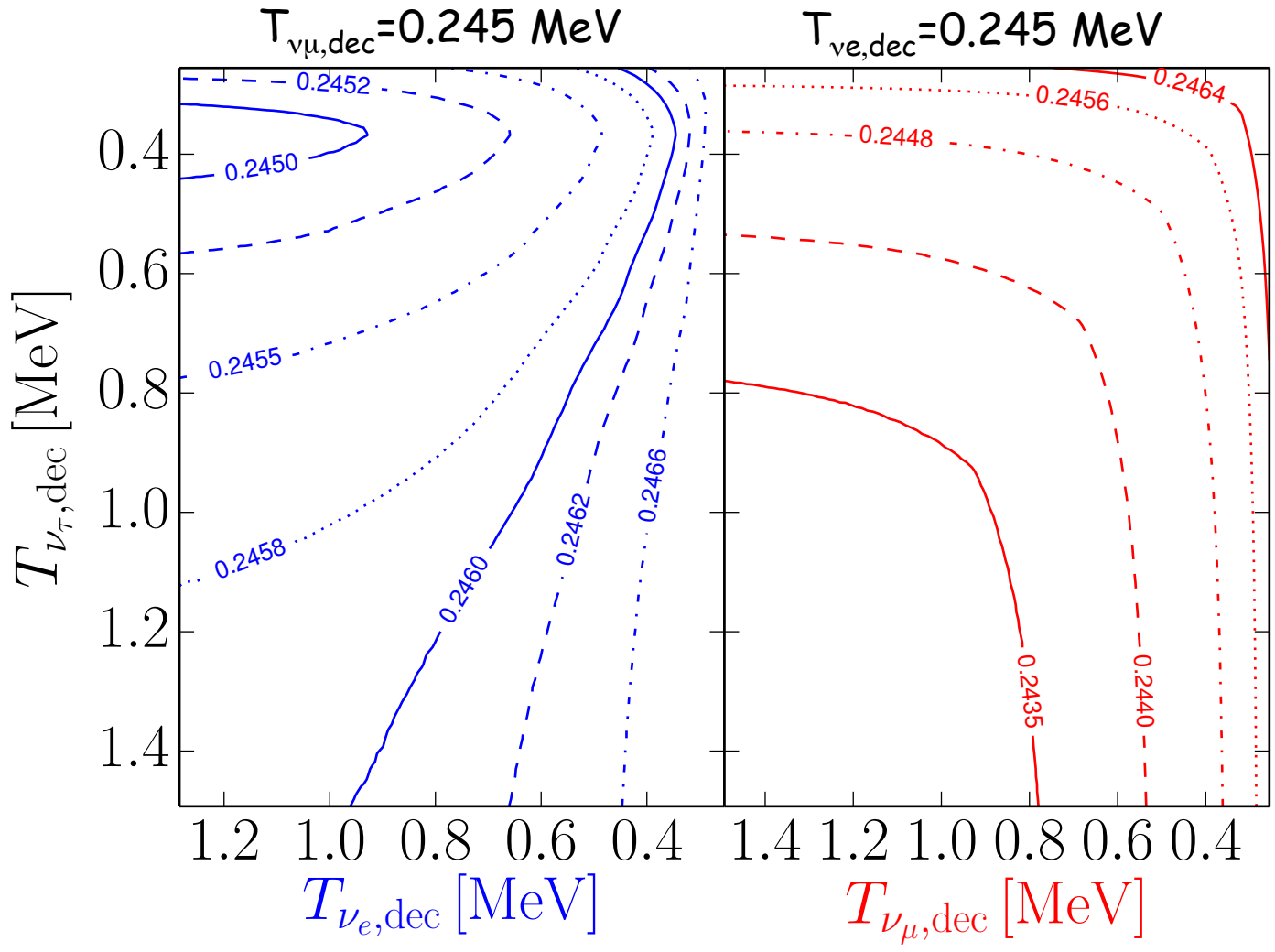


Decoupling temperature of three flavors

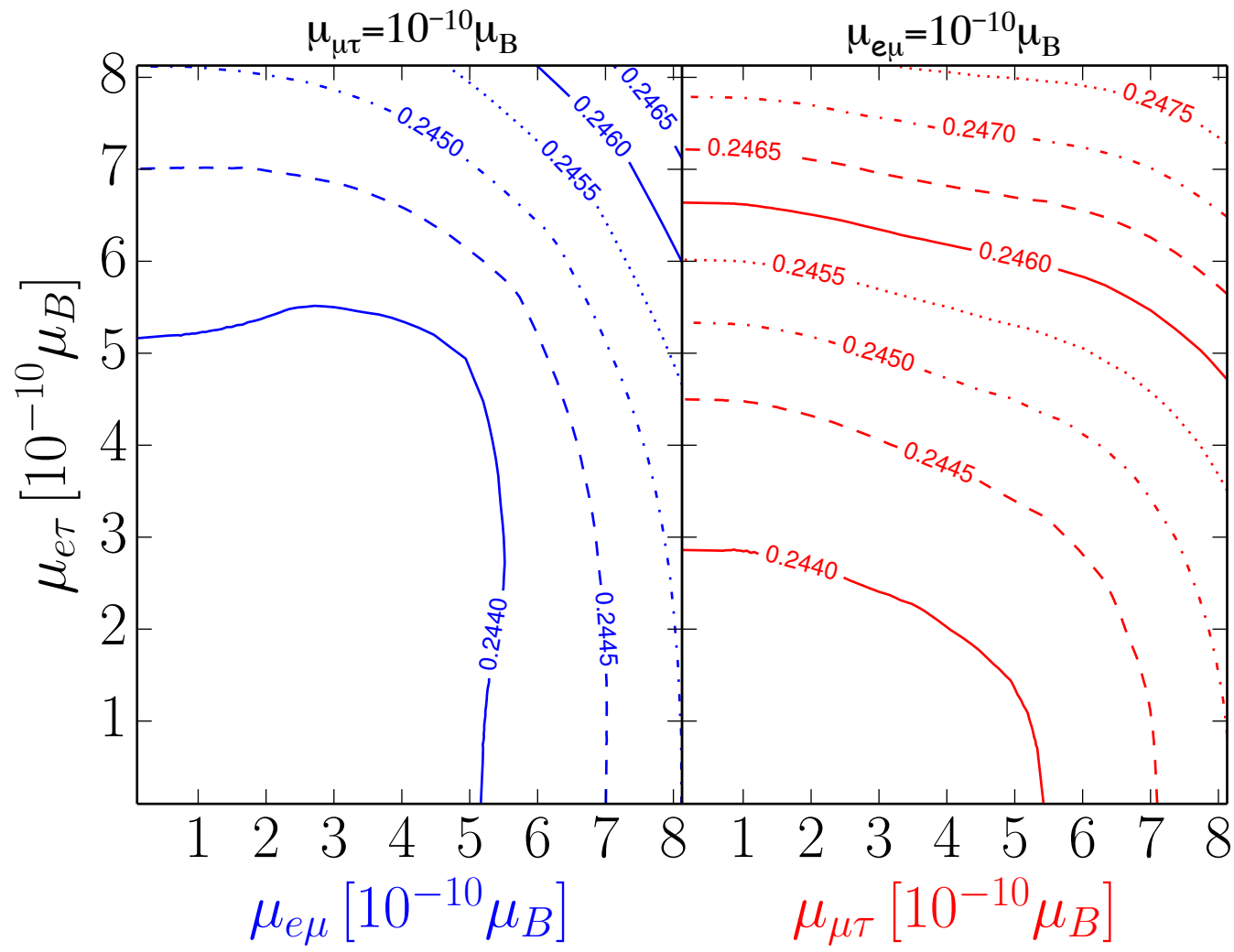


Contours of constant Y_P

$$Y_P \equiv \frac{4n_{He}}{n_p + n_n}$$



Contours of constant Y_p



The change in the BBN abundances due to the neutrino magnetic moment

Solid lines: $\mu_{e\tau} = 10^{-11} \mu_B$

black: $\mu_{\mu\tau} = 10^{-11} \mu_B$

red: $\mu_{\mu\tau} = 4 \times 10^{-10} \mu_B$

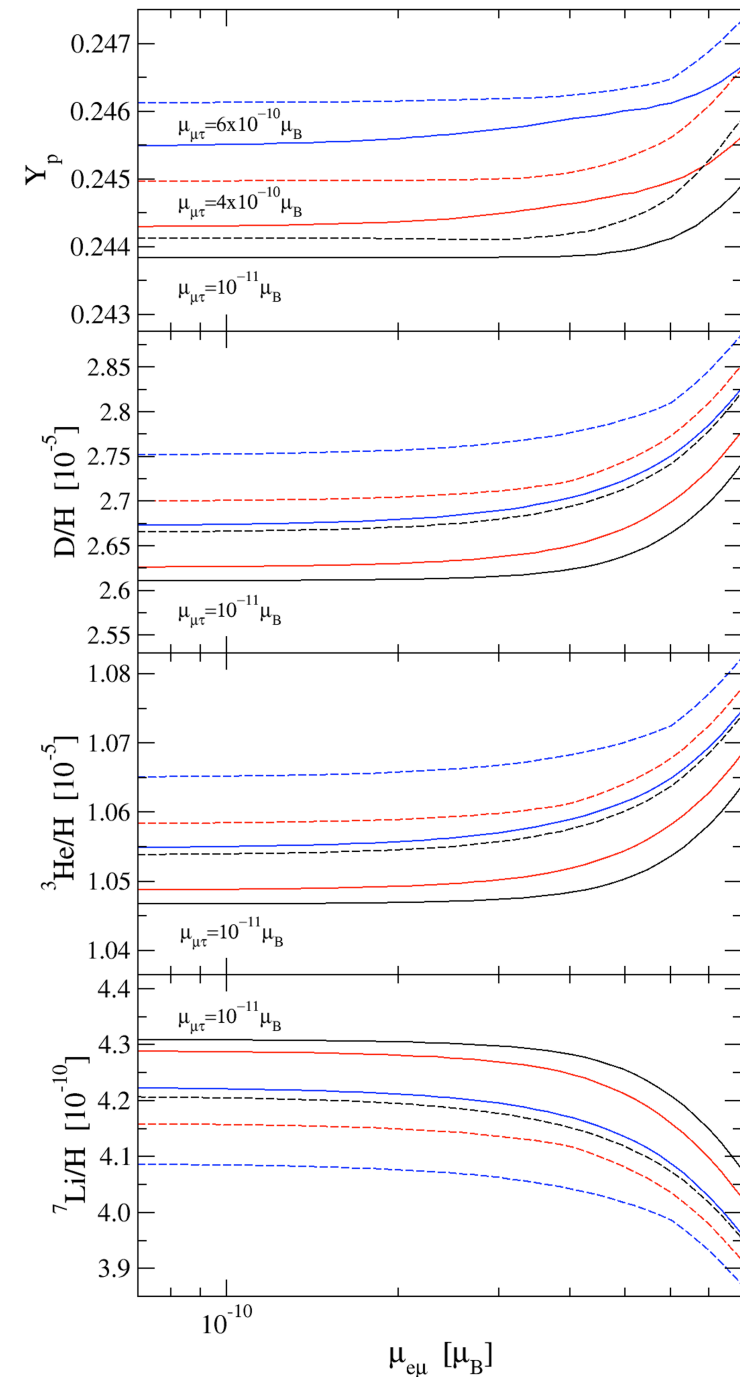
blue: $\mu_{\mu\tau} = 6 \times 10^{-10} \mu_B$

Dashed lines: $\mu_{e\tau} = 6 \times 10^{-10} \mu_B$

black: $\mu_{\mu\tau} = 10^{-11} \mu_B$

red: $\mu_{\mu\tau} = 4 \times 10^{-10} \mu_B$

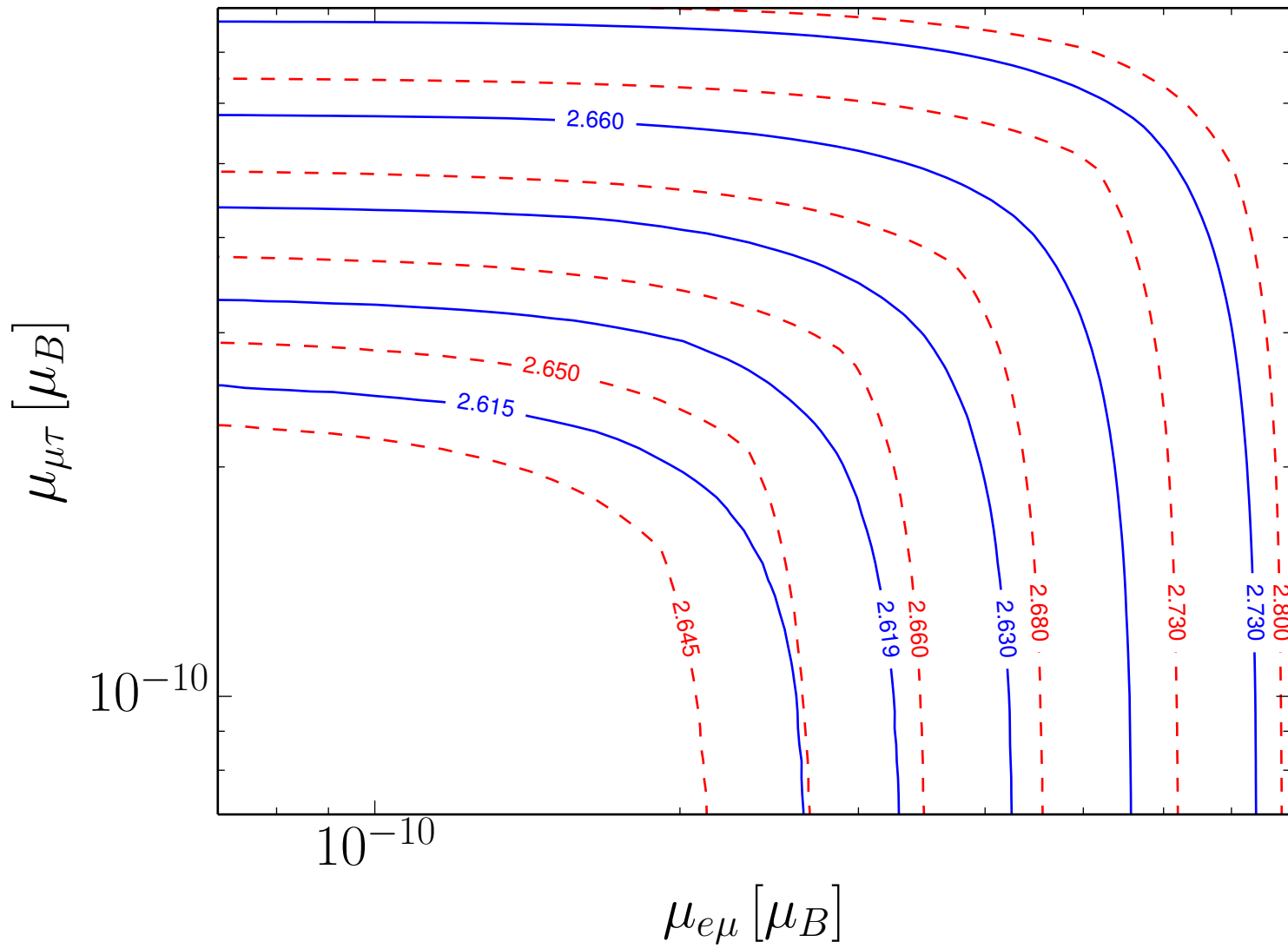
blue: $\mu_{\mu\tau} = 6 \times 10^{-10} \mu_B$



