Event-by-event picture for the medium-induced jet evolution

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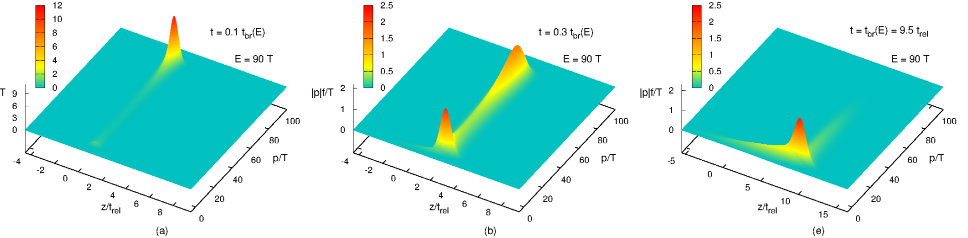
recent work by the “Saclay collaboration” (since 2012)

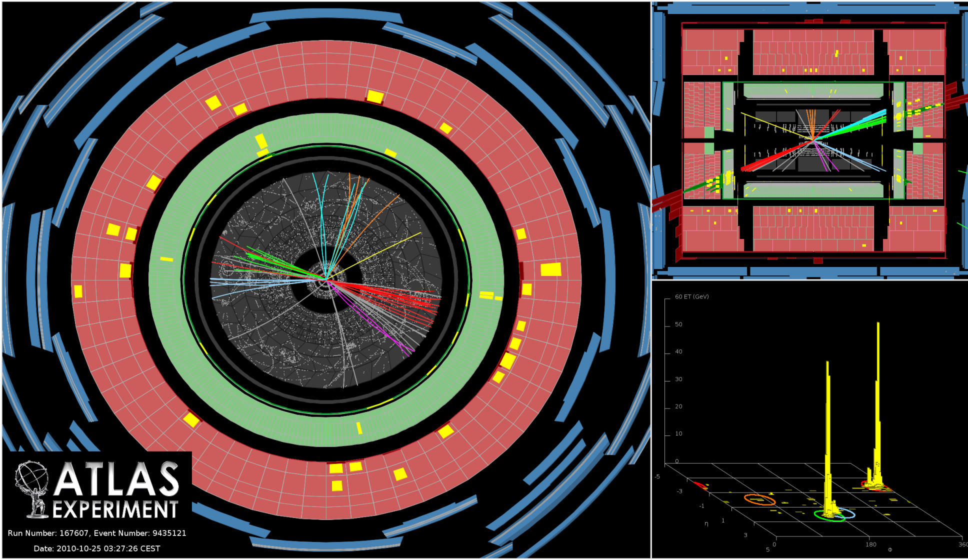
J.-P. Blaizot, F. Dominguez, M. Escobedo, Y. Mehtar-Tani, B. Wu



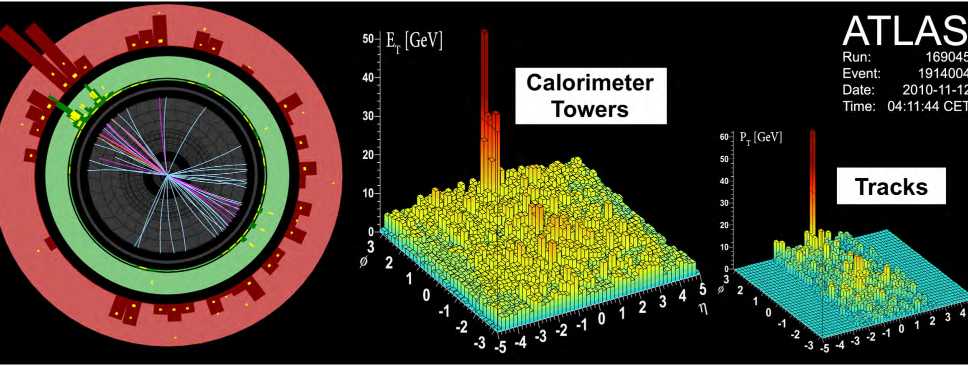
Outline

- Motivation: di-jet asymmetry at the LHC
- Medium-induced radiation: BDMPS-Z
- Multiple branching: physical discussion
- Average gluon distribution & energy loss
- Correlations & fluctuations
- Gluon multiplicities
- Thermalization of mini-jets

From di–jets in p+p collisions ...

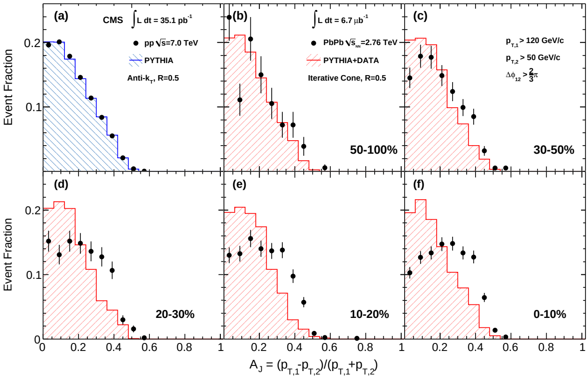


... to mono-jets in Pb+Pb collisions



- Central Pb+Pb: 'mono–jet' events
- The secondary jet can barely be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry : A_{J} (CMS)

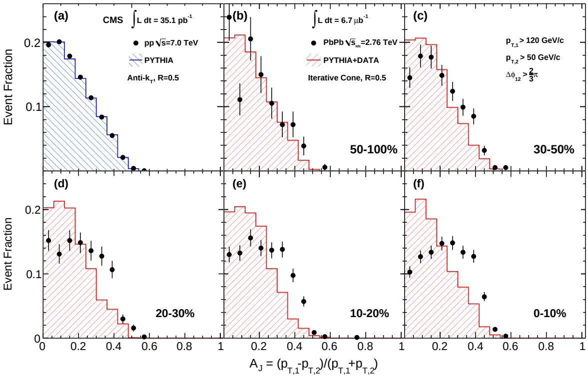


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_{\mathrm{J}} = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in p+p collisions !

