$p_\perp$-broadening and $\hat{q}$

Bin Wu

The Ohio State University

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Outline

1. **Motivations: jets in nucleus collisions**

2. **Radiative correction to jet quenching parameter**

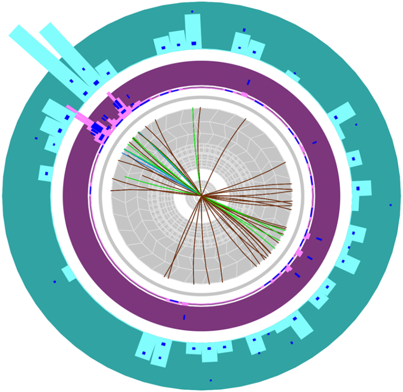
3. **Vacuum vs medium-induced double logs**

4. **Discussion and perspective**

Liou, Mueller and BW, Nucl. Phys. A 916 (2013) 102-125;

BW, JHEP **1110**, 029 (2011); JHEP **1412**, 081 (2014);
Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016);
Phys. Rev. D **95**, 034007 (2017).

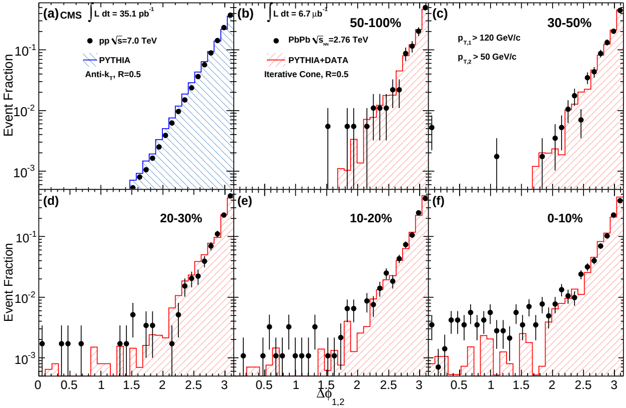
1.1 Jets in nucleus-nucleus collisions



an asymmetric dijet event in a PbPb collision

ATLAS, Phys. Rev. Lett. **105**, 252303 (2010).

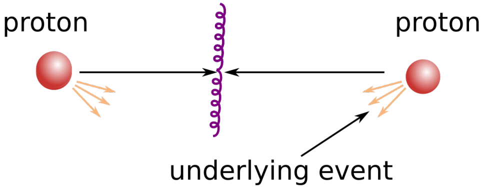
1.1 Jets in nucleus-nucleus collisions



CMS Collaboration, arXiv:1102.1957

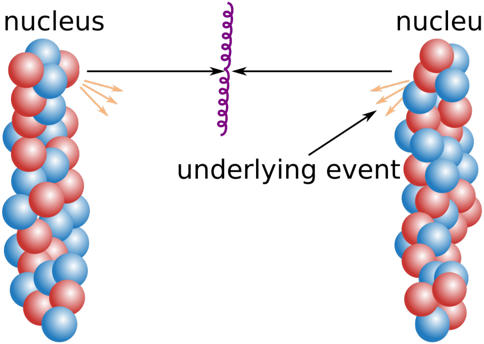
$p_\perp$-broadening $\Rightarrow$ dijet $\Delta\phi_{12}$ distributions

