Microscopic Description of Heavy-Flavor Diffusion in QCD Matter

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INT Program on
"Precision Spectroscopy of Quark-Gluon Plasma
with Jets and Heavy Quarks"
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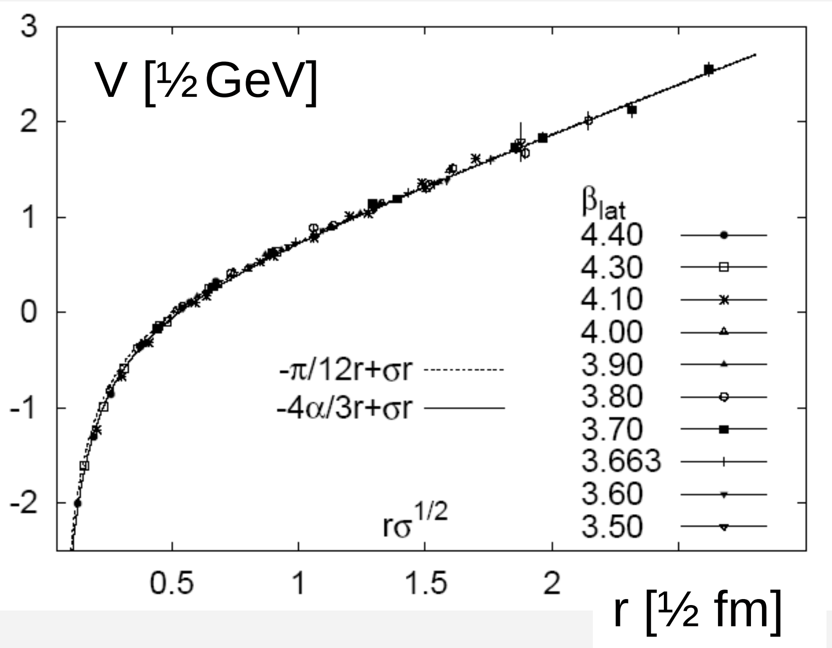
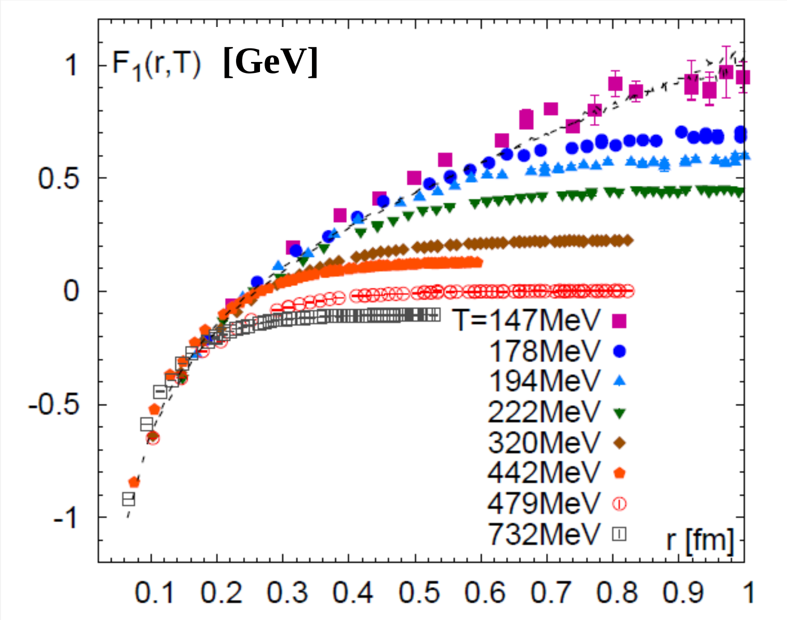
1.) Introduction: A "Calibrated" QCD Force





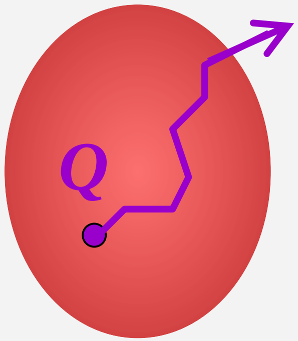
[Bazavov et al '13]

- Vacuum quarkonium spectroscopy well described
- Confinement ↔ **linear part of potential**

Objective: **Determine medium-modifications of QCD force and infer HF transport properties + spectral functions to probe QGP at varying resolution.**

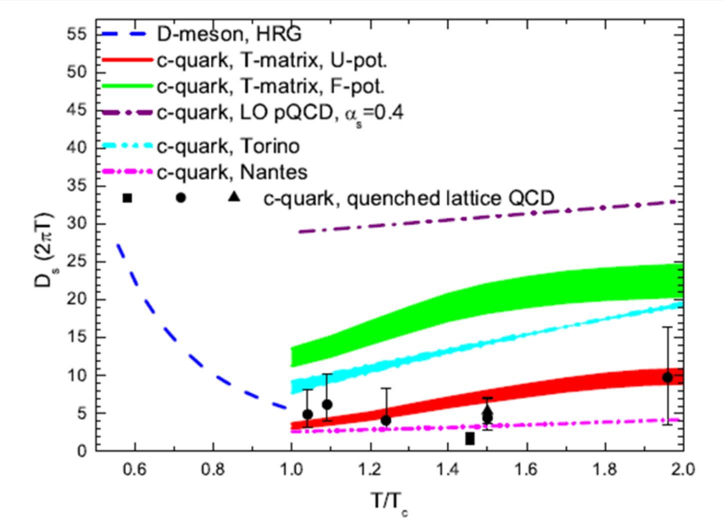
Exploit $m_Q \gg \Lambda_{QCD}$, T_c, $T_{RHIC,LHC}$

1.2 Heavy Quarks in Medium



- **Diffusion: "Brownian motion"**
- Thermalization delayed by $\mathbf{m_Q/T}$
  → memory in URHICs
- Direct access to transport coefficient $\mathcal{D}_s$ **(2πT)** **(~ η/s ~ σ_{EM}/T)**
- Elastic scattering rates (radiation suppressed, $\mathbf{q_0^2 \ll q^2}$)
  → widths; quasiparticles? ($\mathbf{m_Q \gg T}$)

- Probe of hadronization
- Non-perturbative effects until **~ 2T$_{pc}$**
- Potential-type interactions ($\mathbf{q_0 \sim q^2/m_Q \ll q}$)
  → same as for heavy quarkonia!?
- Ample connections to lattice QCD



[He et al '12]

Outline

1.) Introduction

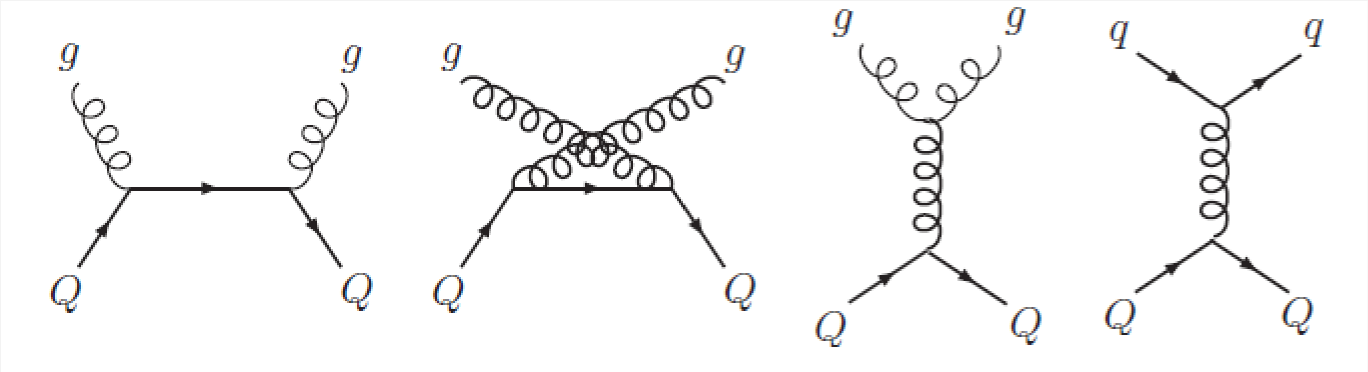
2.) Heavy-Flavor Interactions in QCD Matter

3.) Heavy-Flavor Transport

4.) Phenomenology

5.) Conclusions

2.1 Leading-Order Perturbative QCD



- gluon exchange regularized by Debye mass:

$$G(t) = \frac{1}{t} \to \frac{1}{t-\mu_D^2}\ , \qquad \mu_D = gT$$

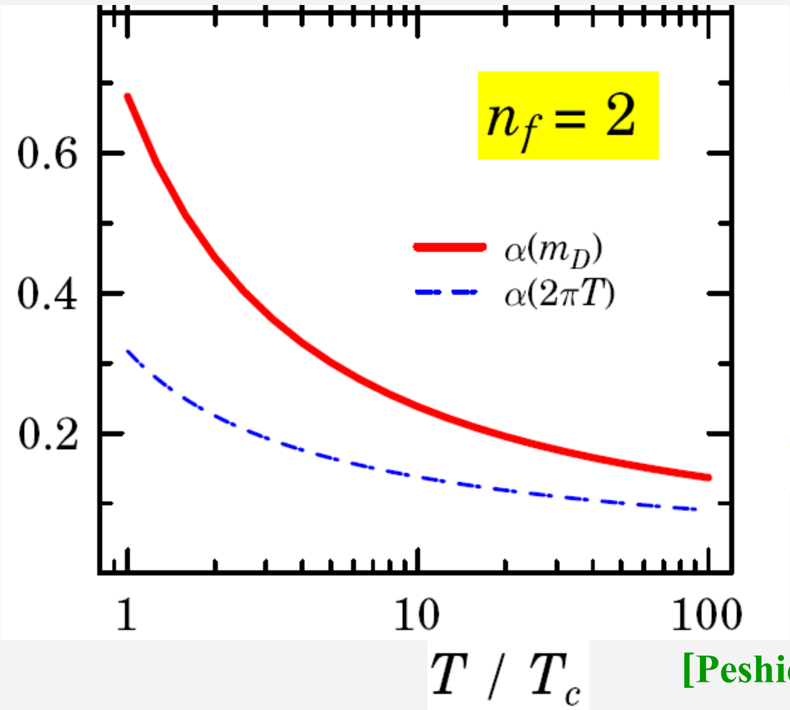
[Svetitsky '88, Mustafa et al '98, Molnar et al '04, Zhang et al '04, Hees+RR '04, Teaney+Moore'04]

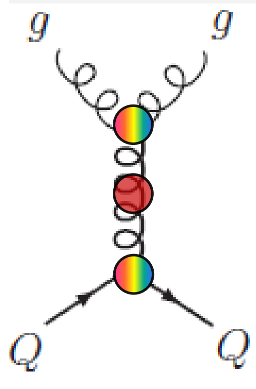
- dominated by forward scattering
- long thermalization time $\gamma^{-1} = \tau_c \geq 20$ fm/c (T$\leq$ 300MeV, α_s=0.4)

2.2 Perturbative QCD with Running Coupling

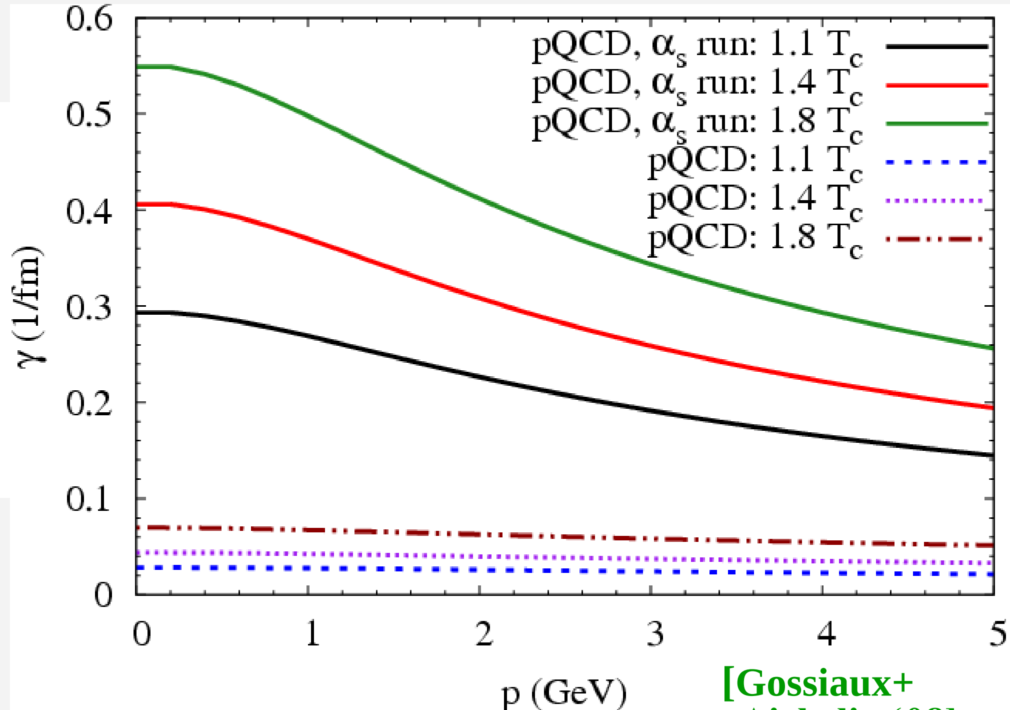
- run α_s to $m_D \sim gT$, rather than $2\pi T$
- reduced Debye mass $\tilde{\mu}^2 = \frac{1}{5}\mu_D^2$

$$G(t) = \frac{\alpha}{t} \rightarrow \frac{\alpha_{\rm eff}(t)}{t - \tilde{\mu}^2}$$





[Peshier '07]