Contours of constant $10^5 \times (D/H)$

— $\mu_{e\tau} = 10^{-10} \mu_B$

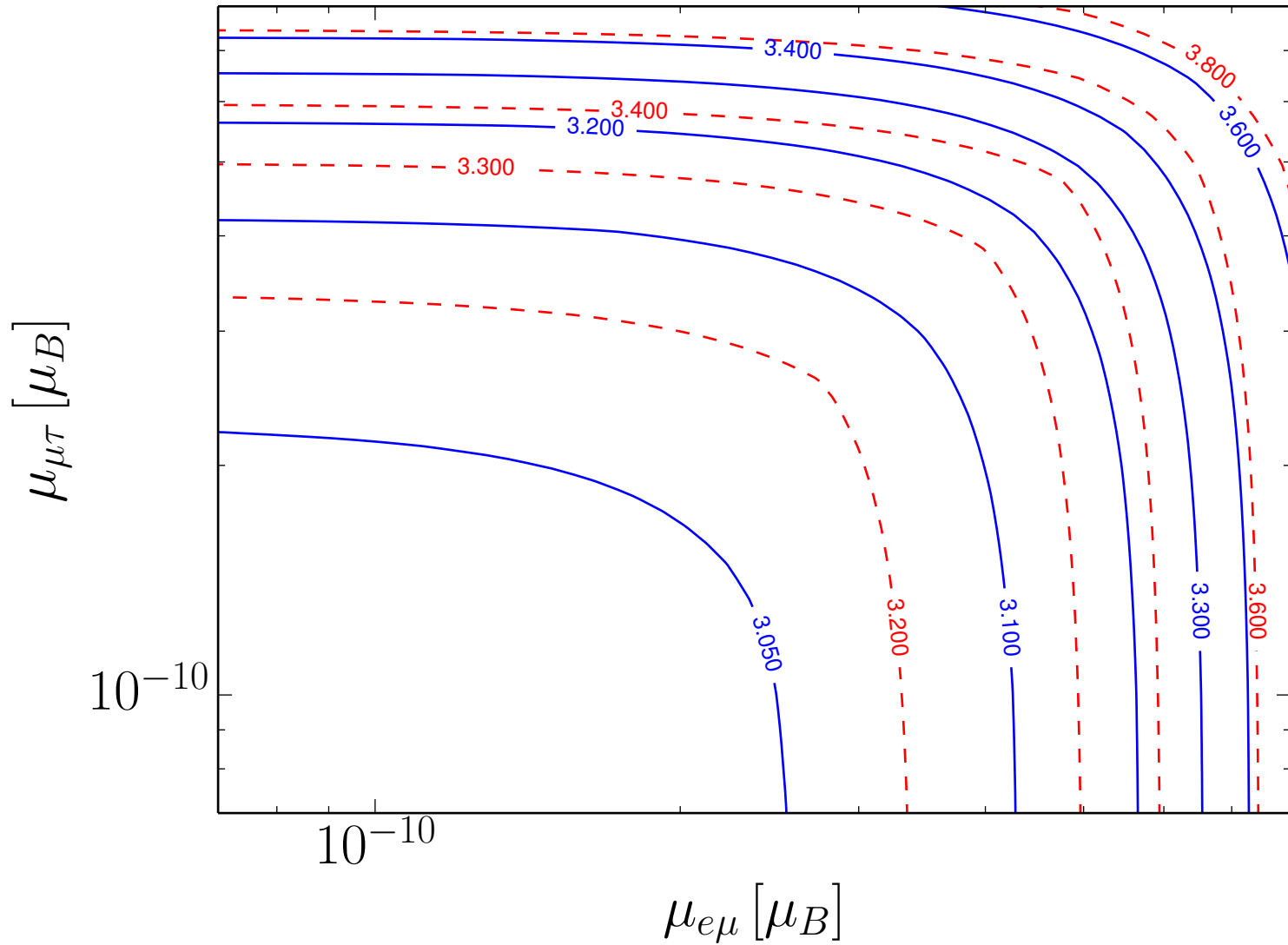
- - $\mu_{e\tau} = 4.9 \times 10^{-10} \mu_B$



$$\rho_{\text{relativistic}} = \frac{\pi^2}{15} T_\gamma^4 \left[1 + \frac{7}{8} N_{\text{effective}} \left(\frac{4}{11} \right)^{4/3} \right]$$

Contours of constant N_{eff}

— $\mu_{e\tau} = 10^{-10} \mu_B$
 - - - $\mu_{e\tau} = 4.9 \times 10^{-10} \mu_B$



Planck: $N_{eff} = 3.30 \pm 0.27 \Rightarrow \mu \leq 6 \times 10^{-10} \mu_B$

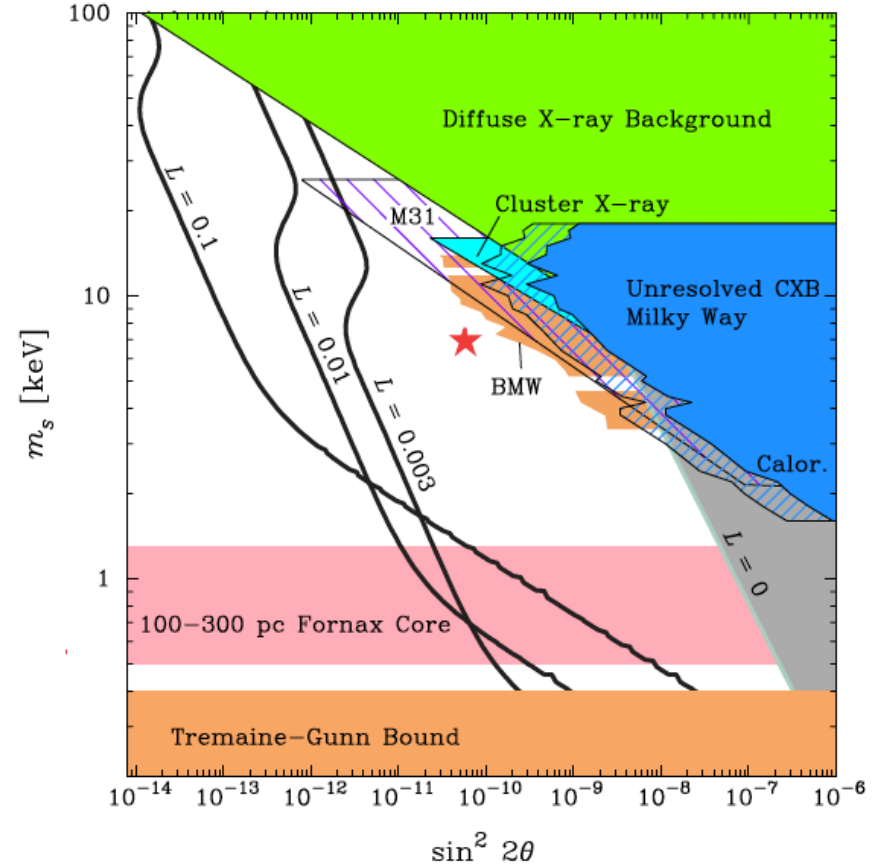
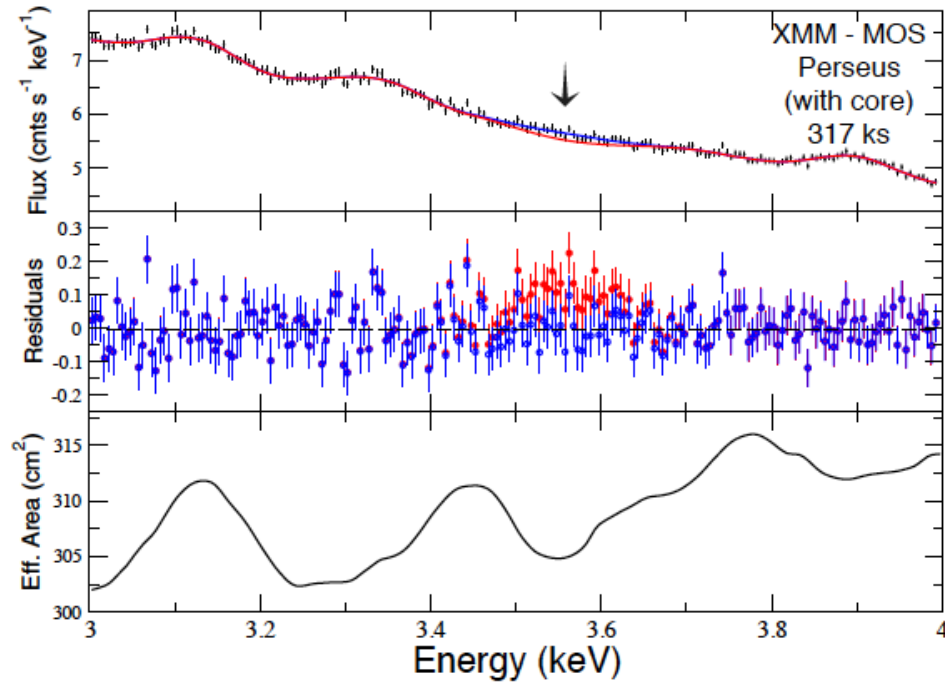
DETECTION OF AN UNIDENTIFIED EMISSION LINE IN THE STACKED X-RAY SPECTRUM OF GALAXY CLUSTERS

ESRA BULBUL^{1,2}, MAXIM MARKEVITCH², ADAM FOSTER¹, RANDALL K. SMITH¹ MICHAEL LOEWENSTEIN², AND SCOTT W. RANDALL¹

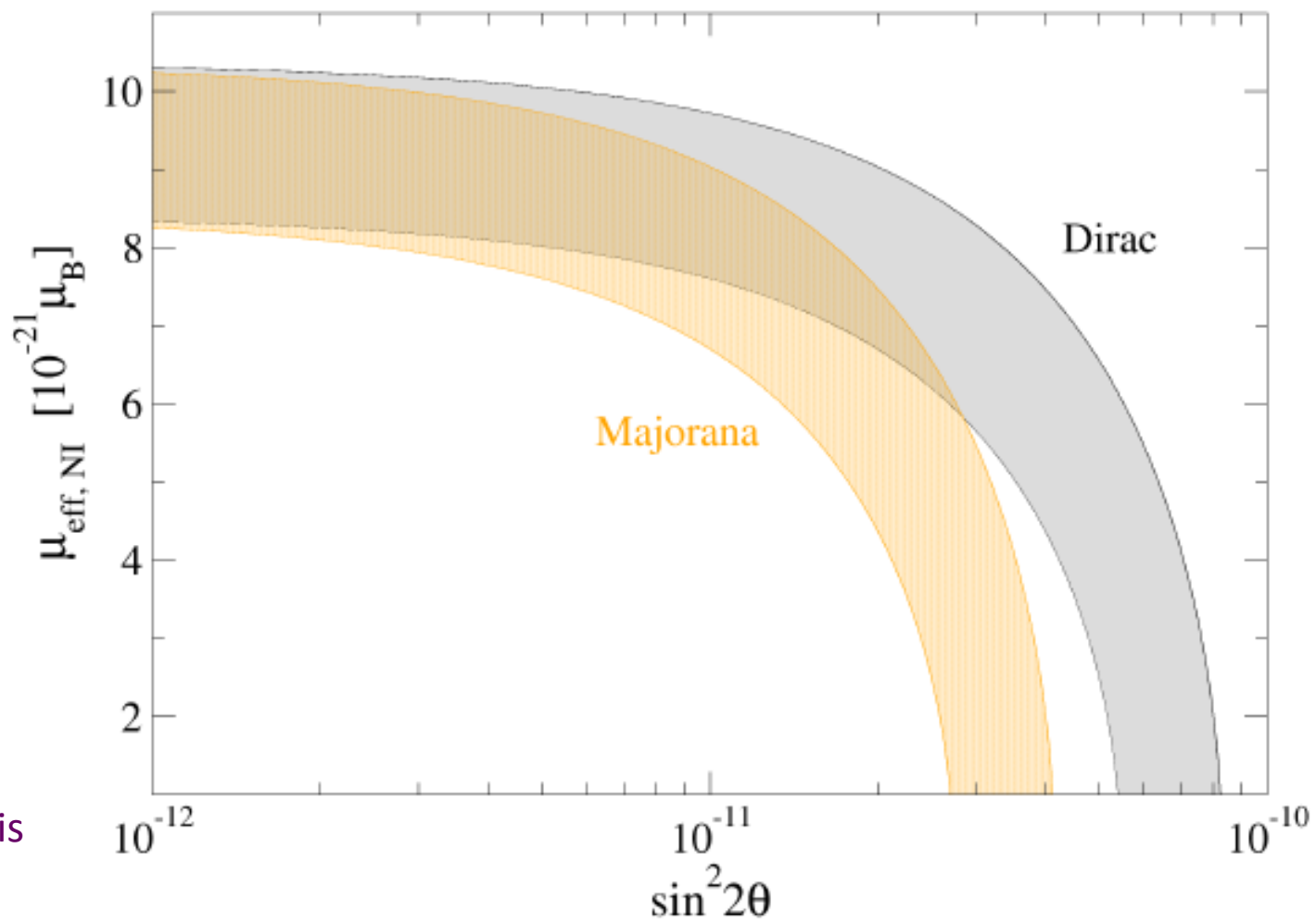
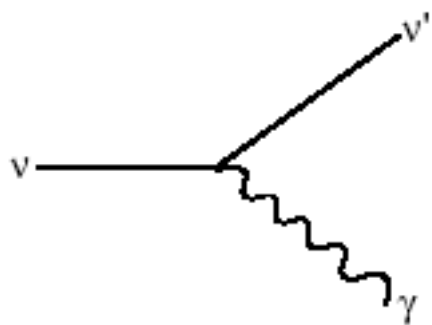
¹ Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138.

