



A Linear Boltzmann Transport Model

Tan Luo

*Central China Normal University
Institute of Particle Physics*



*In collaboration with
Shanshan Cao, Wei Chen, Yayun He, Longgang Pang, Xin-Nian Wang
and Yan Zhu*

Outline

- Introduction
- A Linear Boltzmann Transport (LBT) model
- A Coupled LBT-hydro (CoLBT-hydro) model
- Summary and Outlook

A Linear Boltzmann Transport (LBT) Model

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1 (C_{elastic} + C_{inelastic})$$

Linear Boltzmann jet Transport

Elastic collision + Induced gluon radiation.

Follow the propagation of recoiled parton.

Include recoiled parton in jet reconstruction.

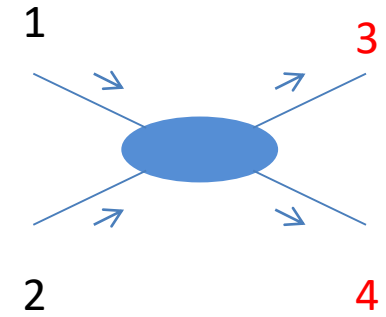
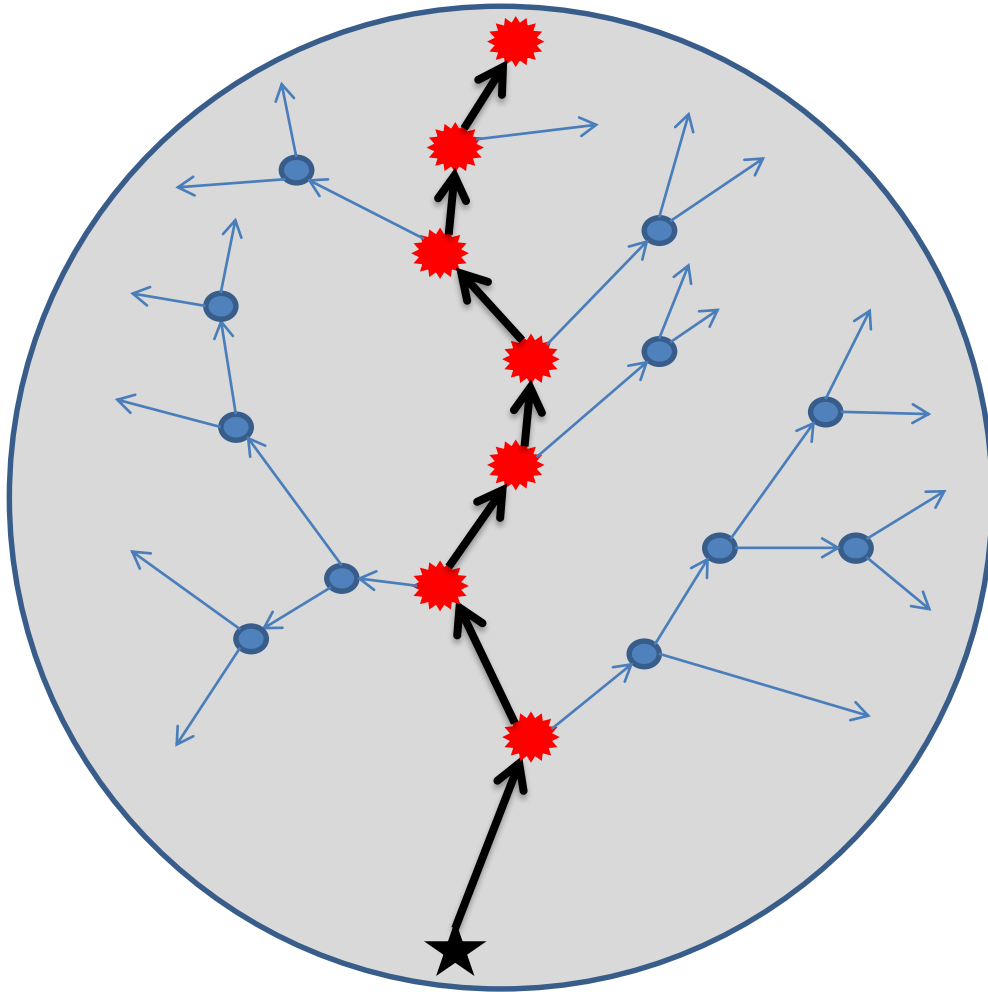
Linear Approximation



It works when the jet induced medium excitation $\delta f \ll f$.

Jet induced medium excitation

("Negative" parton for the **back reaction**)

Jet induced medium excitation: recoiled parton

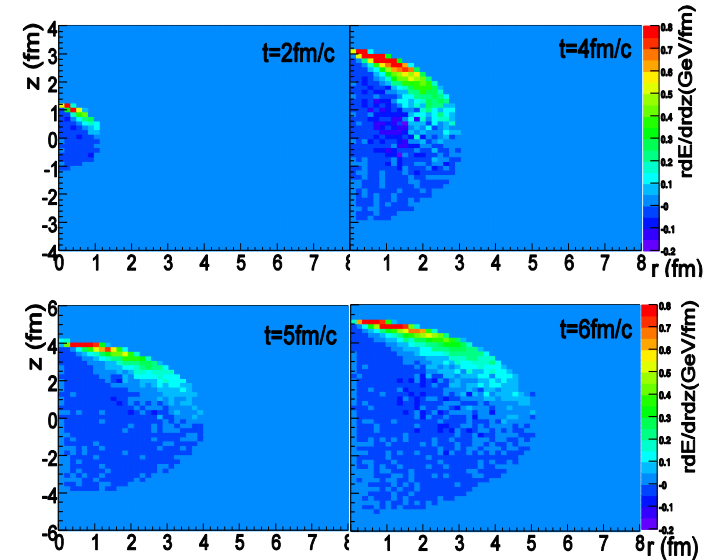
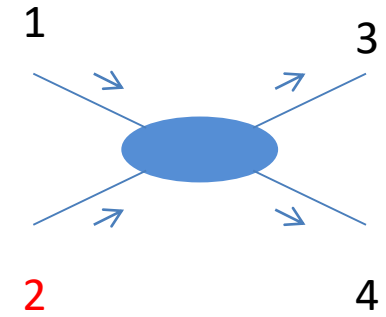
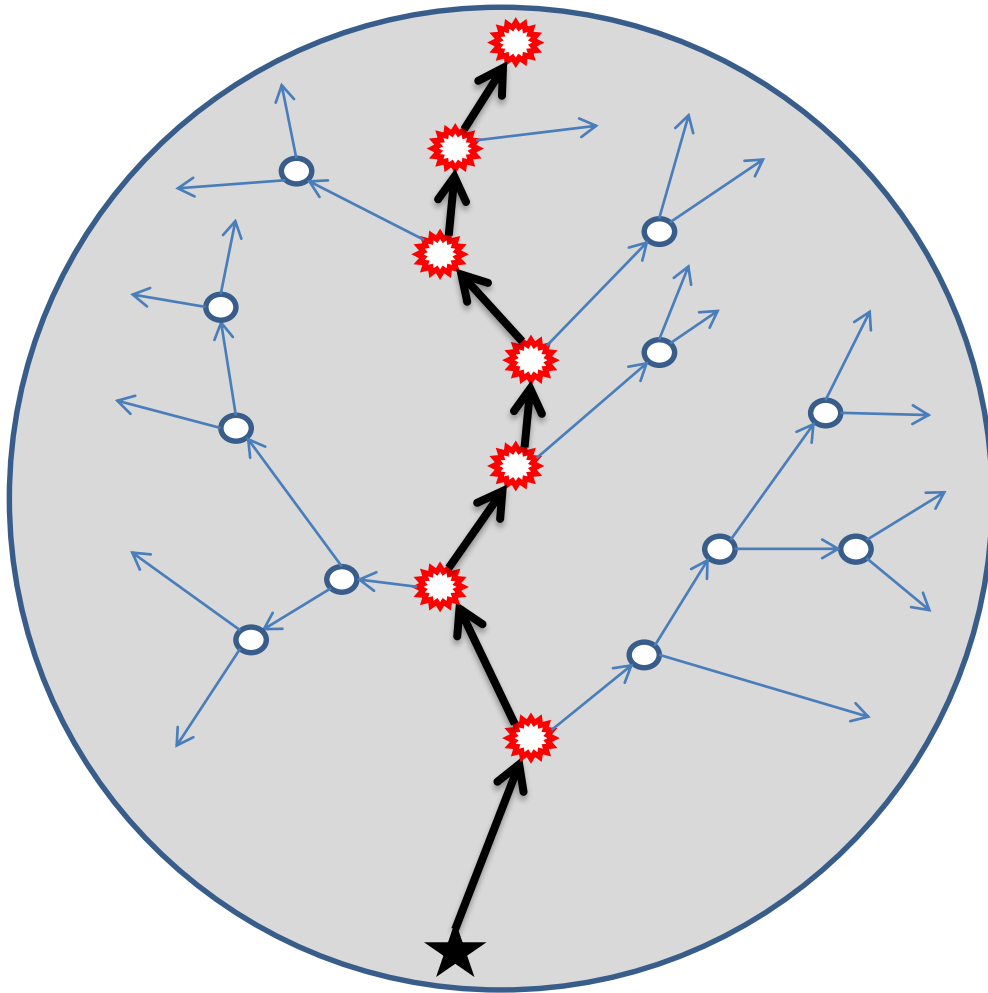


-  Leading parton-----thermal parton scattering
-  recoiled parton-----thermal parton scattering

Linearized Boltzmann jet transport
neglect scatterings between recoiled
medium partons.

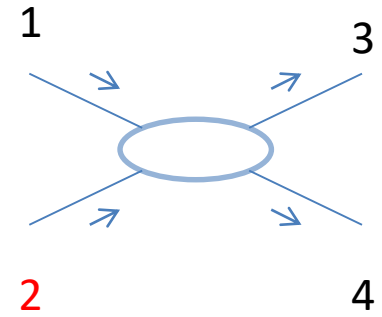
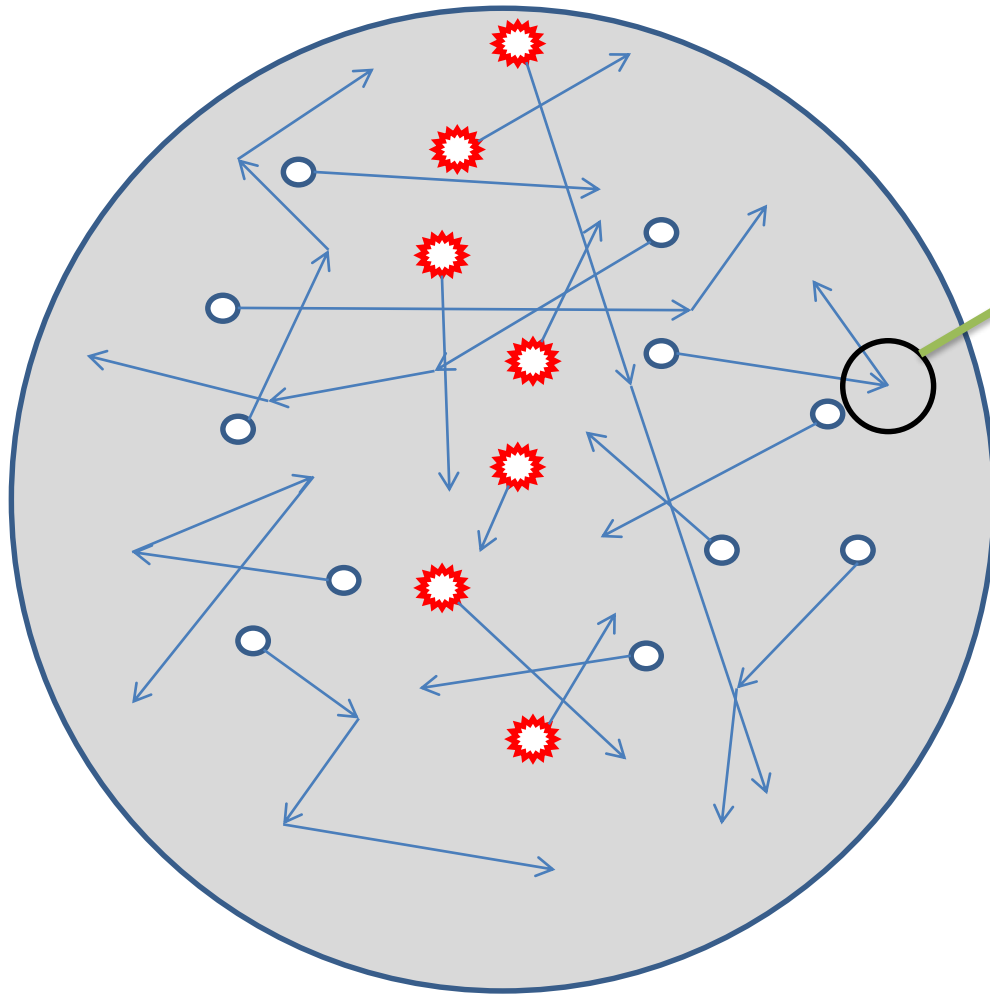
It's a good approximation when the jet
induced medium excitation $\delta f \ll f$.

Jet induced medium excitation: particle hole



One has to subtract the 4-momentum of negative particle when combine it to jet

Jet induced medium excitation: back reaction



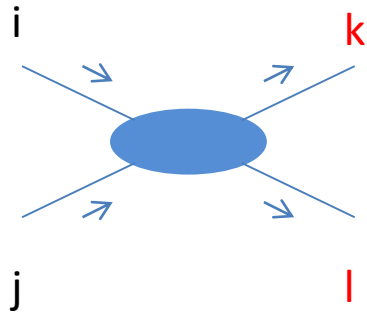
thermal parton-----thermal parton
scattering

the negative particle is also traveling in
the medium

One has to subtract the 4-momentum of
negative particle when combine it to jet

Complete set of elastic processes

Single scattering



$$i, j = g, u, d, s, \bar{u}, \bar{d}, \bar{s}$$

Jussi Auvinen, Kari J. Eskola, Thorsten Renk

Phys.Rev. C82 024906

- Scattering rate for a process $ij \rightarrow kl$ in the local rest frame of the fluid

$$\Gamma_{ij \rightarrow kl} = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} \times f_j(p_2 \cdot u, T)$$

$$\times |M|_{ij \rightarrow kl}^2(s, t, u) \times S_2(s, t, u) \times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)$$

$$S_2(s, t, u) = \theta(s \geq 2\mu_D^2) \theta(-s + \mu_D^2 \leq t \leq -\mu_D^2) \quad \mu_D^2 = \left(\frac{3}{2}\right) 4\pi\alpha_s T^2$$

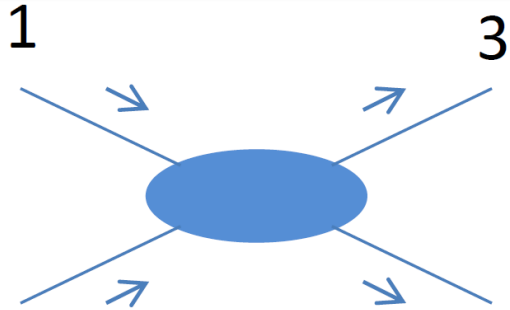
- The mean free path

$$\Gamma_i = \sum_{j,(kl)} \Gamma_{ij \rightarrow kl} = 1/\lambda_0$$

$$P(\Delta t) = 1 - e^{-\Gamma_i \Delta t}$$

$$P(ij \rightarrow kl) = \frac{\Gamma_{ij \rightarrow kl}}{\Gamma_i}$$

Energy distribution of the recoiled parton



Single scattering

Dominance of small angle scattering.

Switch of flavor and species of the leading parton.

2

4

$$gg \rightarrow gg$$

$$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$$

$$gg \rightarrow q\bar{q}$$

$$q_i\bar{q}_i \rightarrow gg$$

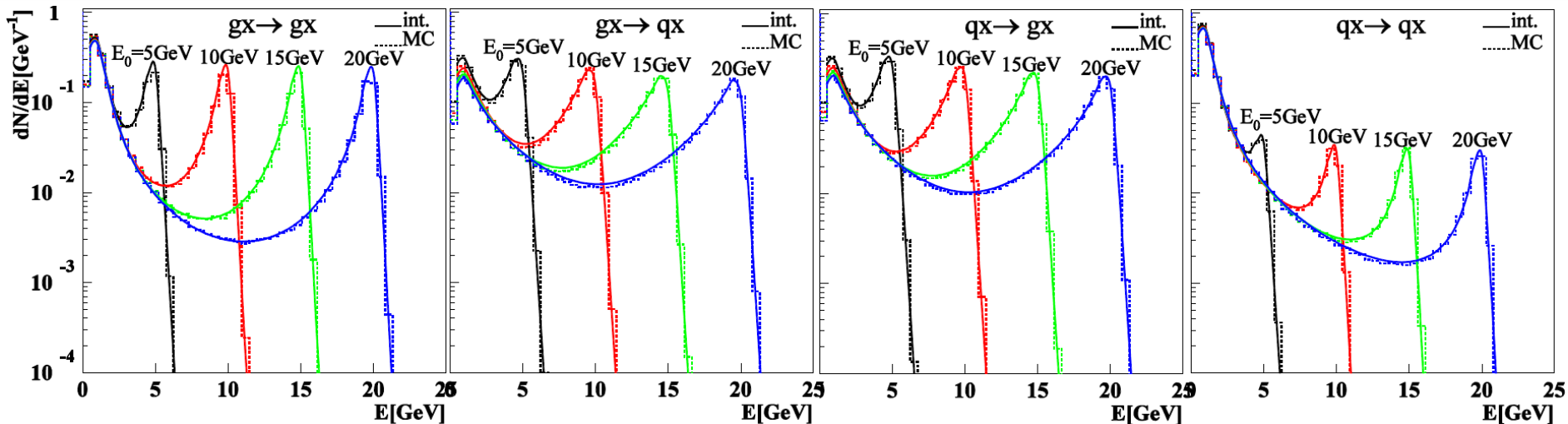
$$q_i\bar{q}_i \rightarrow q_j\bar{q}_j$$

$$q_i\bar{q}_i \rightarrow q_i\bar{q}_i$$

$$q_i g \rightarrow q_i g$$

$$q_i q_j \rightarrow q_i q_j$$

$$q_i q_i \rightarrow q_i q_i$$



Medium-induced gluon radiations

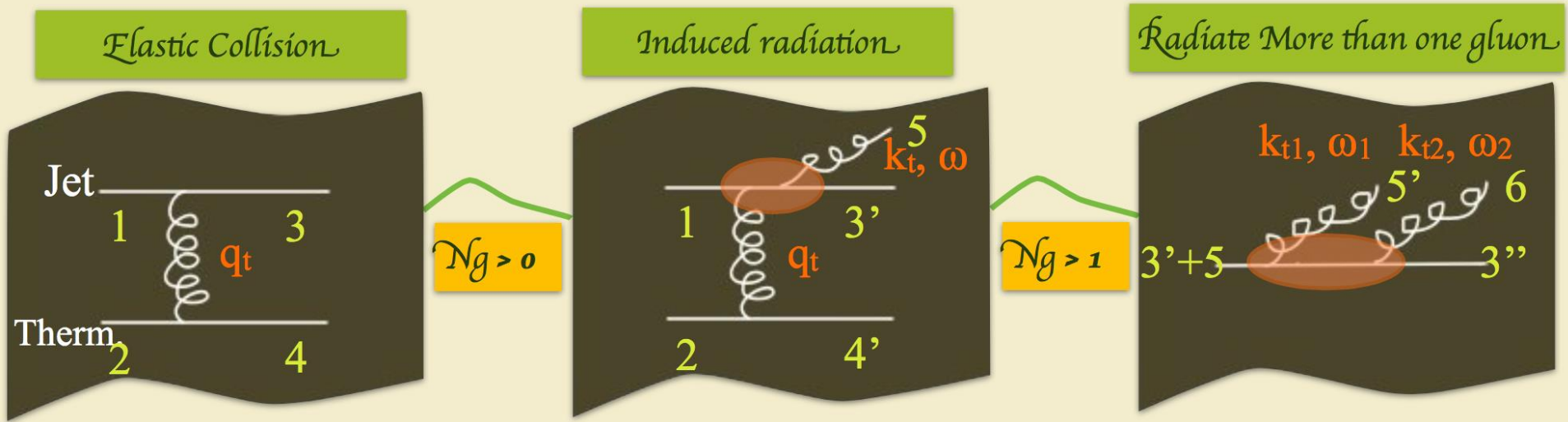
Radiated gluon distribution: Guo and Wang (2000), Majumder (2012); Zhang, Wang and Wang (2004)

$$\frac{dN_g}{dx dk_{\perp}^2 dt} = \frac{2C_A \alpha_s P(x)}{\pi k_{\perp}^4} \hat{q} \left(\frac{k_{\perp}^2}{k_{\perp}^2 + x^2 M^2} \right) \sin^2 \frac{t-t_i}{2\tau_f} \quad \tau_f = 2Ex(1-x) / (k_{\perp}^2 + x^2 M^2)$$

