Lattice QCD for open and hidden heavy flavor probes

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Spatial meson correlation functions
⇒ open charm mesons and charmonia
Bazavov, Karsch, Maezawa, Mukherjee, PP, PRD91 (2015) 054503

Quarkonium correlators and spectral functions from NRQCD
S. Kim, PP, A. Rothkopf, PRD91 (2015) 054511
S. Kim, PP, A. Rothkopf, work in progress

Charm fluctuations and correlations:
⇒ open charm hadrons above T_c ?
Mukherjee, PP, Sharma, PRD 93 (2016) 014502

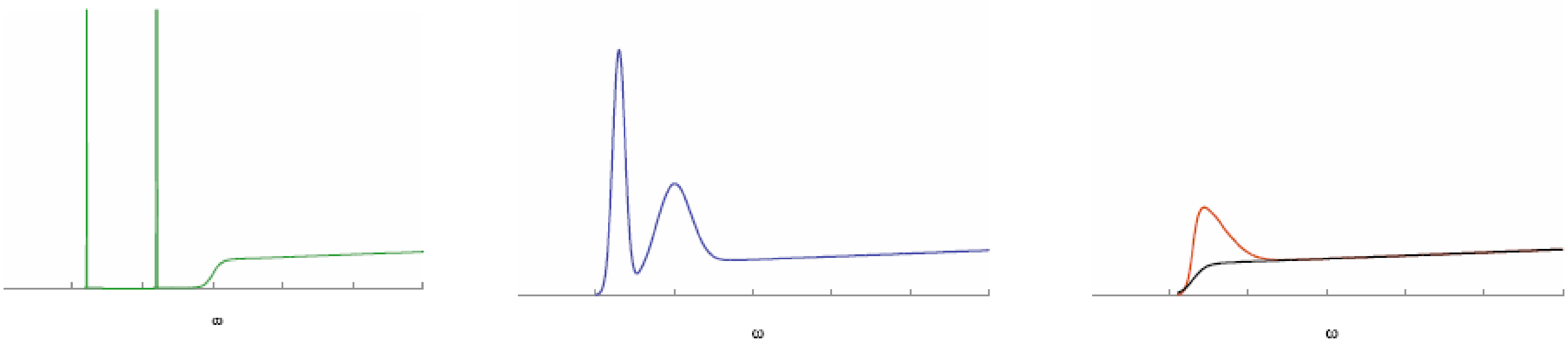
INT Program INT-17-1b , INT, Seattle, May 9, 2017

Meson correlators and spectral functions

In-medium properties and/or dissolution of mesons are encoded in the spectral functions

$$\rho(\omega, p, T) = \frac{1}{2\pi} \mathrm{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [J(x,t), J(0,0)] \rangle_T$$

Melting is see as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$D(\tau, p, T) = \int d^3x e^{ipx} \langle J(x, -i\tau) J(0,0) \rangle_T$$

$$D(\tau, p, T) = \int_0^{\infty} d\omega \rho(\omega, p, T) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

MEM

σ(ω,p,T)

1S charmonium survives to *1.6*T_c ??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Temperature dependence of temporal charmonium correlators

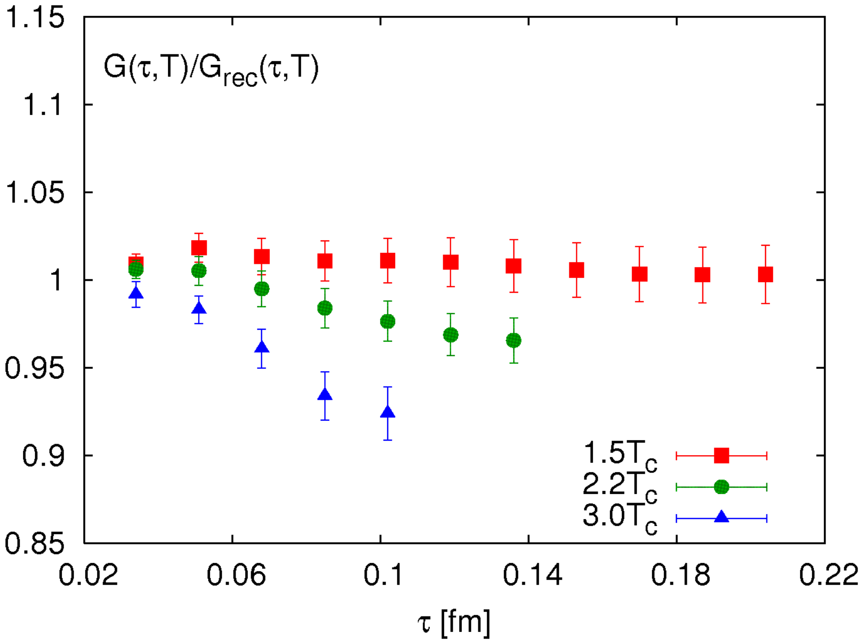
temperature dependene of $D(\tau,T)$

$$D(\tau,T) = \int_0^\infty d\omega \rho(\omega,T) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

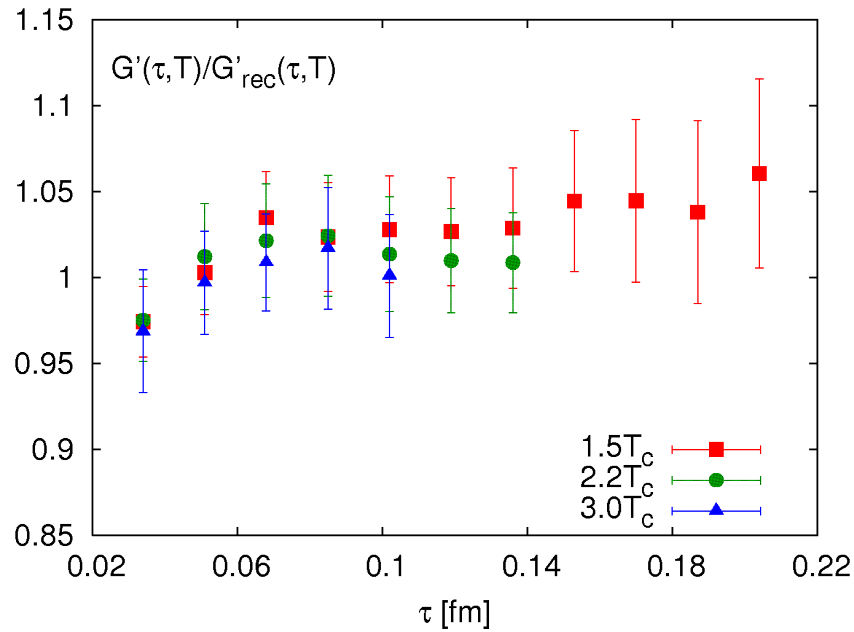
If there is no T-dependence in the spectral function, $D(\tau,T)/D_{rec}(\tau,T)=1$

$$D_{rec}(\tau,T) = \int_0^\infty d\omega \rho(\omega,T=0) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Pseudo-scalar ⇔ $1S$



Scalar ⇔ $1P$



Datta, Karsch, P.P., Wetzorke, PRD 69 (2004) 094507

Temporal vs spatial meson correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x}, -i\tau) J(\mathbf{0}, 0)\rangle_T, \; G(z \to \infty, T) = A e^{-m_{scr}(T) z}$$

but related to the same spectral functions $G(z,T) = 2\int_{-\infty}^{\infty} dp e^{ipz} \int_0^{\infty} d\omega \frac{\rho(\omega,p,T)}{\omega}$

Low T limit :

$$\sigma(\omega,p,T) \simeq A_{mes} \delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T) z}$$

High T limit :

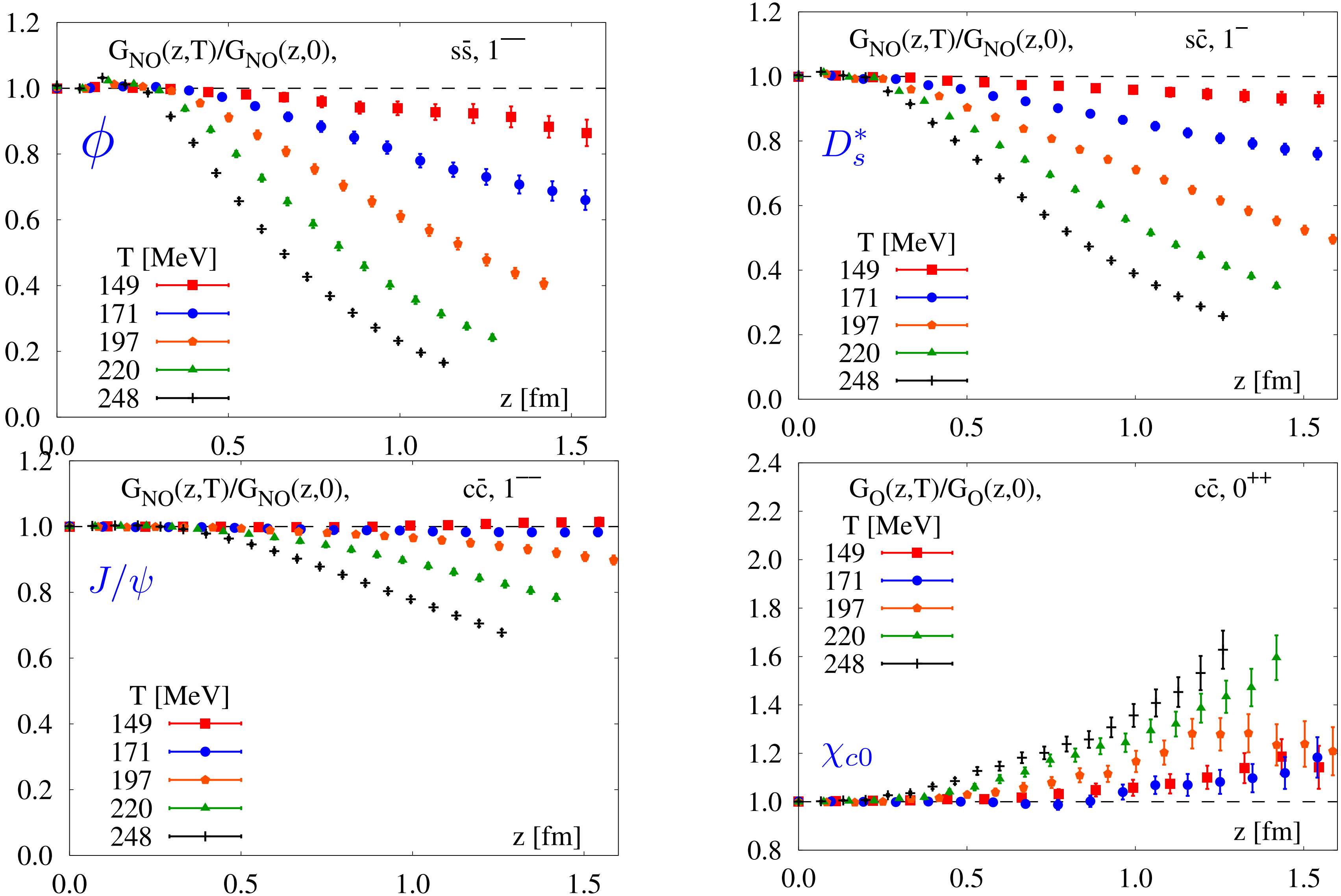
$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

Temporal meson correlator only avalaible for $\tau T < ½$ and thus may not be very sensitive to In-medium modifications of the spectral functions; also require large N_τ (difficult in full QCD)

Spatial correlators can be studied for arbitrarily large separations and thus are more sensitive to the changes in the meson spectral functions; do not require large N_τ (easy in full QCD).

