

Double Charge-Exchange Reactions and Double Beta-Decay

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D.C. Zheng, L. Zamick and NA, Annals of Physics 197, 343 (1990).
Nuclear Structure Studies of Double Gamow-Teller and Double Beta Decay
Strength

Light and heavy ion DCX. Another way to achieve probes that could produce double charge-exchange when scattered is to use as projectiles. An isospin $T \geq 1$ light or heavy ion may undergo a double charge-exchange reaction from a T_z to a $T_z \pm 2$ charge state. The use of $T = 1$ nuclei as probes is the most convenient case. However, the stable $T = 1$ nuclei, such as ^{14}C , ^{18}O , etc. have an excess of two neutrons ($T_z = -1$) and the DCX reaction will involve $\Delta T_z = 2$ transition in the target. Such transitions do not involve the DIAS and for most nuclear targets also the DGT is excluded. Nevertheless, the use of some targets such as ^{12}C , ^{54}Fe , ^{60}Ni may be of interest for reactions such as $(^{14}\text{C}, ^{14}\text{O})$, $(^{18}\text{O}, ^{18}\text{N})$, etc. In these cases DGT strength is allowed. An intriguing possibility developing now is the use of radioactive beams as, for example, of ^{14}O nuclei. DCX reactions of the type $(^{14}\text{O}, ^{14}\text{C})$ may then be used in studying DGT strength, for a wide variety of targets. The choice of a proper energy per nucleon for such ion projectiles will be probably of importance. The problem of internal projectile excitations may be a hampering factor in such experiments. One should try to explore theoretically and experimentally this possibility.

Motivation

The ($\beta\beta$ -decay) matrix element, is very small and accounts for only a 10^{-4} to 10^{-3} of the total DGT sum rule. A precise calculation of such hindered transition is, of course, very difficult and is inherently subject of large percent uncertainties. At present there is no direct way to “calibrate” such complicated nuclear structure calculations involving miniature fractions of the two-body DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and theoretically, one may be able to “calibrate” to some extent, the calculations of $\beta\beta$ -decay nuclear matrix elements.

N. Auerbach, L. Zamick, and D. Zheng, *Annals of Physics* 192, 77 (1989).

New attempts to measure DCX reactions, this time with ions

- ▶ Two places (as far as I know) have now programs to perform DCX reactions with ions, in Catania and Riken.
- ▶ In Catania the project is called NUMEN. The main motivation is to help to determine the nuclear matrix elements for neutrinoless double beta decay. Reaction with ions O18 to Ne18, etc., are planned.
- ▶ The work at RIKEN has more emphasis on new nuclear structure, as for example observing double Gamow-Telle states, tetra-neutron states, so on.

Isotensor giant resonances

- A “model” giant resonance (state) built on a state $|n\rangle$ is :
- $|Q_\alpha; n\rangle = Q_\alpha|n\rangle / |\langle n|Q_\alpha^\dagger Q_\alpha|n\rangle|^{-1/2}$
- Q_α is a one-body operator, $Q_\alpha = \sum_i q_\alpha(i)$
- For $|0\rangle$, the g.s. we have the “usual” giant resonances.
- When $|n\rangle$ is itself a giant resonance state, then the giant state built on it will be a *double giant state*.

Double Giant Resonances

- The operator Q_α might depend on spin and isospin. For example the electric dipole is an isovector and the corresponding operator is:
- $Q_\alpha \equiv D = \sum_i r_i Y_1(\theta_i) \tau_\mu(i)$, with $\mu = 0, \pm 1$
- In addition to the “common” $\mu = 0$ dipole one has also the *charge-exchange analogs* $\mu = \pm 1$
- The action of the operator with $\mu = -1$ (or $\mu = 1$) twice will lead to states with $\Delta T_z = \pm 2$
- These states can be reached in *double* charge-exchange (DCX) processes.
- One of the best examples, are the pion DCX reactions $[(\pi^+, \pi^-), (\pi^-, \pi^+)]$, studied extensively in the past.
- Because we deal with isotensor transitions, the selectivity of the DCX is large and enhances the possibilities to observe double giant resonances (states).

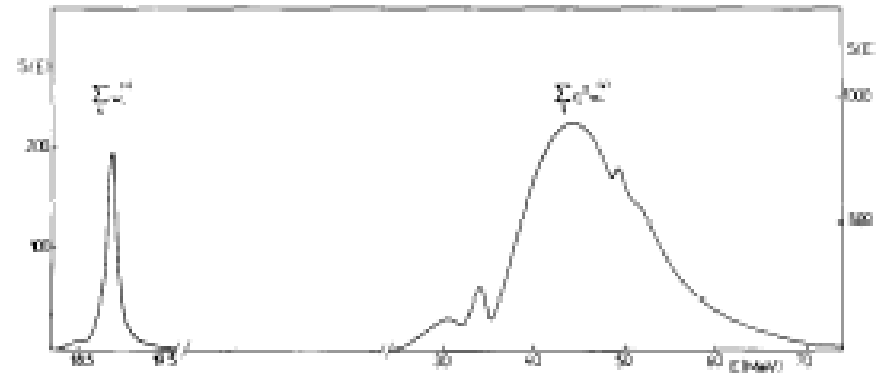
PARTICLE-HOLE CALCULATION OF THE ISOBARIC ANALOG AND ISOVECTOR MONOPOLE RESONANCES

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THE GIANT DIPOLE RESONANCE AND ITS CHARGE-EXCHANGE ANALOGS

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The isovector dipole strength for all three components $\tau_z = \pm 1, 0$ is calculated in the RPA.

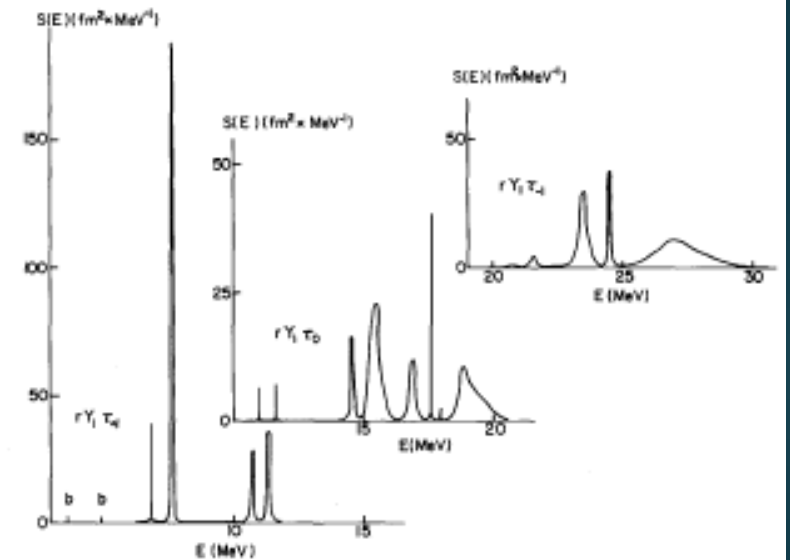
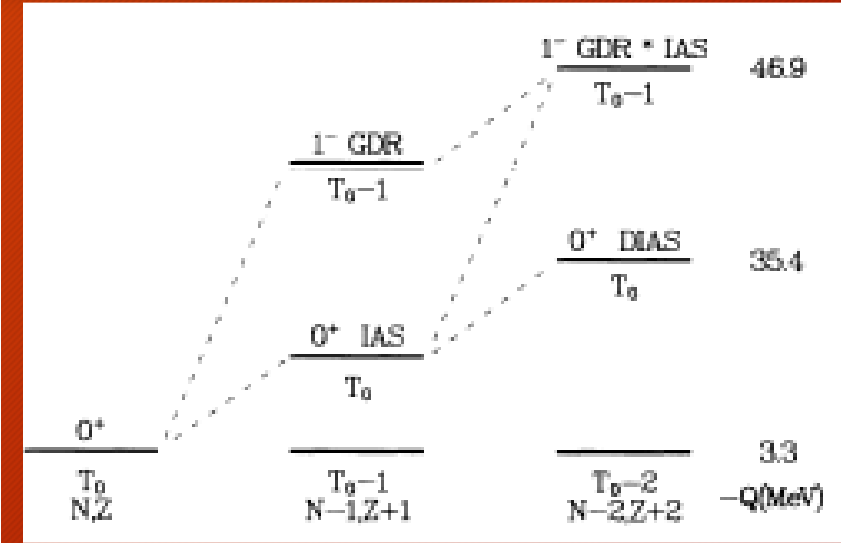
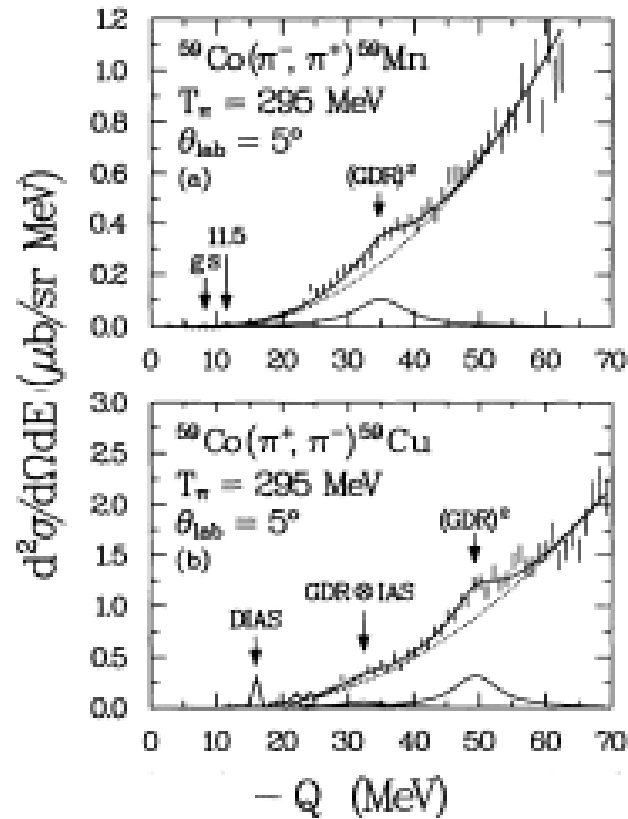
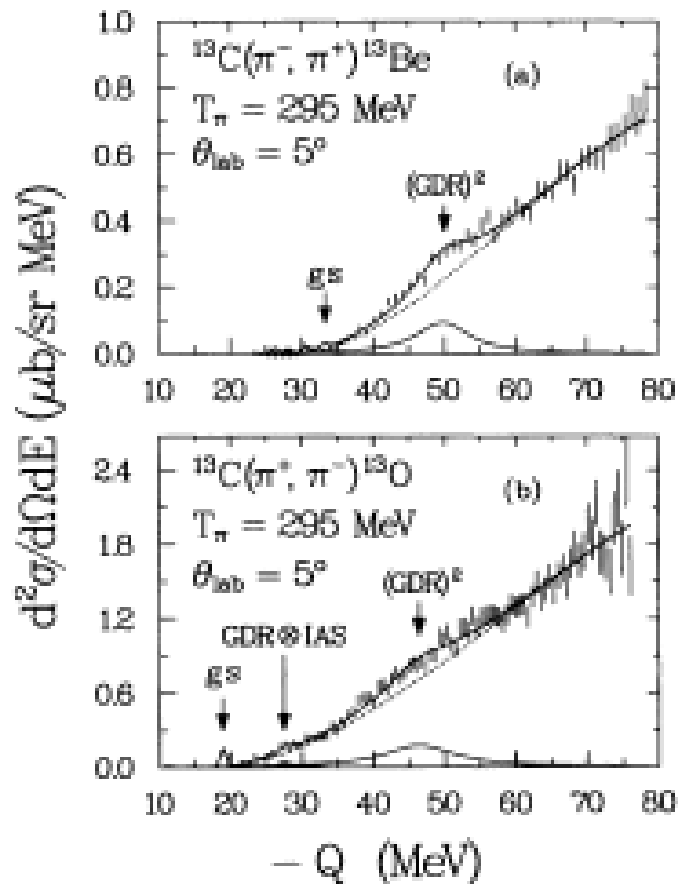


