Errors in Model to Data Comparisons

Data Errors (some examples)
Model Errors (an example)
Discussion …

Workshop on Precision Spectroscopy of QGP
Institute for Nuclear Theory, UW
May 25, 2017

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This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

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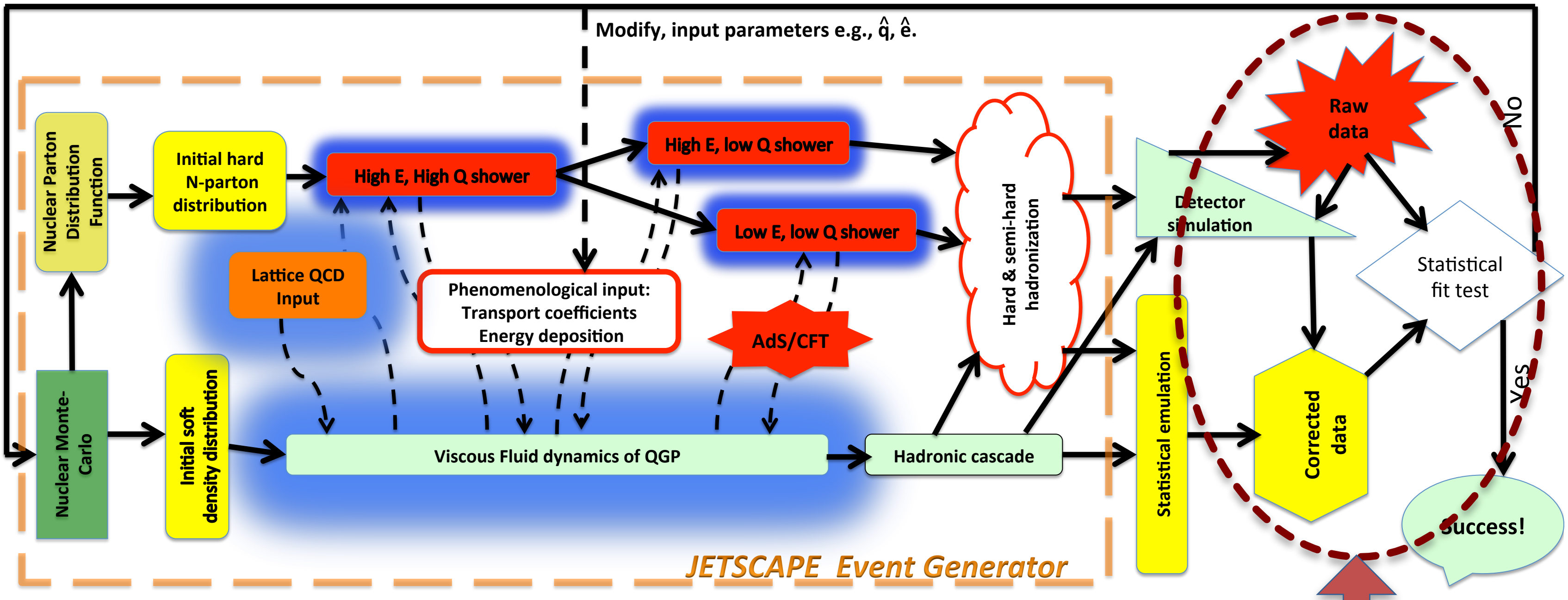
Motivation by JETSCAPE



- This talk is about the last stage of the diagram

MADAI treatment of errors

- Seminal Model-to-Data-Comparison by Novak,…Pratt, et al. (14) http://link.aps.org/doi/10.1103/PhysRevC.89.034917

- But underneath the hood of this Ferrari are some squirrels MADAI errors pegged at 6% (and 3%)

TABLE II. Observables used to compare models to data.