² NASA Goddard Space Flight Center, Greenbelt, MD, USA.

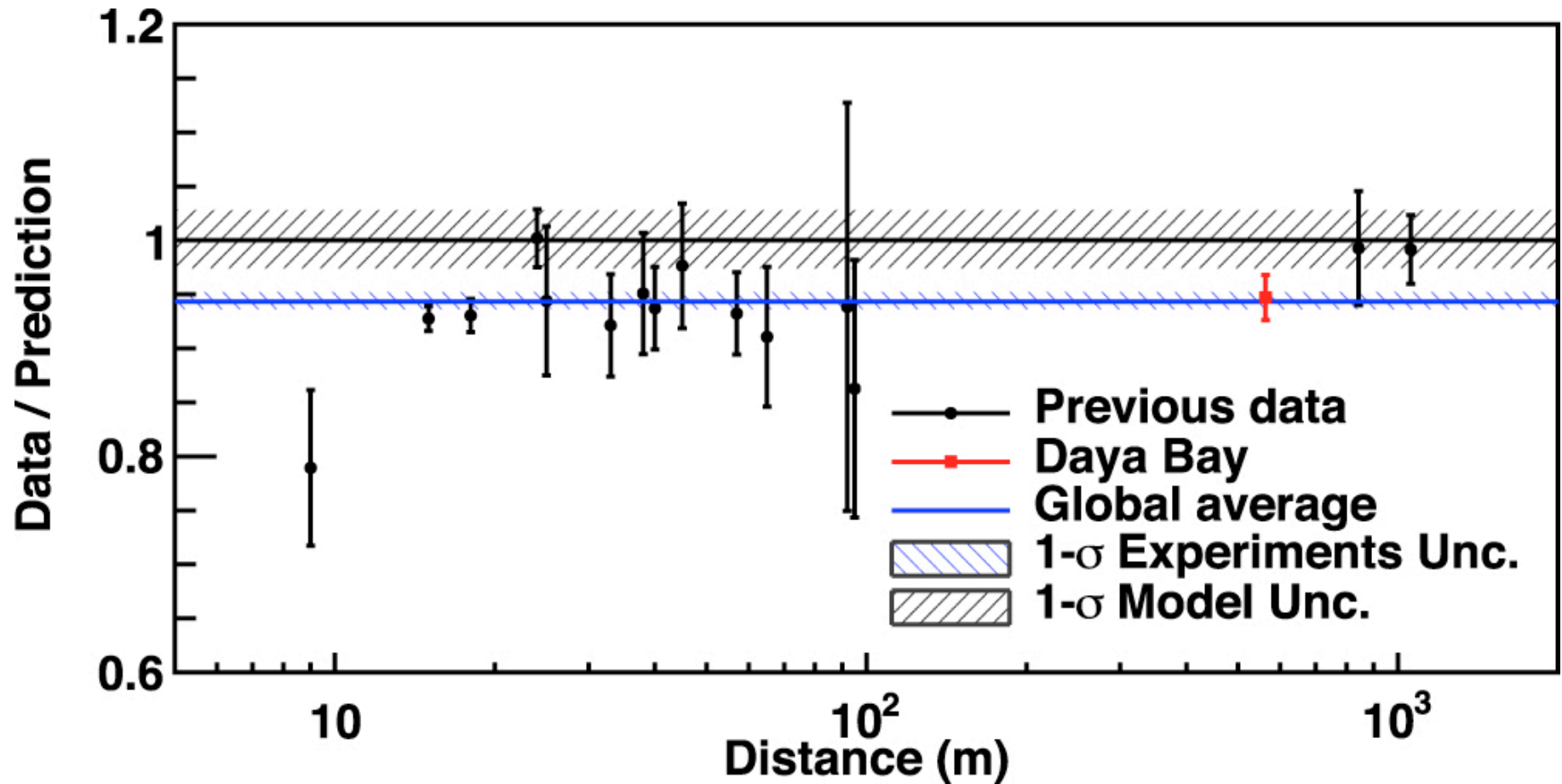
Submitted to ApJ, 2014 February 10



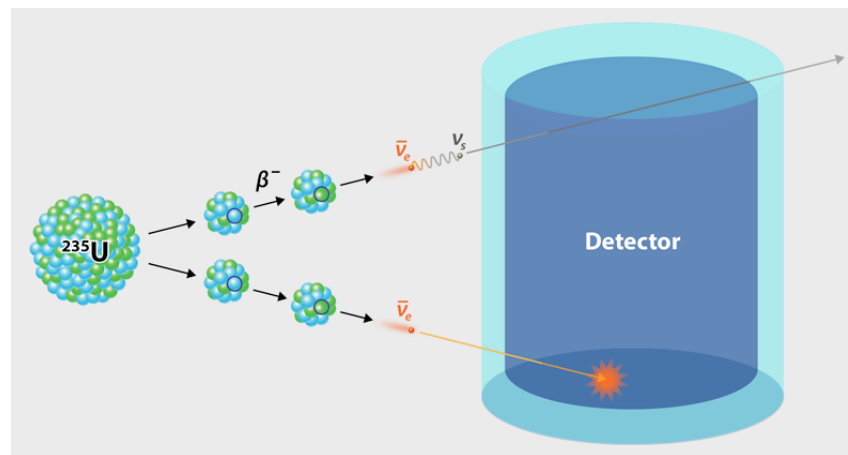
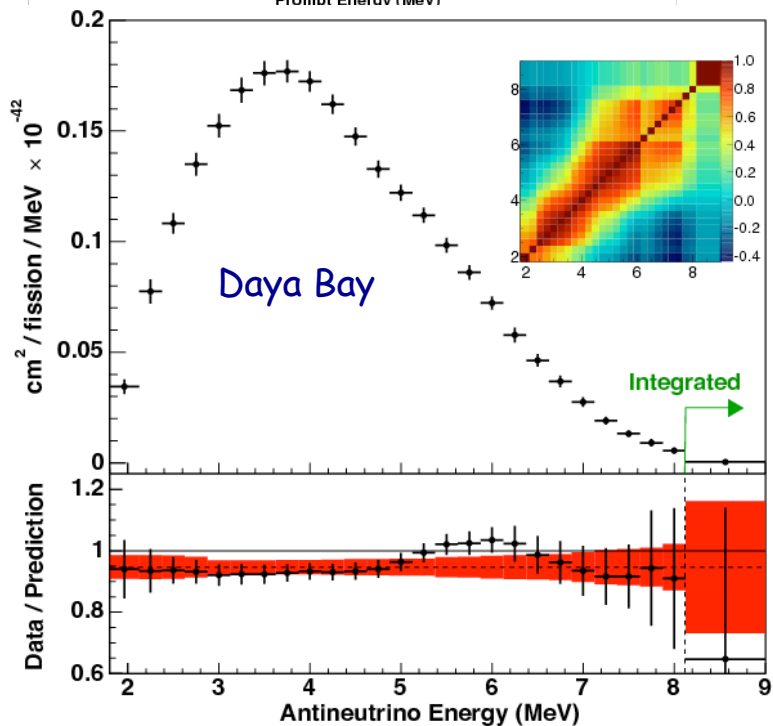
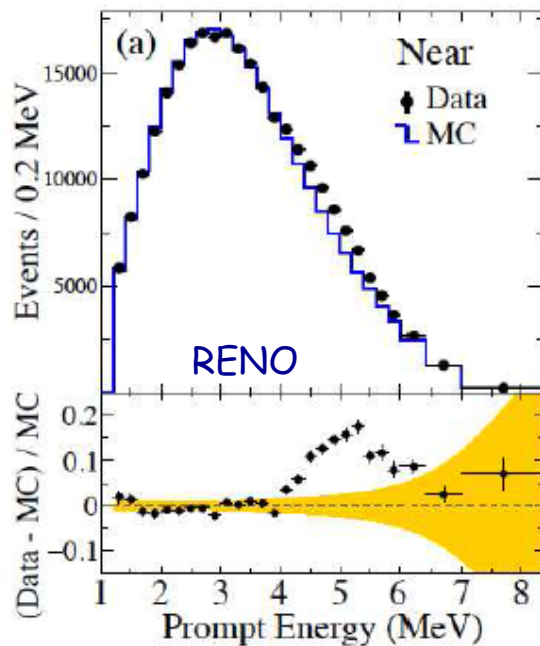
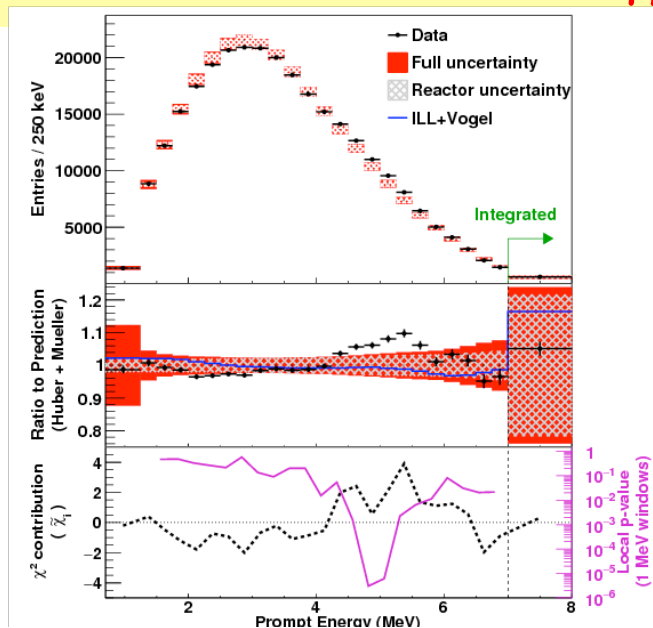
See also : [arXiv:1204.5477](https://arxiv.org/abs/1204.5477) [hep-ph],
[F. Bezrukov](#), [A. Kartavtsev](#), [M. Lindner](#)



"The reactor anomaly"

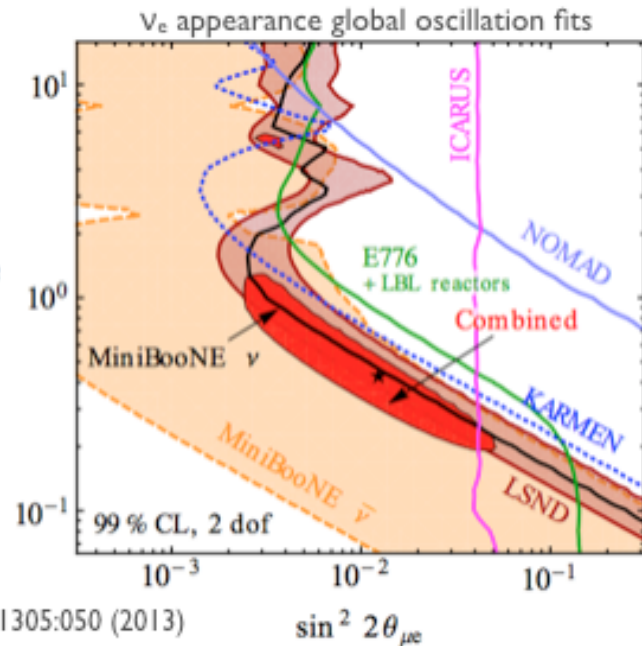
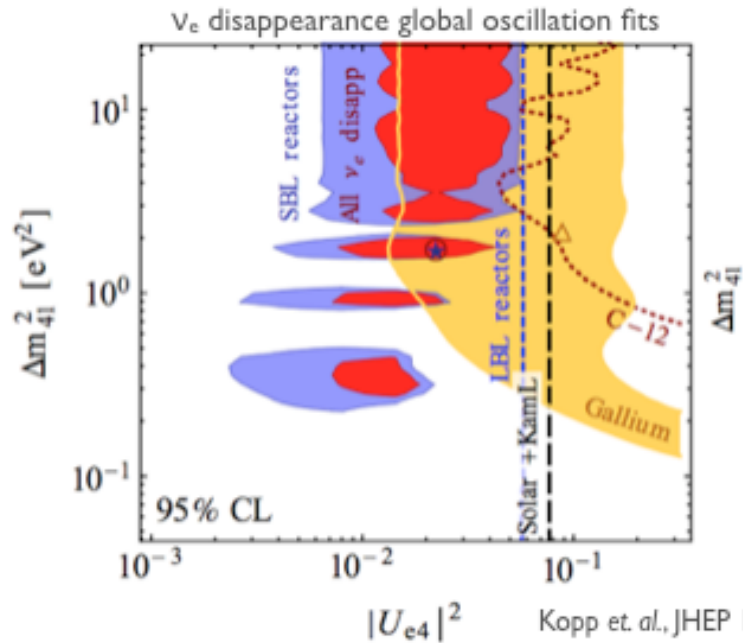
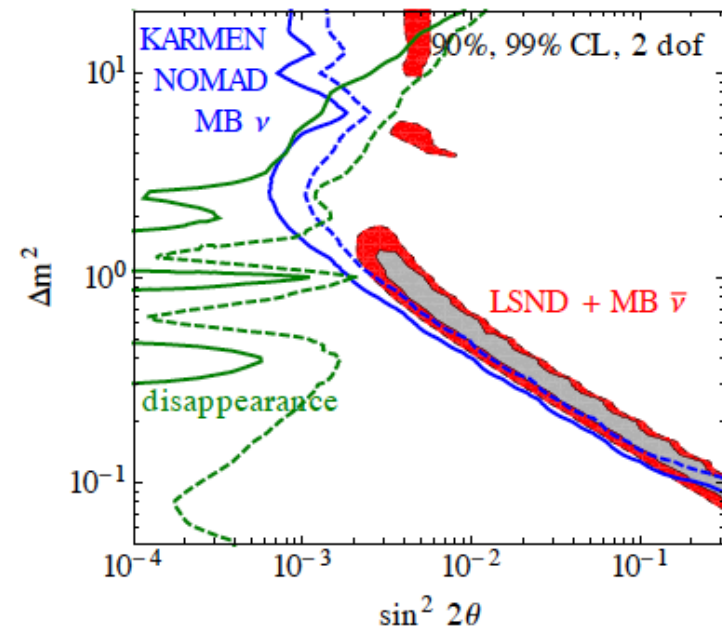
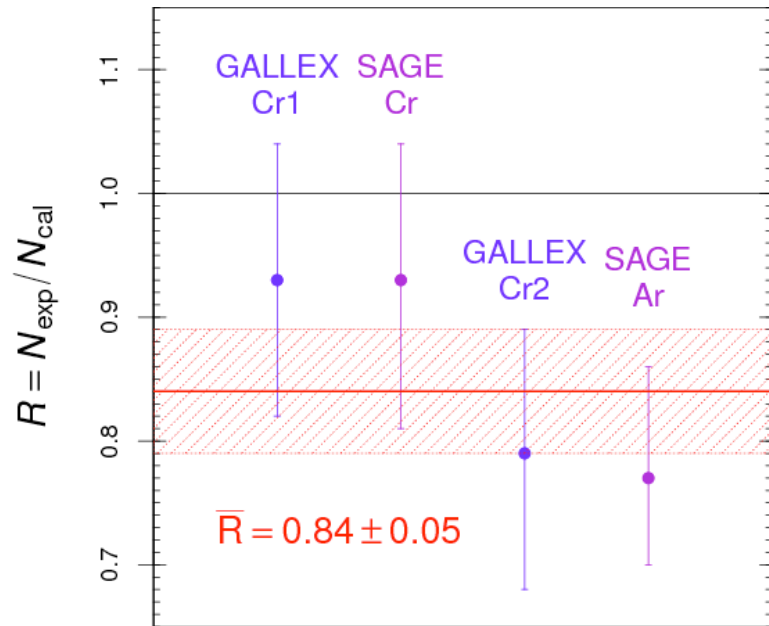


Then comes the bump!