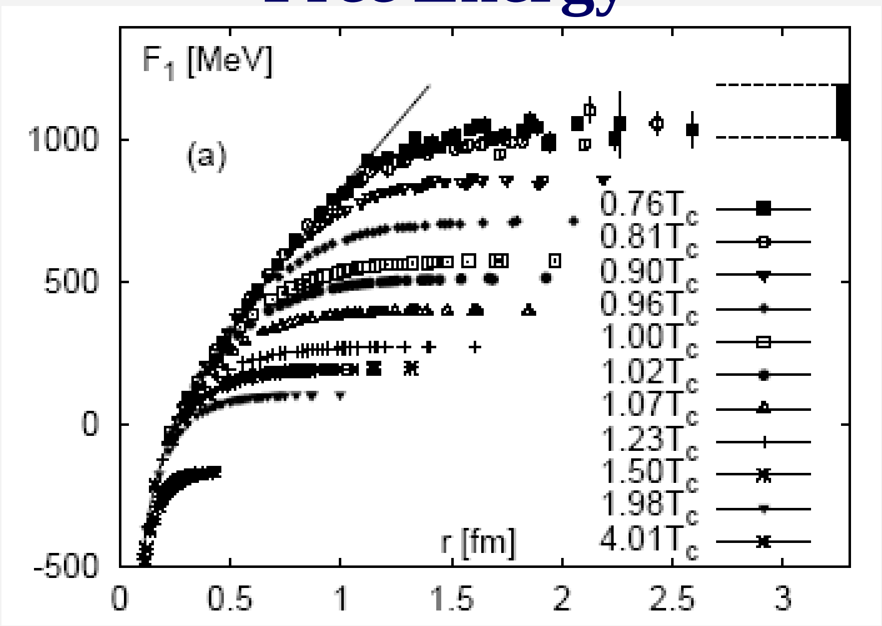
[Gossiaux+ Aichelin '08]

- factor ~10 faster thermalization: $\tau_c \approx$ 2-3 fm/c
- perturbative regime? Need to resum large diagrams…

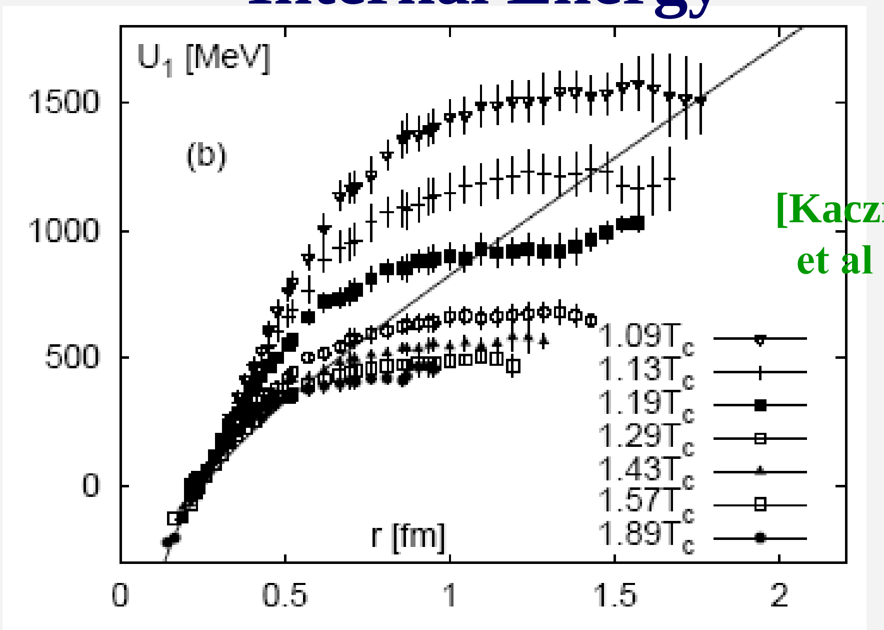
2.3 Heavy-Quark Free and Internal Energies in Lattice QCD

$$F_1(r,T) = U_1(r,T) - T\,S_1(r,T)$$

Free Energy



Internal Energy



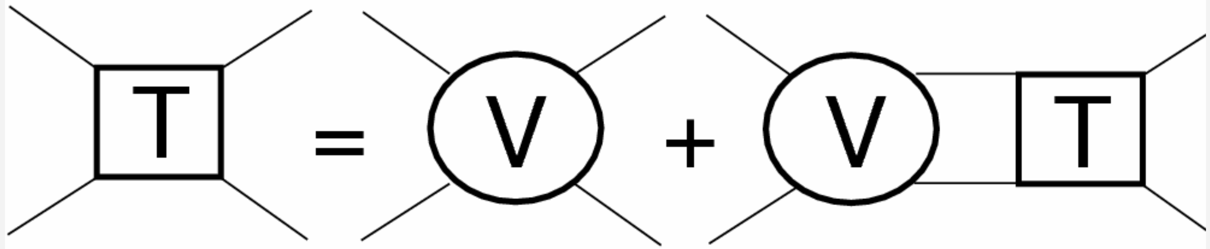
[Kaczmarek et al '05]

- "**weak**" **$Q\bar{Q}$** potential
- "**strong**" **$Q\bar{Q}$** potential, **$U = \langle H_{int} \rangle$**
- **F, U, S thermodynamic quantities**
- **Entropy**: many-body effects

2.4 Thermodynamic T-Matrix in QGP

- **Lippmann-Schwinger equation**

In-Medium
Q-Q̄ T-Matrix:

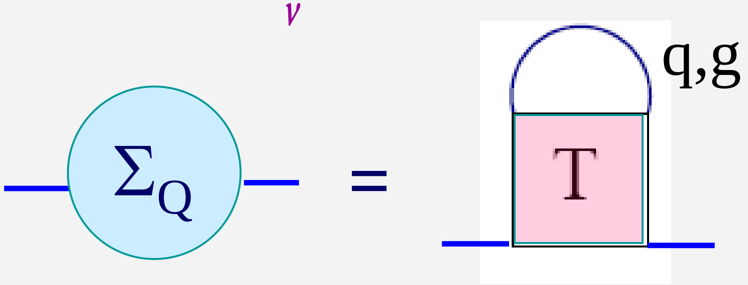


$$T_\alpha(E;q,q') = V_\alpha(q,q') + \int k^2 dk \; V_\alpha(q,k) \; G^0_{Q\bar{Q}}(E,k) \; T_\alpha(E;k,q')$$

- thermal 2-particle propagator: $G^0_{Q\bar{Q}}(E,k;T) = T\sum_\nu D_Q(z_\nu,\vec{k})D_{\bar{Q}}(E-z_\nu,-\vec{k})$

- selfenergy:

$$\Sigma_Q(\omega,k) = \sum_{p=q,g} \int T_{Qp}(\omega+\omega_p)\, f^p(\omega_p)$$



- **In-medium potential V?**

[Cabrera+RR '06,
Riek+RR '10]

2.4.2 Free Energy from T-Matrix

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\,\sigma(\omega, r_1 - r_2)\,e^{-\beta\omega}\right)$

[Beraudo et al ’08]

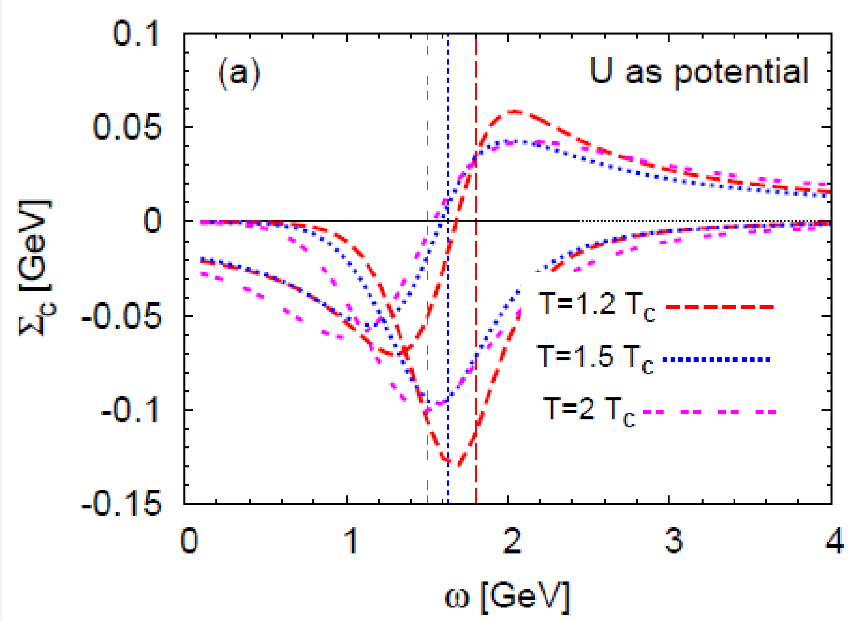
- Euclidean **T-matrix** in static limit

$$\tilde{T}(z_t|r) = V(z_t, r) + V(z_t, r)\,\tilde{G}_0^{(2)}(z_t - v^a, v^a)\,\tilde{T}(z_t|r) = \frac{V(z_t, r)}{1 - V(z_t, r)\,\tilde{G}_0^{(2)}(z_t)}$$

- **Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

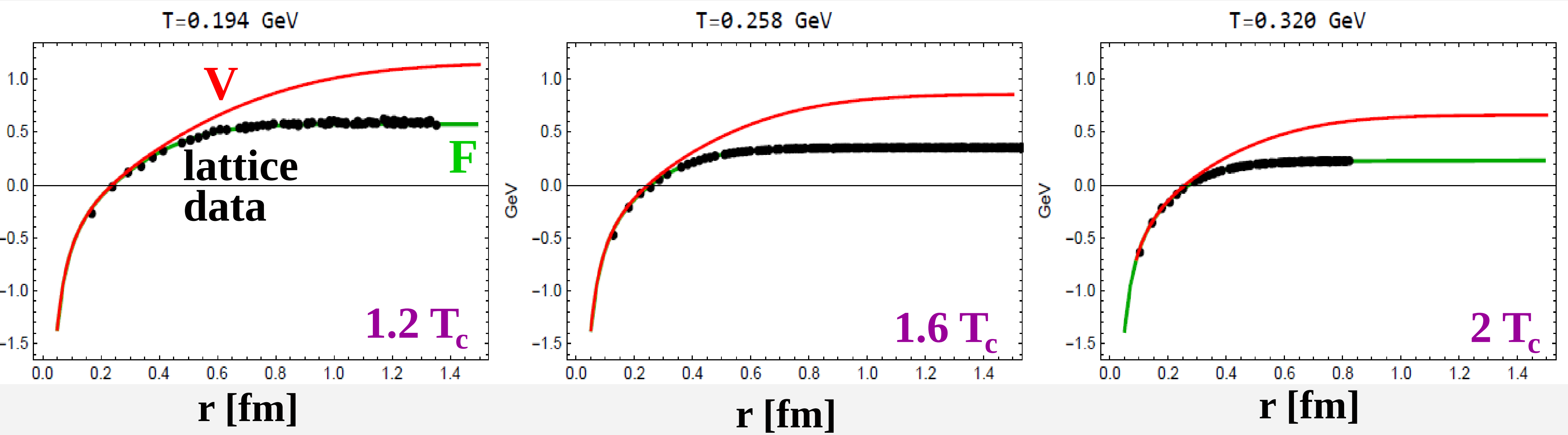
[S.Liu+RR ’15]

- Key ingredients:
  imaginary parts + their ω dependence
- heavy-quark self-energies calculated self-consistently from **Qq T-matrix**



2.4.3. Free Enegy + Potential from T-Matrix

• potential ansatz: $V_R(E,r) = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} - \sigma \frac{e^{-m_s r}}{m_s} - \frac{4}{3}\alpha_s m_D + \sigma \frac{1}{m_s}$

