Viscosity and diffusion coefficients at (almost) g^3

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- Jacopo Ghiglieri, J. Hong, A. Kurkela, G. Moore, DT, JHEP; photons
- Jacopo Ghiglieri, DT, QGP5, arXiv:1502.03730; review
- Jacopo Ghiglieri, G. Moore, DT, JHEP; arXiv:1509.07773; energy-loss
- Jacopo Ghiglieri, G. Moore, DT, arXiv:almost-done; shear viscosity and diffusion coefficients

Computing the shear viscosity

Plasma with small velocity: $\vec{u}(t,x)$

$$\sigma_{ij} = \partial_i u_j + \partial_j u_i - \tfrac{2}{3}\,\delta_{ij}\,\partial \cdot u$$

1. The Boltzmann equation

$$(\partial_t + \boldsymbol{p}\cdot\partial_x)\, f = C[f]$$

2. Then linearize close to the (time-dependent) equilibrium:

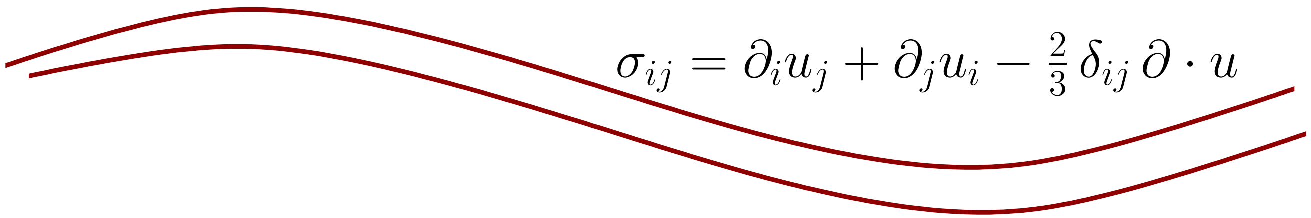
$$f_p = \underbrace{n(P\cdot U)}_{\text{equilibrium } f_0(t,\boldsymbol{x},\boldsymbol{p})} + \underbrace{n_p(1+n_p)\chi(\boldsymbol{p})}_{\text{viscous correction } \delta f(\boldsymbol{p})}$$

where $n_p = 1/(e^{E_p/T} - 1)$.

Linearize the Boltzmann equilibrium, and solve for the viscous correction χ

Computing the shear viscosity

Plasma with small velocity: $\vec{u}(t,x)$



$$\sigma_{ij} = \partial_i u_j + \partial_j u_i - \tfrac{2}{3}\,\delta_{ij}\,\partial \cdot u$$

1. The viscous correction, $\chi(\boldsymbol{p})$, satisfies the steady state linearized equation

$$\underbrace{n_p(1+n_p)\frac{p^i p^j \sigma_{ij}}{2Tp}}_{\text{strain}} = \underbrace{\mathcal{C}\chi(\boldsymbol{p})}_{\text{linearized collision op}}$$

Find: $\chi(\boldsymbol{p}) \propto p^i p^j \sigma_{ij}$

2. The shear viscosity can be found with $\delta f = n_p(1+n_p)\chi(\boldsymbol{p})$

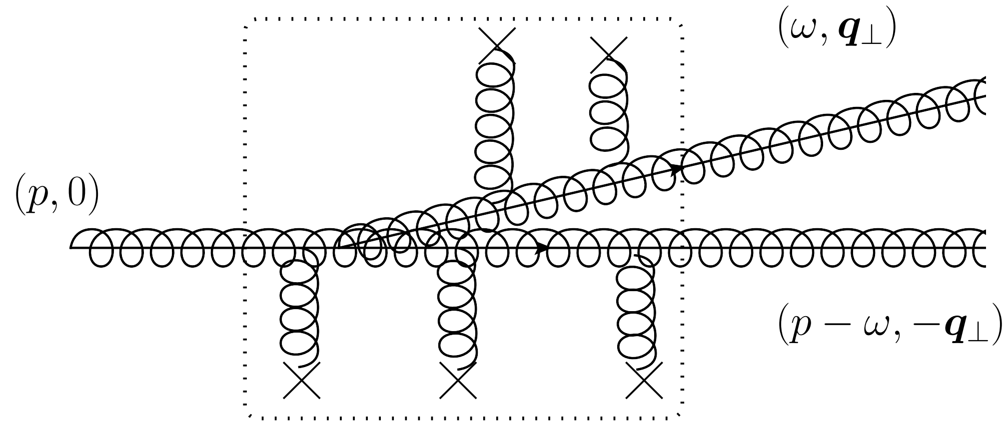
$$T^{ij} = p\delta^{ij} - \eta\sigma^{ij} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{p^i p^j}{E_p}\,(f_0 + \delta f)$$

We will specify the collision operator at NLO, and solve for $\chi(\boldsymbol{p})$ and η!

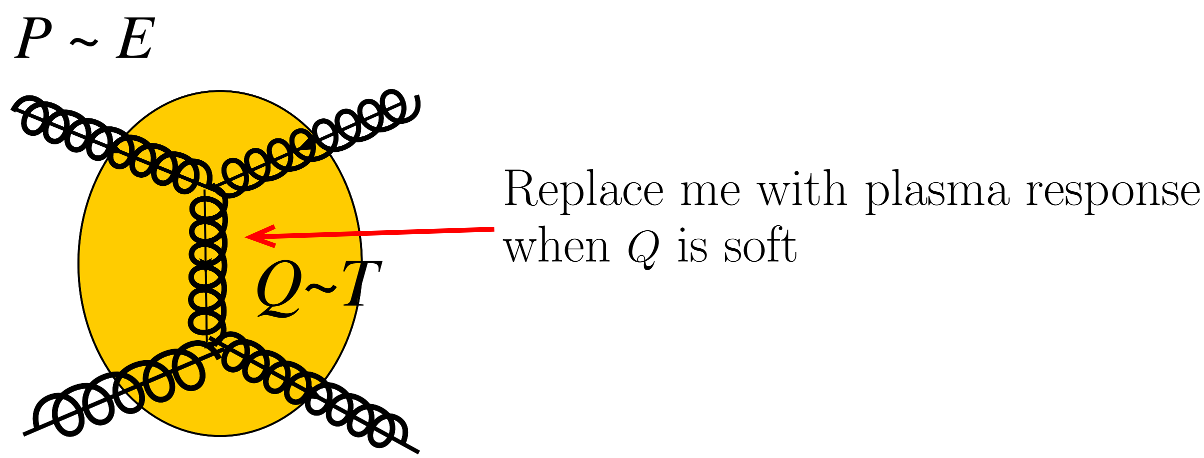
Leading order Collision Operator (AMY)

$$(\partial_t + \mathbf{v}_p \cdot \partial_{\boldsymbol{x}})\, f_p = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

1. Collinear Bremsstrhalung $C^{1\leftrightarrow 2}$:



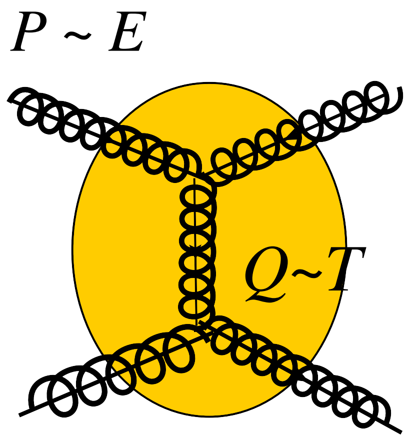
2. Collisions $C^{2\leftrightarrow 2}$:



First we need to separate scales, and treat hard and soft scattering differently

Three mechanisms for transport at LO in QGP

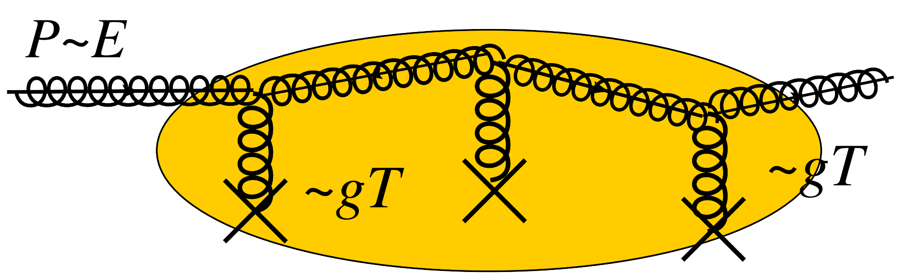
1. Hard Collisions: $2 \leftrightarrow 2$



$$\underbrace{C^{2\leftrightarrow 2}[\mu_\perp]}_{\text{vacuum matrix elements}}$$

2. Drag, longitudinal and transverse, diffusion: collisions with soft random classical field

soft fields have $p \sim gT$ and *large occupation numbers* $n_B \sim \frac{T}{p} \sim \frac{1}{g}$