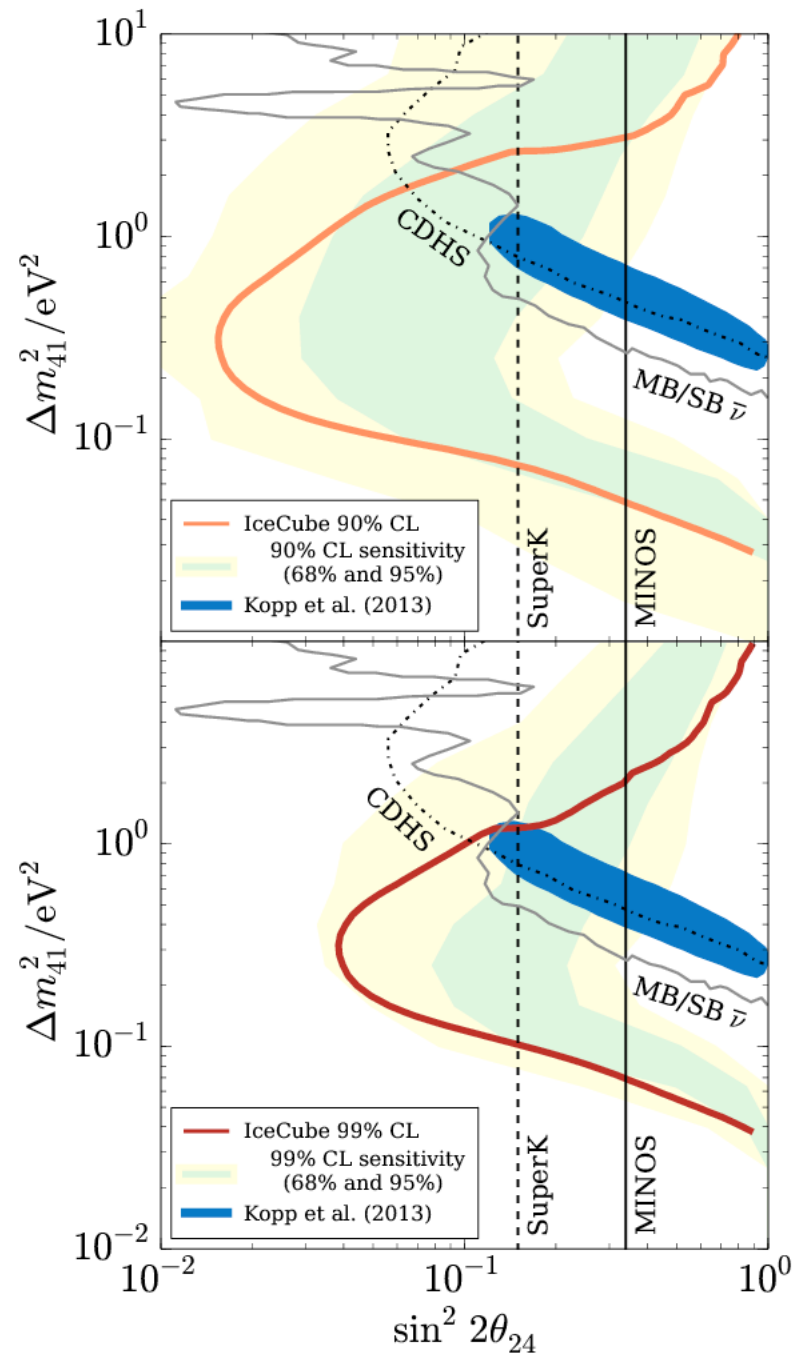
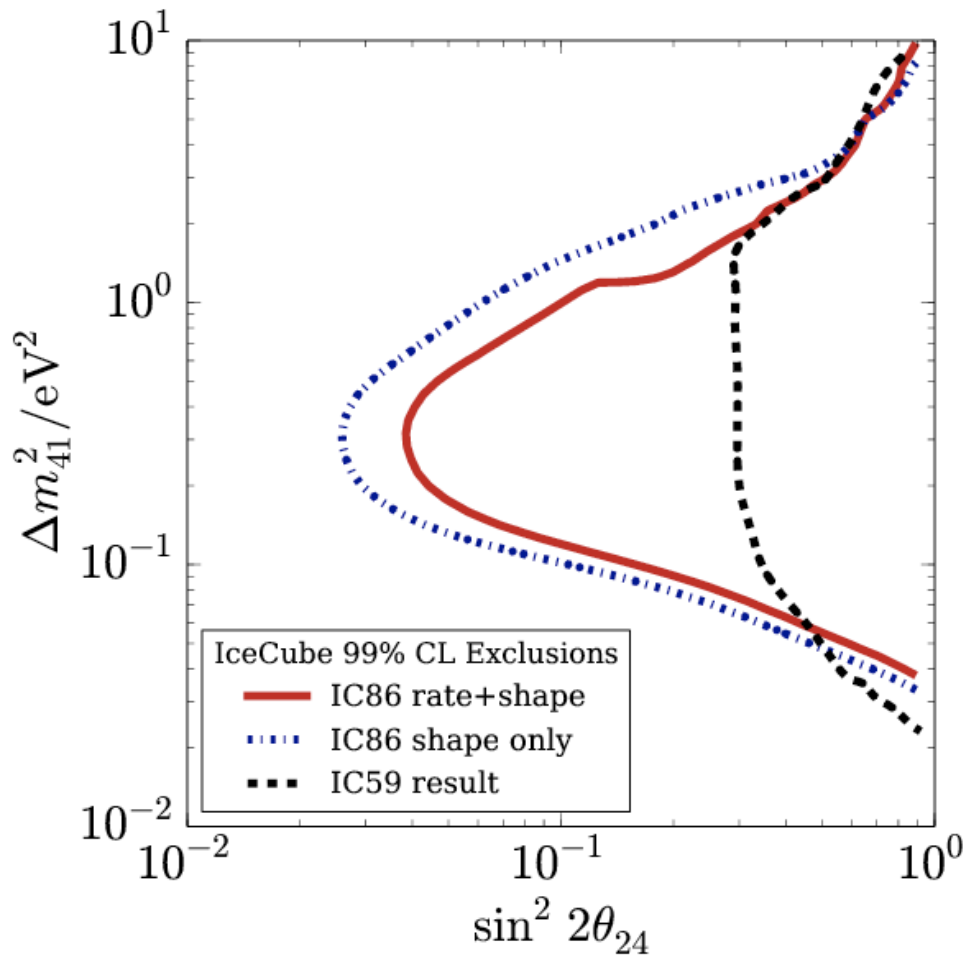


Source: APS

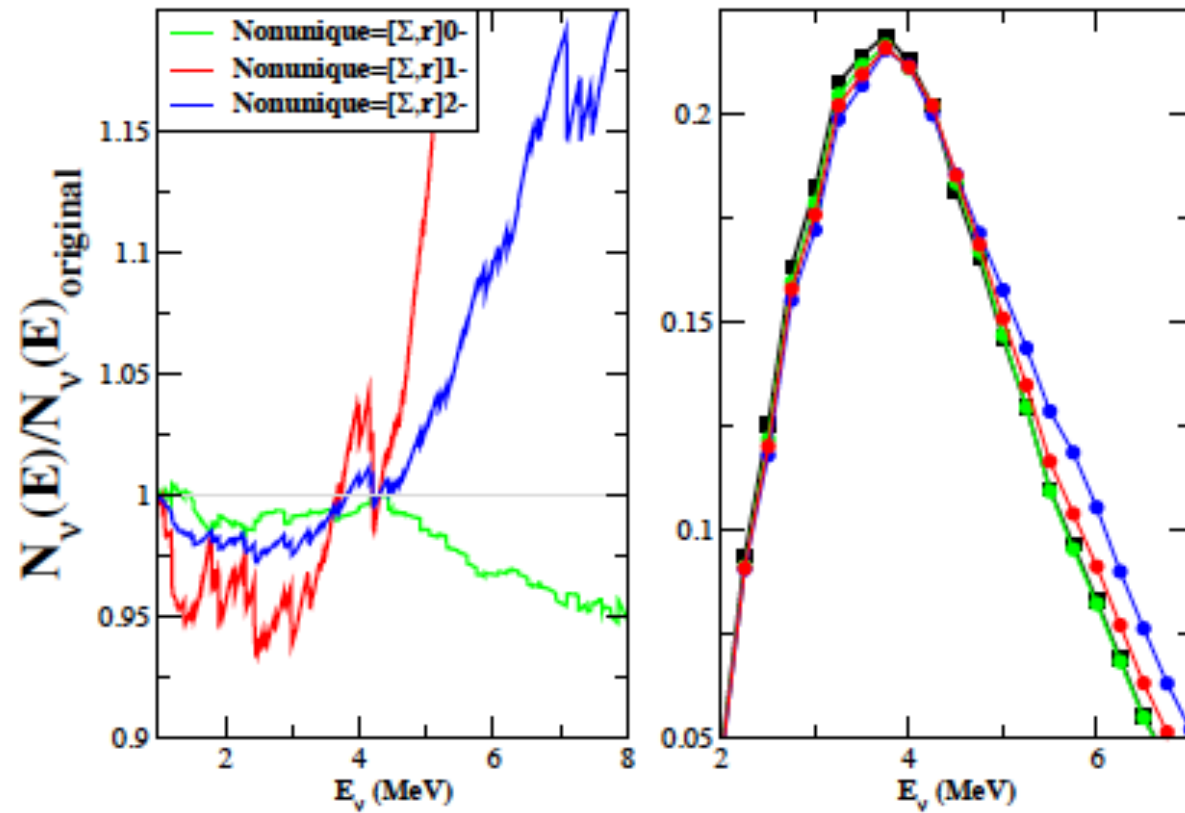
Does the reactor-flux anomaly imply active-sterile neutrino mixing?



Sterile Neutrino Limits
from ICECUBE - θ_{24}



Does the reactor-flux anomaly imply active-sterile neutrino mixing?

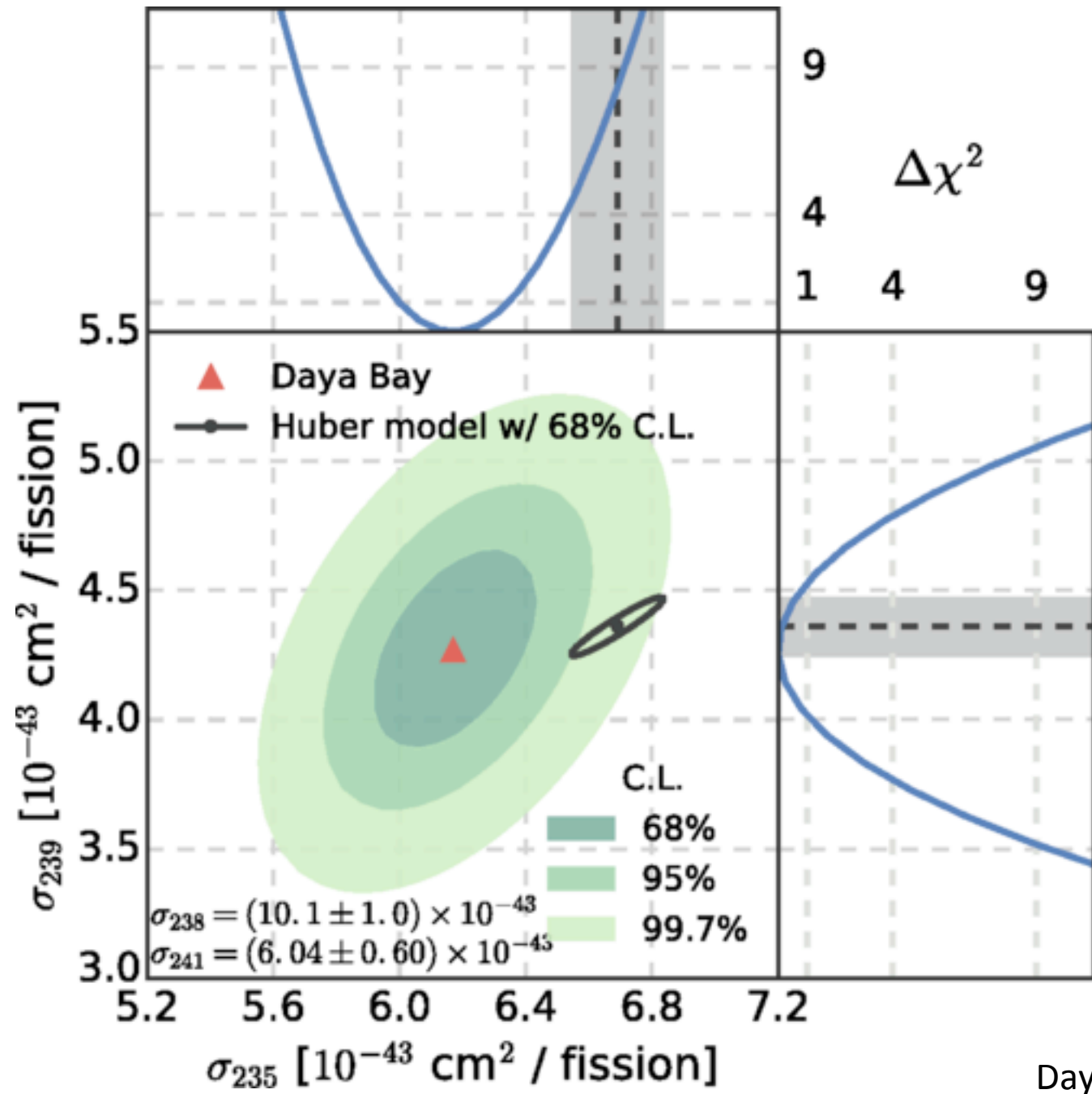


Hayes, et al., arXiv: 1309.4146 [nucl-th]

β -decays of many isotopes in a reactor are more complicated than we assumed:

Neutrino wave function:

$$e^{ikx} = \underbrace{1}_{\text{allowed app.}} + \underbrace{ikx}_{\text{first forbidden}} + \frac{1}{2} \underbrace{(ikx)^2}_{\text{second forbidden}} + \dots$$

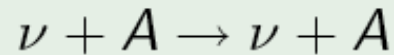


Questions about sterile neutrinos in no specific order

- Is there any $\bar{\nu}_\mu$ disappearance?
- Do both reactor and non-reactor $\bar{\nu}_e$'s disappear?
 - Is there visible oscillatory behavior?
- Can the sterile nature of the new flavors be established without recourse to the Z width?
 - Is there any associated CP violation?