Multiple gluon emissions: $P(N_g, \langle N_g \rangle) = \frac{\langle N_g \rangle^{N_g} e^{-\langle N_g \rangle}}{N_g!}$

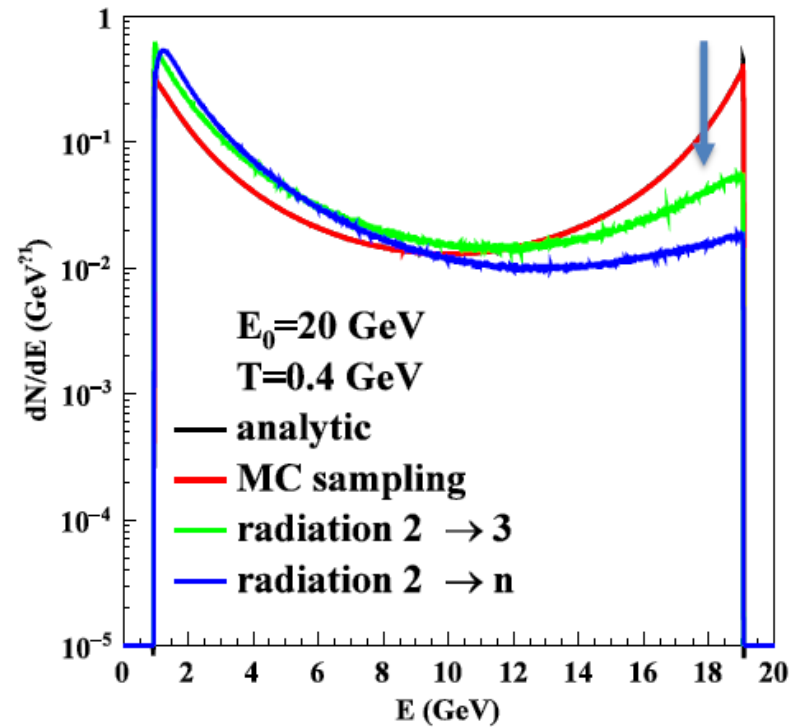
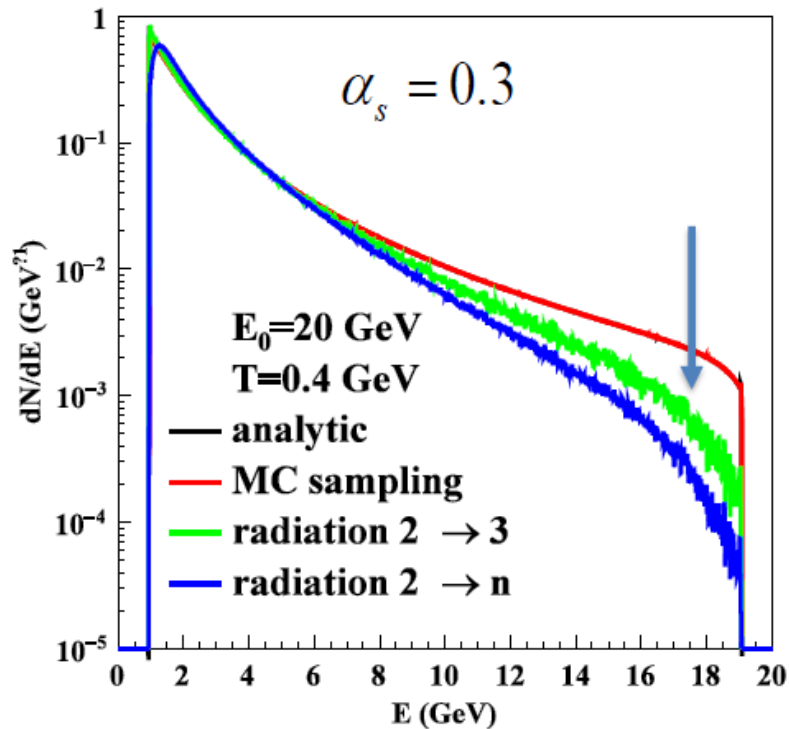
Induced radiations are accompanied by elastic collisions.

Jet medium Interaction:



Energy distribution of the radiated gluon

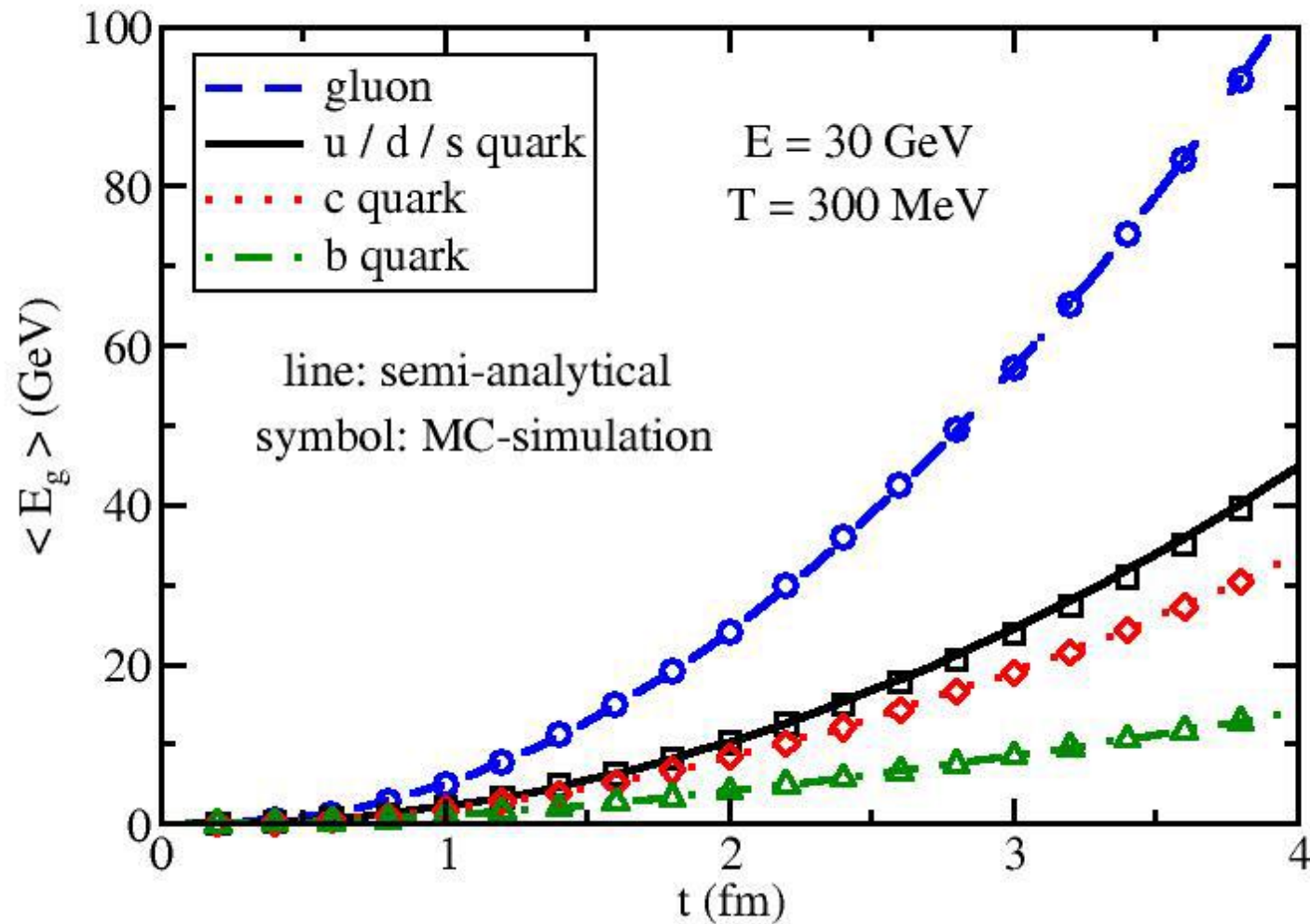
Global energy-momentum conservation in 2->3 and 2->n processes



$$P_{q \rightarrow qg}(x) = \frac{(1-x)(1+(1-x)^2)}{x}$$

$$P_{g \rightarrow gg}(x) = \frac{2(1-x+x^2)^3}{x(1-x)}$$

Energy loss from radiation process in an uniform medium



Jet induced medium excitation (Energy distribution in space)

Initial jet parton: gluon

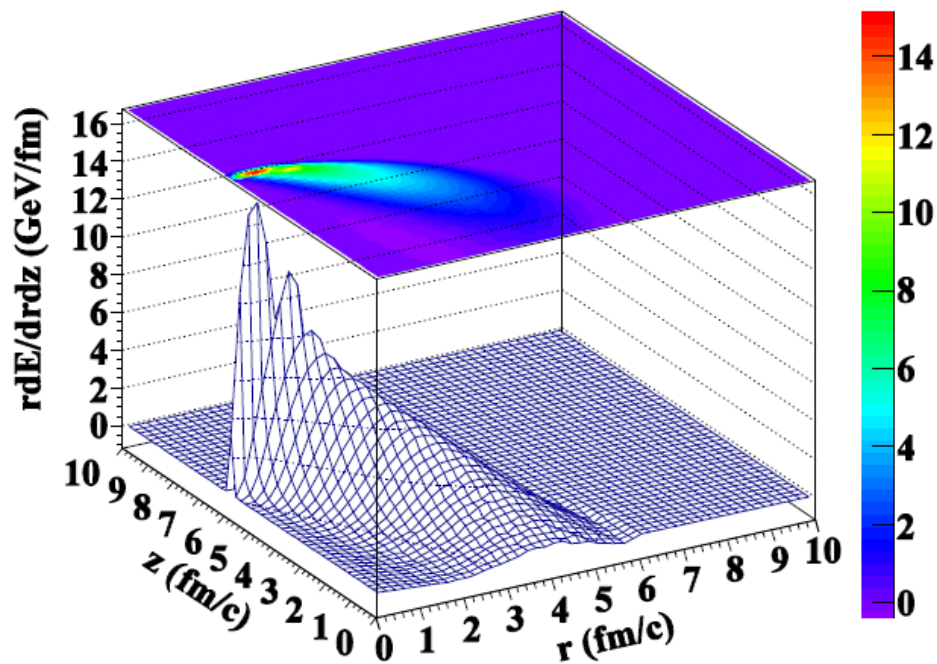
$E = 100 \text{ GeV}$

$T = 0.4 \text{ GeV}$ $\alpha_s = 0.3$

- Mach Cone like shock wave and the diffusion wake.

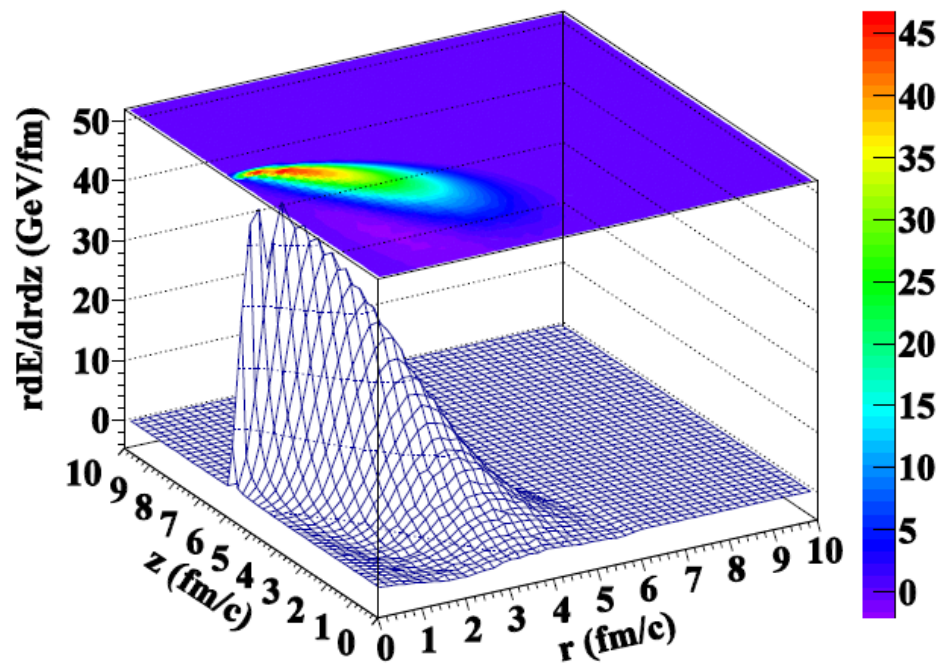
Elastic only

gluon: elastic only at $t=6 \text{ fm/c}$



Elastic + Radiation

gluon: elastic + radiation at $t=6 \text{ fm/c}$

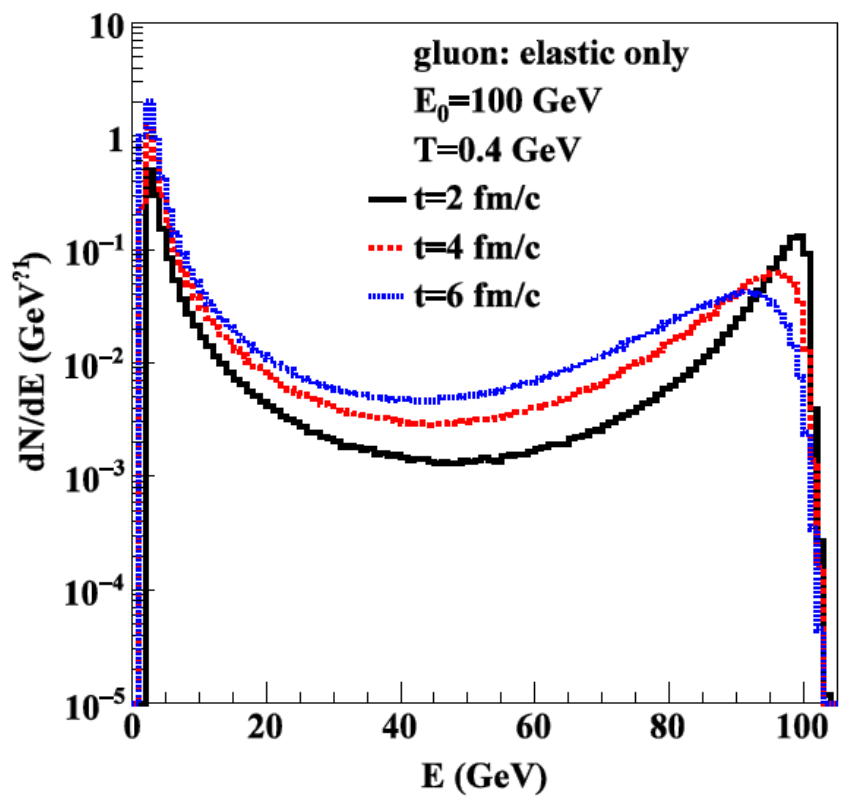


Jet induced medium excitation (Energy distribution at different time)

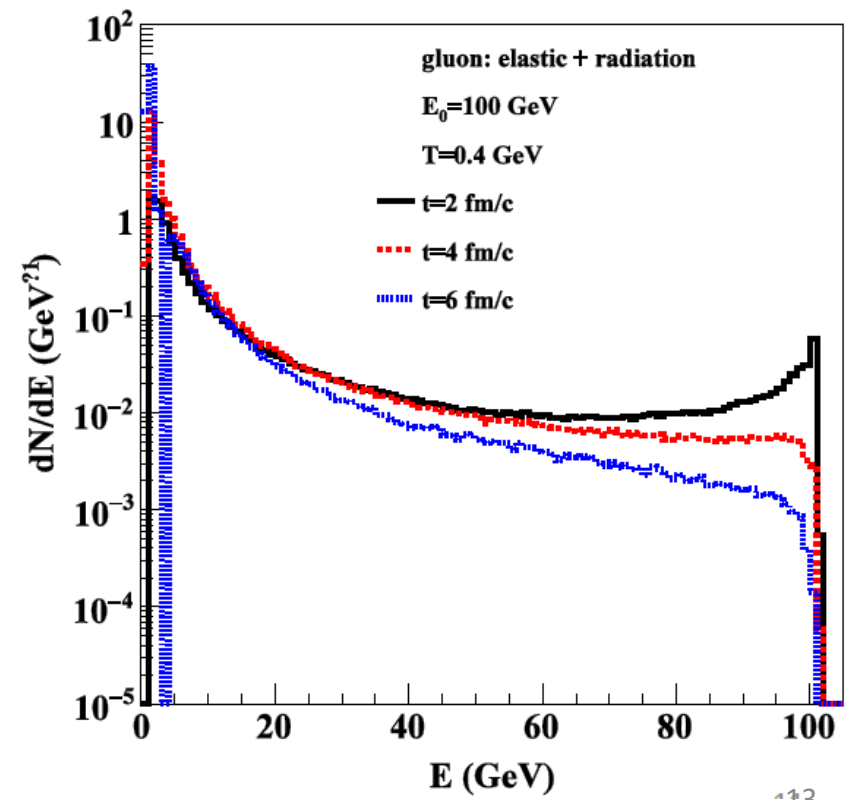
Initial jet parton: gluon
 $E = 100 \text{ GeV}$
 $T = 0.4 \text{ GeV}$ $\alpha_s = 0.3$

- Depletion of the energy of the leading parton.