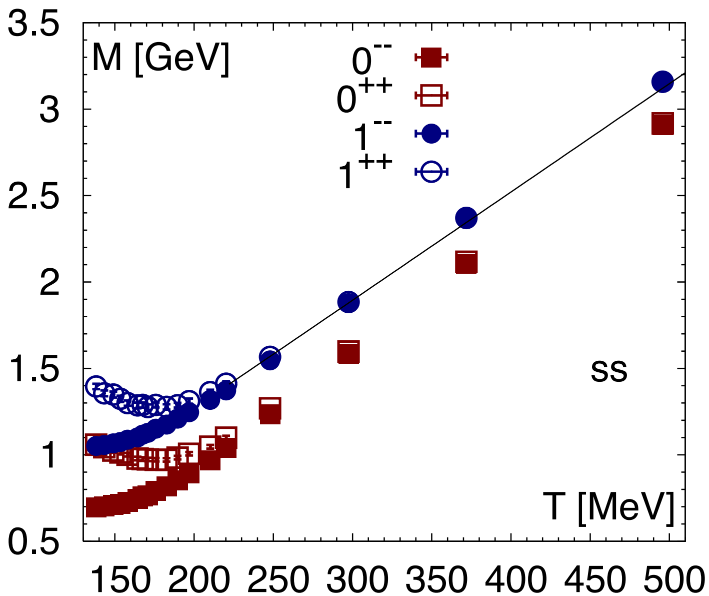
Lattice calculations: spatial meson correlators in 2+1 flavor QCD for ssbar, scbar and ccbar sectors using 48^3x12 lattices and highly improved staggered quark (HISQ) action (HotQCD) , physical m_s and $m_\pi = 161$ MeV.

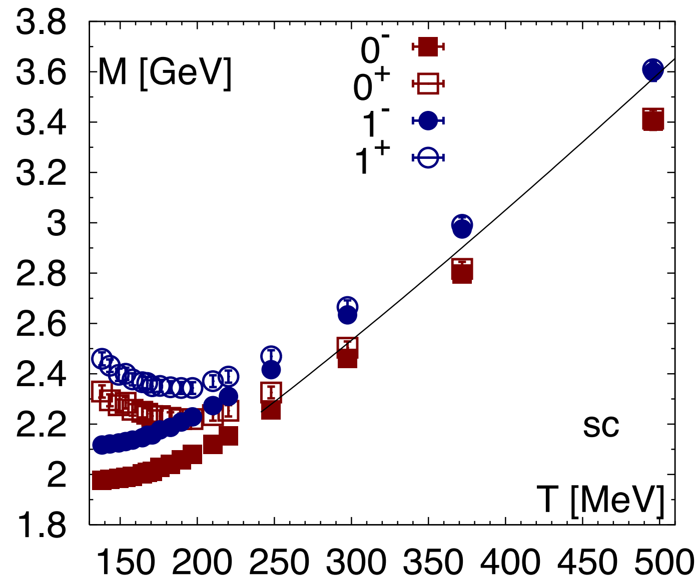
Temperature dependence of spatial meson correlators

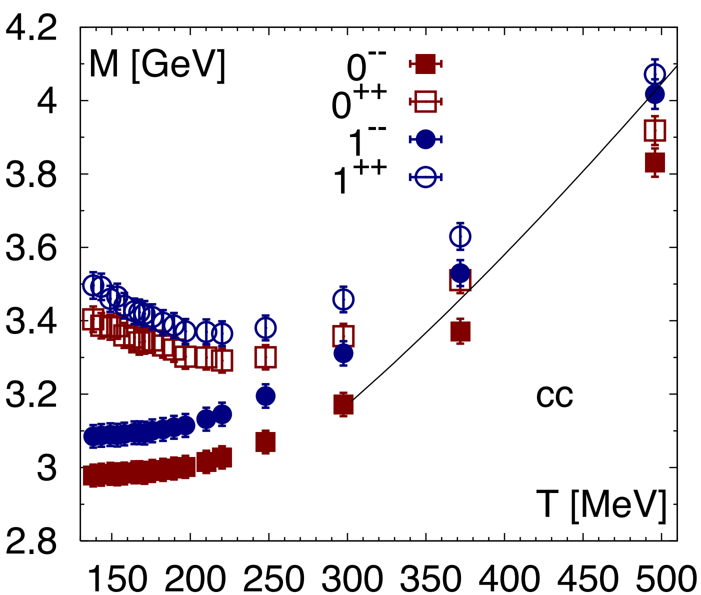


Medium modifications of meson correlators increase with *T*, but decrease with heavy quark content; larger for *1P* charmonium state than for *1S* charmonium state

Temperature dependence of meson screening masses





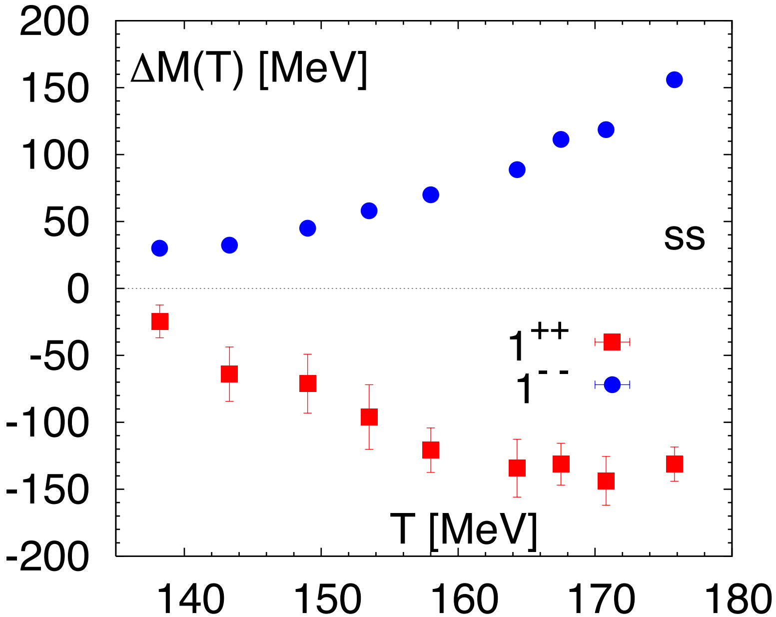


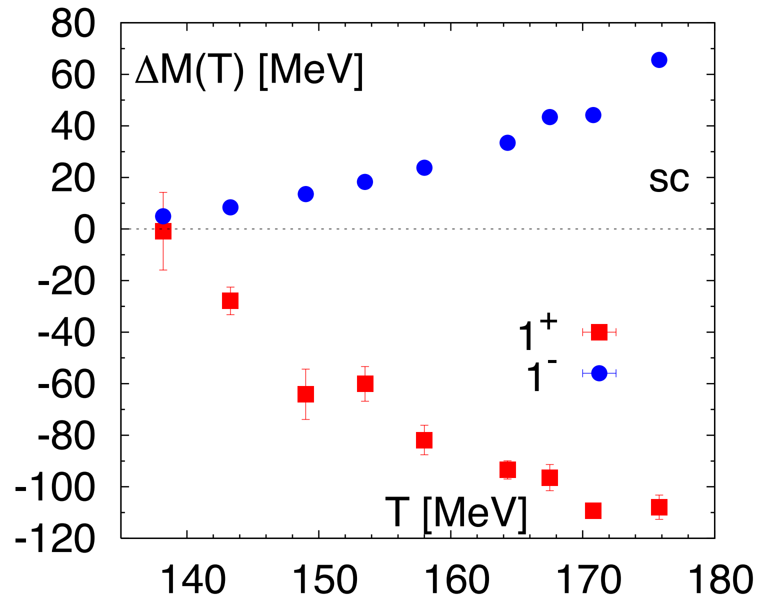
Qualitatively similar behavior of the screening masses for ssbar, scbar and ccbar sectors

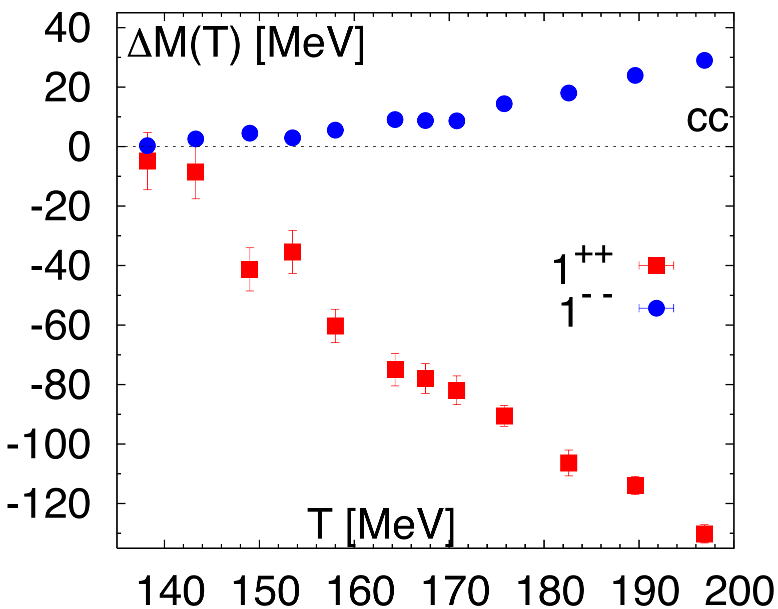
Screening Masses of opposite parity mesons become degenerate at high T (restoration of chiral and axial symmetry)

Screening masses are close to the free limit $2\,(m_q^2+(\pi T)^2)^{1/2}$ at $T>200$ MeV, $T>250$ MeV, $T>300$ MeV for ssbar, scbar and ccbar sectors, respectively.

Temperature dependence of meson screening masses (cont'd)







- At low T changes in the meson screening Masses $\Delta M = M_{scr}(T) - M_{T=0}$ are indicative of the changes in meson binding energies
- ΔM is significant already below T_c
- Above the transition temperature the changes in ΔM are comparable to the meson binding energy and except for *1S* charmonium (sequential melting)

Why NRQCD ?

