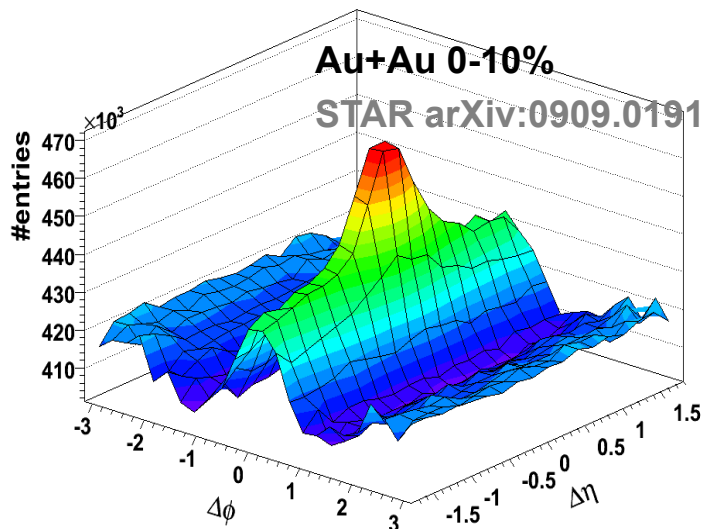
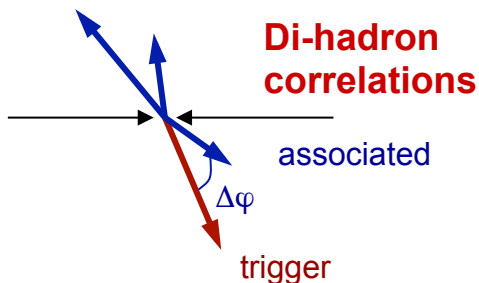


# Probing extreme QCD through ridge-like correlations in small systems: status and problems

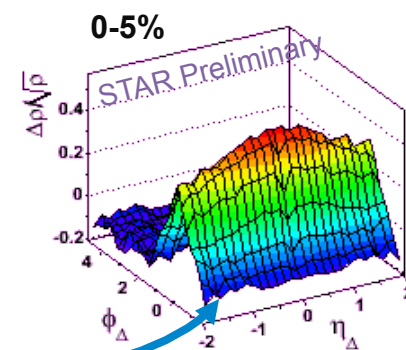
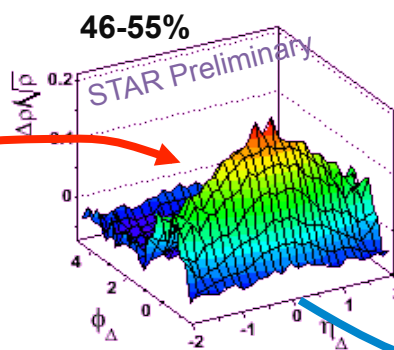
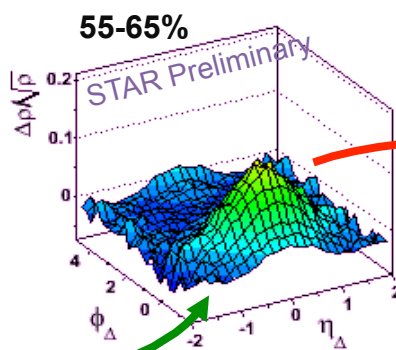
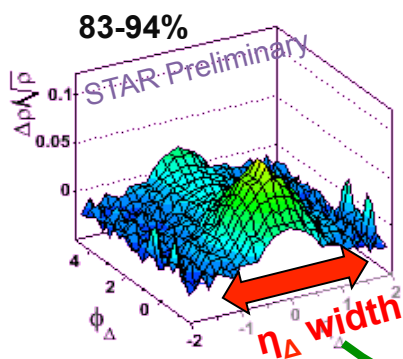
Raju Venugopalan  
Brookhaven National Laboratory

INT workshop, May 15, 2017

# The ridge in A+A collisions



$3 < p_{t,trigger} < 4 \text{ GeV}$   
 $p_{t,assoc.} > 2 \text{ GeV}$

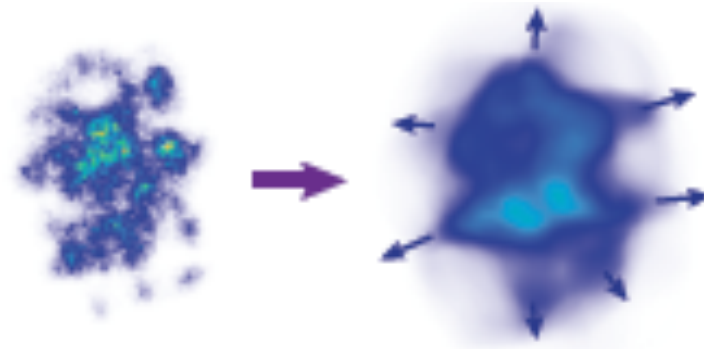
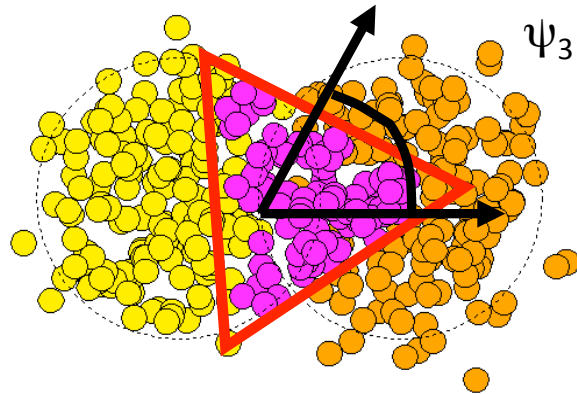


**Collimated, long range rapidity correlations:  
First seen by RHIC Au+Au experiments: STAR, PHOBOS, PHENIX**

# The ridge in A+A collisions

Alver, Roland, PRC81(2010) 054905

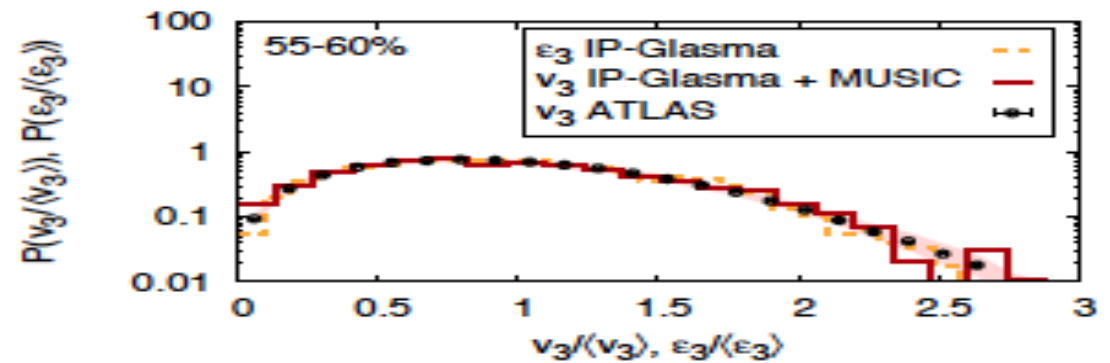
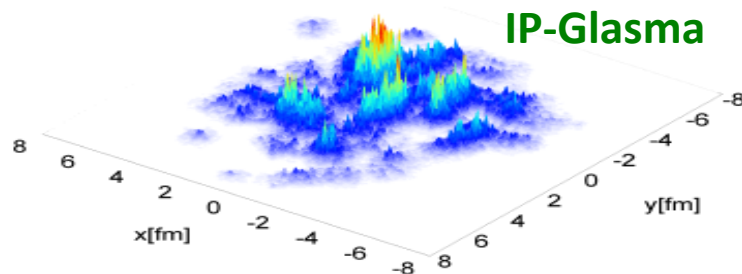
Alver, Gombeaud, Luzum, Ollitrault, PRC82 (2010) 03491



Structure of ridge-correlations can be understood as hydrodynamic flow driven by event-by-event fluctuations in nucleon positions

$$\frac{1}{N_{\text{trig}} N_{\text{assoc.}}} \frac{d^2 N}{d\Delta\Phi} = 1 + V_1 \text{Cos}(\Delta\Phi) + V_2 \text{Cos}(2\Delta\Phi) + \dots$$

Gale, Jeon, Schenke, Tribedy, Venugopalan, PRL110 (2013) 012302



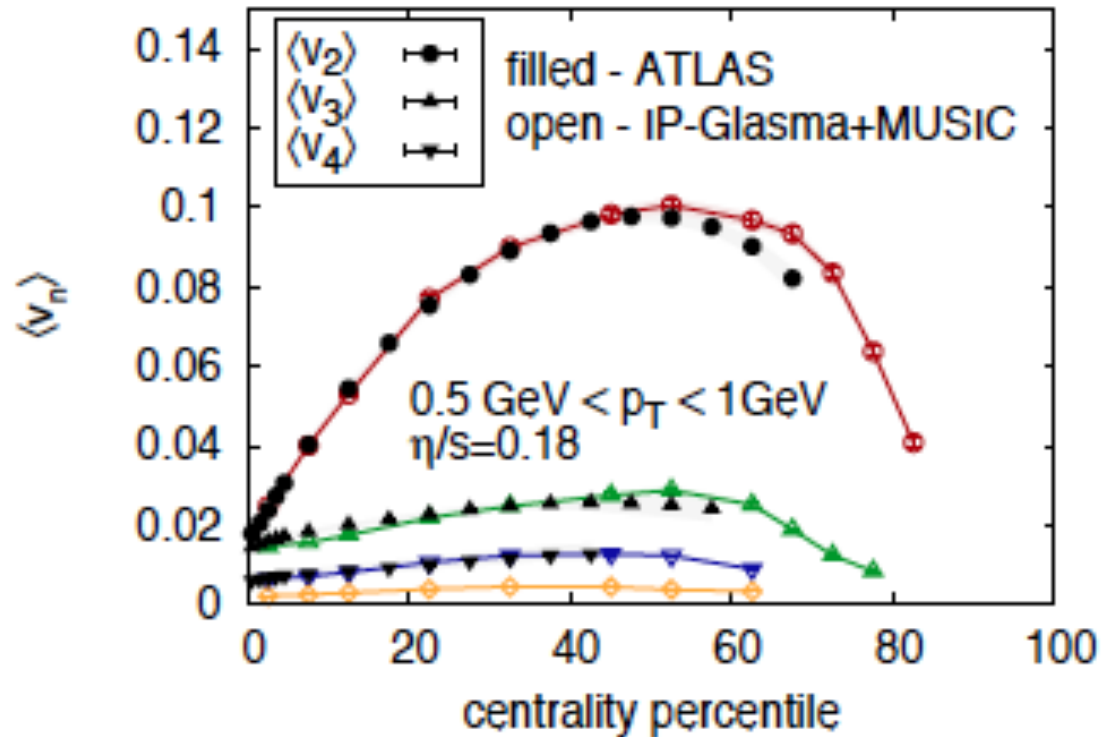
Some evidence of sensitivity of data to sub-nucleon scale fluctuations

# What's the smallest sized QGP droplet?

IP-Glasma= initial state

MUSIC=event.by.event. hydro

Schenke, Venugopalan, PRL 113 (2014) 102301



Where does the hydro paradigm break down?

# Higher cumulants of elliptic flow

m-particle flow  
cumulants

$$c_n \{2m\} = \langle\langle e^{in(\phi_1 + \dots + \phi_m - \phi_{m+1} - \dots - \phi_{2m})} \rangle\rangle$$

Borghini,Dinh,Ollitrault, nucl-th/0105040

$$v_n \{2\}^2 \equiv c_n \{2\} \quad v_n \{4\}^4 \equiv -c_n \{4\} \quad v_n \{6\}^6 \equiv c_n \{6\} / 4$$

Spatial eccentricities: 
$$\epsilon_n = \frac{1}{\langle r_\perp^n \rangle} \int d^2 r_\perp e^{in\phi_r} r_\perp^n \frac{dN}{dy d^2 r_\perp}$$

A number of simple models give  $\epsilon_n \{2\} > \epsilon \{4\} = \epsilon \{6\} = \dots$

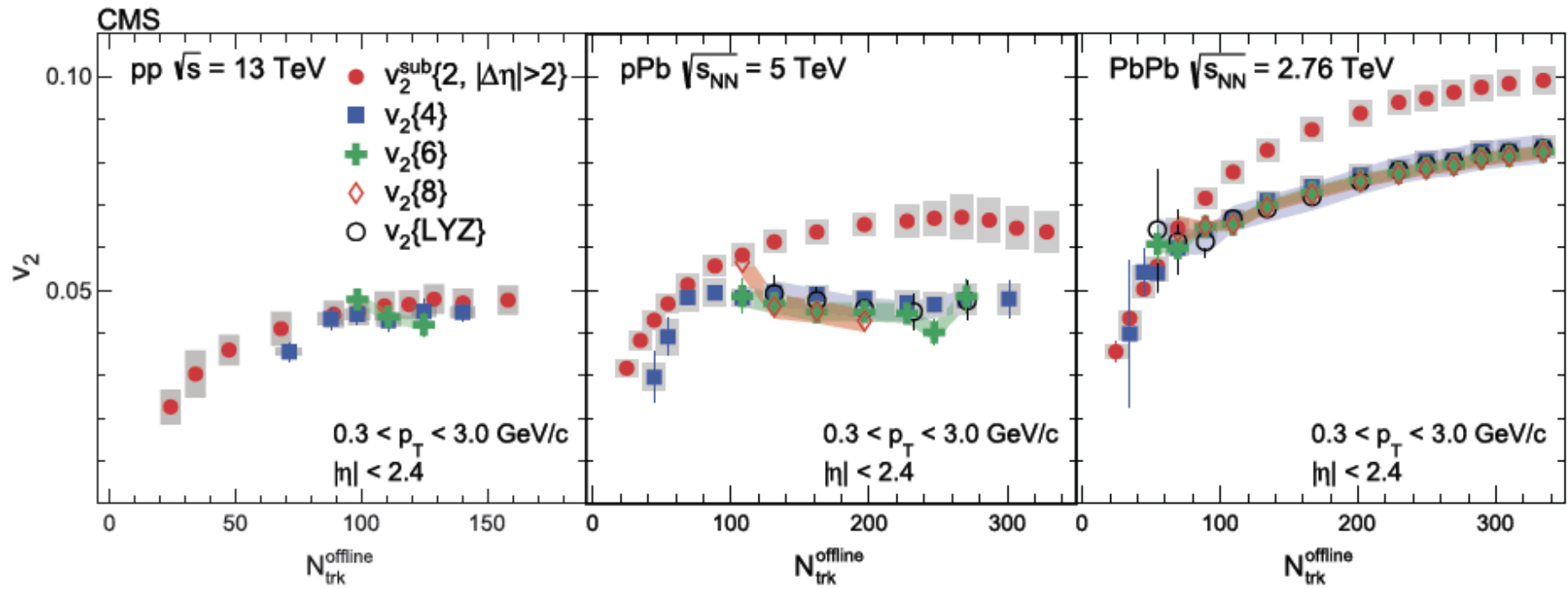
Hydro linear response: 
$$v_n \{m\} \approx c_n \epsilon_n \{m\}$$