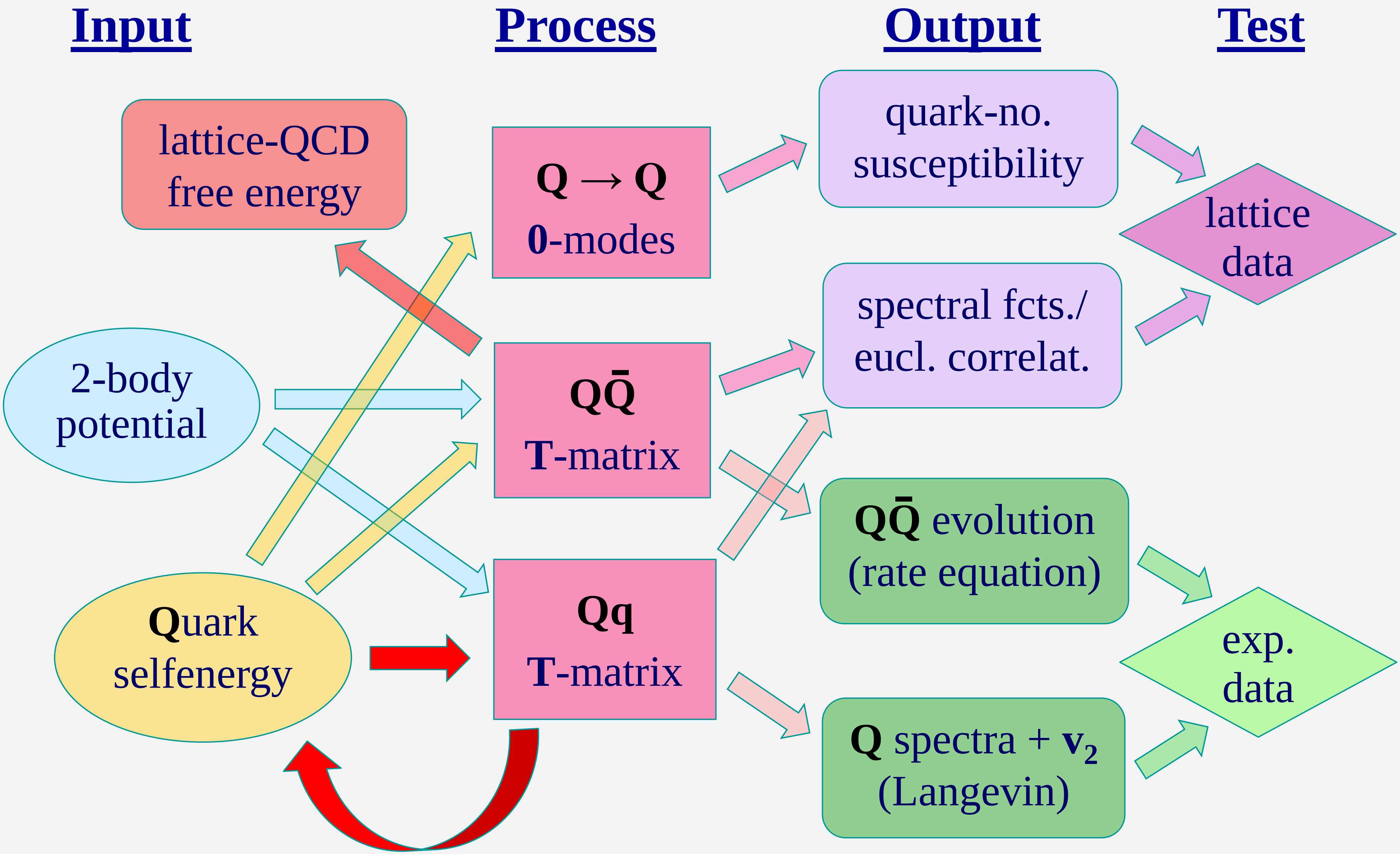


• **remnant of long-range "confining" force in QGP**

• smaller in-medium quark mass relative to internal energy

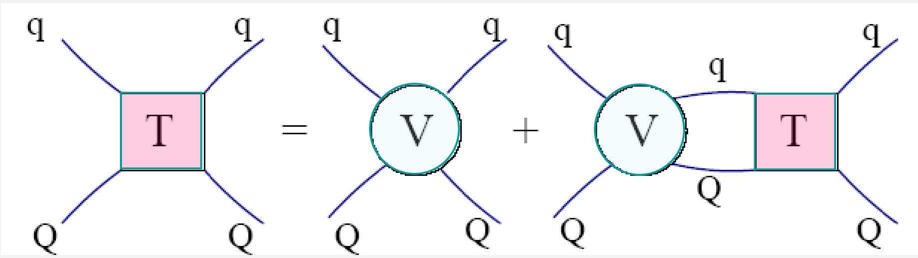
[S.Liu+RR '15]

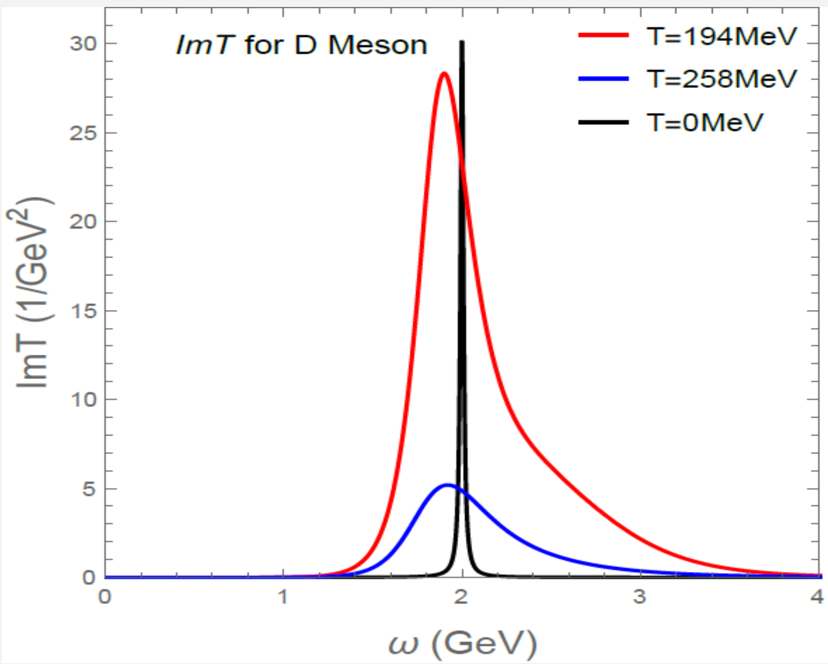
2.5 Brueckner Theory of Heavy Flavor in QGP



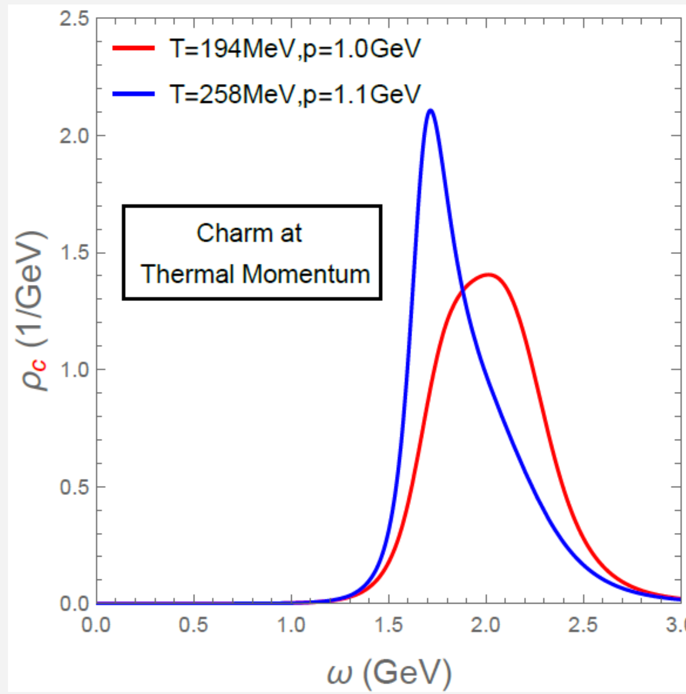
2.6 D-Meson + c-Quark Spectral Functions in QGP

T-matrix w/ "lattice potential" V





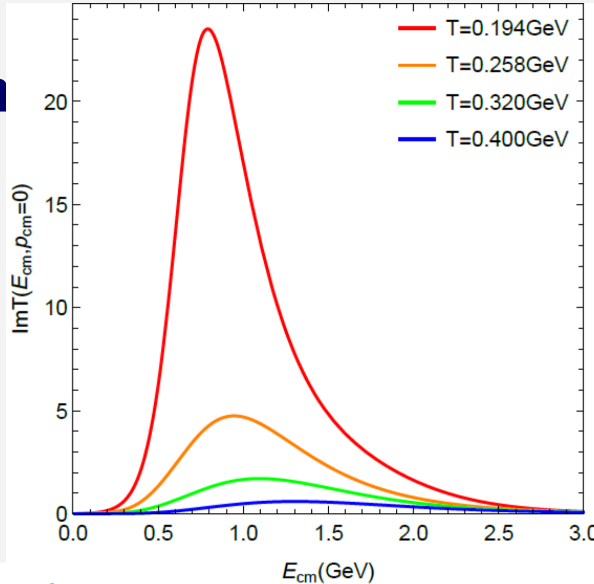
In-Medium c-Quark Selfenergy



D-meson resonances near $\mathbf{T_{pc}}$ ⟶ **c**-quark quasi-particles at high **T**

2.7 Selfconsistent Equation of State for QGP

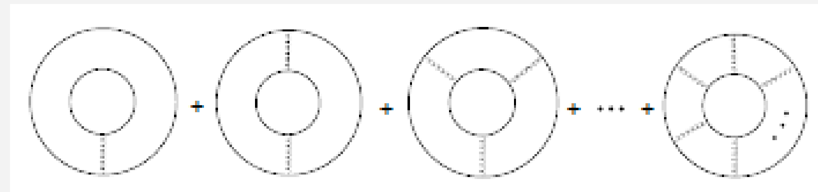
Thermodynan Potential

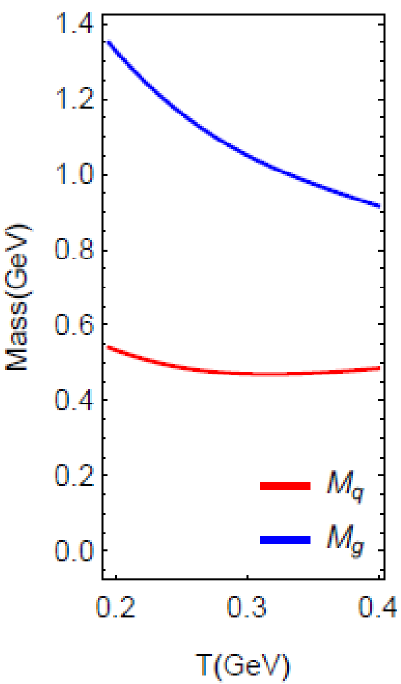


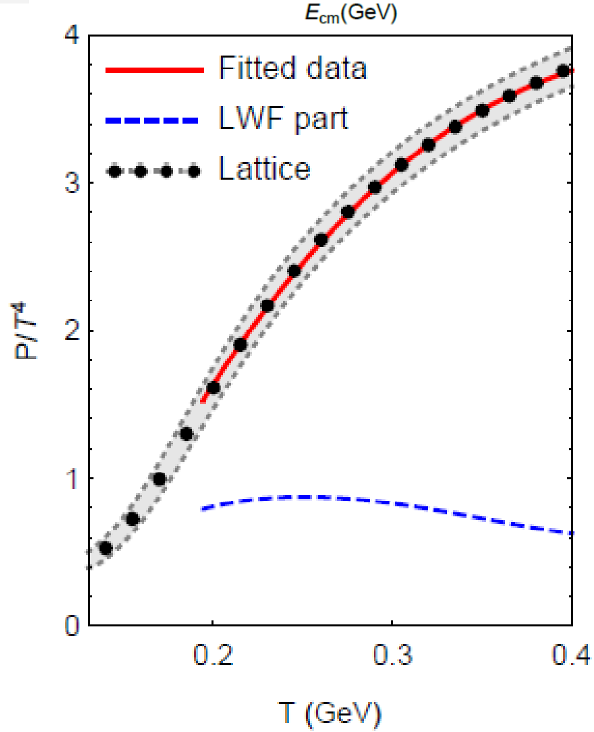
$$\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

$$\frac{1}{\nu}\left(\frac{-1}{\beta}\right)^{\nu}[(-\beta)^{\nu}\Sigma_{\nu}(G)]G\}$$

Skeleton Diagrams





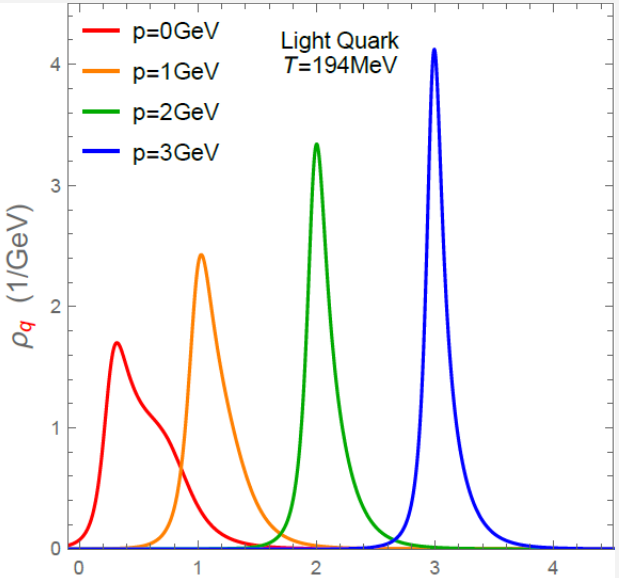


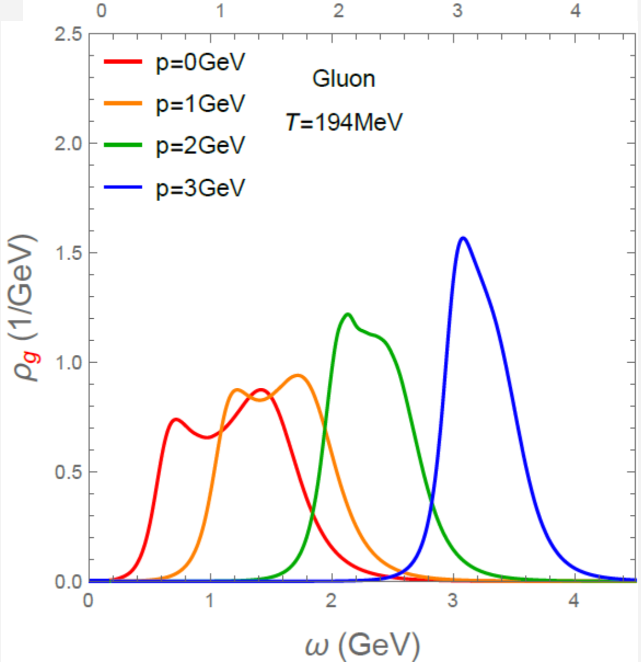
- **Fully resummed Luttinger-Ward functional (!)**
- Fit quark+ gluon "bare" masses
- **Broad hadronic resonances emerge near T_{pc}**

[S.Liu+RR '16]