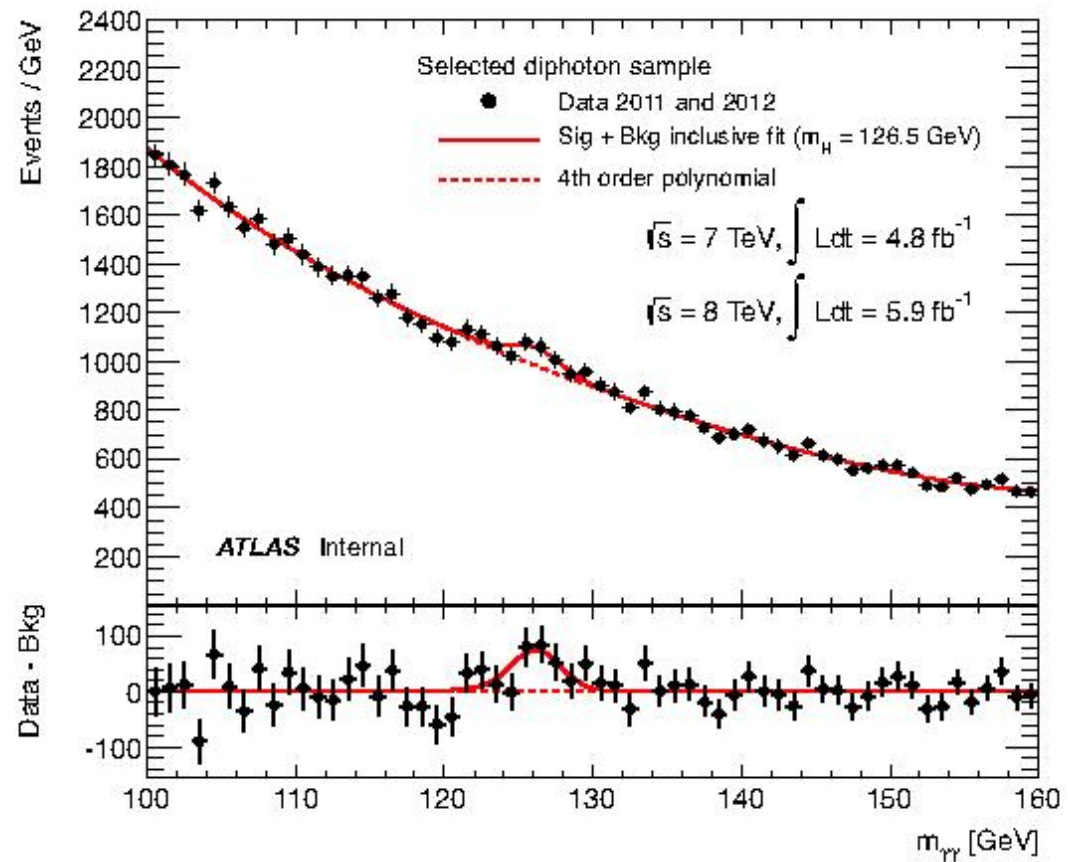
Fig. 1. Distribution of dipole strength in ^{90}Zr and its charge-exchange analogs calculated in the RPA.

Examples of simple isotensor states

- ▶ Double isobaric analog state (DIAS).
- ▶ The IAS is defined as: $|A_1\rangle = T_-|0\rangle/(N - Z)^{1/2}$
- ▶ The DIAS is: $|A_2\rangle = T_-^2|0\rangle/[(N - Z)(N - Z - 1)]^{1/2}$
- ▶ Dipole built on an analog: $|D_{-1}; A_1\rangle = \sum_i r_i Y_1(\theta_i) \tau_{-1}(i) |A_1\rangle / \tilde{N}$
- ▶ Yet another example, the double dipole (for the $\Delta T_z = -2$)
- ▶ $|D_{-1}; D_{-1}\rangle = \sum_i r_i Y_1(\theta_i) \tau_{-1}(i) |D_{-1}\rangle / \tilde{N}$

Pion DCX experiments





Double charge exchange ($^{11}\text{B}, ^{11}\text{Li}$) reaction for double beta decay response

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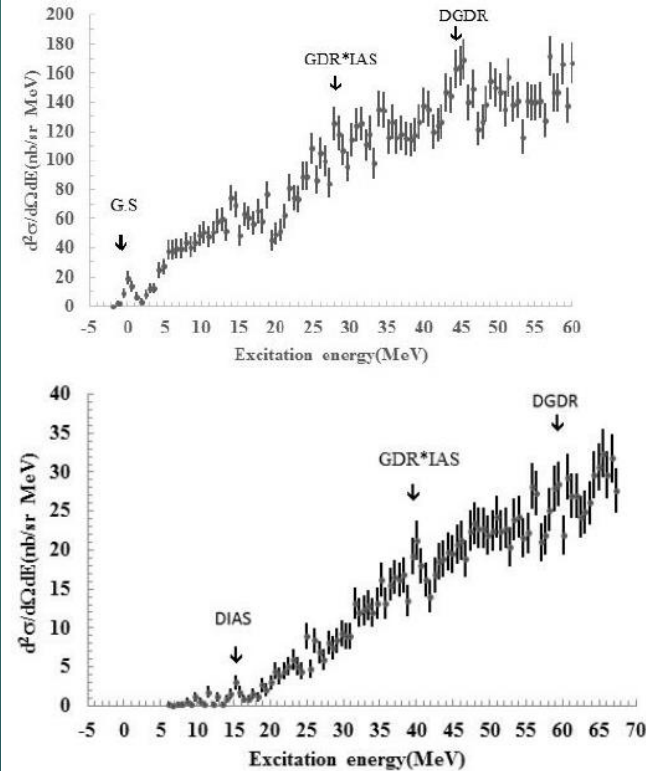


Figure 3. Cross-sections for the $^{13}\text{C}(^{11}\text{B}, ^{11}\text{Li})^{13}\text{O}$ reaction(upper) and the $^{56}\text{Fe}(^{11}\text{B}, ^{11}\text{Li})^{56}\text{Ni}$ reaction(lower).