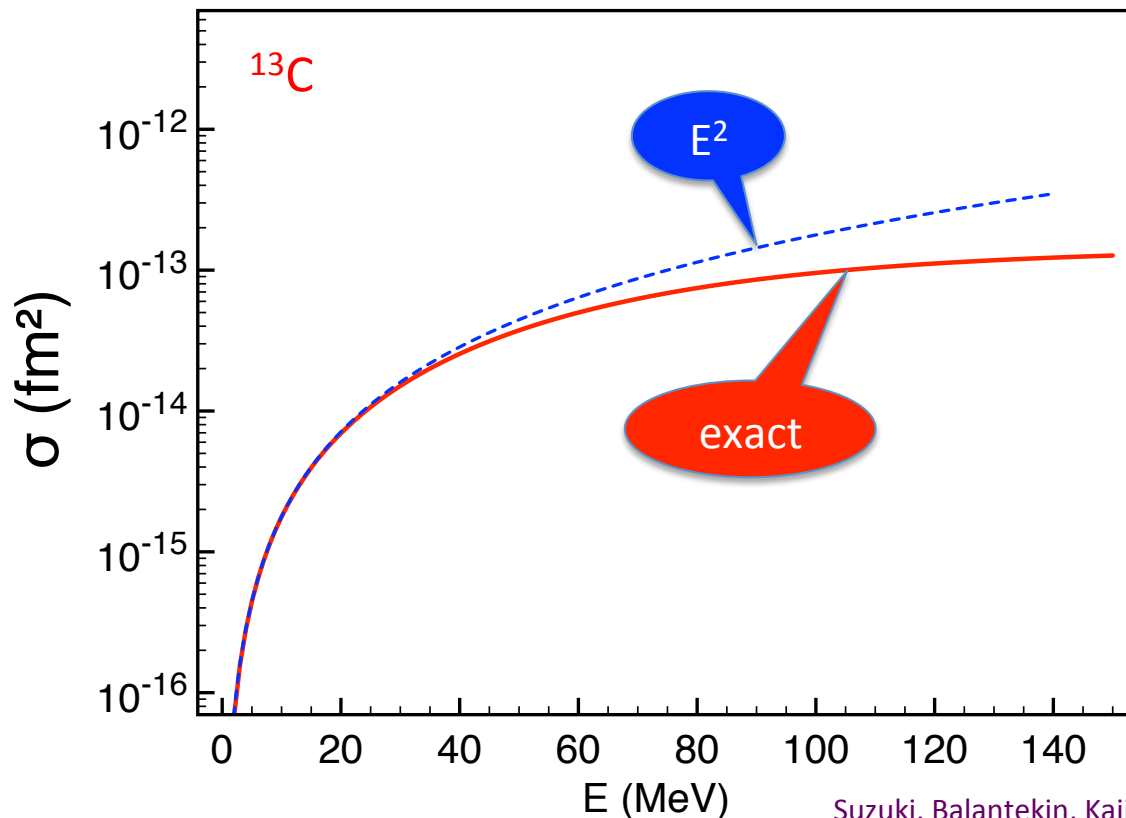
Oscillatory behavior of the neutral-current event rate, would establish, without recourse to the Z-width, oscillation into sterile flavor(s).

Neutrino Coherent Scattering

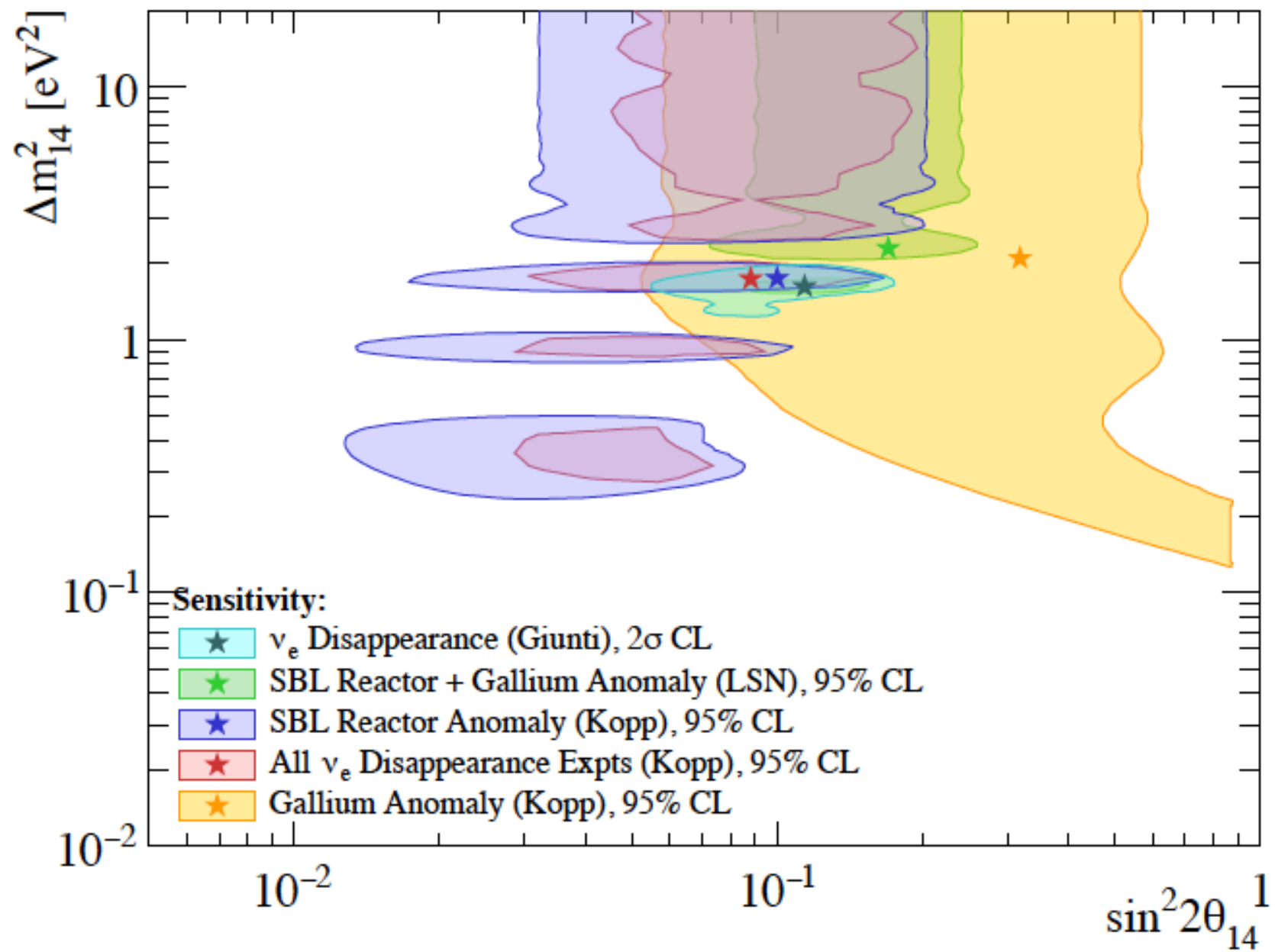


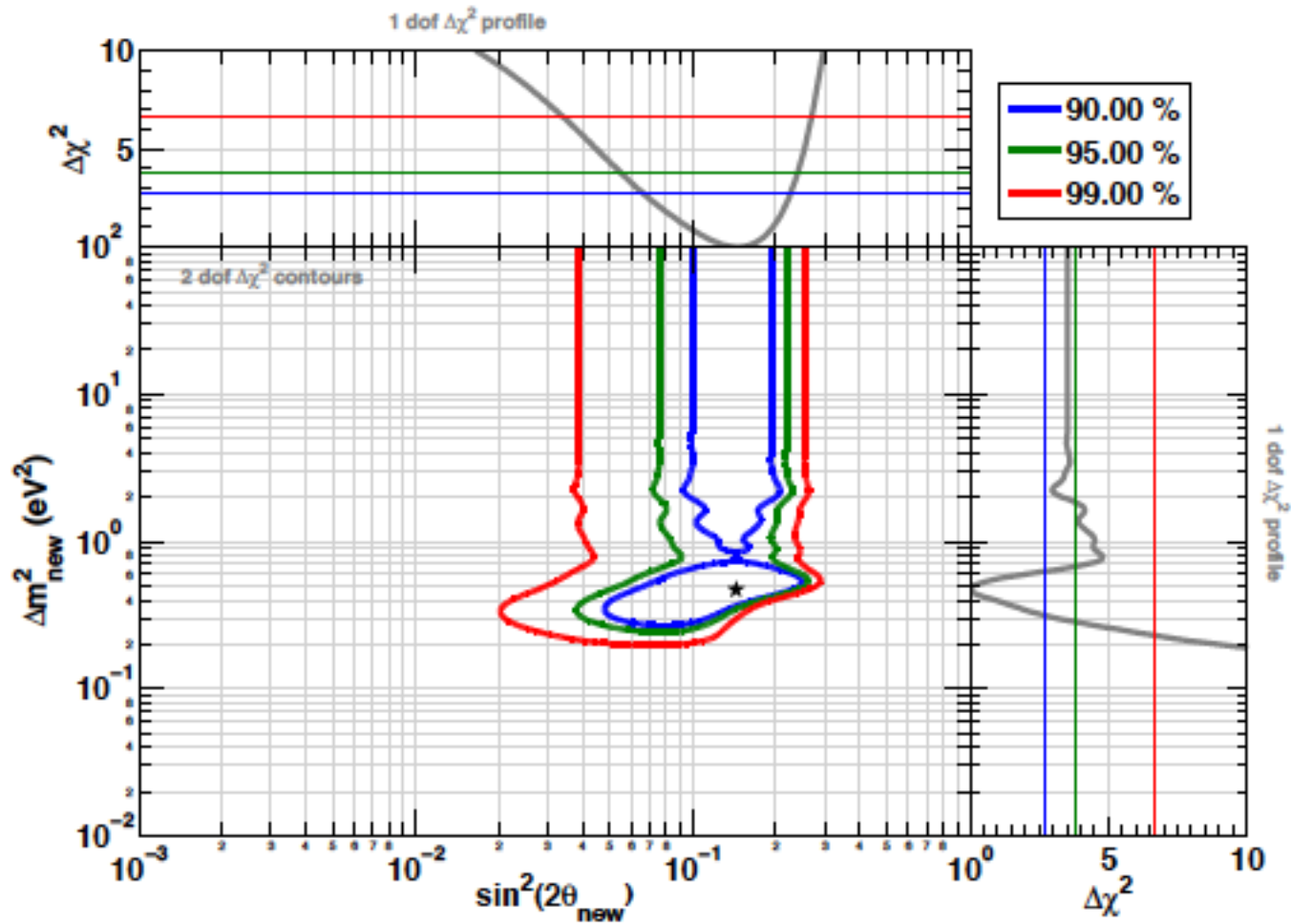
$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{8\pi} \{Z^2 (4\sin^2\theta_W - 1) + N\}^2 E_\nu^2 (1 + \cos\theta)$$

$$T_{\text{av. recoil}} = \frac{2}{3A} \left(\frac{E_\nu}{\text{MeV}} \right) \text{keV}$$



- First calculated by Freedman.
- This reaction is background to the dark matter searches with nuclear targets.
- Nuclear form factors need to be included. *McLaughlin, Engel.*
- A calculation for scintillators with the state-of-the-art nuclear interactions is shown on the left.