Observable	p_t weighting	Centrality (%)	Collaboration	Uncertainty (%)	Reduced uncertainty
$v_{2,\pi^+\pi^-}$	Average over 11 p_t bins from 160 MeV/c to 1 GeV/c	20–30	STAR[1] [52]	12	6%
R_{out}	Average over 4 p_t bins from 150–500 MeV/c	0–5	STAR [53]	6	3%
R_{side}	Average over 4 p_t bins from 150–500 MeV/c	0–5	STAR [53]	6	3%
R_{long}	Average over 4 p_t bins from 150–500 MeV/c	0–5	STAR [53]	6	3%
R_{out}	Average over 4 p_t bins from 150–500 MeV/c	20–30	STAR [53]	6	3%
R_{side}	Average over 4 p_t bins from 150–500 MeV/c	20–30	STAR [53]	6	3%
R_{long}	Average over 4 p_t bins from 150–500 MeV/c	20–30	STAR [53]	6	3%
$\langle p_t \rangle_{\pi^+\pi^-}$	200 MeV/c < p_t < 1.0 GeV/c	0–5	PHENIX [54]	6	3%
$\langle p_t \rangle_{K^+K^-}$	400 MeV/c < p_t < 1.3 GeV/c	0–5	PHENIX [54]	6	3%
$\langle p_t \rangle_{p\bar{p}}$	600 MeV/c < p_t < 1.6 GeV/c	0–5	PHENIX [54]	6	3%
$\langle p_t \rangle_{\pi^+\pi^-}$	200 MeV/c < p_t < 1.0 GeV/c	20–30	PHENIX [54]	6	3%
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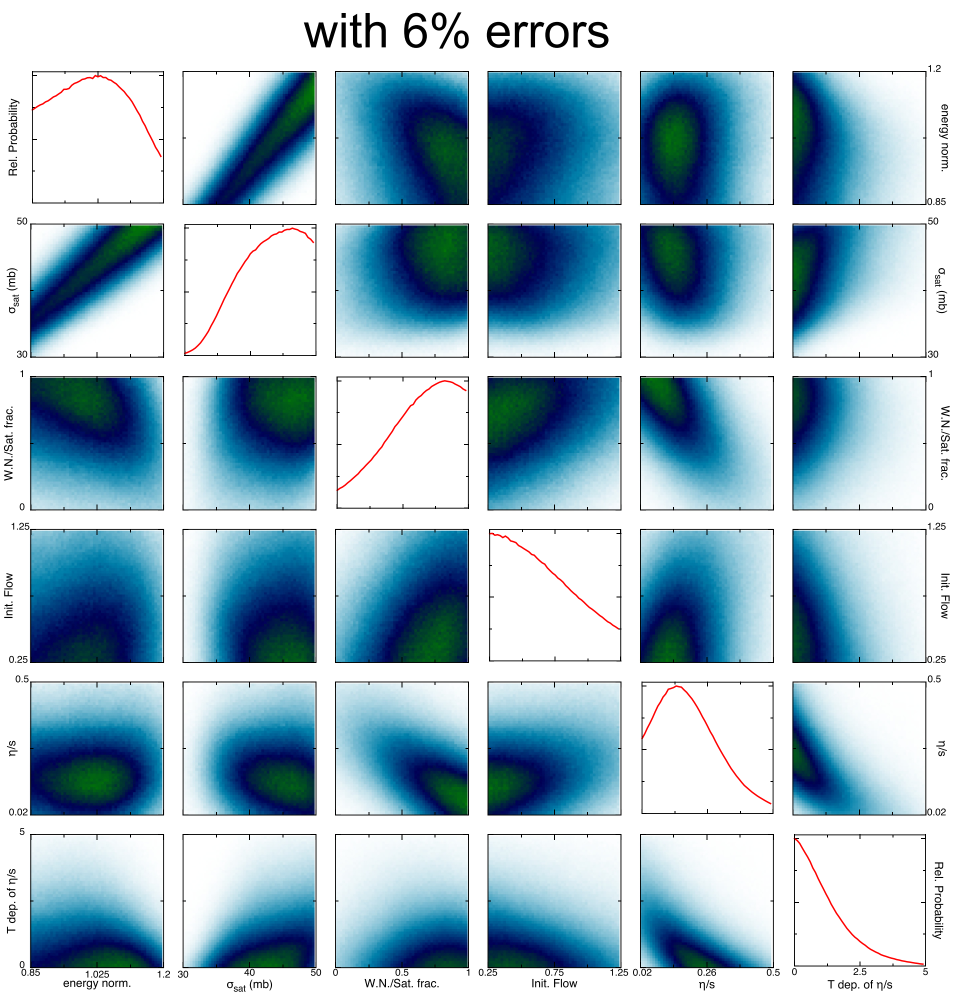
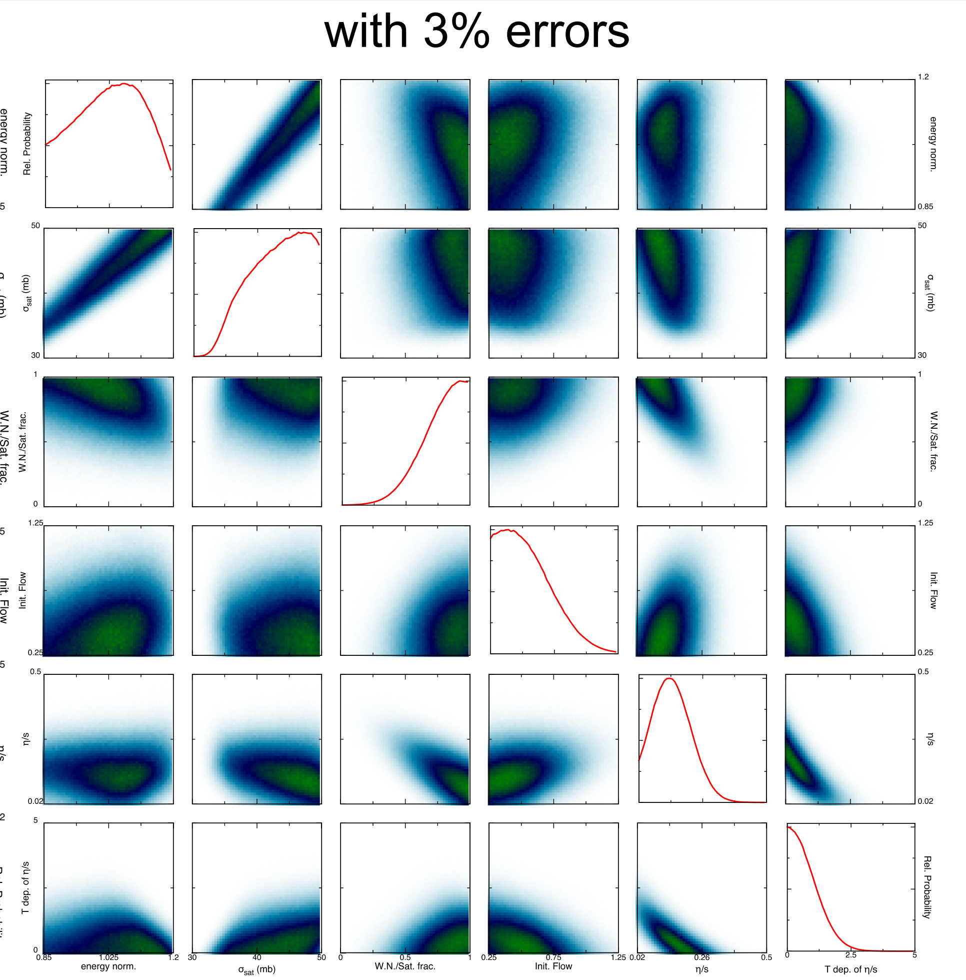
$\langle p_t \rangle_{p\bar{p}}$	600 MeV/c < p_t < 1.6 GeV/c	20–30	PHENIX [54]	6	3%
$\pi^+\pi^-$ yield	200 MeV/c < p_t < 1.0 GeV/c	0–5	PHENIX [54]	6	3%
$\pi^+\pi^-$ yield	200 MeV/c < p_t < 1.0 GeV/c	20–30	PHENIX [54]	6	3%