Gardim,Grassi,Luzum,Ollitrault, PRC (2012)024908; Niemi,Denicol,Holopainen,Huovinen, PRC87 (2013)054901

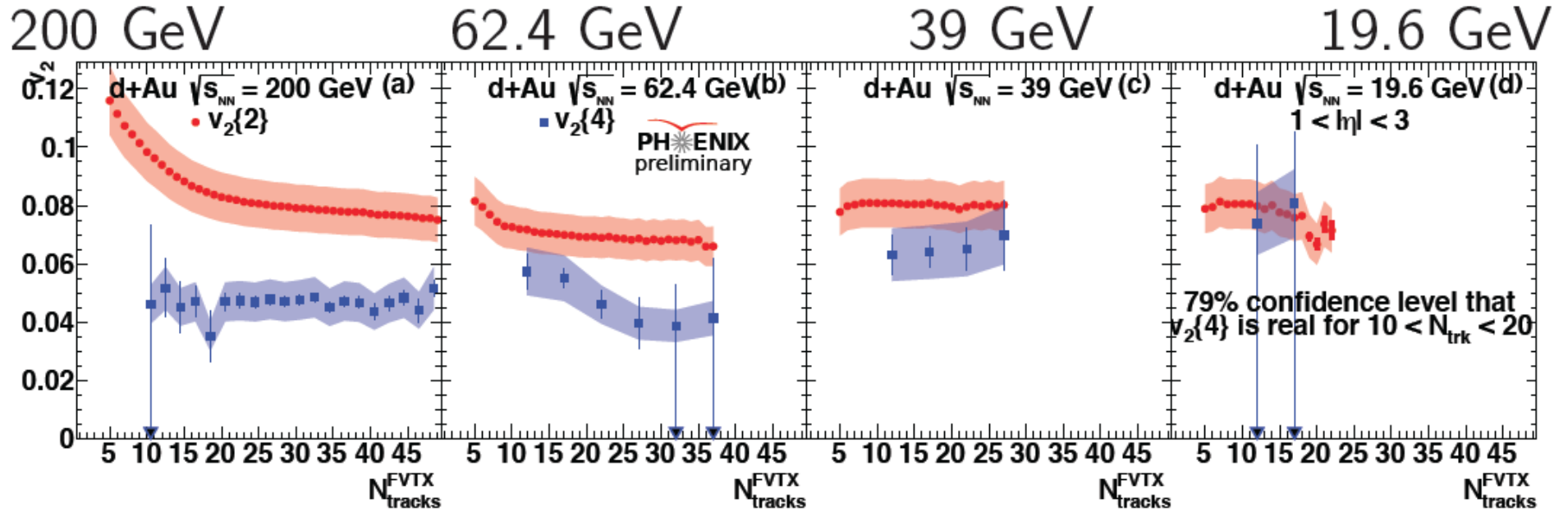
Bzdak,Bozek,McLerran, arXiv:1311.7325, Bzdak, Skokov, arXiv: 1312.7349

Yan, Ollitrault, arXiv:1312.6555, Basar,Teaney, arXiv:1312.6770

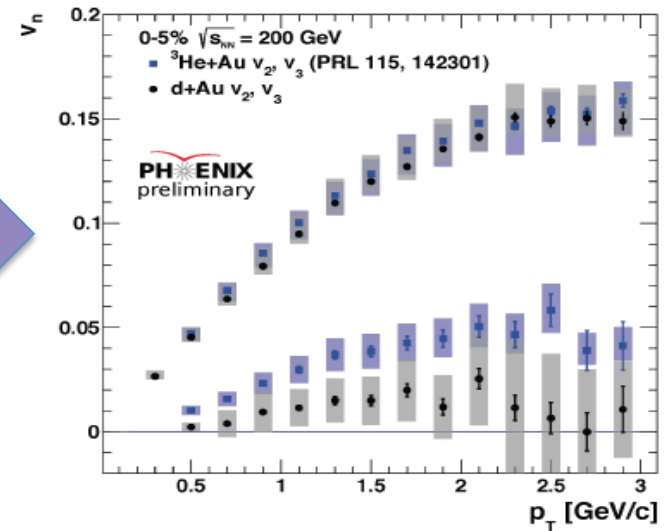
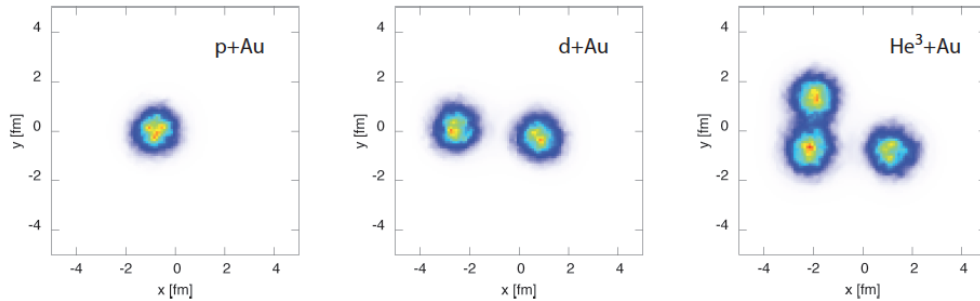
# Collectivity across system size



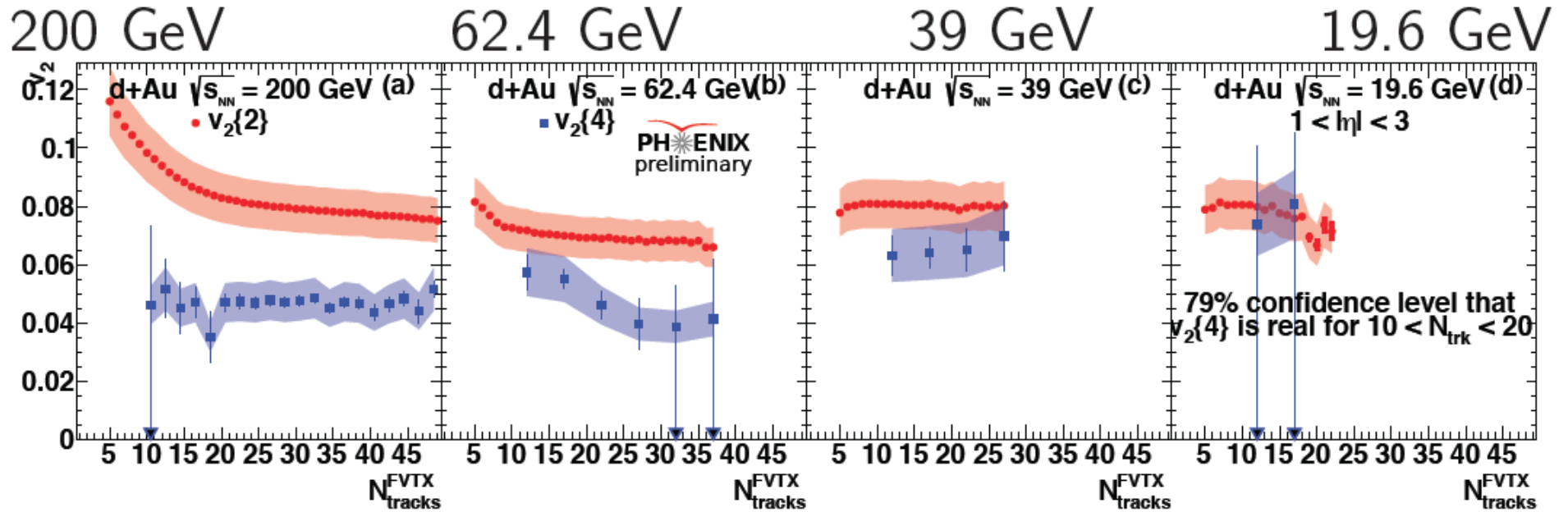
# Collectivity across wide energy scales



Schenke, RV:1407.7557



# Collectivity across wide energy scales





# Panta Rhei?



Heraclitus of Ephesus  
535-475 BC

**Natural in hydro – yet very few ab initio hydro computations of 4-particle cumulants for  $p+A$**

**-- none for  $p+p$**

# Issues with the hydrodynamic paradigm: I

Two frequently used measures: Reynolds # and Knudsen #

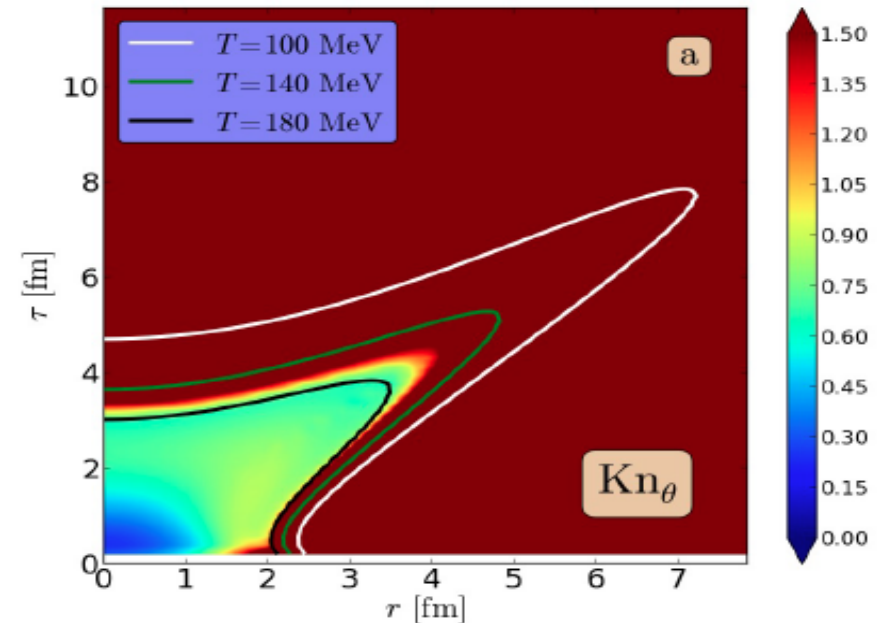
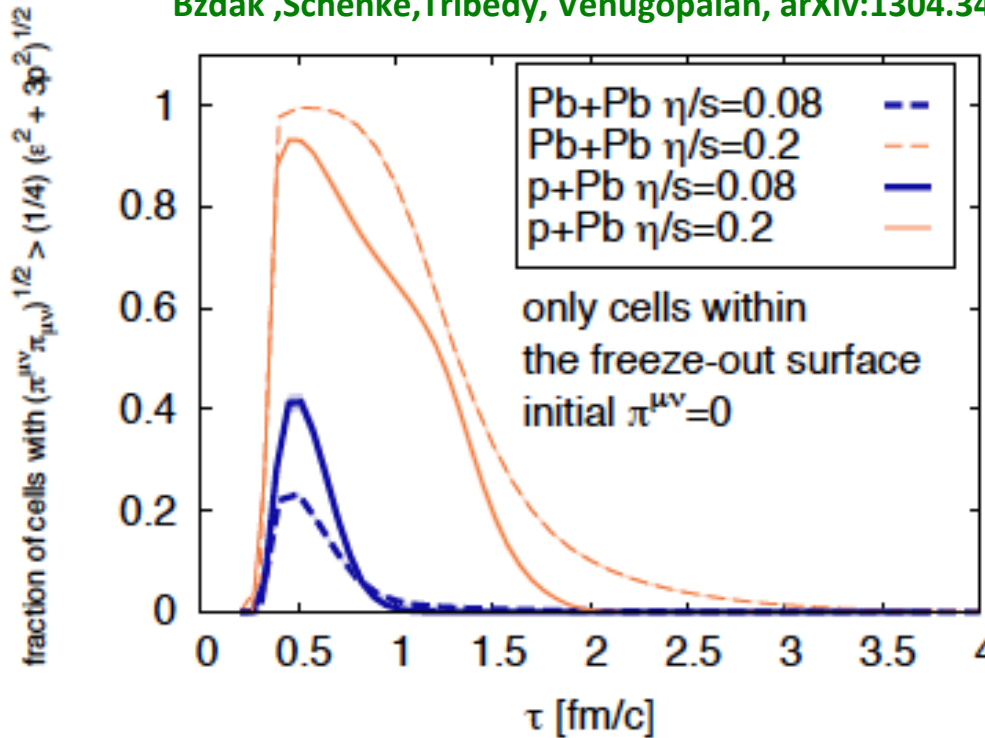
$$R^{-1} \propto (\Pi^{\mu\nu} \Pi_{\mu\nu})^{1/2} / (\epsilon^2 + 3P^2)^{1/2}$$