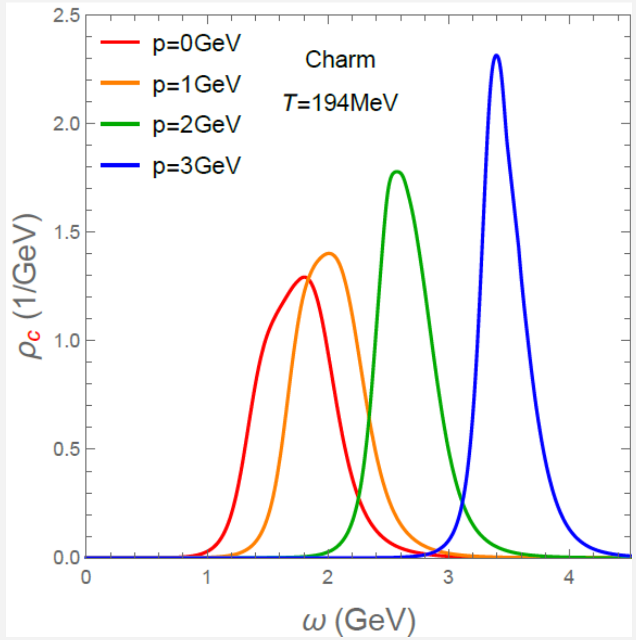
2.8 Parton Spectral Functions in QGP

Up/Down + Gluon

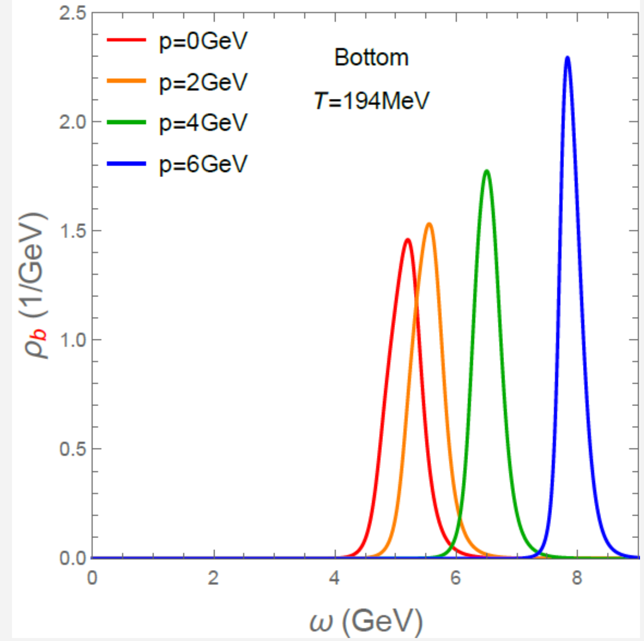




Charm



Bottom



- QGP structure changes with resolution scale
- Quasi-particles re-emerge when increasing M_q, p, T

2.9 D-Mesons in Hadronic Matter

Hadrons	$L_{I,2J}$	γ_D [fm^{-1}]
π	$S_{1/2,0}$, $P_{1/2,2}$, $D_{1/2,4}$, $S_{3/2,0}$	0.0371
$K+\eta$	$S_{0,0}$, $S_{1,0}$	0.0236
$\rho+\omega+K^*$	$S_{1/2,2}$, $S_{0,2}$, $S_{1,2}$	0.0129
$N+\bar{N}$	$S_{0,1}$, $S_{1,1}$	0.0128
$\Delta+\bar{\Delta}$	$S_{1,3}$	0.0144

- effective ***D-h*** scattering amplitudes

  [He,Fries+RR '11, Tolos+Torres-Ricon '13]

- **D**-meson in pion gas:
  - consistent with unitarized HQET **[Cabrera et al '11]**
  - factor **~10** larger in heavy-meson χPT **[Laine '11]**

- hadron gas at **~T_c**: **$\tau_D \approx$ 10fm/c**

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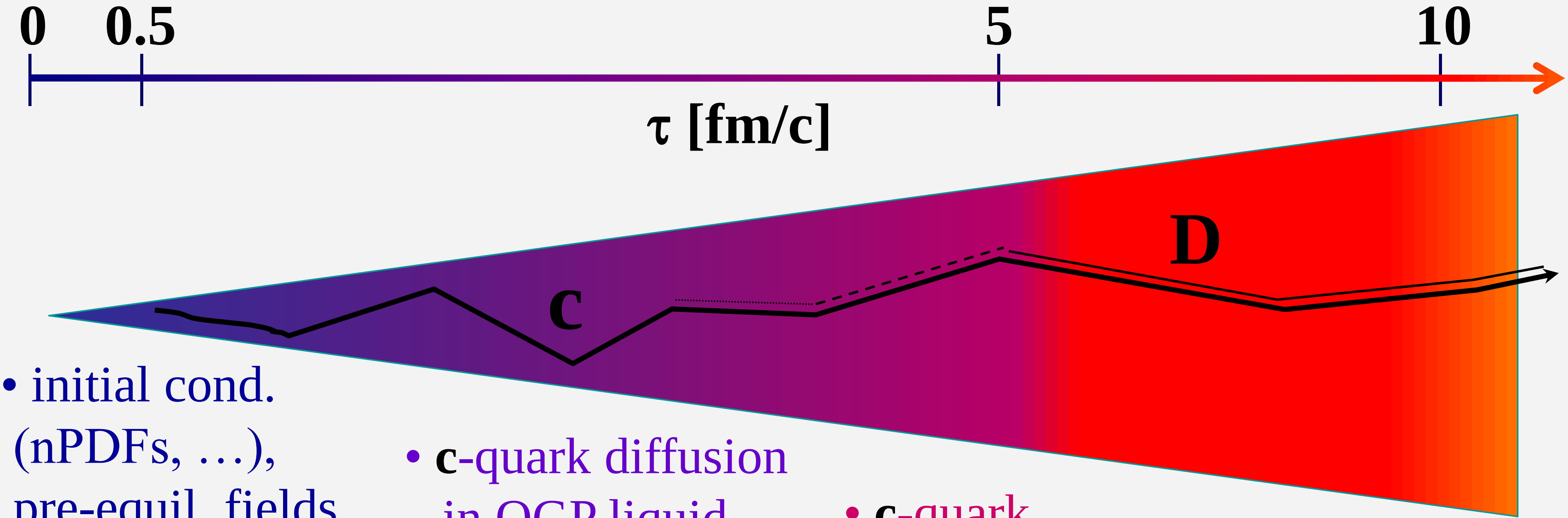
3.1 Heavy-Flavor Transport in URHICs



- initial cond. (nPDFs, …), pre-equil. fields
- **c**-quark diffusion in QGP liquid
- **c**-quark hadronization
- **D**-meson diffusion in hadron liquid

- no “discontinuities” in interaction

⇒ **diffusion toward** T_{pc} and **hadronization** same interaction (**confining**!)

[Moore+Teaney ‘05, van Hees et al ‘05, Gossiaux et al ‘08, Vitev et al ‘08, Das et al ‘09, Uphoff et al ’10, M.He et al ‘11, Beraudo et al ‘11, Cao et al ’13, Bratkovskaya et al ‘14, …]

3.2 Transport Approaches

- **Boltzmann equation for HQ phase-space distribution** f_Q

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{\omega_{\boldsymbol{p}}}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_{\mathrm{Q}}(t,\boldsymbol{x},\boldsymbol{p}) = C[f_{\mathrm{Q}}]$$

- explicit simulation of medium (quasi-) particles in collision term
- semi-classical approximation

- **Fokker-Planck equation**

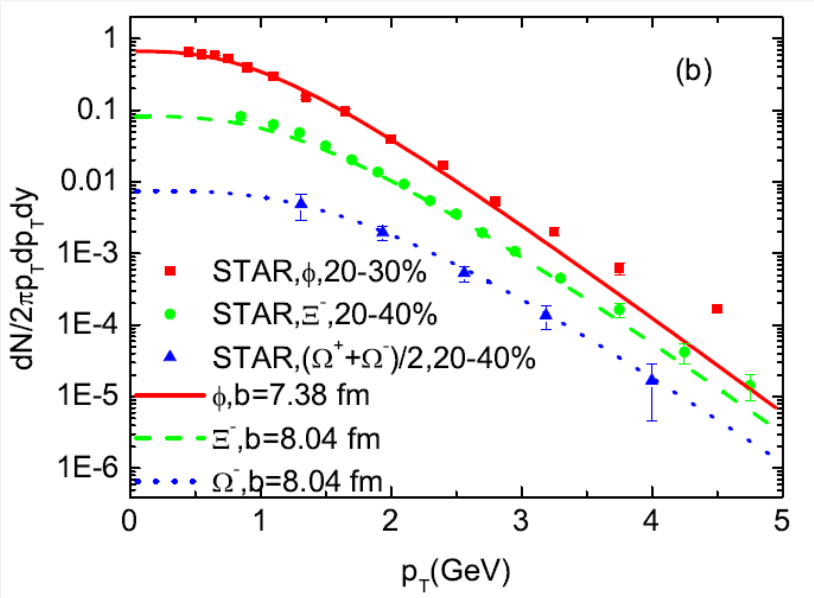
$$\frac{\partial}{\partial t} f_Q(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(t,\boldsymbol{p})]\right\}$$

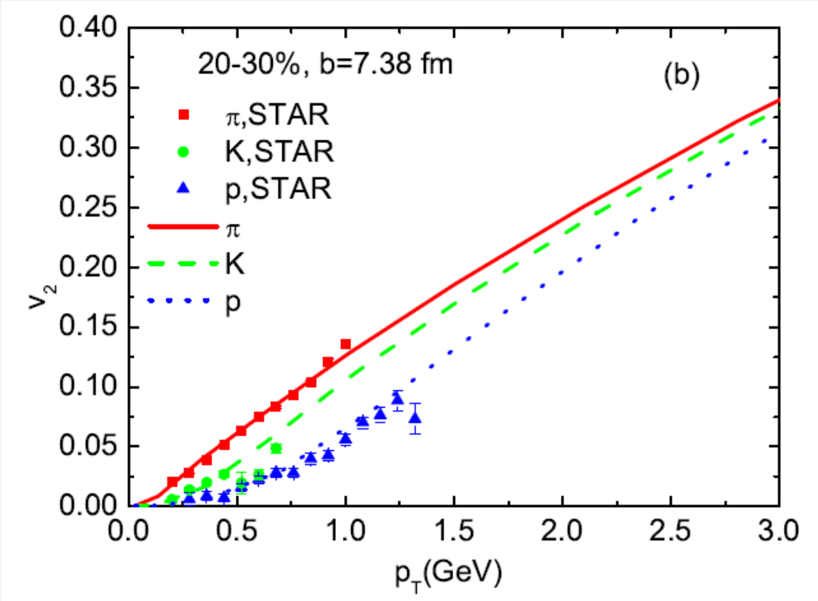
- follows from Boltzmann with $q^2 \ll p^2$ ($T^2 \ll 2m_Q T$); ok for $m_Q/T \geq 5$
- does not require quasi-particle medium
- well suited for strongly coupled medium where $E_{th} \leq \Gamma_{q,Q} < m_Q$

3.3 Quantitative Bulk-Medium Evolution

- initial conditions (compact, initial flow?)
- EoS: lattice (QGP, T_c~170MeV) + chemically frozen hadronic phase
- spectra + elliptic flow: multistrange at T_{ch} ~ 160MeV [He et al '11]

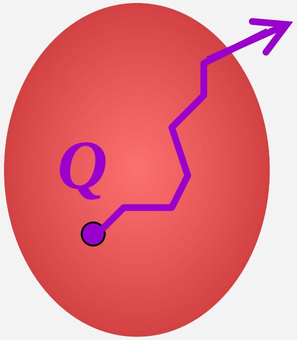
  π, K, p, Λ, … at T_{fo} ~ 110MeV





- v_2 saturates at T_{ch}, good light-/strange-hadron phenomenology

3.4 Heavy-Quark Transport Coefficient



- $p^2 \sim m_Q T >> k^2 \sim T^2 \Rightarrow$ Brownian Motion:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial (pf)}{\partial p} + D_p \frac{\partial^2 f}{\partial p^2}$$

Fokker Planck Eq.

thermalization rate

$$\gamma p = \int d^3k \, w_Q(k,p) \, k$$
$$\sim \int |T_{Qq}|^2 (1 - \cos\theta) \, f^{q,g}$$

diffusion coefficient

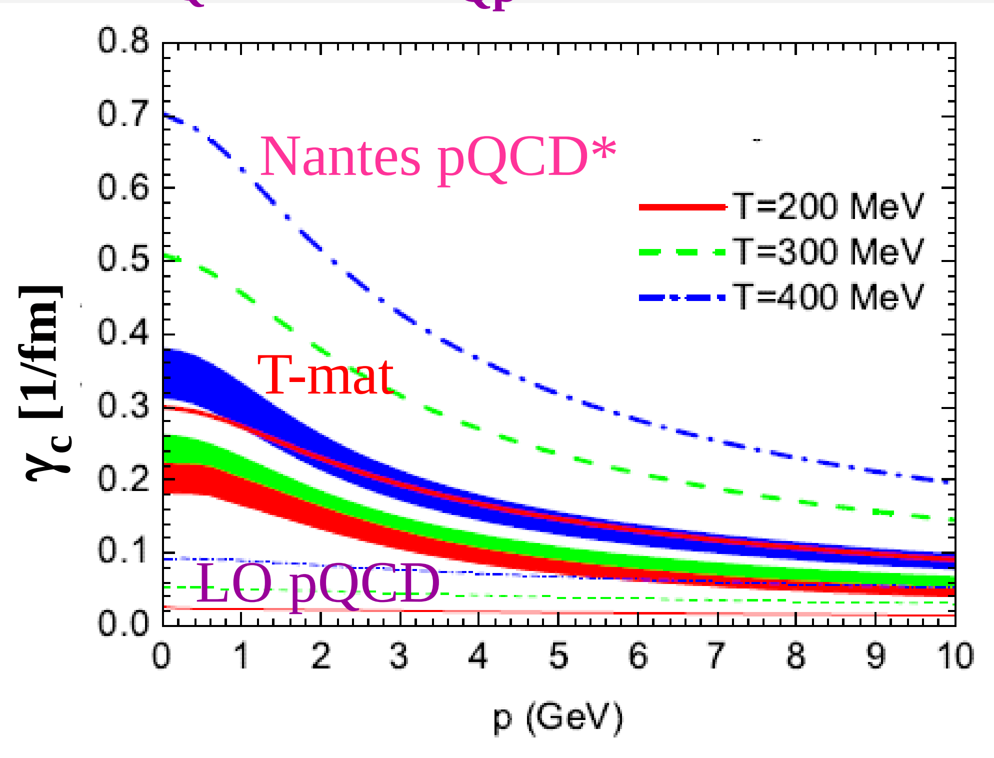
$$D_p = \frac{1}{2} \int d^3k \, w_Q(k,p) \, k^2$$