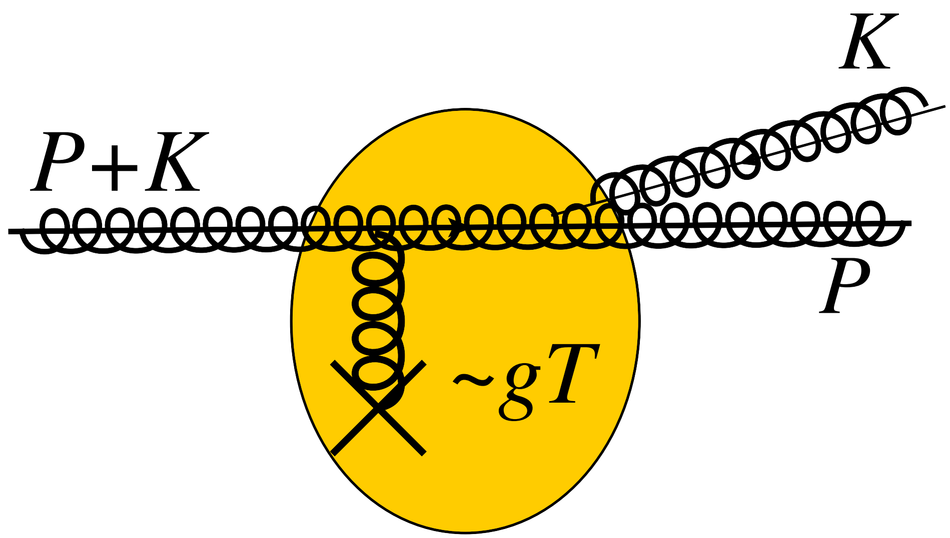


$$\hat{q}^{ij} = \hat{q}_{\parallel}(\mu)\,\hat{p}^i\hat{p}^j + \tfrac{1}{2}\hat{q}(\mu)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

$$C_{\rm diff}[\mu_\perp] = \frac{\partial}{\partial \boldsymbol{p}^i}\left(n_p(1+n_p)q^{ij}(\mu_\perp)\frac{\partial \chi(\boldsymbol{p})}{\partial \boldsymbol{p}^j}\right) + \text{gain}-\text{terms}$$

3. Bremm: $1 \leftrightarrow 2$

- random walk induces collinear bremsstrhalung



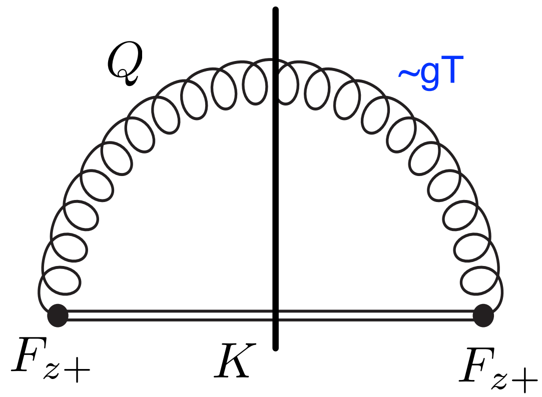
- The rate of a transverse kicks of momentum $\mathbf{q}_\perp$ from soft fields:

$$\hat{C}_{LO}[\mathbf{q}_\perp] = \frac{Tm_D^2}{q_\perp^2(q_\perp^2 + m_D^2)}$$

with

$$\hat{q} = g^2 C_R \int \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 \hat{C}_{LO}[\mathbf{q}_\perp]$$

Drag and long-diffusion: A longitudinal force-force correlator along the light cone



- Evaluate longitudinal force-force with hard thermal loops + sum-rules

$$\hat{q}_{\|}(\mu) \propto g^2 C_A \int^{\mu} \frac{d^2\mathbf{q}_T}{(2\pi)^2} \int \frac{dq_+ dq_0}{(2\pi)^2} \underbrace{\langle F_{z+}(P) F_{z+}\rangle\, 2\pi\delta(q_+)}_{\text{evaluate with sum-rule } q_0 \to \infty}$$

$$\propto g^2 C_A \int^{\mu} \frac{d^2\mathbf{q}_T}{(2\pi)^2} \frac{T m_\infty^2}{q_T^2 + m_\infty^2}$$

$$\propto g^2 C_A \frac{m_\infty^2}{4\pi} \log(\mu^2/m_\infty^2)$$

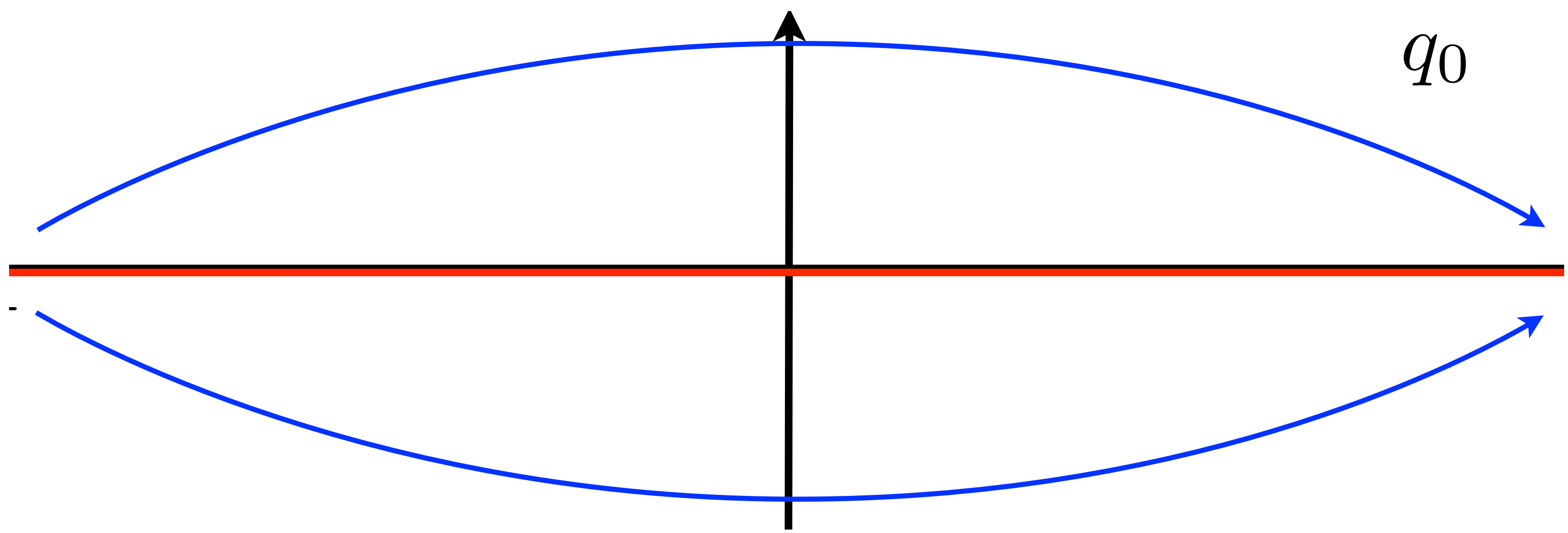
The $\mu-$dependence of the drag cancels against μ-dependence of the $2 \to 2$ rate

To much math??

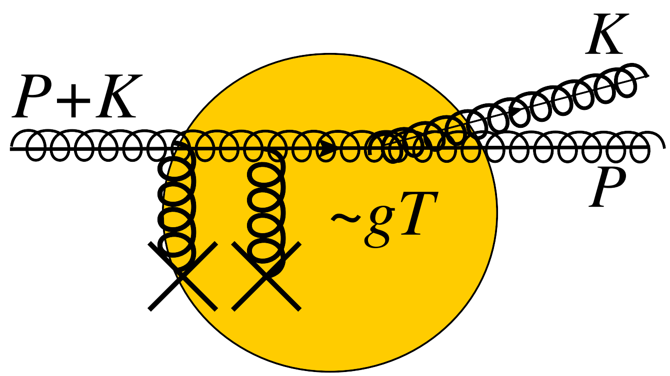
$$\hat{q}_{||}(\mu) \propto \int \frac{dq_+ dq_0}{(2\pi)^2} \langle F_{z+}(P) F_{z+} \rangle \, 2\pi\delta(q_+)$$

$$\int \frac{dq_0}{(2\pi)} q^0 q^0 \qquad \underbrace{\frac{T}{q^0} \left[G_R^{++}(q_0, q_+) - G_A(q_0, q_+) \right]}$$

Use FDT (equilibrium) in an essential way



Transverse momentum diffusion:



The bremsstrhalung rate is proportional to the rate of transverse momentum kicks, $\hat{C}_{LO}[\mathbf{q}_\perp]$:

$$\hat{C}_{LO}[q_\perp] = \text{in medium scattering rate with momentum } \mathbf{q}_\perp$$

- Need to compute transverse force-force correlators along the light cone. Aurenche, Gelis, Caron-Huot

$$q_\perp^2 \hat{C}_{LO}[\mathbf{q}_\perp] = \int \frac{dq_+ dq_0}{(2\pi)^2} \underbrace{\langle F_{i+}(Q) F_{i+} \rangle \, 2\pi\delta(q_+)}_{\text{evaluate with sum rule at } q_0 = 0}$$

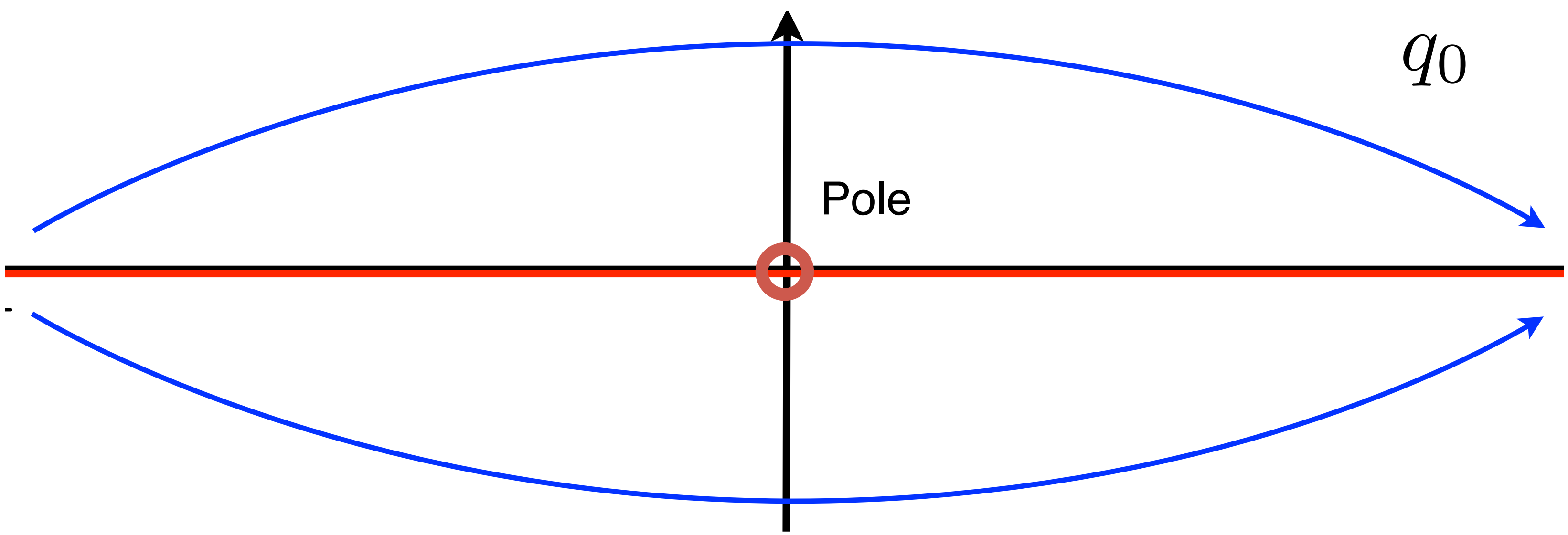
$$= \frac{T m_D^2}{q_\perp^2 + m_D^2}$$

To much math??