Di–jet asymmetry : A_J (CMS)



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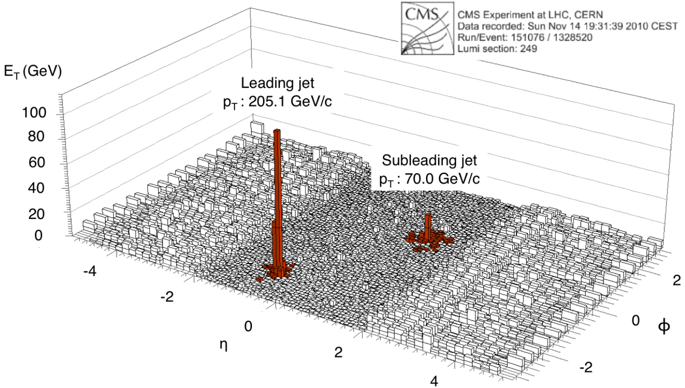
$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

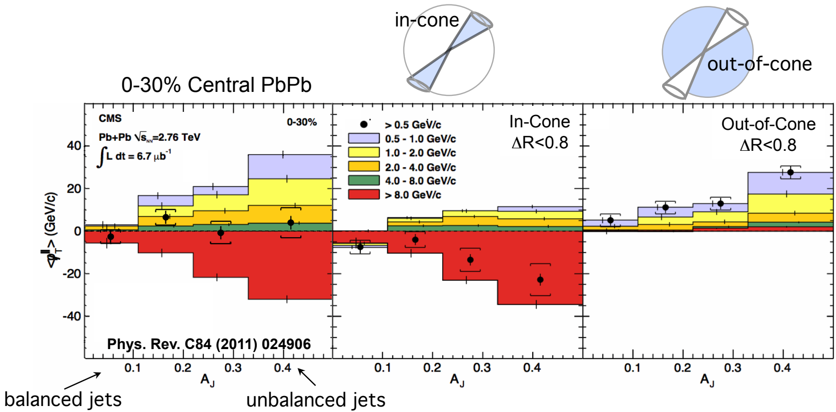
- Central Pb+Pb : the asymmetric events occur more often

Di–jet asymmetry at the LHC *(CMS)*



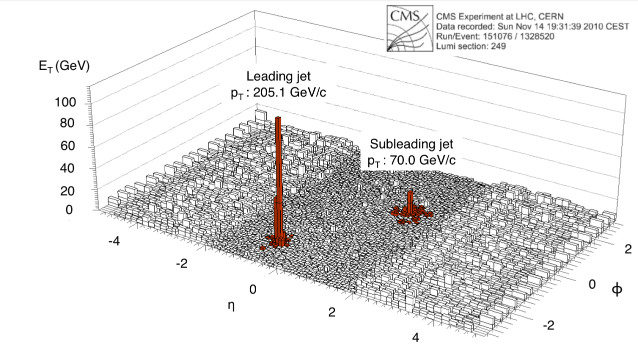
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- The 'missing energy' is actually found in the underlying event:
  - many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

Energy imbalance @ large angles: $R = 0.8$



- No missing energy : $E^{\rm in+out}_{\rm Lead} = E^{\rm in+out}_{\rm SubLead}$
- In-Cone : $E^{\rm in}_{\rm Lead} > E^{\rm in}_{\rm SubLead}$: di-jet asymmetry, hard particles
- Out-of-Cone : $E^{\rm out}_{\rm Lead} < E^{\rm out}_{\rm SubLead}$: soft hadrons @ large angles

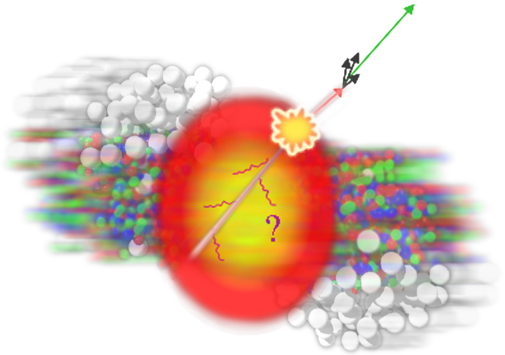
Di–jet asymmetry at the LHC



- Very different from the usual jet fragmentation pattern in the vacuum
  - bremsstrahlung favors collinear splittings ⇒ jets are collimated
- Soft hadrons can be easily deviated towards large angles
  - elastic scatterings with the medium constituents
- A main question: how is that possible that a significant fraction of the jet energy be carried by its soft constituents ?

The generally expected picture

- "One jet crosses the medium along a distance longer than the other"



- Implicit assumption: fluctuations in energy loss are small
  - "the energy loss is always the same for a fixed medium size"

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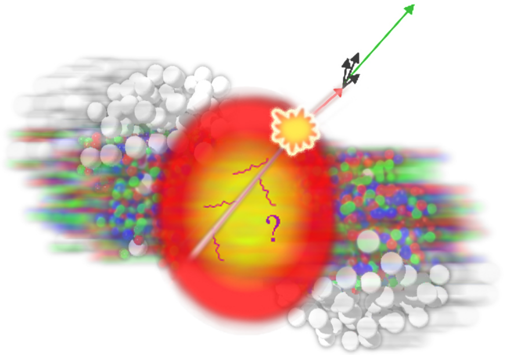
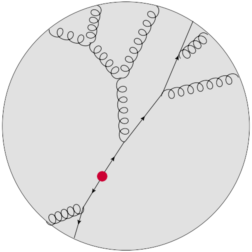


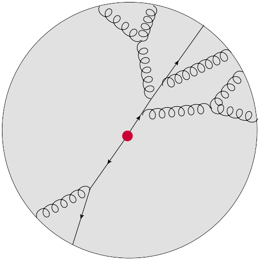
- Implicit assumption: fluctuations in energy loss are small
  - "the energy loss is always the same for a fixed medium size"
- Fluctuations are known to be important for a branching process