Elastic only



Elastic + Radiation

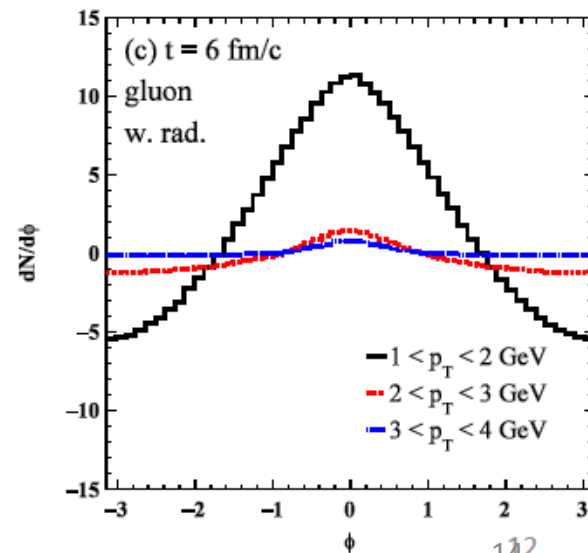
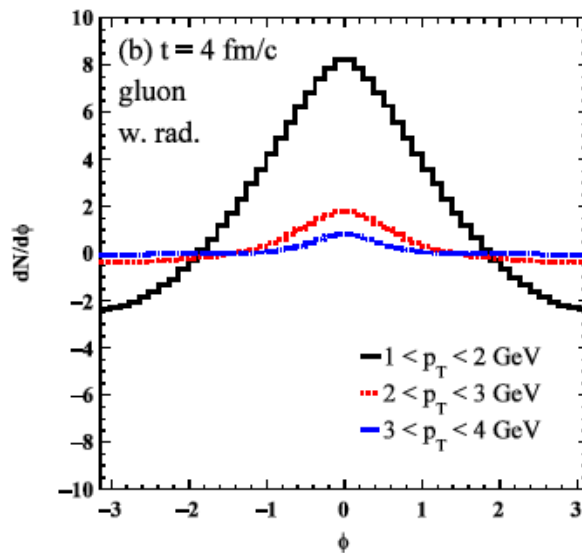
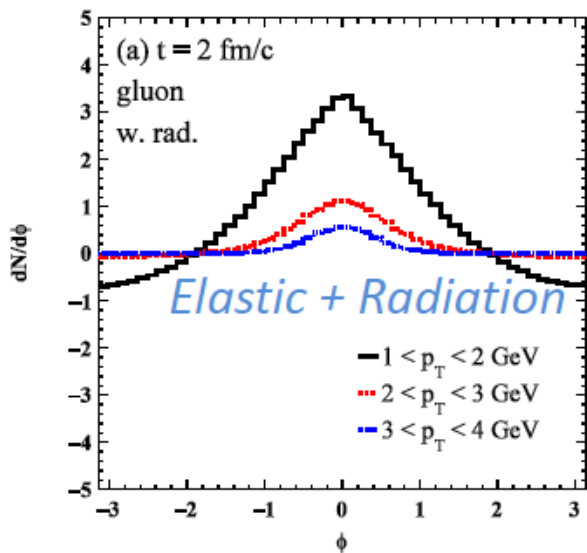
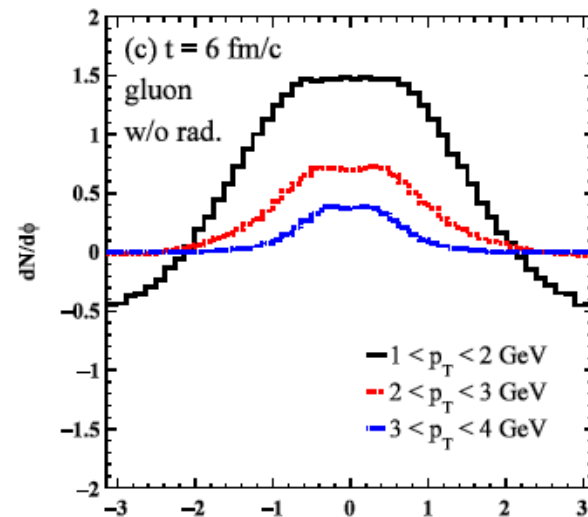
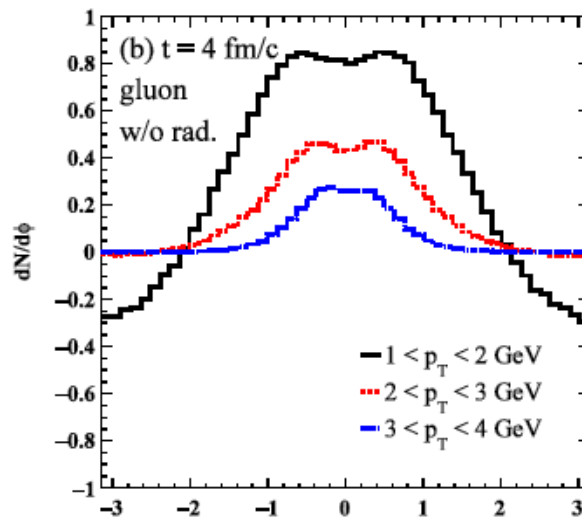
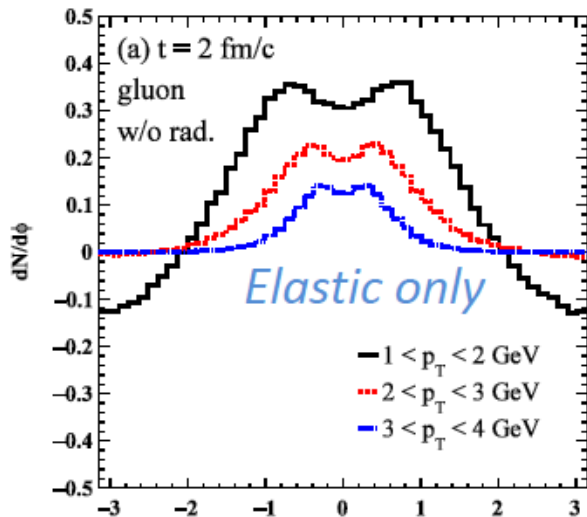


Jet induced medium excitation (Angular distribution)

$t = 2 \text{ fm}$

$t = 4 \text{ fm}$

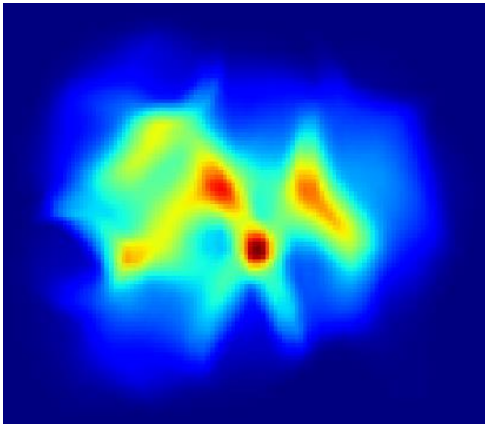
$t = 6 \text{ fm}$



Jets in a 3+1D hydro

Initial jet shower partons from a p+p collision (Pythia or Hijing)

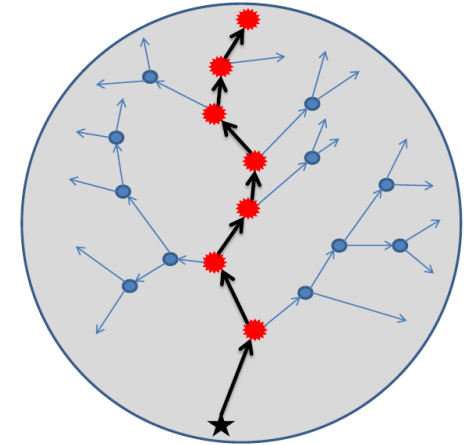
- 3+1D Ideal hydro



$\epsilon T u$

LBT Model

- Location of jets are decided according probability of binary collision.



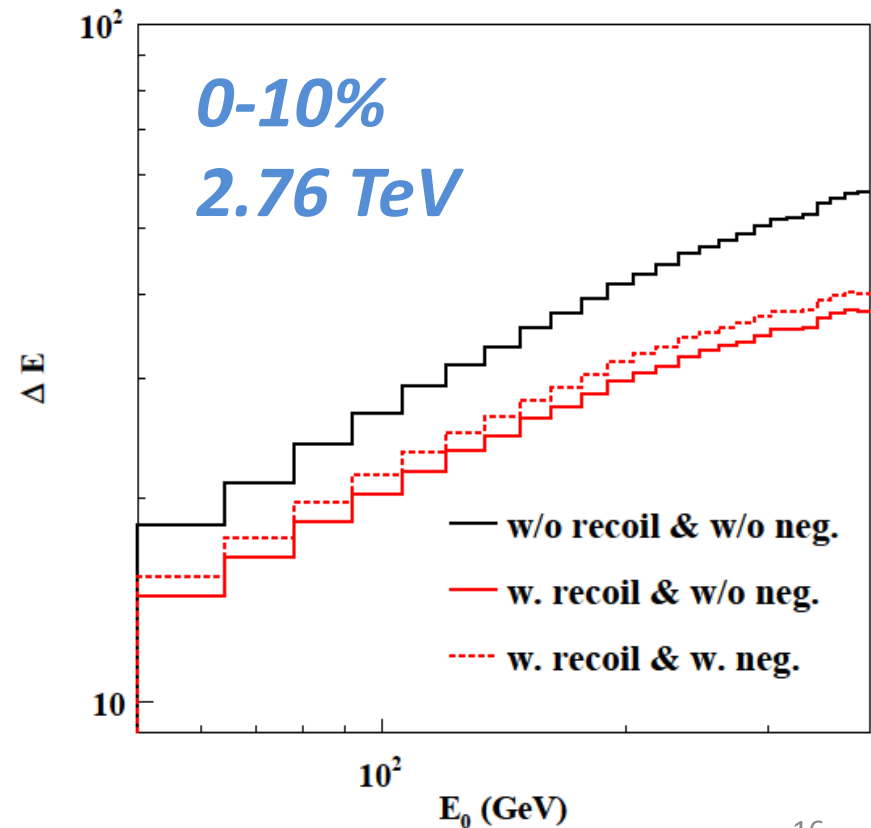
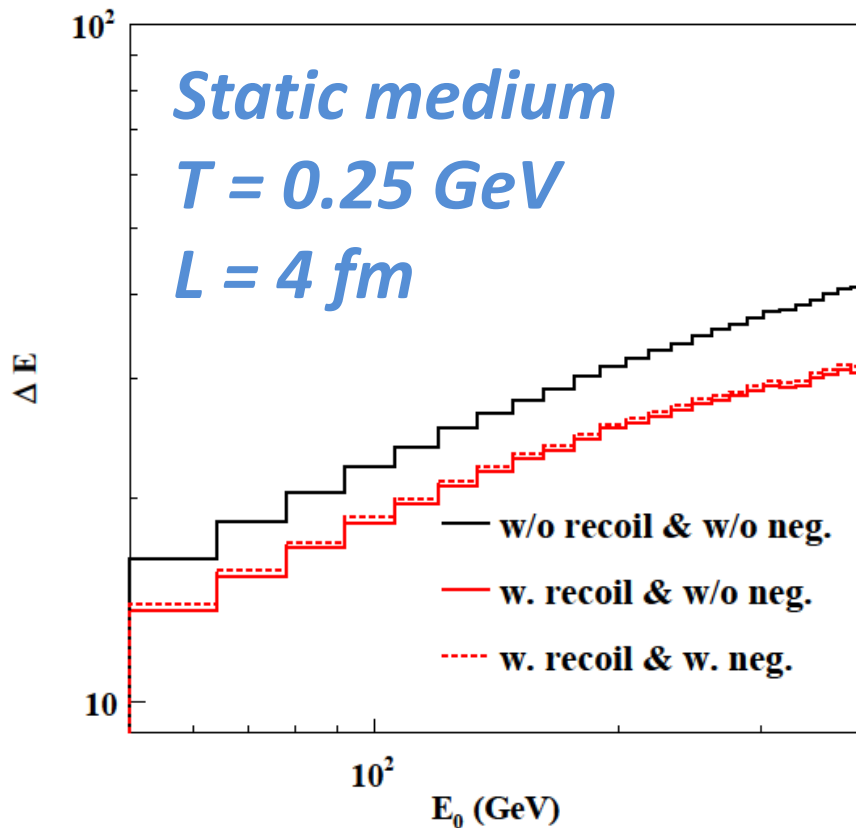
L-G. Pang, Q. Wang, X-N. Wang
Phys.Rev. C86 (2012) 024911

M. Cacciari, G. P. Salam and G. Soyez
Eur. Phys. J. C 72, 1896 (2012).

Jet reconstruction (Fastjet)

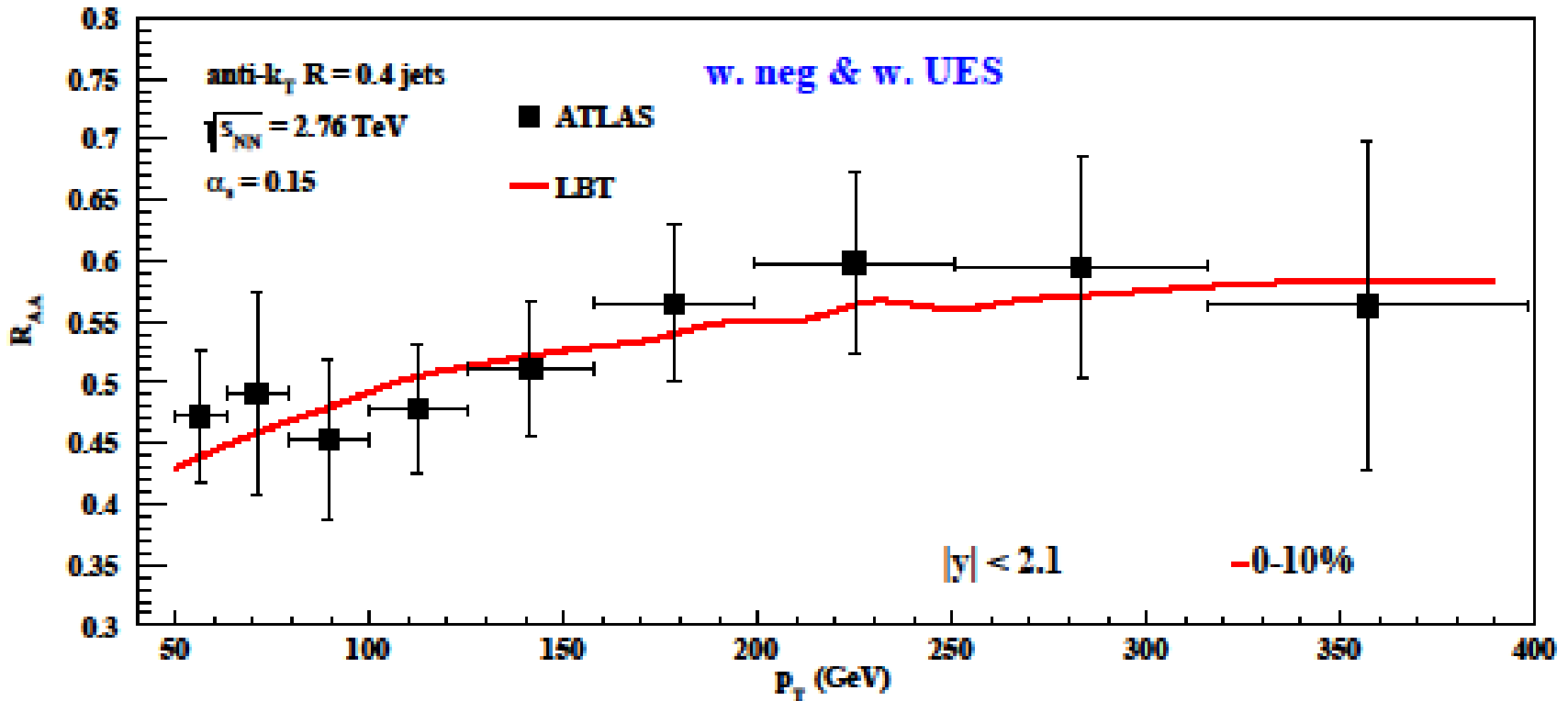
Recoiled effect in the reconstructed jets

- The inclusion of the recoiled parton in the reconstructed jets will reduce the jet energy loss.
- The recoiled effect is more significant in the evolving medium.