1.1 Jets in nucleus-nucleus collisions



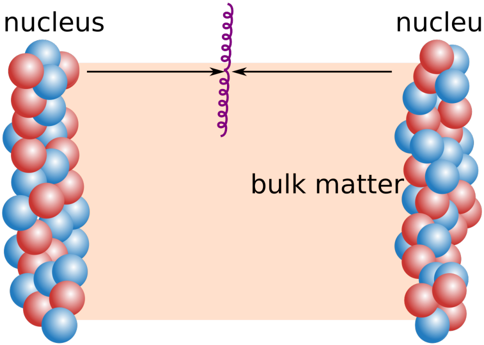
compare to a proton-proton collision

1.1 Jets in nucleus-nucleus collisions



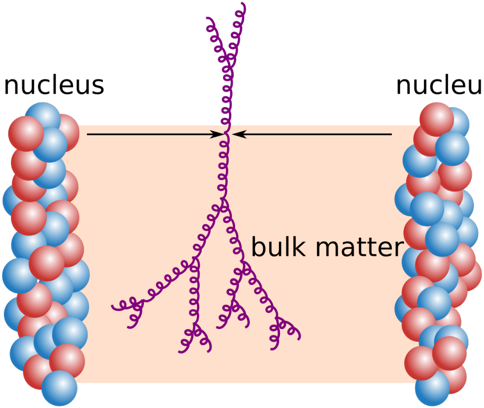
partons in a hard scattering process

1.1 Jets in nucleus-nucleus collisions



underlying event → bulk QCD matter

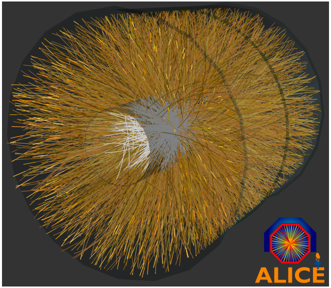
1.1 Jets in nucleus-nucleus collisions



jet evolution in bulk QCD matter

1.2 PQCD picture at leading order

Bulk matter in central AA collisions



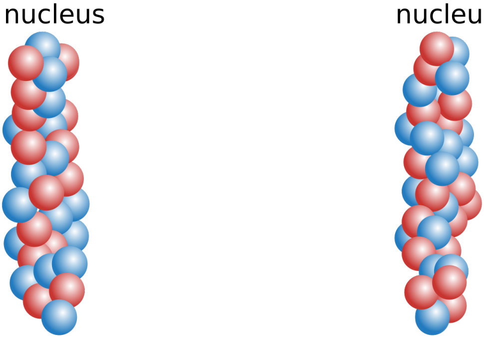
- **Particles produced**

  $\approx$ 25,000 at $\sqrt{s_{NN}} = 2.76$ TeV

- **Modelling**

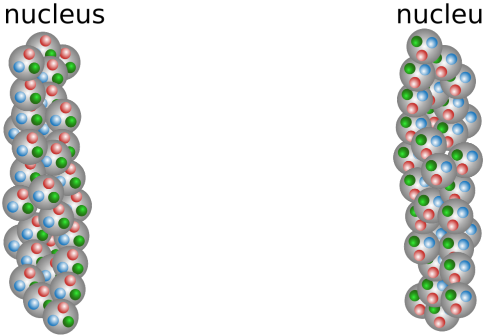
  CGC, hydro, etc.

1.2 PQCD picture at leading order



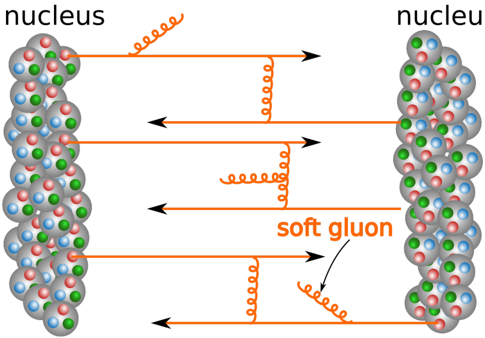
Leading order picture

1.2 PQCD picture at leading order



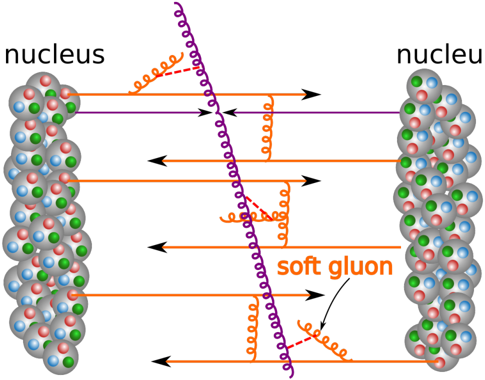
Nuclei→ constituent quarks

1.2 PQCD picture at leading order



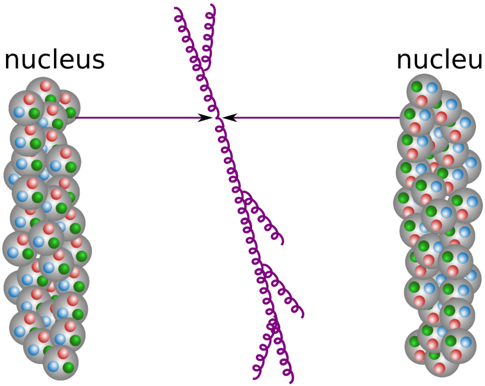
Bulk matter→ small-x gluons→ thermalized QGP (???)

1.2 PQCD picture at leading order



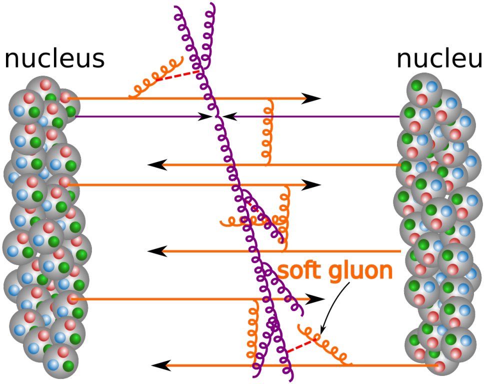
Large x partons→ jets

1.3 The QCD evolution: vacuum radiation



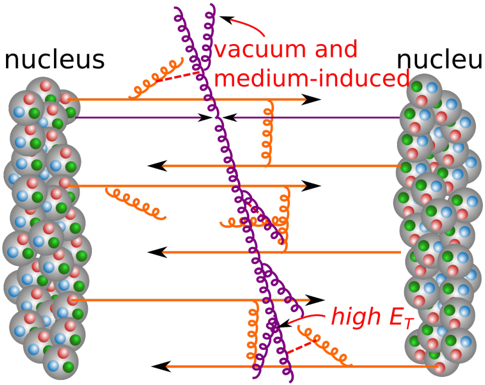
Resummation of Sudakov double log for high-energy jets