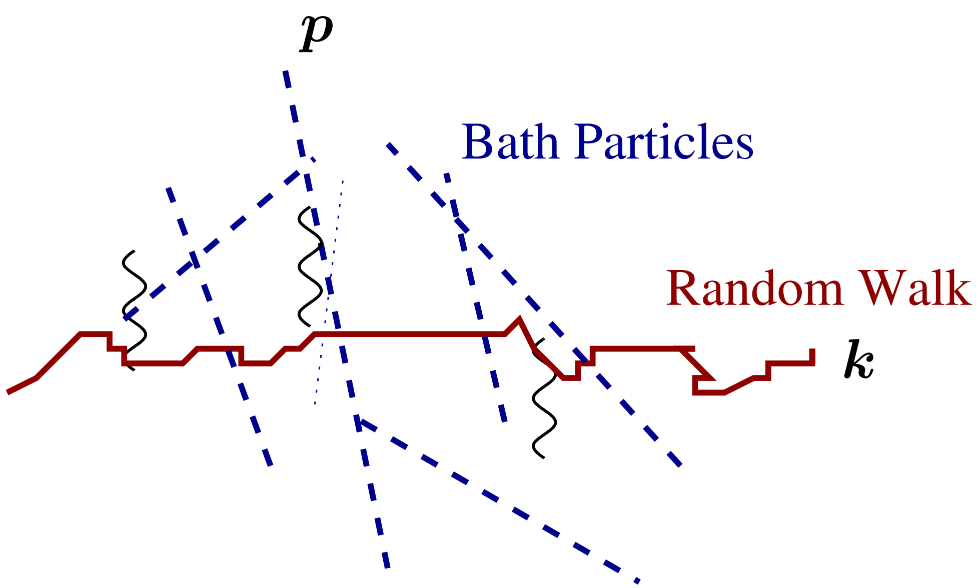
$$\hat{q}(\mu) \propto \int \frac{dq_+ dq_0}{(2\pi)^2} \langle F_{i+}(P) F_{i+} \rangle \, 2\pi\delta(q_+)$$

$$\int \frac{dq_0}{(2\pi)} \, q^i q^i \qquad \underbrace{\frac{T}{q^0} \left[G_R^{++}(q_0, q_+) - G_A(q_0, q_+) \right]}_{\text{Use FDT (equilibrium) in an essential way}}$$

Use FDT (equilibrium) in an essential way



Gain Terms:



Without including the "bath" particles momentum will not be conserved

$$\left(\partial_t + \mathbf{v}_{\boldsymbol{p}} \cdot \frac{\partial}{\partial \boldsymbol{x}}\right) \delta f_p = \frac{\partial}{\partial p^i} \left(n_p (1+n_p) \tfrac{1}{2} \hat{q}^{ij} \frac{\partial \chi}{\partial p^j} \right)$$

$$- \underbrace{\frac{\partial}{\partial p^i} \left(n_p(1+n_p) \nu_g \int_{\boldsymbol{k}} C^{ij}(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{k}})\, n_k (1+n_k) \frac{\partial \chi(\boldsymbol{k})}{\partial k^j} \right)}_{\text{Diffusion of particle-}\boldsymbol{k}\text{ disturbing bath particle-}\boldsymbol{p}\text{ from equilibrium}}$$

General structure of gain terms

1. Energy and momentum conservation:

$$\nu_g \int \frac{d^3k}{(2\pi)^3} n_k(1+n_k)\, \mathcal{C}^{ij}(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{k}}) = \hat{q}_{\parallel}\hat{p}^i\hat{p}^j + \tfrac{1}{2}\hat{q}(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

2. Tensor analysis relates $\hat{q}$ and $\hat{q}_{\parallel}$ to $\ell = 0, 1$ moments of scalar functions:

$$C^{ij} \equiv \underbrace{\mathcal{A}_0(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{k}})\,(\hat{\boldsymbol{p}}+\hat{\boldsymbol{k}})^i(\hat{\boldsymbol{p}}+\hat{\boldsymbol{k}})^j}_{\text{contributes to } \hat{q}_{\parallel},\, \hat{q}} + \underbrace{\mathcal{A}_1(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{k}})\,(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}})^i(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}})^j}_{\text{contributes to } \hat{q}}$$

3. For shear, the particles have an $\ell = 2$ angular distribution in steady state:

$$\chi(\boldsymbol{p}) \propto p^i p^j \sigma_{ij}$$

For computing the shear need only certain higher moments of $\mathcal{A}_0$ and $\mathcal{A}_1$.

Only three finite ($\mu_\perp$ independent) numbers must be computed for η !

$$\left(\chi_{ij}, C^{2\leftrightarrow 2}_{\rm gain}\,\chi_{ij}\right) = \frac{d_A g^4}{8\pi^5 T^3} \sum_{ab} T_{R_a} T_{R_b} \int_0^\infty dp\, p^2\, f_0^a(p)[1 \pm f_0^a(p)] \int_0^\infty dk\, k^2\, f_0^b(k)[1 \pm f_0^b(k)]$$

$$\times \underbrace{\left[-0.1833\frac{\chi^a(p)\chi^b(k)}{pk} - 0.1360\left(\frac{\chi^a(p)\chi^b(k)'}{p} + \frac{\chi^a(p)'\chi^b(k)}{k}\right) - 0.3066\chi^a(p)'\chi^b(k)'\right]}_{\text{three numbers computed using } 2 \leftrightarrow 2 \text{ HTL matrix elements}}.$$

Summary – the full LO Boltzmann equation:

$$[\partial_t + v_{\boldsymbol{k}} \cdot \partial_{\boldsymbol{x}}]\, f_{\boldsymbol{k}} = \frac{\partial}{\partial \boldsymbol{p}^i} \left(n_p (1 + n_p) q^{ij}(\mu_\perp) \frac{\partial \chi(\boldsymbol{p})}{\partial \boldsymbol{p}^j} \right) + \text{gain}-\text{terms}$$
$$+ C_{2\leftrightarrow 2}[\mu] + C_{1\leftrightarrow 2}$$

1) The cutoff dependence of drag/diffusion cancels against the $2 \to 2$ rate!

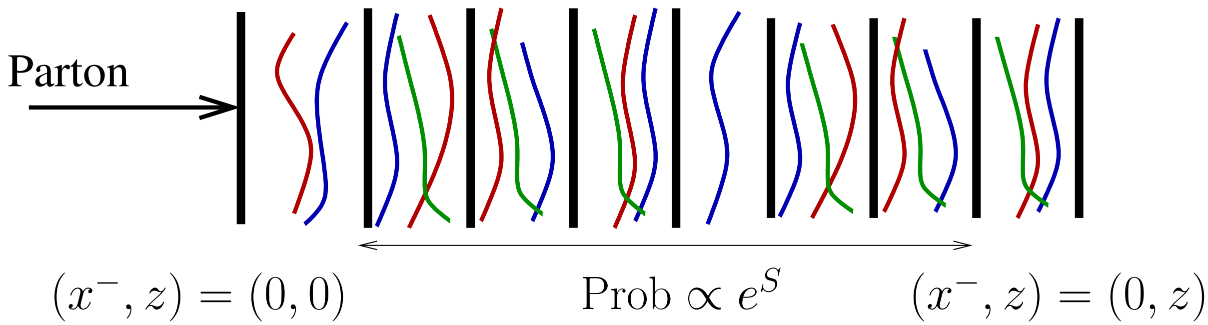
2) Debye sector enters in just a few places.

3) Light cone sum rules.