The role of fluctuations

- Different path lengths



- Fluctuations in the branching pattern



- Fluctuations in the energy loss are as large as the average value

  (M. Escobedo and E.I., arXiv:1601.03629 & 1609.06104)

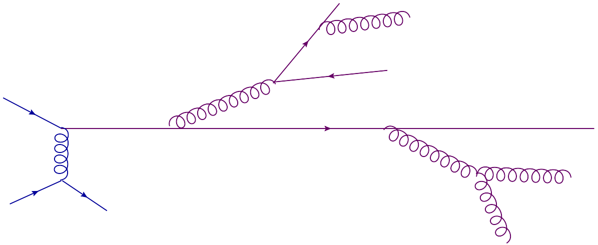
- Similar conclusion independently reached by a Monte-Carlo study

  (Milhano and Zapp, arXiv:1512.08107, "JEWEL")

- One needs a better understanding of the in-medium jet dynamics

Medium-induced jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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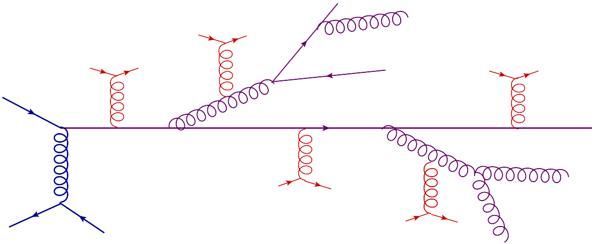

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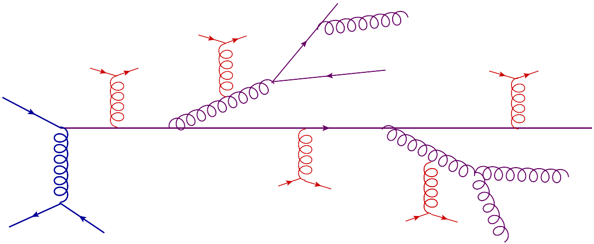
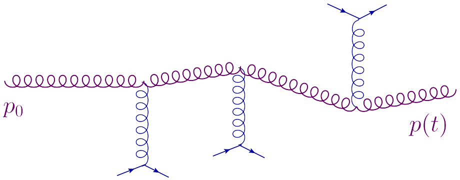


- ... and via collisions off the medium constituents
- Collisions can have two main effects
  - trigger additional radiation ('medium-induced branching')
  - thermalize the products of this radiation
- BDMPSZ mechanism *(Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov)*
  - gluon emission is linked to transverse momentum broadening

Transverse momentum broadening

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$

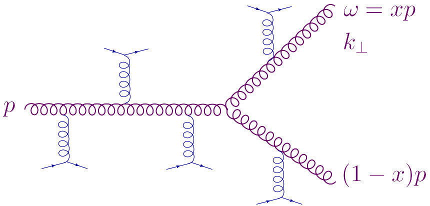


$$\langle p_\perp^2 \rangle \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} \sim \alpha_s^2 T^3 \ln\frac{1}{\alpha_s}$$

$$\hat{q} \simeq 1\,\mathrm{GeV}^2/\mathrm{fm}$$

- Collisions destroy quantum coherence and thus trigger emissions



formation time

$$t_\mathrm{f} \simeq \frac{1}{\Delta E} \simeq \frac{\omega}{k_\perp^2}$$

- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q} t_\mathrm{f}$

Transverse momentum broadening

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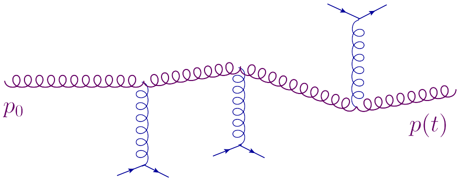

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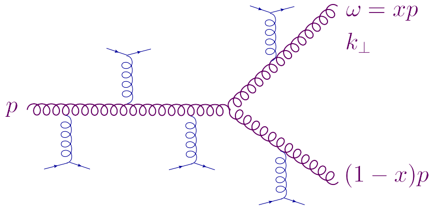

formation time

$$t_\mathrm{f} \simeq \frac{\omega}{\hat{q} t_\mathrm{f}} \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

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Formation time & production angle

$$t_{\rm f} \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q} t_{\rm f} \quad \Longrightarrow \quad t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Maximal ω for this mechanism: $t_{\rm f} \le L \;\Rightarrow\; \omega \le \omega_c \equiv \hat{q} L^2$
- Soft gluons $(\omega \ll \omega_c)$ have ...

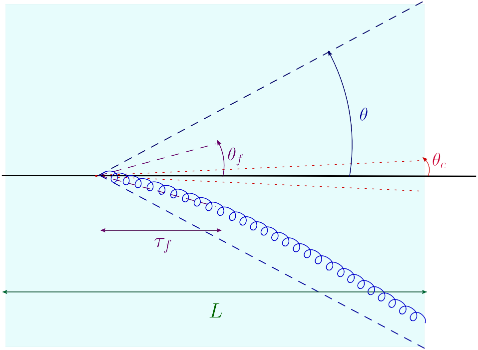
- small formation times:

$$t_{\rm f}(\omega) \ll L$$

- ... and large production angles:

$$\theta(\omega) \simeq \frac{k_\perp}{\omega} \simeq \frac{\sqrt{\hat{q}L}}{\omega}$$

- promising for dijet asymmetry



- Final transverse momentum (roughly) : $k_\perp^2 \sim \hat{q} L$

Multiple branchings

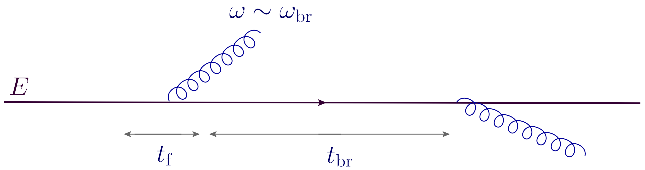
- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