1.4 Topics of this talk



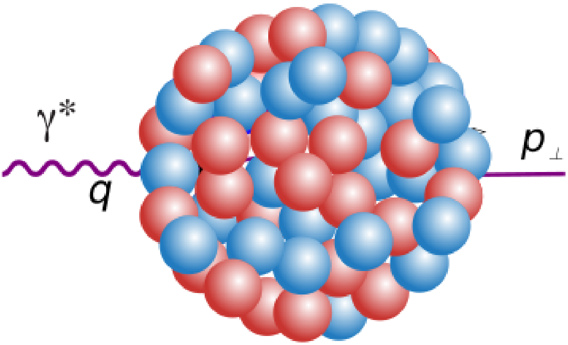
1. medium-induced radiative correction to p_T-broadening ($\hat{q}$)

1.4 Topics of this talk



2. interplay between different leading logs

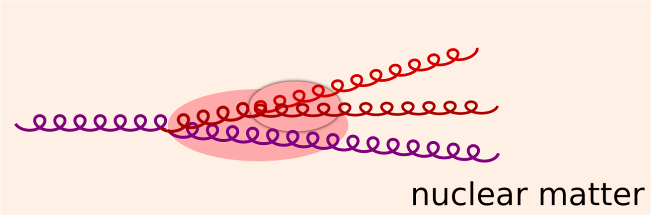
1.4 Topics of this talk



2. interplay between different leading logs

2 Radiative correction to $\hat{q}$

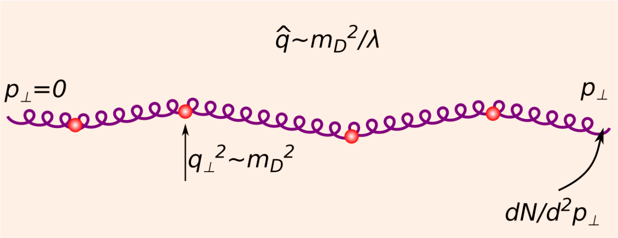
$p_\perp$-broadening: diffusion + the recoil of gluon radiation



Liou, Mueller and BW (2013); BW (2011, 2014).

2 Radiative correction to $\hat{q}$

Setup the problem: calculate typical $\langle p_\perp^2 \rangle$



$$\frac{dN}{d^2p_\perp} = \frac{1}{\pi\langle p_\perp^2\rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2\rangle}} \text{ with } \langle p_\perp^2\rangle = \hat{q}t \text{ with}$$

$$\boxed{\hat{q} = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\rho x G(x)}$$

Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B ***484****, 265 (1997).*

2 Radiative correction to $\hat{q}$

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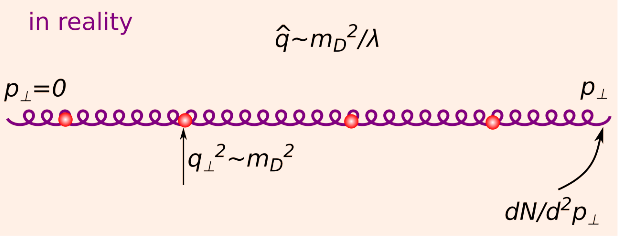
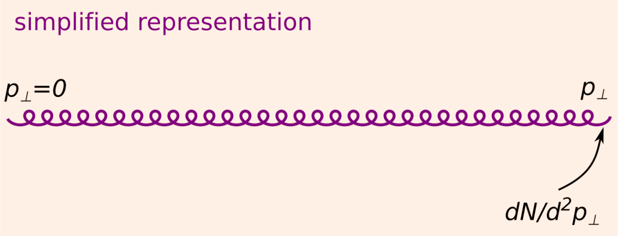


$$\frac{dN}{d^2 p_\perp} = \frac{1}{\pi\langle p_\perp^2\rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2\rangle}} \text{ with } \langle p_\perp^2\rangle = \hat{q}t$$