- thermal relaxation time $\tau_Q = 1/\gamma$
- Einstein relation: $T = D_p / \gamma m_Q$ $\rightarrow$ check FP approximation
- spatial diffusion constant: $D_s = T/\gamma m_Q$
- relation to bulk medium: $D_s (2\pi T) \sim \eta/s$

3.5 Charm Transport

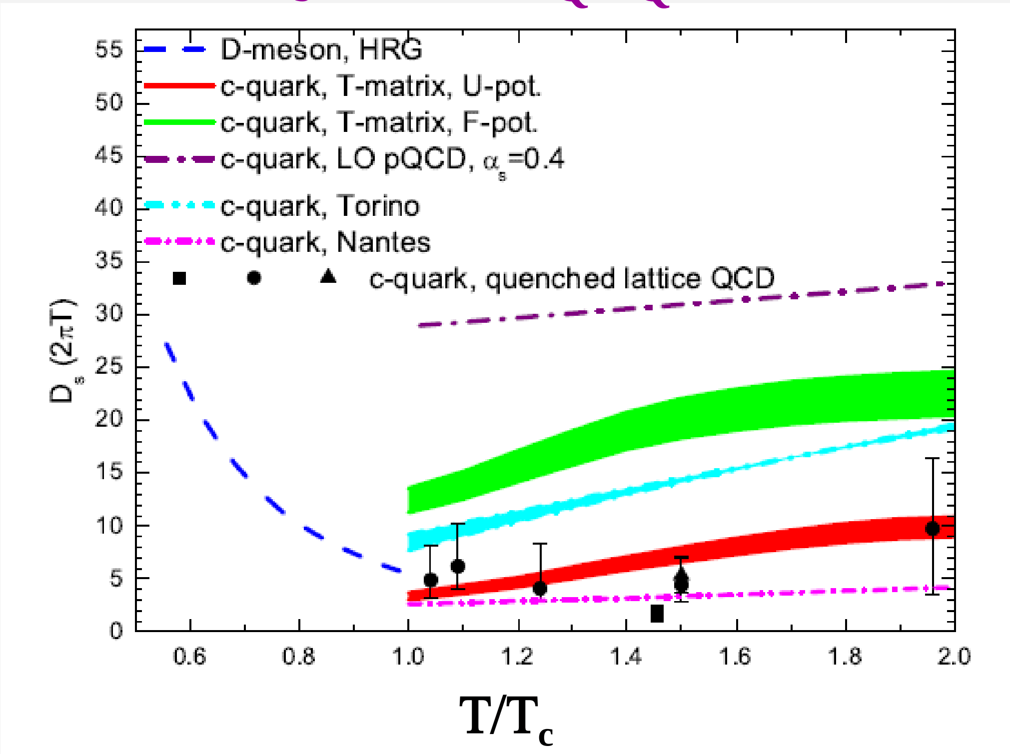
Thermalization Rate

$\gamma_Q\, p = \int |\mathcal{M}_{Qp}|^2\, (1\text{-}\cos\Theta)\, f^p$



Spatial Diffusion Coefficient

$\mathcal{D}_s = T / (m_Q\, \gamma_Q(p=0))$



- **p**- and **T**-dependence reflect core properties of QCD
- suggests minimum of $\mathcal{D}_s(2\pi T) \sim$ **2-4** near T_{pc}
- width: $\Gamma_{coll} \sim 3/\mathcal{D}_s \sim$ **0.5-1 GeV** – **no light quasi-particles!**

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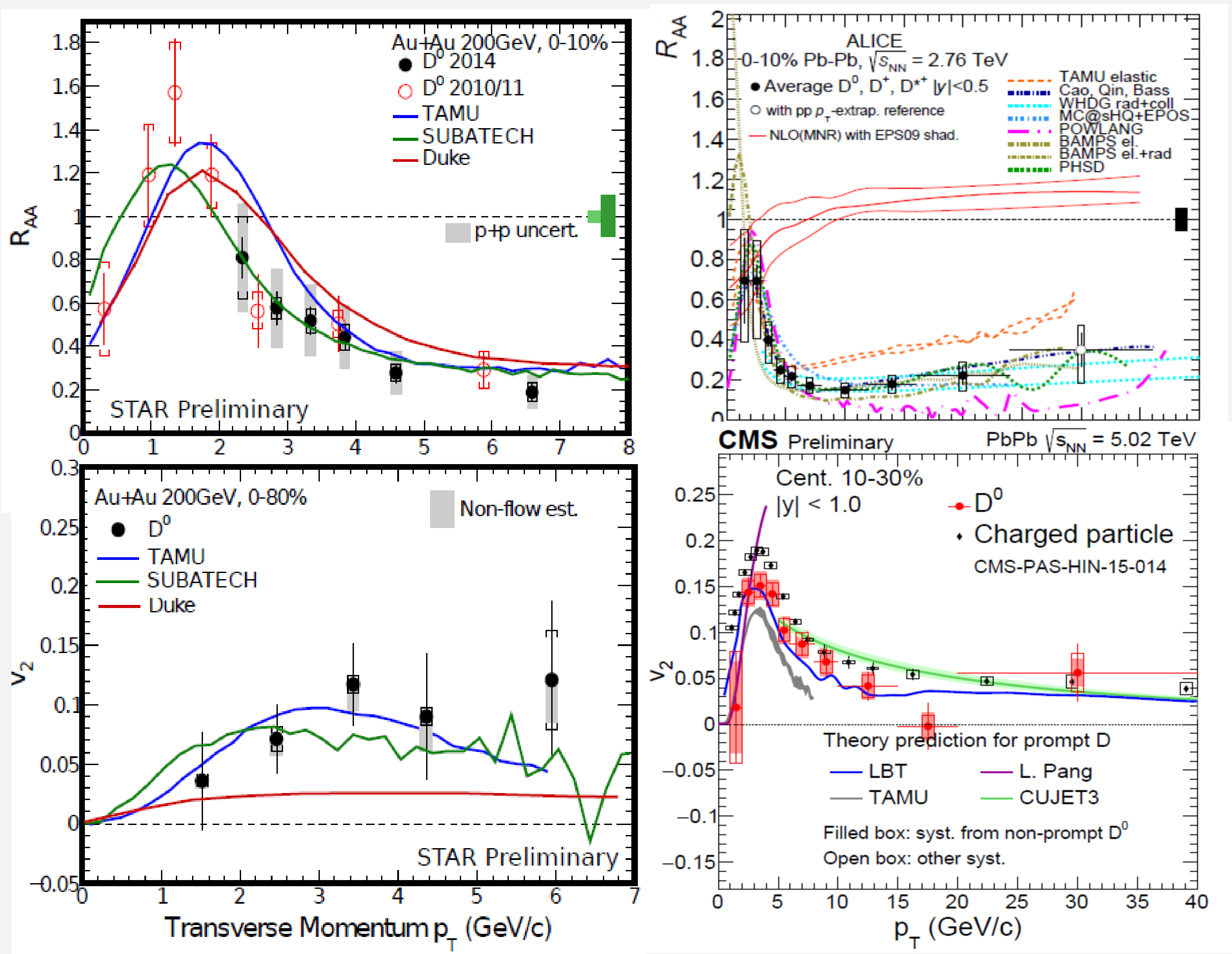
4.1 Heavy-Flavor Transport at RHIC + LHC



- flow bump in $\mathbf{R_{AA}}$ + large $\mathbf{v_2}$ ↔ strong coupling near $\mathbf{T_{pc}}$ (**recombination**)
- high-precision $\mathbf{v_2}$: transition from elastic to radiative regime?

5.) Summary

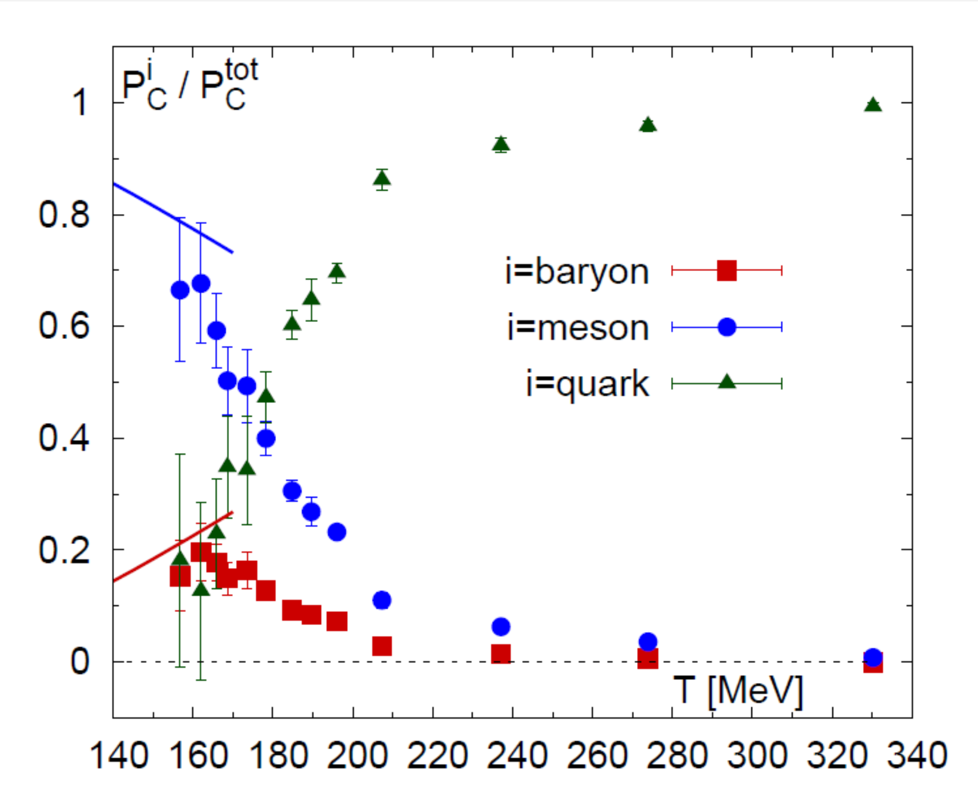
- Extract heavy-quark potential in QGP from lat-QCD F_{QQ} utilizing microscopic many-body approach
- Remnants of long-range confining force survive well above T_{pc}
  → large scattering rates, strong-coupling diffusion (small $\mathcal{D}_s$)
  → extended transition from heavy-quark to -hadron dofs
- Self-consistent QGP equation of state (resummed LWF!)
  → no long-wavelength quasi-partons near T_{pc} (re-emerge at high **p**,**T**,m_Q)
  → broad hadronic bound states emerge near T_{pc}
  → reflected in strong HF coupling to medium
- Heavy-flavor phenomenology at **RHIC** + **LHC**
  - drag force imprinted on **D**-meson R_{AA} + v_2, transition to radiation
  - test hadronization + hadronic phase with Λ_c and D_s

2.5 Charm Susceptibilities from Lattice QCD

- Extract partial charm pressures from susceptibilities

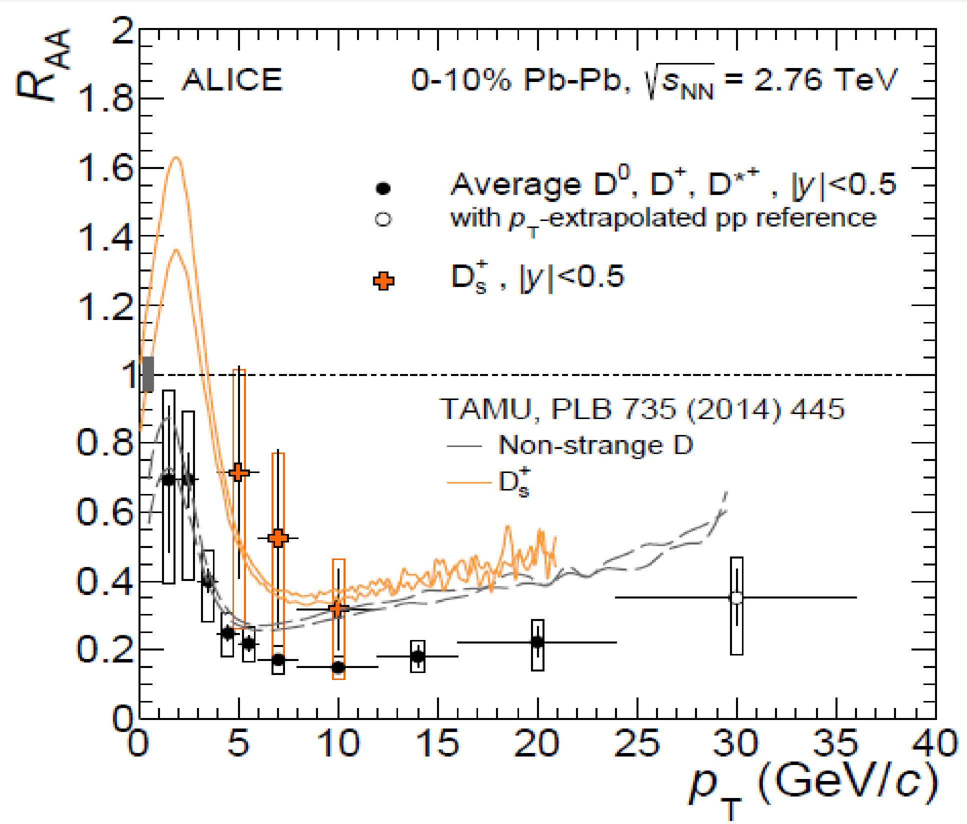
[Mukherjee,Petreczky +Sharma '15]