Quarkonia to a fairly good approximation are non-relativistic bound state

$$p_Q \sim M_Q v \ll M_Q$$

EFT approach: integrate the physics at scale of the heavy quark mass

NRQCD is the EFT at scale << M_Q

Heavy quark fields are non-relativistic Pauli spinors:

$$L_{NRQCD} = \psi^\dagger \left(D_\tau - \frac{D_i^2}{2M_Q} \right) \psi + \chi^\dagger \left(D_\tau + \frac{D_i^2}{2M_Q} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Advantages:

- the large quark is not a problem for lattice calculations, lattice study of bottomonium is feasible (usually $a M_Q << 1$, which is challenging)
- The structure of the spectral function is simpler => more sensitivity to the bound state properties
- Quarkonium correlators are not periodic and can be studied at larger time extent ($=1/T$) => more sensitivity to bound state properties

NRQCD on the Lattice

Inverse lattice spacing provides a natural UV cutoff
for NRQCD provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$S_Q(x,\tau+a) = U_4^\dagger(1 - \frac{p^2}{2M_Q}\Delta\tau)S_Q(x,\tau),\ \Delta\tau = a/n$$

well behaved if $naM_Q < 3$

Davies, Thacker, PRD 45 (1992) 915

$$D(\tau) = \sum_x \langle O(x,\tau)S_Q(x,\tau)O^\dagger(0,0)S_Q^\dagger(x,\tau)\rangle_T,\quad O(^3S_1;x,\tau) = \sigma_i,\quad O(^3P_1;x,\tau) = \Delta_i\sigma_j - \Delta_j\sigma_i$$

Thacker, Lepage, PRD43 (1991) 196

The energy levels in NRQCD are related to meson masses by a constant
lattice spacing dependent shift, e.g.

$$M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$$

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations
from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied
by varying $a \leftrightarrow \beta = 10/g^2$ Bazavov et al, PRD85 (2012) 054503

$\Rightarrow 140\text{MeV} \leq T \leq 407\text{MeV}$

$2.759 \geq aM_b \geq 0.954$ (ok if $n = 2,\ 4$)
$0.757 \geq aM_c \geq 0.427$ (ok if $n \geq 8$)

Bayesian Reconstruction of spectral functions

$$D(\tau) = D(\mathbf{p}=0,\tau) = \sum_{\mathbf{x}} D(\mathbf{x},\tau) = \int_{-2M_q}^{\infty} d\omega\, e^{-\omega\tau}\, \rho(\omega)$$

Discretize the integral $D_i^\rho = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$ and find ρ_l

using Bayesian approach, i.e. maximizing

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

Likelihood:

$$P[D|\rho I] = \exp(-L)$$

$$L[\rho] = \frac{1}{2}\sum_{ij}(D_i - D_i^\rho)C_{ij}(D_j - D_j^\rho)$$

Prior probability:

$$P[\rho|I] = \exp[S]$$

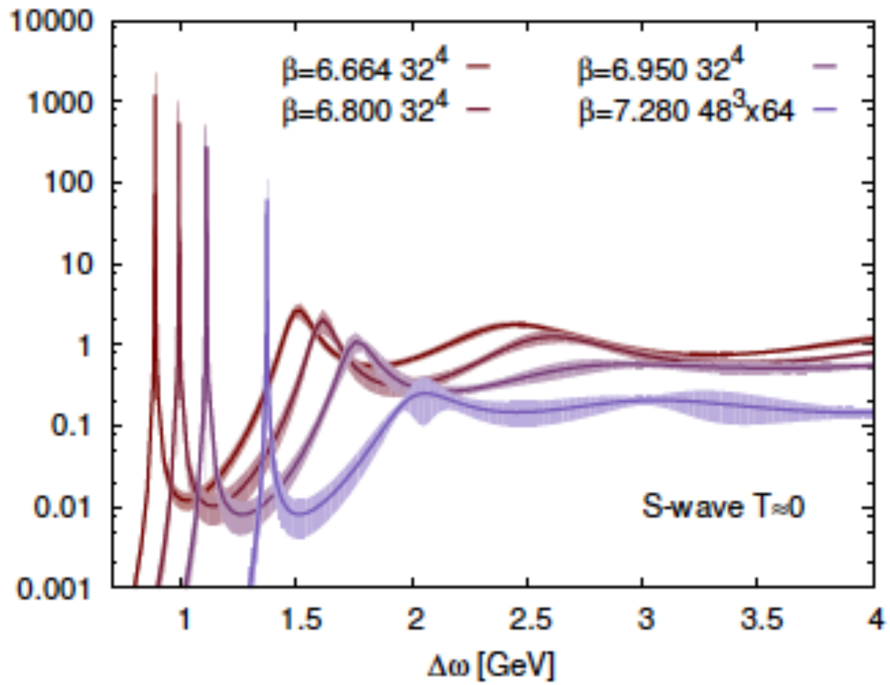
$$S[\rho] = \alpha \sum_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)\Delta\omega_l.$$

no restriction on the search space
no flat directions

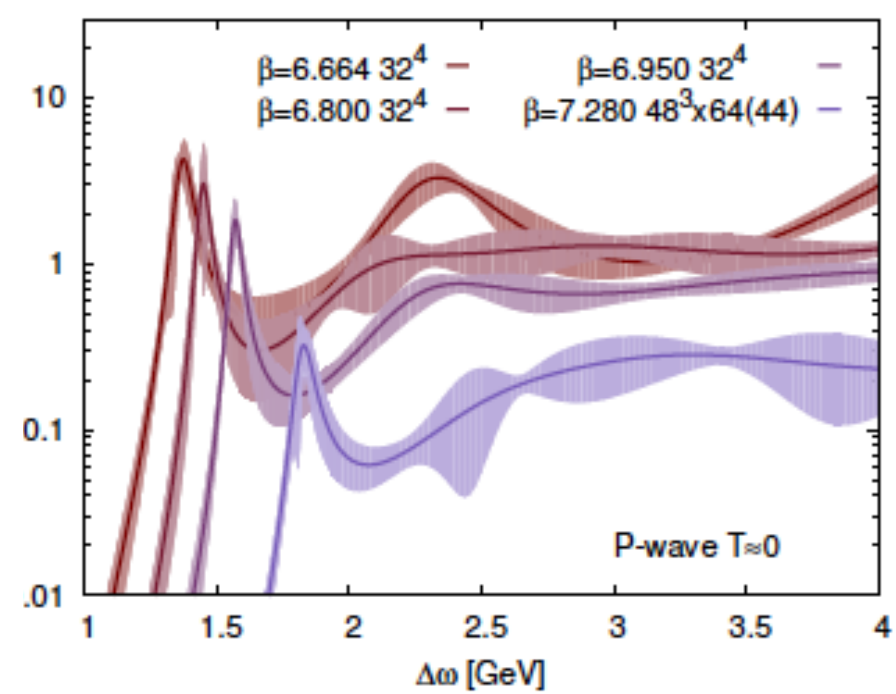
Different from MEM !

Burnier Rothkopf, PRL 111 (2013) 182003

Bottomonium spectral functions at T=0



Well resolved Υ ground state peak



Acceptable resolution for χ_b state

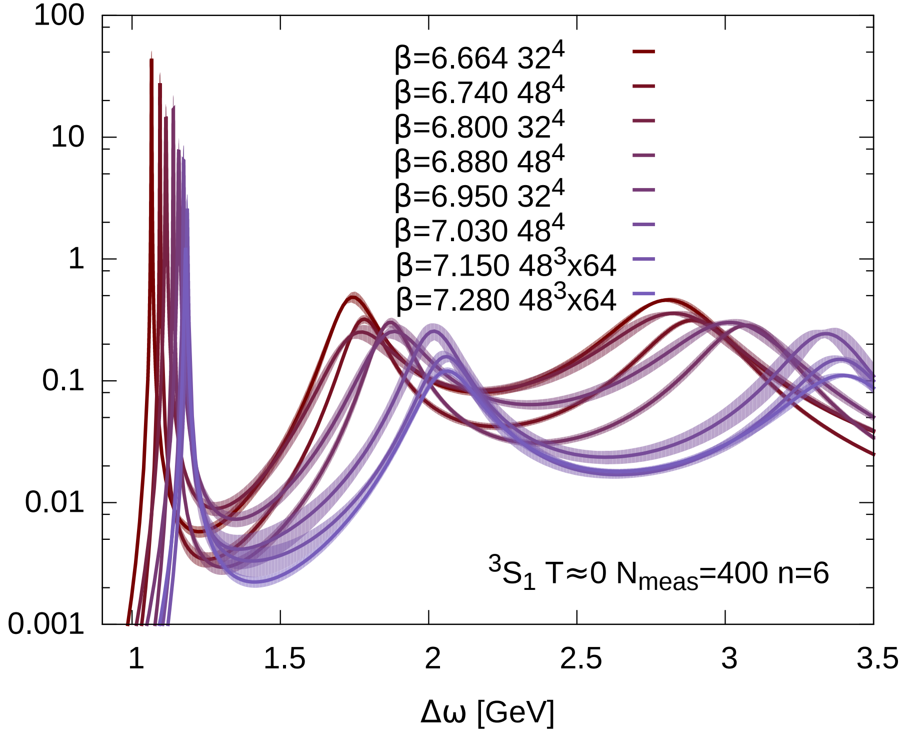
But excited states, Υ', Υ'', χ_b' cannot be resolved well

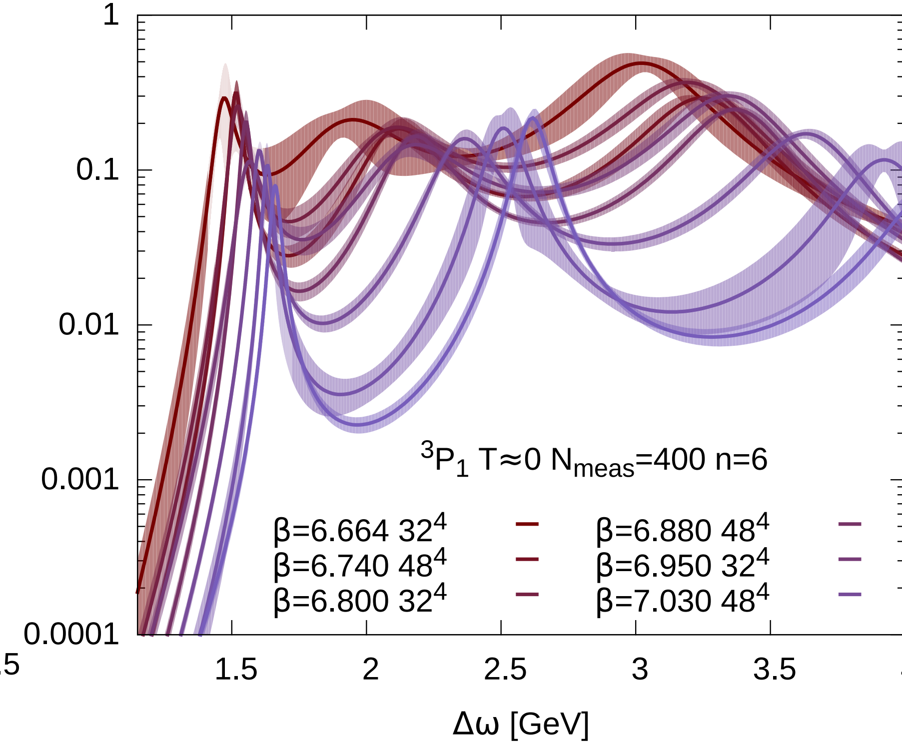
Define the NRQCD energy shift $C_{\text{shift}}(a)$ by fixing the Υ peak to PDG

$$E_\Upsilon + C_{\text{shift}}(a) = 9.46030 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_b, χ_{b0}, χ_{b1}, h_b