Small angle scatterings

2 Radiative correction to $\hat{q}$

Setup the problem: calculate typical $\langle p_\perp^2 \rangle$

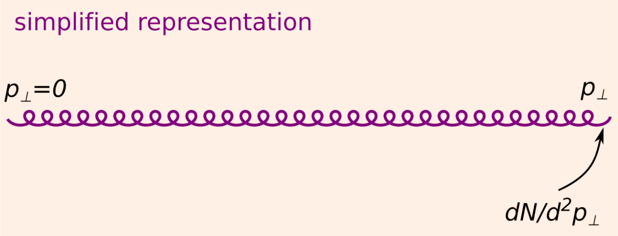


$$\frac{dN}{d^2 p_\perp} = \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} \text{ with } \langle p_\perp^2 \rangle = \hat{q} t$$

Multiple soft scatterings

2 Radiative correction to $\hat{q}$

Setup the problem: calculate typical $\langle p_\perp^2 \rangle$

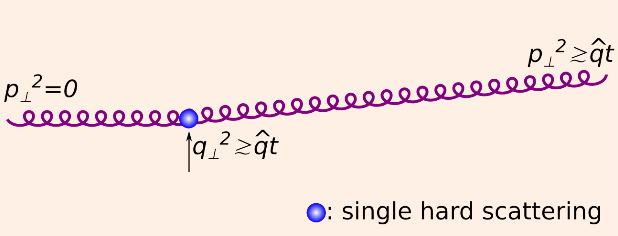


$$\frac{dN}{d^2 p_\perp} = \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} \text{ with } \langle p_\perp^2 \rangle = \hat{q} t$$

What is the contribution from recoil of gluon radiation?

2 Radiative correction to $\hat{q}$

Missing effect: (rare) single hard scattering

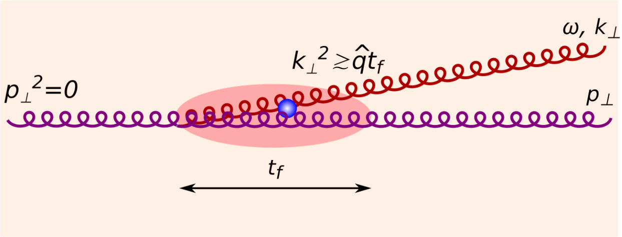


$$\frac{dN}{d^2p_\perp} \approx \frac{t}{\lambda}\frac{1}{\sigma}\frac{d\sigma}{d^2p_\perp} \propto p_\perp^{-4} \qquad \text{for } p_\perp^2 \gtrsim \hat{q}t$$

Does single scattering play any important role?

2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within t_f

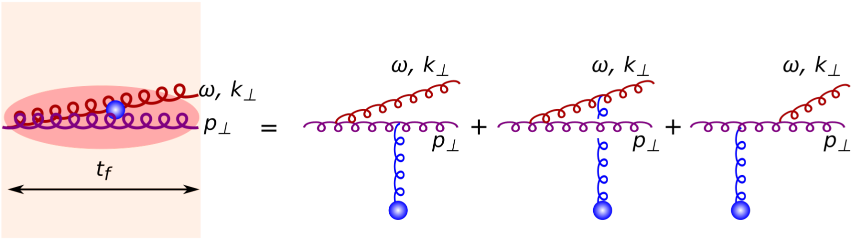


- kinematic region ($t_f = \frac{2\omega}{k_\perp^2}$)

$$k_\perp^2 \gtrsim \hat{q} t_f$$

2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within t_f

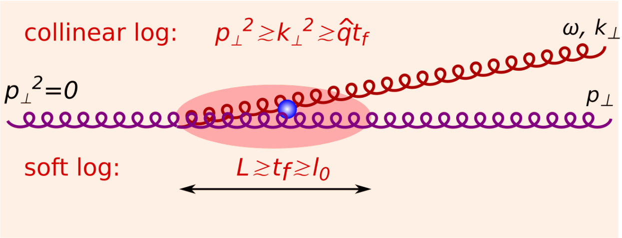


Soft & collinear divergences

$$\langle p_\perp^2 \rangle = \alpha \hat{q} L \underbrace{\int \frac{d\omega}{\omega}}_{\text{soft}} \underbrace{\int \frac{dk_\perp^2}{k_\perp^2}}_{\text{collinear}} \qquad \text{with } \alpha \equiv \frac{\alpha_s N_c}{\pi}.$$

2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within t_f

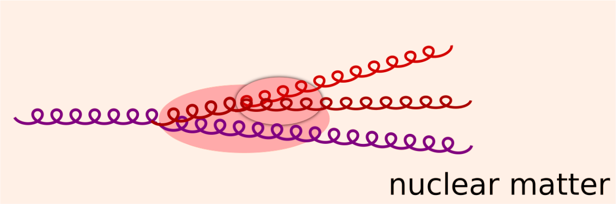


Double logarithmic enhanced contribution

$$\langle p_\perp^2 \rangle_{rad} = \alpha \hat{q} L \int_{l_0}^{L} \frac{dt_f}{t_f} \int_{\hat{q}z}^{\hat{q}L} \frac{dk_\perp^2}{k_\perp^2} = \frac{\alpha}{2} \hat{q} L \ln^2 \frac{L}{l_0}$$