- When $\mathcal{P}(\omega, L) \sim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br}(L) \equiv \alpha_s^2 \hat{q} L^2 \quad \Longleftrightarrow \quad L \gtrsim t_{\rm br}(\omega) \equiv \frac{1}{\alpha_s} t_{\rm f}(\omega)$$

- LHC: the leading particle has $E \sim 100\,{\rm GeV} \gg \omega_{\rm br} \sim 5 \div 10\,{\rm GeV}$



- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$

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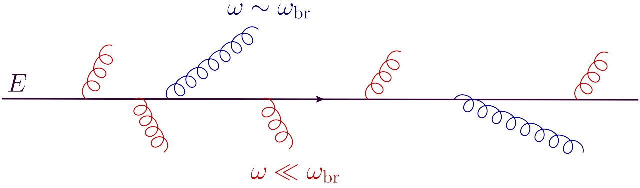

- In a typical event, the LP emits ...
  - a large number of softer gluons with $\omega \ll \omega_{\rm br}$

Multiple branchings

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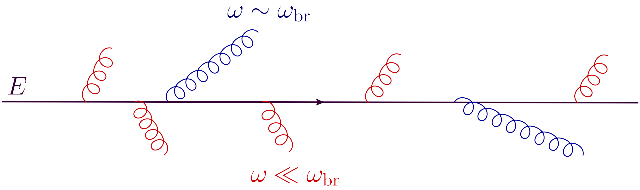
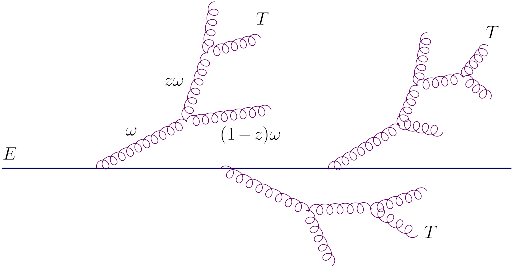


- The energy loss is controlled by the hardest primary emissions

Democratic branchings

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- The primary gluons generate 'mini-jets' via democratic branchings
  - daughter gluons carry comparable energy fractions: $z \sim 1 - z \sim 1/2$



$$\mathcal{P}(z\omega, L) \simeq \frac{L}{t_{\rm br}(z\omega)} \simeq \alpha_s\, L \sqrt{\frac{\hat{q}}{z\omega}}$$

  - when $\omega \sim \omega_{\rm br}$, $\mathcal{P}(z\omega, L) \sim 1$ independently of the value of z

- A mini-jet with $\omega \lesssim \omega_{\rm br}$ decays over a time $t_{\rm br}(\omega) \lesssim L$

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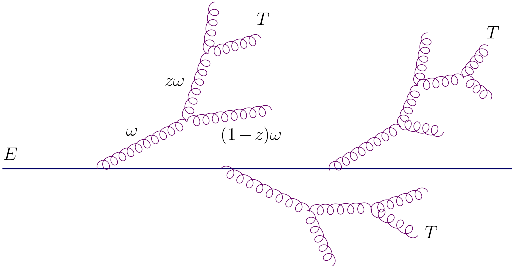

- Via successive democratic branchings, the energy is efficiently transmitted to softer and softer gluons, down to $\omega \sim T$
- The soft gluons thermalize via elastic collisions, thus stopping the branching process *(E.I. and Bin Wu, arXiv:1506.07871)*

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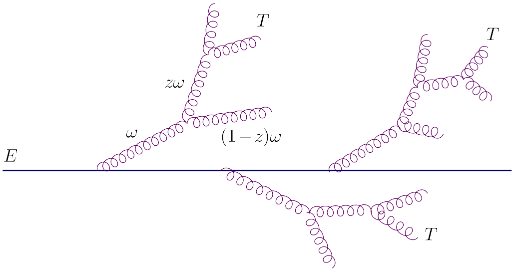


- All the energy taken by primary gluons with $\omega \lesssim \omega_{\rm br}$ ends up in the medium
- This energy appears in many soft quanta propagating at large angles
- What is the average energy loss and its fluctuations ?

Probabilistic picture

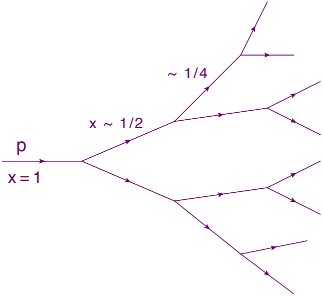
- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - successive branchings are non-overlapping: $t_{\rm br} \sim \frac{1}{\alpha_s}\, t_{\rm f}$
  - interference phenomena could complicate the picture ...
    (in the vacuum, interferences lead to angular ordering)
  - ... but they are suppressed by rescattering in the medium
    (Blaizot, Dominguez, E.I., Mehtar-Tani, 2012)
    (Apolinário, Armesto, Milhano, Salgado, 2014)
- Hierarchy of equations for n-point correlation functions ($x \equiv \omega/E$)

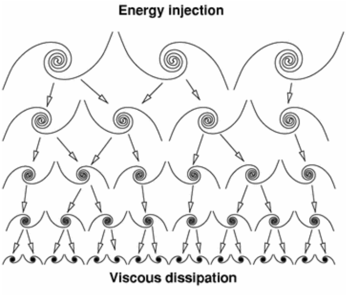
$$D(x,t) \equiv x \left\langle \frac{{\rm d}N}{{\rm d}x}(t) \right\rangle , \qquad D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{{\rm d}N_{\rm pair}}{{\rm d}x\,{\rm d}x'}(t) \right\rangle$$

- Analytic solutions *(Blaizot, E. I., Mehtar-Tani, '13; Escobedo, E.I., '16)*
- Interesting new phenomena:
  - wave turbulence, KNO scaling, large fluctuations

Wave turbulence

- Democratic branchings lead to wave turbulence
  - energy flows from one parton generation to the next one, at a rate which is independent of the generation
  - it eventually dissipates into the medium, via thermalization
  - mathematically: a fixed point $D(x) = \frac{1}{\sqrt{x}}$ (Kolmogorov spectrum)