Charmonium spectral functions at T=0





Well resolved J/ψ peak

Only the position of the χ_{c1} state can be reliably determined

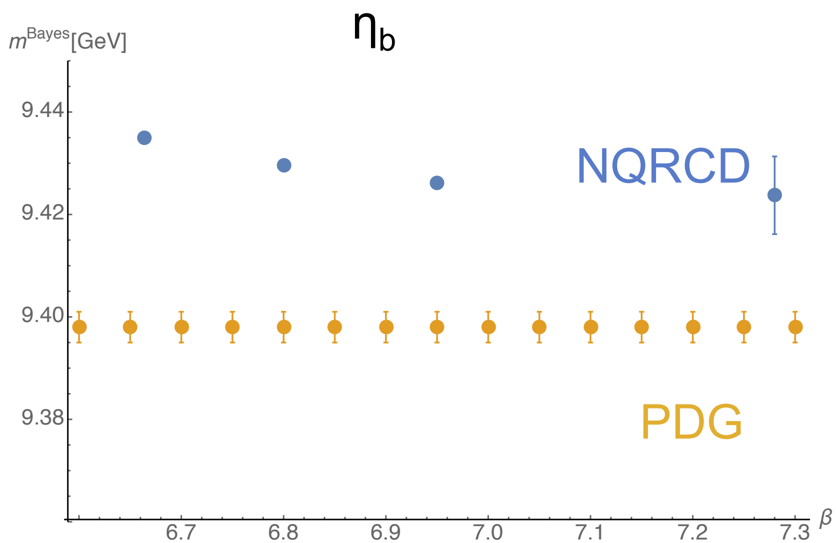
Excited states cannot be determined, artifacts at $\Delta\omega > 3$ GeV

Define the NRQCD energy shift $C_{\text{shift}}(a)$ by fixing the J/ψ peak to PDG

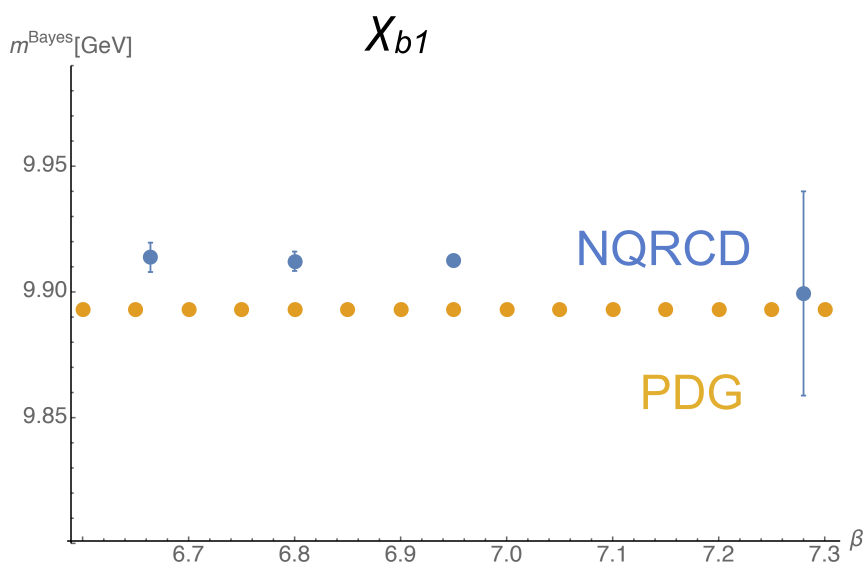
$$E_{J/\psi} + C_{\text{shift}}(a) = 3.097 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_c, χ_{c0}, χ_{c1}, h_c

How Well NQRCD Works for Bottomonium ?

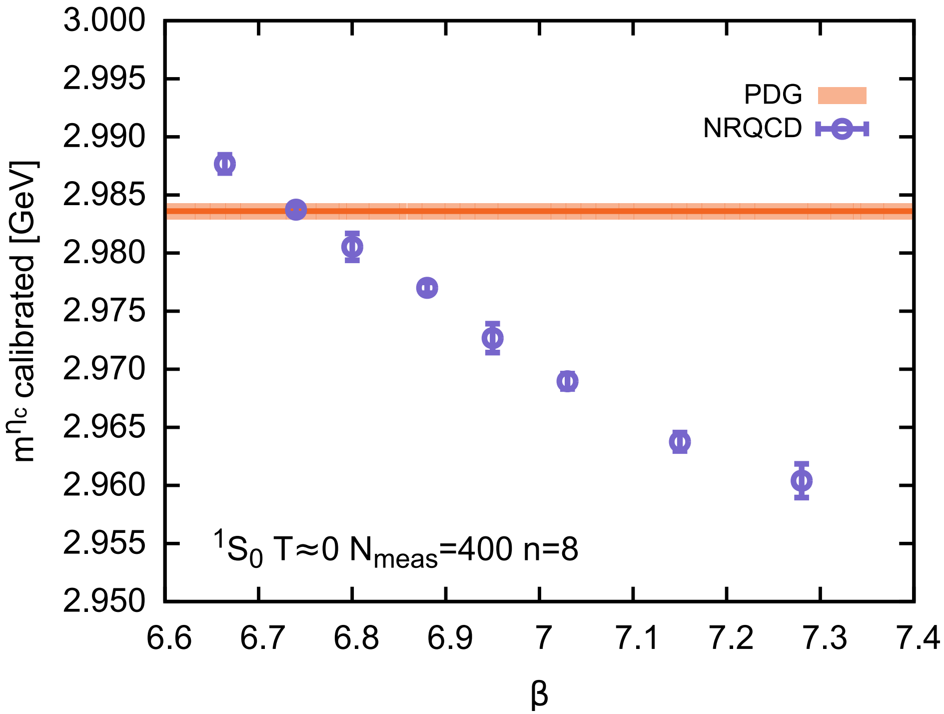


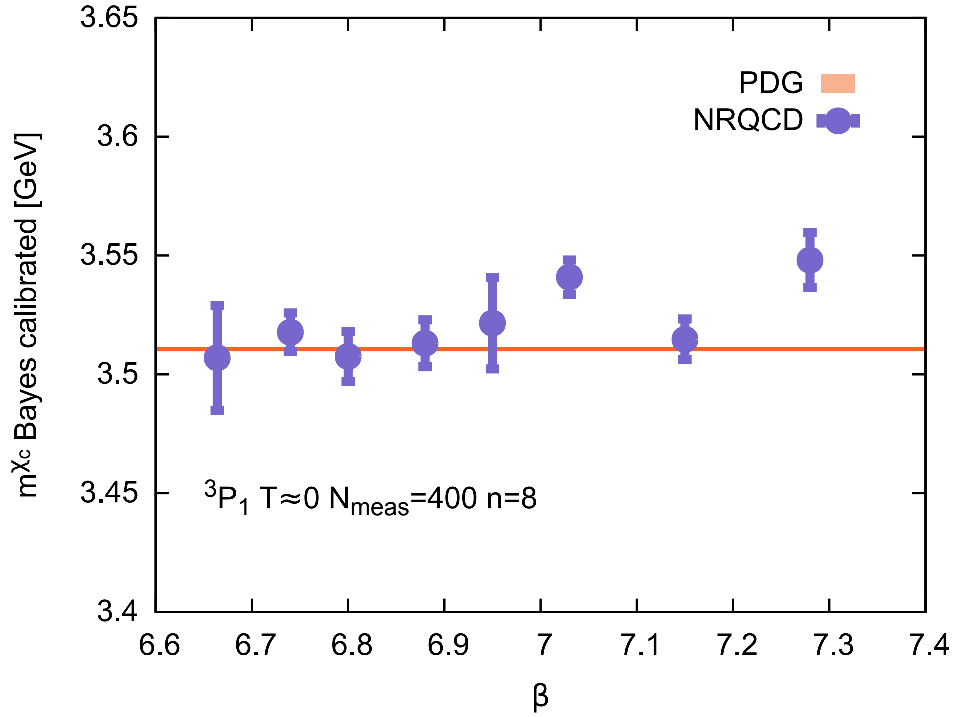
NRQCD can reproduce the hyperfine splitting in bottomonium with accuracy < *20-40* MeV depending on the lattice spacing



NRQCD can reproduce the *1P-1S* splitting in bottomonium with accuracy < *15* MeV

How Well NQRCD Works for Charmonium ?