[a]To account for nonflow correlations, the value of v_2 was reduced by 10% from the value reported in Ref. [52].

MADAI Error Comparison





The Dukes

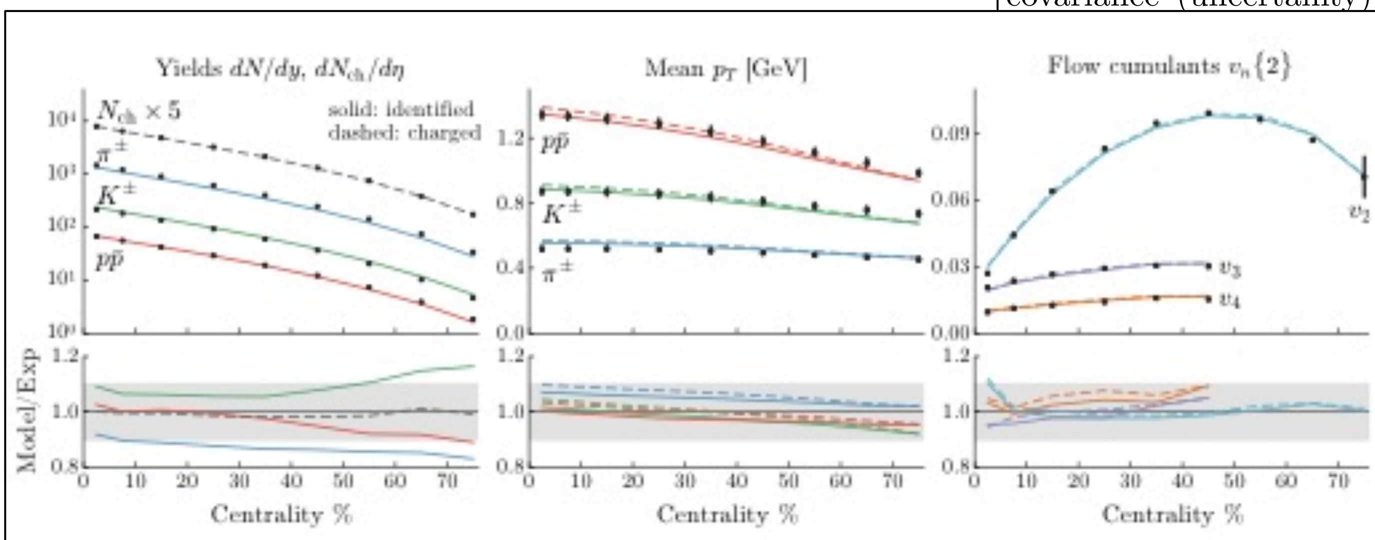
- Bernard, … Bass, et al (16) improved upon work of MADAI
  https://journals.aps.org/prc/abstract/10.1103/PhysRevC.94.024907

- But not with error treatment – their Tesla still has a few squirrels principle component errors pegged at 10%

covariance (uncertainty) matrix. As in previous work
stant fractional uncertainty on
, so that the covariance matrix