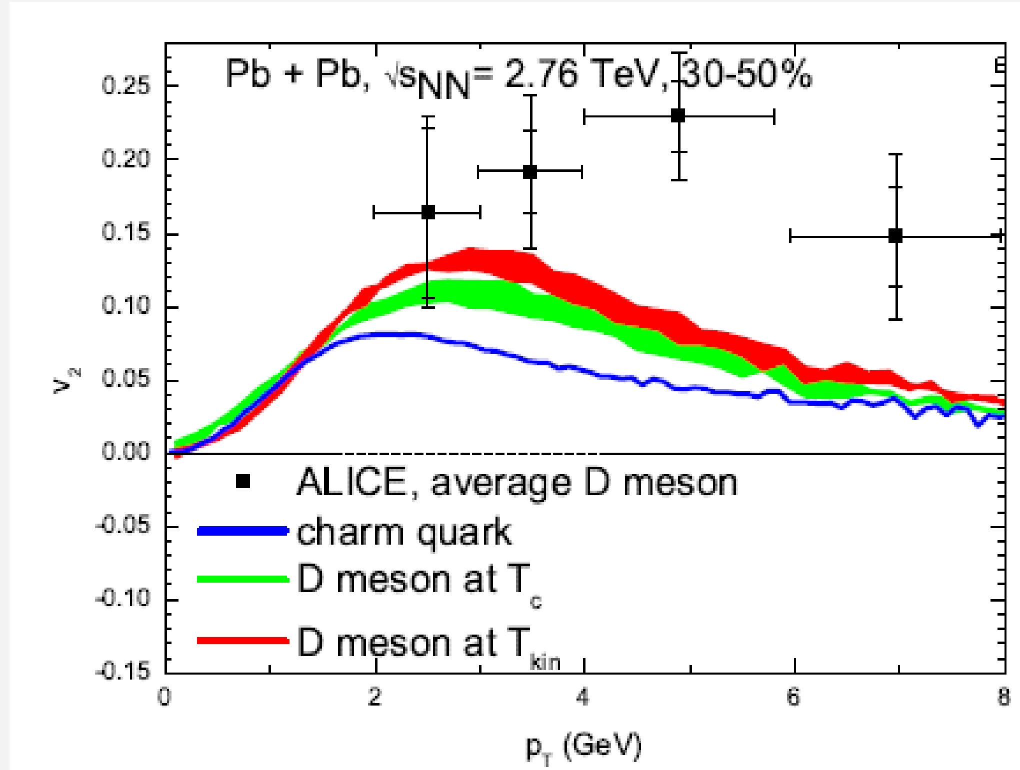
$$p_c(T,\mu_B,\mu_c) = p_M(T)\cosh\left(\frac{\mu_c}{T}\right) + p_B(T)\cosh\left(\frac{\mu_c+\mu_B}{T}\right) + p_Q(T)\cosh\left(\frac{\mu_c+\mu_B/3}{T}\right)$$



- Hadronic (**D**-meson) correlations contribute until **~1.4 T_c**
- Realistic models of heavy-quark diffusion should account for this (induces HQ-v_2 + hadronization)

4.2 Charm Transport at LHC: D-Meson Spectra





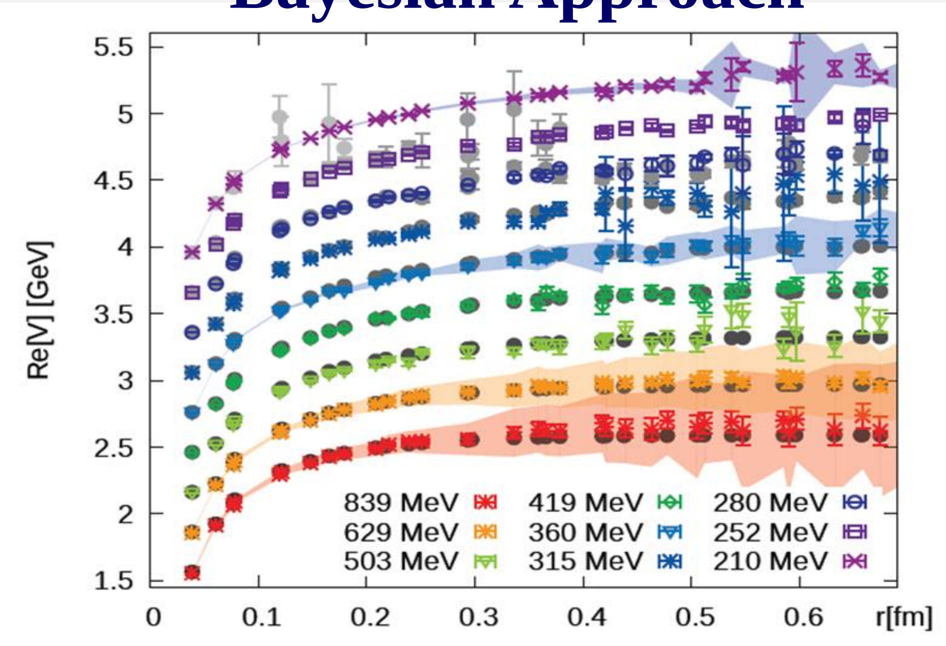
[M.He et al '14]

- **R_{AA}** "bump" from radial flow
- **D_s** meson (**cs**) enhanced from coalescence with strange quarks
- Coalescence + hadronic diffusion increase **v_2**
- similar features at **RHIC**

2.4 Potential Extraction from Lattice Data

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$
- **$Q\bar{Q}$ Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

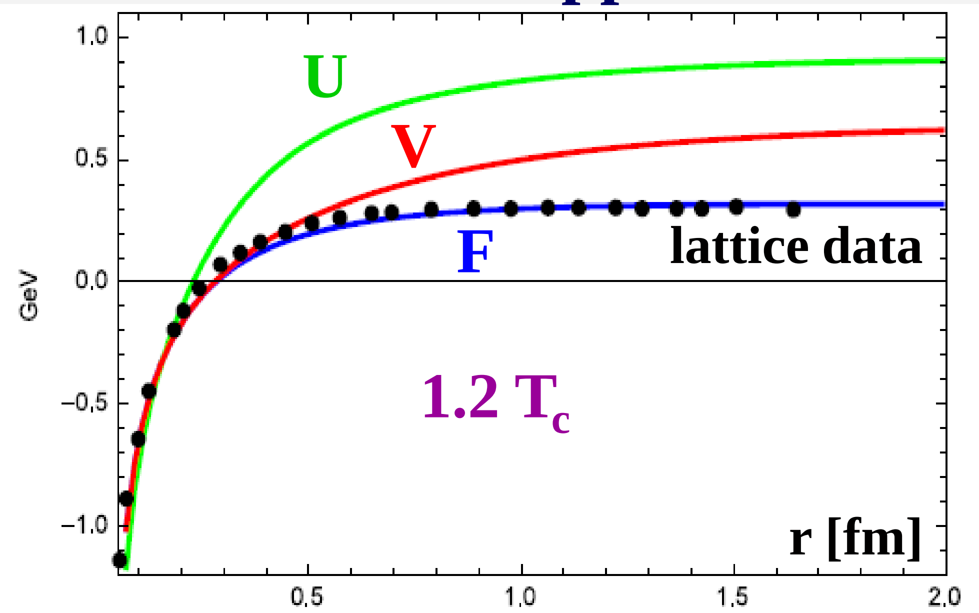
Bayesian Approach



- **Potential close to free energy**

[Burnier et al ’14]

T-Matrix Approach



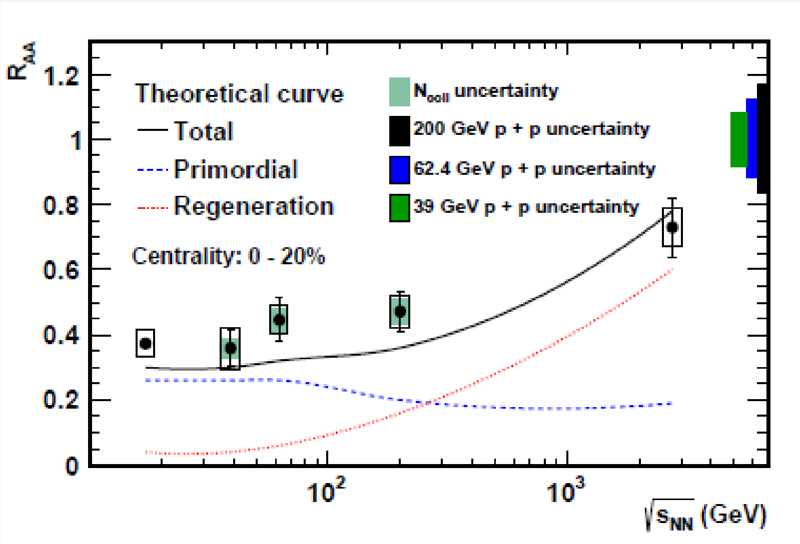
- Account for large imaginary parts
- **Remnant of confining force!**

[S.Liu+RR ’15]

4.1 Upshot of Quarkonium Phenomenology

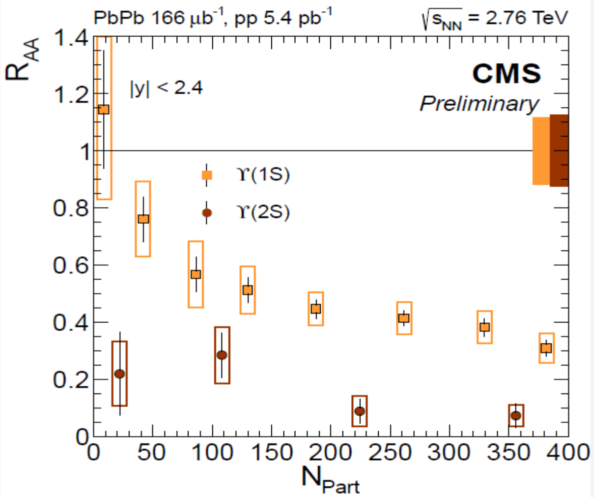
A. Charmonia (J/ψ)

- **SPS**: large cold-nucl.-abs., $R_{AA}^{hot} \sim 0.8$
- **RHIC**: $R_{AA}^{hot} \sim 0.6$
- **LHC**: $R_{AA}^{hot} \sim 0.7$,
  low-p_T excess, sizable v_2



B. Bottomonia

- **RHIC**: $R_{AA}[\Upsilon(1S)] \sim 0.7$
- **LHC**: $R_{AA}[\Upsilon(1S)]{\sim}0.4$, $R_{AA}[\Upsilon(2S)]{\sim}0.1$

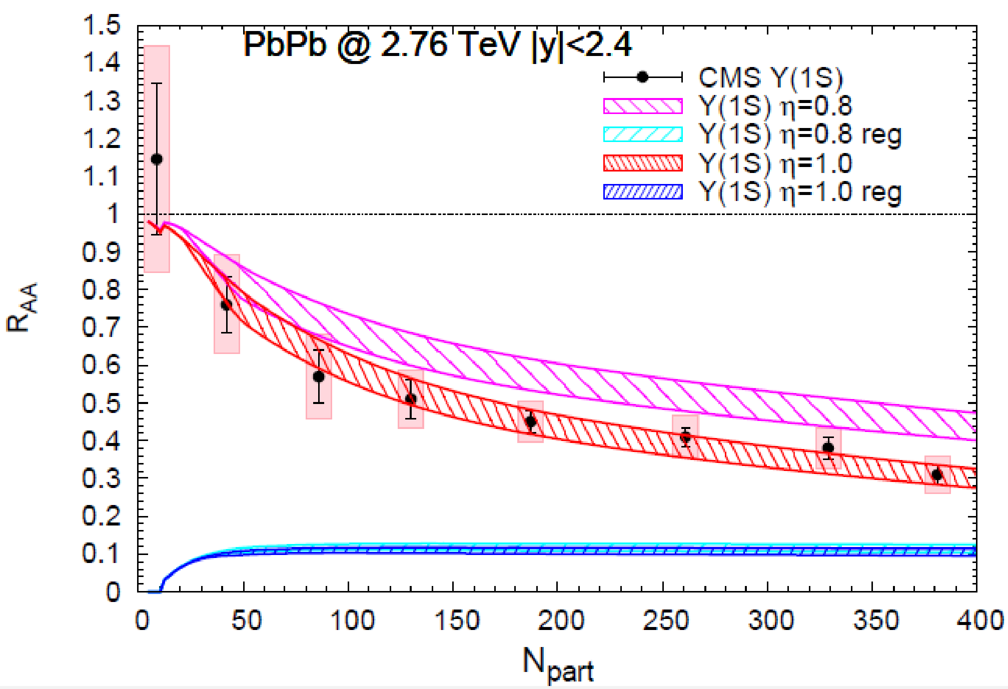


C. Implications

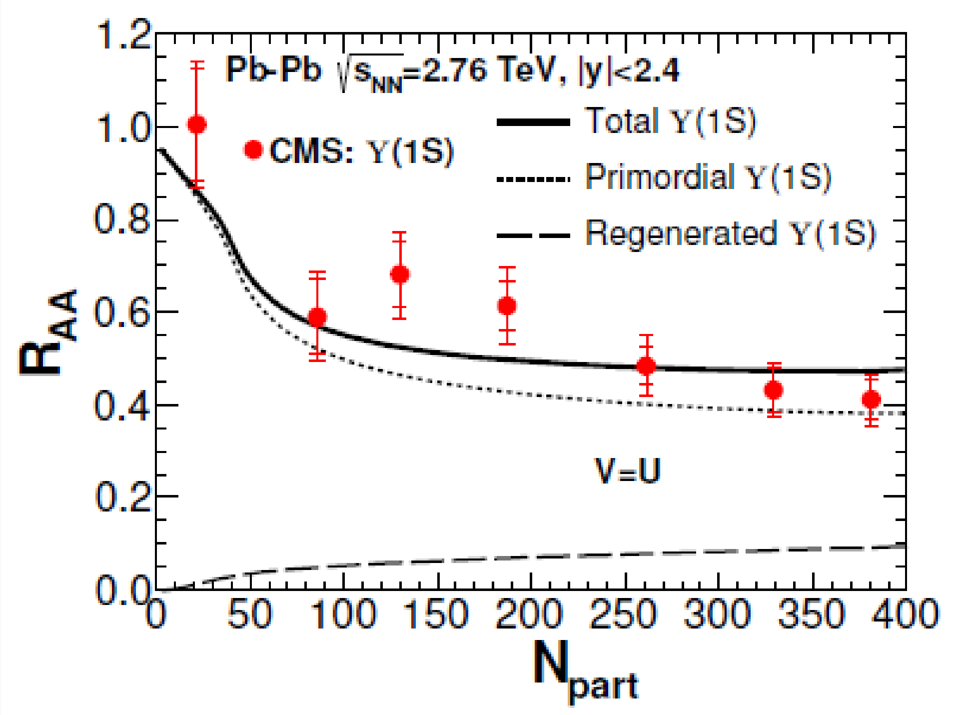
- T_0^{SPS} (~230) < T_{diss}(J/ψ,Υ') < T_0^{RHIC} (~350) < T_0^{LHC}(~550) ≤ T_{diss}(Υ)
- **confining force screened at RHIC+LHC**
- thermalizing(!) charm quarks recombine at **LHC**

4.2 ϒ(1S): Screening and Regeneration?