Spin degrees of freedom

- $G_{\pm} = \sum_i \sigma(i) \tau_{\pm}(i)$
- The giant Gamow-Teller (GT) state: $|GT\rangle = G_-|0\rangle/N$
- The double Gamow-Teller (DGT) state: $|DGT; J\rangle = G_-^2(J)|0\rangle/N(J)$
- J is the total spin of the DGT
- Simple selection rules for the total spins of isotensor transitions.
- For two identical phonons the wave function is symmetric in space-spin-isospin, therefore for $\Delta T = 0, 2$ $J = 0^+, 2^+ \dots$

Some properties

- Excitation energies: $E_{Q_1 Q_2} \approx E_{Q_1} + E_{Q_2}$
- Widths (*spreading widths*) $\Gamma_{n;\lambda} = n\Gamma_{1;\lambda}$. In some theories n is replaced by \sqrt{n}
- Isospin and intensities: for an isotensor excitation $\Delta T_z = -2$
- and $(N - Z) \geq 2$ there are five isospin values $T' = T + 2,$
- $T + 1, T, T - 1, T - 2.$
- For an isotensor of rank k $F_\mu^{(k)}$ the intensities for the various isospin components T' are given by the corresponding Clebsh-Gordan (CG) coefficients and reduced matrix elements.
- $S_{T'} = |\langle T, T, k, \mu | T', T + \mu \rangle|^2 \langle T' || F^{(k)} || T \rangle^2$
- The CG coefficient for $T \gg k$ (in our case $T \gg 2$) are dominated by the
- CG of the aligned isospin $T' = T + \mu$. For $\mu = -k$, $(CG)^2 = \frac{2(T-k)+1}{2T+1}$.

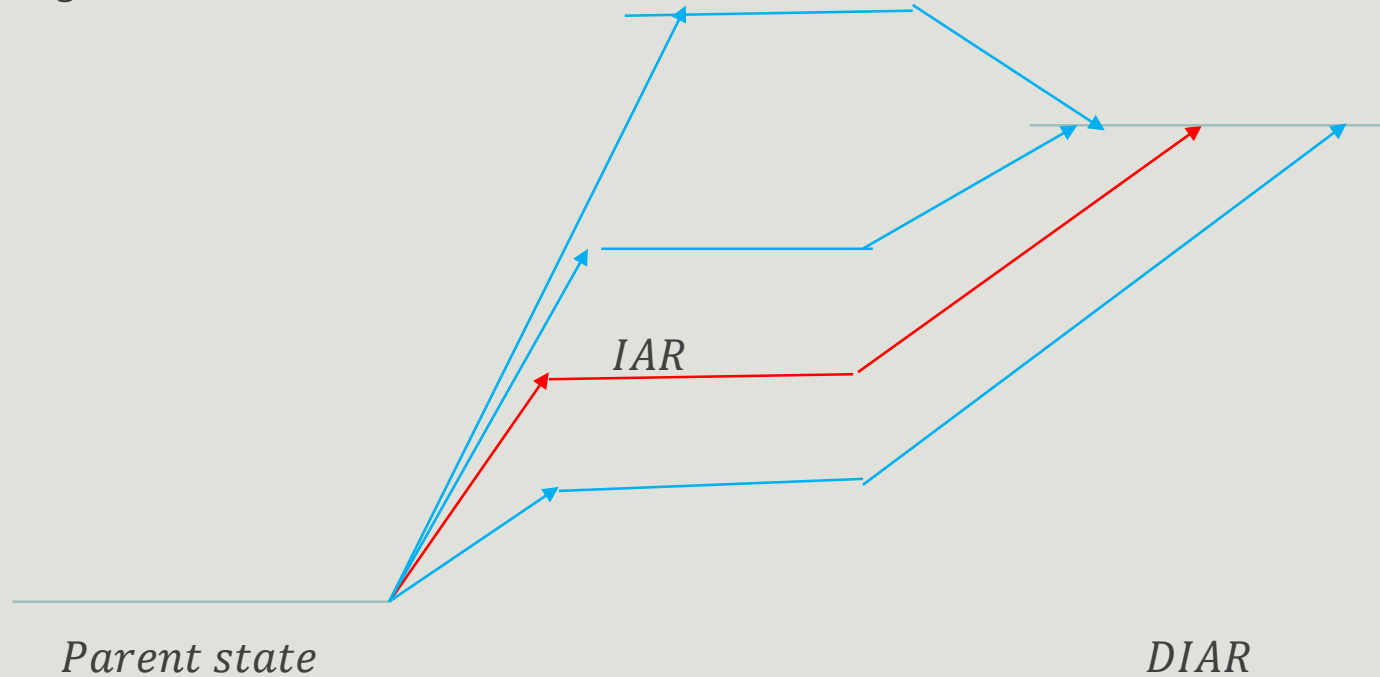
The DCX process.

- The DCX process has held out the hope that it would be a means of probing two-body correlations in nuclei. In the initial studies it was found that uncorrelated nuclear wave functions produced qualitative agreement with experiment at higher energies (pion energies around 300 MeV.) (The cross-sections were proportional to $(N-Z)(N-Z-1)$).
- This was the situation when the double giant resonances were studied. However at low pion energies (around 50 MeV) the disagreement with experiment was very large, (sometimes a factor of 50) when uncorrelated wave functions were used. It was necessary to include wave functions with correlated nucleons.
- The DCX process, as determined in pion charge-exchange reactions involves two basic routes.
- 1. For uncorrelated n particles in the transition to a double analog state the route from the initial state to the final state goes via the excitation of the single analog in the intermediate stage, and from there in a charge-exchange to the double analog. This is termed as the analog route or more generally as the “*sequential*” process. The cross section is proportional to the number of pairs one can form from the n nucleons, $n(n-1)/2$.
- 2. The second process involves correlated nucleons and the process proceeds through intermediate states that are not the analog. This we term the “non-analog route”.

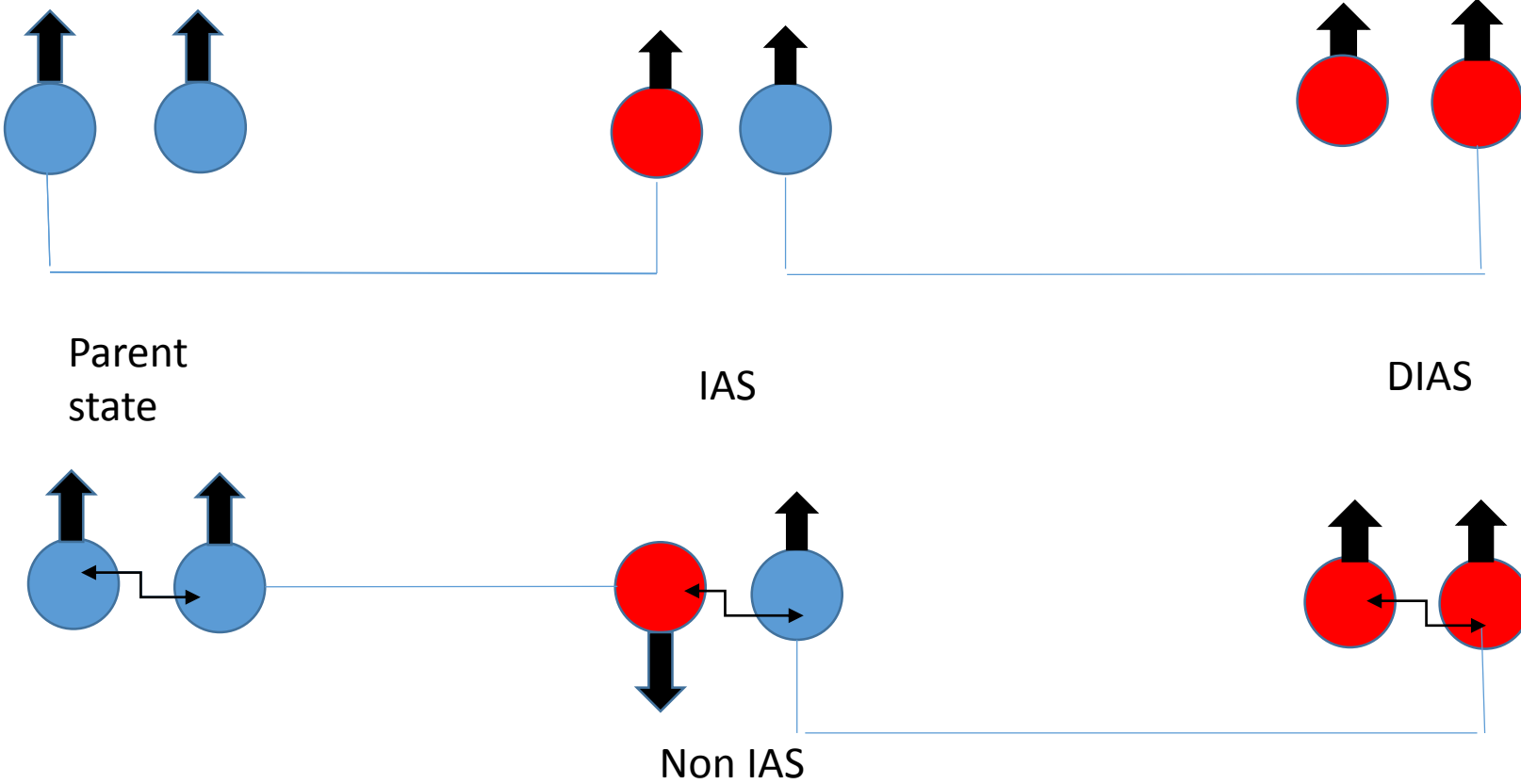
Analog vs non-analog routes

analog route

non - analog route



Correlations and non-analog transitions



The DCX process

In order to obtain the desired relation, we consider the reaction to be caused by a two-body operator between two identical (in the same isospin multiplet) 0^+ states. Thus we need to calculate the matrix element for the transition:

$$M = \langle 0^+ | \sum_{i,j} \theta_{ij}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{k}, \mathbf{k}') | 0^+ \rangle. \quad (1)$$