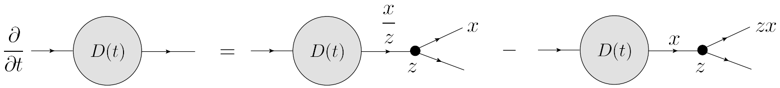




Gluon spectrum: the average energy loss

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- Kinetic equation for $D(x,t)$: 'gain' - 'loss'



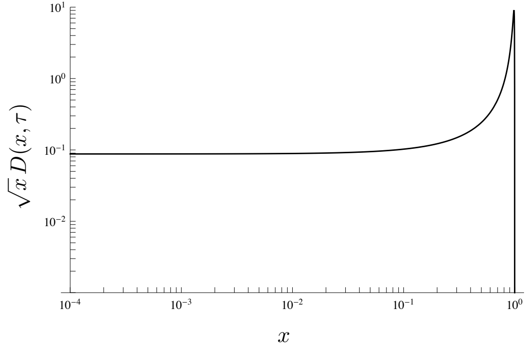
- Exact solution with initial condition $D(x,t=0)=\delta(x-1)$

$$D(x,\tau)\,=\,\frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,,\qquad x\equiv\frac{\omega}{E}\,,\quad \tau\equiv\frac{t}{t_{\rm br}(E)}$$

- $t_{\rm br}(E)$: the lifetime of the LP until its first democratic branching
  - early times $\tau\ll 1$: leading particle peak near $x=1$
  - $\tau\gtrsim 1$: the spectrum is suppressed at all values of x
  - power-law spectrum $D\propto\frac{1}{\sqrt{x}}$ at $x\ll 1$ for any τ

Gluon spectrum

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi \frac{\tau^2}{1-x}}, \qquad x \equiv \frac{\omega}{E}, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$



$\tau = 0.1$

- Pronounced LP peak at small times

Gluon spectrum

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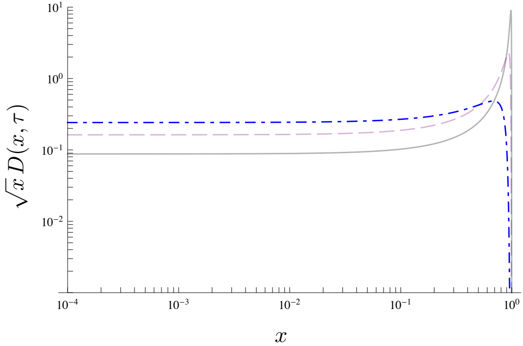
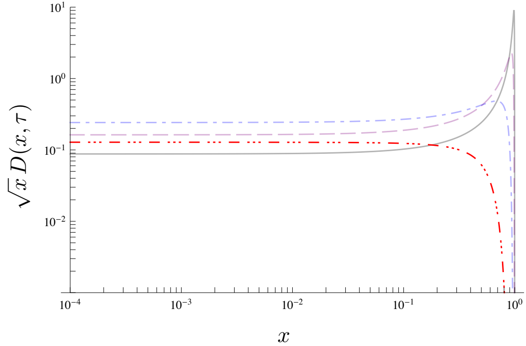


$\tau = 0.1,\ 0.2,\ 0.4$

- Increasing t: the LP peaks decreases, broadens, and moves to the left

Gluon spectrum

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

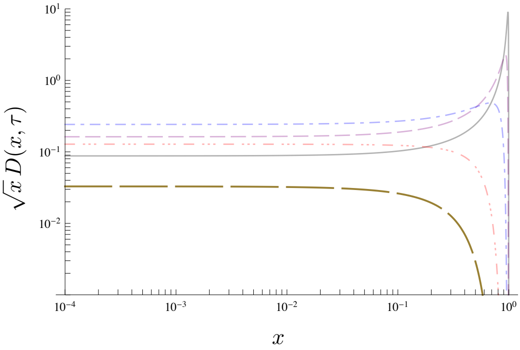


$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8$

- The LP peaks disappears when $\tau \sim 1$

Gluon spectrum

$$D(x,\tau)\,=\,\frac{\tau}{\sqrt{x}(1-x)^{3/2}}\,\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,,\qquad x\equiv\frac{\omega}{E}\,,\quad \tau\equiv\frac{t}{t_{\rm br}(E)}$$

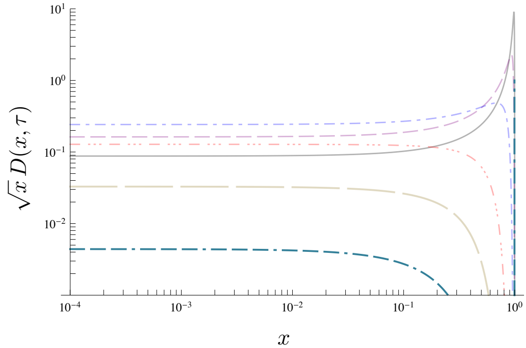


$\tau = 0.1,$ 0.2, 0.4, 0.8, 1

- The shape at small x is not changing: genuine fixed point

Gluon spectrum

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\,\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x\equiv\frac{\omega}{E}\,,\quad \tau\equiv\frac{t}{t_{\rm br}(E)}$$



$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8,\ 1,\ 1.2$

- The energy flows out of the spectrum: $\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2}$

The average energy loss

- The missing energy is transmitted to the medium, via elastic collisions *(E.I. and Bin Wu, 2015)*

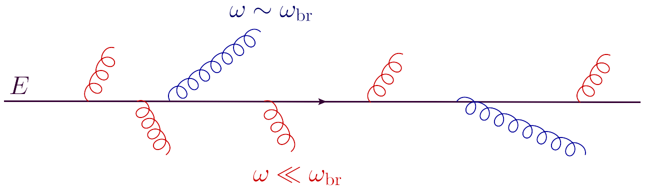
$$\langle \Delta E \rangle = E\left(1 - e^{-\pi\tau^2}\right) = E\left[1 - e^{-\pi \frac{\omega_{\rm br}}{E}}\right]$$