2 Radiative correction to $\hat{q}$

Leading log resummation: arbitrary n-gluon emission



$$\langle p_\perp^2 \rangle_{tot} \approx \hat{q} L \sum_{n=0}^{\infty} \frac{(\alpha \ln^2 \frac{L}{l_0})^n}{(n+1)!n!} = \hat{q} L \frac{I_1(2\sqrt{\alpha} \ln \frac{L}{l_0})}{(\sqrt{\alpha} \ln \frac{L}{l_0})}$$

The lead log result of average energy loss

$$\Delta E_{tot} \approx \frac{\alpha_s N_c}{12} \langle p_\perp^2 \rangle_{tot} L$$

2 Radiative correction to $\hat{q}$

Renormalization of $\hat{q}$

$$\frac{dN}{d^2p_\perp} = \frac{1}{\pi \langle p_\perp^2 \rangle_{tot}} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle_{tot}}} \text{ with } \langle p_\perp^2 \rangle_{tot} \approx \hat{q} L \left[1 + \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{L}{l_0} \right]$$

Formal argument

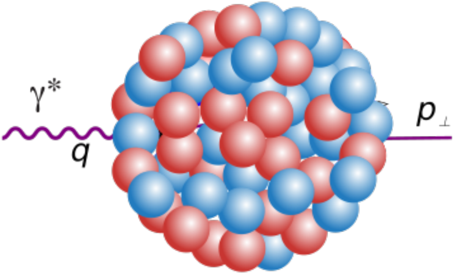
$$\hat{q} \to \hat{q}_{tot} = \frac{\left\langle p_\perp^2 \right\rangle_{tot}}{L}$$

One gluon spectrum shall be modified accordingly.

Blaizot & Mehtar-Tani (2014); Iancu (2014).

3 Vacuum vs medium-induced double logs

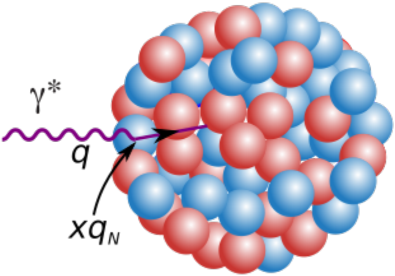
To be discussed: resummation of all the leading logs



$\frac{dN}{d^2 p_\perp}$ in a simple jet production process

3.1 Leading order picture

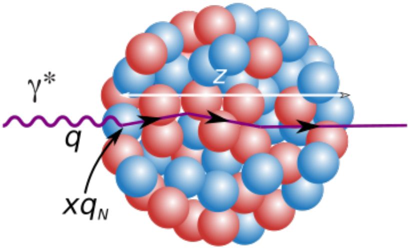
When $\frac{1}{q^-} \ll L$ with $q^2 = -Q^2$:



First, one hard kick by one nucleon

3.1 Leading order picture

When $\frac{1}{q^-} \ll L$ with $q^2 = -Q^2$:



Then, the quark undergoes multiple scattering.

$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho x q_N \left(x, \frac{1}{x_\perp^2 + 1/Q^2} \right)$$

Kovchegov and Mueller, Nucl. Phys. B **529**, 451 (1998).

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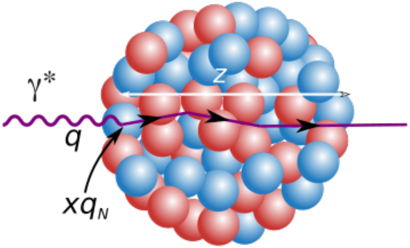

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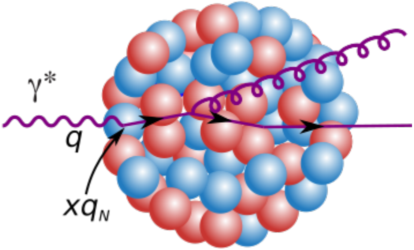
with $S(x_\perp) = \int d^2p_\perp e^{ik_\perp \cdot x_\perp} dN/d^2p_\perp$ and $\hat{q}L = Q_s^2$.

Kovchegov and Mueller, Nucl. Phys. B **529**, 451 (1998).