$$\text{Kn} = \frac{\tau_\pi}{L} ; \tau_\pi \propto \frac{\eta}{sT}$$

Bzdak ,Schenke,Tribedy, Venugopalan, arXiv:1304.3403

Dumitru,Molnar,Nara,arXiv:0706.2233

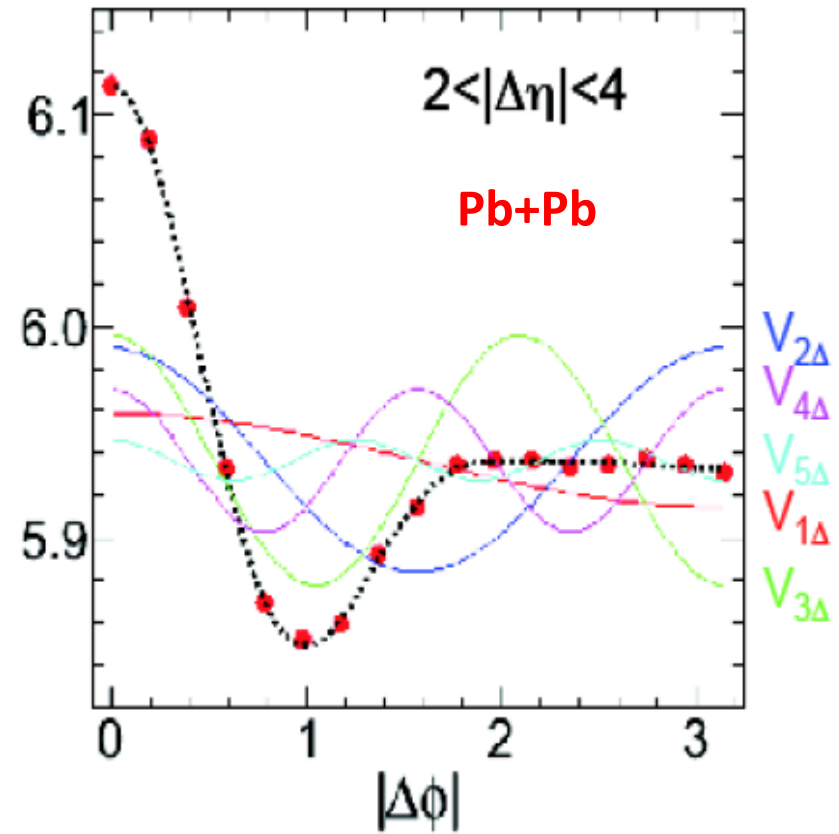
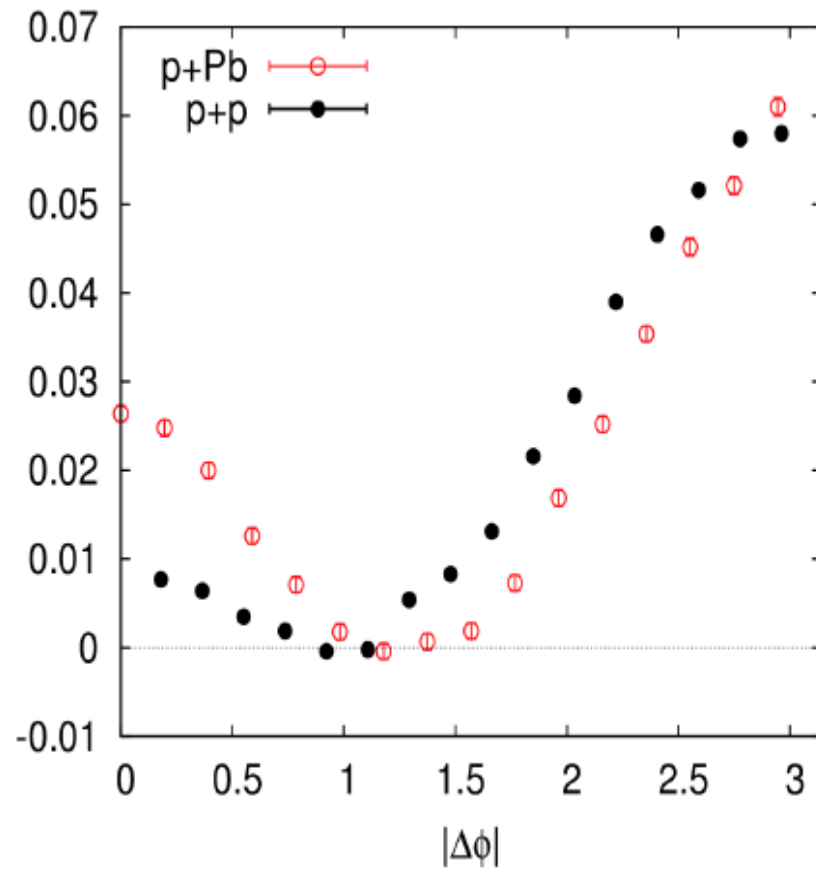
Denicol, Niemi, arXiv:1404.7327



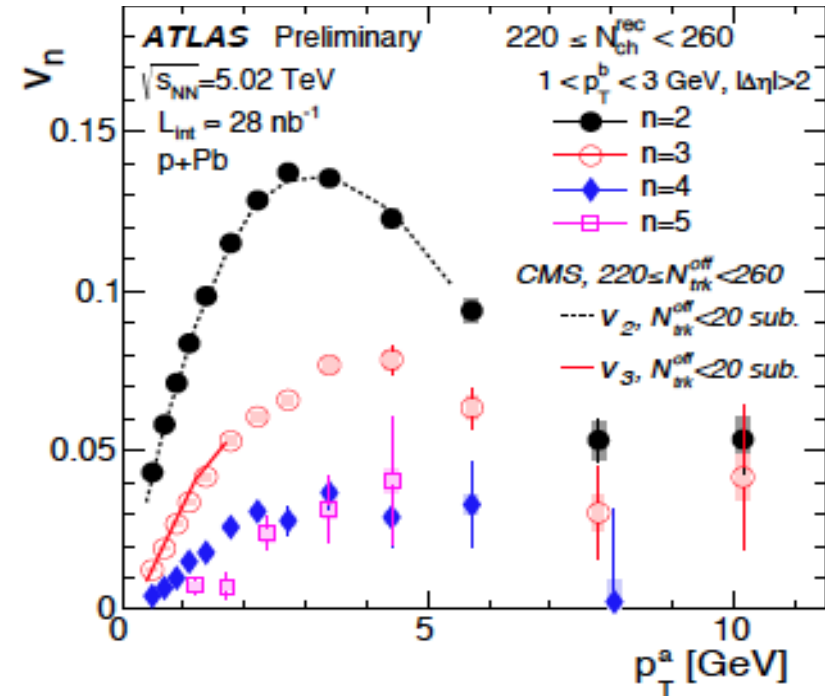
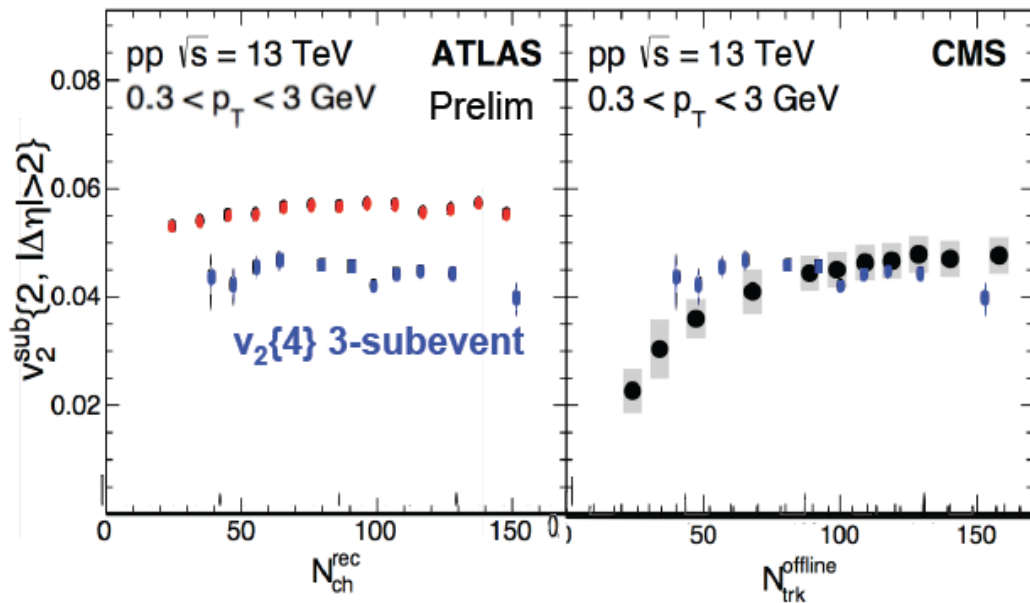
Hydro good for  $\text{Kn} < 0.5$ ,  
 marginal for  $\text{Kn} < 1$  transient regime;  
 $\text{Kn} > 1$  free streaming

# Issues with the hydrodynamic paradigm: II

No (mini-) jet quenching seen in the smaller systems



# Issues with the hydrodynamic paradigm: III



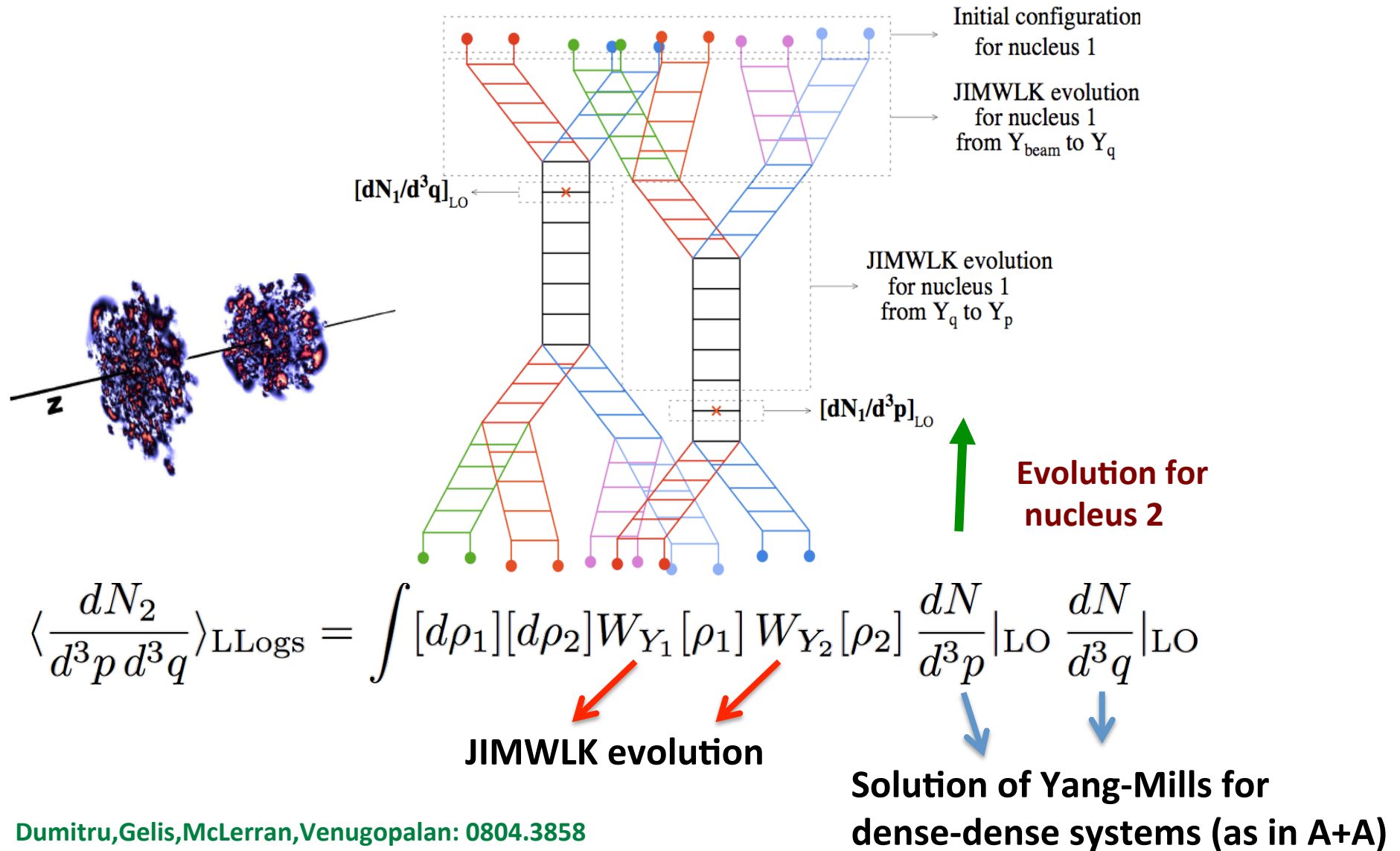
Large anisotropies at larger  $p_T$  and smaller  $N_{\text{ch}}$  than one might reconcile with a hydrodynamic description

Four-particle collectivity seen in minimum bias events...

# Can we understand multiparticle correlations in an *ab initio* approach

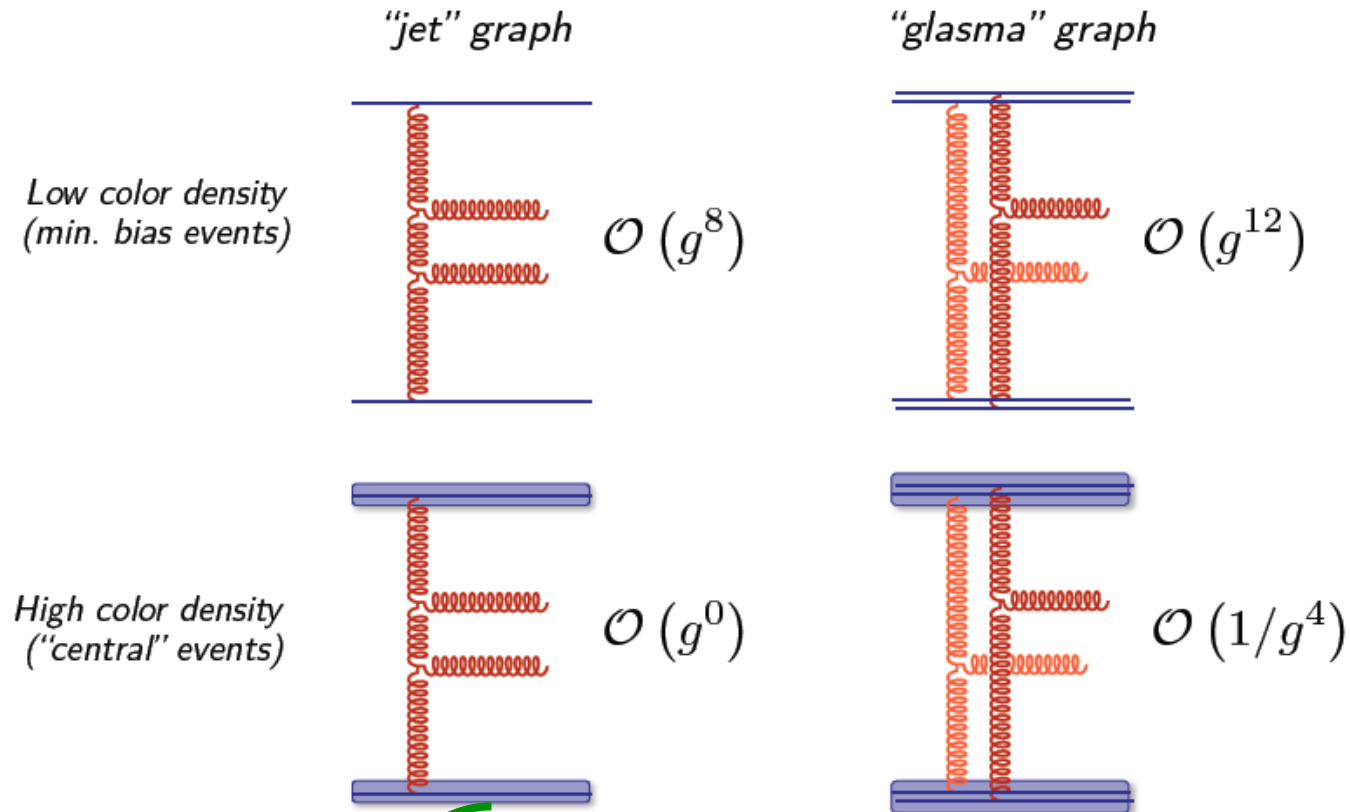
Review: Dusling, Li, Schenke, arXiv:1509.07939