Nuclear modification factor

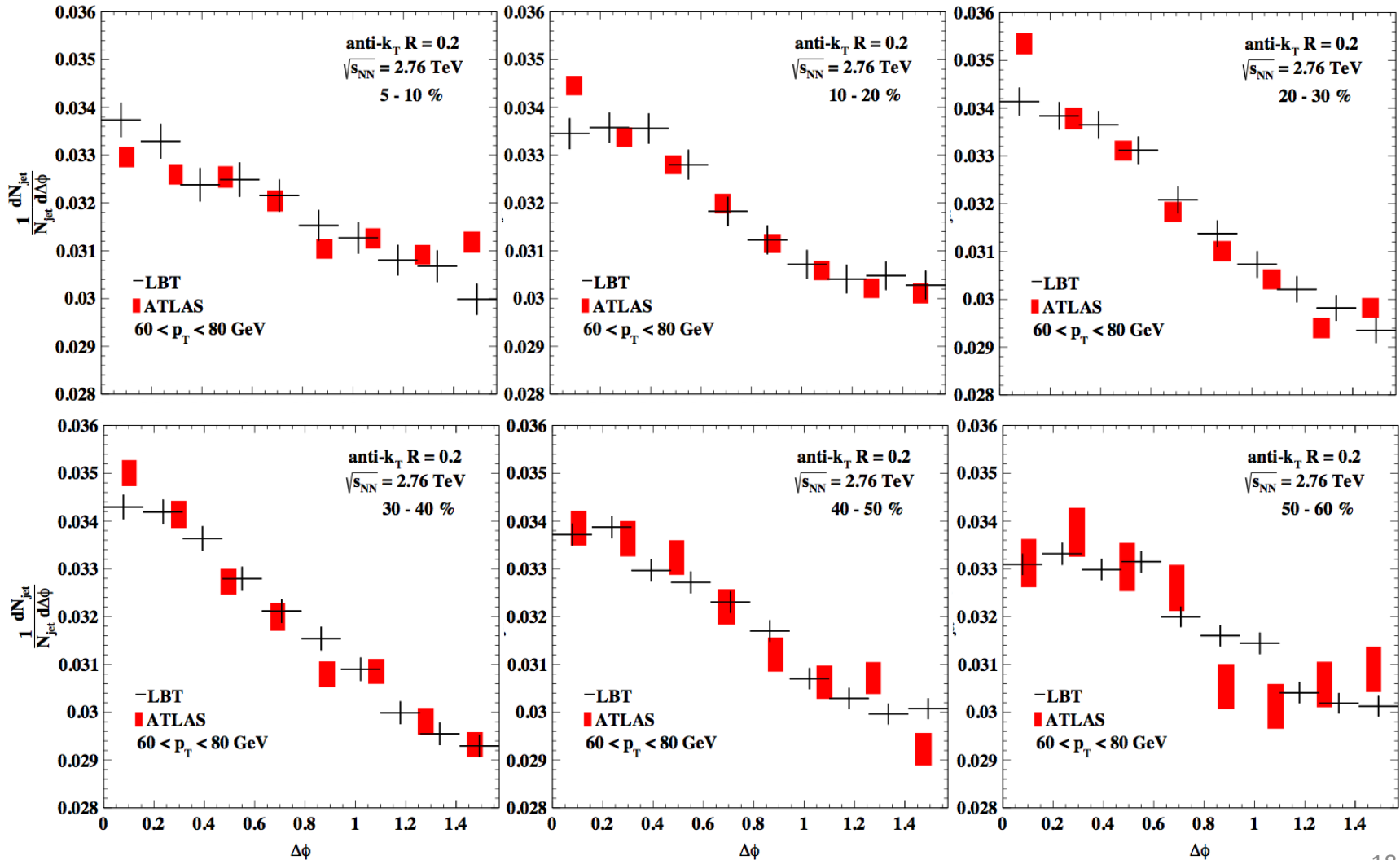
- The only parameter strong coupling constant α_s is fixed.
- We first calculate the single jet R_{AA} to extract the value of α_s .
(fix the strength of jet-medium interaction)



Jet azimuthal distribution with different centralities

Anisotropy shows up

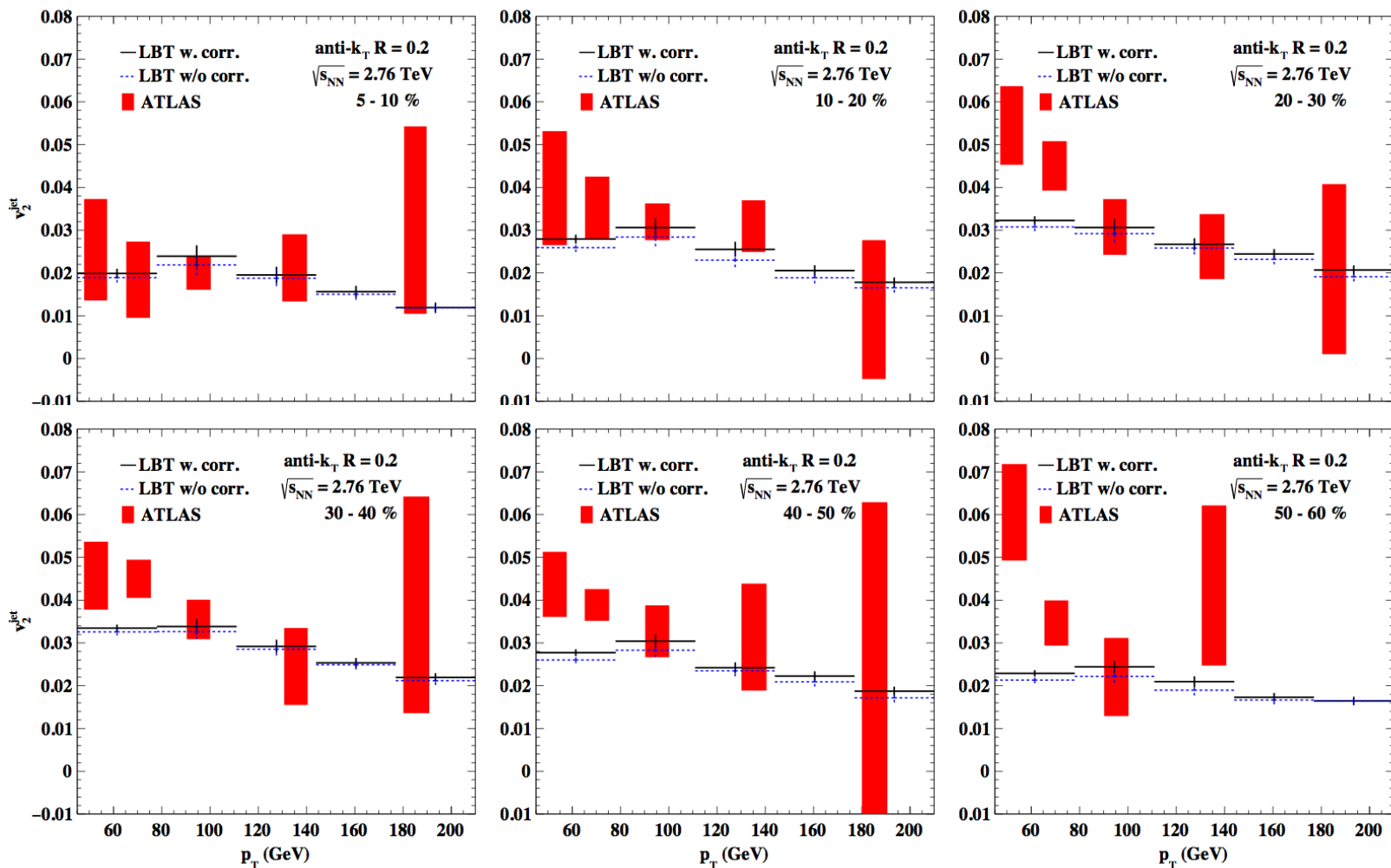
$$\Delta\phi = \phi^{jet} - \Psi_2$$



Jet v_2 with different centralities

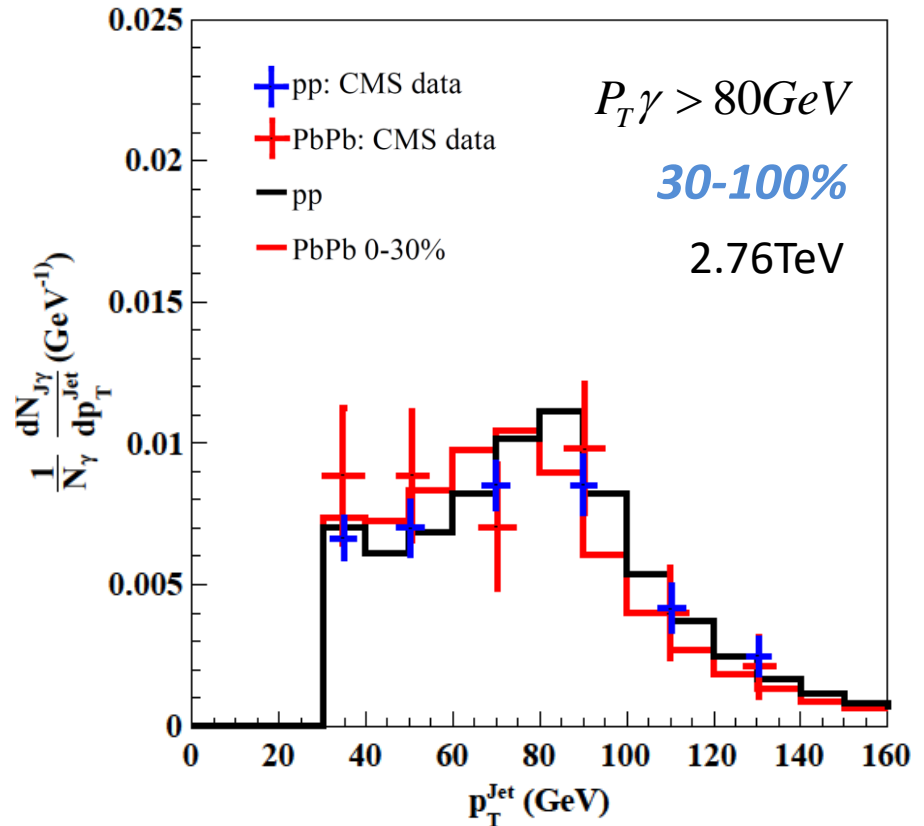
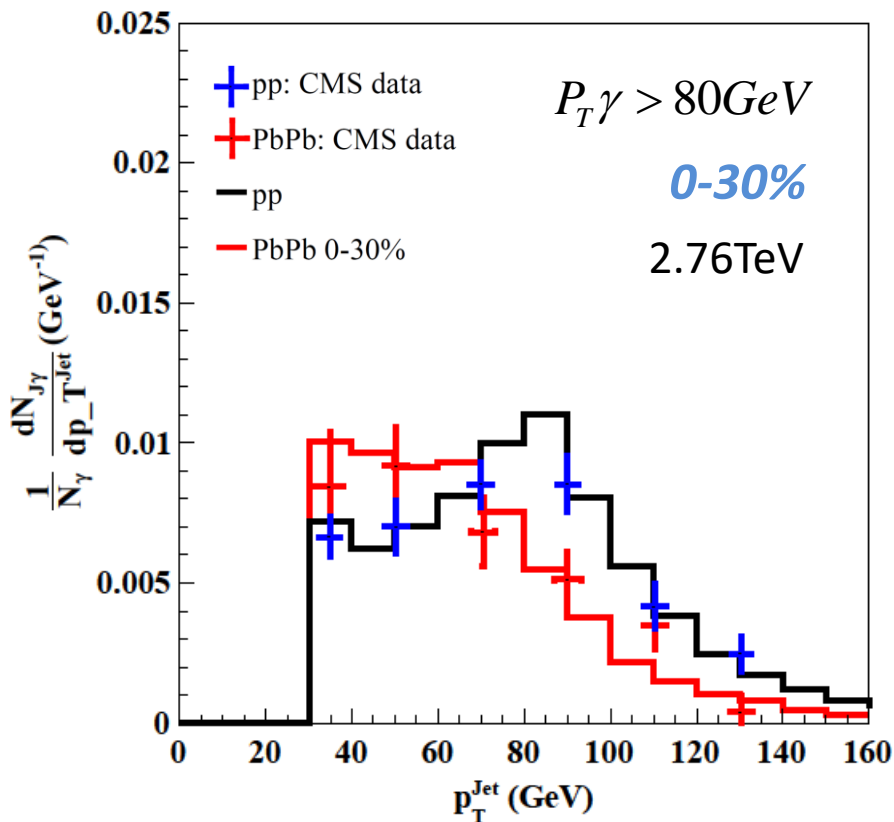
$$v_2^{jet} = \langle \cos(2[\phi^{jet} - \Psi_2]) \rangle$$

$$v_2^{jet} = \frac{\langle v_2^{soft} \cos(2[\phi^{jet} - \Psi_2]) \rangle}{\sqrt{\langle (v_2^{soft})^2 \rangle}}$$



p_T distribution of gamma-jet in heavy-ion collisions

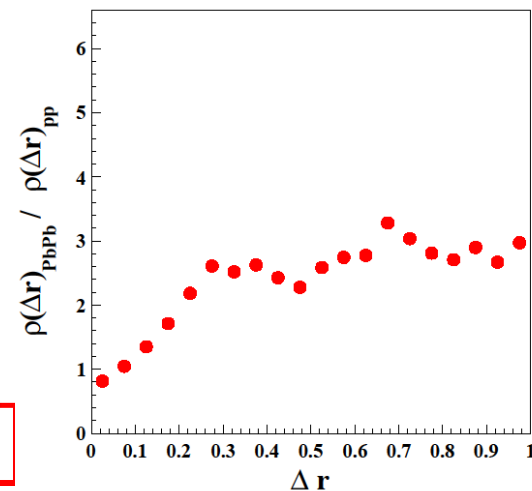
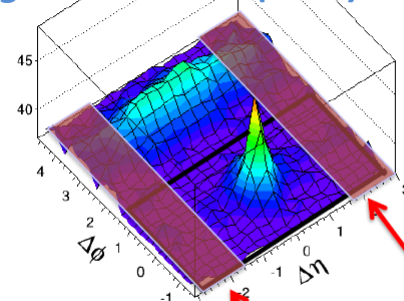
- Shift of the peak of the p_T distribution
- Path length dependence of the energy loss



Jet shape of gamma-jets in heavy-ion collisions

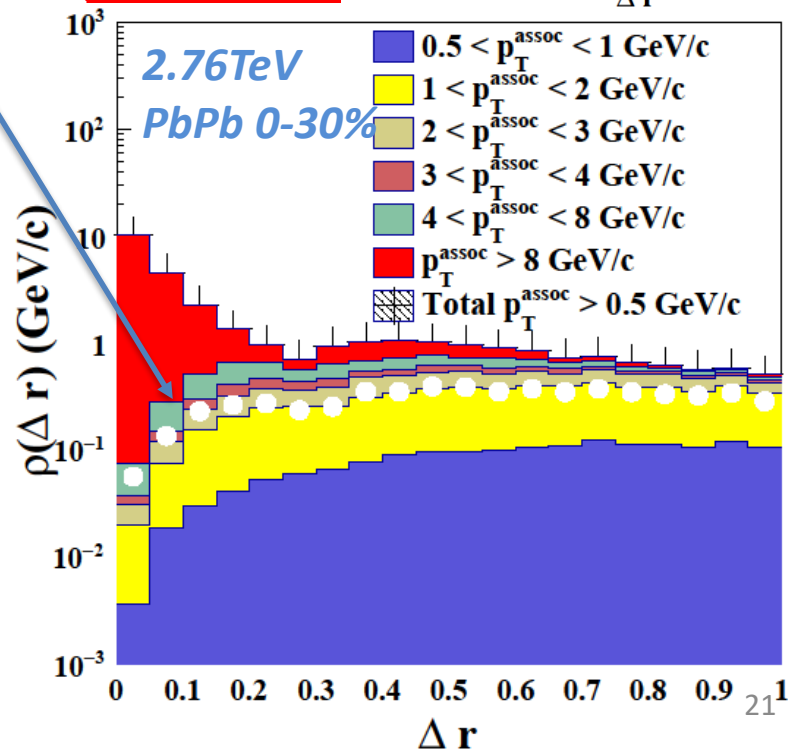
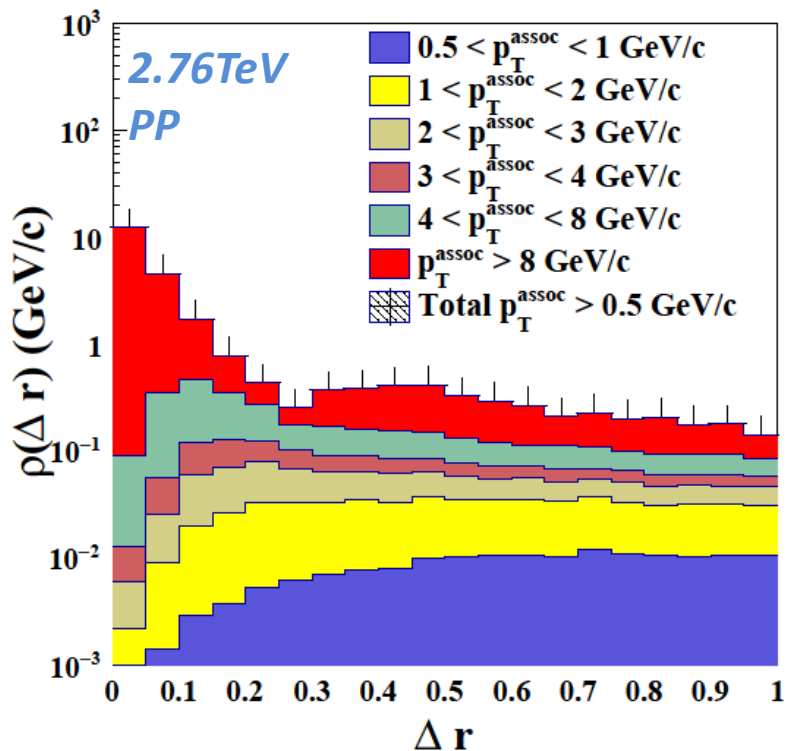
- Energy lost by the hard parton is transport out of the jet cone by the soft parton.