3.2 Leading log resummation

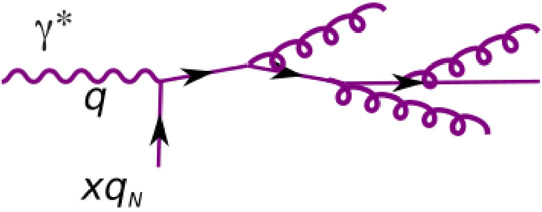
To calculate vacuum and medium-induced radiation:

For $\frac{1}{q^-} \ll L$



3.2.1 Vacuum radiation

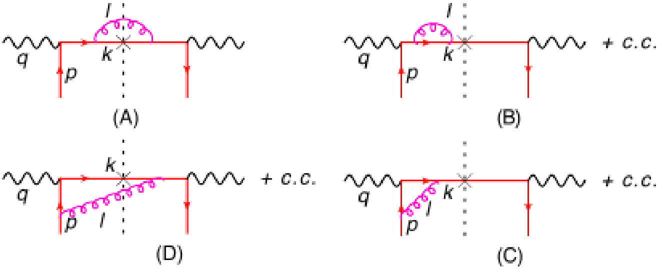
The Sudakov form factor:



$$\frac{dN}{d^2k_\perp} \propto e^{-\frac{\alpha_s C_F}{2\pi}\ln^2\frac{k_\perp^2}{Q^2}}.$$

3.2.1 Vacuum radiation

The Sudakov double log ($A^+ = 0$ gauge):



Diagrams cancel if

1. $k_\perp^2 > l_\perp^2$: real and virtual (A&B, C&D)
2. $\frac{1}{q^-} = \frac{2q^+}{Q^2} < \frac{1}{l^-} = \frac{2l^+}{l_\perp^2}$: $C\&C$ and $B\&D$

Mueller, BW, Xiao and Yuan, arXiv:1608.07339.

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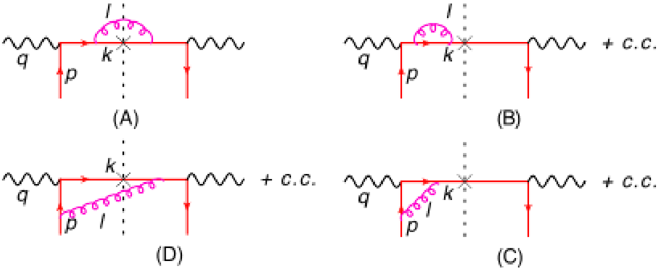


$$A + B \approx -\frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_0^{q^+} \frac{dl^+}{l^+},$$

$$C + D \approx \frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_{l^- > q^-} \frac{dl^+}{l^+} = \frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_0^{q^+ \frac{l_\perp^2}{Q^2}} \frac{dl^+}{l^+},$$

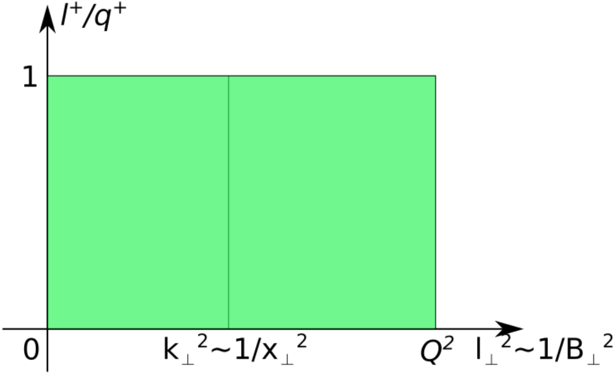
$$A + B + C + D \approx -\frac{\alpha_s C_F}{2\pi} \rho x q_N \ln^2 \frac{Q^2}{k_\perp^2}.$$

3.2.2 Sudakov double log and multiple scattering

Conclusion:

Multiple scattering does not modify the Sudakov factor.

Reason:



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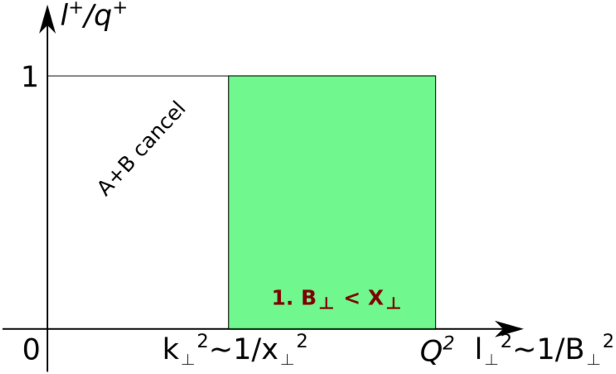

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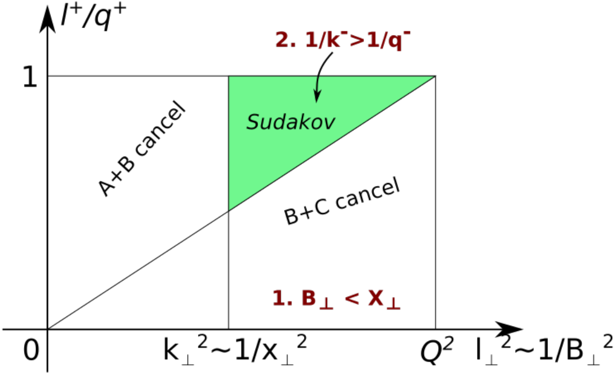
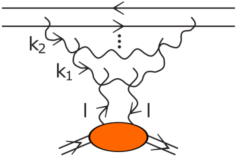


The medium can not "see" the radiated gluon.