At very close distances to the reactor
and for $m_4^2 \geq 1 \text{ eV}^2$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 2|U_{e4}|^2 + 2|U_{e4}|^4$$

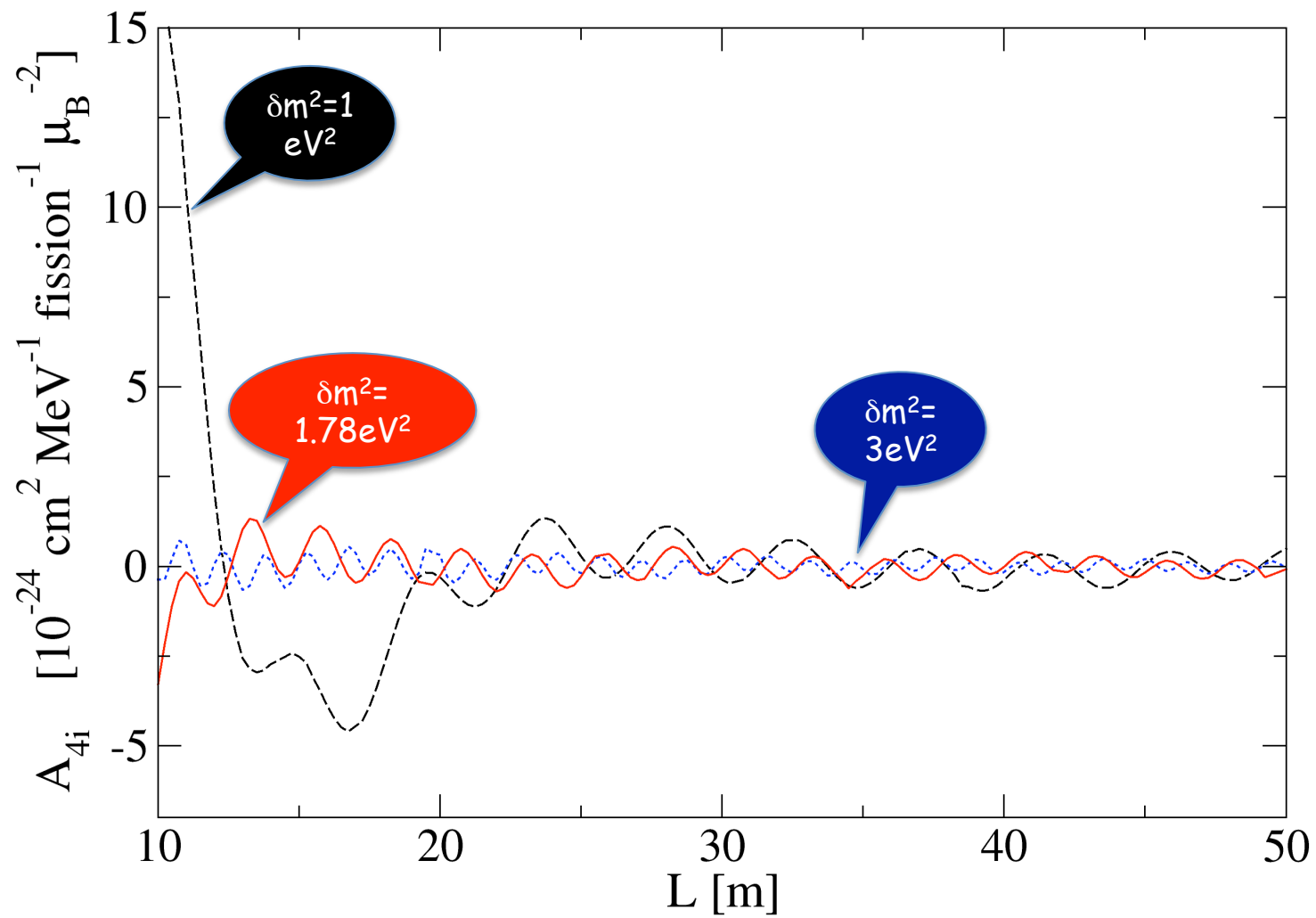
Kopp

$$\frac{d\sigma}{dT_e} = \frac{\alpha^2 \pi}{m_e^2} \mu_{\text{eff}}^2 \left[\frac{1}{T_e} - \frac{1}{E_\nu} \right]$$

$$\mu_{\text{eff}}^2 = \sum_i \left| \sum_j U_{ej} e^{-iE_j L} \mu_{ji} \right|^2$$

For a sufficiently heavy sterile neutrino the phases with $(E_4 - E_i)L$ average to zero

$$\mu_{\text{eff}}^2 = \sum_{i,j=1}^3 \left[U_{ei} (\mu\mu^+)_{ij} U_{je}^+ \right] + U_{e4} (\mu\mu^+)_{44} U_{4e}^+$$



$$A_{4i} = \int_{E_{\nu, \min}}^{\infty} \frac{2\alpha^2 \pi}{m_e^2} \left[\frac{1}{T_e} - \frac{1}{E_{\nu}} \right] \left[\cos \left(\frac{\delta m_{4i}^2 L}{2E_{\nu}} \right) \right] \left(\frac{dN}{dE_{\nu}} \right) dE_{\nu}$$

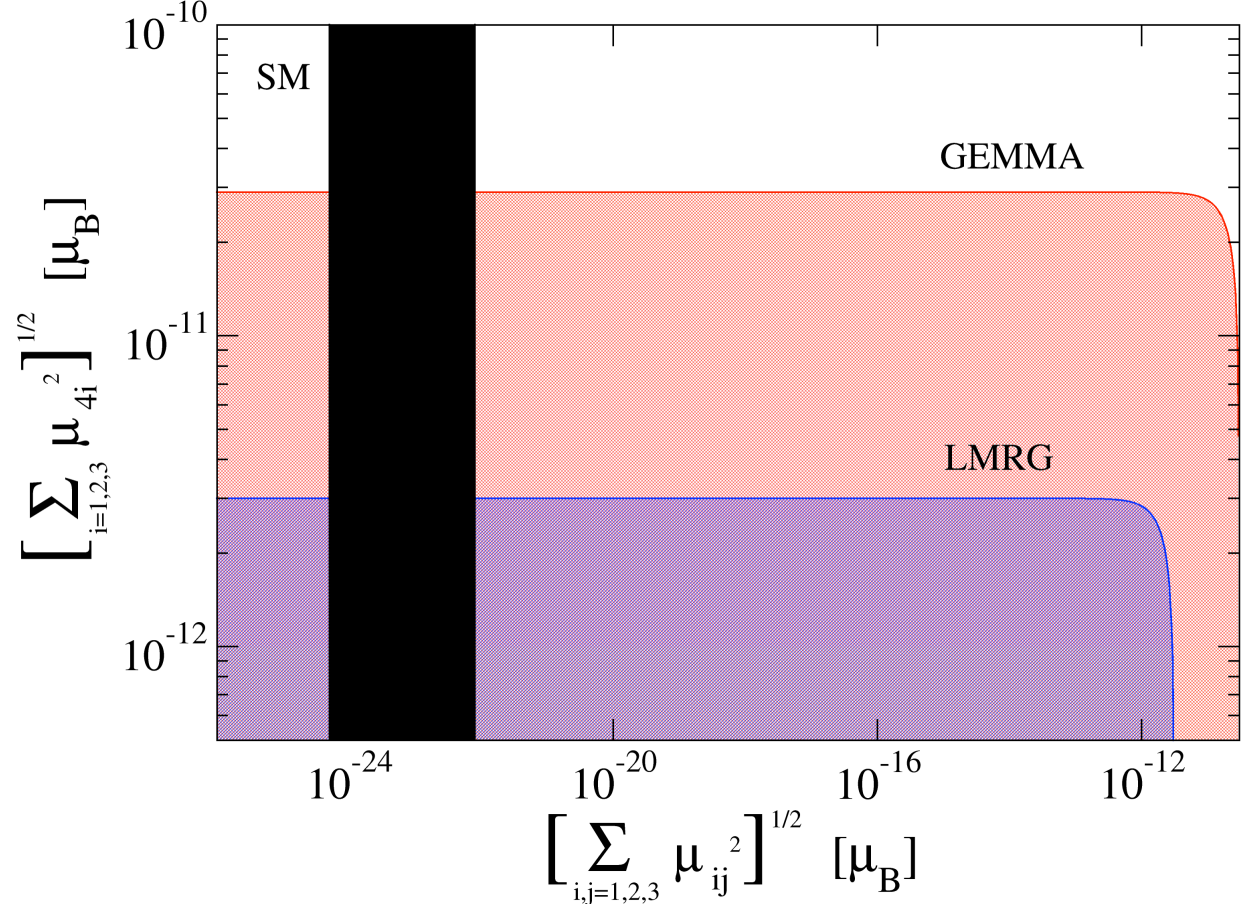
For a sufficiently heavy sterile neutrino the phases with $(E_4 - E_i)L$ average to zero

$$\mu_{\text{eff}}^2 = \sum_{i,j=1}^3 \left[U_{ei} (\mu\mu^+)_{ij} U_{je}^+ \right] + U_{e4} (\mu\mu^+)_{44} U_{4e}^+$$

$$\Rightarrow \mu_{\text{eff}}^2 \leq \sum_{i=1}^3 \mu_{i4}^2 + \left(1 - |U_{e4}|^2\right) \sum_{i,j=1}^3 \mu_{ij}^2$$

$$\frac{d\sigma}{dT_e} = \frac{\alpha^2 \pi}{m_e^2} \mu_{\text{eff}}^2 \left[\frac{1}{T_e} - \frac{1}{E_\nu} \right]$$

$$\mu_{\text{eff}}^2 = \sum_i \left| \sum_j U_{ej} e^{-iE_j L} \mu_{ji} \right|^2$$



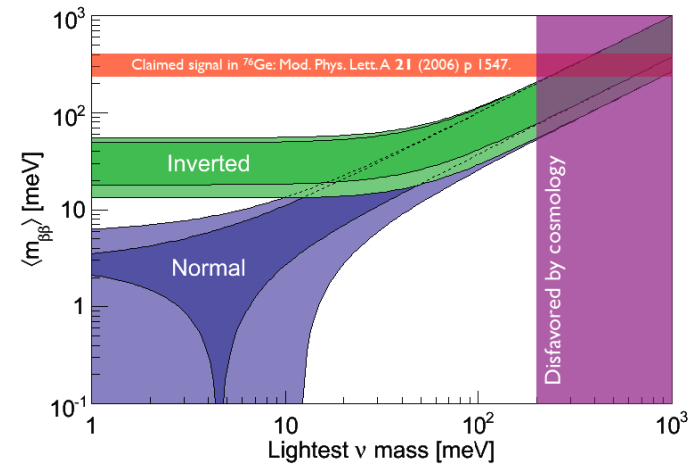
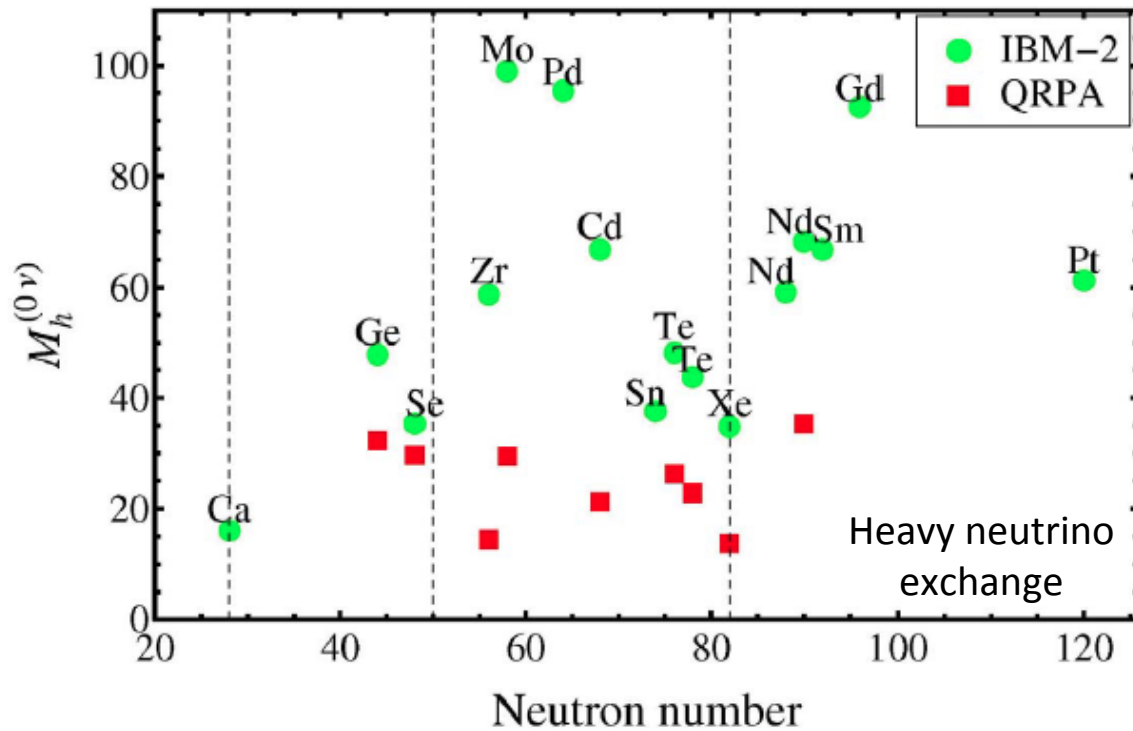
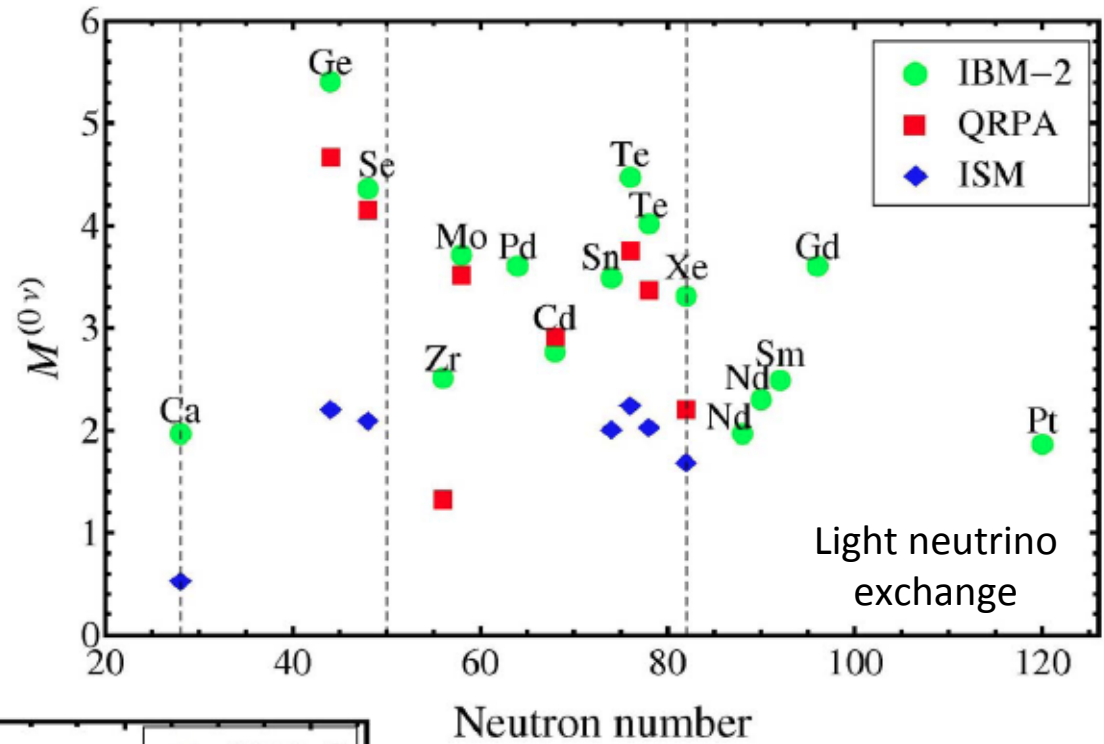
0ν double beta decay