Leading
Olga Evdokimov (CMS)



Recoiled effect

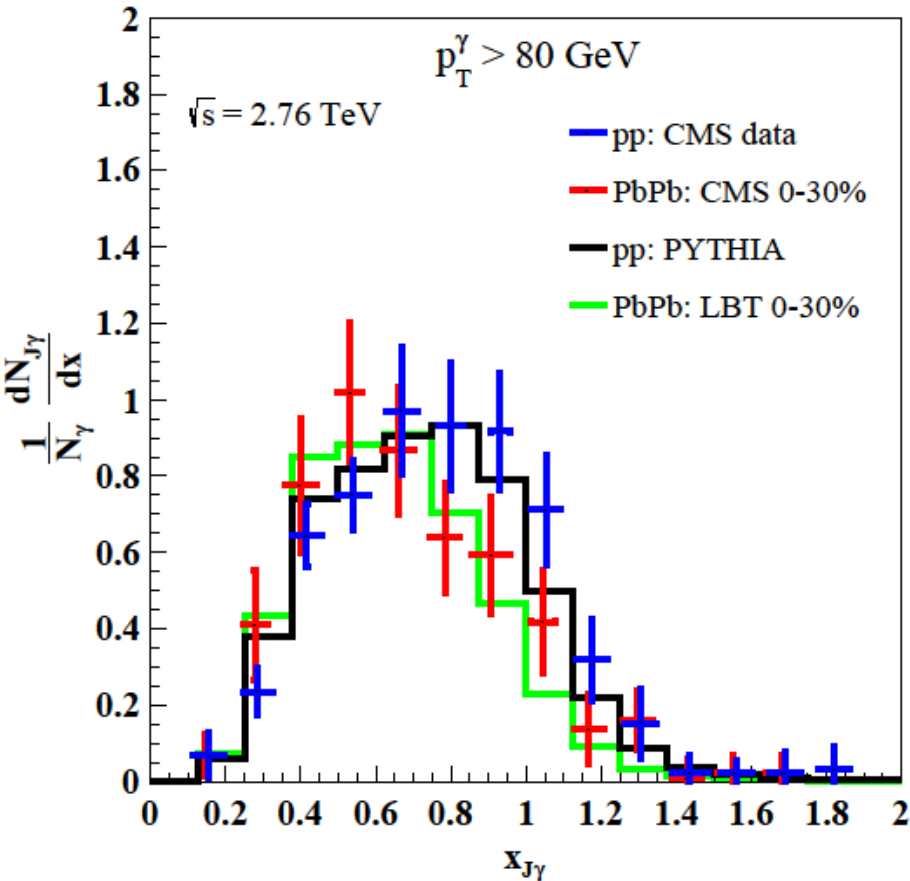
“Sideband” region
 $1.5 < |\Delta\eta| < 2.5$



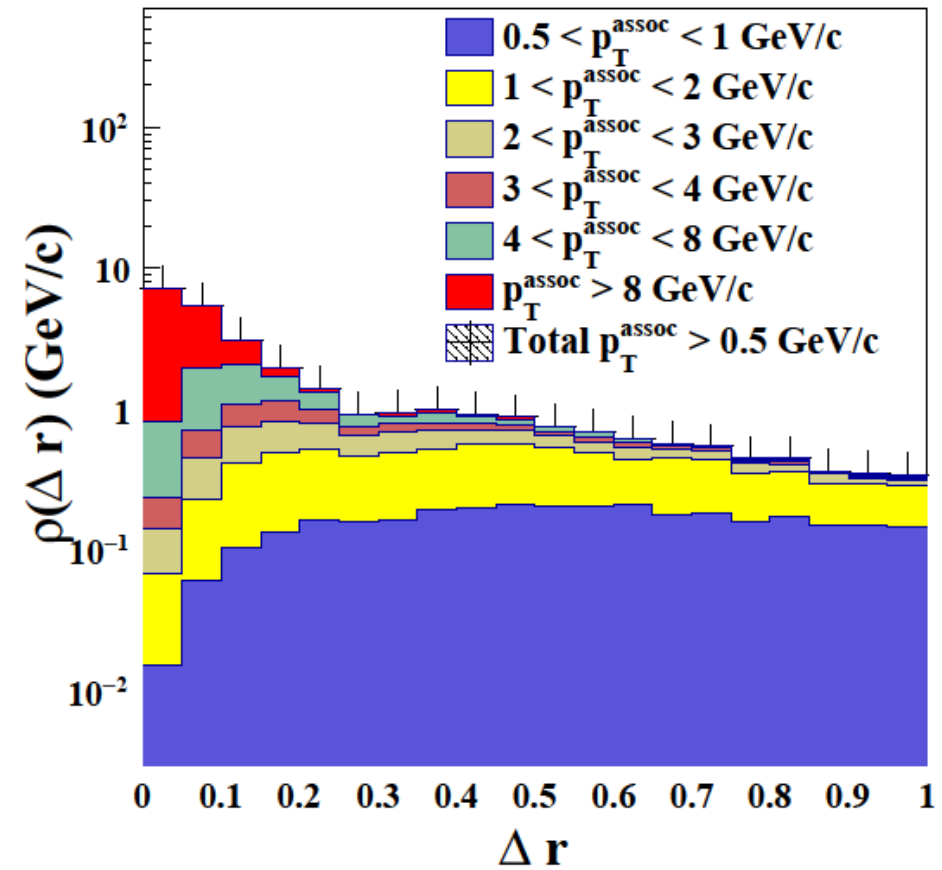
Jet reconstruction with recombination model

Han, Fries and Ko, Phys. Rev. C93 (2016) 045207

Gamma-jet asymmetry



Jet shape



preliminary

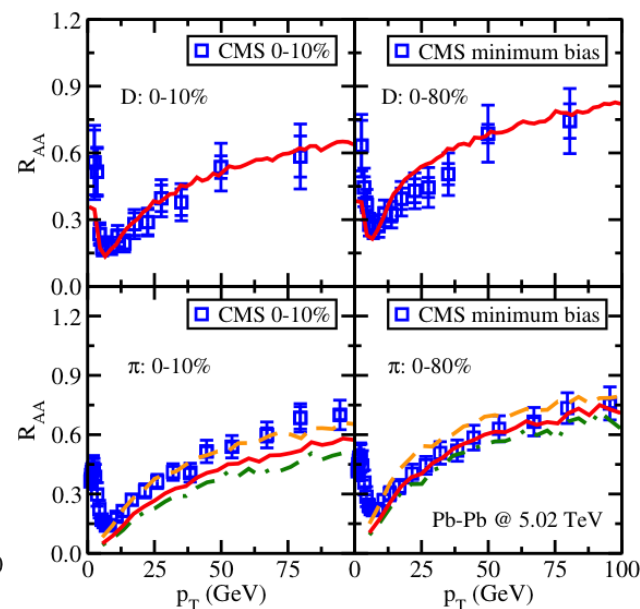
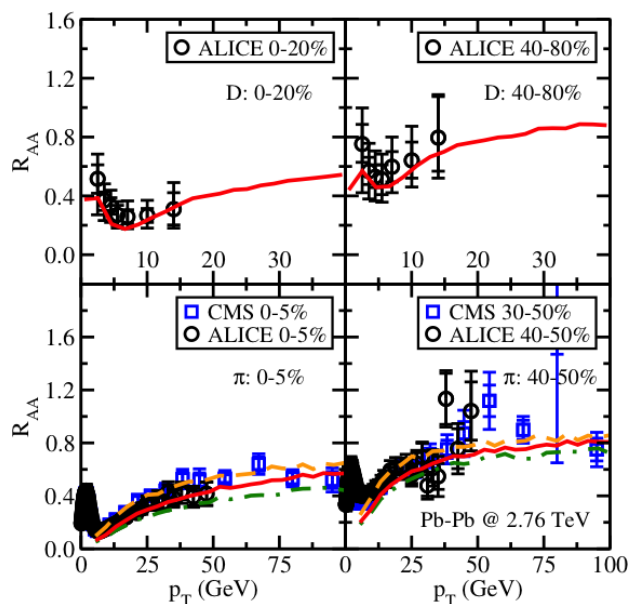
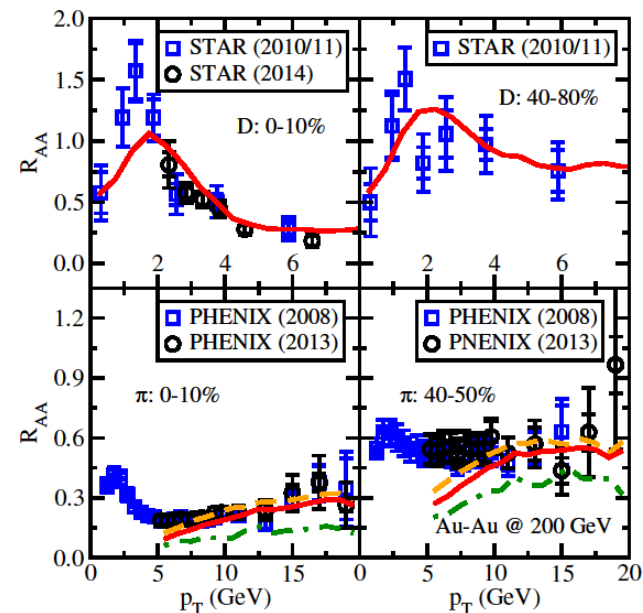
Hadron R_{AA} in heavy ion collision

- Simultaneous description of single hadron R_{AA} from RHIC to LHC (AuAu@200GeV, PbPb@2760GeV and PbPb@5020GeV, 2 centrality bins for each system, 6 data sets in total)

AuAu@200GeV

PbPb@2760GeV

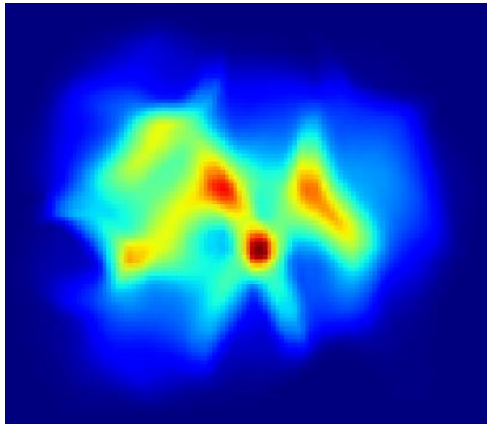
PbPb@5020GeV



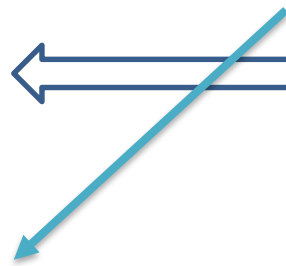
Beyond LBT model (modified medium background)

- Linear approximation : jet induced medium excitation $\delta f \ll f$.
- Jet-Medium interaction : Where is the modification of the thermal background ?

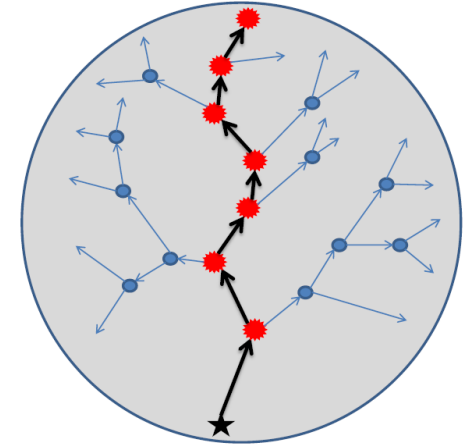
Modified medium background



$\epsilon T u$



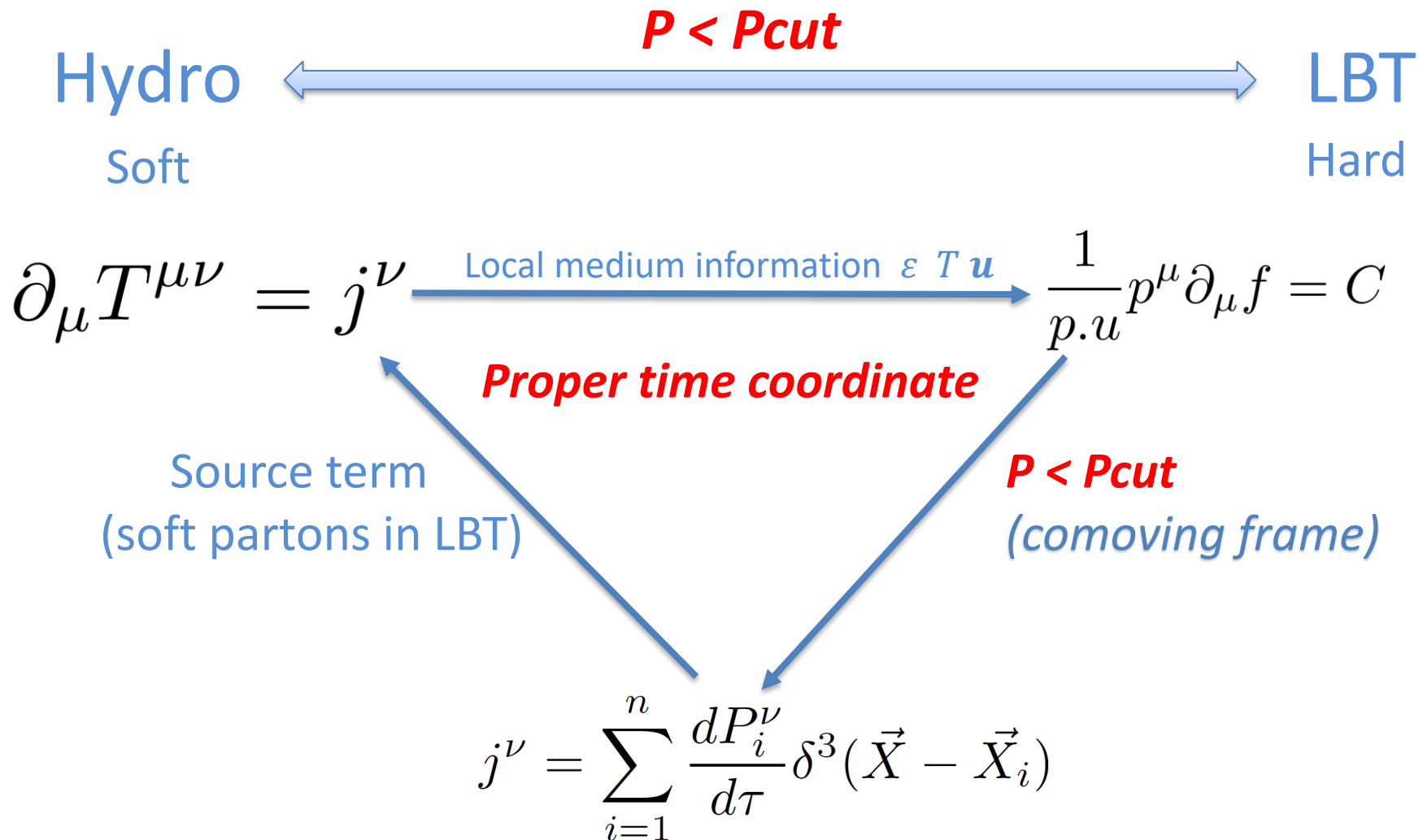
JET



Energy and momentum deposited from the jets as source terms into hydro

CoLBT-Hydro model
(A coupled LBT Hydro (3+1D) Model)