- The operator depends also on the spins of the two particles

Take the simple case when the shell-model state is described by a single j^n configuration of identical particles coupled to total angular momentum $J=0^+$ and seniority $v=0$. In this case, the diagonal matrix element (for even n) is given by⁶

$$\begin{aligned} \langle j^n, v=0, J=0^+ | \sum \theta_{ij} | j^n, v=0, J=0^+ \rangle \\ = \frac{1}{2} n(n-1)\alpha + \frac{1}{2} n\beta, \end{aligned} \quad (2)$$

where α and β are constants independent of n . This strikingly simple formula holds for any two-body $\sum \theta_{ij}$ interaction. The DCX transition operator is not necessarily, of course, a scalar in space; however, for matrix elements involving a transition from a $J=0^+$ to $J=0^+$ state only the scalar part of θ_{12} survives and one can use Eq. (2) to calculate the DCX. In calculating the DCX cross section, one has to take into account the fact that we go from the ground state (g.s.) to the DIAS, i.e., from $T_Z = T$ to $T_Z = T-2$. This introduces an additional factor $[n(n-1)/2]^{-1/2}$ in the amplitude. The DCX cross section is then

$$\sigma_{\text{DCX}}(\theta) = \frac{1}{2} n(n-1) | \alpha + \beta/(n-1) |^2. \quad (3)$$

This formula is remarkable in its simplicity. The DCX cross section to the DIAS contains now two, and in the case of a pure j^n configuration only two, amplitudes. The first one represents transitions which occur in the absence of shell-model correlations in the nuclear wave function. The cross section due to this term is simply proportional to the number of neutron pairs to be made into pairs of protons when going from the ground state to the DIAS. This counting rule is independent of the mutual location of the two nucleons in each pair, and therefore this term will dominate the cross section if the tran-

sition DCX operator is of long range. In the case of a transition operator which is constant (over the volume of the nucleus), only this term will contribute. (Note, however, that a short-range interaction will contribute also to the α term.) *The second term, proportional to β , is a new term that did not receive much attention in the past.* It represents DCX transitions which take place when the nuclear wave function is more than just that of independent particles. The shell-model state in the seniority scheme has a correlated wave function formed by the combination of pairs of nucleons coupled to $J=0^+$. Each time such a pair is added, one has to antisymmetrize the wave function. The β term will contribute when the DCX transition operator is of short range. By examining Eq. (3), one sees that the relative contribution of the pairing term is largest when n is the smallest, i.e., when $n=2$ or, equivalently, $T=1$. As one increases the number of excess neutrons, the field effects, represented by the parameter α , become more important in comparison with the two-body pairing term represented by $\beta/(n-1)$. If one prefers to discuss the DCX reaction in terms of a sequential process, the term proportional to α corresponds to the transition in which the intermediate state is the single isobaric analog state (IAS), while the second term in Eq. (3) would correspond in the case of sequential transitions to processes in which the intermediate states are nonanalog states.

The DCX for the even Ca isotopes (the AGP formula) (Single- j, n-even). (NA, W.R. Gibbs, E. Piasezky, PRL, 59, 1076 (1987))

$$\sigma_{\text{DCX}}(\theta) = \frac{n(n-1)}{2} \left| A + \frac{(2j+3-2n)}{(n-1)(2j-1)} B \right|^2,$$

$$A = \alpha + [2/(2j+1)]\beta, \quad B = [(2j-1)/(2j+1)]\beta,$$

$$\sigma_{\text{DCX}}(\theta) = \frac{1}{2} n(n-1) \left| \alpha + \beta/(n-1) \right|^2.$$


We see that the counting rule given by the $n(n-1)/2$ independent pairs is completely destroyed, indicating that the correlations play a very important role in the DCX process at these low pion energies. Fitting the data, one finds that the ratio $|B|/|A| \simeq 3$. For higher energies⁴⁾ $E_\pi \geq 150$ MeV this ratio is found to be $\simeq 1$.

Nucleus	σ_{DCX}
⁴² Ca	$ A+B ^2$
⁴⁴ Ca	$6 A+\frac{1}{9}B ^2$
⁴⁶ Ca	$15 A-\frac{1}{15}B ^2$
⁴⁸ Ca	$28 A-\frac{1}{7}B ^2$

Nucleus	$n(n-1)/2$ Scaling	Exp.
⁴² Ca	1	1
⁴⁴ Ca	6	0.5
⁴⁶ Ca	15	—
⁴⁸ Ca	28	1

The DCX process (cont'd)

- In general, the DCX cross section will contain the sequential term and the correlation term. Both amplitudes could be complex.
- The above formula is extended to the generalized seniority scheme when several orbits are involved. Then:
- $\sigma_{DCX} = \frac{n(n-1)}{2} \left| A + \frac{(\Omega+1-n)}{(\Omega-1)(n-1)} B \right|^2$; with $\Omega = \sum_i (j_i + 1/2)$
- This formalism was applied to the Te 128 and 130 nuclei, relevant to the double beta-decay, (H.C. Wu, et.al., Phys. Rev. C54, 1208 (1996)).
- See also N. Auerbach, et. al., Phys. Rev. C38, 1277(1988),
- J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

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- ▶ The characteristic behavior of the DCX amplitude to the DIAS as a function of the number of valence neutrons for a single j orbit persists also when there is rather weak configuration mixing. The change is absorbed by the altered values of the A and B coefficients in such a way that the ratio $\frac{B}{A}$ is increased, meaning that the effective DCX operator is of shorter range.
 - ▶ But even in the case of stronger mixing the effect could be similar.
 - ▶ This may suggest why certain models in which a limited space is used can provide satisfactory results when effective operators are used. (An example is the IBM description of the double beta-decay)

Spin

- The two-body DCX operator is also a function of the spin of the two nucleons σ_1 and σ_2 . In fact one of the important non-analog routes leading to the DIAS is the route with intermediate state being the Gamow-Teller (GT).
- [In the case of good SU_4 symmetry the Double GT (DGT) strength obeys the relation:
 - $\frac{S_{DGT(DIAS)}}{\sum_f S_{DGT}(all\ 0_f^+)} = \frac{3}{(N-Z)^2-1}$. For $N - Z = 2$ all the $J = 0^+$ DGT strength
 - is in the DIAS. As N-Z increases most of the DGT strength is contained in states that are not the DIAS. For example in Ca48 the DIAS contains only 1/21 of DGT strength.]

“Pairing” property and short range operators.

In terms of the matrix elements of a two-body DCX transition operator \hat{O}_{12} one can write [4]

$$\begin{aligned} A &= \frac{1}{2j+1} (2j\bar{O} + O_0) , \\ B &= \frac{2j}{2j+1} (O_0 - \bar{O}) , \end{aligned} \quad (4)$$

where the following definitions were used:

$$\begin{aligned} O_0 &= \langle j^2J=0 | \hat{O}_{12} | j^2J=0 \rangle , \\ \bar{O} &= \frac{\sum_{J \text{ even}} (2J+1) \langle j^2J | \hat{O}_{12} | j^2J \rangle}{\sum_{J \text{ even}} (2J+1)} . \end{aligned} \quad (5)$$

One sees from these expressions that the relative size of A and B depends on the range of \hat{O}_{12} . Consider a real DCX transition operator which possesses the “pairing” property [4], i.e., an operator that for even n satisfies

$$\langle j^nJ=0 | \hat{O}_{12} | j^nJ=0 \rangle = \frac{n}{2} O_0 , \quad (6)$$

where $n/2$ is the number of neutron pairs. Operators such as $\delta(r_1 - r_2)$ and $\sigma_1 \cdot \sigma_2$ fall into this category. For such an operator, $\bar{O} = O_0 / (2j)$, $\alpha = 0$, $\beta = O_0$, and

$$B / A = j - \frac{1}{2} . \quad (7)$$

Double β –decay

▶ $(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}$

▶ $(A, Z) \rightarrow (A, Z + 2) + 2e^-$

2-beta decay amplitudes

The amplitudes A in these expressions contain the nuclear structure ingredients.