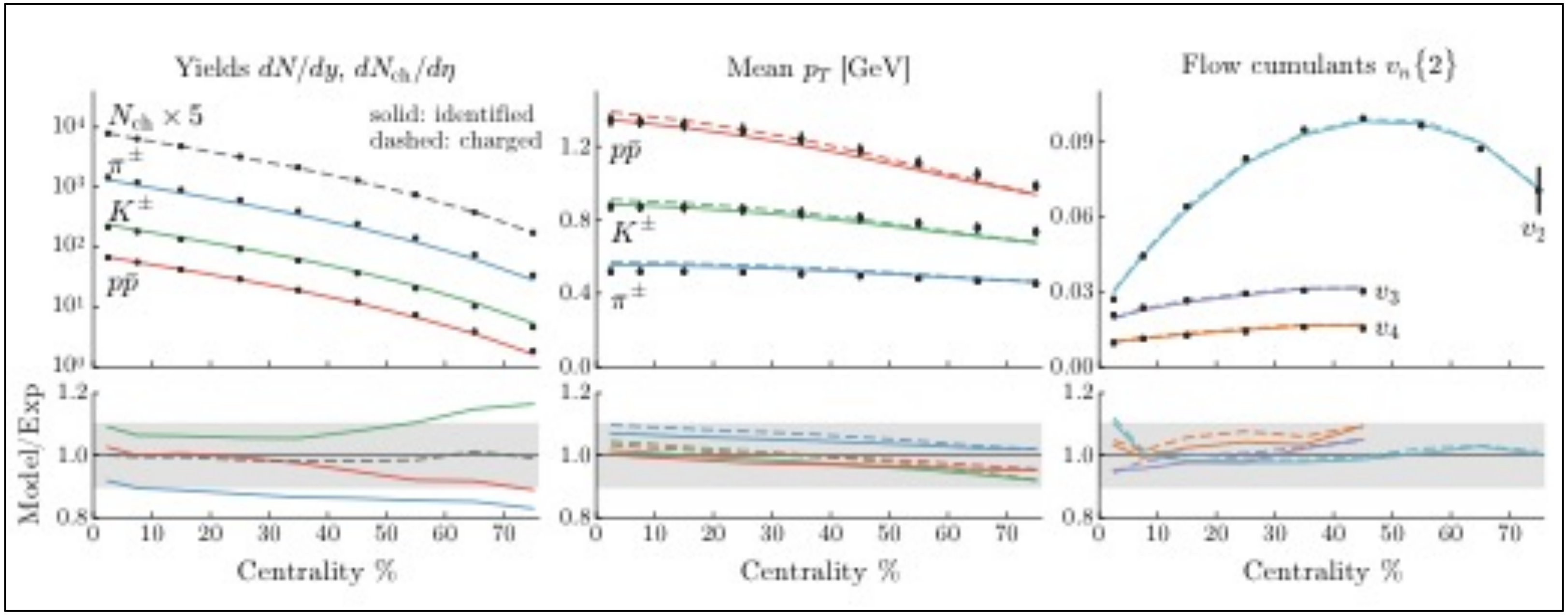
$$\text{iag}(\sigma_z^2\,\mathbf{z}_{\text{exp}}), \qquad (30)$$

resent study. This is a simple
vatively account for the various
the experimental data, model
r predictions. It certainly lim-
tative uncertainties in the final
d is an obvious target for im-
provement in future studies.



The Dukes

- Q: Why were relative errors set to 10% ?
- A: That's what was needed !



Comparison of most probable model results to ALICE data

An Earlier Foray into Model-2-Data-Comparisons

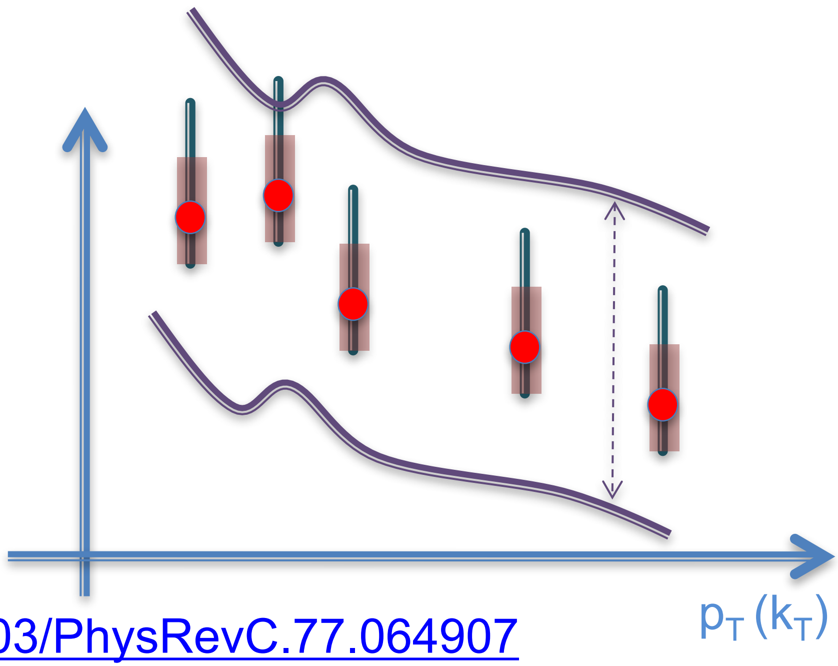
- CHIMERA = Comprehensive Heavy Ion Model Evaluation & Reporting Algorithm
- Non-Bayesian generation χ^2 map in T-η/s space with simultaneous comparison PHENIX and STAR spectra, flow, HBT
  https://dx.doi.org/10.1103/PhysRevC.87.044901 (RAS 2013)
- Just a Subaru, but no squirrels under the hood
- χ^2 evaluations performed using full statistical and systematic errors *as reported by PHENIX and STAR*