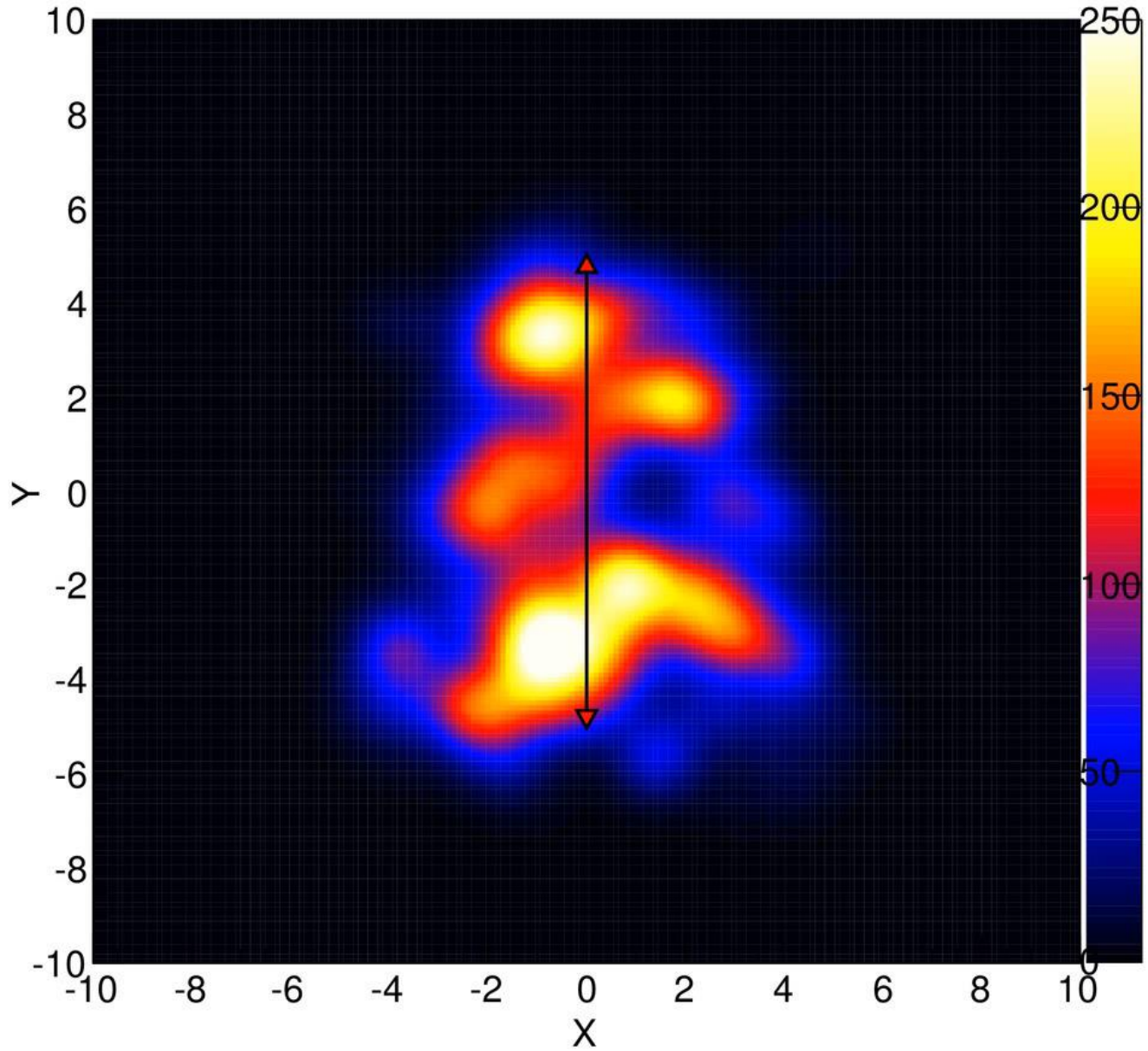
A coupled LBT hydro (3+1D) model



event-by-event (3+1D) hydrodynamics

Longgang Pang, Qun Wang, Xin-Nian Wang *Phys.Rev. C86 (2012) 024911*

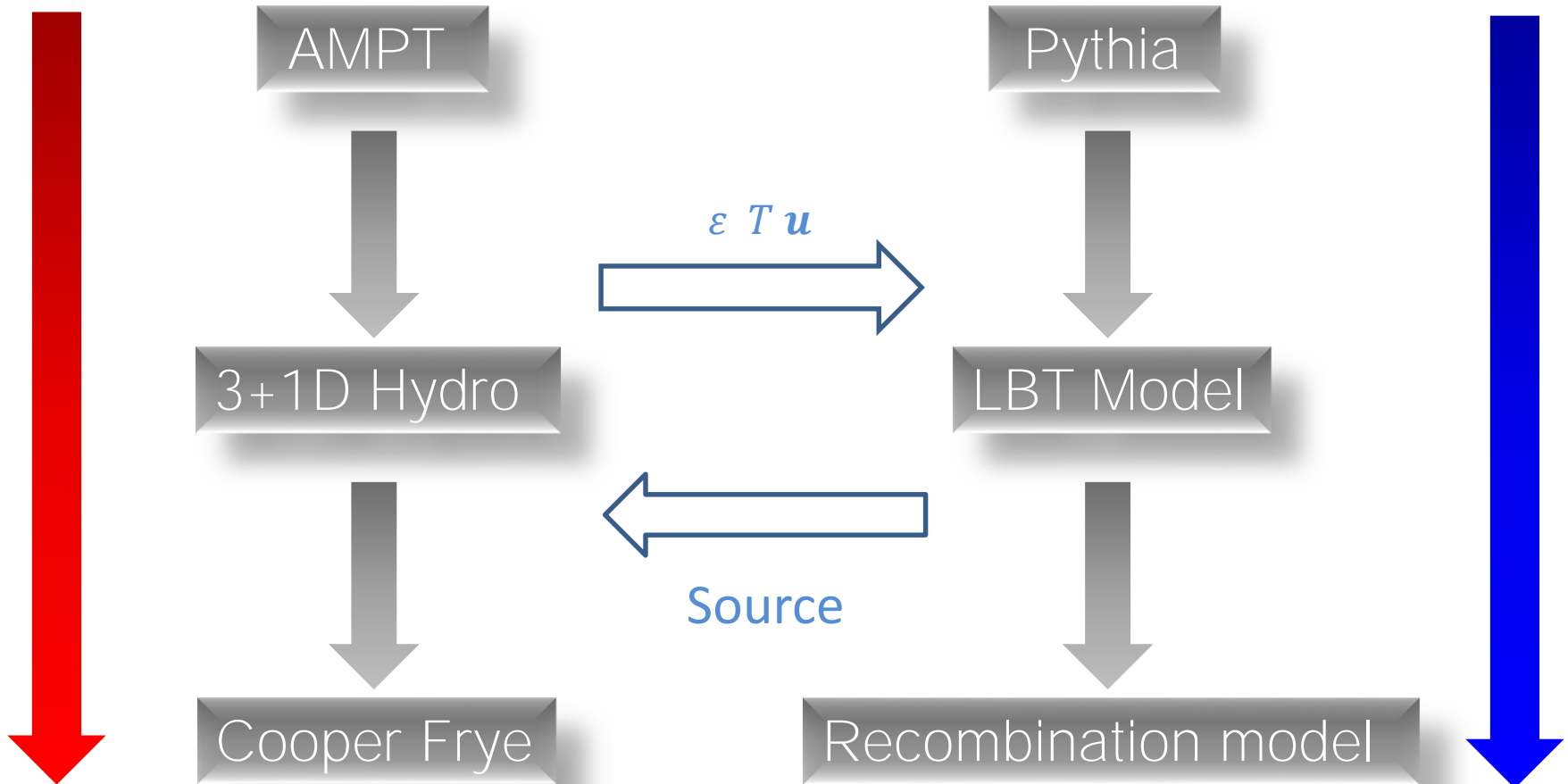
Ed at $\eta=0$ and $t= 0.2$ fm



A coupled LBT hydro (3+1D) model

Medium evolution

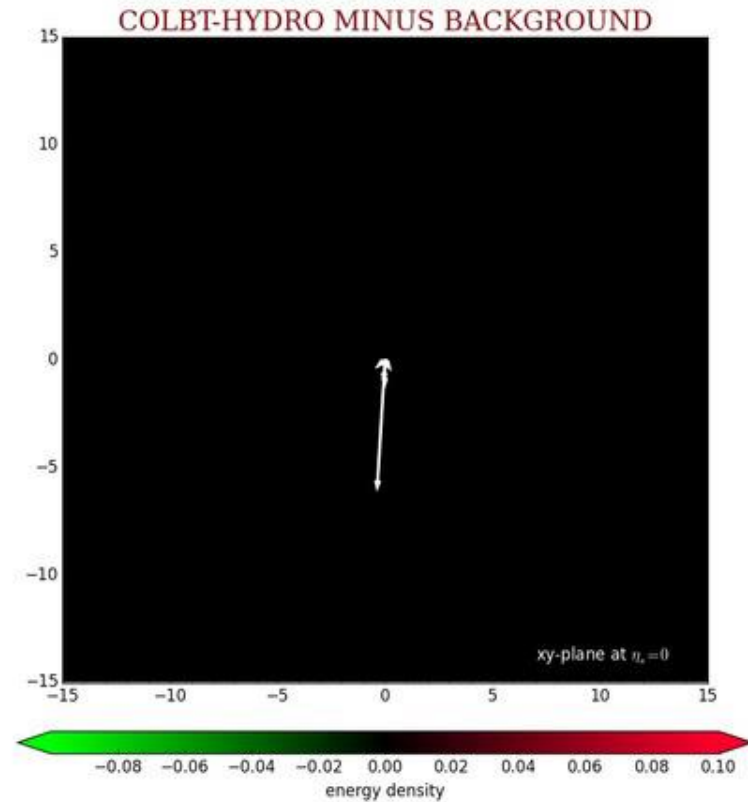
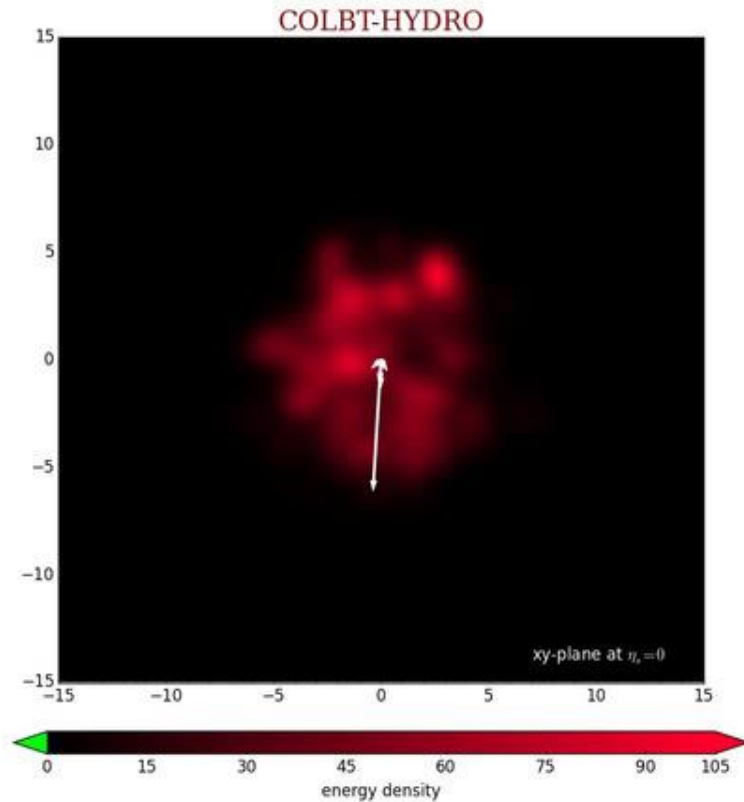
Jet propagation



(Jet+medium) - (medium without jet) in the end

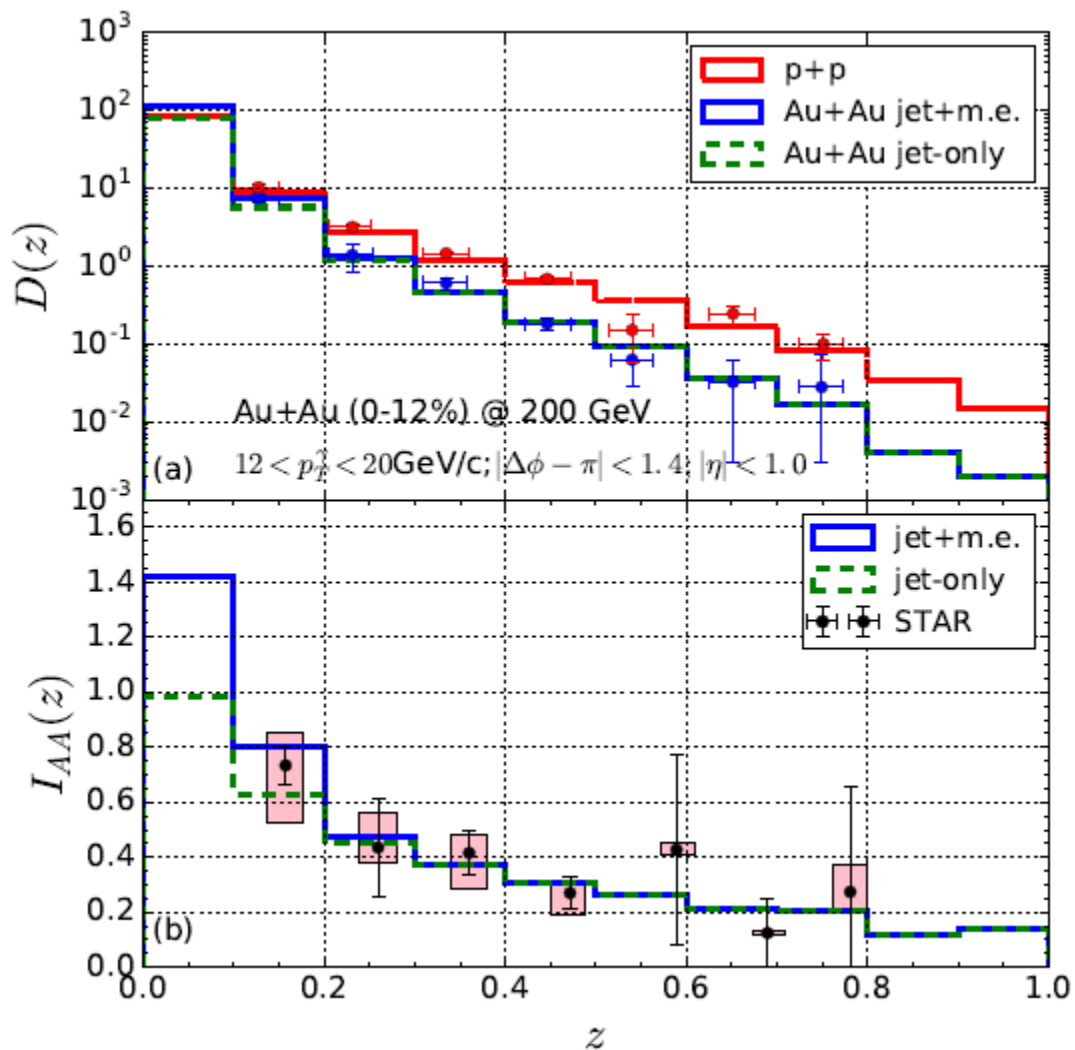
Gamma-jet evolution within CoLBT-hydro model

Jet propagation in hot medium at $\tau=0.4fm$



Medium modification of gamma-triggered hadron yields in RHIC energy

CoLBT-hydro:
$$D(z) = \left. \frac{dN_h}{dydz} \right|_{LBT} + \left. \frac{dN_h}{dydz} \right|_{hydro}^{w./jet} - \left. \frac{dN_h}{dydz} \right|_{hydro}^{w.o./jet} \quad z = p_T^h / p_T^\gamma$$



- The suppression of leading hadrons at intermediate and large z

LBT:

hard parton energy loss

- The enhancement of soft hadrons at small z

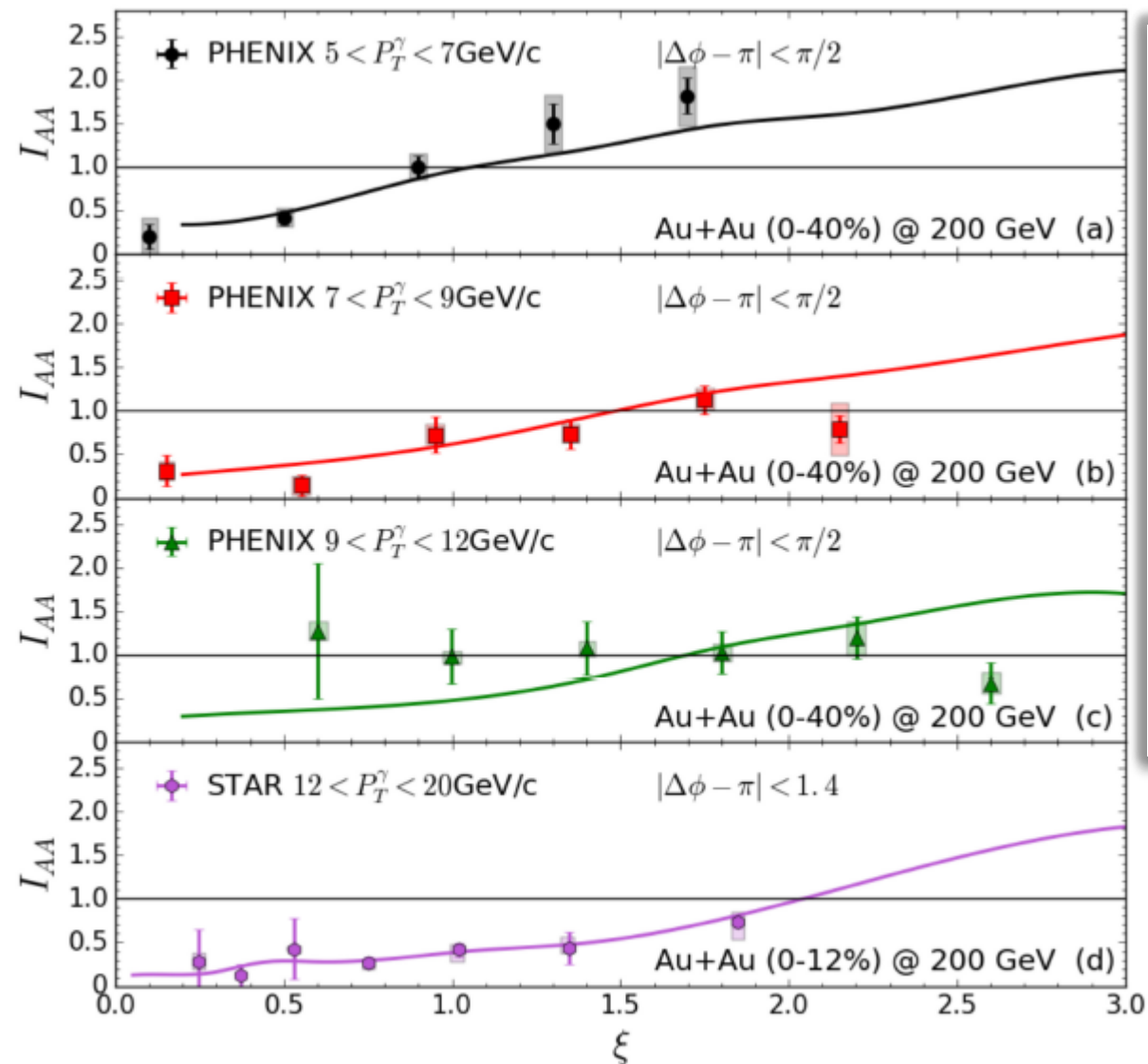
Hydro:

medium excitation

$$p_{cut} = 2 \text{ GeV}$$

$$I_{AA}(z) = D_{AA}(z) / D_{pp}(z)$$

Medium modification of gamma-triggered hadron yields in RHIC energy



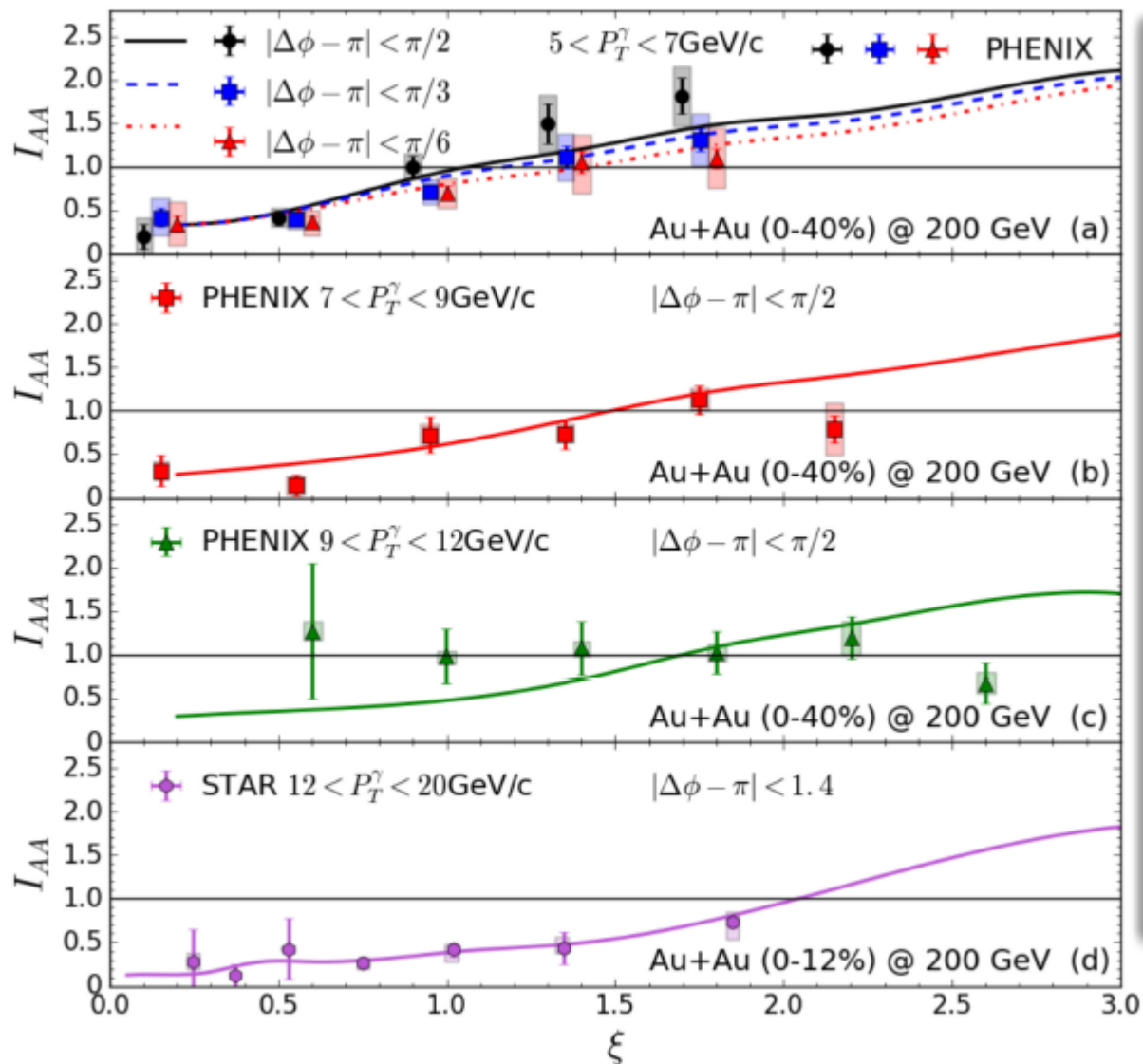
With increasing p_T -gamma

- Transition point from suppression to relative enhancement shifts to larger ξ .
- This transition point corresponds to a fixed p_T range.

$$z = p_T^h / p_T^\gamma$$

$$\xi = \log \frac{1}{z}$$

Medium modification of gamma-triggered hadron yields in RHIC energy

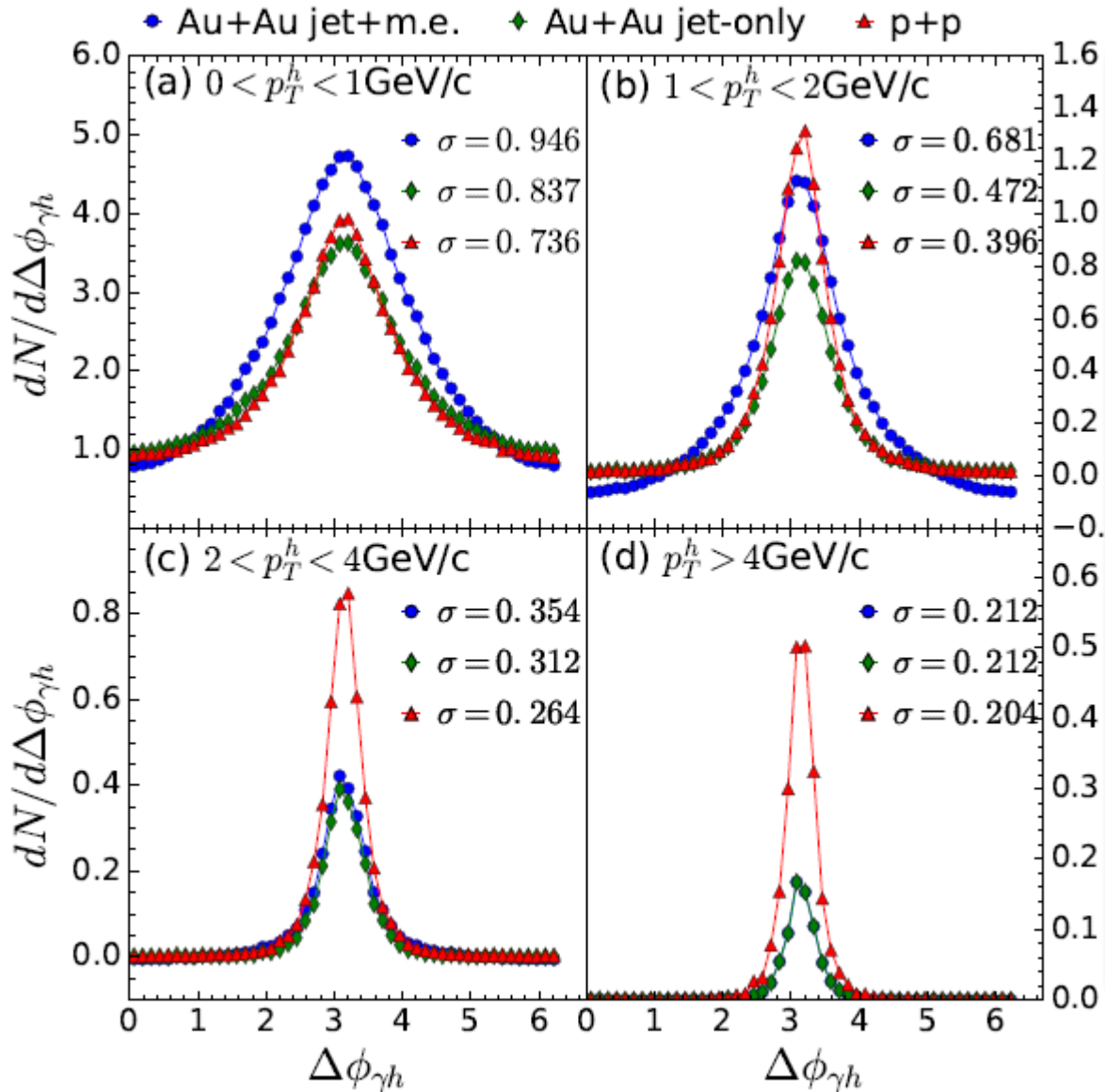


With decreasing azimuthal angle range

- The enhancement at large ξ becomes smaller.
- The transition point shifts to the large ξ .