# Two-parton azimuthal correlations in the CGC



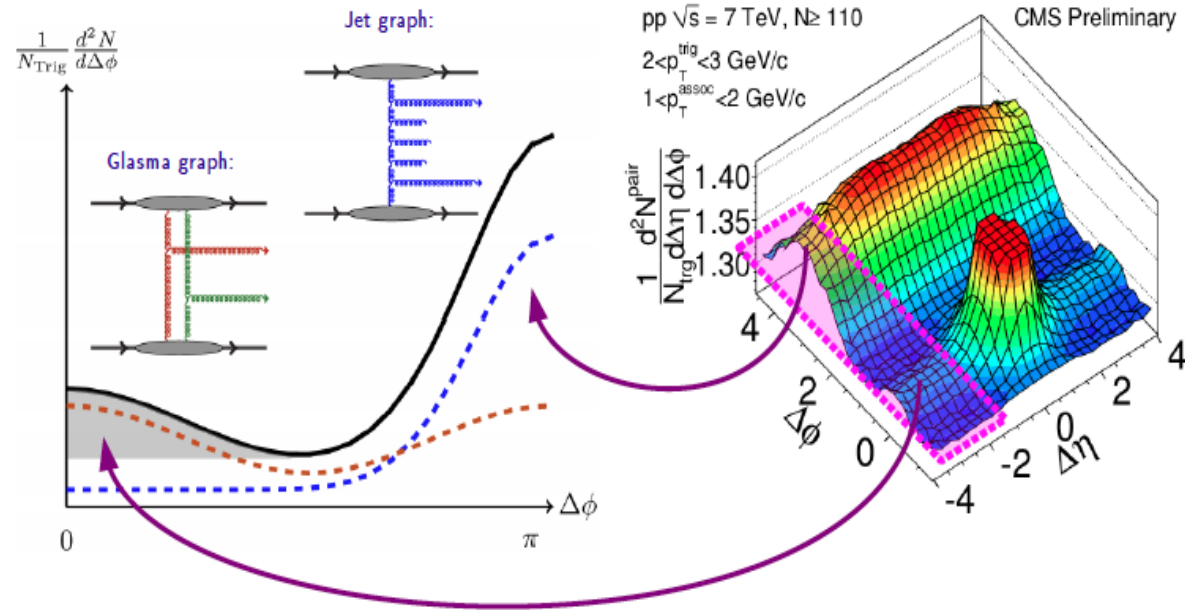
Dumitru, Gelis, McLerran, Venugopalan: 0804.3858  
 Gelis, Lappi, Venugopalan, arXiv: 0807.1306  
 Dusling, Gelis, Lappi, Venugopalan, arXiv: 0911.2720

# Glasma graph approximation: power counting



Gluons with  $k_T \sim Q_S$  resolve  
 $n \sim 1/g^2$  color sources  
 Effective coupling:  $g*n \sim 1/g$

# Anatomy of long range collimations



**RG evolution of Glasma graphs:**

$$C(\mathbf{p}, \mathbf{q}) \propto \frac{g^4}{\mathbf{p}_\perp^2 \mathbf{q}_\perp^2} \int d^2 \mathbf{k}_{1\perp} \Phi_{A_1}^2(y_p, \mathbf{k}_{1\perp}) \Phi_{A_2}(y_p, \mathbf{p}_\perp - \mathbf{k}_{1\perp}) \Phi_{A_2}(y_q, \mathbf{q}_\perp - \mathbf{k}_{1\perp})$$

+ permutations

**RG evolution of the mini-jets:**  $C_{\text{dijet}}(\mathbf{p}, \mathbf{q}) \propto \Phi_A \otimes \Phi_B \otimes G_{\text{BFKL}}$

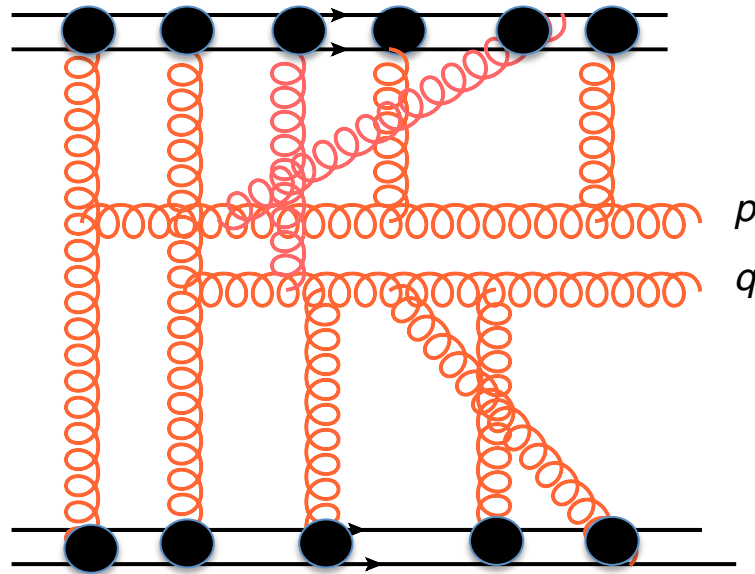
**Good agreement with data for  $p_T > Q_s$**

**However no odd harmonics  $v_3, v_5$  for gluons**

**because  $C(\mathbf{p}, \mathbf{q}) = C(\mathbf{p}, -\mathbf{q})$**



# Beyond glasma graphs



Coherent multiple scattering is of the same order in the coupling:  
power suppressed for  $p_T \gg Q_s$ , important for  $p_T < Q_s$

Compute (numerically) by solving Yang-Mills equations In presence of two  
light cone sources: IP-Glasma model

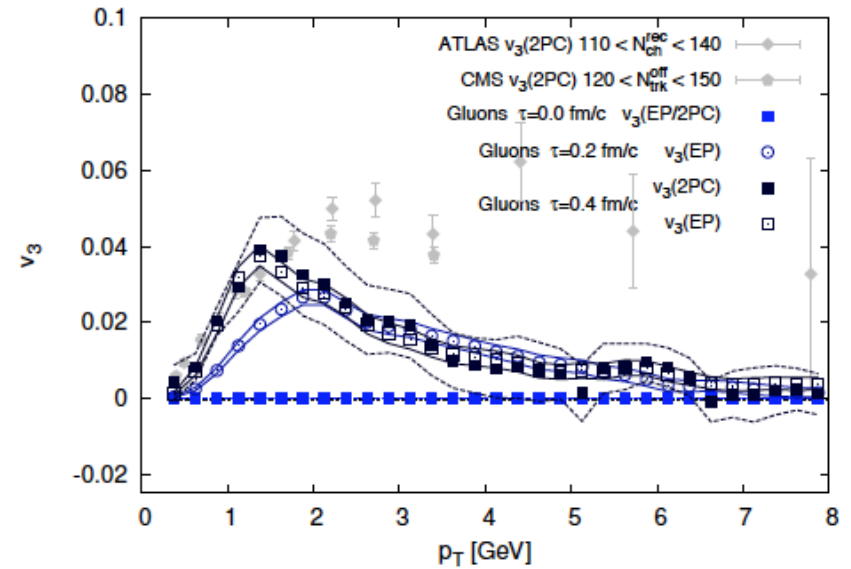
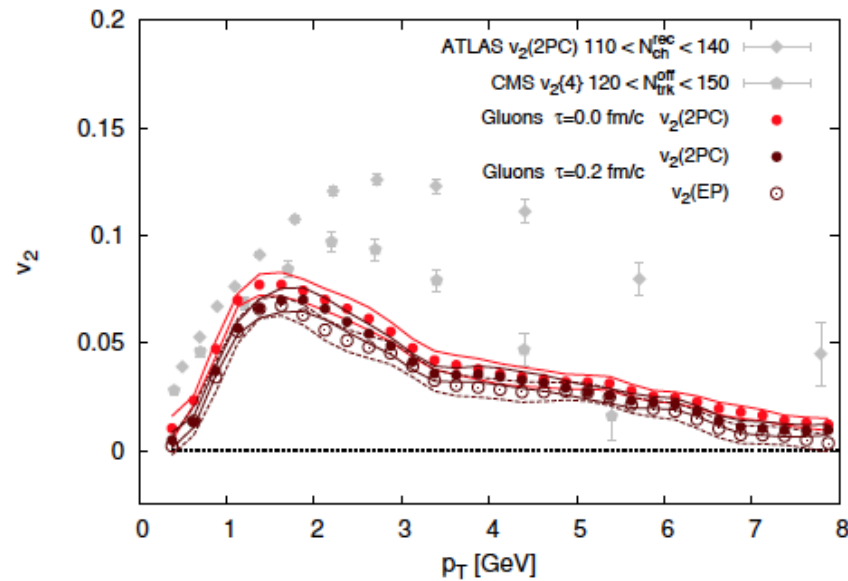
[Schenke, Tribedy, Venugopalan, arXiv:1202.6646, 1206.6805](#)

$$\left\langle \frac{d^2 N}{d^2 p_T d^2 q_T} \right\rangle = \int D\rho_A D\rho_B e^{-\int d^2 x_T \rho_A^2 / Q_{s,A}^2} e^{-\int d^2 x_T \rho_B^2 / Q_{s,B}^2} \frac{dN}{d^2 p_T} [\rho_A, \rho_B] \frac{dN}{d^2 q_T} [\rho_A, \rho_B]$$

$C(p,q) \neq C(p,-q)$  -- all harmonics contribute

[Lappi, Srednyak, Venugopalan, arXiv:0911.2068](#)

# Azimuthal anisotropy from Yang-Mills dynamics



Schenke,Schlichting,RV, PLB747(2015)76

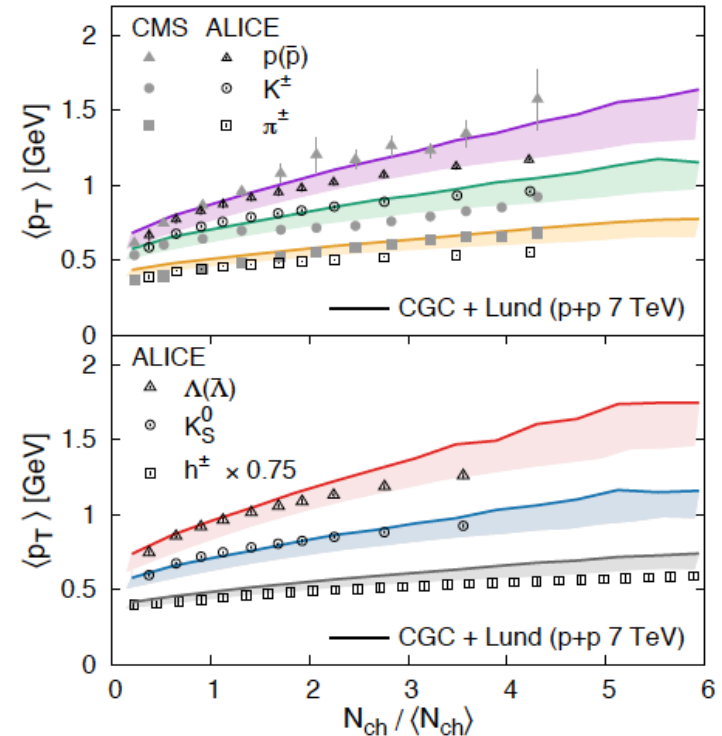
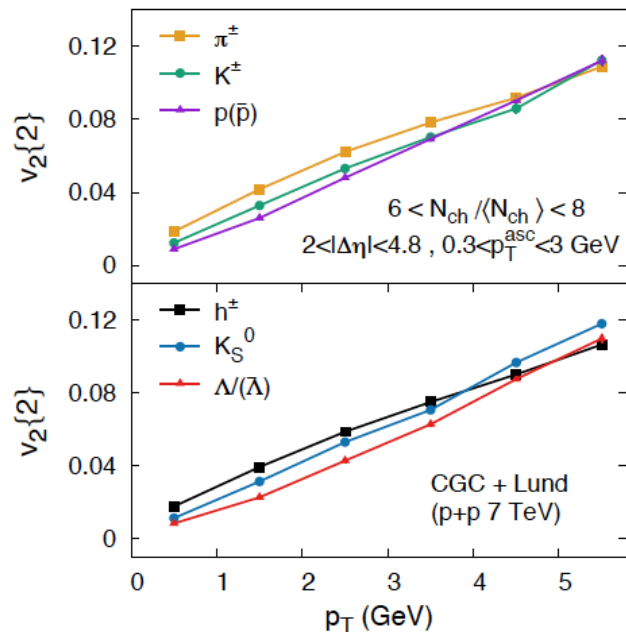
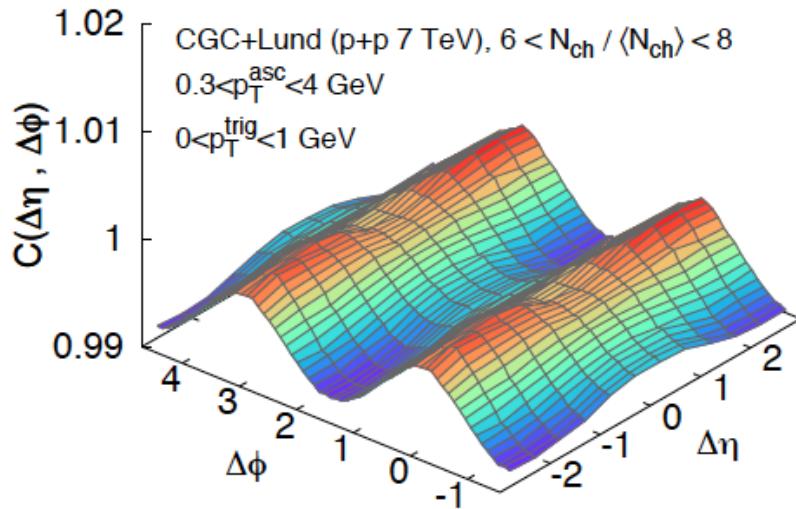
Recent analytical work in dilute-dense approx:

[Kovchegov,Wertepny,NPA906 \(2013\)50](#)

[McLerran, Skokov arXiv:1611.09870](#)

[Kovner,Lublinsky,Skokov, arXiv:1612.07790](#)

# IP-Glasma+Lund fragmentation



Pattern of mass splitting of  $\langle p_T \rangle$  and  $v_2$  seen in high multiplicity events is reproduced [Schenke,Schlichting,Tribedy,RV, PRL117\(2016\)162301](#)

What about 4-particle collectivity?  
 Numerically very challenging-in progress

[Schenke,Schlichting,Tribedy,RV](#)

# Tracing azimuthal initial state correlations

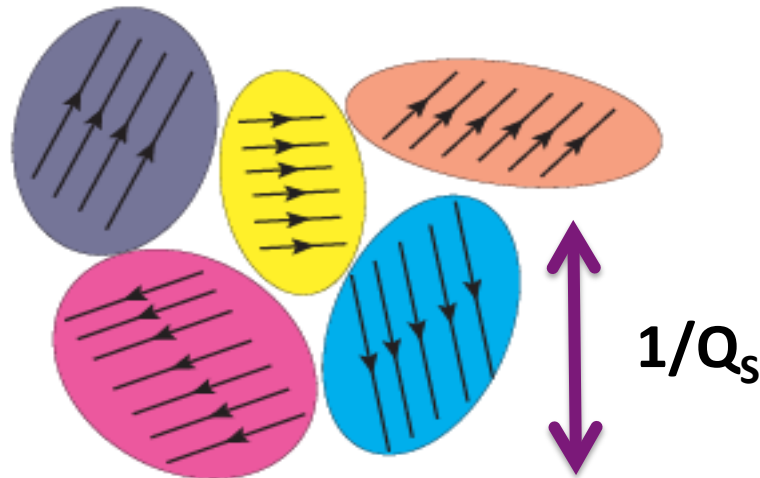
Simple ab initio initial state model:

Multi-particle correlations from Eikonal scattering of partons  
off color domains in a nuclear target

Lappi, arXiv:1501.05505

Lappi,Schenke,Schlichting,RV, arXiv:1509.03499

Dusling,Mace,RV, arXiv:1705.00745



Color domain model:

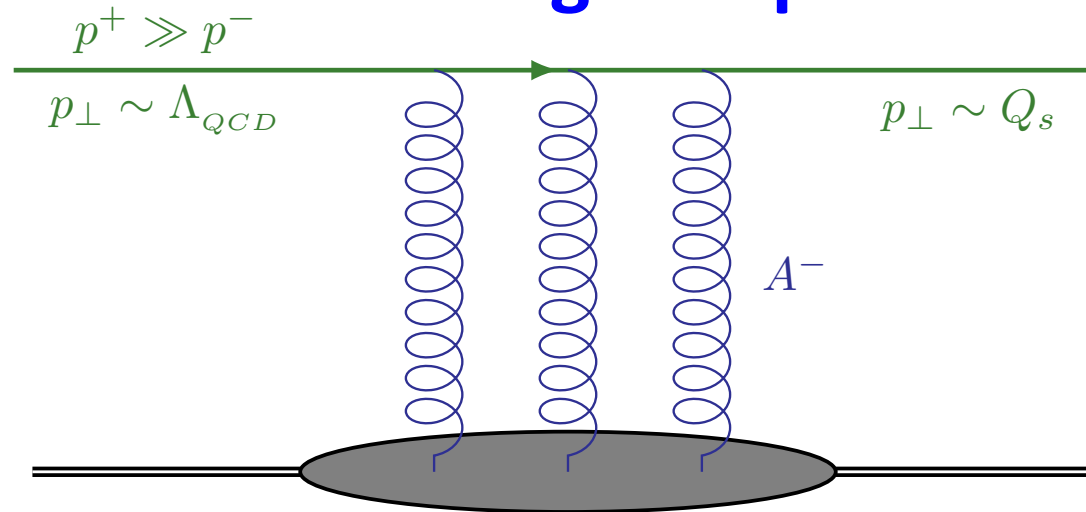
Kovner,Lublinsky,arXiv:1012.3398,1109.0347

Dumitru,Gianini, arXiv:1406.5781

Dumitru,Skokov,arXiv:1411.6630,

Dumitru,McLerran,Skokov,arXiv:1410.4844

# Eikonal scattering: the parton model



Color rotation of parton in external field by a lightlike Wilson line

$$W[A](x) = \mathcal{P} \exp \left[ ig \int dz^+ A_a^-(z^+, x) \right]$$

Parton distribution after coherent multiple scattering off nucleus:

$$\frac{dN}{d^2p} \simeq \frac{1}{\pi B_p} \int_{x\bar{x}} e^{-(x^2 + \bar{x}^2)/2B_p} \left\langle \frac{1}{N_c} \text{Tr} [W(x)W^\dagger(\bar{x})] \right\rangle e^{ip \cdot (x - \bar{x})}$$

$B_p$  is the transverse area of the proton

Bjorken, Kogut, Soper, Phys. Rev., D3:1382, (1971)

Dumitru, Jalilian-Marian, Phys. Rev. Lett., 89:022301, (2002)

# Multiparton Eikonal scattering

**Two partons:**

$$\frac{d^2 N}{d^2 p_1 d^2 p_2} \simeq \frac{1}{(\pi B_p)^2} \int_{x\bar{x}y\bar{y}} e^{-(x^2+\bar{x}^2)/2B_p} e^{-(y^2+\bar{y}^2)/2B_p} e^{ip_1 \cdot (x-\bar{x})} e^{ip_2 \cdot (y-\bar{y})}$$
$$\times \left\langle \frac{1}{N_c} \text{Tr} [W(x)W^\dagger(\bar{x})] \frac{1}{N_c} \text{Tr} [W(y)W^\dagger(\bar{y})] \right\rangle$$
$$\propto \langle D D \rangle$$

**Dipole correlator:**  $D(x, \bar{x}) = \frac{1}{N_c} \text{Tr} [W(x)W^\dagger(\bar{x})]$

**Four partons:**

$$d^4 N \sim \int \langle \text{Tr} [W(w)W^\dagger(\bar{w})] \text{Tr} [W(x)W^\dagger(\bar{x})] \text{Tr} [W(y)W^\dagger(\bar{y})] \text{Tr} [W(z)W^\dagger(\bar{z})] \rangle$$
$$\propto \langle D D D D \rangle$$

**and so on ...**

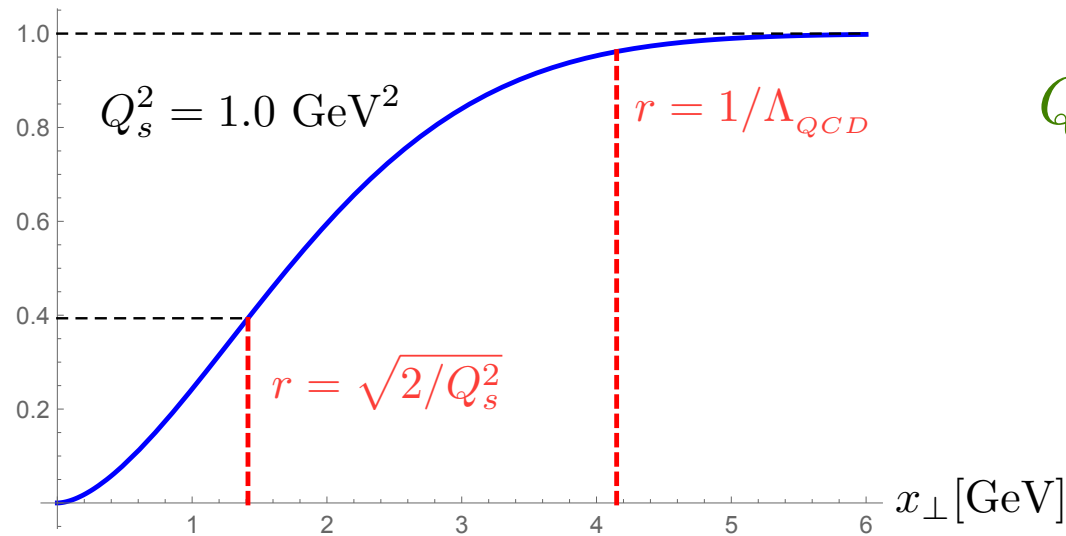
# Averaging over color representations in the target

Dipole correlator is evaluated in the MV model where color correlations in the target are Gaussian (random walk in color)

$$g^2 \langle A_a^-(x) A_b^-(y) \rangle = \delta^{ab} L_{xy} \quad L_{xy} = -\frac{g^4 \mu^2}{16\pi} |x - y|^2 \ln \frac{1}{\Lambda |x - y|}$$

**gives**  $D(x, \bar{x}) = \frac{1}{N_c} \text{Tr} [W(x) W^\dagger(\bar{x})] = \exp(C_F L_{x\bar{x}})$

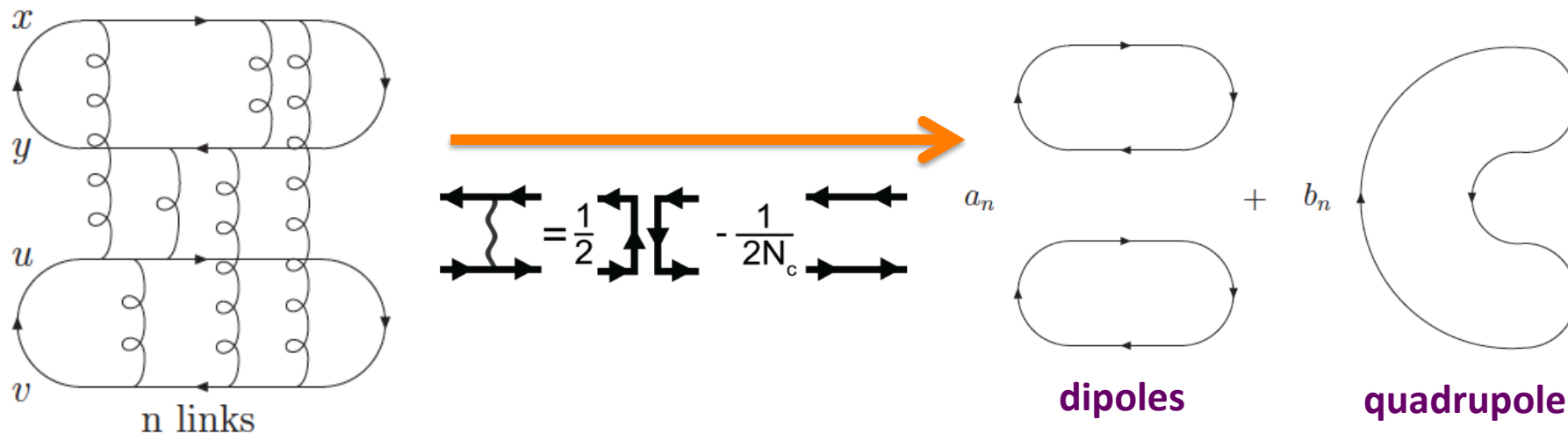
$$N(x_\perp) = 1 - D(x_\perp)$$



$$Q_s \propto g^2 \mu$$

# Averaging over multi-point dipole correlators

To compute the 2-dipole correlator:



**Use**  $W(x) \equiv \mathcal{P} \exp \left[ ig \int dz^+ A_a^-(z^+, x) \right] \simeq V(x) [1 + ig A_a^-(\xi, x) T^a + \dots]$

**This gives**  $\langle D_{x\bar{x}} D_{y\bar{y}} \rangle_W \simeq \alpha_{x\bar{x}y\bar{y}} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle_V + \beta_{xy\bar{x}\bar{y}} \langle Q_{x\bar{y}y\bar{x}} \rangle_V$

**Equivalently,**  $\begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle \\ \langle Q_{x\bar{y}y\bar{x}} \rangle \end{pmatrix}_W = \begin{pmatrix} \alpha_{x\bar{x}y\bar{y}} & \beta_{xy\bar{x}\bar{y}} \\ \beta_{xy\bar{y}\bar{x}} & \alpha_{x\bar{y}y\bar{x}} \end{pmatrix} \begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle \\ \langle Q_{x\bar{y}y\bar{x}} \rangle \end{pmatrix}_V$

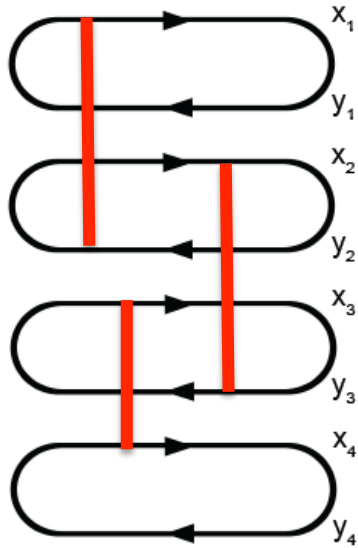
Kovner, Wiedemann, *Phys. Rev.*, D64:114002, (2001)  
 Fujii, *Nucl. Phys.*, A709:236 (2002).  
 Blaizot, Gelis, Venugopalan. *Nucl. Phys.*, A743:57, (2004)  
 Dominguez, Marquet, Wu, *Nucl. Phys.*, A823:99, (2009)

**Iterate, diagonalize, exponentiate,  
to compute correlator**

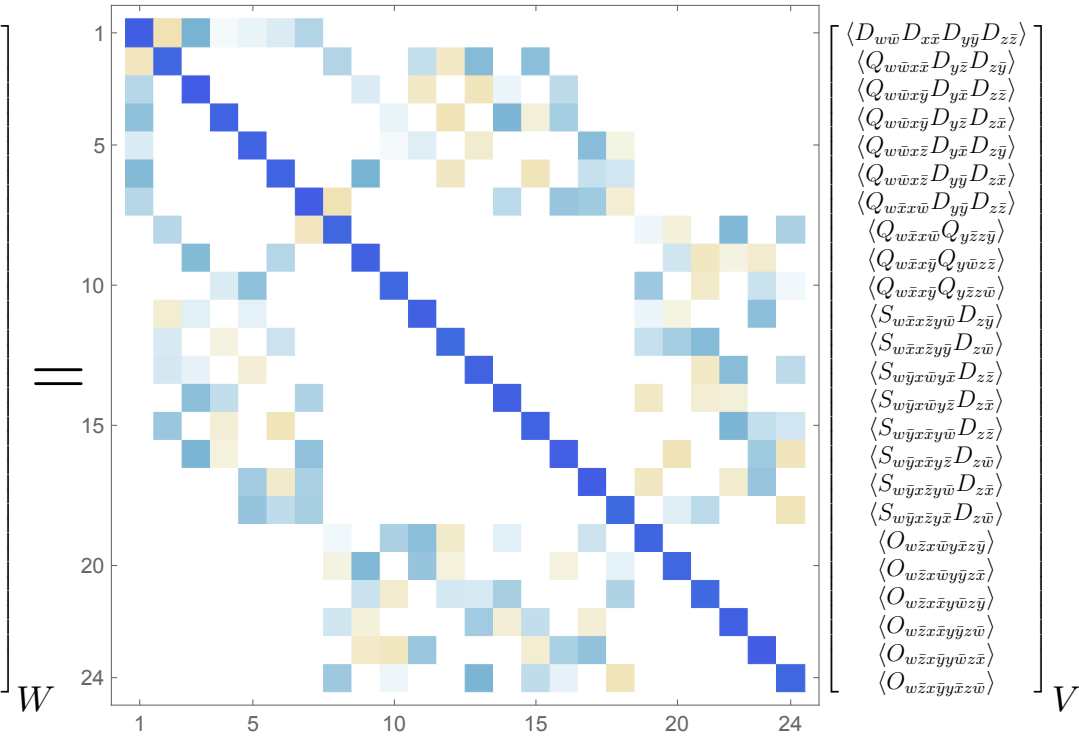


# Averaging over multi-point dipole correlators

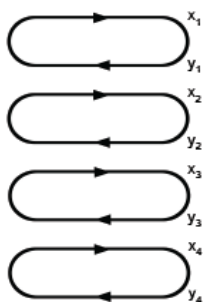
4-dipole correlator:



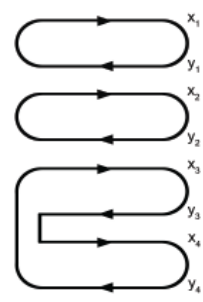
- $\langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle$
- $\langle Q_{w\bar{w}x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle$
- $\langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle$
- $\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle$
- $\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle$
- $\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle$
- $\langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle$
- $\langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle$
- $\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle$
- $\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle$
- $\langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle$
- $\langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{w}} \rangle$
- $\langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle$
- $\langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle$
- $\langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{z}} \rangle$
- $\langle S_{w\bar{y}x\bar{y}z\bar{w}} D_{z\bar{w}} \rangle$
- $\langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle$
- $\langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{w}} \rangle$
- $\langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle$
- $\langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle$
- $\langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{y}} \rangle$
- $\langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{w}} \rangle$
- $\langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{x}} \rangle$
- $\langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{w}} \rangle$



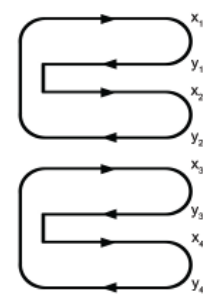
To compute n-gluon exchange, diagonalize 24x24 matrix and exponentiate



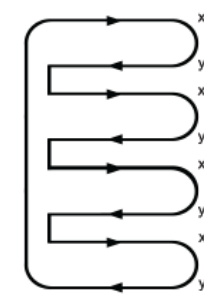
dipoles



quadrupoles



sextupole



octupole

# Results for azimuthal anisotropies from multiparton eikonal scattering

Mace,Dusling,Venugopalan,arXiv:1705.00745

# Objects to be computed

**N-particle distributions:**

$$\frac{d^n N}{d^2 p \dots} \sim \int \overbrace{e^{-(x^2 + \bar{x}^2)/2B_p} \dots}^{B_p = 4 \text{ GeV}^{-2}} \left\langle \overbrace{\frac{1}{N_c} \text{Tr} [W(x)W^\dagger(\bar{x})] \dots}^{Q_s^2 \sim 1 - 3 \text{ GeV}^2} \right\rangle e^{ip \cdot (x - \bar{x}) \dots}$$

**Two-particle cumulants:**

$$c_n\{2\} = \frac{\kappa_n\{2\}}{\kappa_0\{2\}}, \quad v_n\{2\} = \sqrt{c_n\{2\}}$$

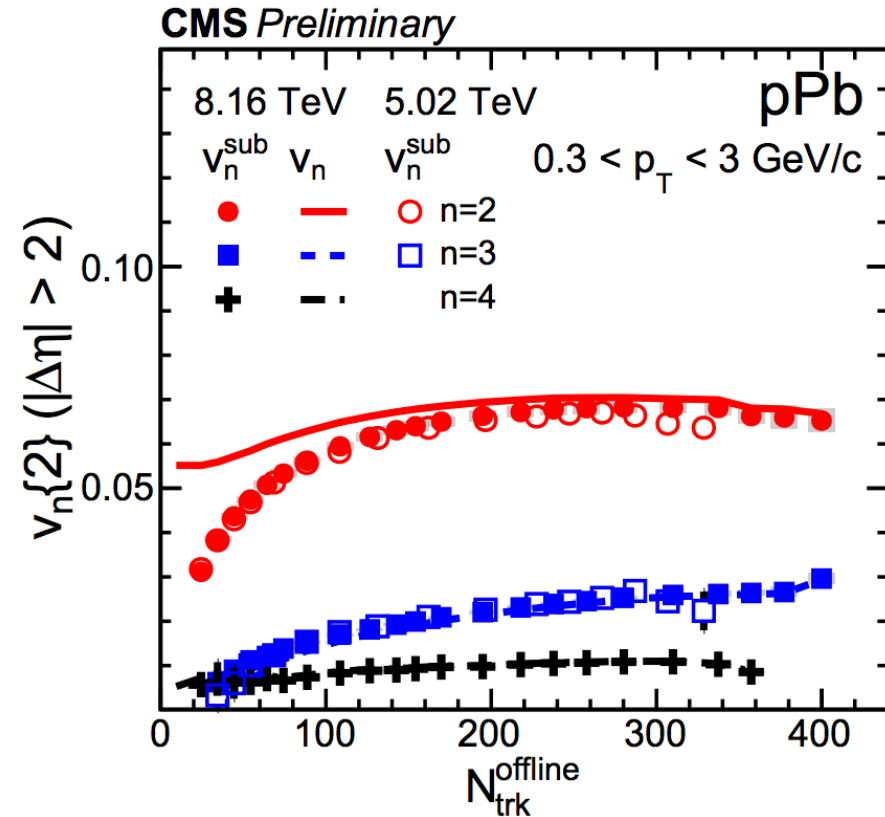
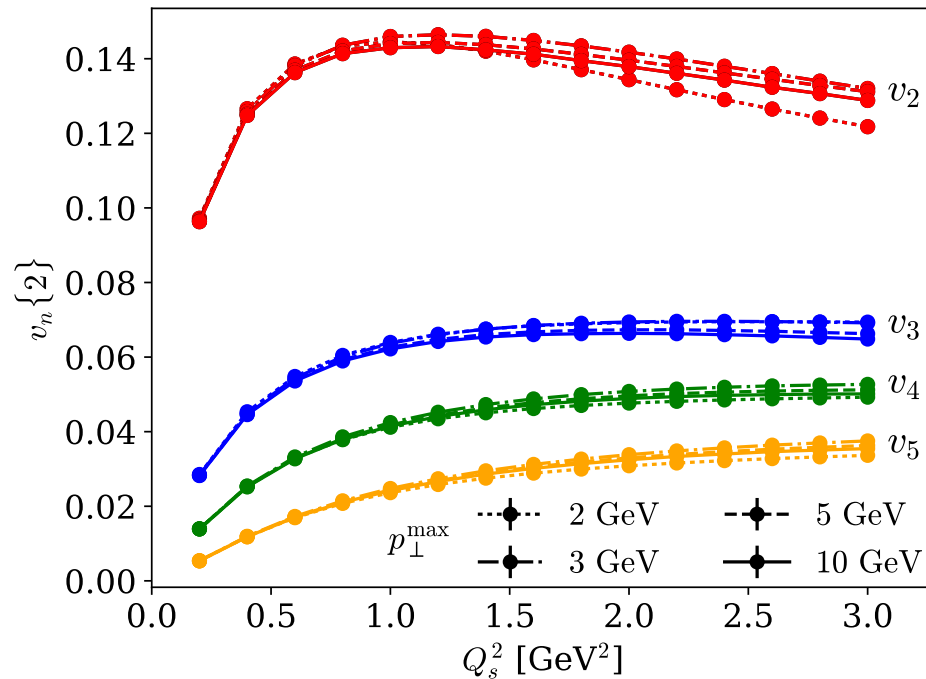
$$\kappa_n\{2\} = \int d^2 p_1 d^2 p_2 \cos[n(\phi_{p1} - \phi_{p2})] \frac{d^2 N}{d^2 p_1 d^2 p_2}$$

**Four-particle cumulants:**

$$c_n\{4\} = \frac{\kappa_n\{4\}}{\kappa_0\{4\}} - 2 \left( \frac{\kappa_n\{2\}}{\kappa_0\{0\}} \right)^2, \quad v_n\{4\} = (-c_n\{4\})^{1/4}$$

$$\kappa_n\{4\} = \int d^2 p_1 d^2 p_2 d^2 p_3 d^2 p_4 \cos[n(\phi_{p1} + \phi_{p2} - \phi_{p3} - \phi_{p4})] \frac{d^4 N}{d^2 p_1 d^2 p_2 d^2 p_3 d^2 p_4}$$

# Integrated anisotropy coefficients



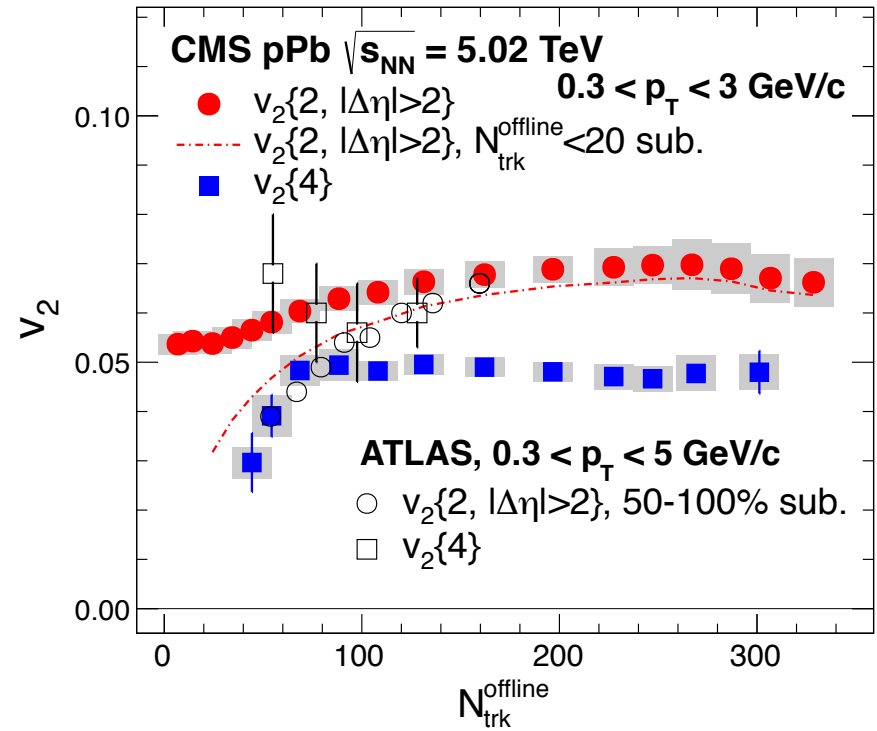
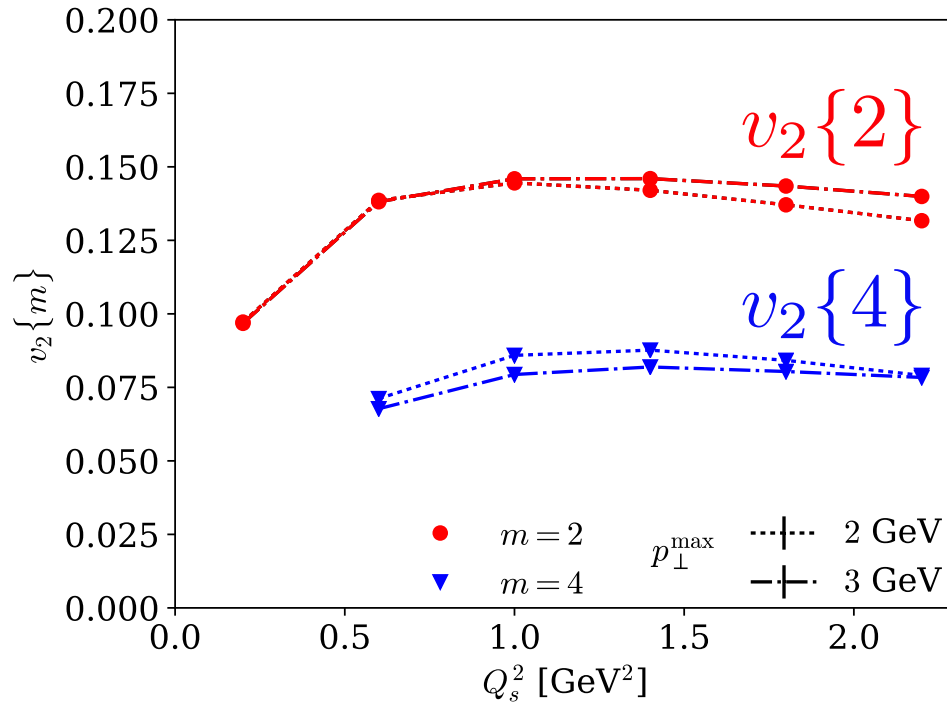
Similar ordering of “Flow” coefficients as seen in the data

Important caveat: No simple map between theory and experiment  
 Theory results are for quarks,  $Q_s^2$  is the saturation scale in the target

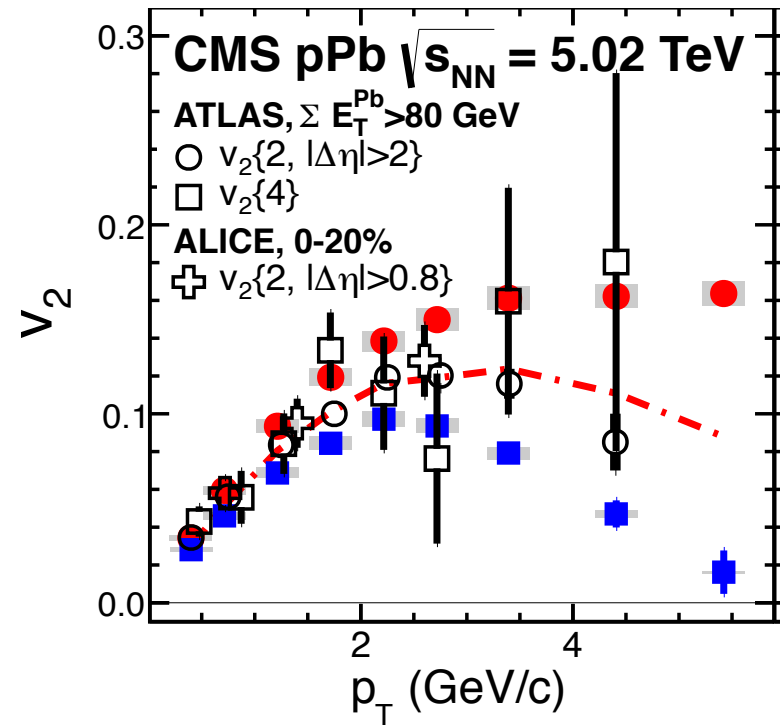
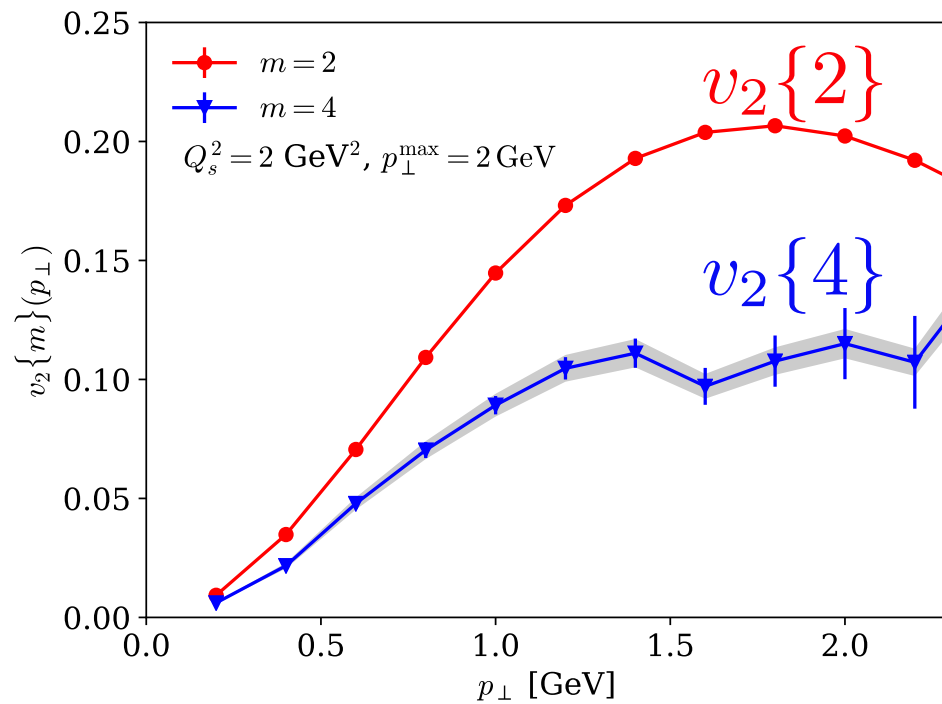
Lappi, Phys. Lett., B744:315, (2015)