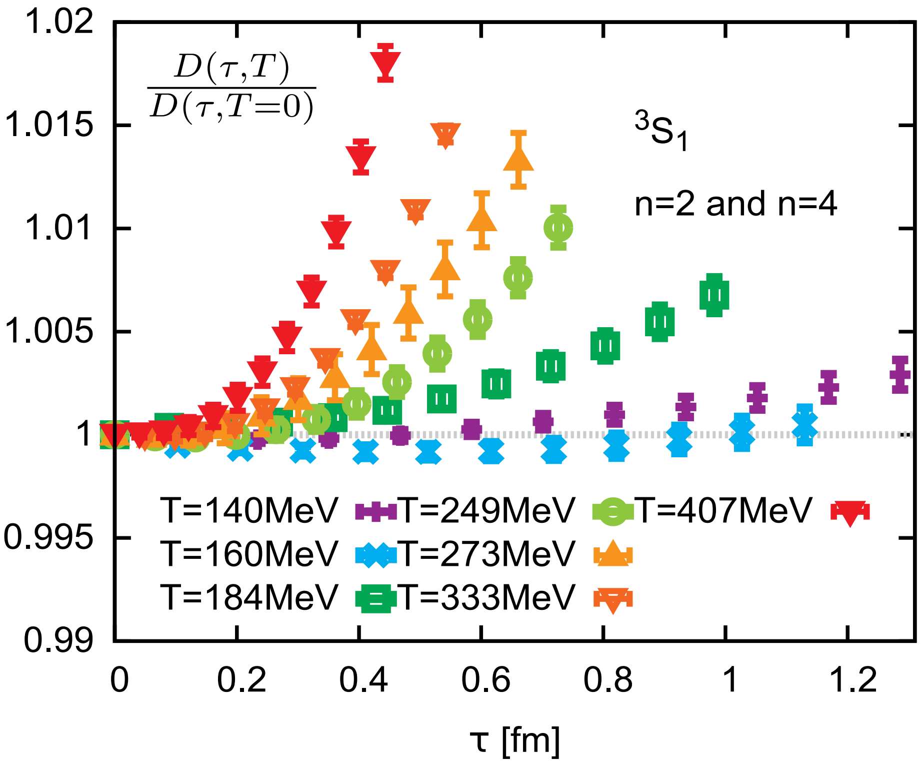


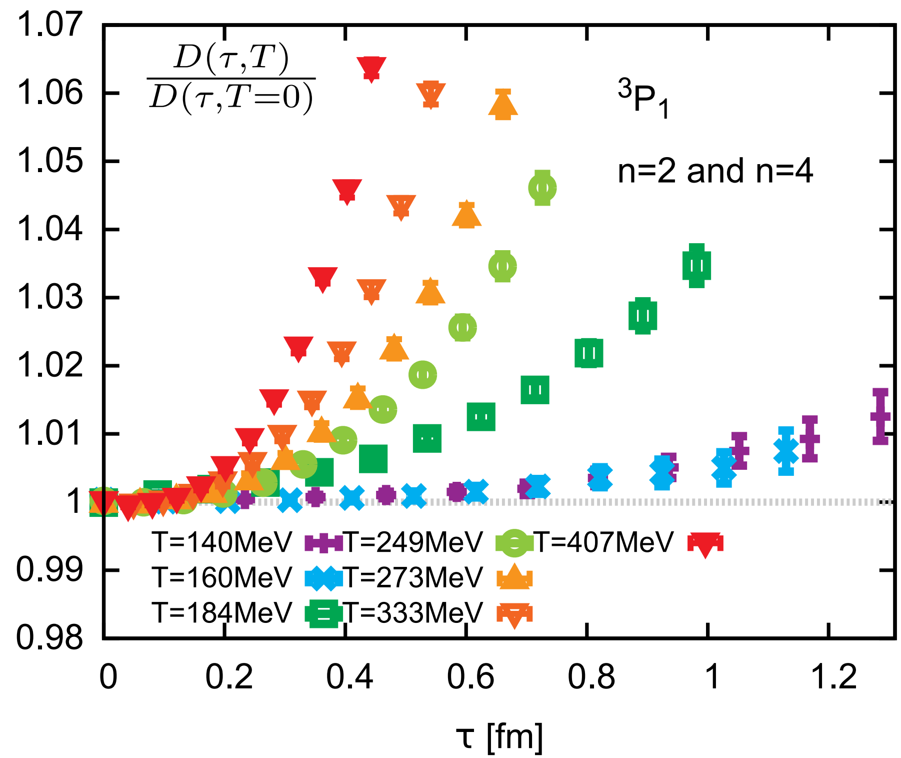


NRQCD can reproduce the hyperfine splitting in charmonium with an accuracy < *40* MeV

NRQCD can reproduce the *1P-1S* splitting in charmonium well for lattice spacing a > 0.08fm

Temperature dependence of the bottomonium correlators



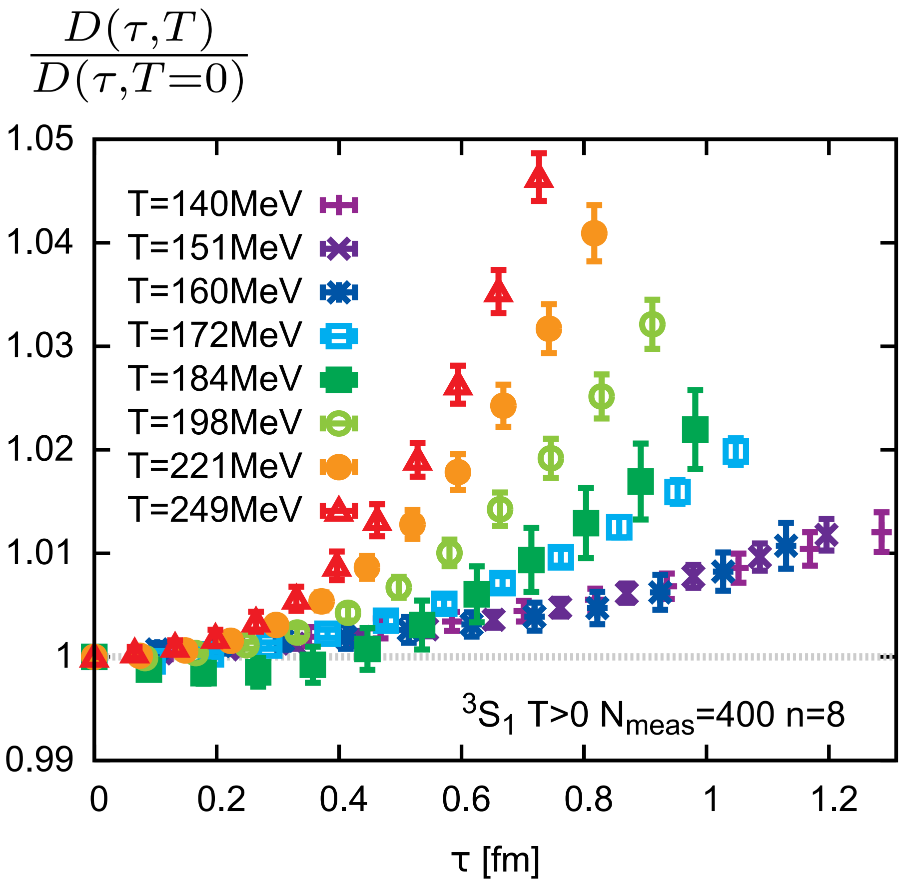


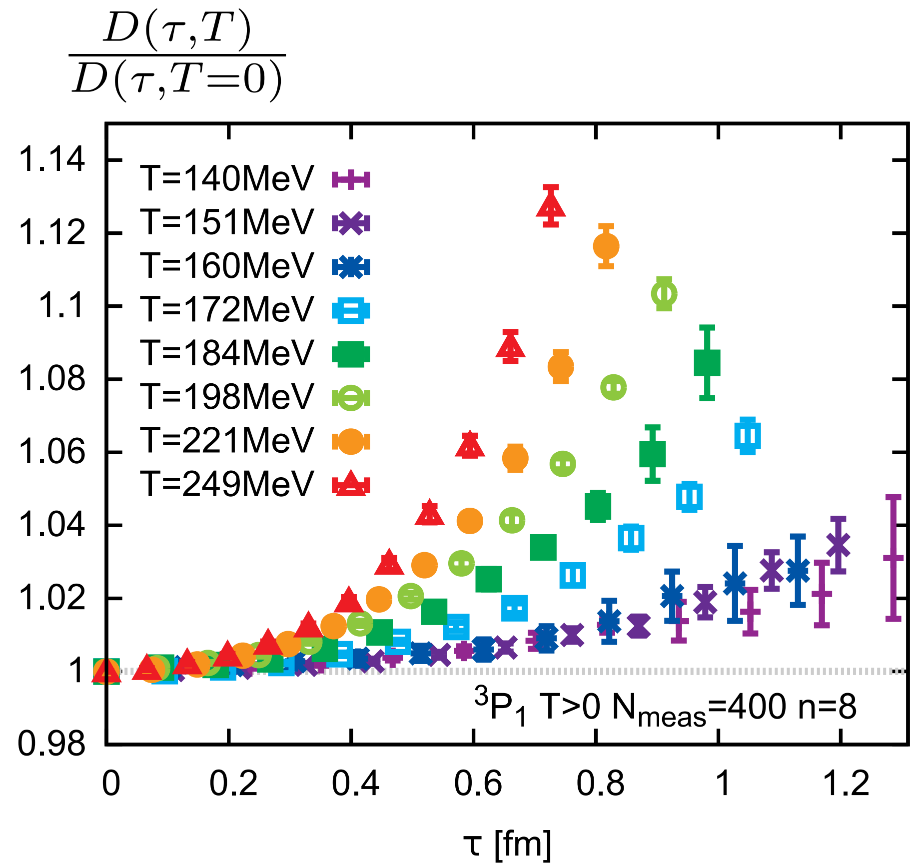
change in Υ correlator $< 2\%$

change in χ_{b1} correlator $< 7\%$

$\Rightarrow$ hints for sequential melting pattern: stronger medium modification of χ_{b1} spectral function than for Υ spectral function

Temperature dependence of the charmonium correlators





change in J/ψ correlator $< 5\%$ change in χ_{c1} correlator $< 12\%$

$\Rightarrow$ hints for sequential melting pattern:

changes in the J/ψ correlator are about the same as in the χ_b correlator (same size); changes in the χ_c correlators are factor of two larger

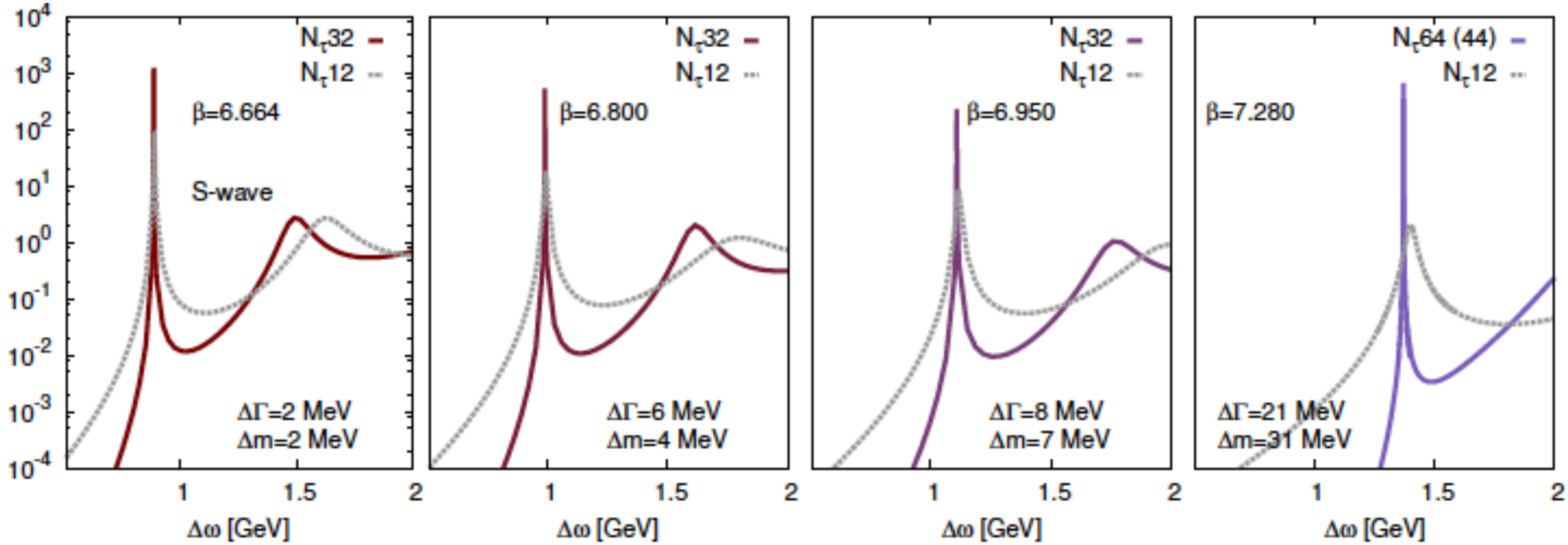
Reconstructing Spectral Functions at T>0