Light cone and associated dimensional reduction in thermal QCD (S. Caron-Huot)

- Heuristic reason:

Hard Parton "sees" undisturbed soft modes (on light cone), which sample the statistical weight



- Use coordinates (Weldon)

$$\underbrace{x^-,\ z,\ x_\perp}_{\text{coordinates}} \qquad \underbrace{p_0,\ p_+,\ p_\perp}_{\text{momenta}} \qquad \underbrace{P^2 = 2p_0p_+ + p_+^2 + p_\perp^2}_{\text{squared four momentum}}$$

- Computing the correlator with euclidean formulation, $p_0 \to \omega_n = (2\pi T)n$

$$G(x^-{=}0, p_+, p_z) = T\sum_n \frac{1}{2\omega_n p_+ + p_+^2 + p_\perp^2} \approx \frac{T}{p_\perp^2 + p_+^2} \Leftarrow \text{3D propagator}$$

Effective 3D Lagrangian, EQCD, for light cone physics: S. Caron-Huot

$$L_{EQCD} = \frac{1}{4} F^a_{ij} F^a_{ij} + \mathrm{tr}\left((D_i A_0)^2 \right) + m_D^2 \, \mathrm{tr}\left(A_0^2 \right) + \ldots$$

- It is not difficult to compute to the order of interest:

$$g^2 C_R \langle F_{i+}(q_+, q_\perp) F_{i+} \rangle \Big|_{q_+=0} = g^2 C_R \Big(\underbrace{\frac{-T}{q_\perp^2 + m_D^2}}_{\text{3D } \langle A^0 A^0 \rangle} + \underbrace{\frac{T}{q_\perp^2}}_{\text{3D } \langle A^z A^z \rangle} \Big)$$

- 3D Lattice simulations of the effective theory can provide non-perturbative input
  - First start: Marco Panero, Kari Rummukainen, Adreas Schafer, PRL, arxiv:1307.5850

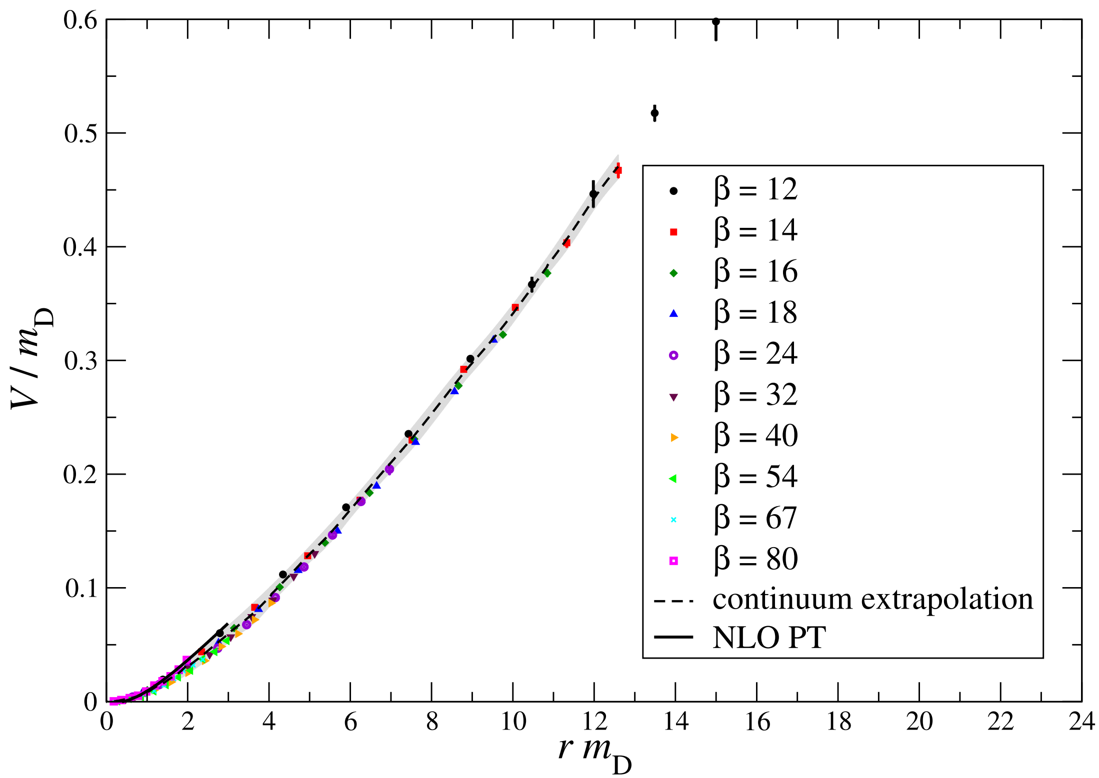
Many things need to be checked before this a useful non-perturbative tool!

But still its a great idea . . .

Lattice computation of $C(q_\perp)$:

M. Panero et al, PRL

$$V(r_\perp) = g^2 C_F \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{q}_\perp \cdot \boldsymbol{r}_\perp}\right) \hat{C}[q_\perp]$$



Estimate $\hat{q}_{\rm soft} \simeq 2T^3$ for a quark jet

Next-to-Leading Order

Use the Boltzmann equation for shear viscosity:

Plasma with small velocity: $\vec{u}(t, x)$

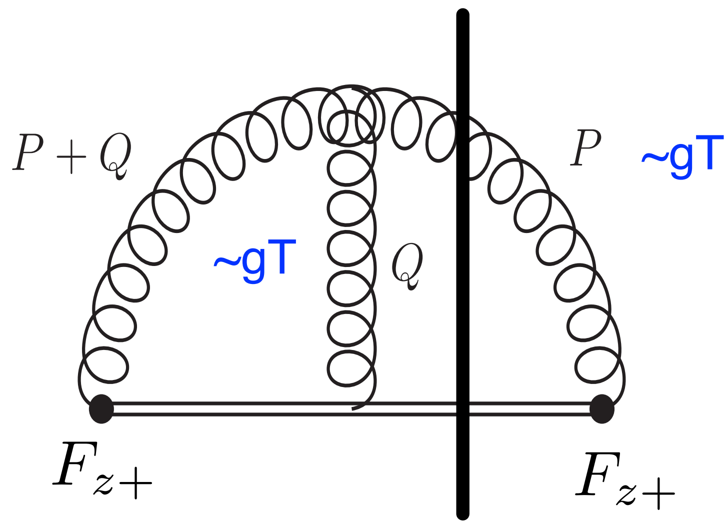
$$\sigma_{ij} = \partial_i u_j + \partial_j u_i - \tfrac{2}{3}\,\delta_{ij}\,\partial \cdot u$$

$$\frac{\eta}{s} \propto \frac{1}{g^4}\Big[\underbrace{C + \log(1/g)}_{\text{LO Boltzmann (AMY)}} + \underbrace{O(g\log) + O(g)}_{\text{“NLO”, from soft } gT \text{ gluons, } n_B \simeq \frac{T}{\omega} \simeq \frac{1}{g}} + \dots \Big]$$

$O(g)$ Corrections to Hard Collisions, Drag/Diffusion, Bremm:

1. No corrections to Hard Collisions:

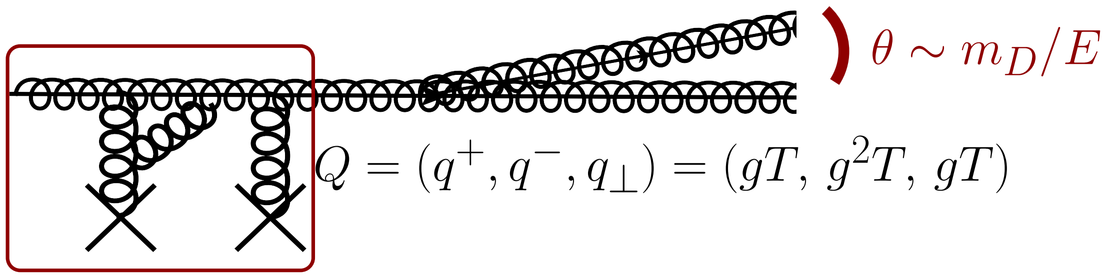
2. Corrections to Longitudinal diffusion:



- Nonlinear interactions of soft classical fields changes the force-force correlator
- Doable because of HTL sum rules (light cone causality)

3. Corrections to Bremm:

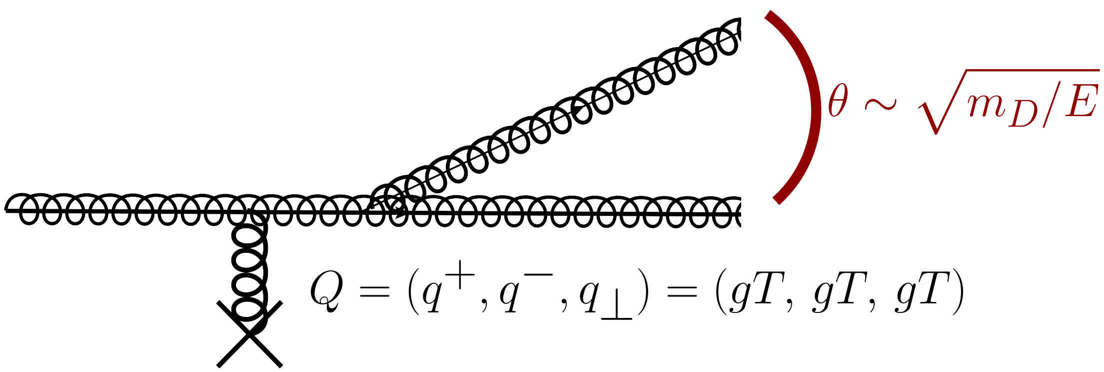
(a) Small angle bremm. Corrections to AMY coll. kernel. (Caron-Huot)



$$\hat{C}_{LO}[q_\perp] = \frac{T g^2 m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$ → A complicated but analytic formula

(b) Large angle brem and collisions with plasmons.

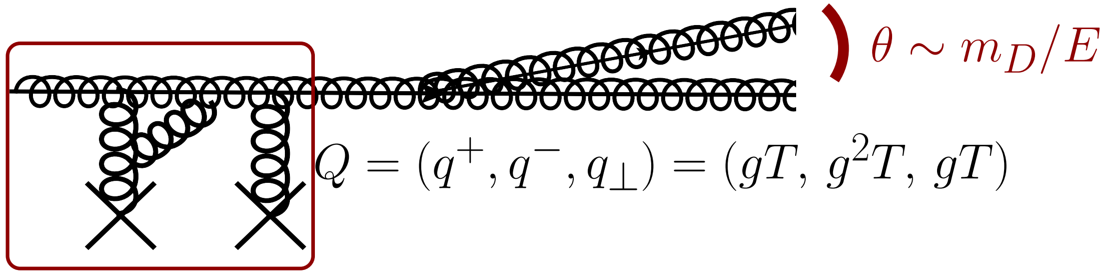
- Include collisions with energy exchange, $q^- \sim gT$.