Lappi, Schenke, Schlichting, Venugopalan. JHEP, 01:061, (2016)

# Integrated anisotropy coefficients

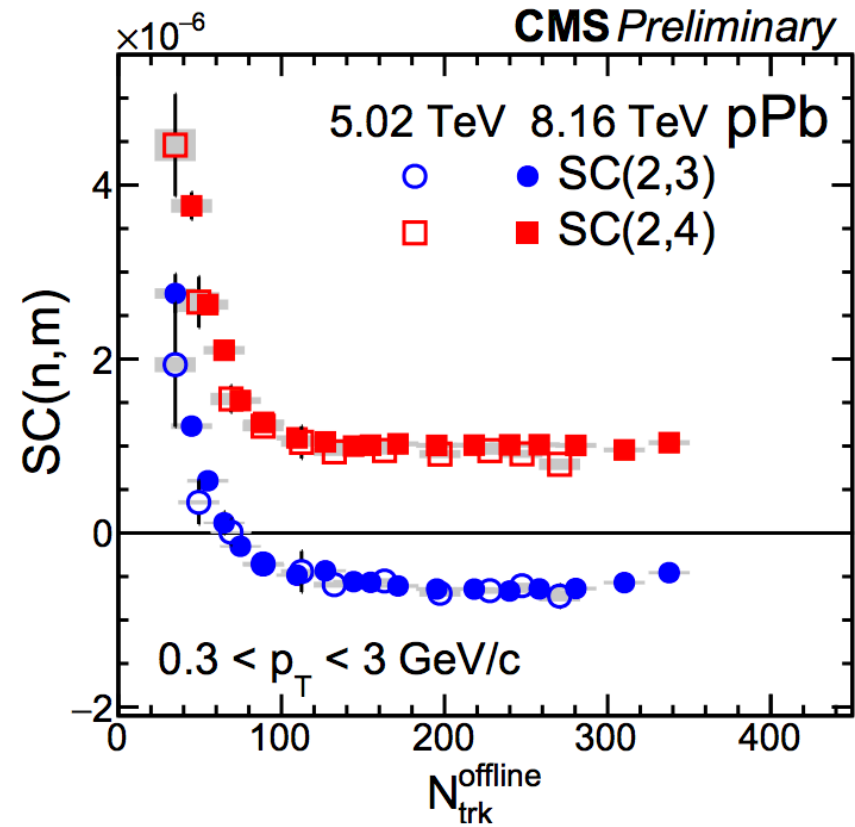
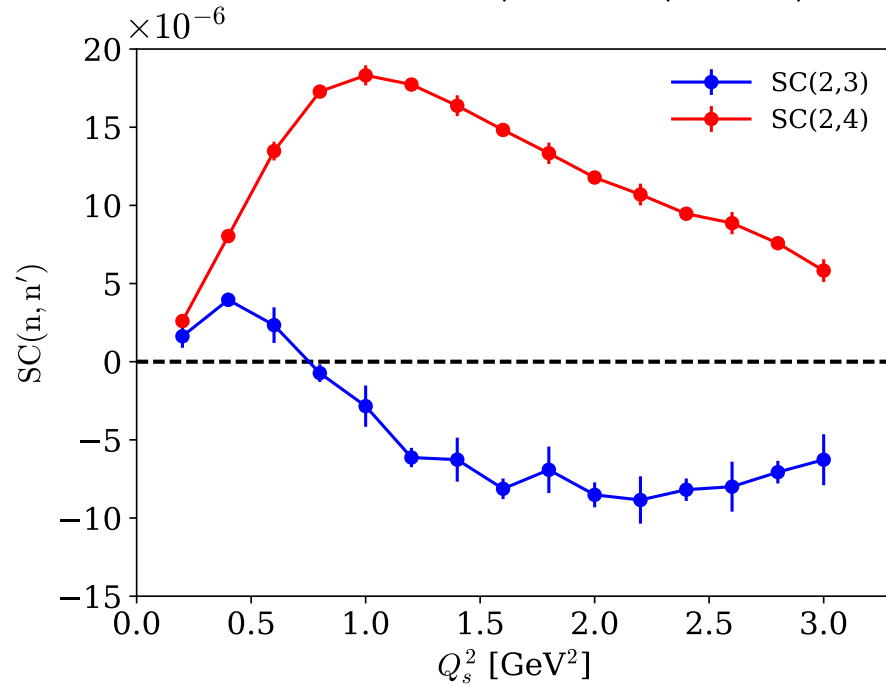


# $p_T$ dependence of anisotropy coefficients



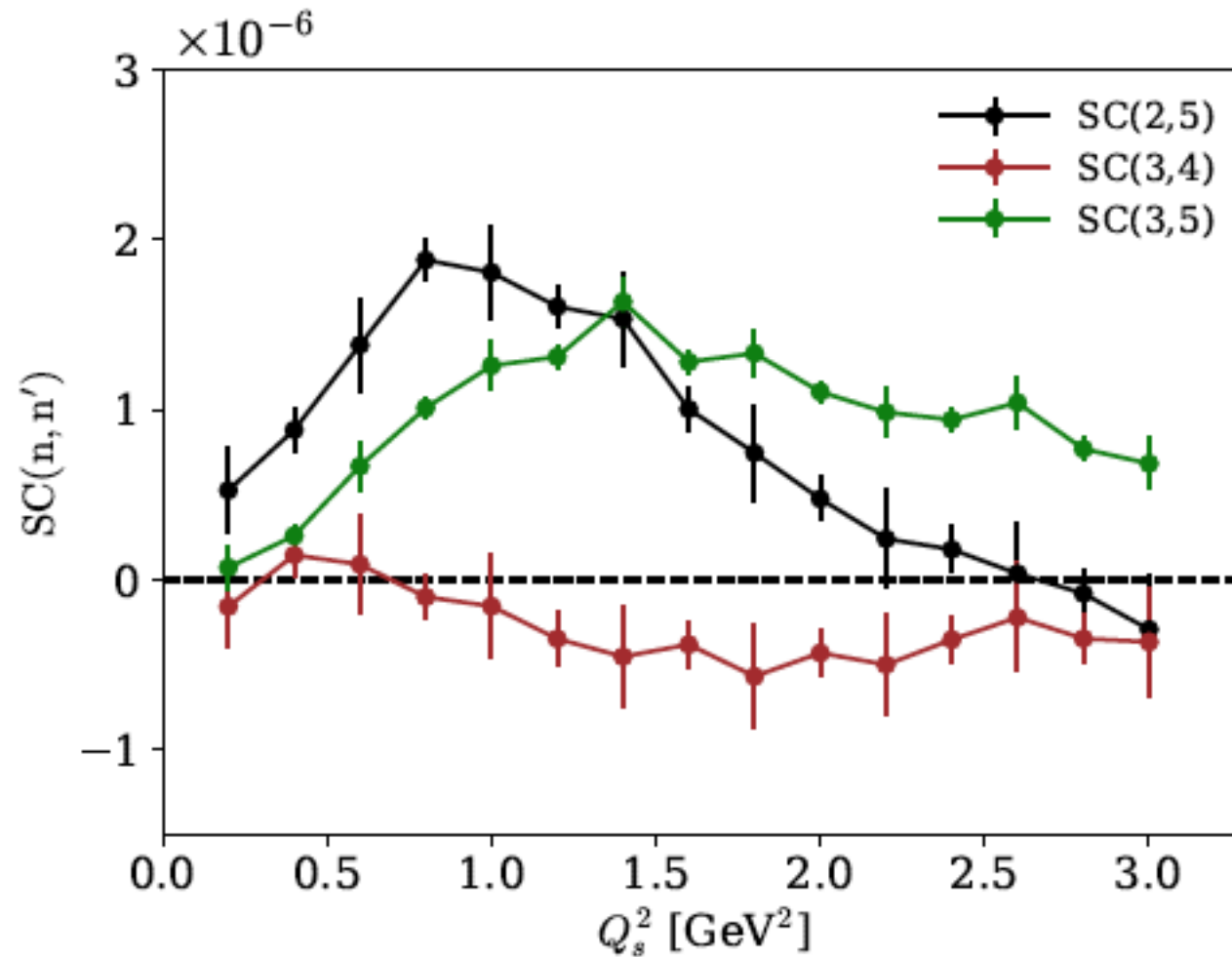
# Symmetric cumulants

$$SC(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$



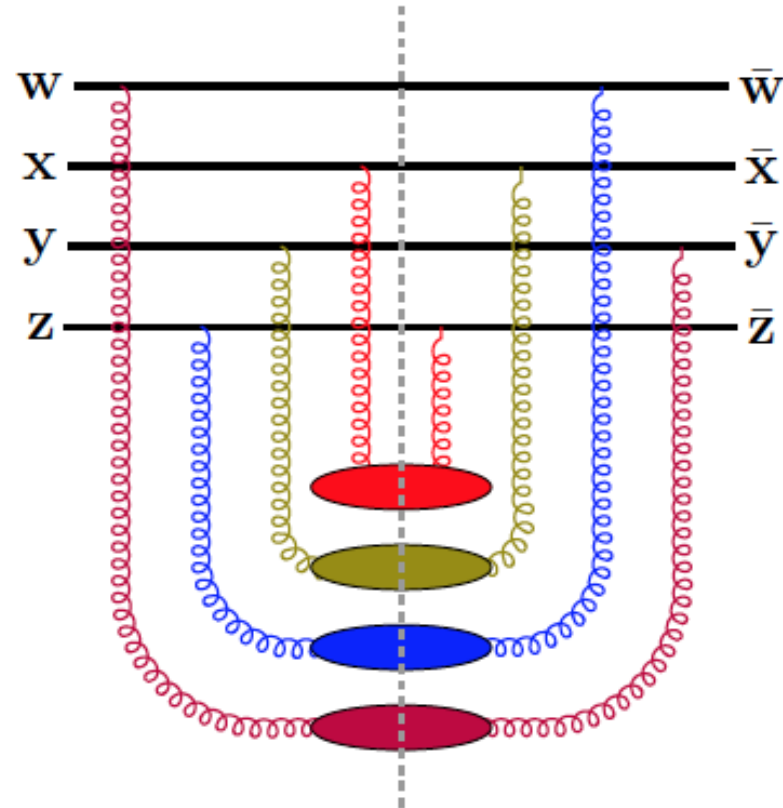
In hydro,  $SC(m, n)$  are a measure of the nonlinear response of the system

# Predictions for p+A symmetric cumulants





# Back to glasma graphs



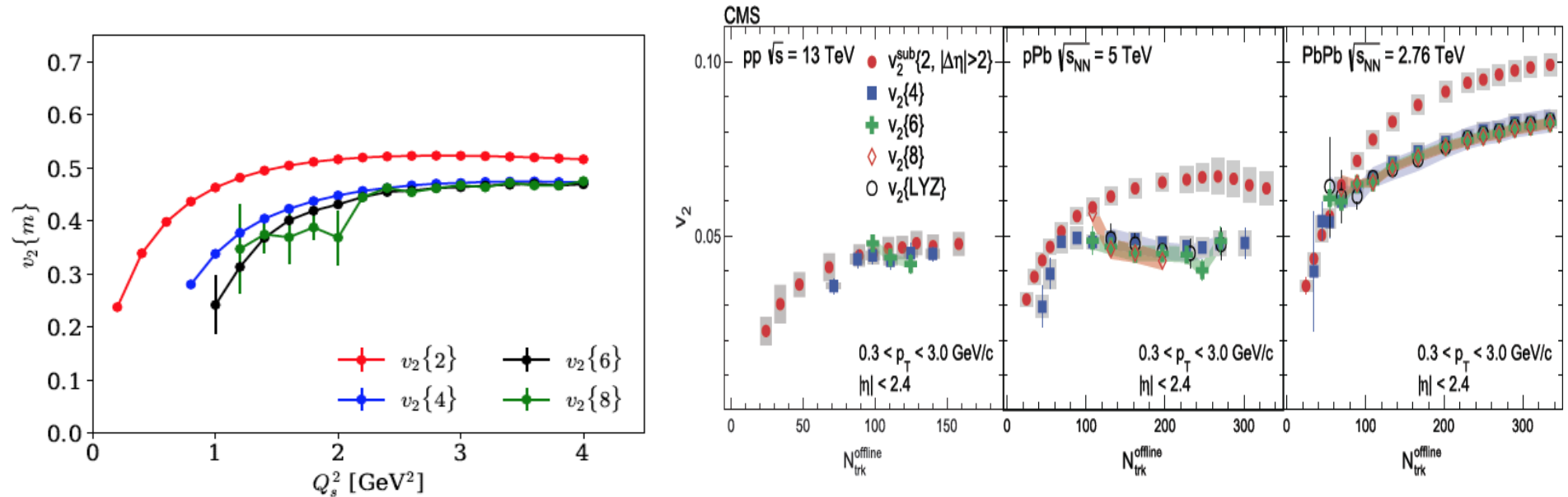
Glasma graph (single scattering) correlations are very strong  
– the n-particle distribution is close to a Bose distribution

Gelis,Lappi,McLerran, arXiv:0905.3234

But  $v_2\{4\}$  is imaginary...