- LHC: $E \sim 100\,\mathrm{GeV} \gg \omega_{\rm br} \sim 5 \div 10\,\mathrm{GeV}$

$$\langle \Delta E \rangle \simeq \pi\omega_{\rm br} = \pi\alpha_s^2 \hat{q} L^2$$

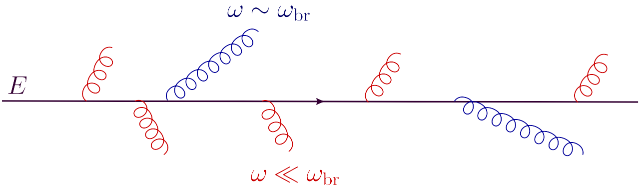
- Consistent with our general physical picture:

  ▷ energy loss is controlled by the primary emissions with $\omega \sim \omega_{\rm br}$



Fluctuations in the energy loss

- Recall: the probability for a primary emission with $\omega \sim \omega_{\rm br}$ is of $\mathcal{O}(1)$

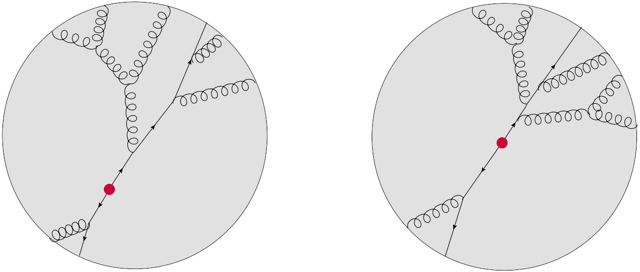


  - the average number of such emissions is of $\mathcal{O}(1)$ (indeed, it is π)
  - successive such emissions are quasi-independent $(E \gg \omega_{\rm br})$

- Fluctuations in the number of such emissions must be of $\mathcal{O}(1)$ as well
- The fluctuations in the energy loss are comparable with the average value
- Confirmed by exact calculations *(M. Escobedo and E. I., arXiv:1601.03629)*

$$\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2 \simeq \frac{\pi^2}{3}\,\omega_{\rm br}^2 = \frac{1}{3}\langle \Delta E \rangle^2$$

Di-jet asymmetry from fluctuations



- Average asymmetry is controlled by the difference in path lengths

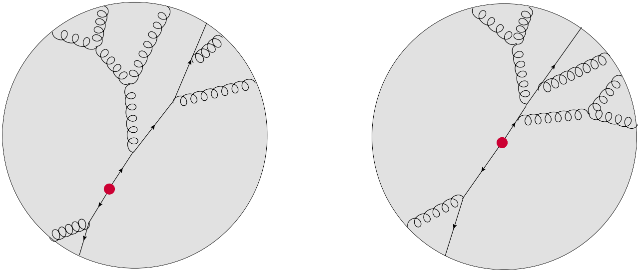
$$\langle E_1 - E_2 \rangle = \langle \Delta E_2 - \Delta E_1 \rangle \propto \langle L_2^2 - L_1^2 \rangle$$

- In experiments though, one rather measures $|E_1 - E_2|$

$$\langle (E_1 - E_2)^2 \rangle - \langle E_1 - E_2 \rangle^2 = \sigma_1^2 + \sigma_2^2 \propto \langle L_1^4 + L_2^4 \rangle$$

- Fluctuations dominate whenever $L_1 \sim L_2$ (the typical situation)

Di-jet asymmetry from fluctuations



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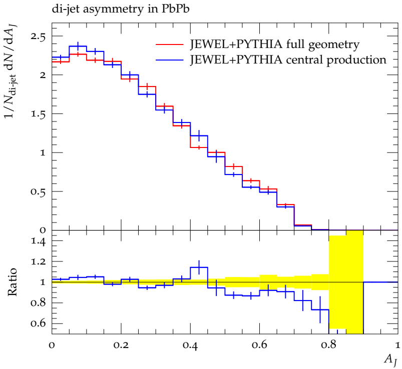
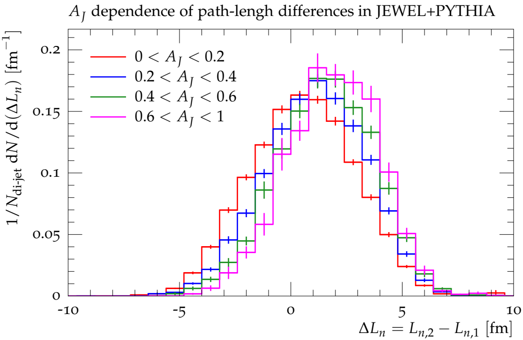
- Difficult to check: no direct experimental control of L_1 and L_2

Monte-Carlo studies (JEWEL)

(Milhano and Zapp, arXiv:1512.08107)





- Left: Central production $(L_1 = L_2)$ vs. randomly distributed production points ("full geometry")
- Right: Distribution of $\Delta L \equiv L_1 - L_2$ for di-jet events in different classes of asymmetry (A_J)
  - the width of the distribution is a measure of fluctuations

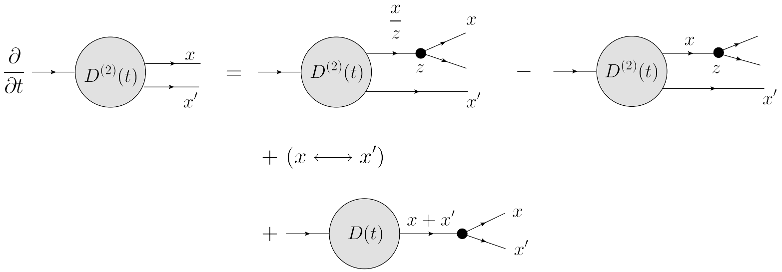
Correlations & fluctuations

M.A. Escobedo and E. I., arXiv:1601.03629, arXiv:1609.06104

- The variance is related to the density $D^{(2)}(x, x', t)$ of gluon pairs:

$$D^{(2)}(x, x', t) \equiv xx' \left\langle \frac{\mathrm{d}N_{\mathrm{pair}}}{\mathrm{d}x\,\mathrm{d}x'}(t) \right\rangle$$