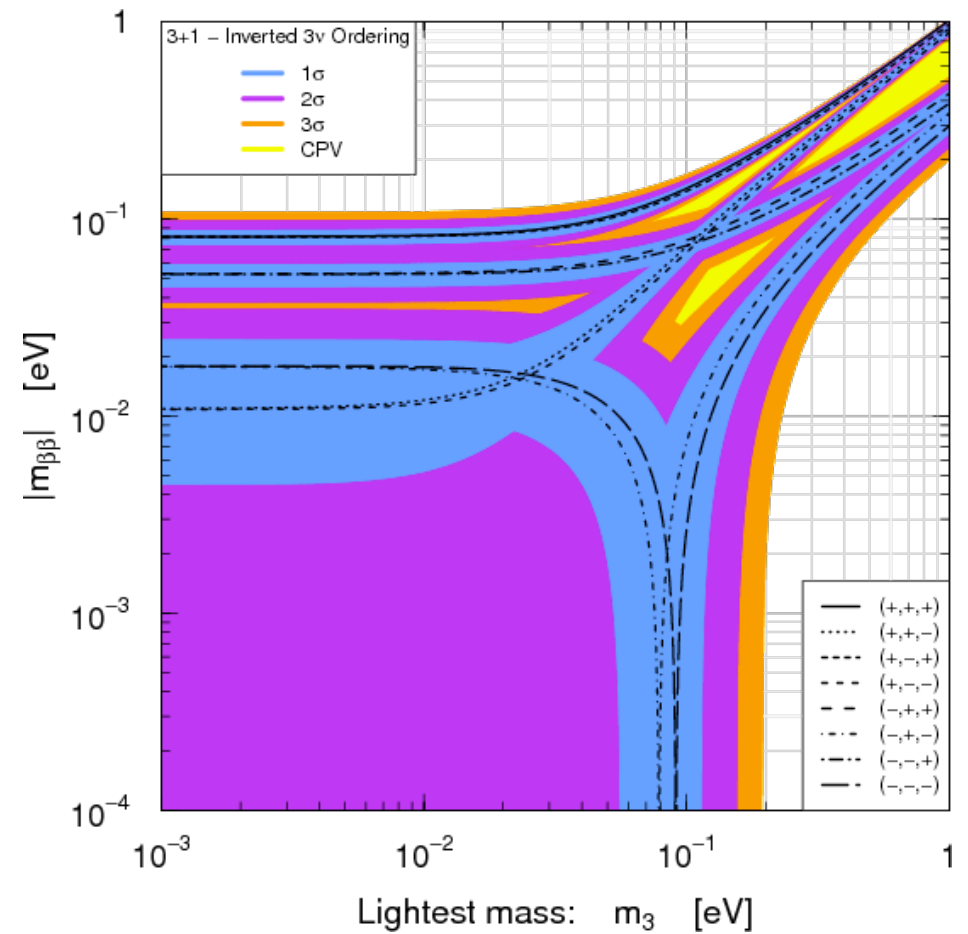
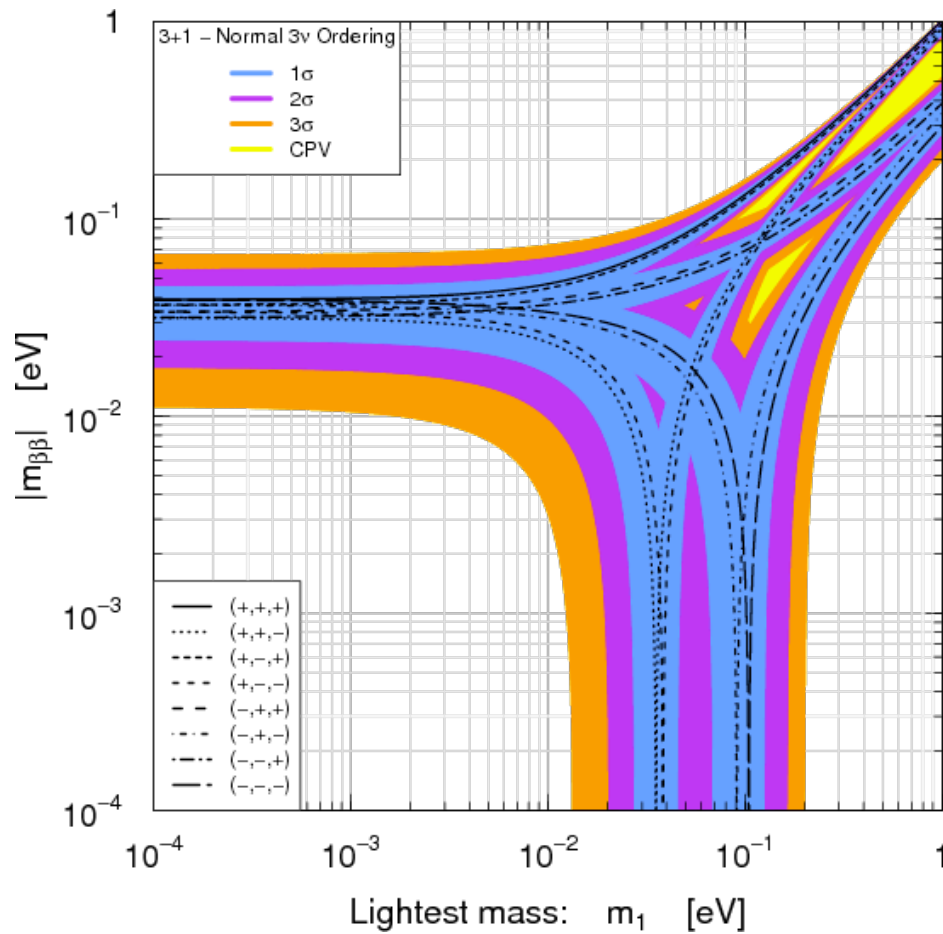
$$(1/T_{1/2}) = G(E,Z) M^2 \langle m_{\beta\beta} \rangle^2$$

$G(E,Z)$: phase space

M : nuclear matrix element

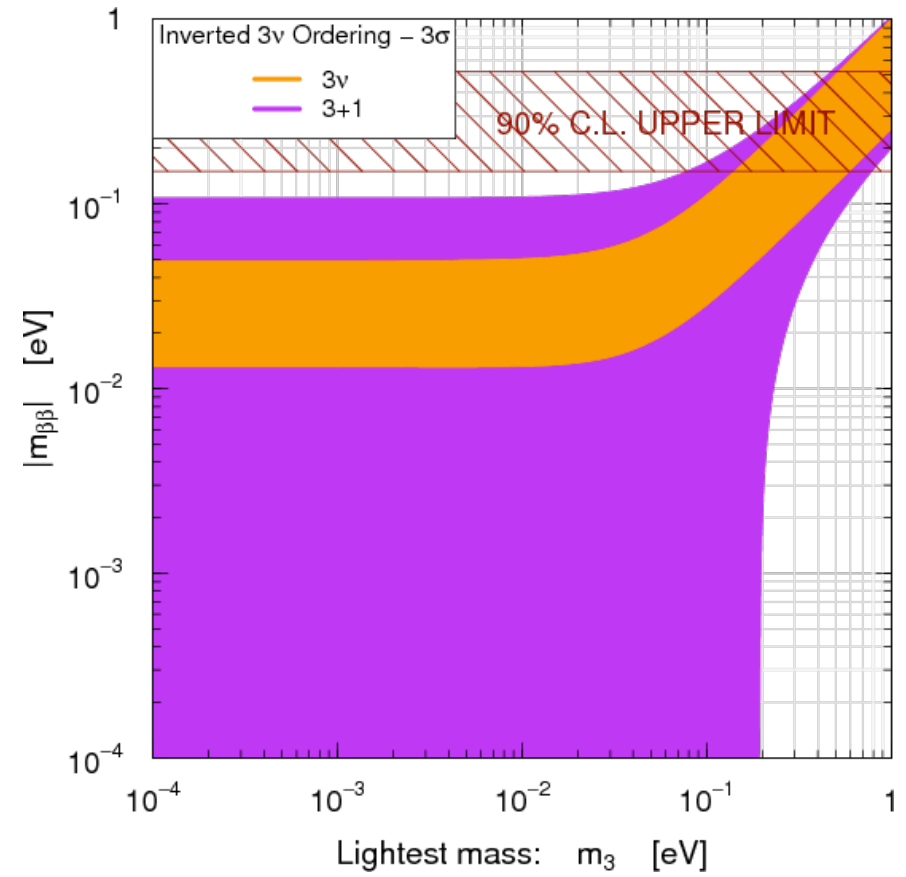
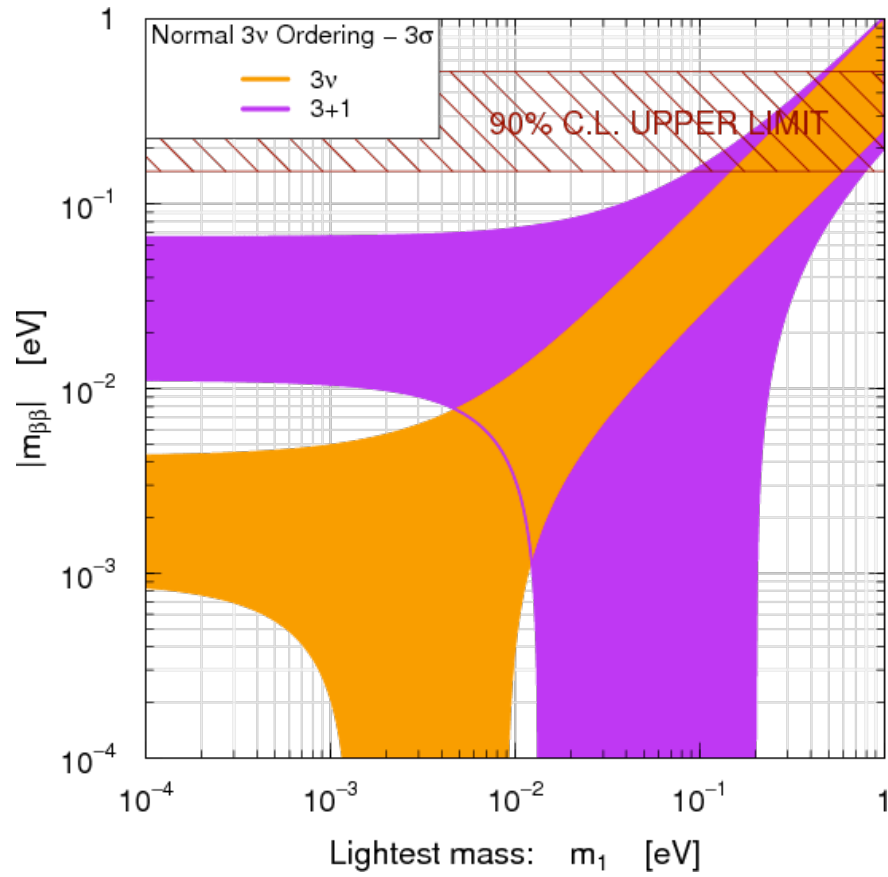
$$\langle m_{\beta\beta} \rangle = \left| \sum_j |U_{ej}|^2 m_j e^{i\delta(j)} \right|$$





Giunti and Zavanin

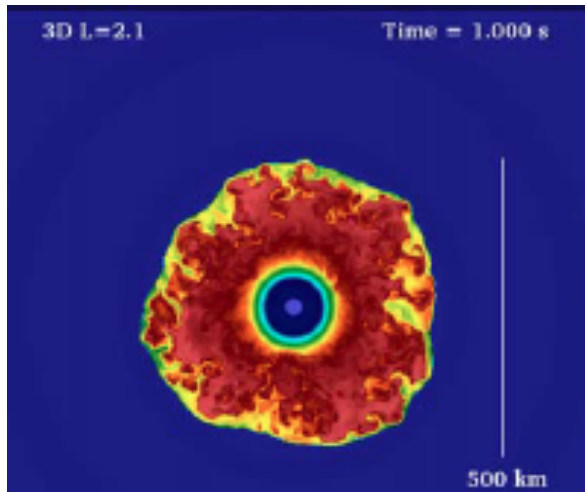
A positive result would be consistent with 3+1 light active neutrinos and NH, IH, and quasi-degenerate scenario, but not definitive as to mechanism



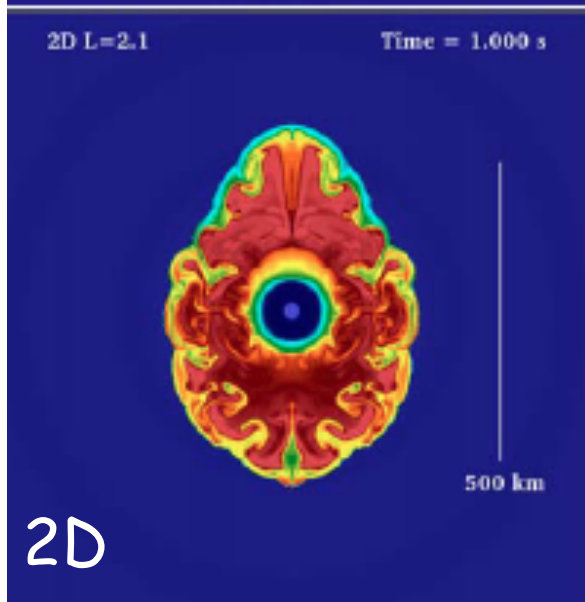
Giunti

$$|m_{\beta\beta}| = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3 + |U_{e4}|^2 e^{i\alpha_4} m_4 \right|$$

Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.