Two main problems:

1) $\tau < 1/T \Rightarrow$ limited temporal extent at high T

2) relatively small number of time slices ($N_\tau = 12$ in our study)

Study these effects at $T = 0$ by using only the first 12 data points:

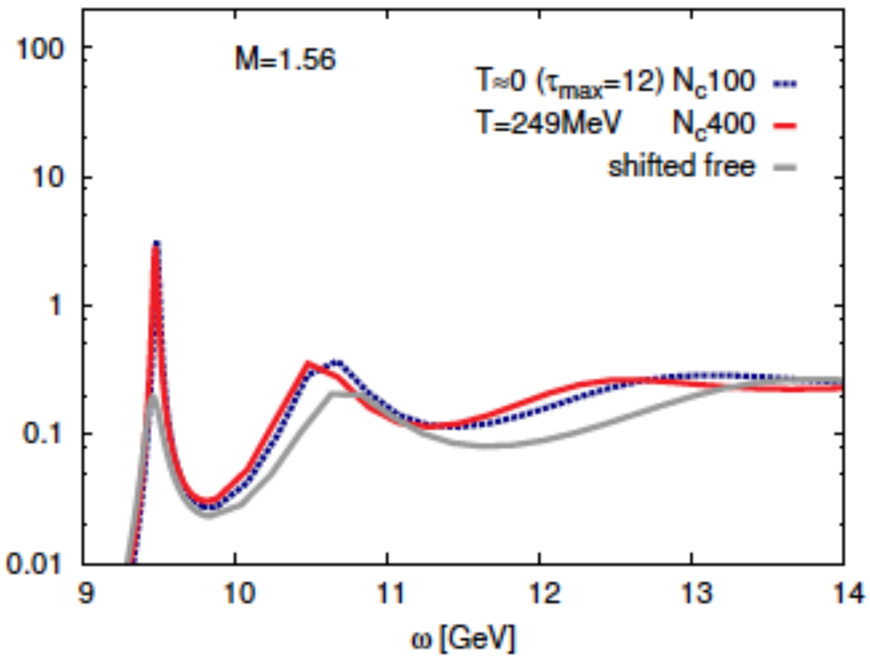


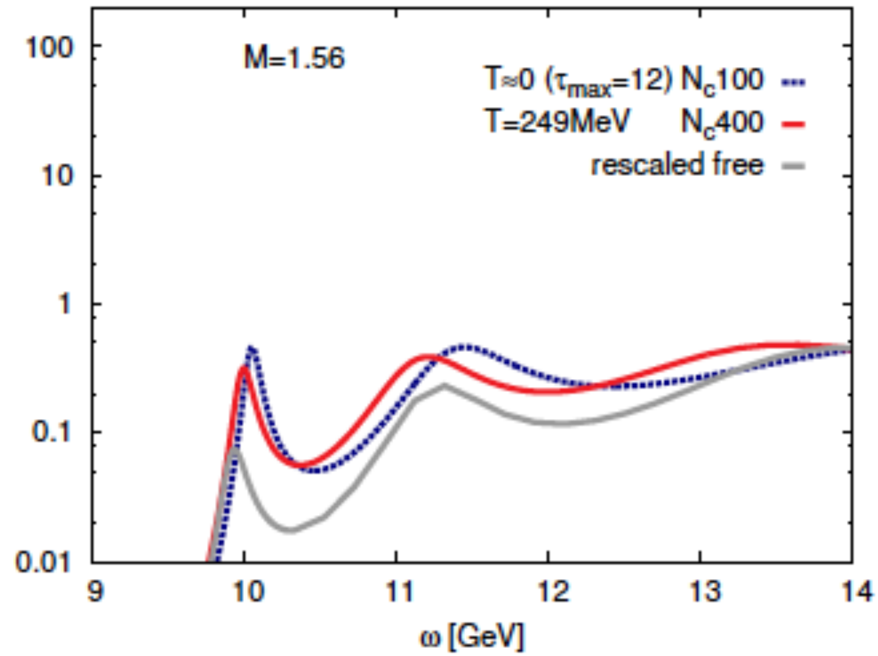
Decreasing $\tau_{max} = 1/T$ leads to broadening of the bound state peak

(to be taken into account in comparison $T = 0$ and $T > 0$ spectral functions)

Bottomonium Spectral Functions at T>0

Compare $T = 0$, $T > 0$ and free spectral functions reconstructed using the same systematics ($\tau_{max} = 1/T$ and $N_{data} = 12$)

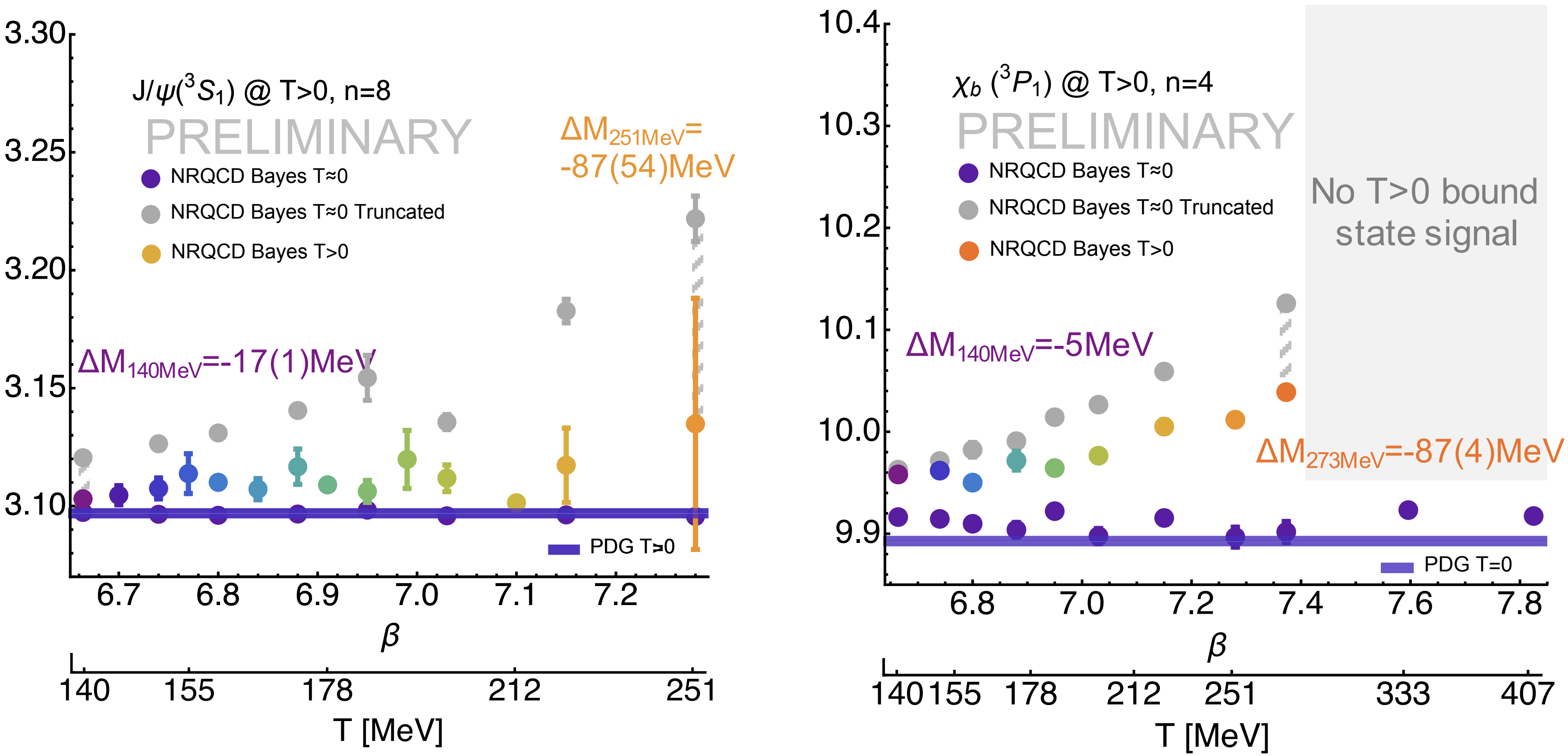




Both Υ and χ_b survive up to temperature $T > 249$ MeV

Onia masses at T>0

Onia masses from the peak positions:



Shifts in the peak location is smaller at *T>0* than in the vacuum for the same temporal extent ➔ the actual onia masses decrease with increasing temperature

Kim, PP, Rothkopf, arXiv:1704.05221

Summary of NRQCD results

- Both charmonium and bottomonium properties at $T > 0$ can be studied reliably using lattice NRQCD
- Systematic effects in the reconstructed spectral functions at $T > 0$ should be carefully studied through comparison to the $T = 0$ spectral functions and free spectral functions obtained under the same conditions
- Combined analysis of the T-dependence of the correlation functions and study of the spectral functions suggests a sequential dissociation (melting) pattern

$$T_d(\chi_c) < T_d(J/\psi) \simeq T_d(\chi_b) \ll T_d(\Upsilon)$$

$$T_d(J/\psi) \simeq 250\text{MeV},\ T_d(\chi_b) \simeq 270\text{MeV},\ T_d(\Upsilon) > 407\text{MeV}$$

in agreement with potential model calculations

PP, Miao, Mocsy, NPA855 (2011) 125

and the study of spatial charmonium correlators

Bazavov et al, PRD91 (2015) 054503

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4}=\sum_{ijkl}\frac{1}{i!j!k!l!}\chi_{ijkl}^{BQSC}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k\left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4}=\sum_{ijkl}\frac{1}{i!j!k!l!}\chi_{ijkl}^{udsc}\left(\frac{\mu_u}{T}\right)^i\left(\frac{\mu_d}{T}\right)^j\left(\frac{\mu_s}{T}\right)^k\left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd}=T^{i+j+k+l}\frac{\partial^i}{\partial\mu_b^i}\frac{\partial^j}{\partial\mu_b^j}\frac{\partial^k}{\partial\mu_c^i}\frac{\partial^l}{\partial\mu_d^l}\ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d)\,|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X=\chi_X=\frac{1}{VT^3}(\langle X^2\rangle-\langle X\rangle^2) \qquad \chi_{11}^{XY}=\frac{1}{VT^3}(\langle XY\rangle-\langle X\rangle\langle Y\rangle)$$