The large-angle (semi-collinear radiation) interpolates collisional and rad. loss

3. Corrections to Bremm:

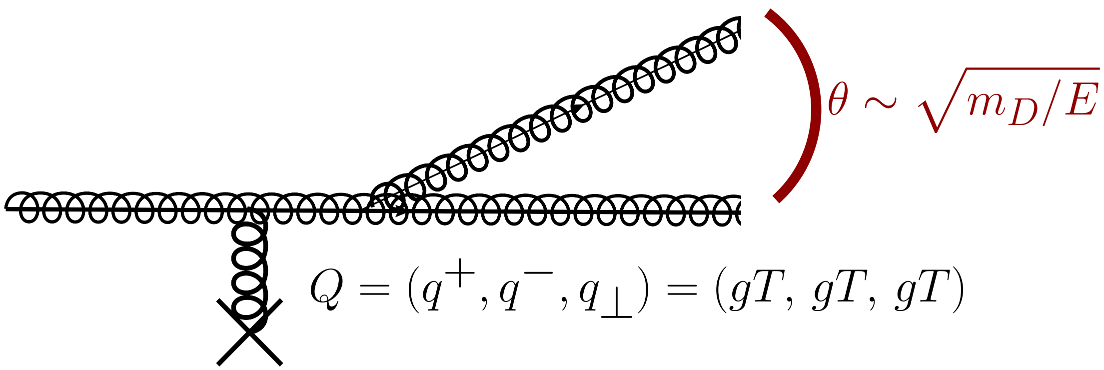
(a) Small angle bremm. Corrections to AMY coll. kernel. (Caron-Huot)



$$\hat{C}_{LO}[q_\perp] = \frac{Tg^2m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \to \text{A complicated but analytic formula}$$

(b) Large angle brem and collisions with plasmons.

- Include collisions with energy exchange, $q^- \sim gT$.



The semi-collinear radiation is done with the replacement, $\hat{C}_{LO}[q_\perp] \to \hat{C}_{LO}[\hat{q}, \delta E]$

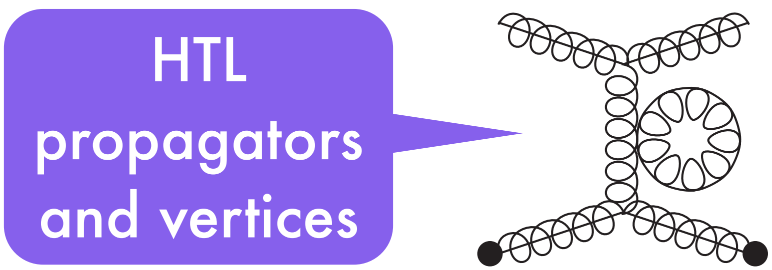
4. Gain Terms:

(a) The structure of the gain terms is the same as LO – three numbers:

$$\left(\chi_{ij}, C^{2\leftrightarrow 2}_{\rm gain}\,\chi_{ij}\right) = \frac{d_A g^4}{8\pi^5 T^3}\sum_{ab} T_{R_a} T_{R_b}\int_0^\infty dp\, p^2\, f_0^a(p)[1\pm f_0^a(p)]\int_0^\infty dk\, k^2\, f_0^b(k)[1\pm f_0^b(k)]$$

$$\times\left[-0.1833\,\frac{\chi^a(p)\chi^b(k)}{pk} - 0.1360\left(\frac{\chi^a(p)\chi^b(k)'}{p} + \frac{\chi^a(p)'\chi^b(k)}{k}\right) - 0.3066\chi^a(p)'\chi^b(k)'\right]$$

(b) It is about a two-year long calculation to do at NLO



(c) Anticipate it to be small, and thus we make an ansatz:

$$C^{NLO}_{\rm gain} = C^{LO}_{\rm gain}\times\frac{m_D}{T}\times\underbrace{c_{\ell=2}}_{\text{We will vary this coefficient!}}$$

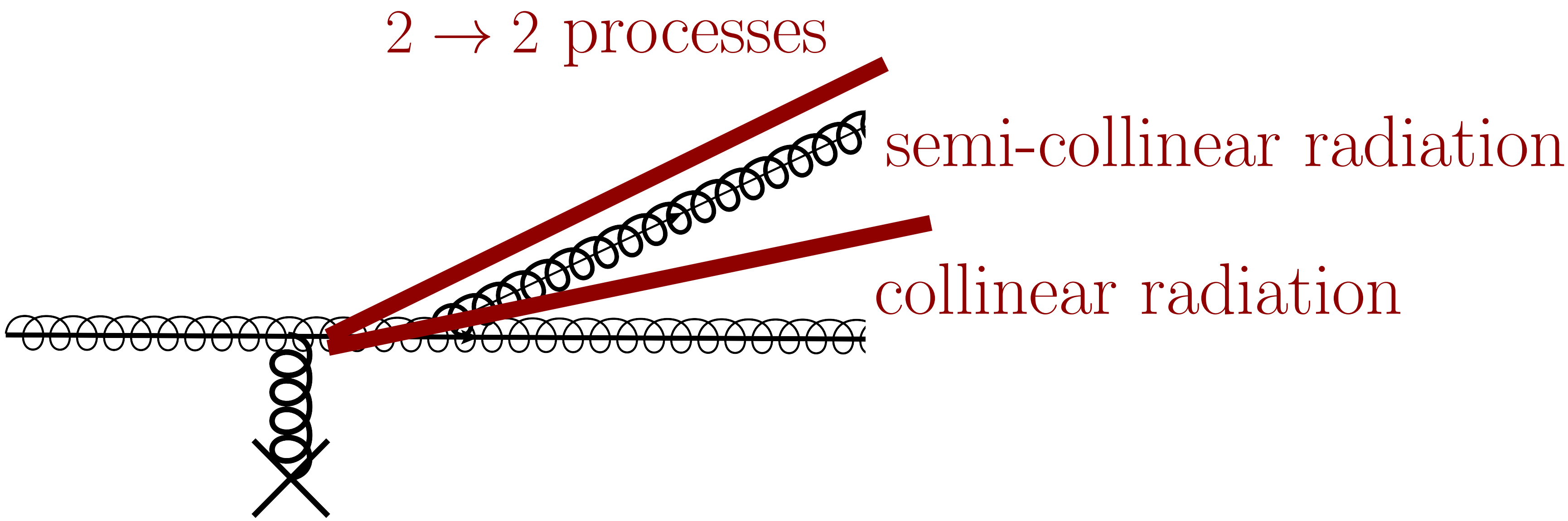
The NLO Boltzmann equation – a preview:

Cutoff dependence cancels

$$[\partial_t + v_{\boldsymbol{k}} \cdot \partial_{\boldsymbol{x}}] f_{\boldsymbol{k}} = \left(C_{\rm diff}^{\hat{q}_{\parallel}(\mu)} + C_{\rm diff}^{\delta\hat{q}_{\parallel}(\mu)}\right) + \left(C_{\rm diff}^{\hat{q}(\mu)} + C_{\rm diff}^{\delta\hat{q}}\right) + C_{\rm gain} + C_{2\leftrightarrow 2}[\mu] + C_{1\leftrightarrow 2} + \delta C_{1\leftrightarrow 2} + C_{\rm semi-coll}[\mu]$$

The μ-dependence of the drag at NLO cancels the μ-dependence of semi-collinear radiation

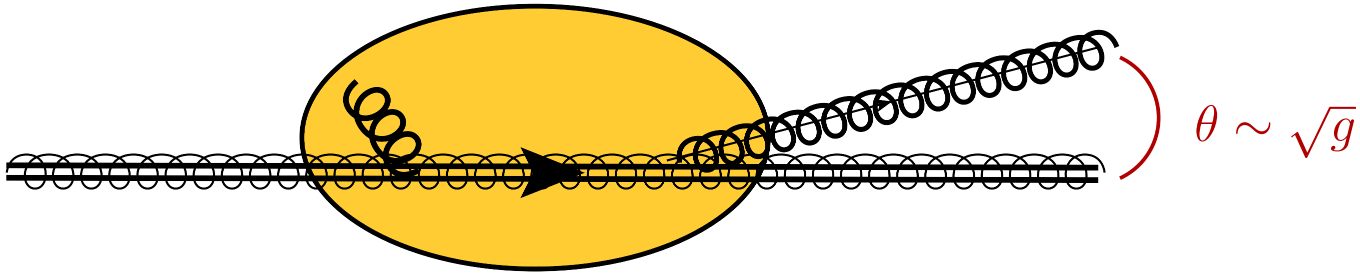
Semi-collinear radiation – a new kinematic window



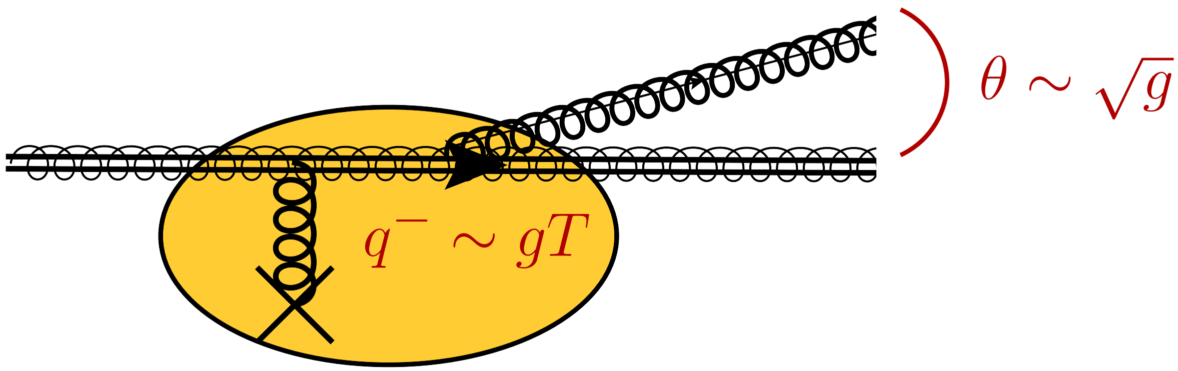
The semi-collinear regime interpolates between brem and collisions