The inverse half-lives of the $2\nu\beta\beta$ decay and the $0\nu\beta\beta$ decay (due to the massive neutrino) can be written [5, 18, 19]:

$$\frac{1}{T_{2\nu}} = G^{2\nu} |A^{2\nu}|^2, \quad (1)$$

$$\frac{1}{T_{0\nu}} = G^{0\nu} |R_0 A^{0\nu}|^2 \left(\frac{m_\nu}{m_e}\right)^2, \quad (2)$$

$$\begin{aligned}
\mathcal{O}_{GT} &= \tau_{1-}\tau_{2-} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) H_{GT}(r, E_\kappa), \\
\mathcal{O}_F &= \tau_{1-}\tau_{2-} H_F(r, E_\kappa), \\
\mathcal{O}_T &= \tau_{1-}\tau_{2-} S_{12} H_T(r, E_\kappa),
\end{aligned} \tag{5}$$

with $S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $r = |\mathbf{r}|$, and $\mathbf{n} = \mathbf{r}/r$. The neutrino potentials, $H_\alpha(r, E_\kappa)$, are integrals over the neutrino exchange momentum q ,

$$H_\alpha(r, E_\kappa) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(qr)h_\alpha(q^2)q dq}{q + E_\kappa - (E_i + E_f)/2}, \tag{6}$$

1.1 Two-neutrino double beta decay $2\nu\beta\beta$

$$\frac{1}{T_{1/2}^{2\nu}} = \Gamma^{2\nu} = G^{2\nu} |A^{2\nu}|^2, \quad (1)$$

where $G^{2\nu}$ is the phase space factors multiplied by some fundamental constants. The $2\nu\beta\beta$ decay nuclear matrix element $A_{GT}^{2\nu}$ is

$$A_{GT}^{2\nu} = \langle 0_f^+ | \sum_{i,j} \sigma_i \cdot \sigma_j t_+(i) t_+(j) | 0_i^+ \rangle, \quad (2)$$

where $t_+|n\rangle = |p\rangle$. If $|0_F^+\rangle$ is the double isobaric analog state (DIAS) of the initial state $|0_I^+\rangle$, one can obtain an analytical result for $A_{GT}^{2\nu}$ within the

single $f_{7/2}$ shell model:

$$A_{GT}^{2\nu}(\text{DIAS}) = -\frac{j+1}{j} \sqrt{\frac{2n}{n-1}} = -\frac{9}{7} \sqrt{\frac{2n}{n-1}}, \quad (3)$$

where $j = 7/2$ and $n = N - Z$.

1.2 Neutrinoless double beta decay $0\nu\beta\beta$

$$\frac{1}{T_{1/2}^{0\nu}} = \Gamma^{0\nu} = G^{0\nu} |R_0 A^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2, \quad (4)$$

where the phase space factors multiplied by some fundamental constants is $G^{0\nu} = 6.369 \times 10^{-14}/\text{year}$ and $m_e = 0.511 \times 10^6$ eV is the electron mass. So the $T_{1/2}^{0\nu}$ is determined by

$$T_{1/2}^{0\nu} = \frac{4.1 \times 10^{24}}{|R_0 A^{0\nu}|^2} \frac{1}{\langle m_{\beta\beta} \rangle^2}. \quad (5)$$

$\langle m_{\beta\beta} \rangle$ being the effective Majorana mass of the electron neutrino (which is unknown)

$$\langle m_{\beta\beta} \rangle = \left| \sum_k m_k U_{ek}^2 \right|. \quad (6)$$

m_k is the neutrino mass eigenvalues and U_{ek} is the neutrino mixing matrix. $R_0 A^{0\nu} = M^{0\nu}$ is the nuclear matrix element (NME) with R_0 is the nuclear radius. The $0\nu\beta\beta$ decay NME $A_{GT}^{0\nu}$ is given by

$$A^{0\nu} = A_{GT}^{0\nu} - \left(\frac{g_V}{g_A} \right)^2 A_F^{0\nu}, \quad (7)$$

$g_V/g_A = 1/1.25$.

$$A_{GT}^{0\nu} = \langle 0_1^+ | \sum_{i,j} H(|\mathbf{r}_i - \mathbf{r}_j|) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j t_+(i) t_+(j) | 0_i^+ \rangle \quad (8)$$

$$A_F^{0\nu} = \langle 0_1^+ | \sum_{i,j} H(|\mathbf{r}_i - \mathbf{r}_j|) t_+(i) t_+(j) | 0_i^+ \rangle \quad (9)$$

$H(|\mathbf{r}_i - \mathbf{r}_j|)$ is “the neutrino potential”.

$$H(r_{ij}) = \int \frac{d^3q}{2\pi^2} \frac{e^{iq \cdot (\mathbf{r}_i - \mathbf{r}_j)}}{q_0(q_0 + \langle \Delta E \rangle)} \quad (10)$$

(q_0, \mathbf{q}) is the four-momentum with $q_0 = \sqrt{\mathbf{q}^2 + m_\mu^2}$, and $\Delta E = E_N - E_I + E_e$ with E_I and E_N the initial and intermediate nuclear state energies and E_e the energy of the electron present in the intermediate state.

If we neglect the neutrino mass m_μ

$$H(r) = \frac{g(\alpha r)}{r}, \quad (11)$$

where $\alpha = \langle \Delta E \rangle / (\hbar c)$, and

$$g(\alpha r) = \frac{2}{\pi} \sin(\alpha r) \text{Ci}(\alpha r) + \cos(\alpha r) \left[1 - \frac{2}{\pi} \text{Si}(\alpha r) \right], \quad (12)$$

with $\text{Si}(x) = \int_0^x \frac{\sin t}{t} dt$, and $\text{Ci}(x) = -\int_x^\infty \frac{\cos t}{t} dt$ $H(r) \approx 1/r$.

Using the decomposition into intermediate states, we have

$$M_\alpha^{0\nu} = \sum_\kappa \sum_{1234} \langle 13 | \hat{O}_\alpha | 24 \rangle \langle 0_f^+ | \hat{c}_3^\dagger \hat{c}_4 | J_\kappa^\pi \rangle \langle J_\kappa^\pi | \hat{c}_1^\dagger \hat{c}_2 | 0_i^+ \rangle, \quad (13)$$

$\alpha = \{GT, F, T\}$. $|J_\kappa^\pi\rangle$ is a intermediate nuclear state.

DGT strength and the double β -decay

- The 2-neutrino amplitude can be written as:

- $$A_{GT}^{2\nu} = \sum_n \frac{\langle f|G_-|n\rangle\langle n|G_-|i\rangle}{E_i - E_n - \varepsilon}$$

- (The 0-neutrino amplitude includes also a Fermi amplitude.)

- The total coherent DGT strength is:

- $$S_{DGT} = \sum_f \underbrace{|\sum_n \langle f|G_-|n\rangle\langle n|G_-|i\rangle|^2}$$

- (Coherent sum)

- $S_{2\beta} = |\sum_n \langle f_0|G_-|n\rangle\langle n|G_-|i\rangle|^2$ is the double beta decay strength (in the closure approximation) and is a *coherent* sum. Thus: $S_{2\beta} \sim |\langle f|G_-(1)G_-(2)|i\rangle|^2$

- The incoherent DGT strength for a given final state f_0 :

- $$S_{DGT}^{inc} = \sum_n |\langle f_0|G_-|n\rangle|^2 |\langle n|G_-|i\rangle|^2$$

D.C. Zheng et.al., *Annals of Physics*, 197, 343 (1990)
Calculation of DGT strength

- We approached this problem of the distribution of DGT strength in the most
- straightforward way. We calculate the shell-model states in an extended model
- space in the parent nucleus, (N, Z) , in the intermediate nucleus $(N - 1, Z + 1)$, and
- in the final nucleus $(N - 2, Z + 2)$. Having determined the nuclear wave functions
- we then evaluate directly the *incoherent* sum and *coherent* sum.

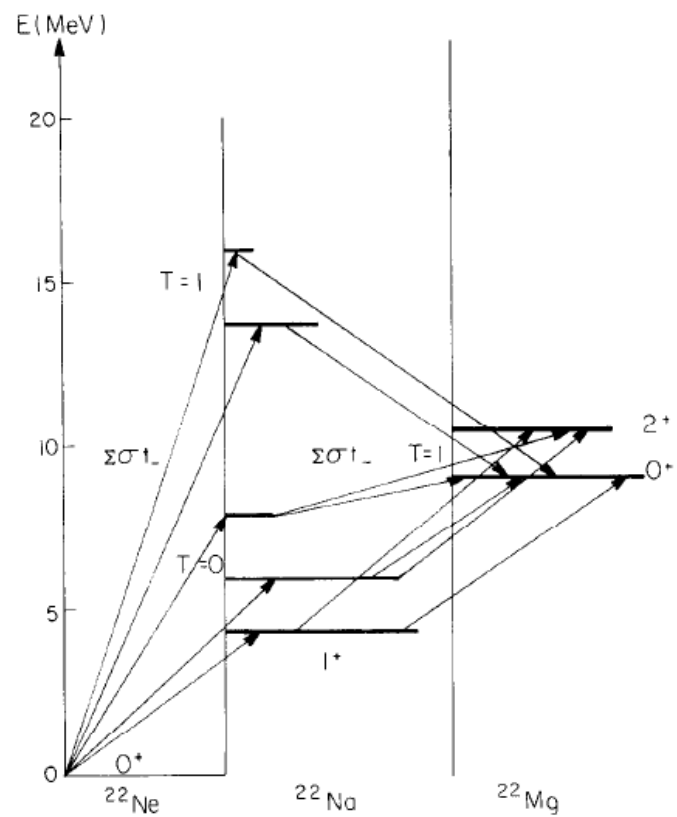


FIG. 2. The calculated important G_- and G_-^2 transitions in the $A=22$ system. The length of each bar represents the relative G_- and G_-^2 strength.