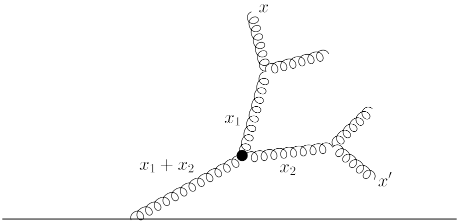
- Kinetic equation for $D^{(2)}(x, x', t)$: correlations due to common ancestors



- The 1-body density $D(x + x', t)$ acts as a source for the 2-body density

The gluon pair density

- The 2 measured gluons x and x' have a last common ancestor (LCA) $x_1 + x_2$



$$D^{(2)}(x, x', \tau) = \frac{1}{2\pi} \frac{1}{\sqrt{xx'(1-x-x')}} \left[\mathrm{e}^{-\frac{\pi\tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi\tau^2}{1-x-x'}} \right]$$

  - 1st term: the splitting of the LCA occurs at late times $\tau' \sim \tau$
  - 2nd term: the splitting of the LCA occurs at early times $\tau' \sim 0$

- All the n-body correlations $D^{(n)}$ have been similarly computed
  - fluctuations are stronger than for jets in the vacuum

Fluctuations in the gluon distribution

- Soft gluons $(x \ll 1)$ and small medium/high energy jet $(\tau \ll 1)$

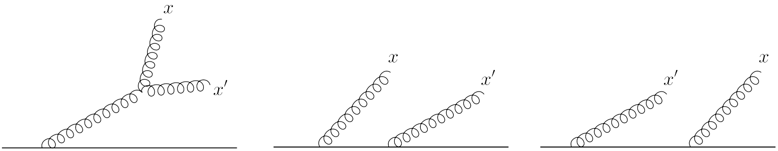
$$D(x,\tau) \,=\, \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\,\mathrm{e}^{-\pi\frac{\tau^2}{1-x}} \,\simeq\, \frac{\tau}{\sqrt{x}}$$

$$D^{(2)}(x,x',\tau) \,\simeq\, \frac{3}{2}\frac{\tau^2}{\sqrt{xx'}} \,\simeq\, \frac{3}{2}D(x,\tau)D(x',\tau)$$

- Factorization ... but large fluctuations

$$D^{(2)}(x,x',\tau) \,-\, D(x,\tau)D(x',\tau) \simeq \frac{1}{2}D(x,\tau)D(x',\tau)$$

- This difference would vanish for a Poissonian distribution



- Huge fluctuations in the multiplicities of the soft gluons

Multiplicities: Koba-Nielsen-Olesen scaling

- Number of gluons with $\omega \geq \omega_0$, where $\omega_0 \ll E$:

$$\langle N(\omega_0)\rangle = \int_{\omega_0}^{E} \mathrm{d}\omega \, \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq 1 + 2\left[\frac{\omega_{\rm br}}{\omega_0}\right]^{1/2} \quad \text{(LP + radiation)}$$

  - $\langle N(\omega_0)\rangle \simeq 1$ when $\omega_0 \gg \omega_{\rm br}$: just the LP
  - $\langle N(\omega_0)\rangle \gg 1$ when $\omega_0 \ll \omega_{\rm br}$: multiple branching

- All the higher moments $\langle N^p\rangle$ have been similarly computed

$$\frac{\langle N^2\rangle}{\langle N\rangle^2} \simeq \frac{3}{2}\,, \qquad \frac{\langle N^p\rangle}{\langle N\rangle^p} \simeq \frac{(p+1)!}{2^p}$$

- KNO scaling : the reduced moments are pure numbers
- A special negative binomial distribution (parameter $r = 2$)
  - huge fluctuations (say, as compared to a Poissonian distribution)

$$\frac{\sigma_N}{\langle N\rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N\rangle} = \frac{1}{\sqrt{\langle N\rangle}}$$

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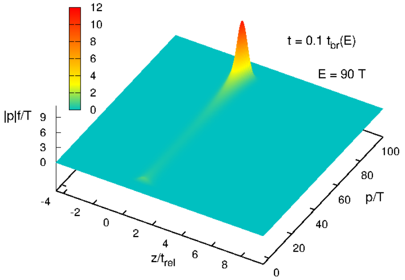
  - fluctuations are stronger than for jets in the vacuum ($r = 3$)

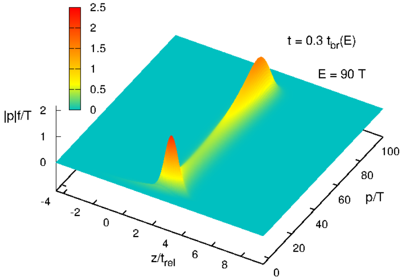
$$\frac{\sigma_N}{\langle N\rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N\rangle} = \frac{1}{\sqrt{3}}$$

Jet thermalization *(E.I. and Bin Wu, arXiv:1506.07871)*

- Kinetic equation for longitudinal dynamics: branchings + elastic collisions (Fokker-Planck approximation: drag and diffusion)
- Gluon distribution in energy (p) and longitudinal coordinate (z)
- Initial condition at $t = z = 0$: $E = 90\,T$



(a)



(b)

- $t_{\rm br}(E)$: the lifetime of the leading particle
- the time before the LP undergoes a first democratic branching