The soft hadron enhancement has significant contributions from both small and large angle azimuthal angle relative to the jet direction.

Gamma-hadron azimuthal correlation in RHIC energy



σ : gaussian width

- Large suppression at large p_T range.
- Enhancement at small p_T range.
- A broaden peak at small p_T range.
- Suppression of hadron yield at small p_T range in the near side.

Summary

- We present a computation of jets modification in QGP within the Linear Boltzmann Transport (**LBT**) model in which both the elastic and inelastic processes are included.
- We develop a coupled LBT hydro (**CoLBT-hydro**) model for the further study of jet-medium interaction.

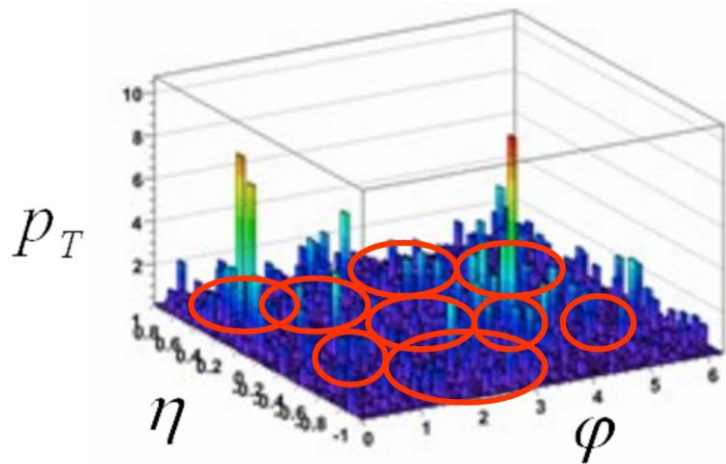
Outlook

- *Jet in hadron level. (Hardon jet, Heavy flavor jet)
(with the **recombination model** developed by Texas A&M group)*
- *Ideal hydro to viscous hydro.*
- *Heavy quarkonium.*

Thanks

Underlying Event Subtraction (UES)

UE: collisions of beam remnant, fluctuation of the background, non-perturbative effects. Subtraction is needed to exclude the soft particles.



Seed jet: $E_T > 3$ GeV for at least one parton, and

$$E_T^{max} / E_T^{ave} > 4$$

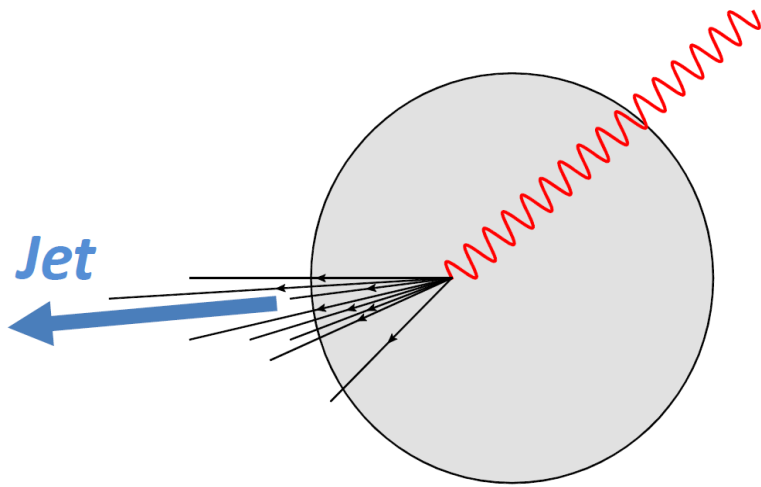
ATLAS Collaboration, Phys. Lett. B 719, 220 (2013).

$$E_T^{UES} = E_T^{seedjet} - A^{seedjet} \rho (1 + 2v_2 \cos[2(\phi_{jet} - \Psi_2)])$$

We only subtract the energy of seed jets,
and count all the final jets!

Asymmetry distribution of gamma-jet in heavy-ion collisions

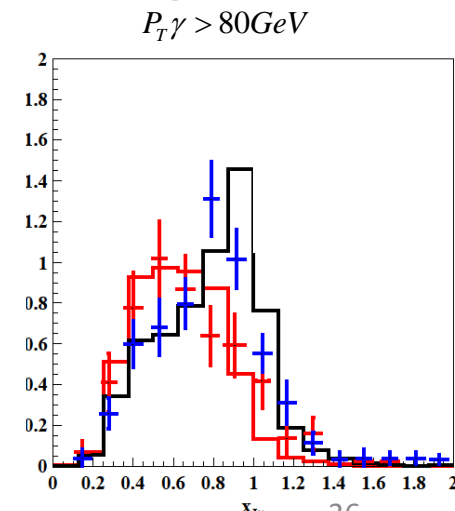
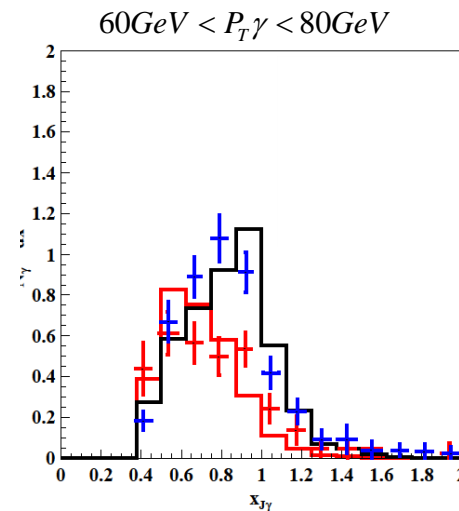
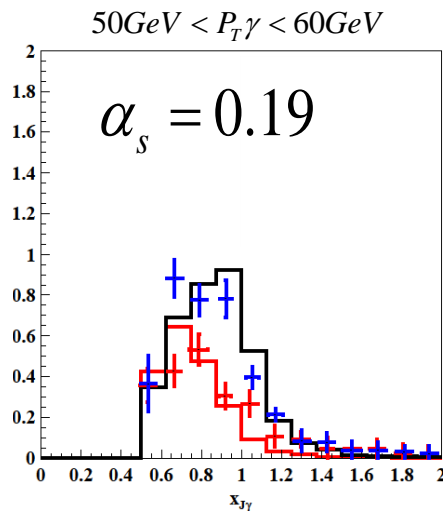
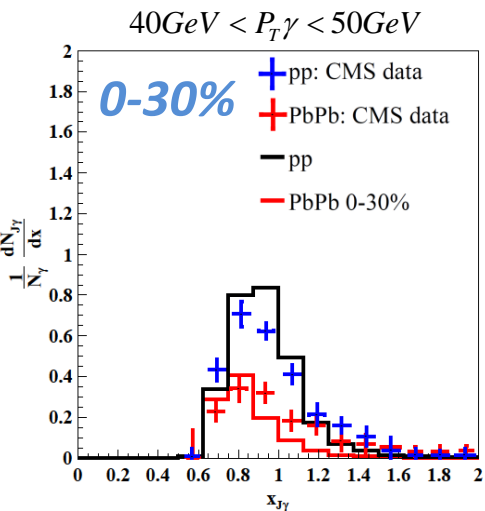
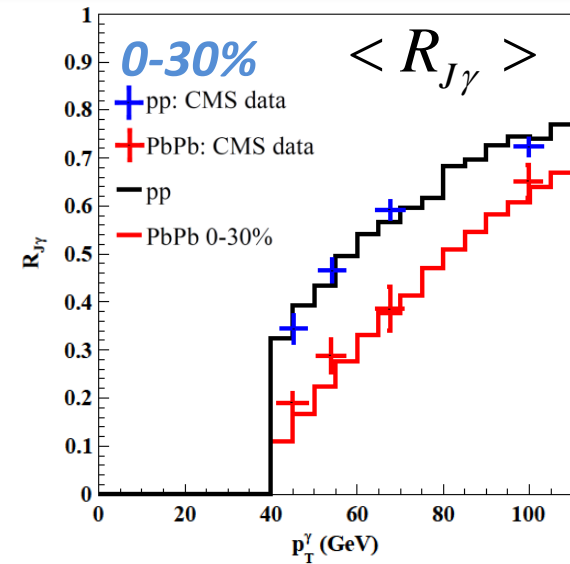
- fix the parameter α_s via the comparison with the γ -jet asymmetry



$$|\eta_\gamma| < 1.44$$

$$P_{Tjet} > 30 GeV$$

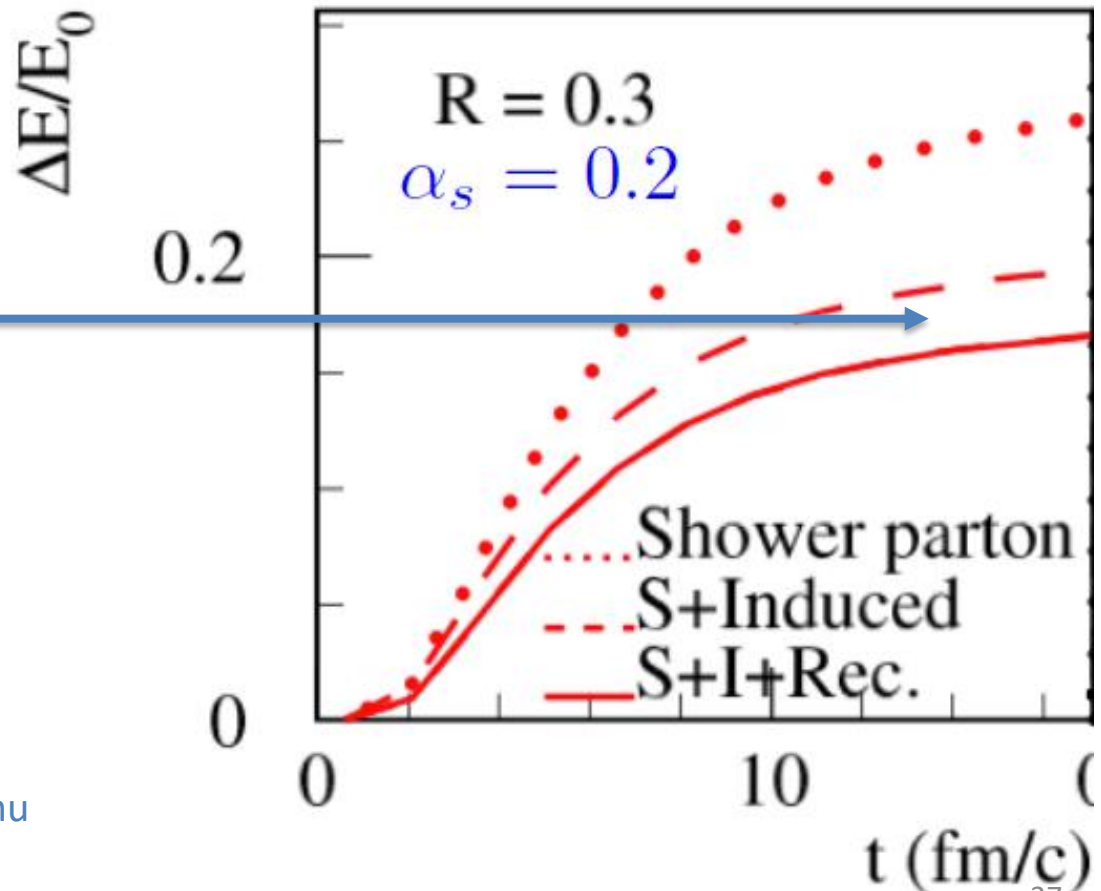
$$|\eta_{jet}| < 1.6$$



Jets in a 3+1D hydro

- 3+1D Ideal hydro Longgang Pang, Qun Wang, Xin-Nian Wang Phys.Rev. C86 (2012) 024911
- Location of gamma-jet is decided according probability of binary collision.