Matching collisions to brem

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



is not physically distinct from the wide angle brem

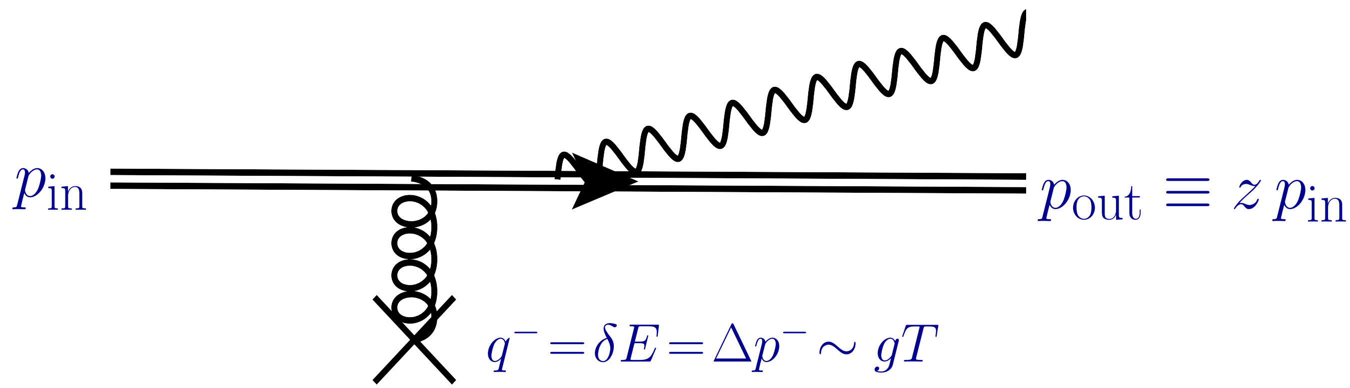


Need both processes

- For harder gluons, $q^- \to T$, bremm becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, wide angle bremm matches onto collinear limit.

Brem and collisions at wider angles (but still small!)

- Semi-collinear emission:



- The matrix element is:

$$|\mathcal{M}|^2 (2\pi)^4 \delta^4(P_{\rm tot}) \propto \underbrace{\frac{1+z^2}{z}}_{\text{QCD splitting fcn}} \int_Q \frac{1}{(q^-)^2} \underbrace{\langle F_{i+}(Q)\, F_{i+} \rangle}_{\text{scattering-center}} \; 2\pi\delta(q^- - \delta E)$$

All of the dynamics of the scattering center in a single matrix element $\langle F_{i+}(Q) F_{i+} \rangle$,

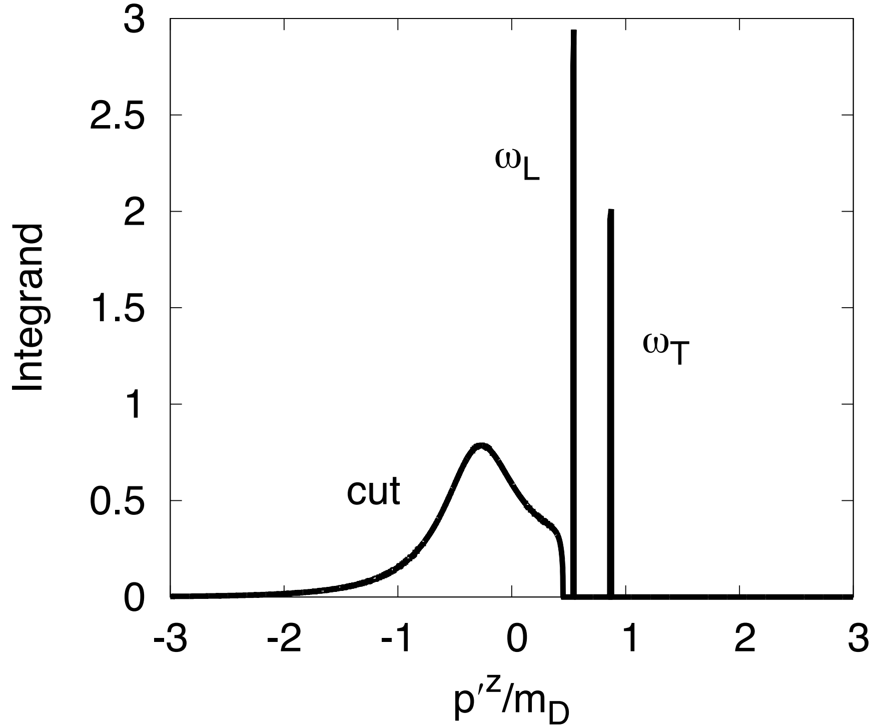
– a transverse force-force correlator + energy exchange

The scattering center:

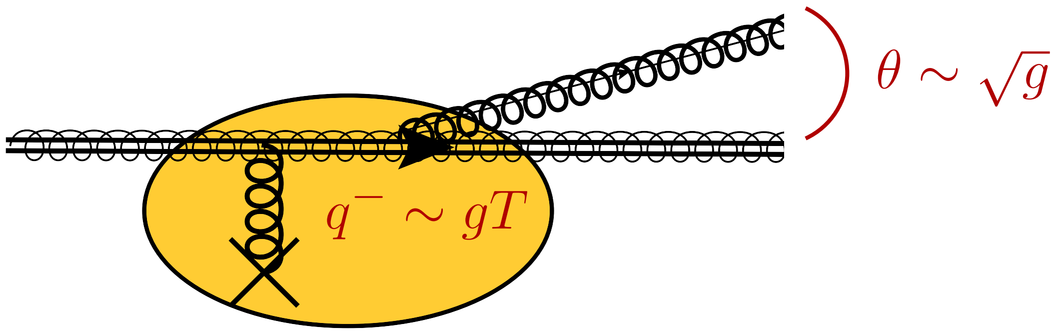
$$\hat{C}[\mathbf{q}_\perp, \delta E] = \int_Q \frac{1}{(q^-)^2} \langle F_{i+}(Q) F_{i+} \rangle \; 2\pi\delta(q^- - \delta E)$$

1. Soft-correlator has wide angle brem = cut

2. And plasmon scattering = poles



Finite energy transfer sum-rule



- The small angle bremm rate involves a transverse force force correlator $\sim \hat{q}$

$$\underbrace{q_\perp^2 \hat{C}_{LO}[q_\perp] = \int_{-\infty}^{\infty} \frac{\mathrm{d}q_0}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle|_{q_+=0} = \frac{T m_D^2}{q_T^2 + m_D^2}}_{\text{Rate of transverse kicks of } q_\perp}$$

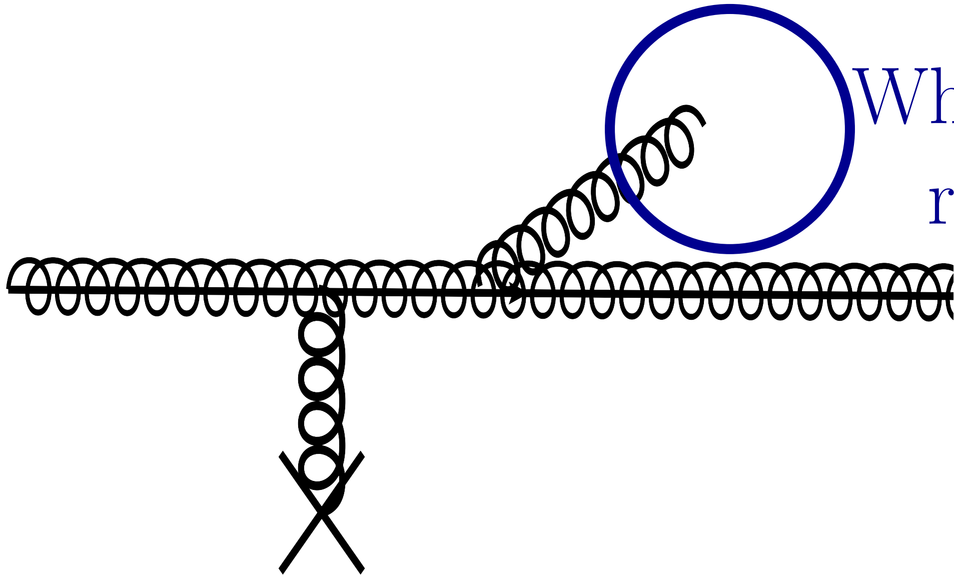
- The wide angle bremm rate involves a finite $q_+ = \delta E$ generalization $\sim \hat{q}(\delta E)$

$$\underbrace{\int_{-\infty}^{\infty} \frac{\mathrm{d}q_0}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle|_{q_+=-\delta E} = T \left[\frac{2(\delta E)^2(\delta E^2 + q_\perp^2 + m_D^2) + m_D^2 q_\perp^2}{(\delta E^2 + q_\perp^2 + m_D^2)(\delta E^2 + q_\perp^2)} \right]}_{\text{Rate of transverse kicks of } q_\perp \text{ and energy transfer } q_+ = \delta E}$$

At NLO the collision kernel for $\hat{q}$ gets is replaced with $C[q_\perp] \to C[q_\perp, \delta E]$

Matching between brem and drag

semi-collinear radiation



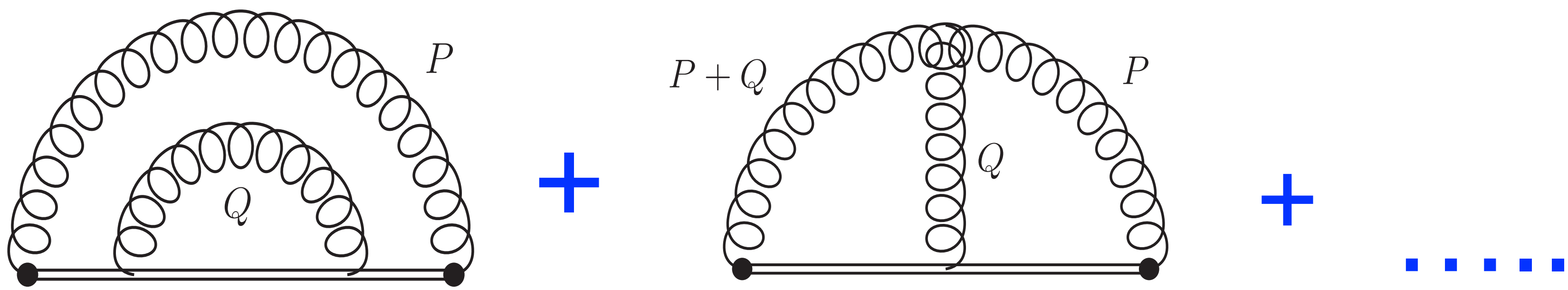
What happens when the radiated gluon is soft?