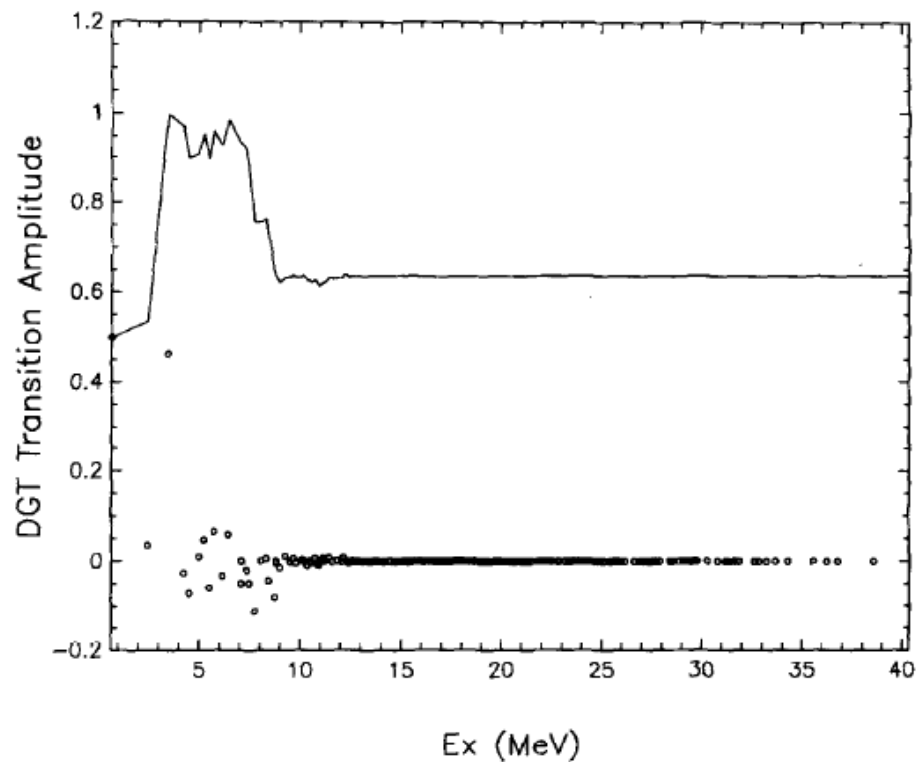
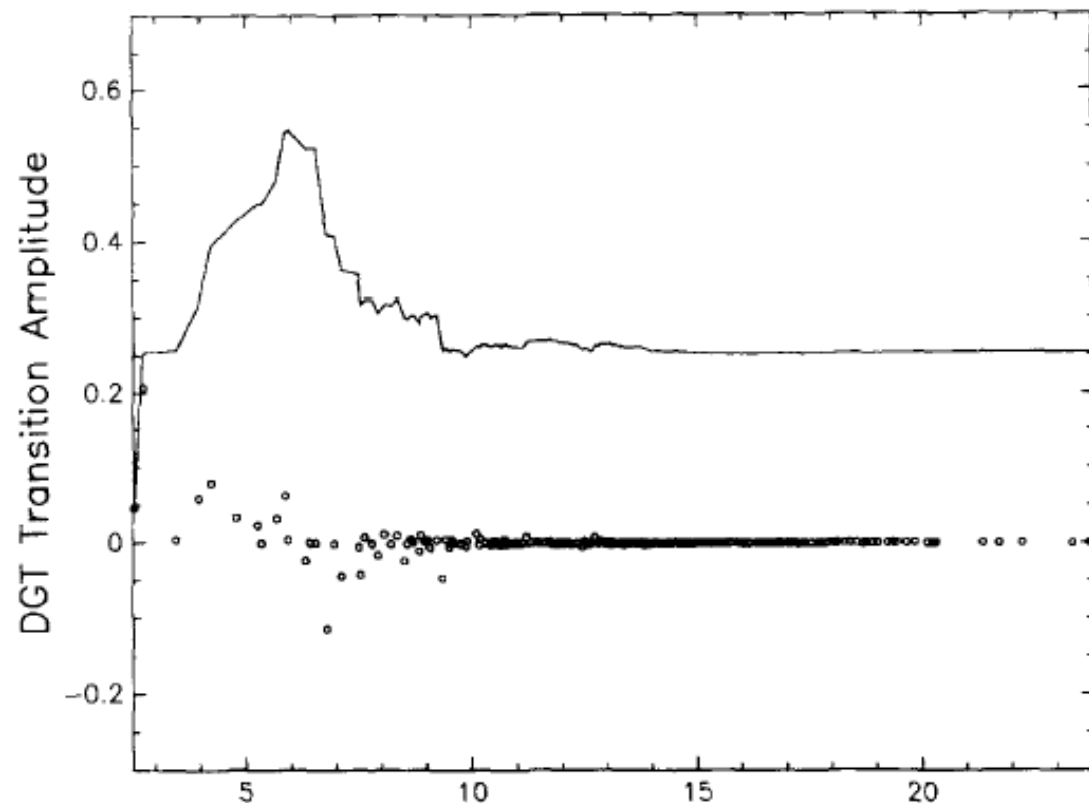
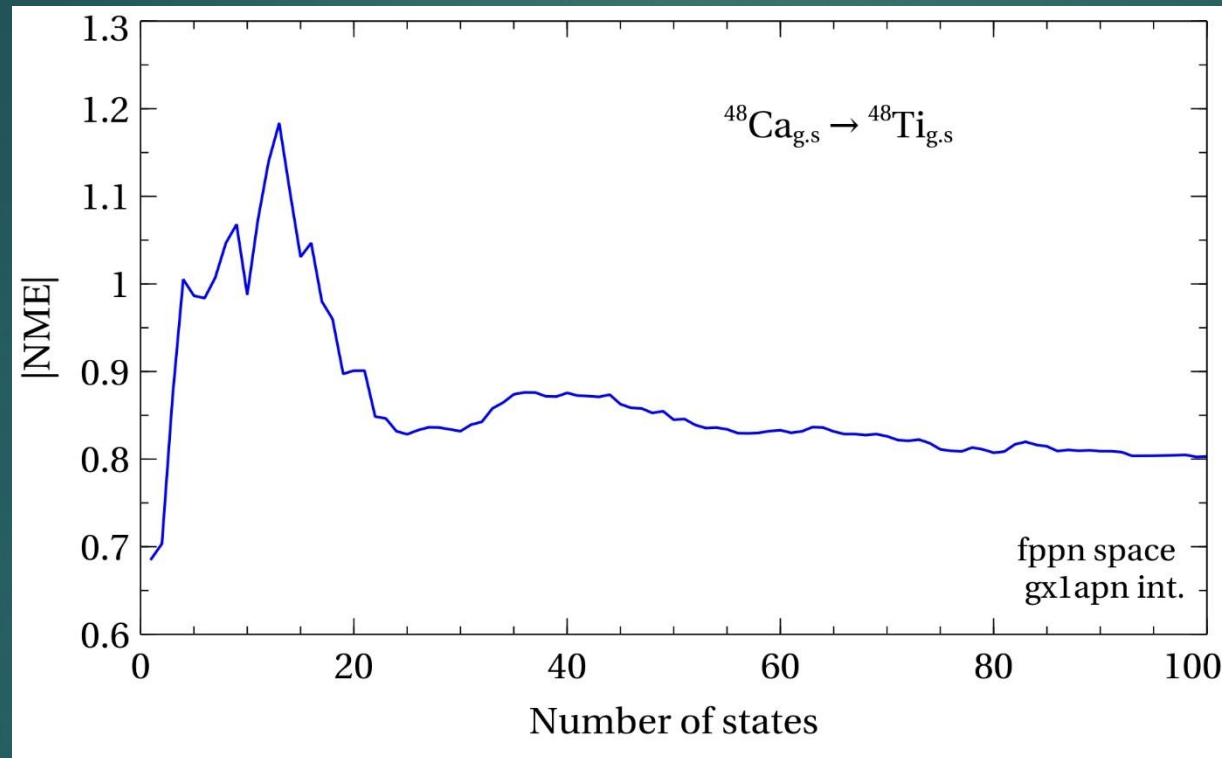
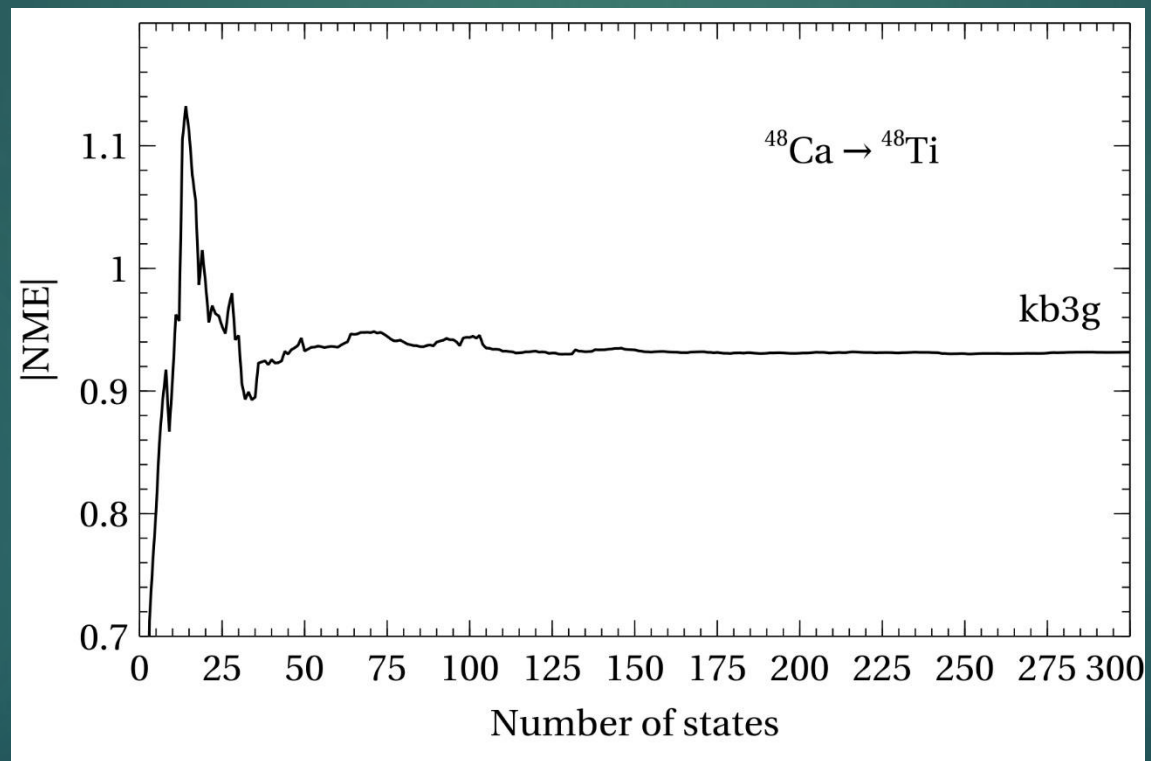
${}^{44}_{20}\text{Ca}$ ${}^{48}_{20}\text{Ca}$ 