… as implemented in current transport approaches



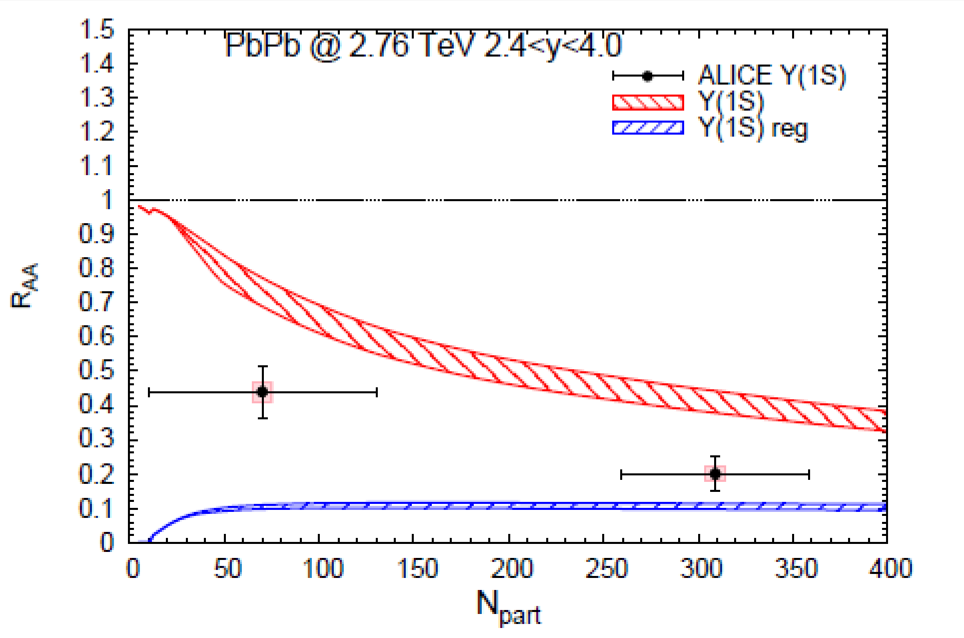
[TAMU]

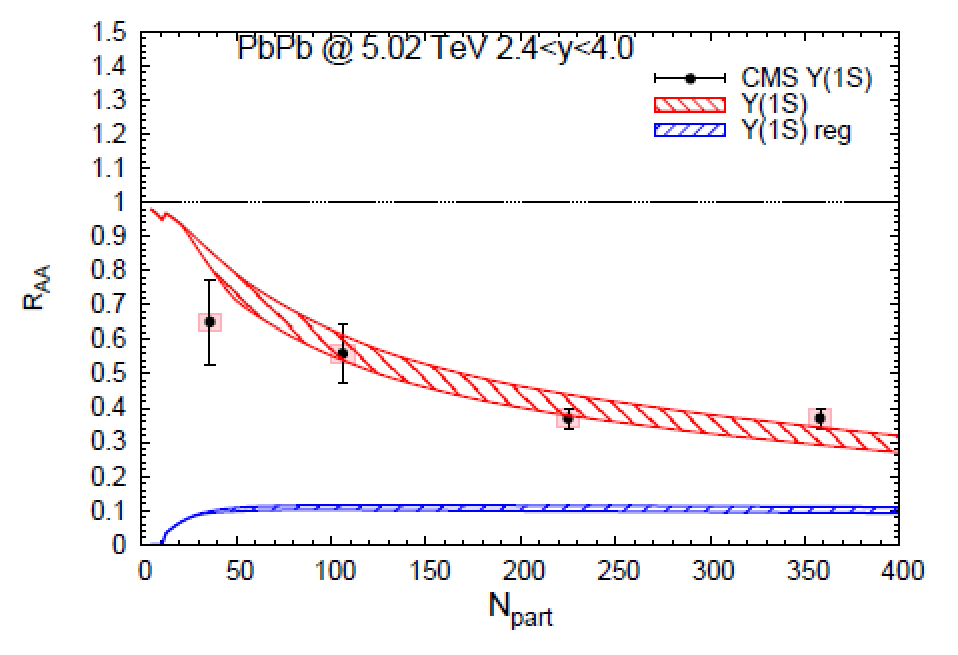


[Tsinghua]

- sensitive to color-screening; prefer strong binding (**U**-pot)
- role of regeneration for **ϒ(1S)**? (larger for **ϒ(2S)**)

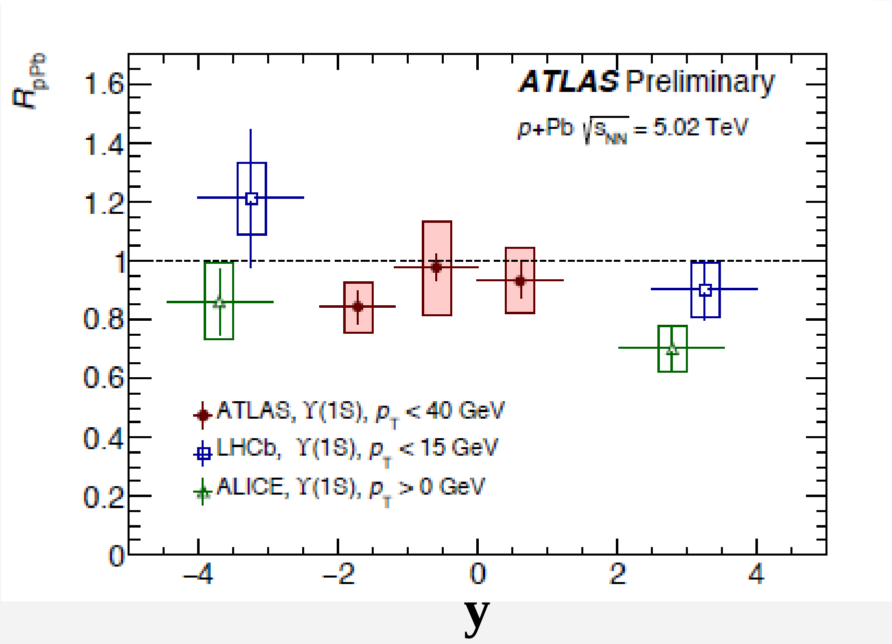
4.3 ϒ(1S): Rapidity Puzzle



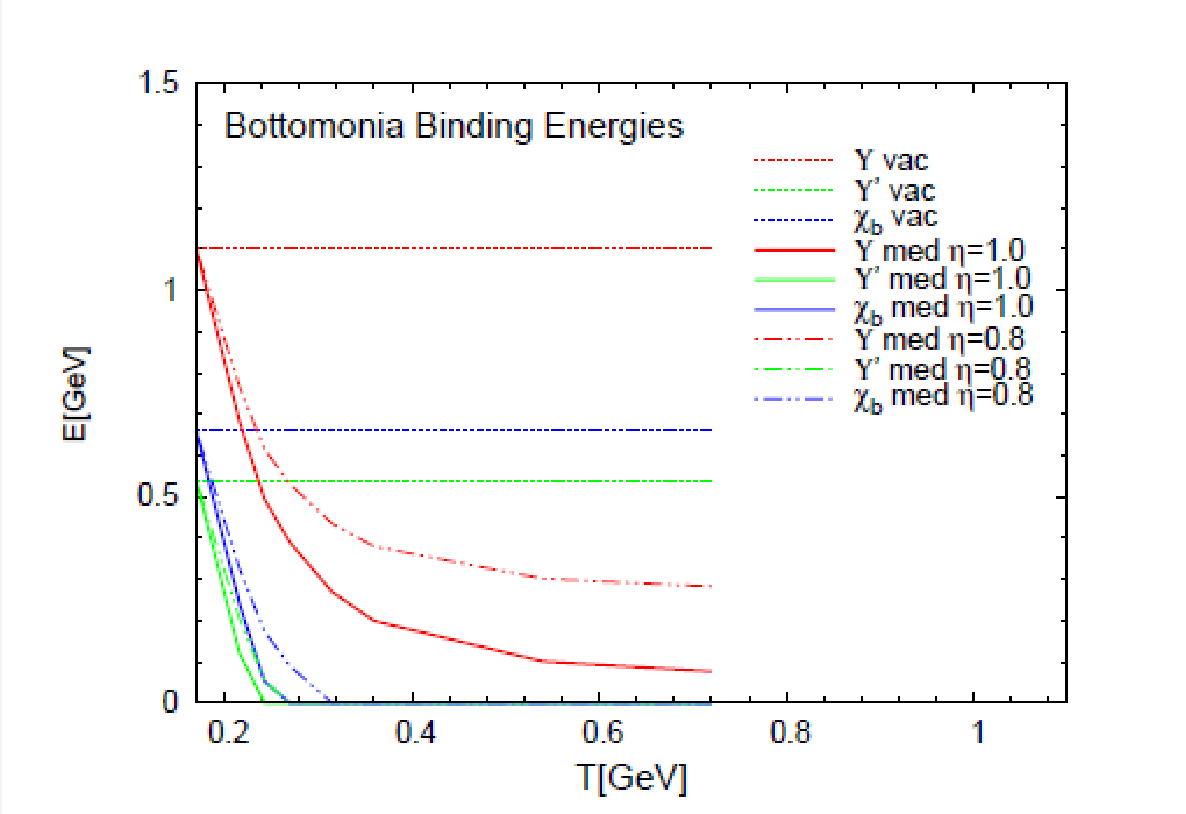


- problem of large(r) suppression in 2.76 TeV **ALICE** data
- beware of cold nuclear matter effects
- Regeneration: $\mathbf{N_{bb} \sim 1}$ for central PbPb