Systematic Errors in CHIMERA

- Evaluate χ^2 from model & data, accounting for type A & B errors

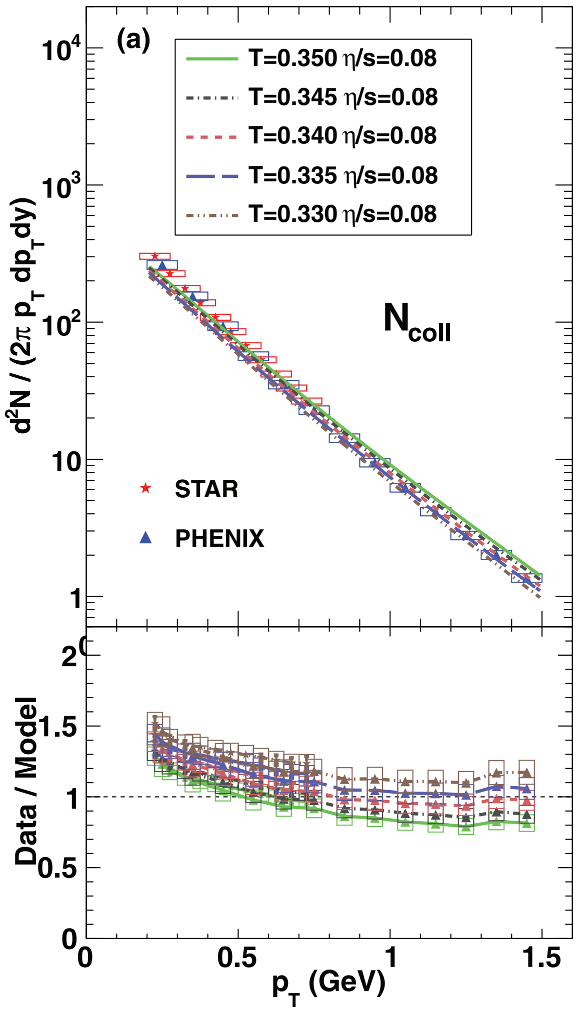
- A type: uncorrelated (σ_i)
- B type: correlated frac.(σ_b)
- C type: normalization (σ_c)
- D type: correlated tilt (not considered)

$$\tilde{\chi}^2(\epsilon_b, \epsilon_c, p) = \left[\left(\sum_{i=1}^{n} \frac{(y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c - \mu_i(p))^2}{\tilde{\sigma}_i^2}\right) + \epsilon_b^2 + \epsilon_c^2\right]$$



Error definitions based on https://dx.doi.org/10.1103/PhysRevC.77.064907

How well did it work?

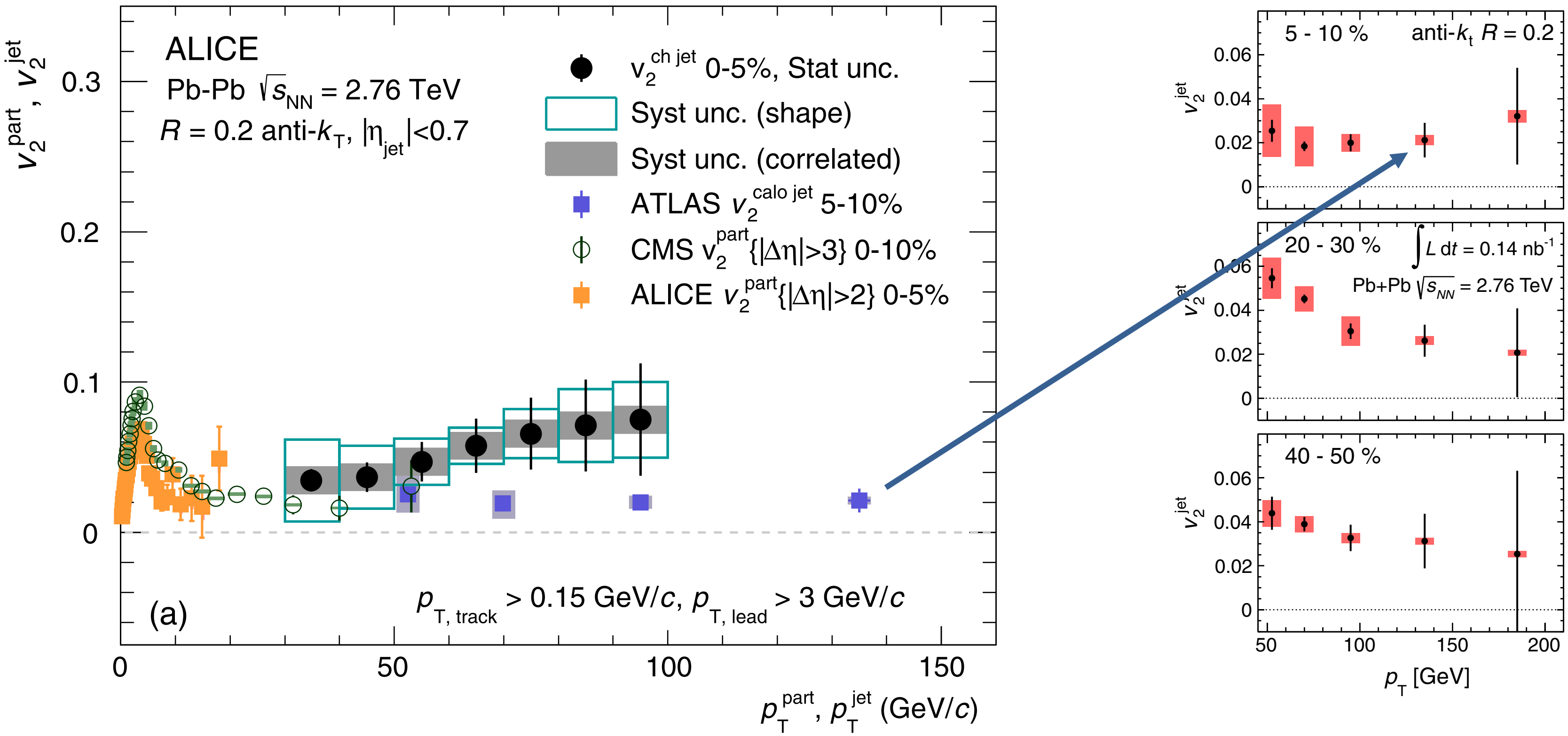


$T_{\rm cent}$	χ^2_{ndf} $N_{\rm coll}$	
(GeV)	PHENIX	STAR
0.350	13.8	2.30
0.345	2.77	1.75
0.340	15.7	8.15
0.335	75.9	6.87
0.330	60.3	6.94