FIG. 4. The DGT ${}^{44}\text{Ca}$ g.s. to ${}^{44}\text{Ti}$ g.s. transition amplitudes $M_{+\beta\beta}^C(i)$ (the individual contribution from the i th intermediate state, open circles) and $\sum M_{+\beta\beta}^C(n)$ (sum of $M_{+\beta\beta}^C(i)$ for $i = 1$ to n , solid line) vs excitation energy (Ex) of the intermediate 1^+ state in ${}^{44}\text{Sc}$. The modified renormalized Kuo-Brown interaction is used in the $0f-1p$ shell.



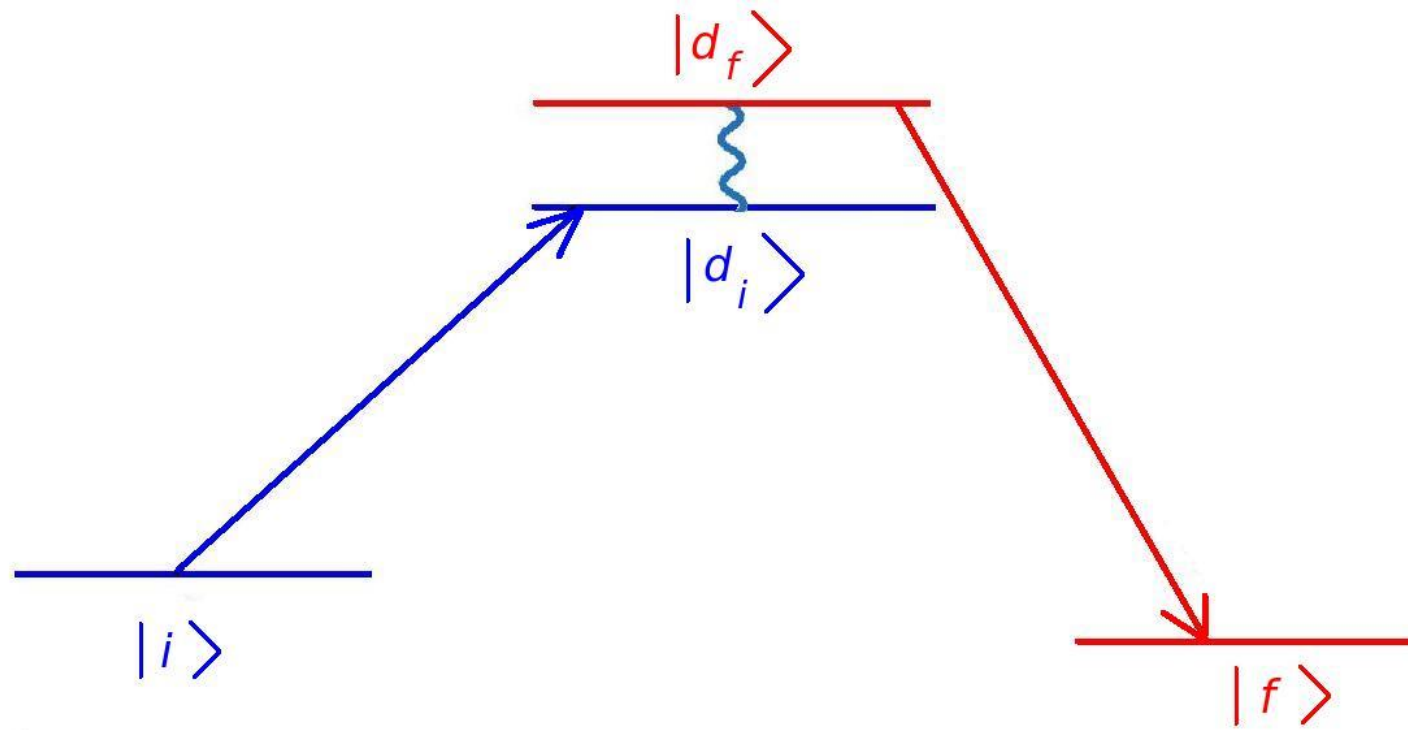
Neutrinoless nuclear matrix element





Cancellations in DCX or double beta-decay contributions

- Let O be a one-body operator (for example the Fermi or Gamow-Teller). We are interested in computing the expression:
- $$T = \sum_n \frac{\langle f|O|n\rangle\langle n|O|i\rangle}{E-E_n}$$
- Let us choose among the intermediate states $|n\rangle$ a state that strongly couples to the initial state $|i\rangle$ via the operator O and denote it as $|d_i\rangle$. The same for the final state $|d_f\rangle$. Usually the initial and final states are quite different and therefore the two intermediate states are different as well. The contribution of either state to the above sum will be very small. However, the nuclear force mixes the two intermediate states and the two mixed states will be:
- $|n_1\rangle = \alpha|d_i\rangle + \beta|d_f\rangle$ at energy E_1
- $|n_2\rangle = -\beta|d_i\rangle + \alpha|d_f\rangle$ at energy E_2
- For the sake of simple presentation we will assume that the matrix element $\langle f|O|d_i\rangle = 0$ and $\langle d_f|O|i\rangle = 0$
- Using the two states above and these assumptions for the matrix elements, rearranging terms we will find that the contribution of the first state $|n_1\rangle$ in T to be:



- $T = \alpha\beta \frac{\langle f|O|d_f\rangle\langle d_i|O|i\rangle}{E-E_1}$

- When we take into account the second state $|n_2\rangle$ we obtain

- $T = -\alpha\beta \frac{\langle f|O|d_f\rangle\langle d_i|O|i\rangle}{E-E_2}$

- We see that the second state cancels partially the contribution of the first one. The amount of cancellation depends on the energy difference between the two states. If these two states are close in energy there will be a very strong cancellation.

SCX and 2β decay

- Some (n, p) studies were initiated in which one chooses as the target states the final states of the relevant double beta decay process. In these (n, p) experiments one measures the GT strength to the various intermediate single-charge exchange states. One can then take and multiply the two GT strengths obtained in (p,n) and (n,p) to the same intermediate states obtained in each reaction and try to sum over the intermediate states observed. For example, let us take the $A = 48$ nuclei. The (p, n) and (n, p) reactions are performed correspondingly on the ^{48}Ca and ^{48}Ti targets. The intermediate states measured are in both reactions located in the ^{48}Sc nucleus. The actual experiments were performed at the initial nucleon energies of 200 MeV. At these energies the spin states and in the forward direction GT states in particular are excited predominantly.
- Using the DWIA analysis, one is able to extract the GT strength for the various intermediate states observed in these two reactions. Note that only the squares of the matrix elements of the type $\langle J_i | \sigma\tau | n \rangle$ and $\langle n | \sigma\tau | J_f \rangle$ can be deduced from these two one-body experiments. The relative signs are not determined in such processes. By measuring transitions to a great number of intermediate states, one in principle can determine the incoherent sum for the g.s. to g.s. transition. There are several difficulties in this procedure even when one tries to determine the incoherent decay sum. First, because of final experimental resolution on the one hand and large density of intermediate states on the other, one is not always sure that it is the same matching intermediate state that is excited when the (p,n) and (n, p) experiments are compared. Second, and related to it is the fact that when a strong GT transition in (p, n) is observed to a given intermediate state, the (n, p) transition to the same intermediate state might be (and often is) weak, and vice versa. This of course makes the experimental exploration of the double-beta decay amplitude difficult. The ground states in the two target nuclei are not related by any simple transformation and therefore the action of the $\sigma\tau_{\pm}$ operators leads to mismatched distributions of strength in the intermediate nucleus.