↓

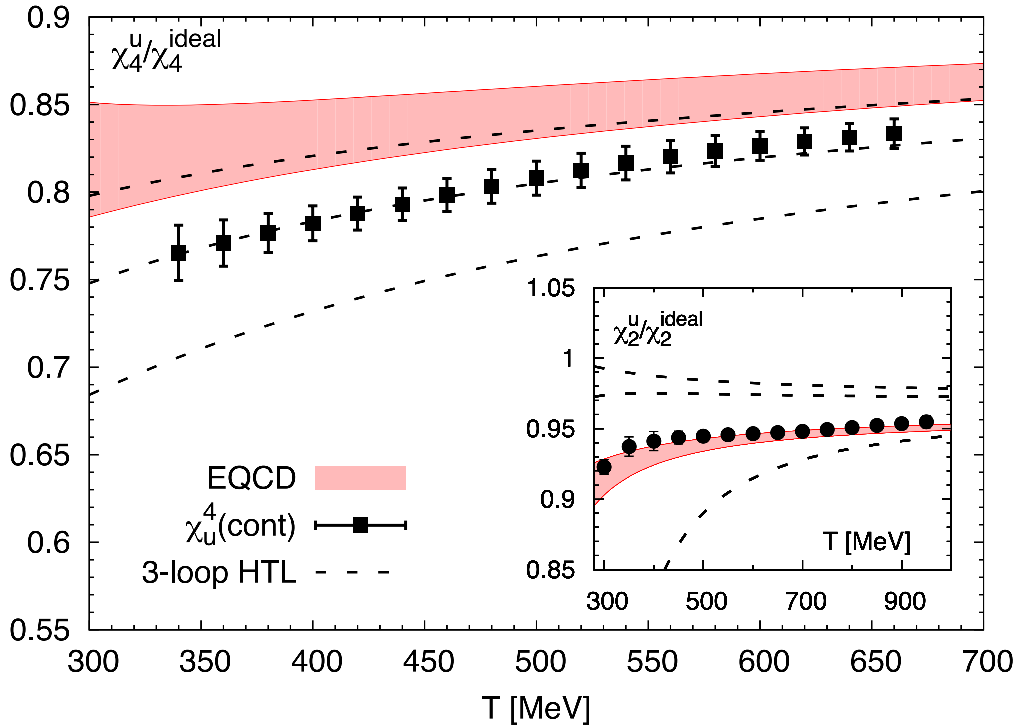
information about carriers of the conserved charges (hadrons or quarks)

↓

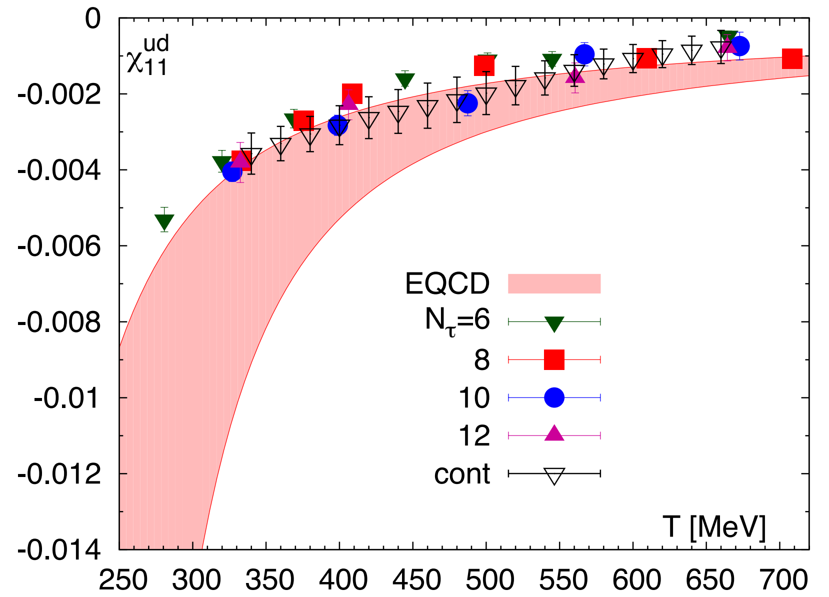
probes of deconfinement

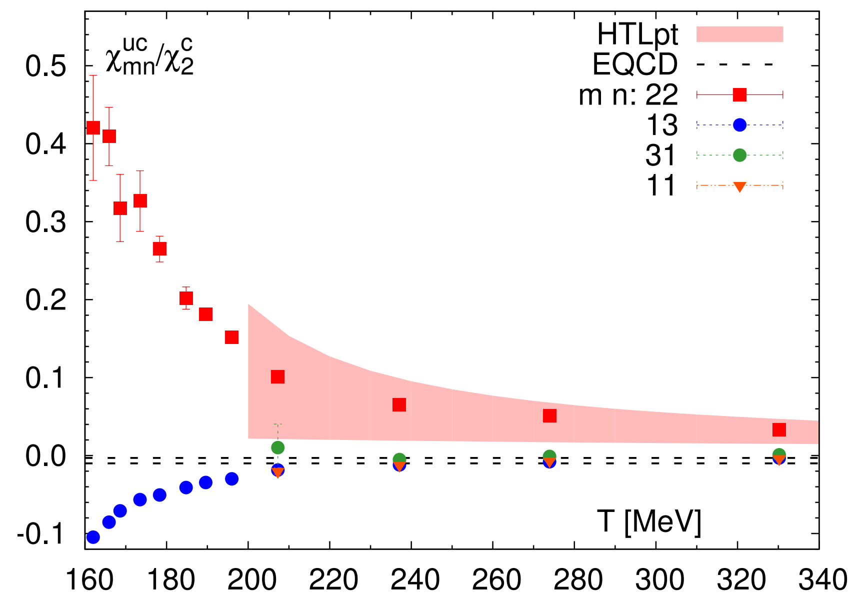
Quark number fluctuations at high T

quark number fluctuations



quark number correlations





Good agreement between lattice and the weak coupling approach for 2nd and 4th order quark number fluctuations

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

Correlations are large for *T<200* MeV but agree with weak coupling expectations for *T>300* MeV, e.g.

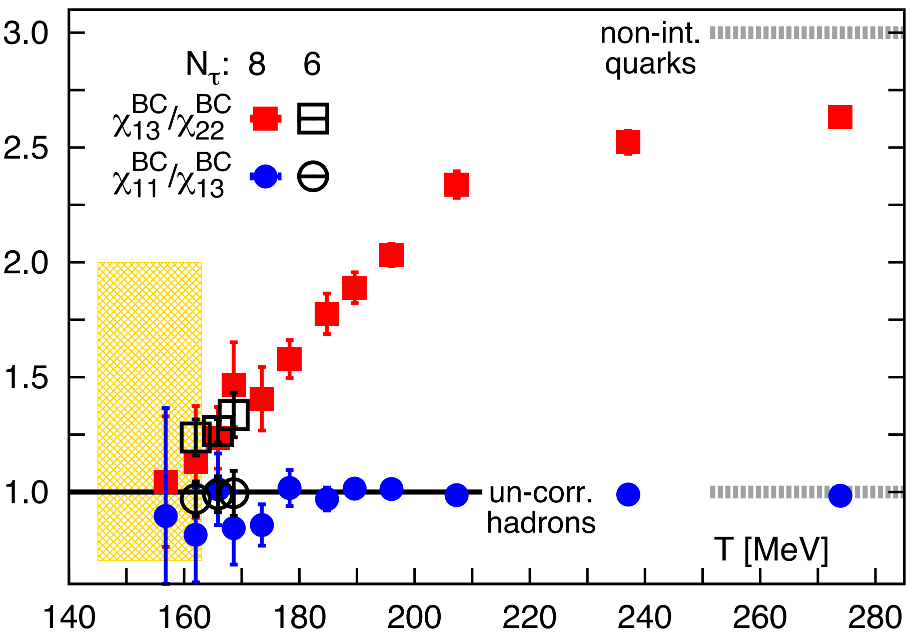
$$\chi_{22}^{uc} \gg \chi_{13}^{uc} \sim \chi_{31}^{uc} \sim \chi_{11}^{uc}$$

Fluctuation and correlations and deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C)/T^4}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l}$$

$m_c \gg T$ ⇒ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !

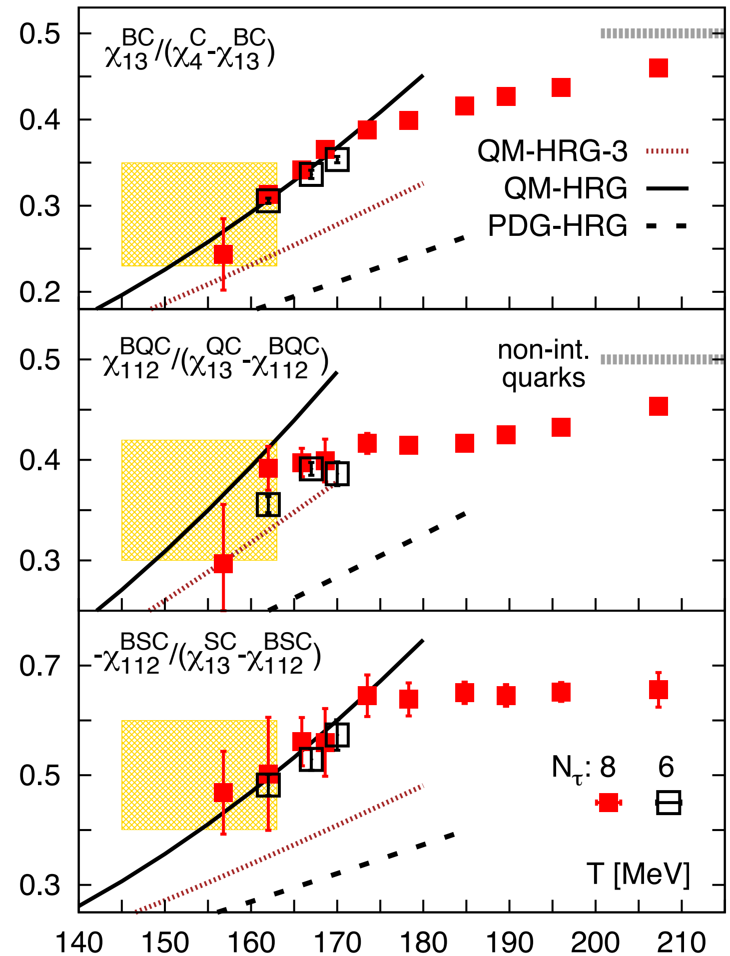


Hadronic description breaks down just above T_c

⇒ open charm deconfines above T_c

Bazavov et al, PLB 737 (2014) 210

Charm baryon to meson pressure



The charm baryon spectrum is not well known (few states in PDG), HRG works only if the “missing” states are included