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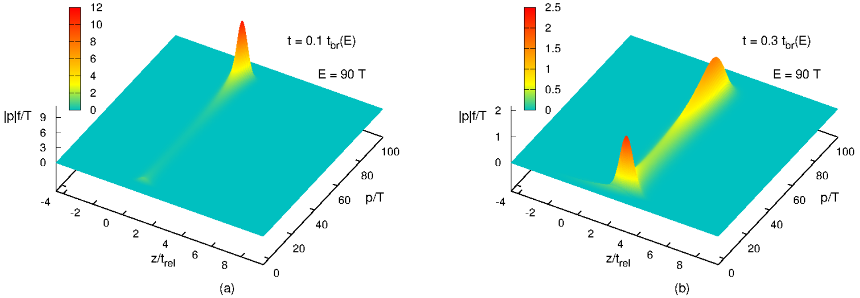
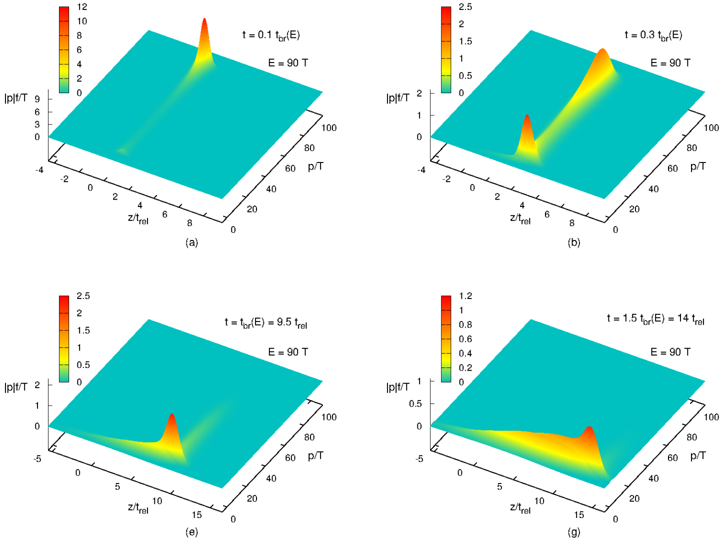


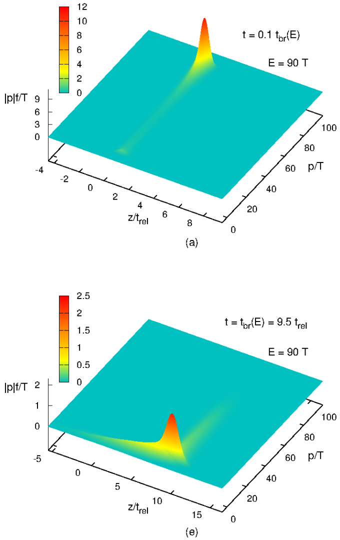
- With increasing time, the jet substructure is softening (mostly via branchings) and broadening (via drag and diffusion)

Jet thermalization



- $t = t_{\rm br}(E)$: the leading particle disappears (democratic branching)

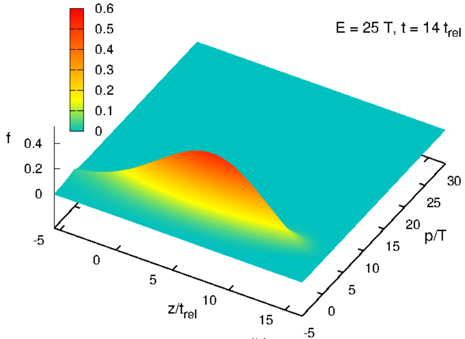
Jet thermalization



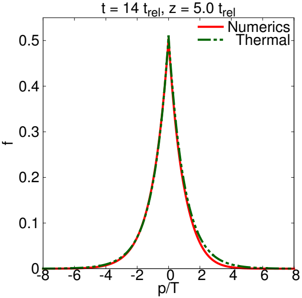
- $t = 1.5\, t_{\rm br}(E)$: the jet is fully quenched

The late stages: thermalizing a mini-jet

- At late times $t \gg t_{\rm br}(E)$, the jet is 'fully quenched'
  - no trace of the leading particle, just a thermalized tail
  - the typical situation for a mini-jet : $E \lesssim \omega_{\rm br}(L)$

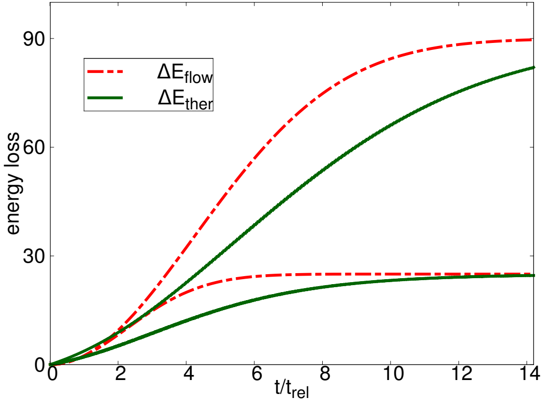


(h)



$$f(t,z,p) \simeq \mathrm{e}^{-\frac{|p|}{T}}\,\mathrm{e}^{-\frac{z^2}{4tt_{\rm rel}}}$$ (spatial diffusion $\Longrightarrow$ hydro)

Energy loss towards the medium



- Upper curves: $E = 90\,T$; lower curves: $E = 25\,T$
- $\Delta E_{\rm ther}$: the energy carried by the thermalized tail $(t - z \geq t_{\rm rel})$
- $\Delta E_{\rm flow} = \pi \omega_{\rm br}(t) \propto t^2$: only branchings

Conclusions

- Effective theory and physical picture for jet quenching from pQCD
  - event-by-event physics: multiple branching
  - democratic branchings leading to wave turbulence
  - thermalization of the soft branching products with $p \sim T$
  - efficient transmission of energy to large angles
  - wide probability distribution, strong fluctuations, KNO scaling
- Fluctuations compete with the difference in path lengths in determining the di-jet asymmetry
- Qualitative and semi-quantitative agreement with the phenomenology of di-jet asymmetry at the LHC
- Important dynamical information still missing: vacuum-like radiation (parton virtualities), medium expansion ...