3.2.3 Medium-induced radiation

Resummation of medium-induced double log:



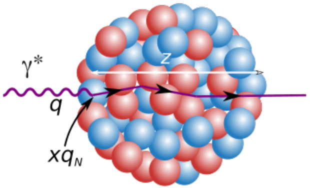
$$\hat{q}_t = \begin{cases} \hat{q} & \text{if } x_\perp^2 > 1/\hat{q} r_0, \\ \hat{q} \frac{1}{\sqrt{\bar{\alpha}_s K}} I_1(2\sqrt{\bar{\alpha}_s} K) & \text{if } 1/\hat{q} r_0 \geq x_\perp^2 \geq 1/\hat{q} L, \\ \hat{q} \left[\frac{1}{\sqrt{\bar{\alpha}_s K \tau}} I_1(2\sqrt{\bar{\alpha}_s K \tau}) + \left(1 - \frac{\tau}{K}\right) I_2(2\sqrt{\bar{\alpha}_s K \tau}) \right] & \text{if } x_\perp^2 \leq 1/\hat{q} L. \end{cases}$$

with $K = \ln \frac{1}{\hat{q} r_0 x_\perp^2}$ and $\tau = \ln \frac{L}{r_0}$.

Liou, Mueller and BW (2013); Iancu & Triantafyllopoulos (2014).

3.3 Resummation of leading logs

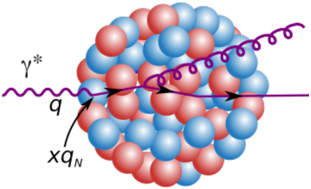
For $\frac{1}{q^-} \ll L$:



$$\frac{dN}{d^2b\,d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[-\frac{1}{4}\hat{q}\; x_\perp^2 z \qquad\qquad\qquad\qquad\right].$$

3.3 Resummation of leading logs

For $\frac{1}{q^-} \ll L$:



$$\frac{dN}{d^2bd^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_t x_\perp^2 z}_{\text{medium-induced}}\right].$$

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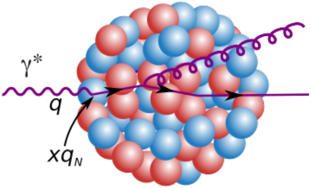
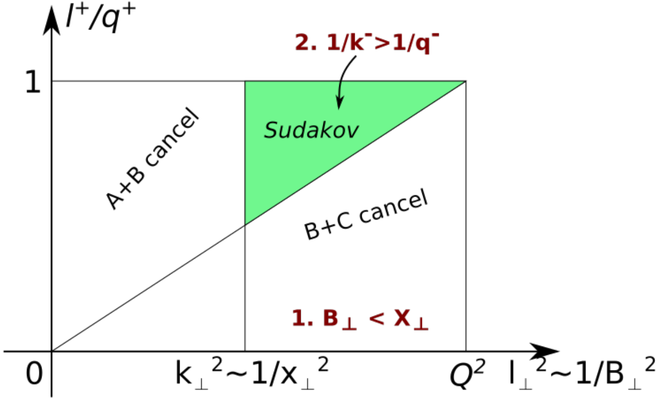


$$\frac{dN}{d^2b\,d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_t x_\perp^2 z}_{\text{medium-induced}} \underbrace{-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}}\right].$$

See, for resummation, Collins, Soper & Sterman (1985); Mueller, Xiao & Yuan (2013).

3.3 Resummation of leading logs

Two double logs are factorized:



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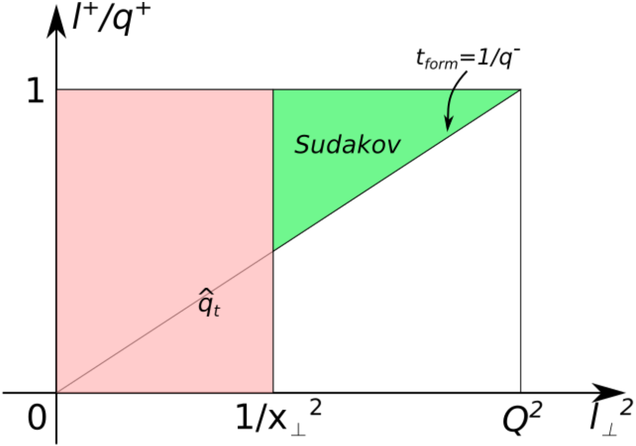