Quasi-particle model for charm degrees of freedom

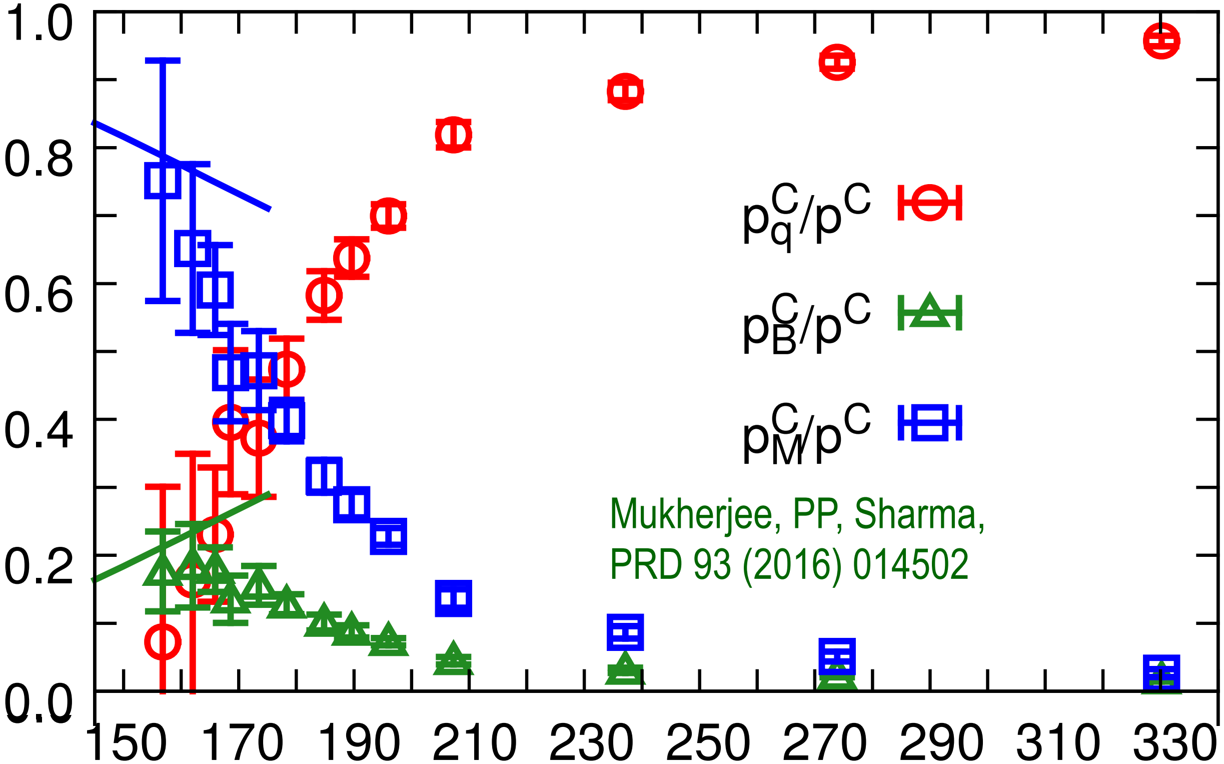
Charm dof are good quasi-particles at all T because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T > 200$ MeV or $\varepsilon > 6\ \mathrm{GeV/fm^3}$



Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See
Jakovác, PRD88 (2013), 065012
Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

Summary

- Temporal meson correlators are not very sensitive to the changes in the spectral functions because of the limited time extent temporal extent at $T>0$

- Spatial meson correlators and NRQCD correlators are sensitive to the temperature to the changes in the spectral functions and are consistent with sequential melting picture: where mesons more heavy quarks dissolve at higher temperatures and *1S* onia survive till higher temperature than *1P* onia

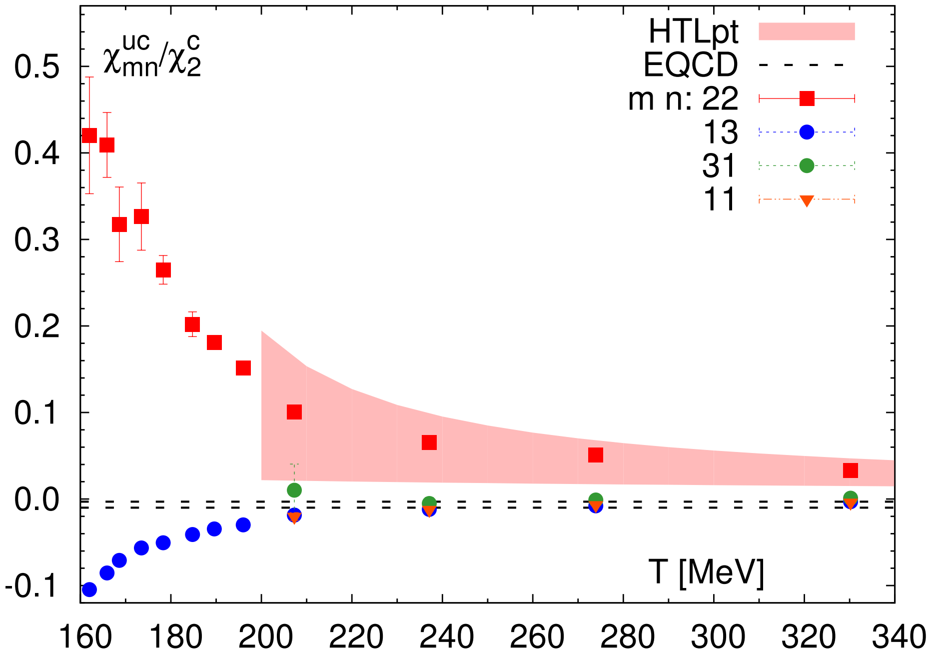
$$T_d(J/\psi) \simeq 250\text{MeV},\ T_d(\chi_b) \simeq 270\text{MeV},\ T_d(\Upsilon) > 407\text{MeV}$$

Charm correlations and fluctuations carry information about charm hadrons:

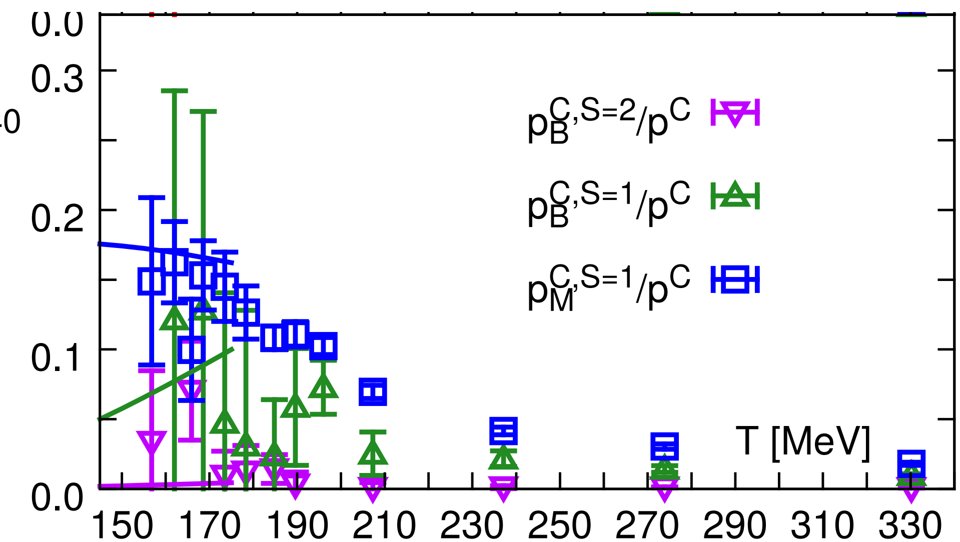
1) For $T<T_c$ fluctuations and correlations are described by hadron resonance gas
2) For $T>1.3T_c$ fluctuations and correlations are described by charm quark gas
3) In-medium open heavy flavor hadron may exist for $T_c < T < 1.3\ T_c$

- Need a link between spatial meson propagators and charm fluctuations to establish the existence and nature of open charm hadrons above T_c

Back-up:



Strange – charm hadrons:



High T (*T>250* MeV) :

$$\chi^{uc}_{22} \gg \chi^{uc}_{13} \sim \chi^{uc}_{31} \sim \chi^{uc}_{11}$$

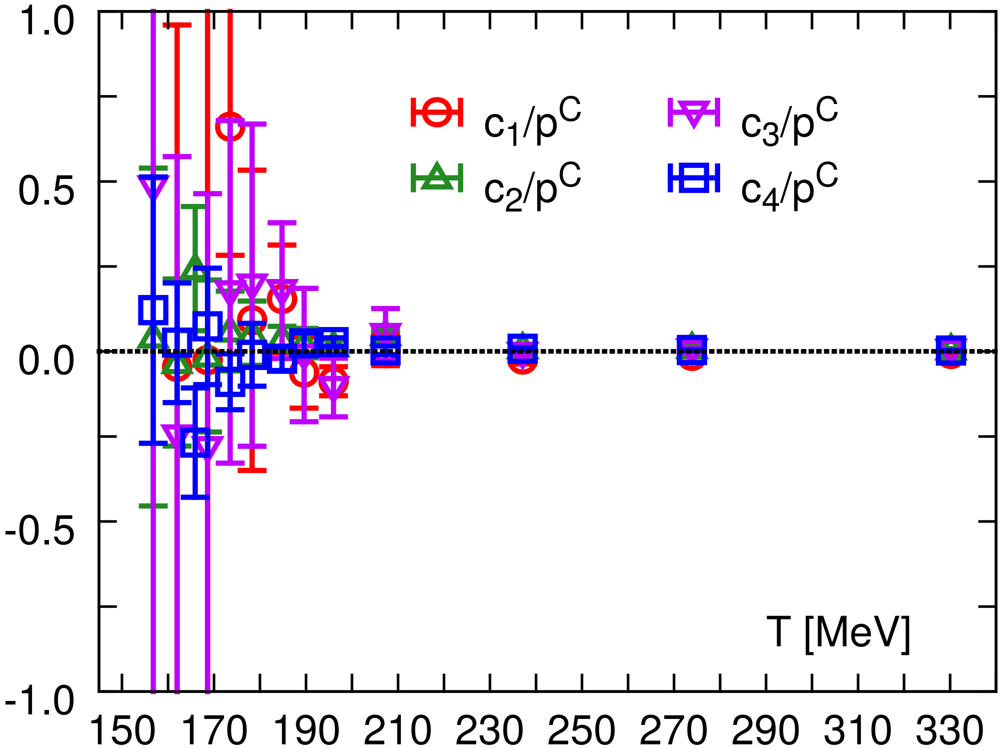
Low T: correlations are large
(bound states ?)

Does the quasi-particle model makes sense ?

4 non-trivial constraints on the model provided by : $\chi_{31}^{BC}, \chi_{31}^{SC}, \chi_{121}^{BSC}, \chi_{211}^{BSC}$

$$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0,$$
$$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} + 2\chi_{13}^{SC} - \chi_{31}^{SC} = 0$$
$$c_3 \equiv 6\chi_{112}^{BSC} + 6\chi_{121}^{BSC} + \chi_{13}^{SC} - \chi_{31}^{SC},$$
$$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC}.$$

← Diquark pressure is zero !



Models with charm quark only:
correlations from an effective mass

$$m_c = m_c(T, \mu_C, \mu_S, \mu_B)$$

Taylor expand the effective mass
in chemical potential

c_n
⇒ Un-natural fine tuning of
the expansion coefficients