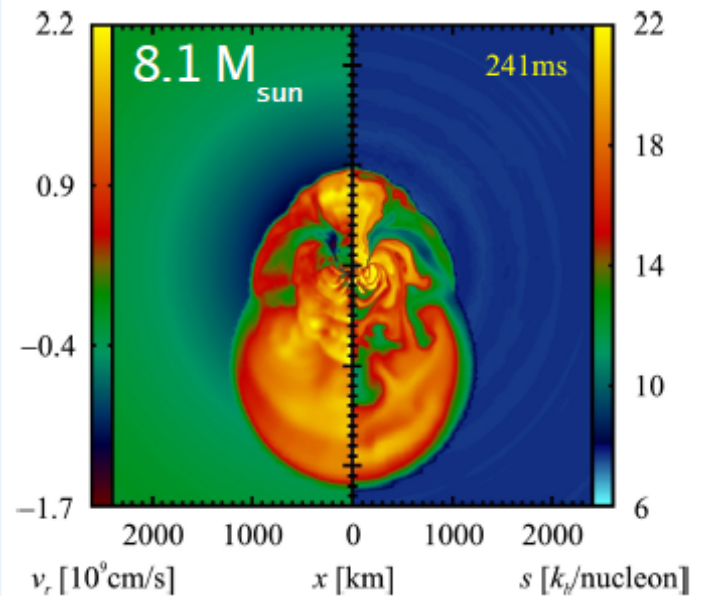
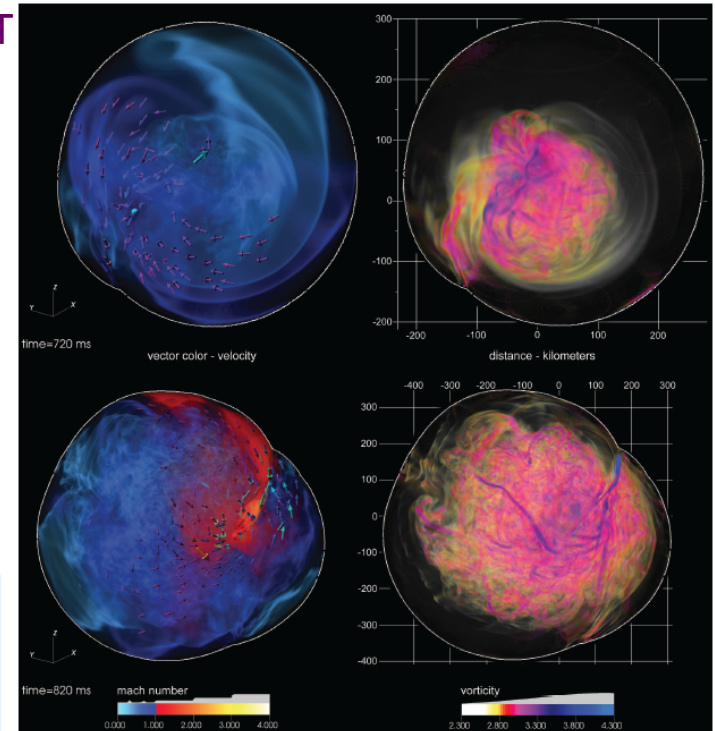
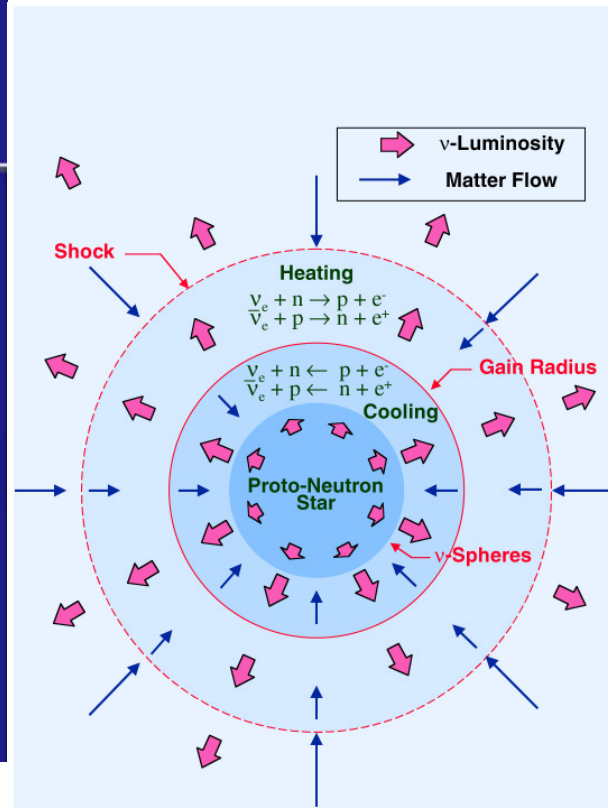


3D



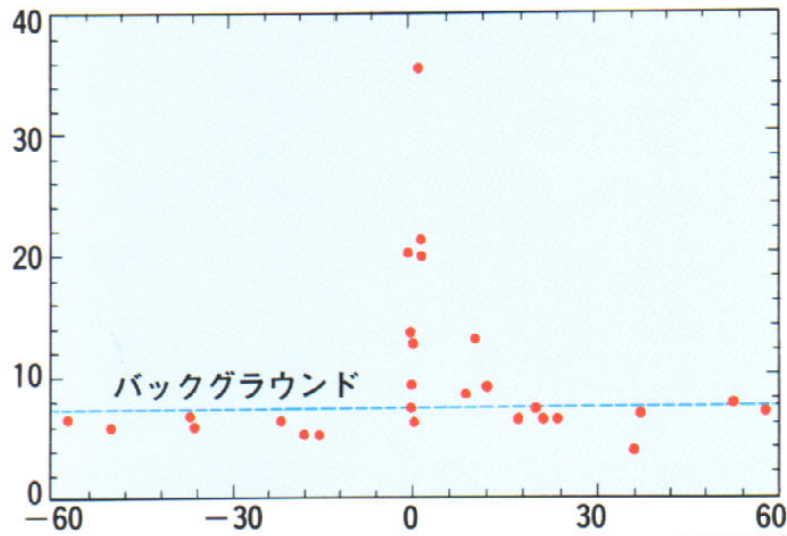
2D

Princeton



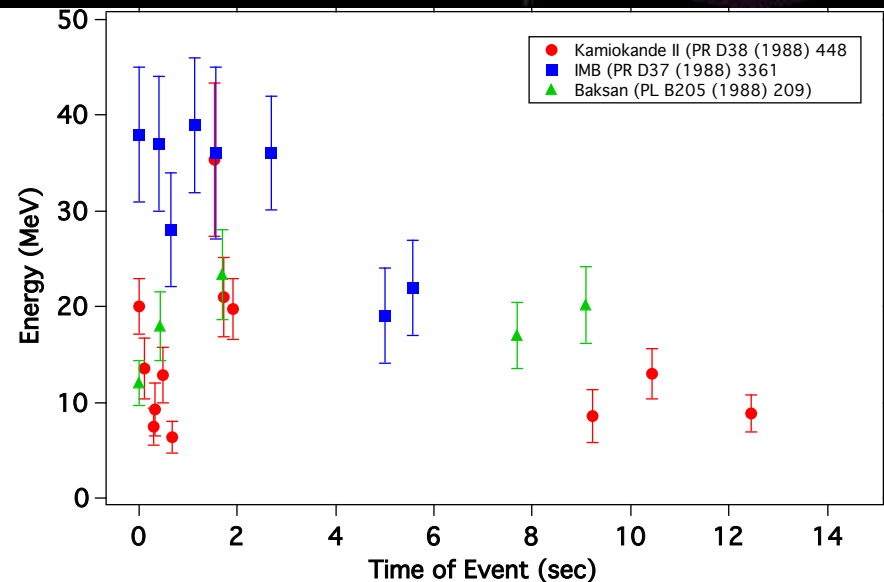
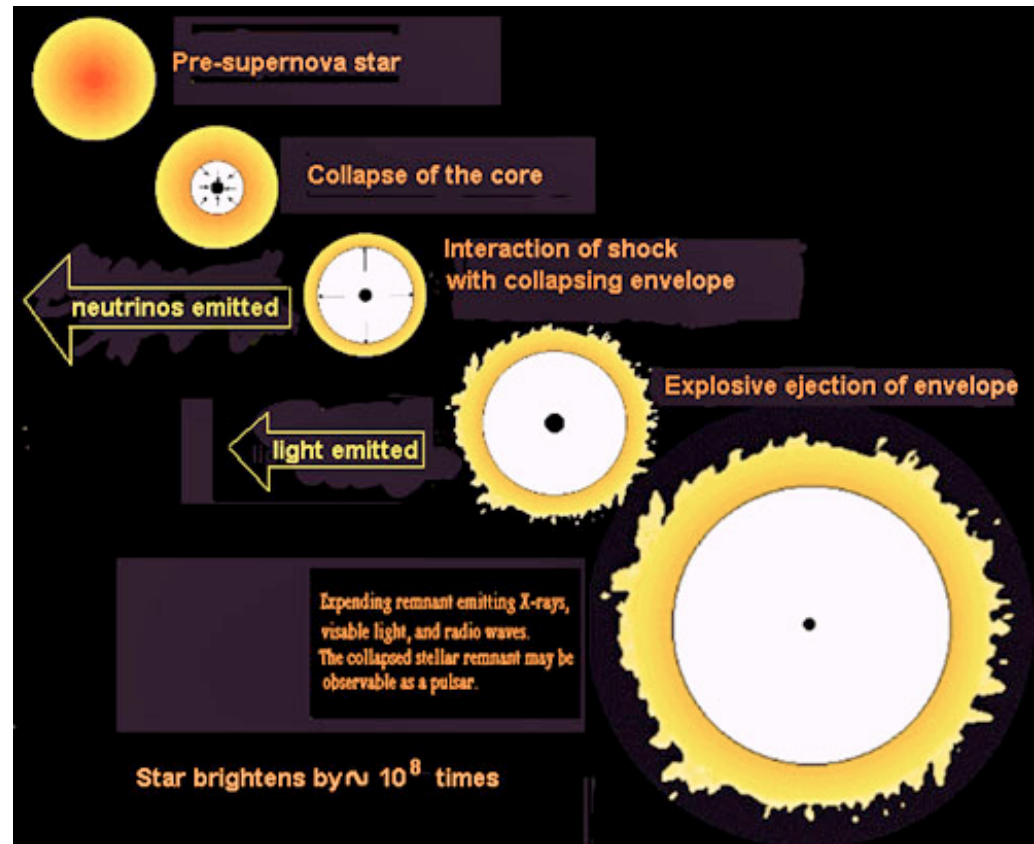
Munich

Neutrinos from core-collapse supernovae



• $M_{\text{prog}} \geq 8 M_{\text{sun}} \Rightarrow \Delta E \approx 10^{53} \text{ ergs} \approx 10^{59} \text{ MeV}$

• 99% of the energy is carried away by neutrinos and antineutrinos with $10 \leq E_{\nu} \leq 30 \text{ MeV} \Rightarrow 10^{58}$ neutrinos



CP-violation

$$T_{23}T_{13}T_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}$ $s_{ij} = \sin \theta_{ij}$

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \tilde{\psi}_\mu \\ \tilde{\psi}_\tau \end{pmatrix} = \left[T_{13}T_{12} \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger + \begin{pmatrix} V_{e\mu} & 0 & 0 \\ 0 & s_{23}^2 V_{\tau\mu} & -c_{23}s_{23} V_{\tau\mu} \\ 0 & -c_{23}s_{23} V_{\tau\mu} & c_{23}^2 V_{\tau\mu} \end{pmatrix} \right] \begin{pmatrix} \psi_e \\ \tilde{\psi}_\mu \\ \tilde{\psi}_\tau \end{pmatrix}$$

$$\tilde{\psi}_\mu = \cos \theta_{23} \psi_\mu - \sin \theta_{23} \psi_\tau$$

$$\tilde{\psi}_\tau = \sin \theta_{23} \psi_\mu + \cos \theta_{23} \psi_\tau$$

$$V_{e\mu} = 2\sqrt{2}G_F N_e \left[1 + O\left(\alpha \frac{m_\mu}{m_W}\right)^2 \right]$$

$$V_{\tau\mu} = -\frac{3\sqrt{2}\alpha G_F}{\pi \sin^2 \theta_W} \left(\frac{m_\tau}{m_W}\right)^2 \left[(N_p + N_n) \log \frac{m_\tau}{m_W} + \left(\frac{N_p}{2} + \frac{N_n}{3}\right) \right]$$

We need to solve an evolution equation

$$i \frac{\partial}{\partial t} U = H U$$

If we ignore $V_{\tau\mu}$ it is easy to show that the CP-violating phase factorizes:

$$U(\delta) = S U(\delta = 0) S^\dagger \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

This factorization still holds when collective oscillations are included, but breaks down if there is spin-flavor precession

This factorization implies that neither

$$P(\nu_e \rightarrow \nu_e)$$

nor

$$P(\nu_\mu \rightarrow \nu_e) + P(\nu_\tau \rightarrow \nu_e)$$

depend on the CP-violating phase δ .

If the ν_μ and ν_τ luminosities are the same at the neutrinosphere of a core-collapse supernova, this factorization implies that ν_e and $\bar{\nu}_e$ fluxes observed at terrestrial detectors will not be sensitive to the CP-violating phase! To see its effects you need to measure ν_μ and ν_τ luminosities separately!

If you see the effects of δ in either charged- or neutral current scattering that may mean any of the following:

- There are new neutrino interactions beyond the standard model operating either within the neutron star or during propagation.
- Standard Model loop corrections (very easy to quantify) are seen.
- There are sterile neutrino states.

Factorization of the CP-violating phase if there are no sterile neutrinos

$$H(\delta) = H_\nu + H_{\nu\nu} = \mathbf{S}H(\delta = 0)\mathbf{S}^\dagger$$
$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

Holds if neutrino magnetic moment is ignored.

- MSW Hamiltonian: Balantekin, Gava, Volpe, Phys. Lett B662, 396 (2008).
- Collective Hamiltonian in the mean-field approximation: Gava, Volpe, Phys. Rev. D78, 083007 (2008).
- Exact collective Hamiltonian: Pehlivan, Balantekin, Kajino, Phys. Rev. D90, 065011 (2014).

Collective oscillations of three flavors with magnetic moment

Neutrinos: $T_{ij}(|\mathbf{p}|, \mathbf{p}) = a_i^\dagger(\mathbf{p})a_j(\mathbf{p})$

Antineutrinos: $T_{ij}(-|\mathbf{p}|, \mathbf{p}) = -b_j^\dagger(\mathbf{p})b_i(\mathbf{p})$

$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}V} \sum_{i,j=1}^3 \sum_{E,\mathbf{p}} \sum_{E',\mathbf{p}'} (1 - \cos\theta_{\mathbf{p}\mathbf{p}'}) T_{ij}(E, \mathbf{p}) T_{ji}(E', \mathbf{p}')$$

$$\underbrace{H_\nu + H_{\nu\nu}}_{\text{with } \delta \neq 0} = S_\tau^\dagger \underbrace{(H_\nu + H_{\nu\nu})}_{\text{with } \delta = 0} S_\tau$$

$$\underbrace{H_\nu + H_{\nu\nu} + H_{\text{SFP}}(\mu)}_{\text{with } \delta \neq 0} = S_\tau^\dagger \left(\underbrace{H_\nu + H_{\nu\nu} + H_{\text{SFP}}(\mu_{\text{eff}})}_{\text{with } \delta = 0} \right) S_\tau$$

$$\mu_{\text{eff}} = S_\tau^\dagger \mu S_\tau = \begin{pmatrix} 0 & \mu_{12} & \mu_{13} e^{i\delta} \\ -\mu_{12} & 0 & \mu_{23} e^{i\delta} \\ -\mu_{13} e^{i\delta} & -\mu_{23} e^{i\delta} & 0 \end{pmatrix}$$

Pehlivan *et al.*, Phys. Rev.D 90, 065011 (2014)

Thank you!