For real  $v_2\{4\}$ , must have dominance of first two moments of distribution

# Higher cumulants from scattering off coherent Abelian fields



Replace  $N_C \times N_C$  trace with simple path ordered exponentials ( $N_C=1$ )

# Summary-I

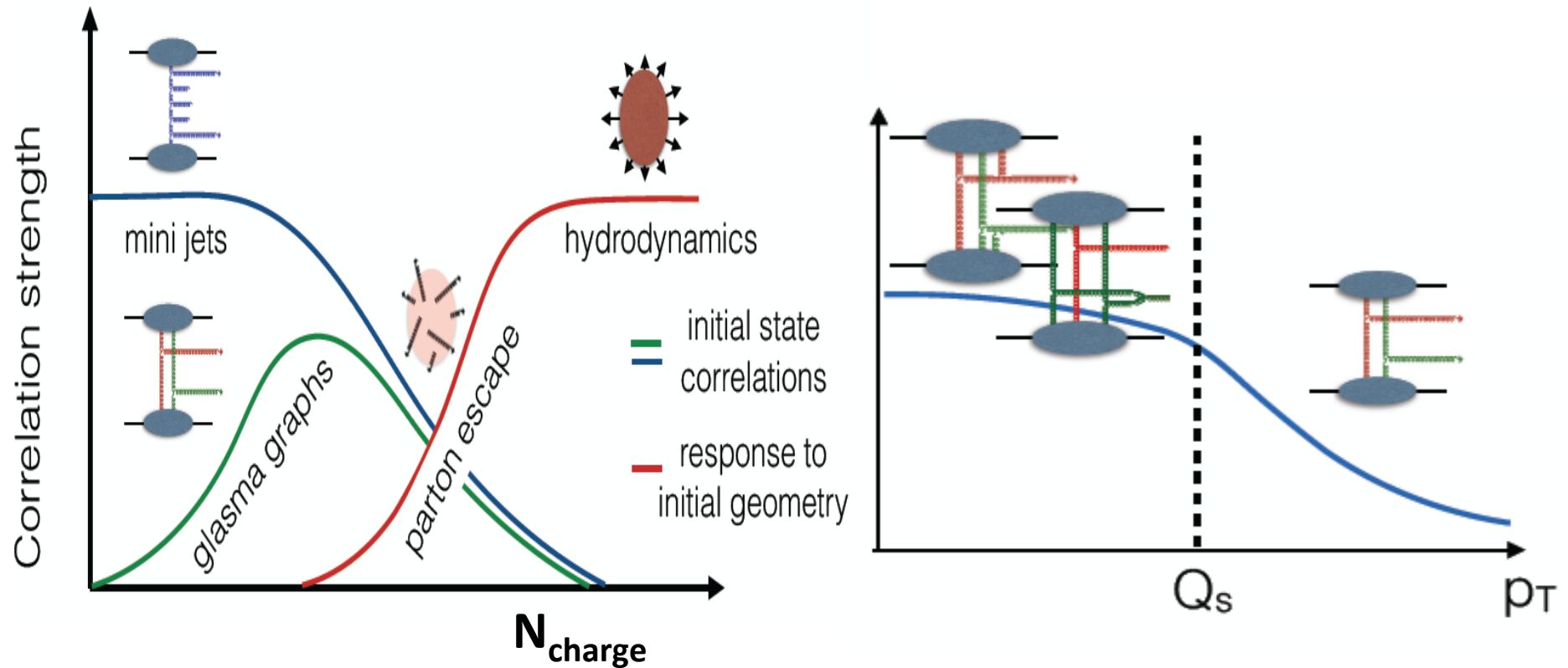
**Hydrodynamic paradigm appears to describe multi-particle correlations even in the smallest systems**

**There are however puzzling features of the data, questions about the the validity of hydro, fine tuning of initial conditions (requiring implicitly strong initial state correlations), ... and explanation of anisotropies for  $p_T > \text{few GeV}$**

**Initial state QCD frameworks now also able to explain many features of the data but systematic treatments are still in their infancy**

**Despite much progress no satisfactory explanation of the data -- the problem is still wide open**

## Summary-II



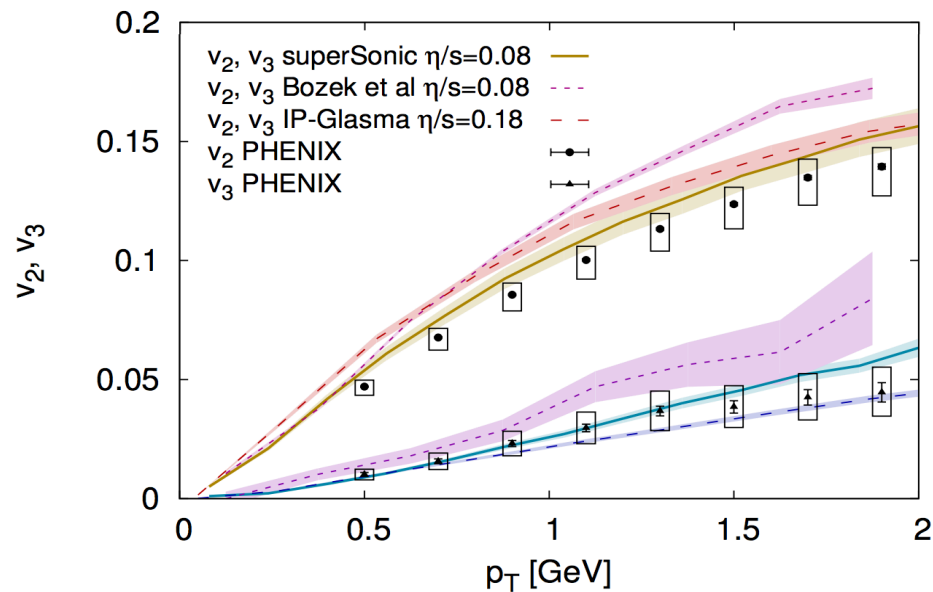
Event engineering across system sizes, energies, and varieties of probes, offers the exciting possibility of exploring dynamical evolution of strongly correlated quark-gluon matter from high occupancy, out of equilibrium, dynamics... to hydrodynamics

Figures: S. Schlichting at Quark Matter 2015

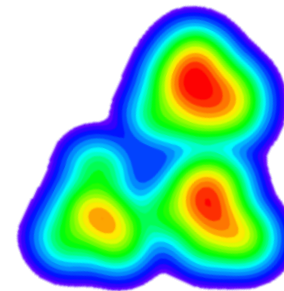
**Thanks for listening!**

# Collectivity in 3He+Au collisions

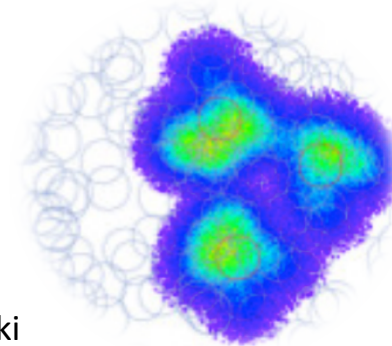
A. Adare et al. (PHENIX Collaboration)  
Phys. Rev. Lett. 115, 142301 (2015)



Schenke, Venugopalan  
Nucl. Phys. A931 (2014) 1039-1044



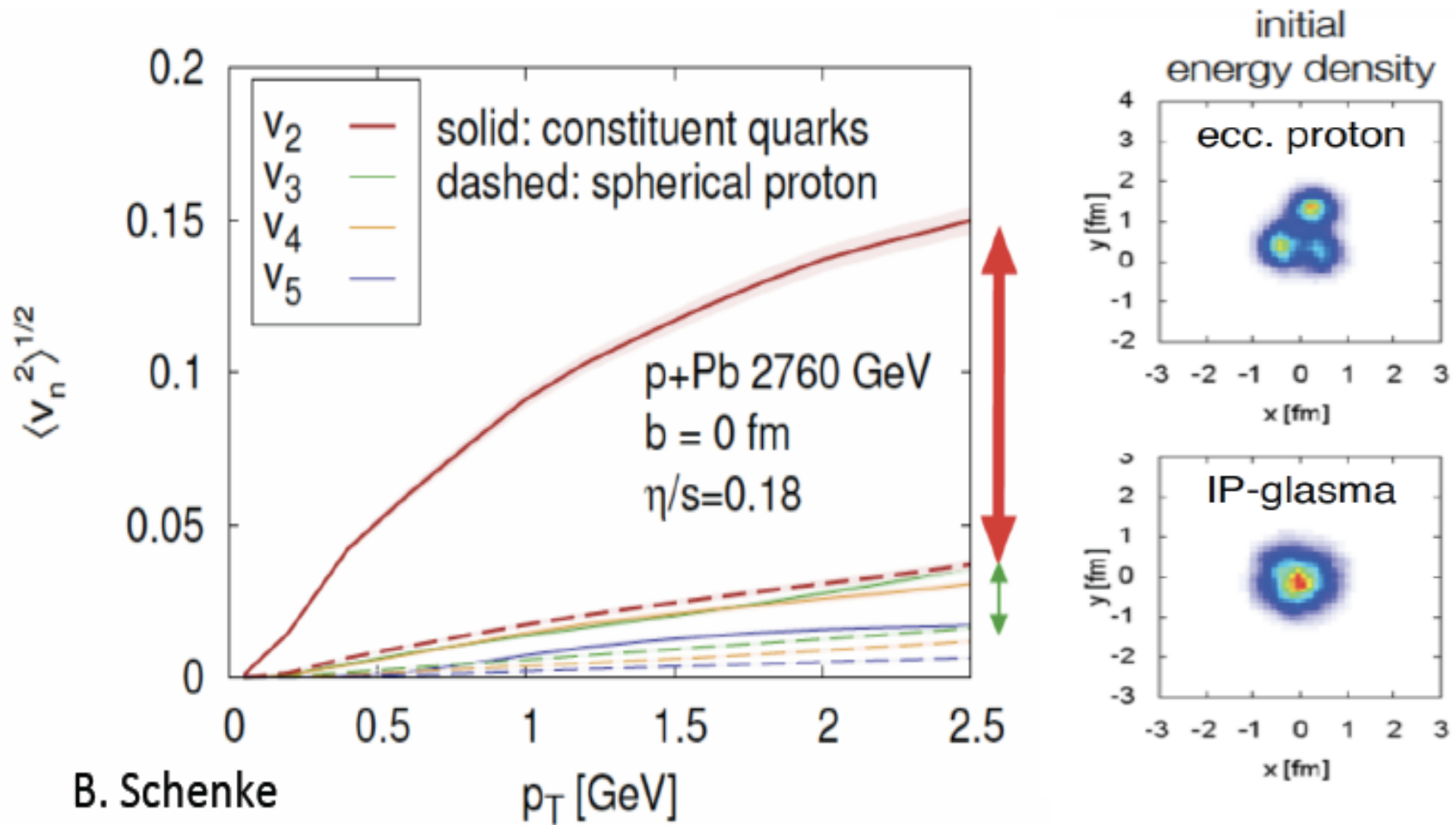
J.L. Nagle, et al.  
Phys. Rev. Lett. 113, 112301



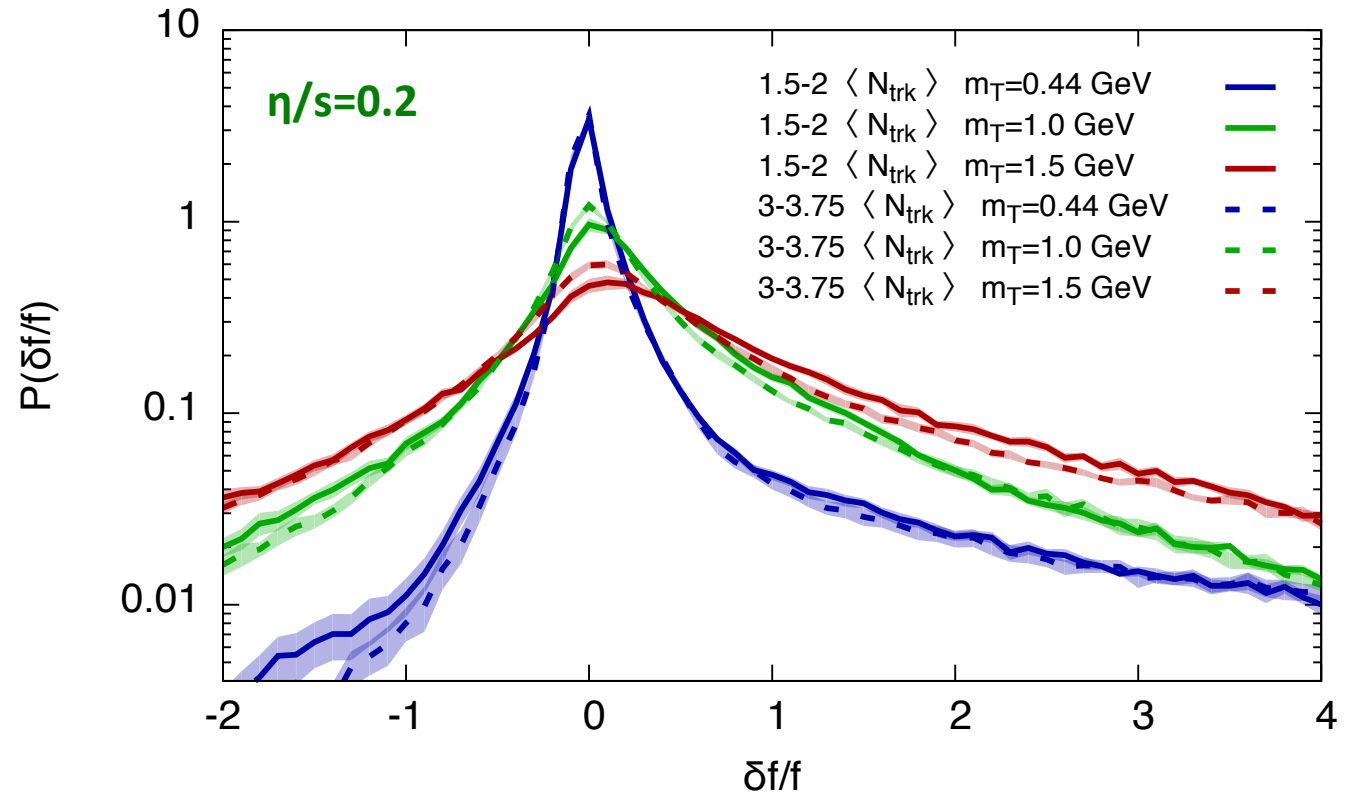
Bożek, Broniowski  
Physics Letters B 747 (2015) 135–138

# Shape matters ?

Schenke, Schlichting, 1407.8458



# Freeze-out corrections in p+Pb as function of $p_T$



Plot by Bjoern Schenke

For  $m_T = 1$  GeV, 26% of hydro cells have a 100% correction

For  $m_T = 1.5$  GeV, 43% have a 100% correction



# Higher cumulants in the color domain model

Dumitru, McLerran, Skokov, 1410.4844

**Color domain model: express intrinsic higher point correlators as correlators of produced particles with a target field in a color domain, averaged over all orientations of the field.**

$$c_2\{2\} = \frac{1}{N_D} \left( \mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right) ;$$
$$c_2\{4\} = -\frac{1}{N_D^3} \left( \mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right)$$

**“A” term is the correlation induced between projectile particles due to color field orientation of target (more generically, non-Gaussian correlations)**

**The  $N_c$  term is the “connected Glasma graph” (Gaussian correlations)**

**$N_D$  is # of color domains – few in p+A, several in A+A**