- The semi-collinear emission rate diverges logarithmically when the gluon gets soft

$$\Gamma_{\rm semi-coll} \sim g^2 C_A \overbrace{\frac{\delta m_\infty^2}{4\pi}}^{\sim\, g^3 T^2} \log\left(\frac{2Tm_D}{\mu}\right)$$

When the gluon becomes soft need to relate radiation and drag.

Computing the NLO drag:



- Evaluate NLO longitudinal force-force with hard thermal loops + sum-rules
- Only change relative to LO is the replacement $m_\infty^2 \to m_\infty^2 + \delta m_\infty^2$

$$\eta(\mu) \propto g^2 C_A \int^{\mu} \frac{d^2\boldsymbol{p}_T}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2}$$

$$\propto \text{leading order} + \underbrace{g^2 C_A \frac{\delta m_\infty^2}{4\pi}\left[\log\left(\frac{\mu_\perp^2}{m_\infty^2}\right) - 1\right]}_{\text{NLO correction to drag}}$$

The cutoff dependence of the drag cancels against the semi-collinear emission rate

The NLO Boltzmann equation review:

Cutoff dependence cancels

$$[\partial_t + v_{\boldsymbol{k}} \cdot \partial_{\boldsymbol{x}}]\, f_{\boldsymbol{k}} = \left(C_{\rm diff}^{\hat{q}_{\|}(\mu)} + C_{\rm diff}^{\delta\hat{q}_{\|}(\mu)}\right) + \left(C_{\rm diff}^{\hat{q}(\mu)} + C_{\rm diff}^{\delta\hat{q}}\right) + C_{\rm gain}$$
$$+ C_{2\leftrightarrow 2}[\mu] + C_{1\leftrightarrow 2} + \delta C_{1\leftrightarrow 2} + C_{\rm semi-coll}[\mu]$$

Further reorganization:

- Semi-collinear corrections + a bit more can be incorporated into $C_{1\leftrightarrow 2}$ with:

$$\hat{C}_{LO}[q_\perp] \to \hat{C}_{LO}[q_\perp, \delta E]$$

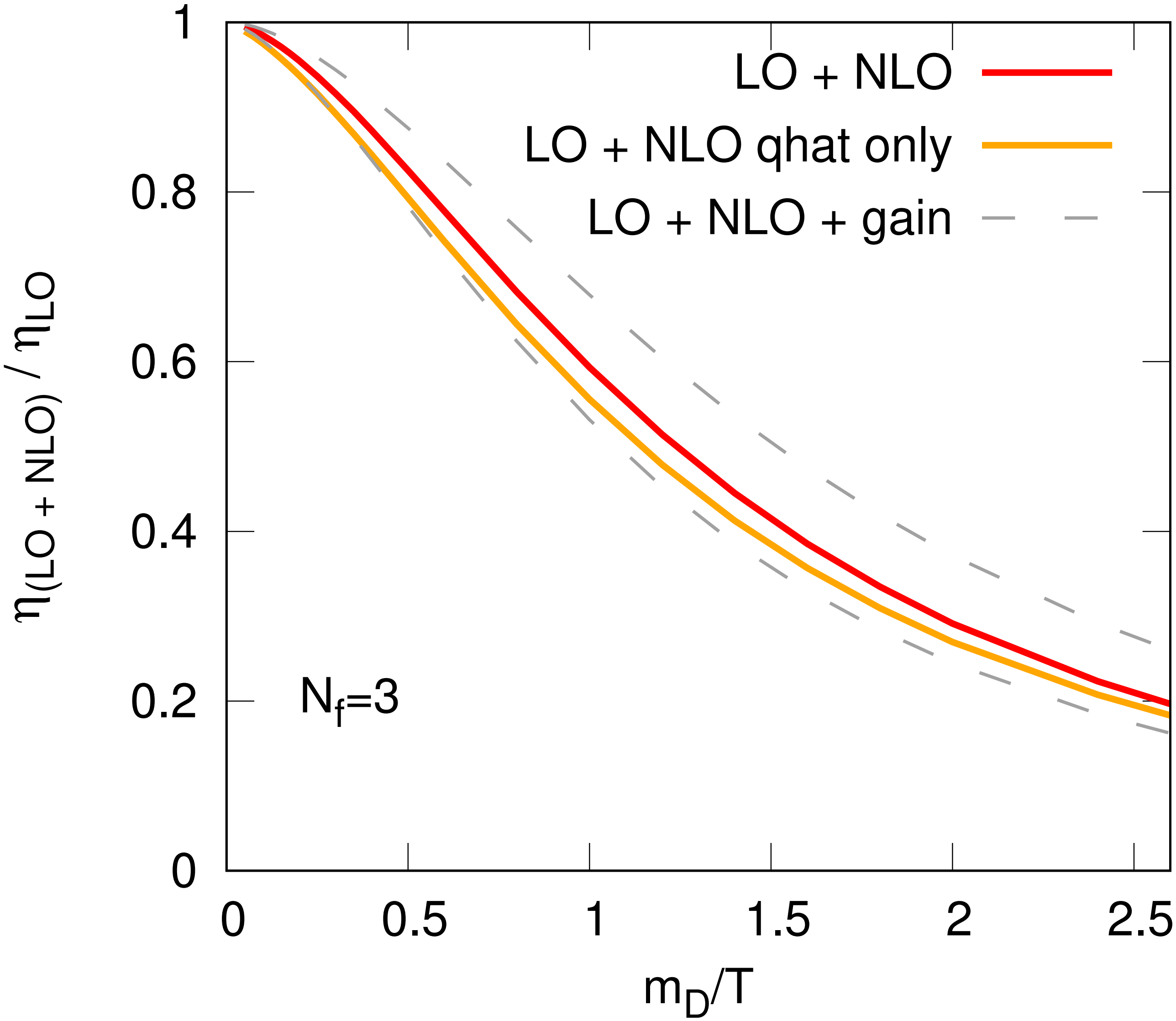
- The bit more is precisely the NLO longitudinal diffusion coefficient

$$[\partial_t + v_{\boldsymbol{k}} \cdot \partial_{\boldsymbol{x}}]\, f_{\boldsymbol{k}} = C_{\rm diff}^{\hat{q}_{\|}(\mu)} + \left(C_{\rm diff}^{\hat{q}(\mu)} + C_{\rm diff}^{\delta\hat{q}}\right) + C_{\rm gain}$$
$$+ C_{2\leftrightarrow 2}[\mu] + \underbrace{\text{souped up } C_{1\leftrightarrow 2}}_{\text{uses } \hat{C}[q_\perp, \delta E]} + \delta C_{1\leftrightarrow 2}$$

Results:

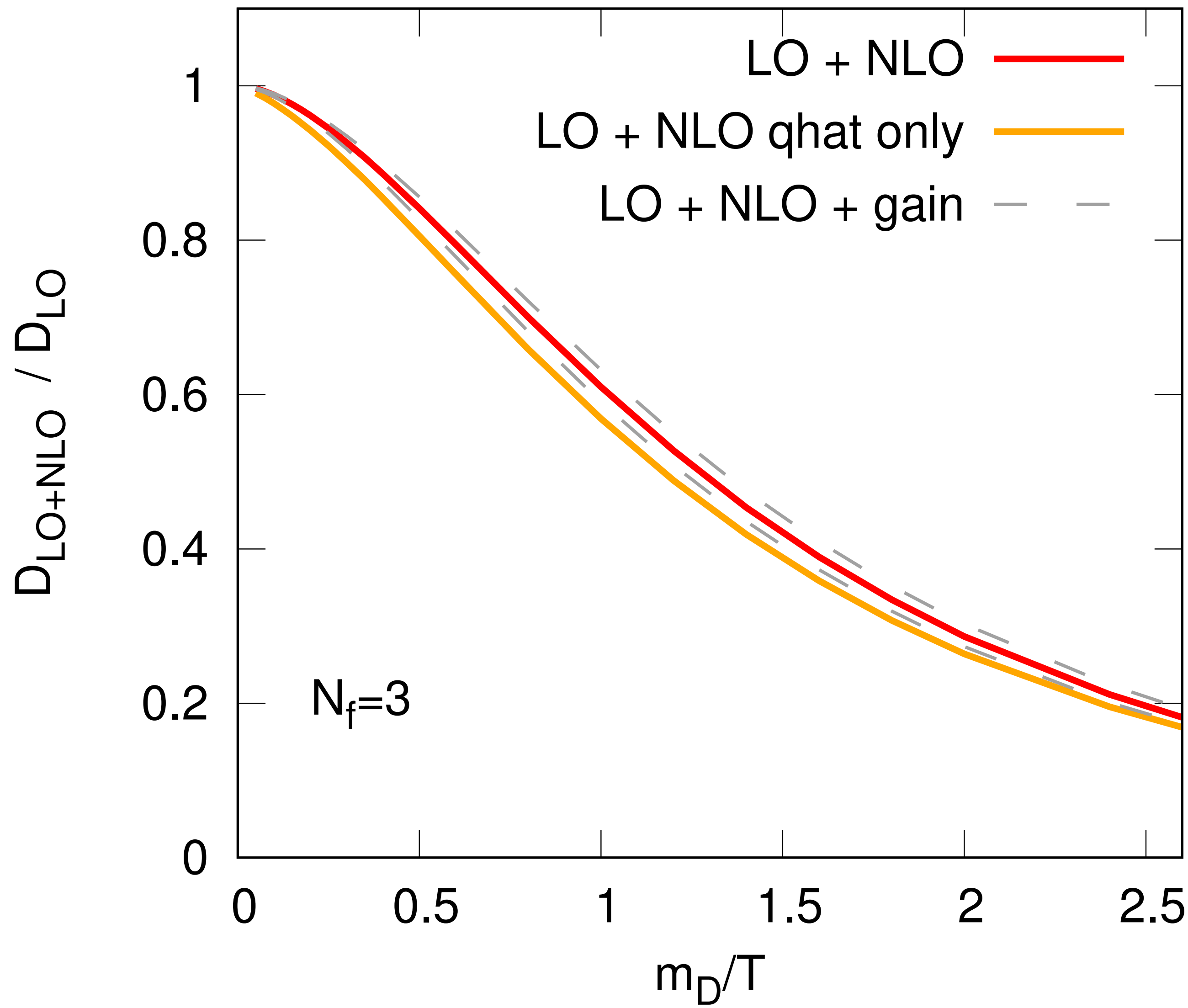
I'll skip details of quarks . . .