- Separate systematic errors allowed STAR and PHENIX data to shift independently to achieve reasonable χ^2 values

Jet errors will be more challenging?

http://dx.doi.org/10.1016/j.physletb.2015.12.047   https://doi.org/10.1103/PhysRevLett.111.152301



- One cannot directly compare ALICE results to ATLAS (without a model)
- But one can ask whether deviation from zero is significant

ALICE $v_2^{ch,jet}$ Error Analysis

	p_T^{chjet} (GeV/c)	Uncertainty on $v_2^{ch\ jet}$					
		30–40	60–70	80–90	30–40	60–70	80–90
	Centrality (%)	0–5			30–50		
Shape	Unfolding	0.017	0.012	0.016	0.016	0.011	0.015
	p_T^{chjet}-measured	0.013	$\ll$ stat	$\ll$ stat	0.024	$\ll$ stat	$\ll$ stat
	$\rho_{ch}(\varphi)$ fit	0.015	$\ll$ stat	0.016	$\ll$ stat	$\ll$ stat	$\ll$ stat
Total		0.027	0.012	0.023	0.029	0.011	0.015
Correlated	Tracking	0.009	0.009	0.009	0.007	0.007	0.007
	p_T^{chjet}-unfolded	$\ll$ stat	$\ll$ stat	$\ll$ stat	$\ll$ stat	$\ll$ stat	$\ll$ stat
Total		0.009	0.009	0.009	0.007	0.007	0.007

χ^2 for the hypothesis $v_2^{ch\ jet} = \mu_i$ is calculated by minimizing

$$\tilde{\chi}^2(\varepsilon_{corr}, \varepsilon_{shape}) = \left[\left(\sum_{i=1}^{n} \frac{(v_{2,i} + \varepsilon_{corr}\sigma_{corr,i} + \varepsilon_{shape} - \mu_i)^2}{\sigma_i^2}\right) + \varepsilon_{corr}^2 + \frac{1}{n}\sum_{i=1}^{n} \frac{\varepsilon_{shape}^2}{\sigma_{shape,i}^2}\right] \quad (16)$$

also based on https://dx.doi.org/10.1103/PhysRevC.77.064907, yields p(m=0) = 0.009, ask Redmer for more details...

Question for the rest of us: is there a better approach?

Why not produce the full co-variance matrix?

- See upcoming publication by NIFFTE = Neutron Induced Fission Fragment Tracking Experiment http://niffte.calpoly.edu
- Co-variant inputs include
  - Beam pile-up
  - Target contamination
  - Tracking efficiency
  - Analysis cuts
- But what would we do with it?

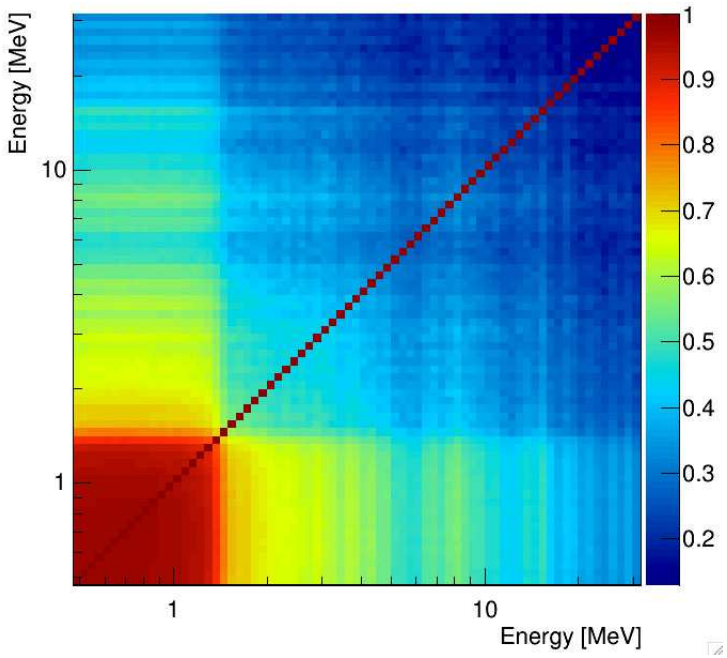
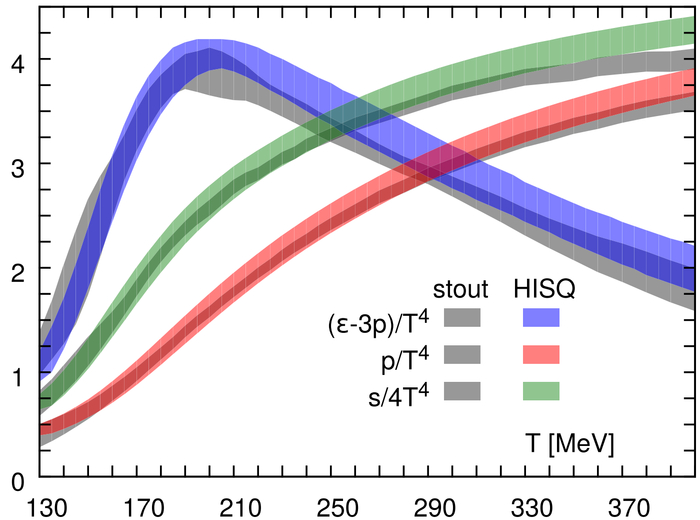