Recoiled effect in the reconstructed jets



The contribution of the recoiled parton in the reconstructed jets

HL Li, FM Liu, GL Ma, XN Wang, Y Zhu

Phys.Rev.Lett. 106, 012301

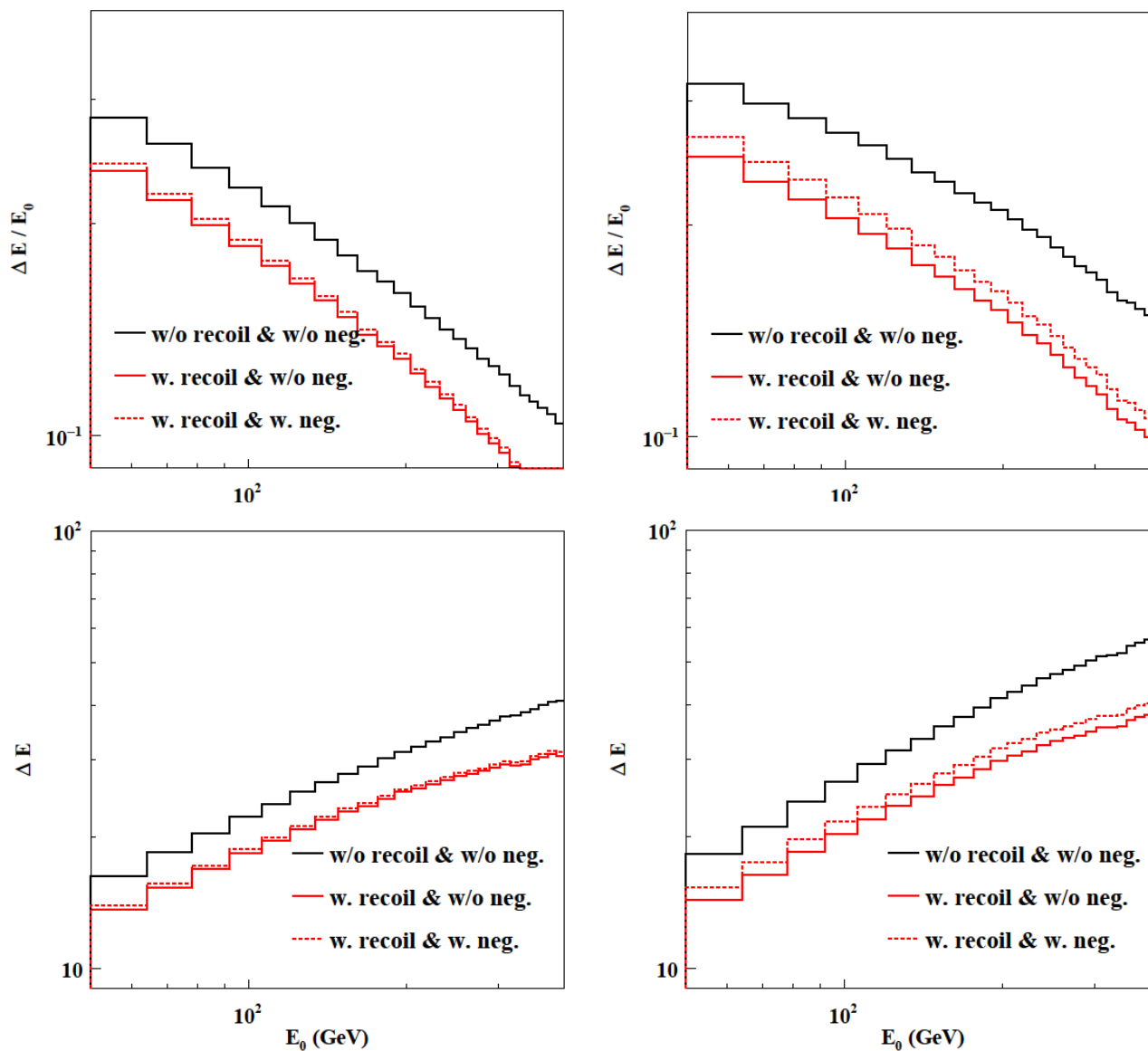
Xin-Nian Wang, Yan Zhu

Phys.Rev.Lett. 111, 062301

Yayun He, Tan Luo, Xin-Nian Wang, Yan Zhu

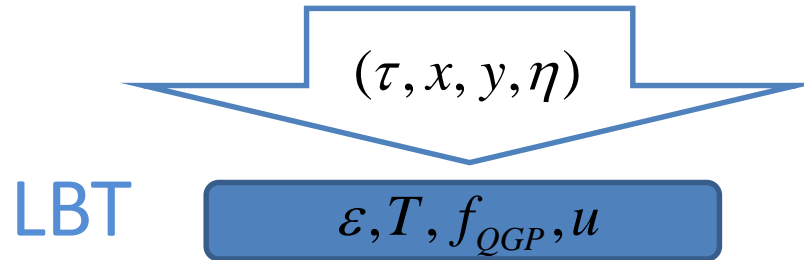
Phys.Rev. C91 (2015) 054908

Recoiled effect in the reconstructed jets

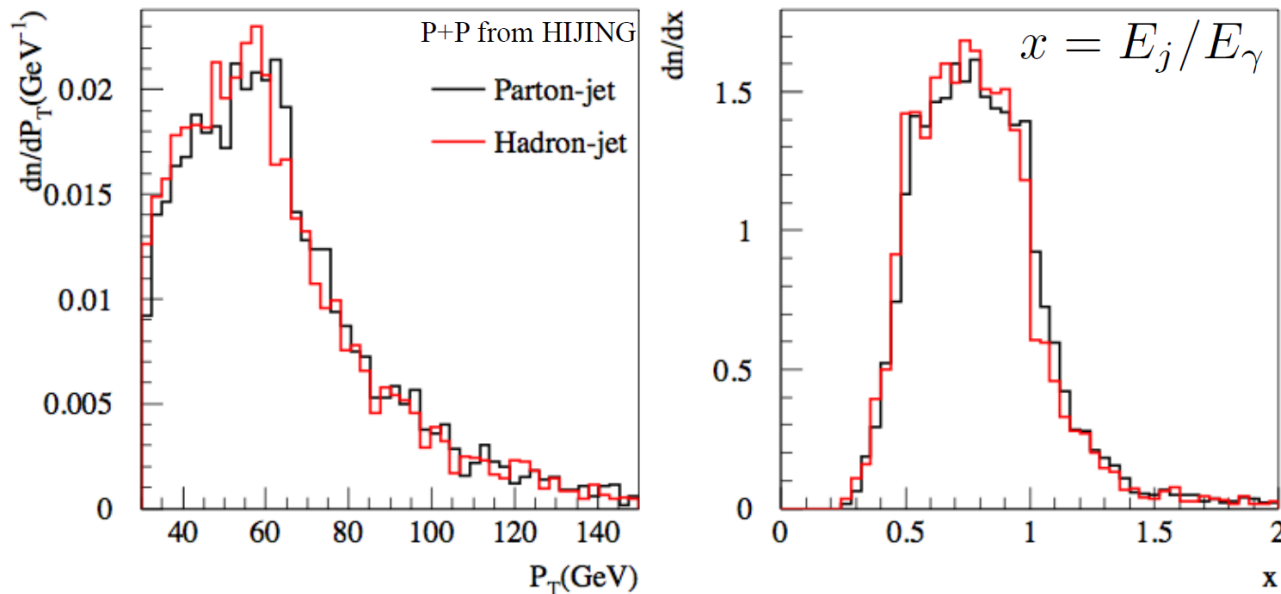


Gamma-jets in a 3+1D hydro

- 3+1D Ideal hydro Longgang Pang, Qun Wang, Xin-Nian Wang Phys.Rev. C86 (2012) 024911



- Location of gamma-jet is decided according probability of binary collision.
- Small difference between parton-jet and hadron-jet.



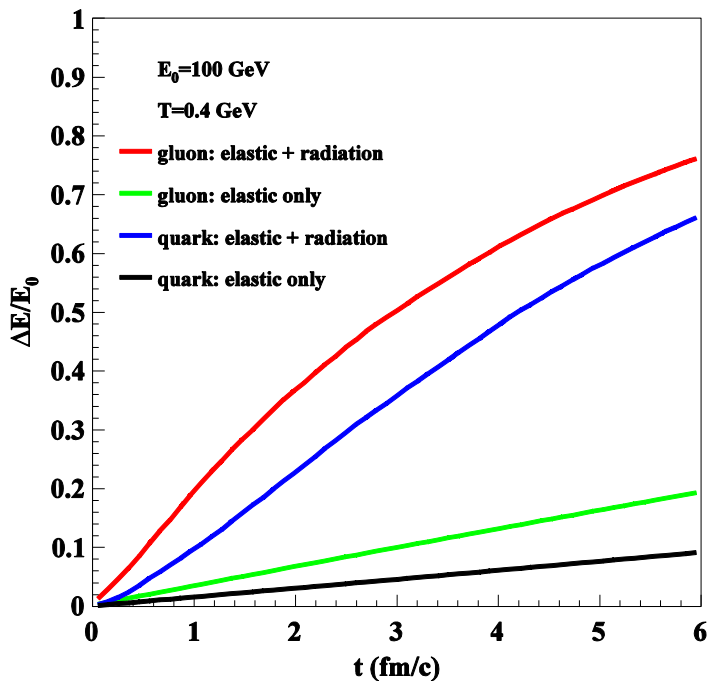
Nontrivial path length dependence on parton energy loss

Leading parton energy loss

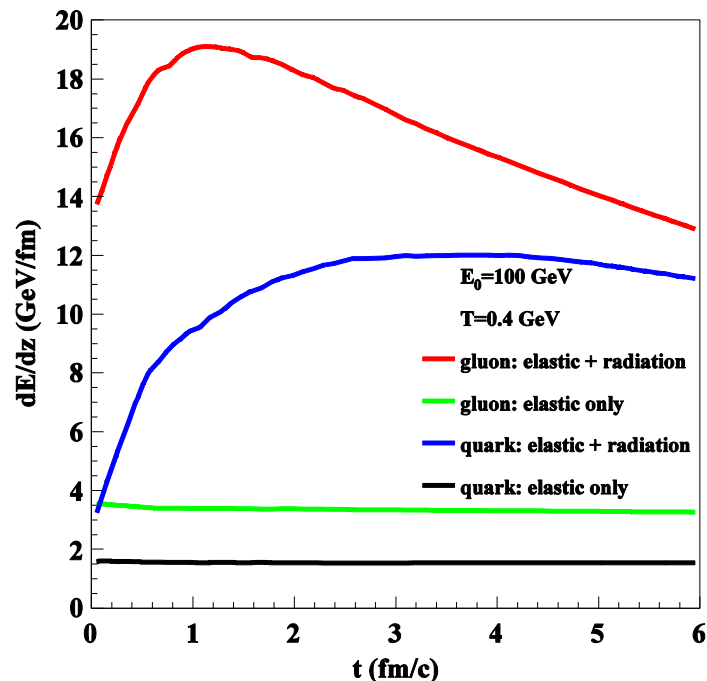
Propagation of a single initial jet parton in a uniform medium

$$\alpha_s = 0.3 \quad E = 100 \text{ GeV} \quad T = 0.4 \text{ GeV}$$

Fractional energy loss



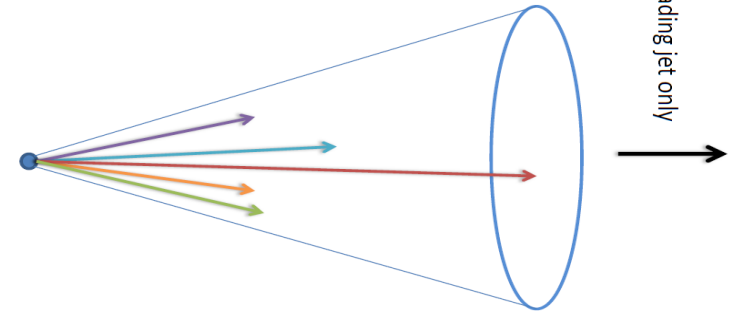
Energy loss per unit length



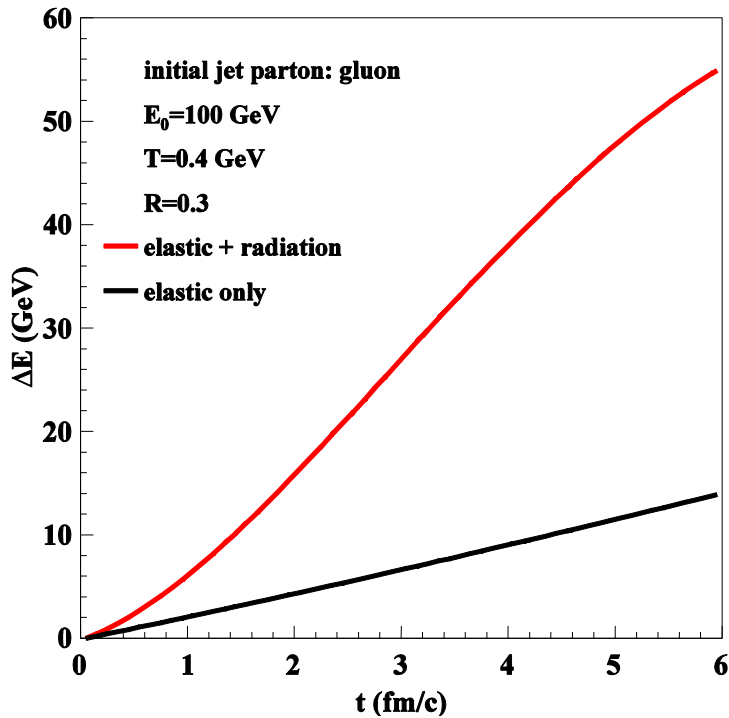
Path length dependence on parton energy loss

Leading jet energy loss

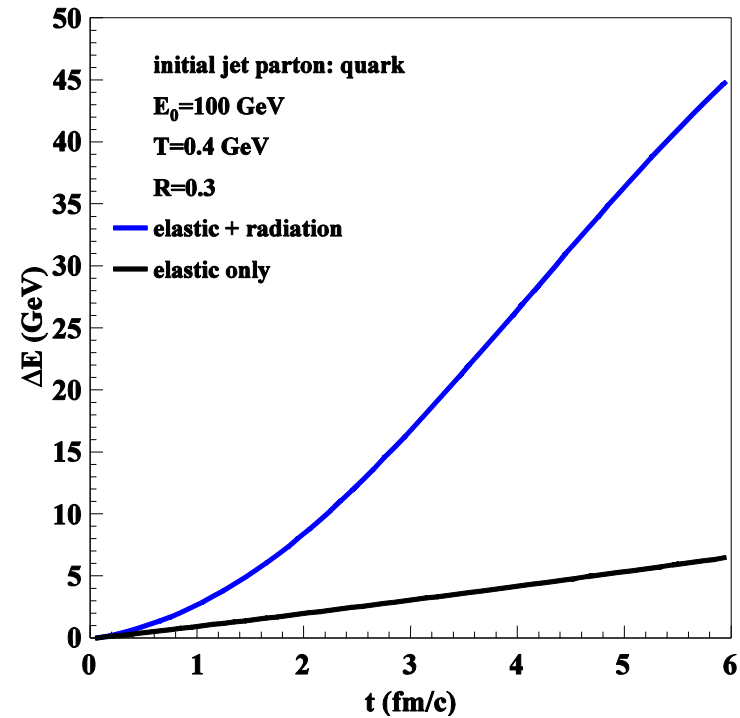
- Leading jet recover some of the energy lost by the leading parton.



Initial jet parton: gluon



Initial jet parton: quark



Inclusive jet in pp collisions

p_T distribution in pp collision within Pythia8

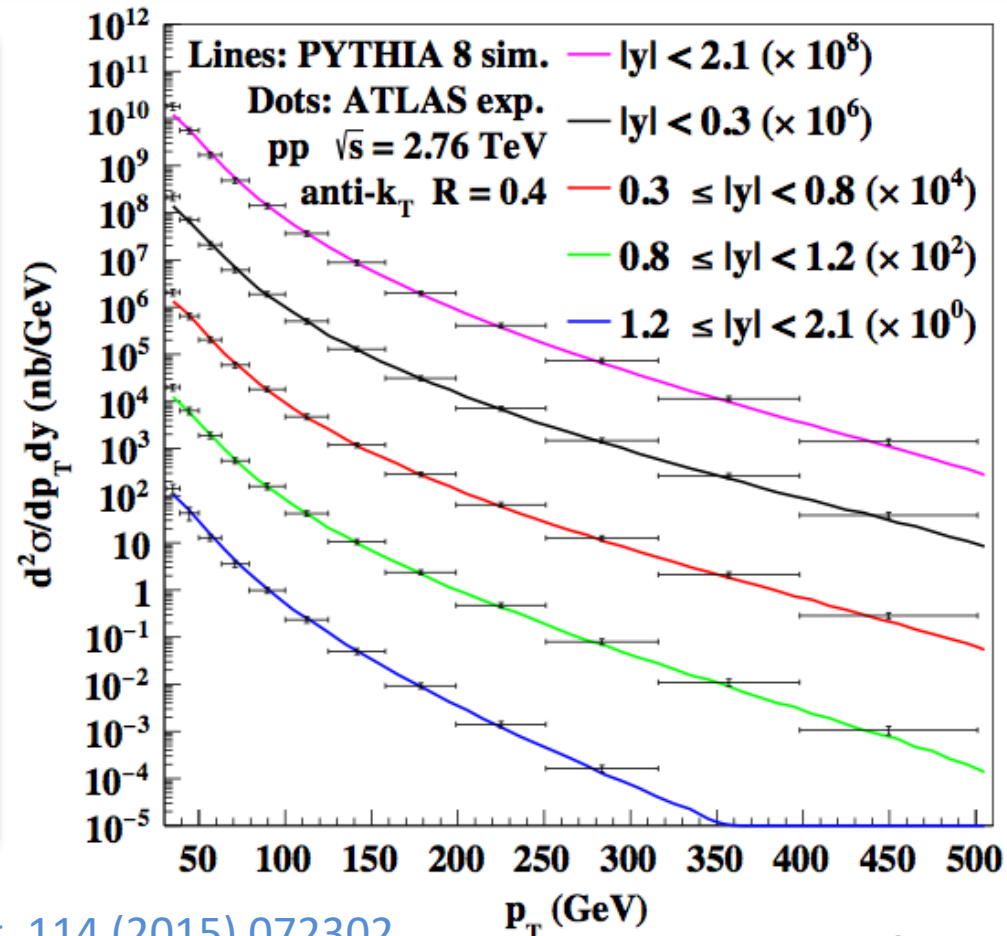
Weighted sampling in triggered p_T bins to increase the efficiency of MC simulations

In PYTHIA 8 one can obtain the cross section for each triggered p_T bin

$$\frac{d^2\sigma}{dp_T dy} = \sum_i \sigma_i \frac{1}{N_{events}} \frac{dN_i^{jets}}{dp_T \Delta y}$$

For each bin, the weight is

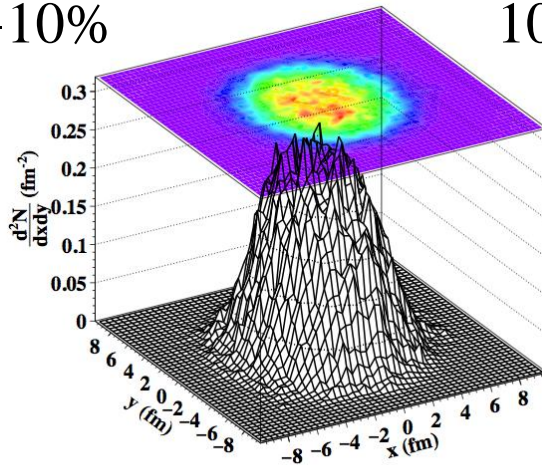
$$\omega_i = \sigma_i / \sum_i \sigma_i$$



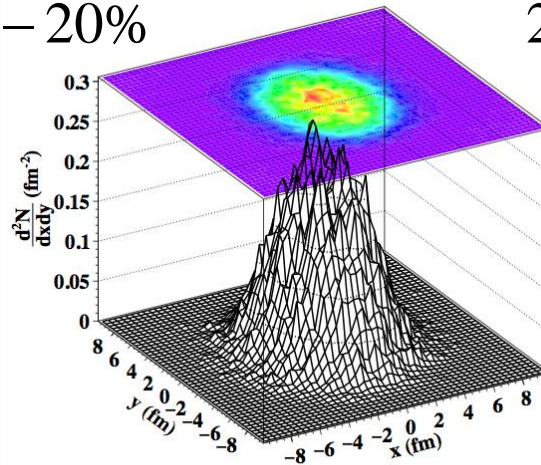
Initial geometry

Averaged over 200 event-by-event hydro profiles

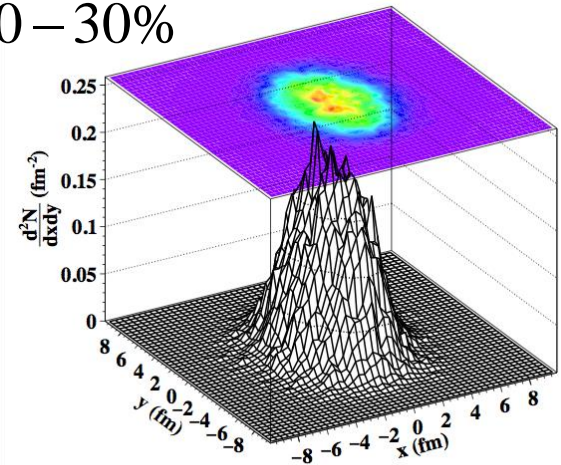
5–10%



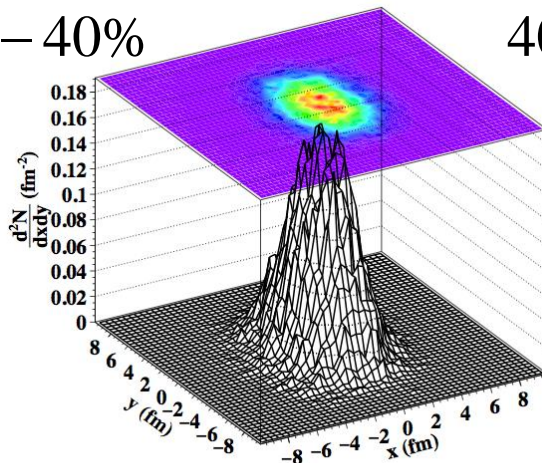
10–20%



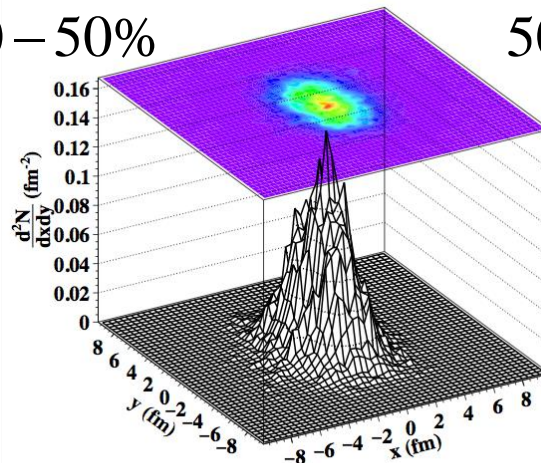
20–30%



30–40%



40–50%



50–60%