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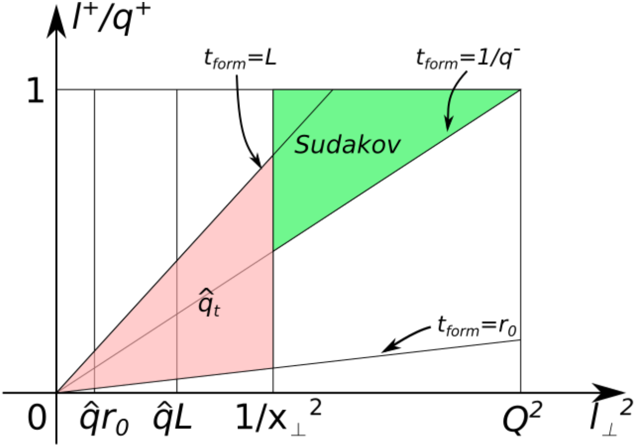

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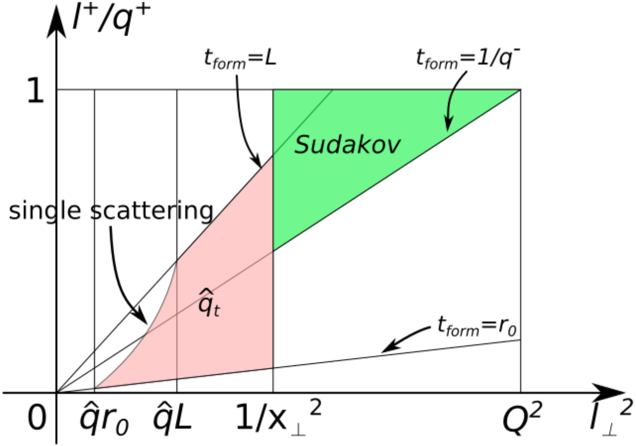
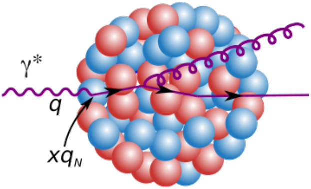


3.3 Resummation of leading logs

To study medium effects, Q should not be large!

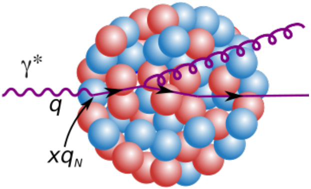


$$\frac{dN}{d^2b\,d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right)$$

$$\times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_t x_\perp^2 z}_{\text{medium-induced}} \underbrace{-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}}\right].$$

3.3 Resummation of leading logs

If $Q \gg Q_s$ or $\hat{q}L$, barely see medium effect!



Otherwise,

$$\frac{dN}{d^2b\,d^2k_\perp} \approx \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{\alpha_s C_F}{2\pi} \ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}} \right].$$

Discussion

Dijet azimuthal angular distributions ($Q \to p_{\perp,1}, p_{\perp,2}$)

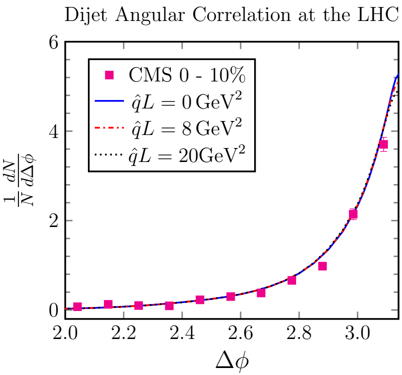


Figure: $p_{\perp,1}$=120 GeV and $p_{\perp,2}$=50 GeV in central (0-10%) collisions at the LHC.

Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016).

Discussion

Ask for the same measurement at RHIC

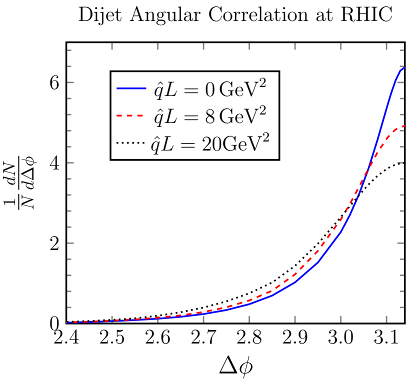


Figure: We take $p_{\perp,1}$=35 GeV.

Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016).

Perspecitve

What to do next?

1. Radiative energy loss at large Q
2. Other observables related to $p_\perp$-broadening
3. Can one study jet and medium altogether from first principles in QCD?