FIG. 16. The ^{238}U(n,f)/^{235}U(n,f) correlation matrix measured in this work. At low neutron energy, the contaminant correction becomes the largest source of uncertainty, resulting in a large correlated region in the correlation matrix. The contaminant correction is a fixed value at all energies and as the ratio becomes small at low energy, a large relative uncertainty results. The z-axis represents the value of the correlation matrix elements.

Modeling Errors : Example from the Lattice

- From 2008—2014 HotQCD and Wuppertal-Budapest Collaborations spent 100s of millions of core hours calculating the QGP EoS for insertion into hydrodynamic code

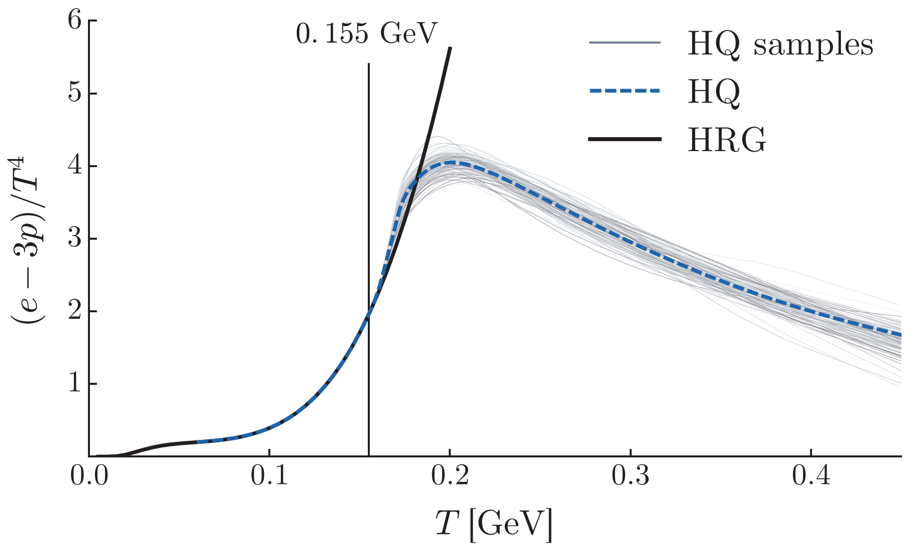


https://dx.doi.org/10.1103/PhysRevD.90.094503
http://linkinghub.elsevier.com/retrieve/pii/S0370269314000197

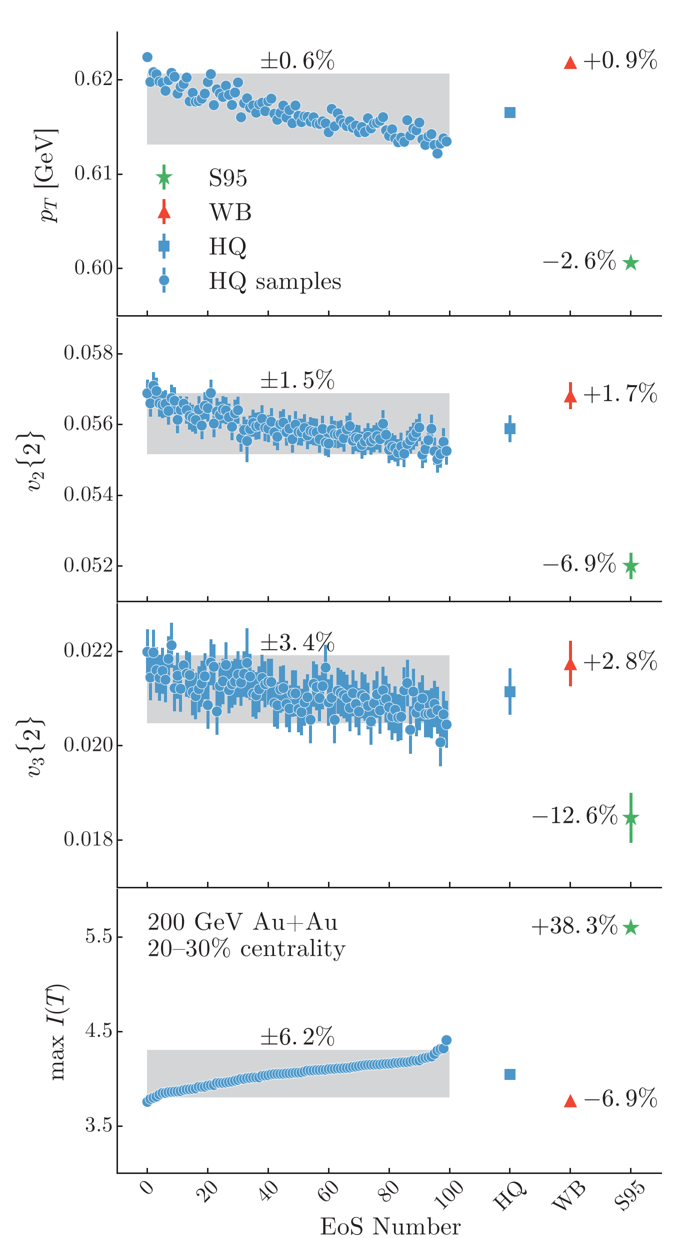
- In 2016, S. Moreland and RAS sought to answer
  - Does it matter which EoS result is used for hydro?
  - How much variation is within the systematic error bands?
  - Will we ever need to repeat these calculations on finer lattices?