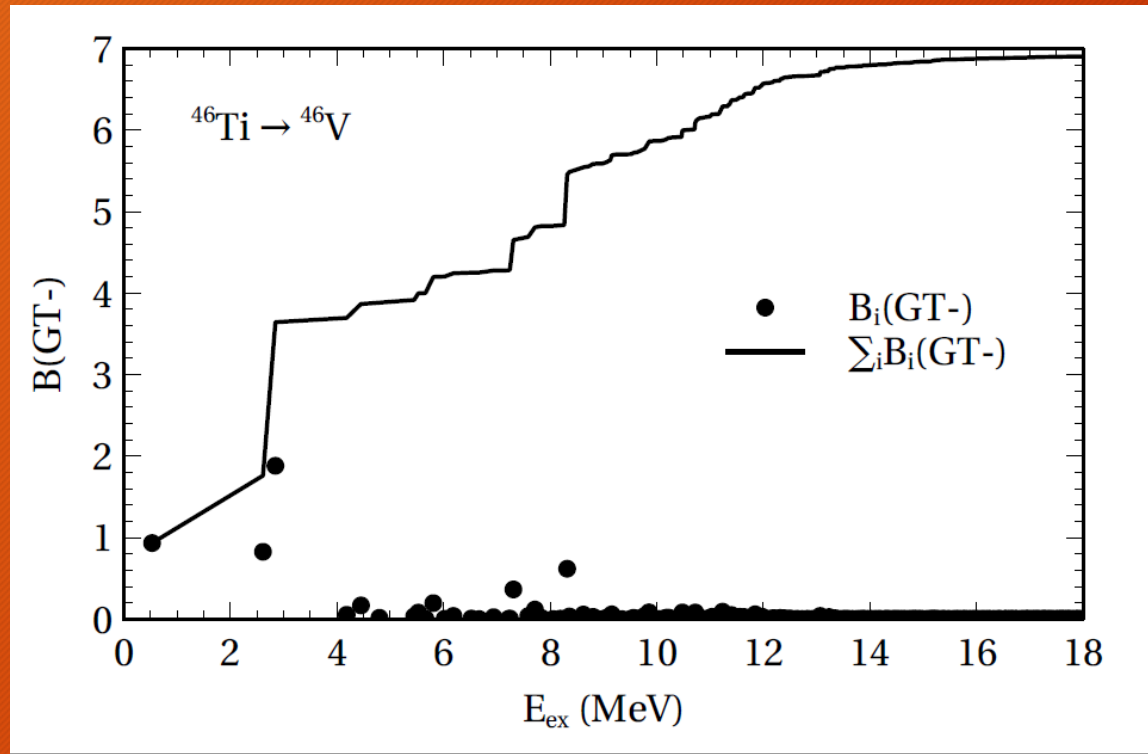
DCX and double beta decay

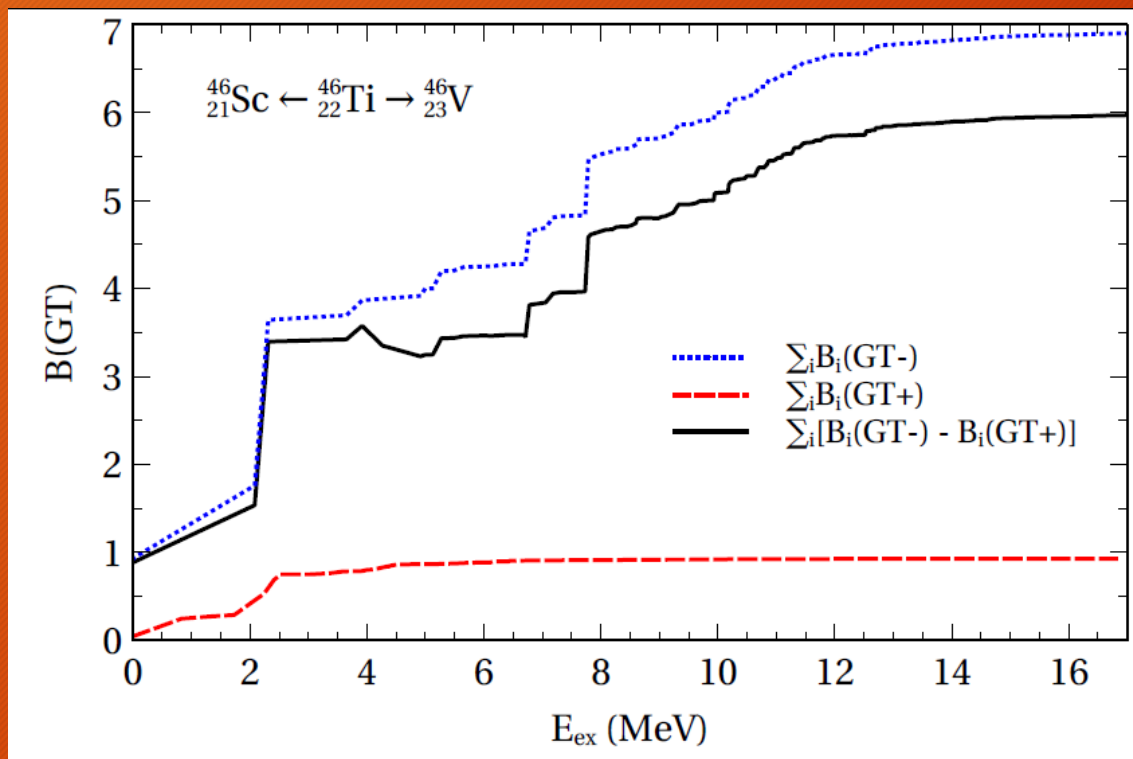
- It is clear that the combined studies of (p, n) and (n, p) may
- provide some information concerning the nuclear structure aspects of the 2β decay
- matrix elements. It is also clear that because of the coherence of the 2β decay
- matrix elements, the use of (n, p) and (p, n) strength cannot determine the value of
- such matrix elements. Also, quite obviously in such experiments, one cannot excite
- all DGT strengths. In order to be able experimentally to study the DGT strength
- (including the 2β decay g.s. to g.s. transition), one must employ processes in which
- the leading terms are genuine two-body transitions. Double charge-exchange
- (DCX) reactions are the natural candidates for such a study.

Comment

- One of the difficult problems in the theoretical studies of 2-beta decay is the “universal” quenching of the single Gamow-Teller strength.
- Experiments show that 30-40% of GT strength is missing in the main GT peak. This affects the 2-beta decay transition matrix element. The source of the above quenching is not certain. There are several ideas. One is that the missing strength is due to the $\Delta - h$ configurations interacting with the GT states and the missing strength is 300 MeV above the GT peak. Another theory is that the GT strength is fragmented and the missing strength is several tens of MeV above the main GT peak. These two different theories will affect the 2-beta decay matrix element differently. The uncertainty, (because of this quenching), could be as large as a factor of 2 for double beta decay.
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and recent calculations, May 2017





Nuclear structure properties of the double-charge-exchange transition amplitudes

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Nuclear structure aspects of the double-charge-exchange (DCX) reaction on nuclei are studied. Using a variety of DCX-type two-body transition operators, we explore the influence of two-body correlations among valence nucleons on the DCX transition amplitudes to the isobaric analog state and to other nonanalog $J^\pi=0^+$ states. In particular, the question of the spin dependence and of the range of the DCX transition operators is explored and the behavior of the transition amplitudes as a function of the valence nucleon number is studied. It is shown that the two-amplitude DCX formula derived by Auerbach, Gibbs, and Piasetzky for a single j^n configuration holds also in some cases when configuration mixing is strong. DCX-type transitions from the Ca and Ni isotopes to the Ti and Zn isotopes and from ^{56}Fe to ^{56}Ni are the subject of this study.

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Interesting problems (circa 1990)

Interesting problems.

1. Can one observe other types of isosensor DGRs in DCX reactions.
2. In particular the DGT R.
3. The reaction theory. Is the 3-channel approach sufficient? Role of correlations.
4. More specific questions, (why only the $J=2^+$ (?) of the DGD R is observed, etc).
5. DCX reactions with IONS.

The stable $T=1$ nuclei that can be used as probes such as ^{14}C , ^{18}O , etc. have an excess of two neutrons ($T_z=+1$) and the DCX will involve $\Delta T_z=2$ transitions in the target. Such transitions do not involve the DIAR and usually also not ^{have} the DGT R. Nevertheless the use of targets such as ^{12}C , ^{54}Fe , ^{60}Ni may be of interest. (In these nuclei the DGT transitions are allowed). An intriguing possibility is the use of radioactive beams such as ^{14}O .

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