  $\Rightarrow$ canonical limit $\mathbf{N_{\Upsilon} \sim (N_{bb})^1}$

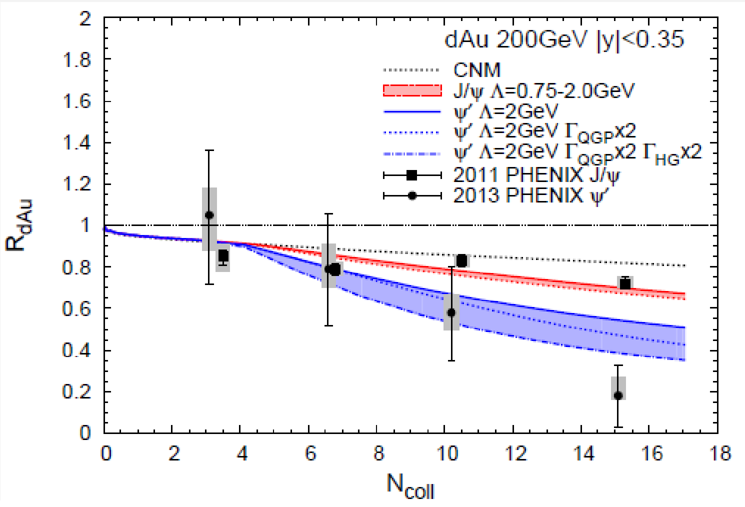


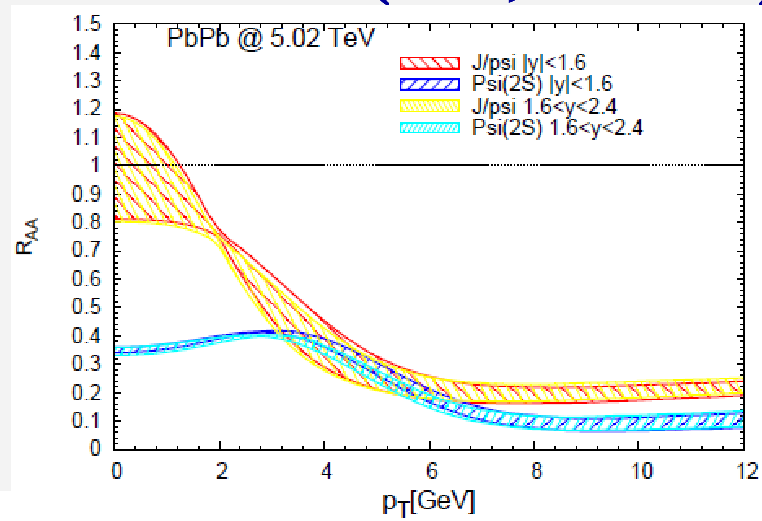
4.5 ϒ Binding Energies

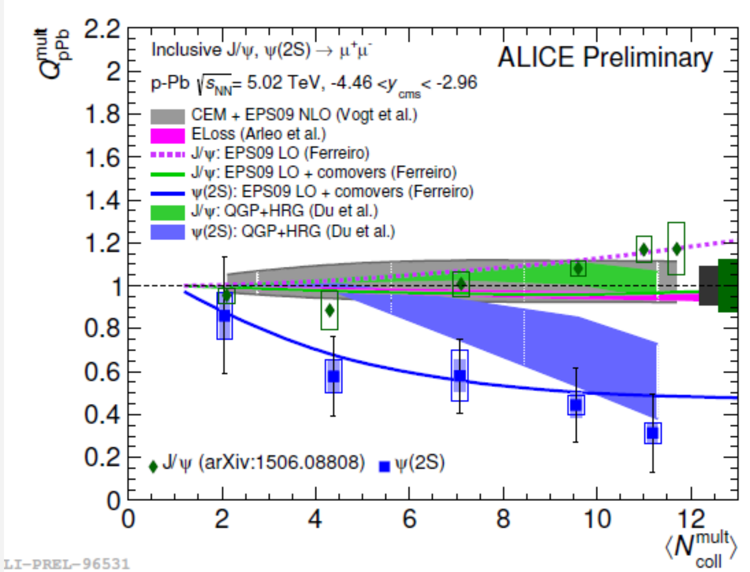


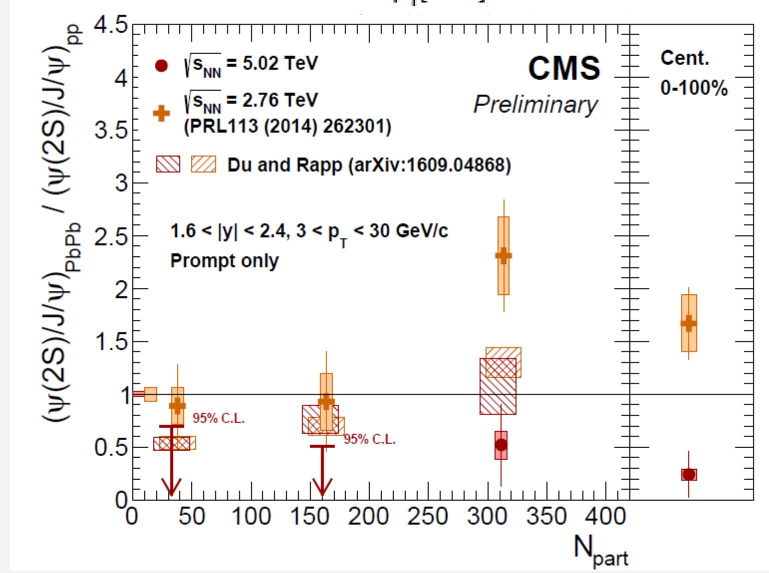
4.6 ψ(2S): Sequential Regeneration?

d-Au(0.2TeV) + p-Pb(5TeV) **Pb-Pb(2.76,5.02TeV)**



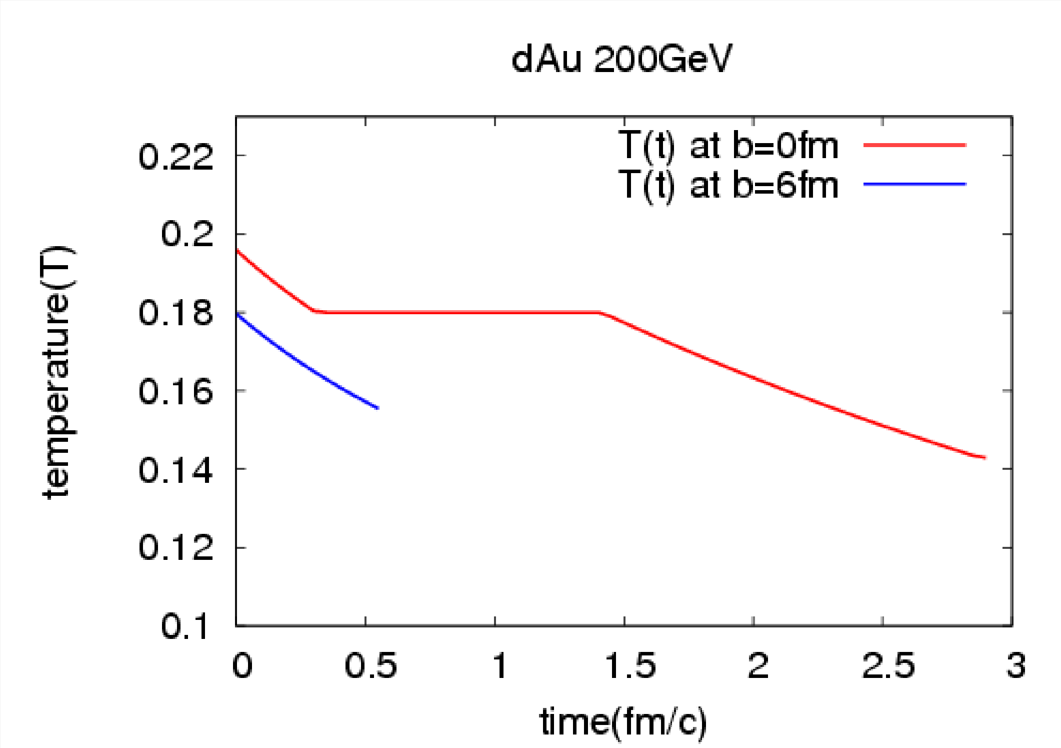


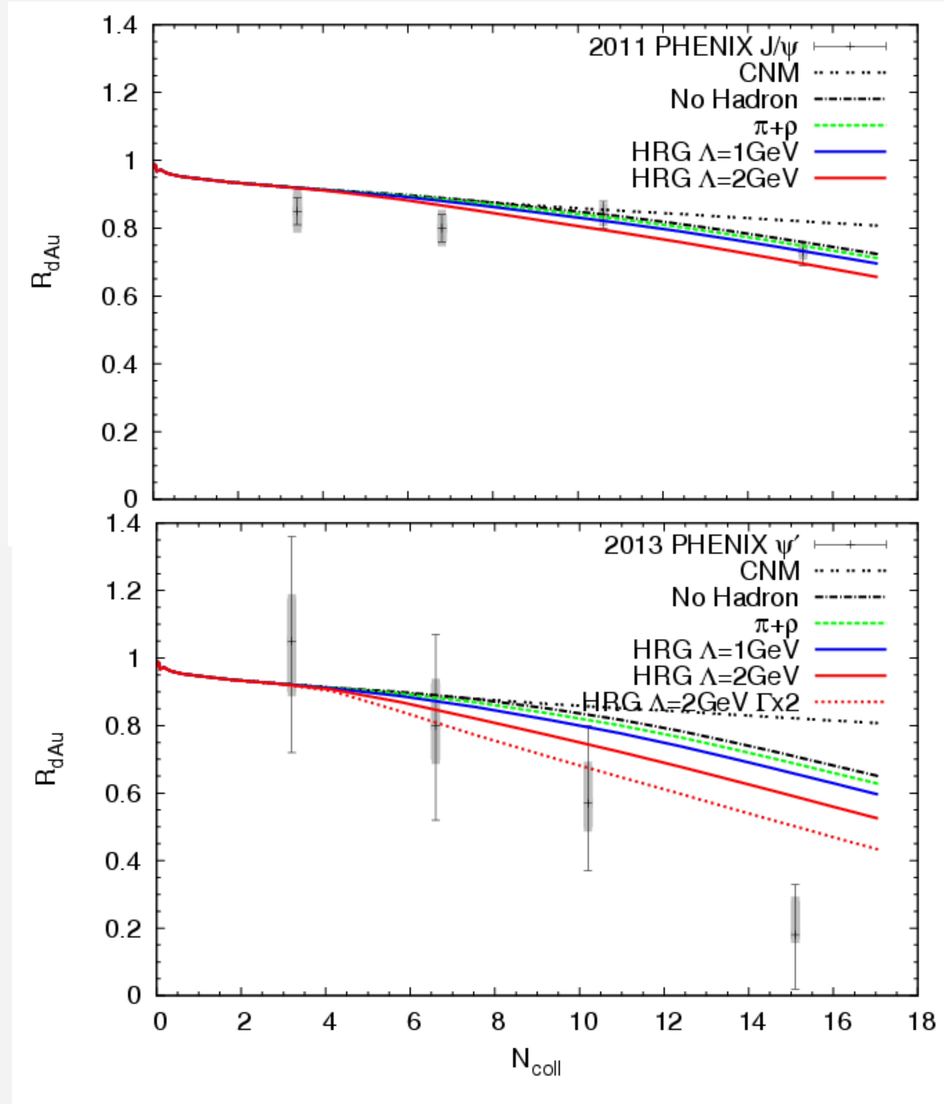




- ψ' suppression dAu, pPb → regenerated later in AA (larger flow than **J/ψ**)

4.2 Charmonia in d+Au Fireball





- construct fireball + evolve rate equat.
  → ψ′ suppression from hot medium

- similar in spirit to comover approach
  [Ferreiro '14]

- formation time effects?!

[X.Du+RR, in prep]

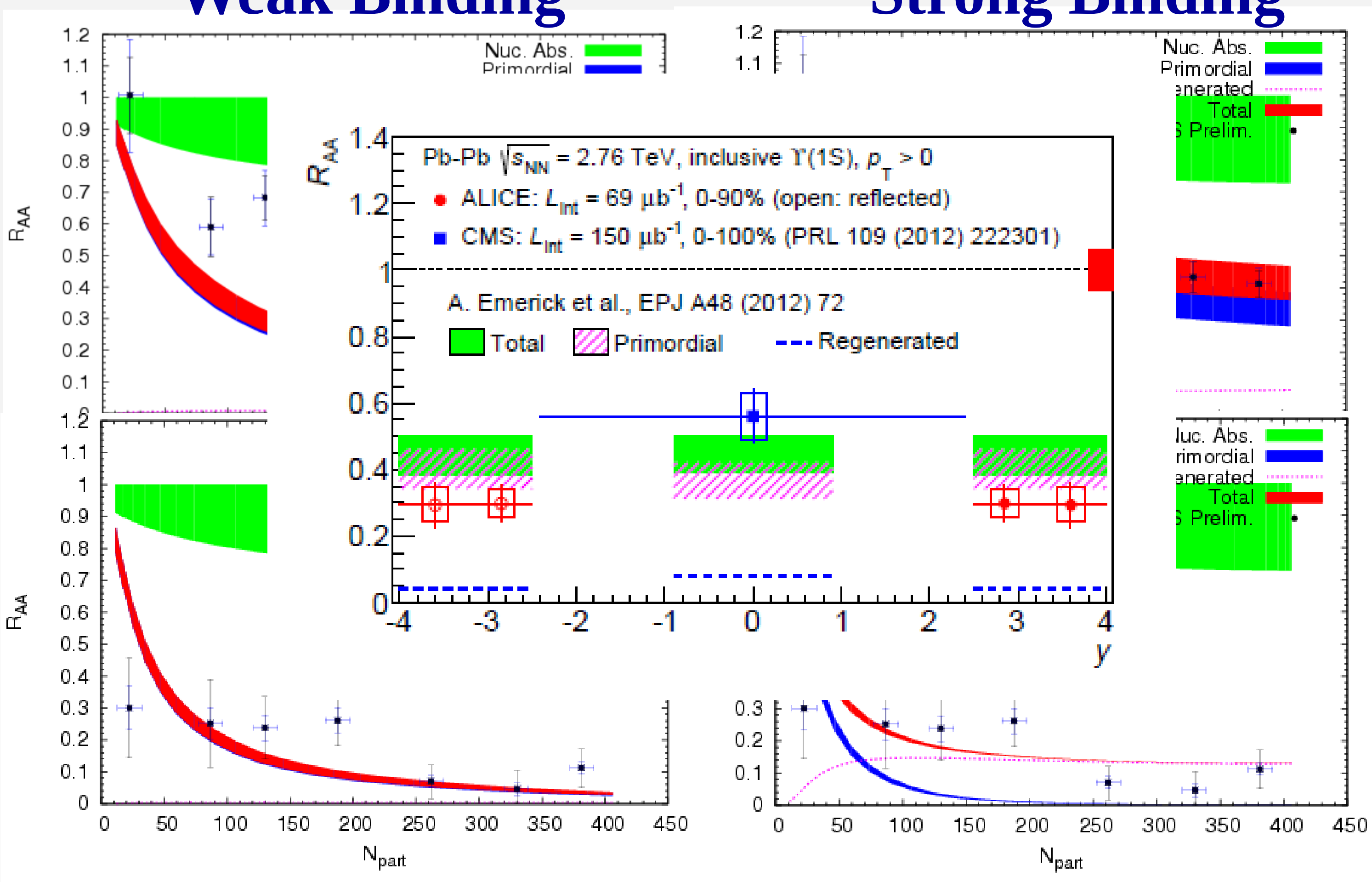
[Y.Liu, Ko et al '14]

3.6 Υ(1S) and Υ(2S) at LHC

Weak Binding **Strong Binding**

Υ(1S) →

Υ(2S) →



[Grandchamp et al '06, Emerick et al '11]

- sensitive to color-screening + early evolution times
- clear preference for strong binding (**U** potential)
- similar results by [Strickland '12]
- possible problem in rapidity dependence