Propagating Errors in the EoS

http://link.aps.org/pdf/10.1103/PhysRevC.93.044913

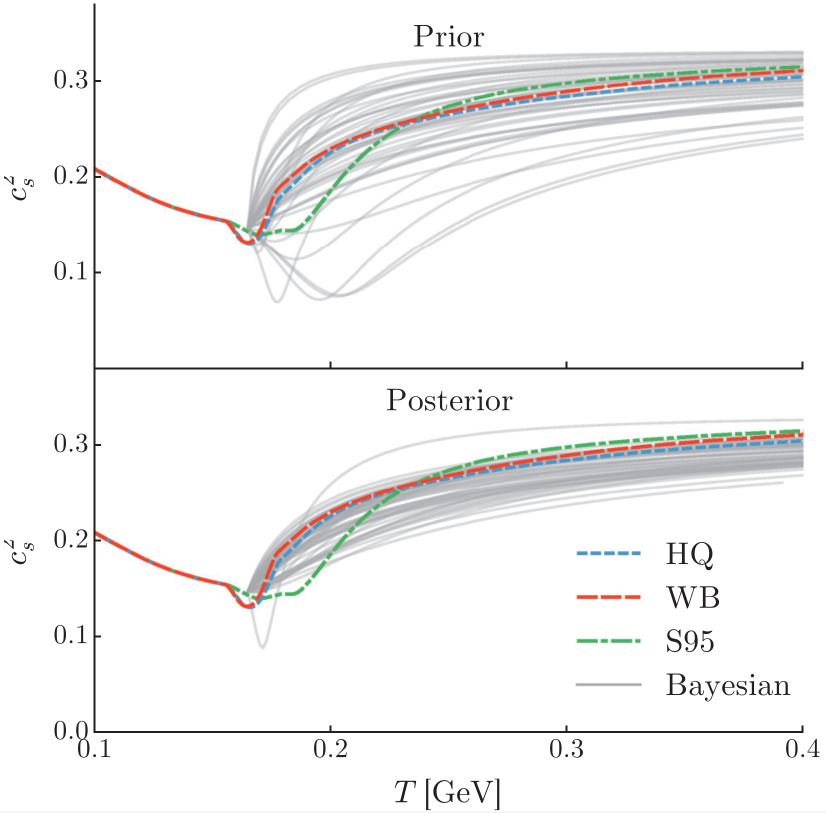


- HotQCD sys. errors calculated with simultaneous spline interpolation with continuum extrapolation. Errors impact observables by 3% or less.



And for Dinesh …

- Bayesian determination of EoS,
- Pratt, Sangaline, Sorenson, Wang (2016)
  https://doi.org/10.1103/PhysRevLett.114.202301



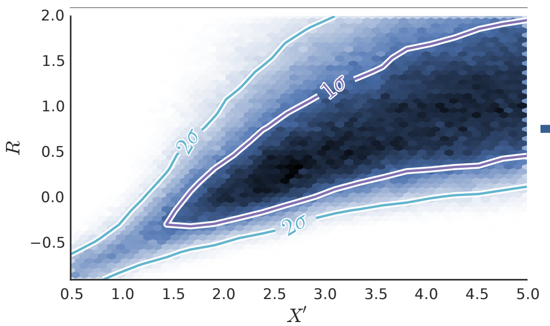


FIG. 4. The posterior likelihood for the two parameters that describe the equation of state, X' and R, have a preference to be along the diagonal. This shows that experiment constrains some integrated measure of the overall stiffness of the equation of state, i.e. a softer equation of state just above T_c is consistent with the data if it is combined with a more rapid stiffening at higher temperature.

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right)\frac{X_0 x + x^2}{X_0 x + x^2 + X'^2}, \quad (2)$$
$$X_0 = X' R c_s(\epsilon)\sqrt{12}, \quad x \equiv \ln \epsilon/\epsilon_h,$$

where ϵ_h is the energy density corresponding to $T = 165$ MeV. The two parameters R and X' describe the behavior of the speed of sound at energy densities above ϵ_h. Whereas R describes how the speed of sound rises or falls for small x, X' describes how quickly the speed of sound eventually approaches 1/3 at high temperature. Once given $c_s^2(\epsilon)$, thermodynamic relations provide all other representations of the equation of state. Runs were performed for $0.5 < X' < 5$, and with $-0.9 < R < 2$. In the limit $R \to -1$ the speed of sound will have a minimum of zero.

Conclusions

- Application of Bayesian methods to determine properties of QGP has progressed rapidly, treatment of errors has not
- Proper treatment of experimental errors is necessary if we are to compare to similar observables from different experiments
- Approach defined in PhysRevC.77.064907 appears to be the one most followed
- Can we (must we) do better ?
- Assigning epistemic uncertainty to models is another challenge
- Future INT workshop for theory, experiment, and statistics ?