Shear viscosity:



NLO corrections are large and dominated by $\hat{q}$!

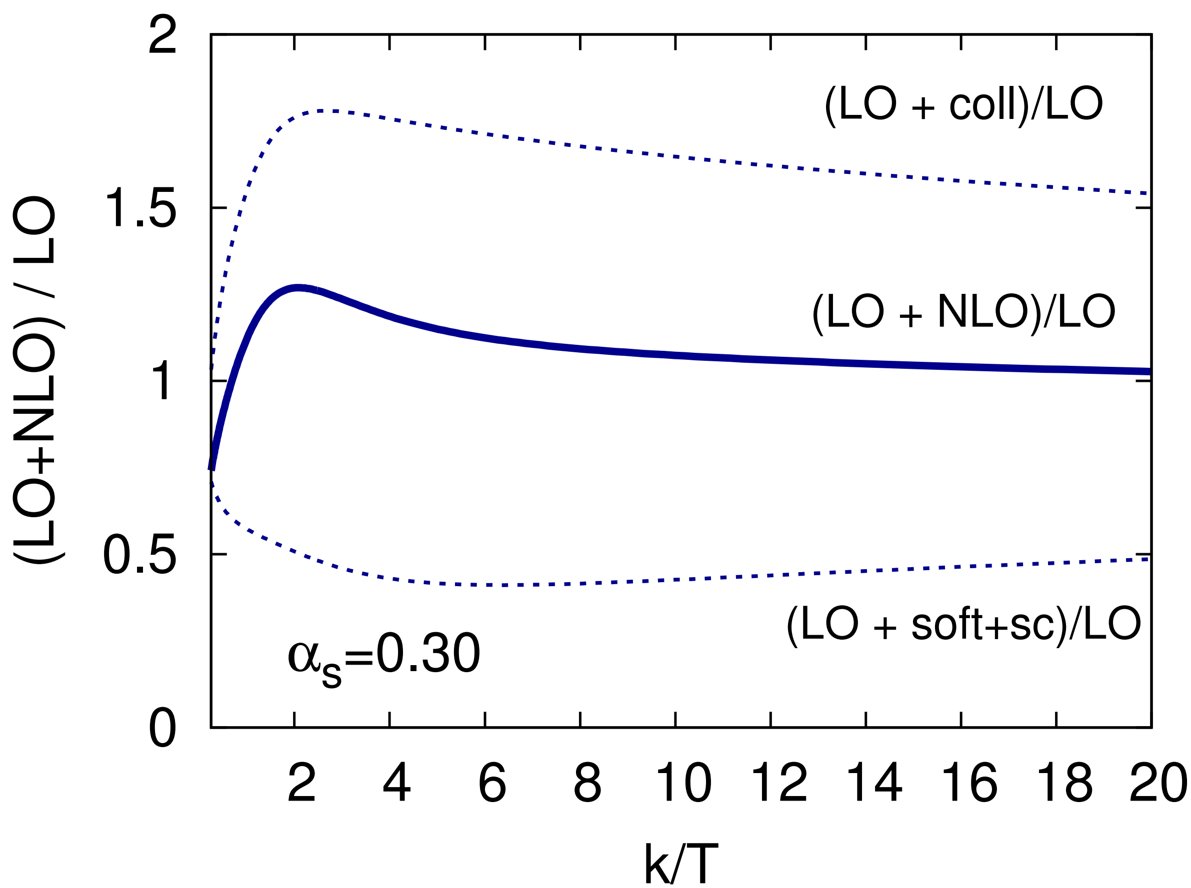
Baryon number diffusion:



NLO corrections are large and dominated by $\hat{q}$!

Comparison with “NLO” results on the photon emission rate

$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} = \text{Photon emission rate per phase-space}$$



“NLO” corrections are modest and roughly momentum independent

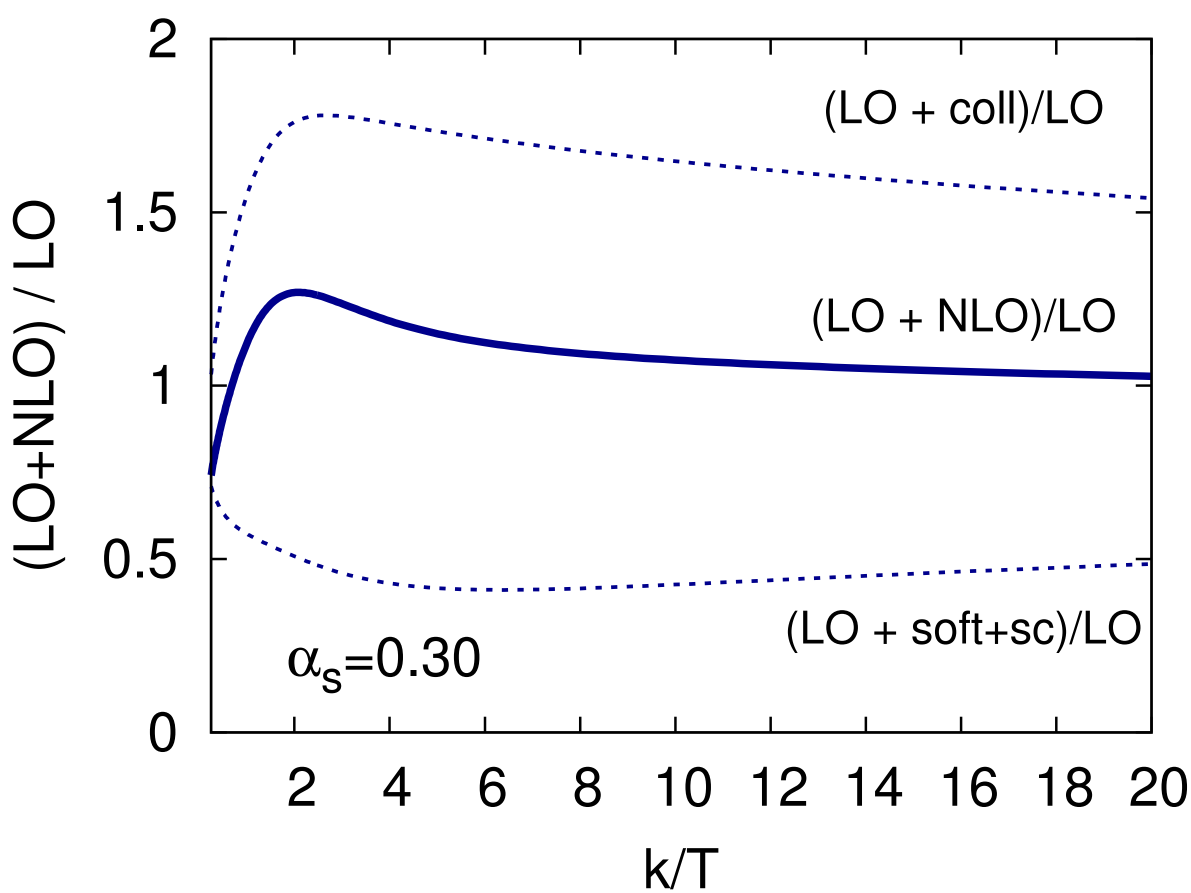
Summary

1. QCD Kinetics = collisions, drag-diffusion, bremm
2. We have constructed a Boltzmann equation valid to NLO
   - Use for energy loss, shear viscosity, photon emission, . . .
3. Close relation between drag, wide angle emissions, quasi-particle mass shift.
   - Use a euclidean formalism to compute δm_∞ and $\hat{q}^{ij}(\mu)$ and $\hat{C}[q_\perp, \delta E]$
4. 3D lattice simulations can give non-perturbative inputs for QCD kinetics
   - The δm_∞ and $\hat{C}[q_\perp, \delta E]$ can be computed
5. Almost all of the NLO modifications of η/s and D arise from modifications of $\hat{q}$

Thank you!

Comparison with "NLO" results on the photon emission rate

$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} = \text{Photon emission rate per phase-space}$$



"NLO" corrections are modest and roughly momentum independent

The different contributions at NLO for photons: