

Improving Strong and Weak Coupling Energy Loss

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May 12, 2017

WAH, Phys.Rev. D91 (2015) [arXiv:1501.04693]

Isobel Kolbé and WAH, arXiv:1511.09313

R. W. Moerman and WAH, arXiv:1605.09285

Abdullah Khalil and WAH, arXiv:1701.00763

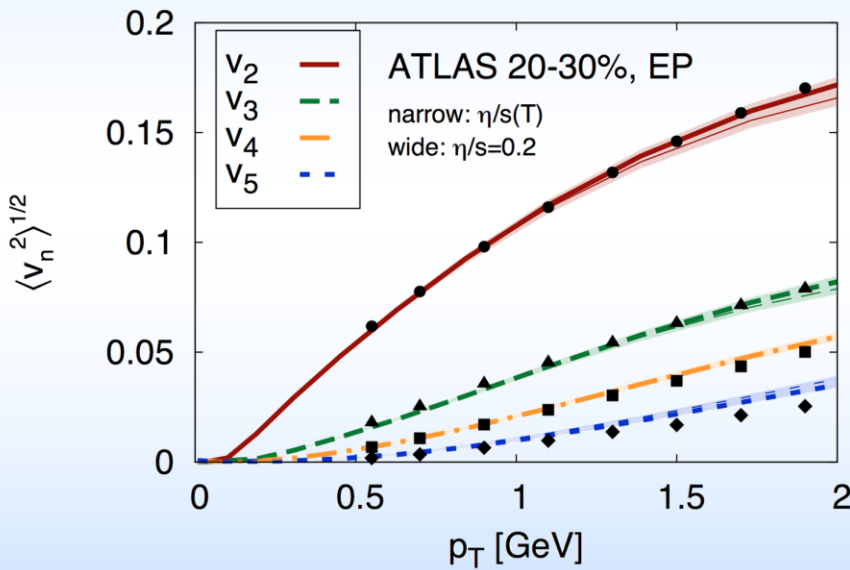
In collaboration with Nadia Barnard, Robert Hambrock,
Abdullah Khalil, Isobel Kolbé, Robert Moerman, and Andri Rasoanaivo,



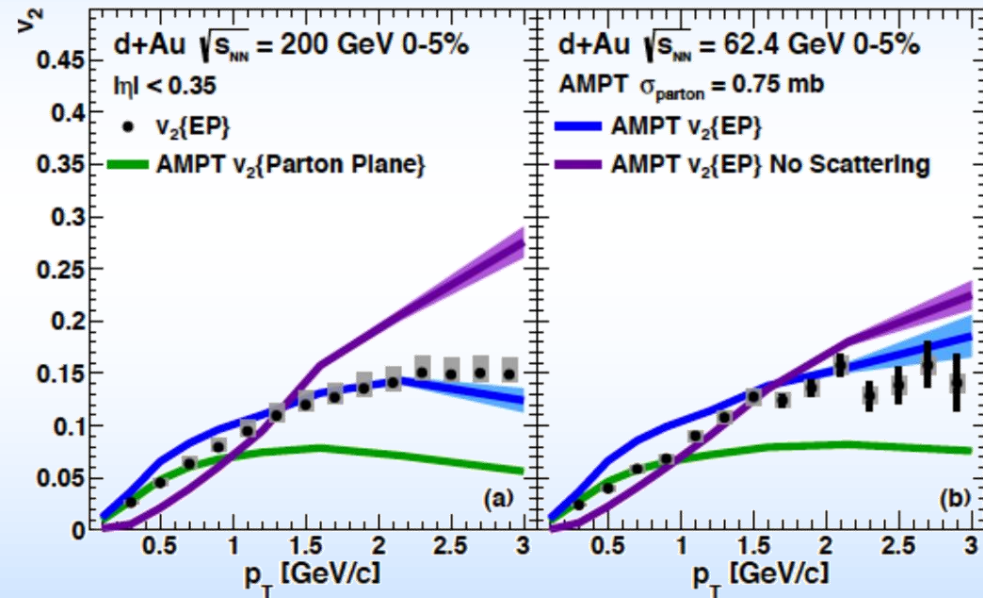
Low- p_T Obs: Strong or Weak?

- Hydro: Strong

- AMPT: Weak



Gale et al., PRL110 (2013)

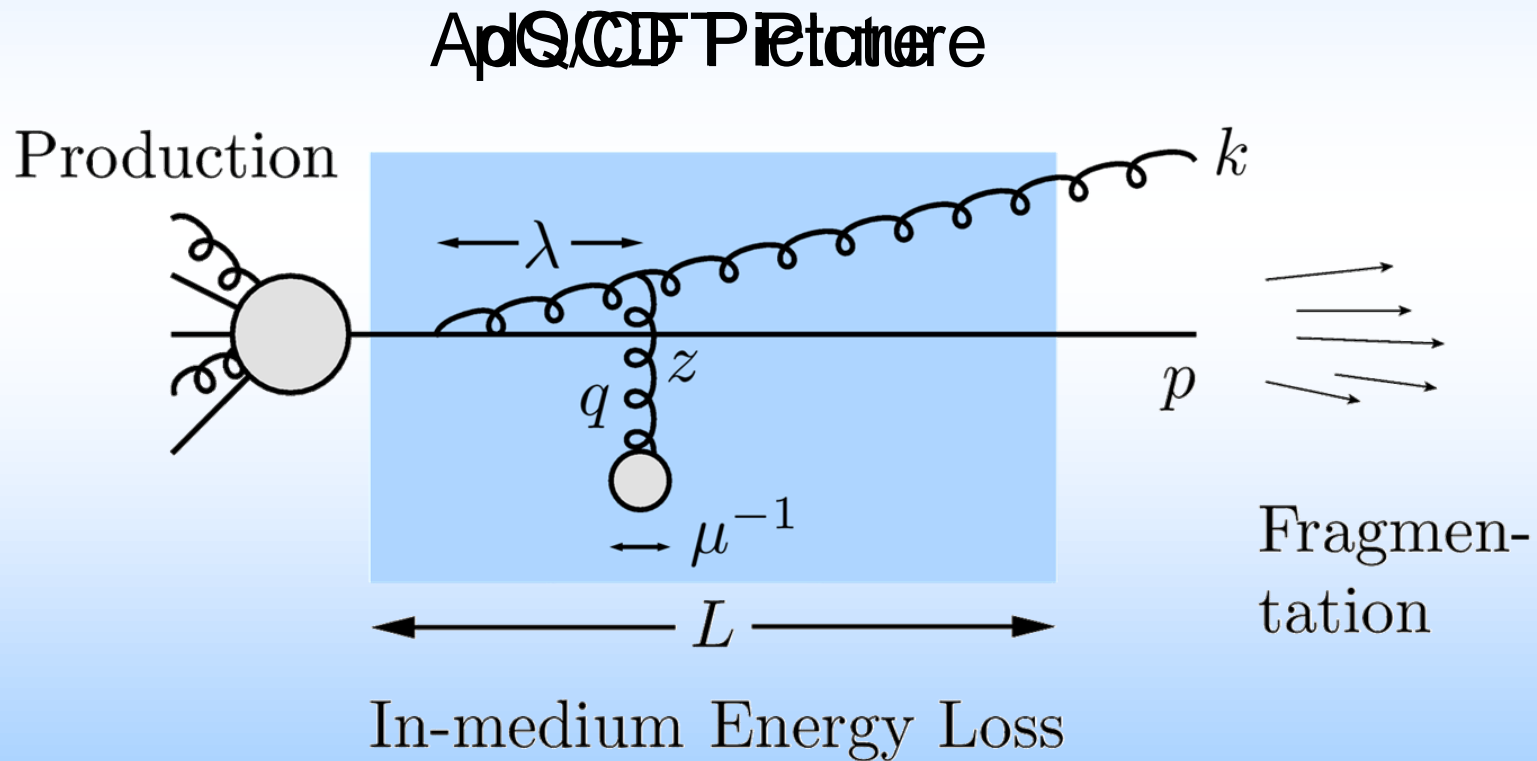


AMPT (from Zajc QM17)



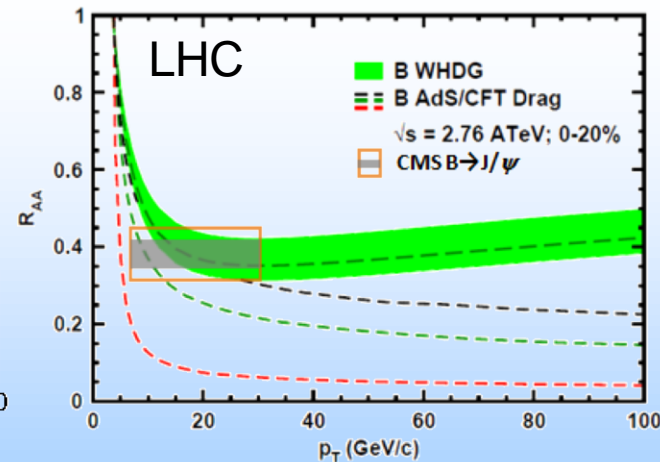
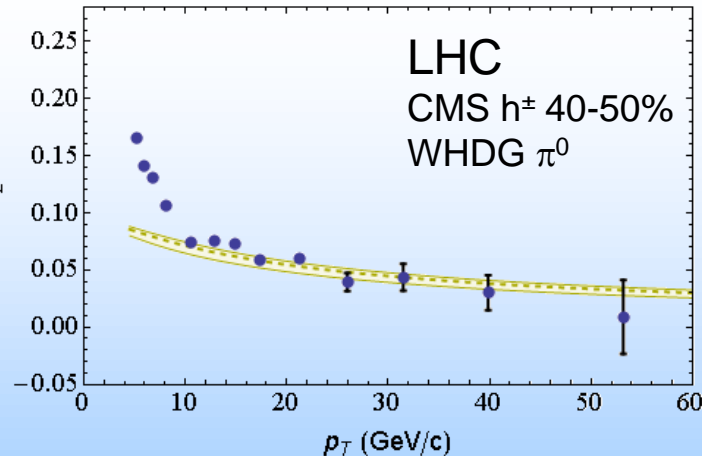
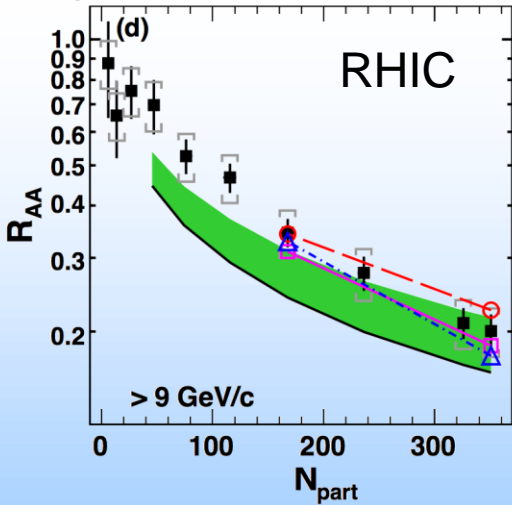
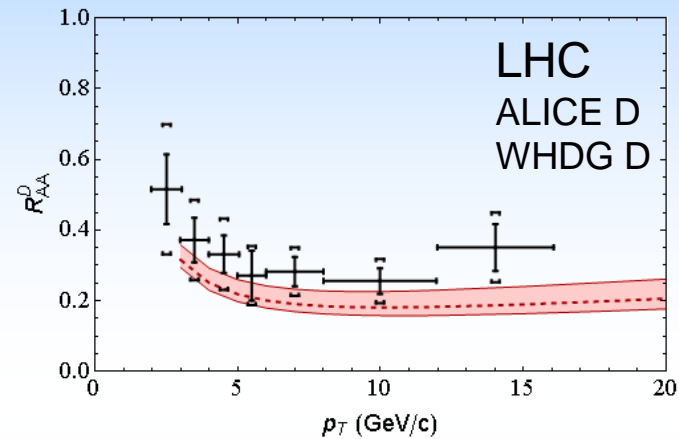
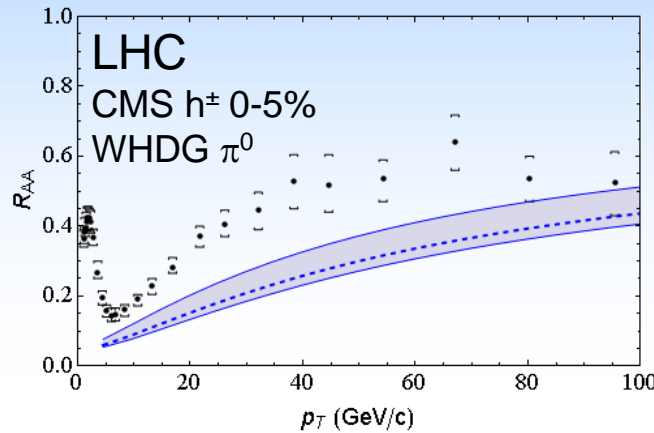
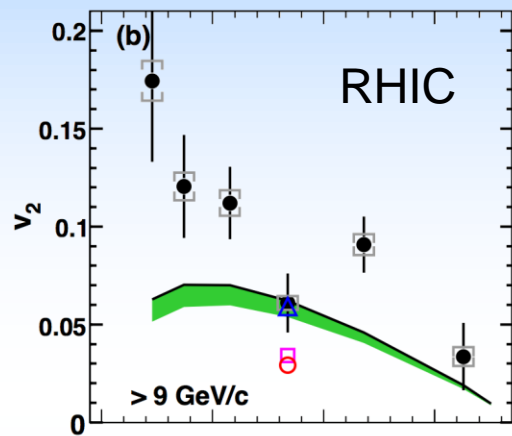
Use High- p_T Femtoscope to Differentiate

Most direct probe of DOF of QGP



pQCD E-loss Describes RHIC/LHC

– Constrained by RHIC, LO pQCD predictions strikingly similar to LHC data



PHENIX PRL105 (2010)

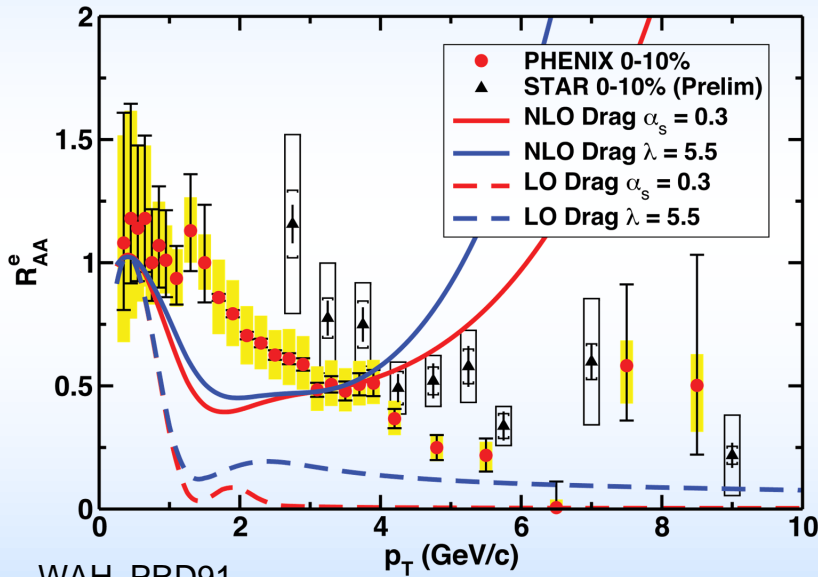
CMS, Eur.Phys.J. C72 (2012)
CMS, PRL109 (2012)

ALICE, JHEP1209 (2012) 112
CMS, JHEP 1205 (2012) 063



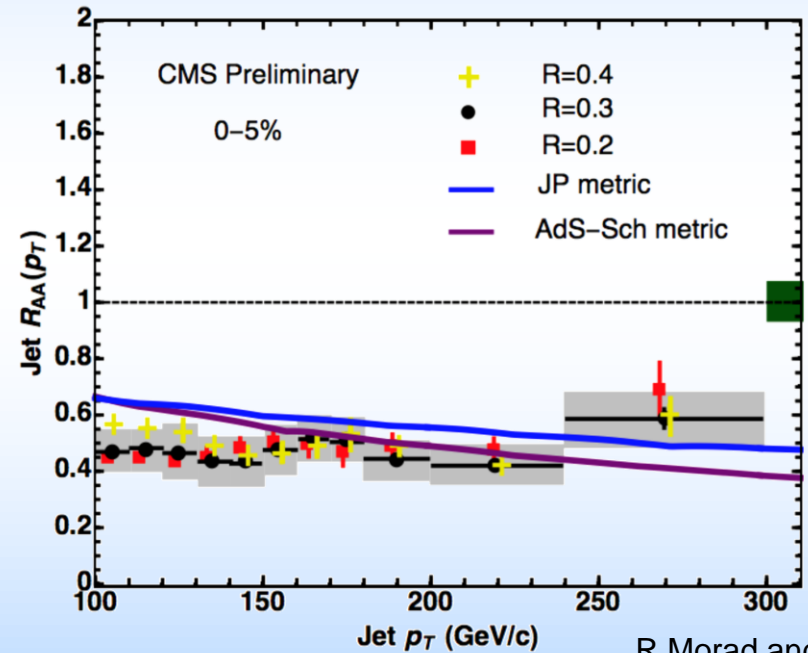
AdS/CFT Describes RHIC/LHC

- RHIC HF e-



WAH, PRD91
[1501.04693]

- LHC Jets



R Morad and WAH,
JHEP11(2014)017



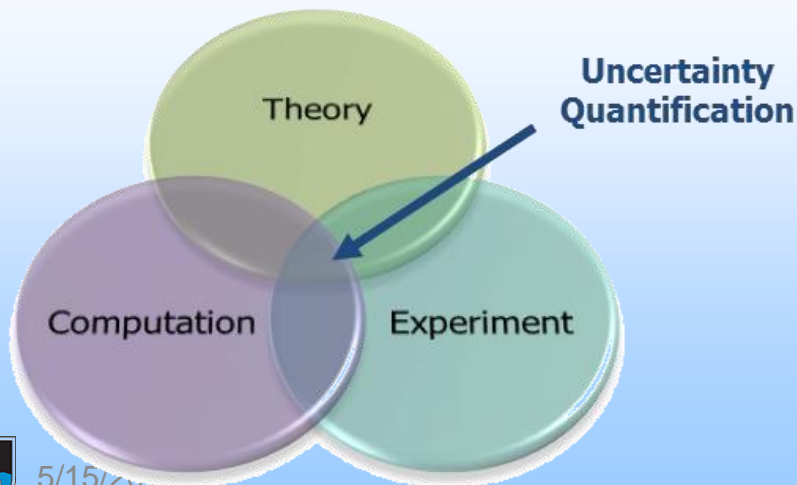
One Approach

- Error bars are infinite
- For all of it, let's all become bankers

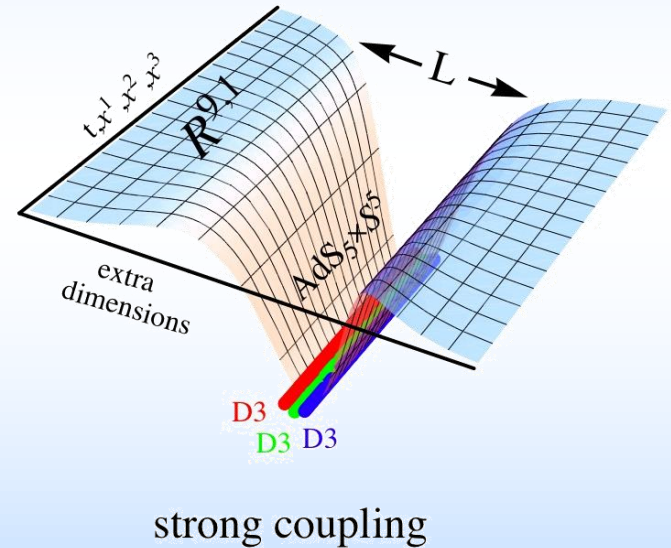
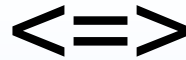
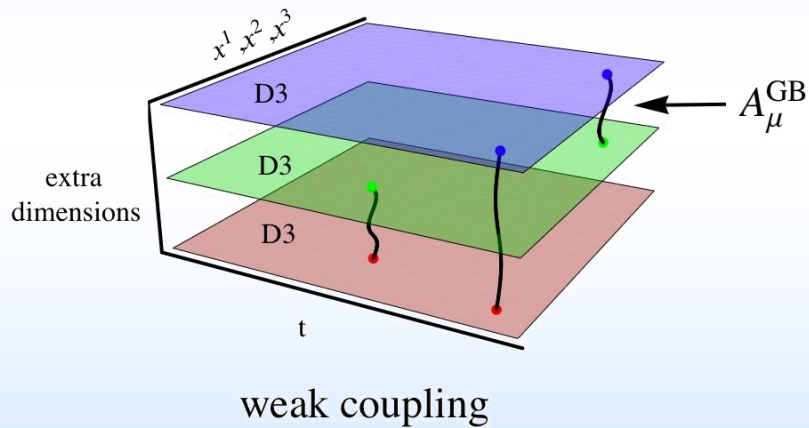


Another Approach

- Seek for both pQCD and AdS/CFT to
 - reduce theoretical uncertainties
 - extend regime of applicability
 - find differentiating observables



Start with AdS/CFT



From When We Had Sensible, Rational US Leadership...



There are known knowns. These are things we know that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know.

(Donald Rumsfeld)

izquotes.com



Rummy Says We Have Some

- **Known knowns:**
 - LO energy loss for an infinitely massive dragged, time invariant string in static background $N = 4$ SYM; some generalizations of $N = 4$
 - ...
- **Known unknowns:**
 - Correct dual for light flavor/jets
 - Unforced heavy quarks
 - Momentum fluctuations
 - Connection to QCD
 - ...
- **Unknown unknowns: ???**



LO AdS for Heavy Quark E-Loss

- Result is a drag:

$$dp_T/dt = -\mu p_T$$

$$\mu = \pi\lambda^{1/2} T^2/2M_q$$

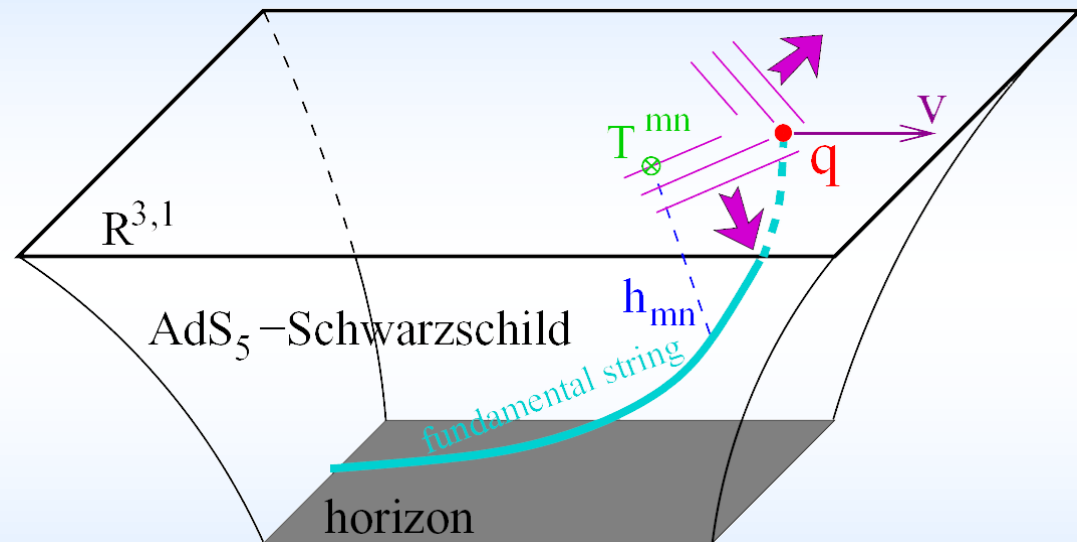
Herzog et al., JHEP 0607 (2006)
Gubser, PRD74 (2006)

Similar to Bethe-Heitler

$$dp_T/dt \sim -(T^3/M_q^2) p_T$$

Very different from usual pQCD and LPM

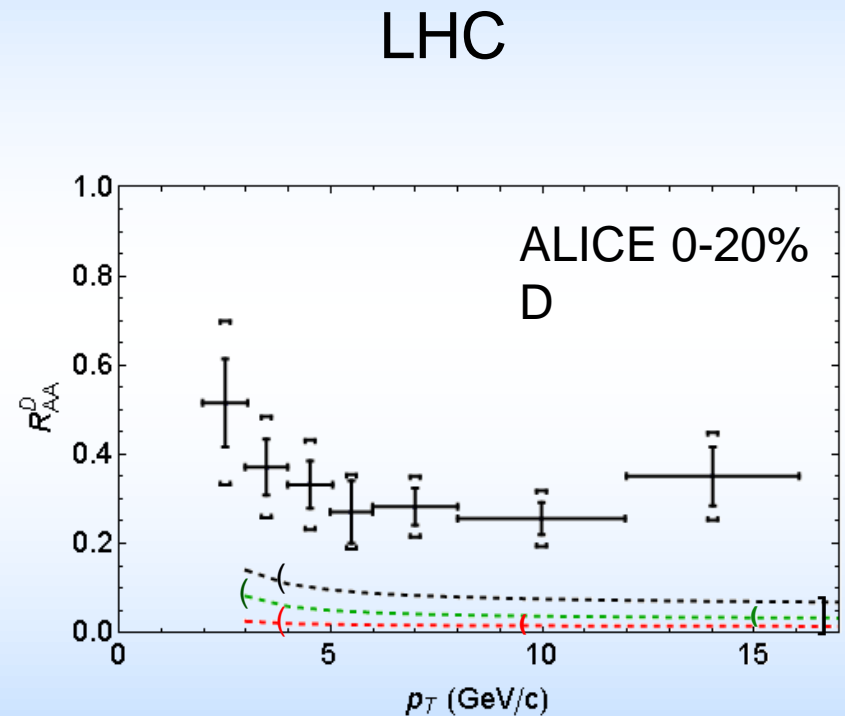
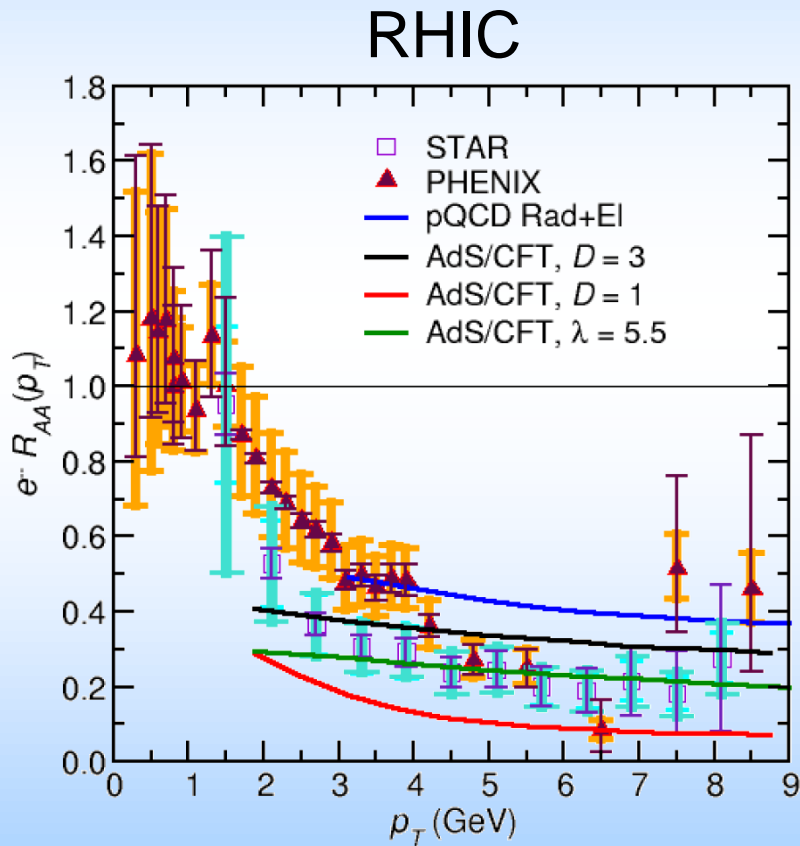
$$dp_T/dt \sim -LT^3 \log(p_T/M_q)$$



J Friess, et al., PRD75 (2007)

Failure of LO AdS for Heavy High- p_T

- Constrained by RHIC, oversuppress LHC



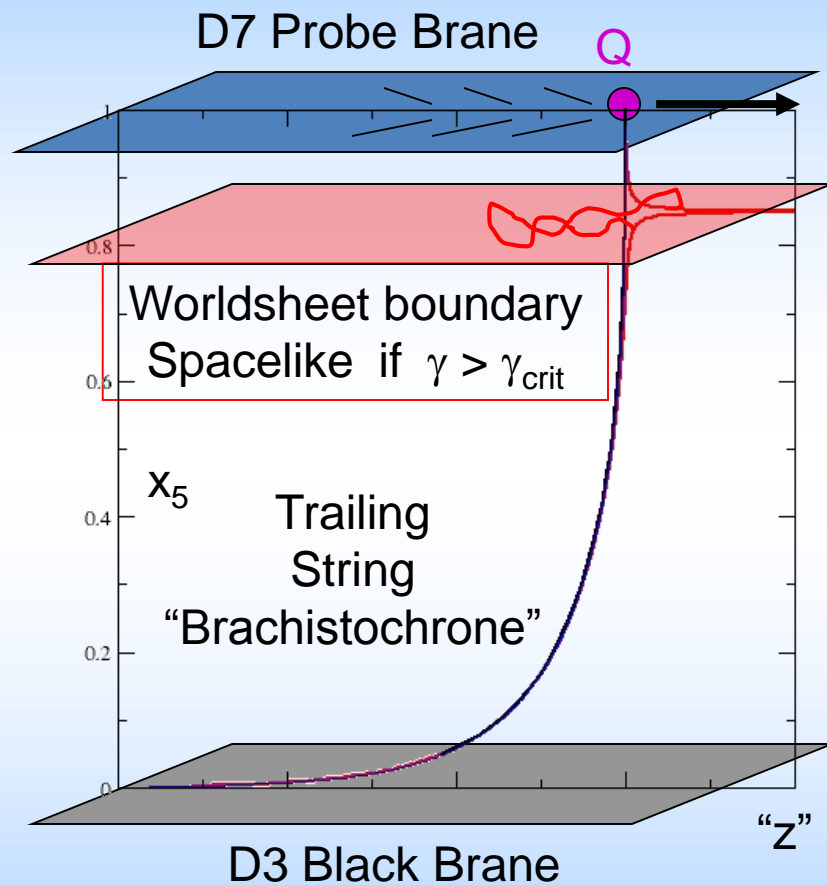
WAH, PANIC11 (arXiv:1108.5876)
ALICE, JHEP 1209 (2012)

WAH, PhD Thesis, arXiv:1011.4316



Limits on Heavy Flavor AdS Setup

- For LO AdS:
 - Space-like quark endpoint
 - $\gamma_{\text{crit}} = (1 + 2M_q/\lambda^{1/2} T)^2$
 $\sim 4M_q^2/(\lambda T^2)$
 - Equiv. due to Schwinger
 - Mom. Loss Fluctuations
 - $\gamma_{\text{crit}} = M_q^2/(4T^2)$
- Speed limit from fluct parametrically larger, but numerically smaller



Fluctuations a la Gubser/Teaney

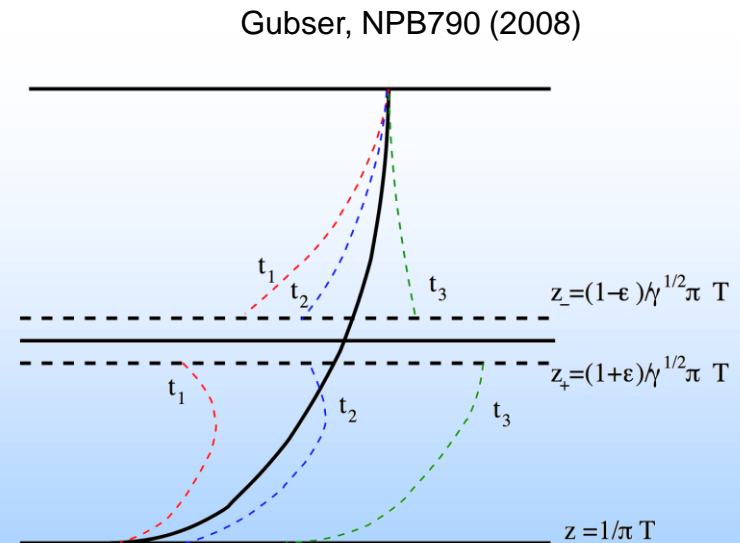
$$\frac{dp_i}{dt} = -\eta_D + F_i^L + F_i^T$$

$$\langle F_i^L(t_1) F_j^L(t_2) \rangle = \kappa_L \hat{p}_i \hat{p}_j g(t_1 - t_2)$$

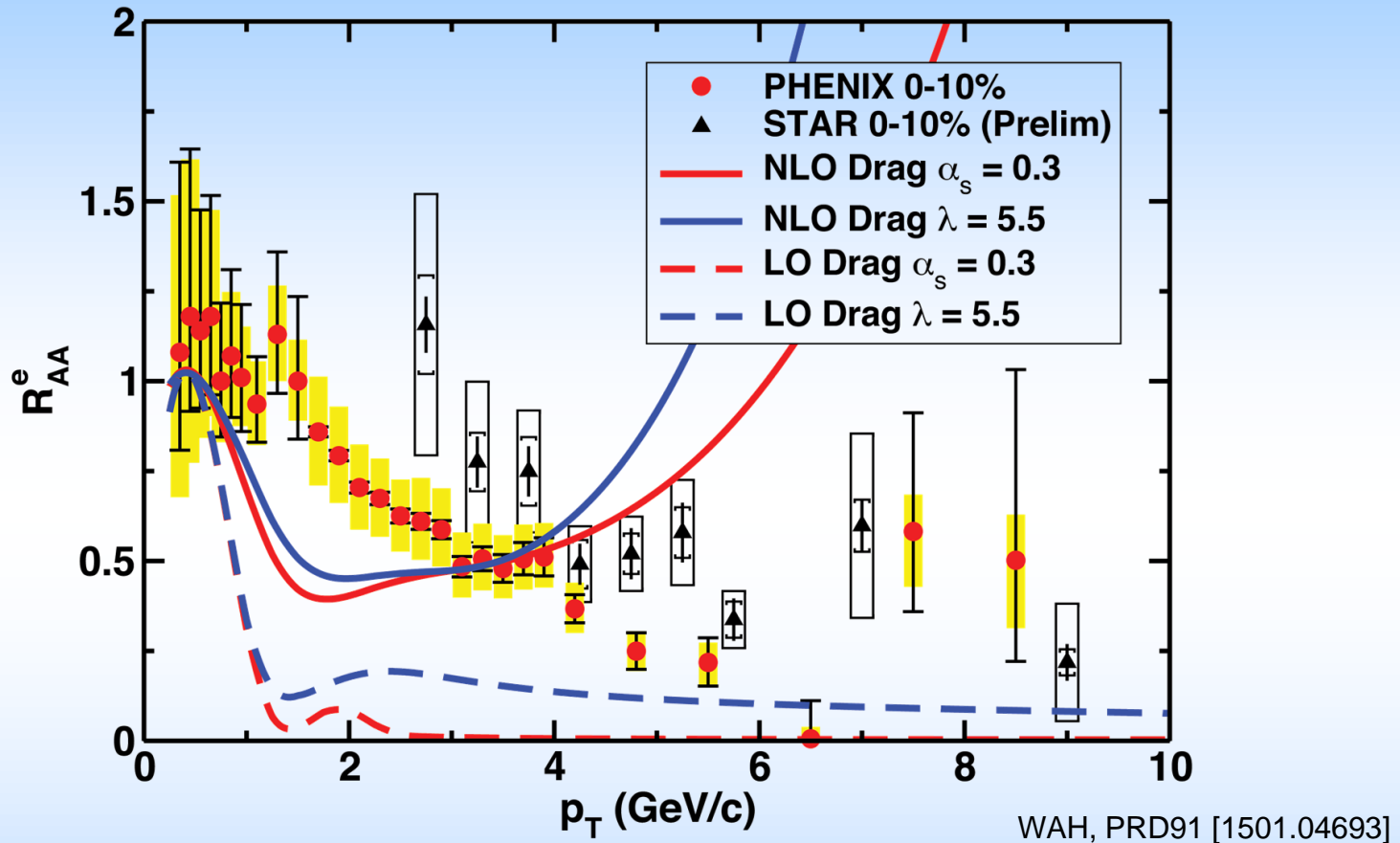
$$\langle F_i^T(t_1) F_j^T(t_2) \rangle = \kappa_T (\delta_{ij} - \hat{p}_i \hat{p}_j) g(t_1 - t_2)$$

$$\kappa_T = \pi \sqrt{g^2 N_c T^3} \sqrt{\gamma}; \quad \kappa_L = \pi \sqrt{g^2 N_c T^3} \gamma^{5/2}$$

- Obeys Einstein's relations *only* at $v = 0$. Thermal in origin?
- Multiplicative Langevin problem!
 - Results depend on time within timestep kicks are evaluated
 - Ito, Stratonovich, Hänggi-Klimontovich
- Non-Markovian:
 - Colored (not white) noise
 - Momentum kicks have a memory



Compare to RHIC HF Electrons

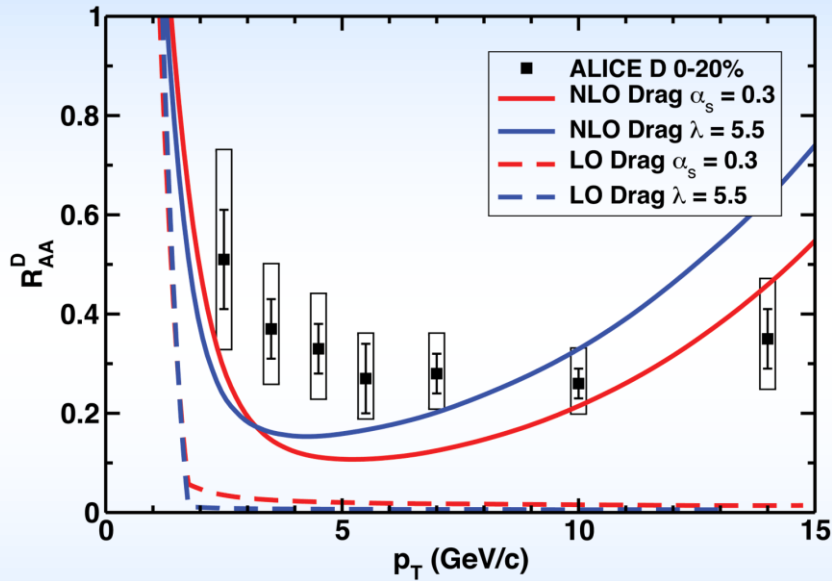


- Agreement in sweet spot $p_T \sim 3 - 4 \text{ GeV}/c$
 - Below 3 GeV production unreliable
 - Above 4 GeV theory corrections necessary (col. noise, non-const p)
- NB: VISHNU medium hotter than from previous calc => larger LO supp.

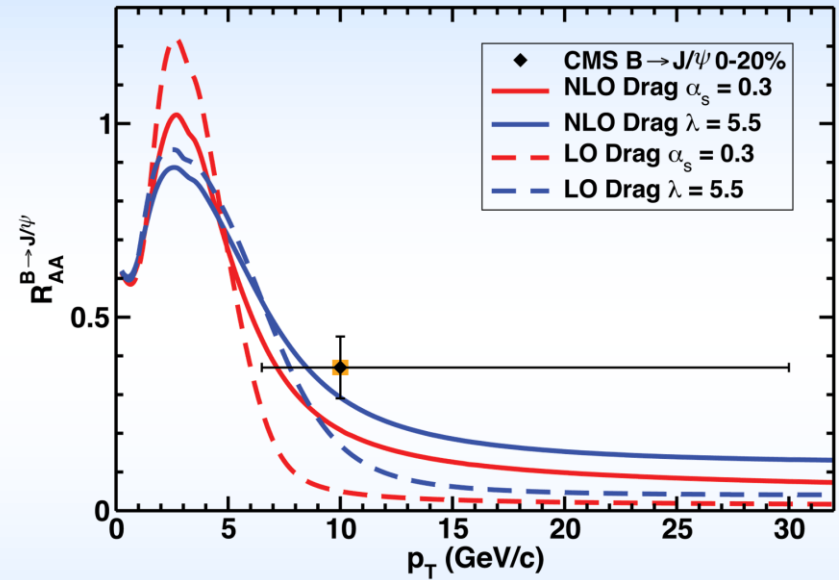


Compare to LHC: R_{AA}

- D Mesons



- B Mesons



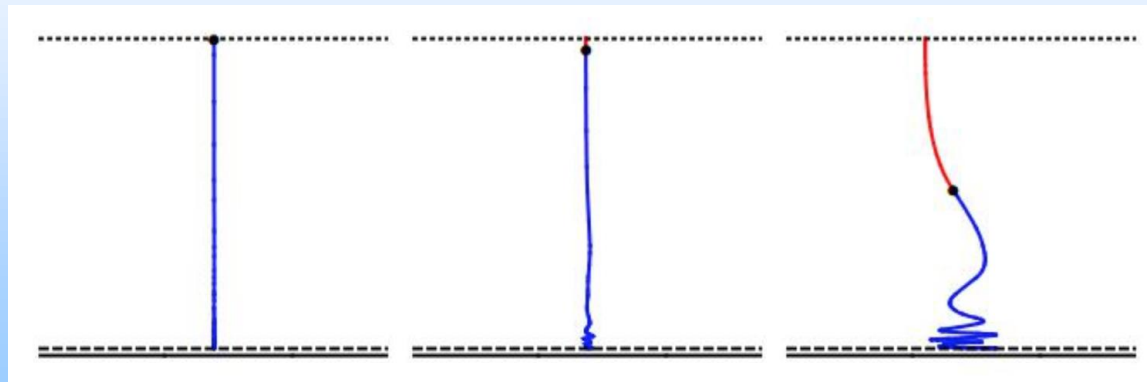
WAH, PRD91 [1501.04693]

- Predictions qualitatively similar to data
 - D harder than e; $m_B \Rightarrow$ valid to higher p_T



Extending Fluctuating E-Loss

- What happens when dragging string picture breaks down at high- p_T ?
 - Equivalently, how do fluctuations affect light flavor?
- Extremely difficult problem
 - Solve simpler initial $v = 0$ in AdS_3
 - Compute mean distance squared travelled by the endpoint as it falls



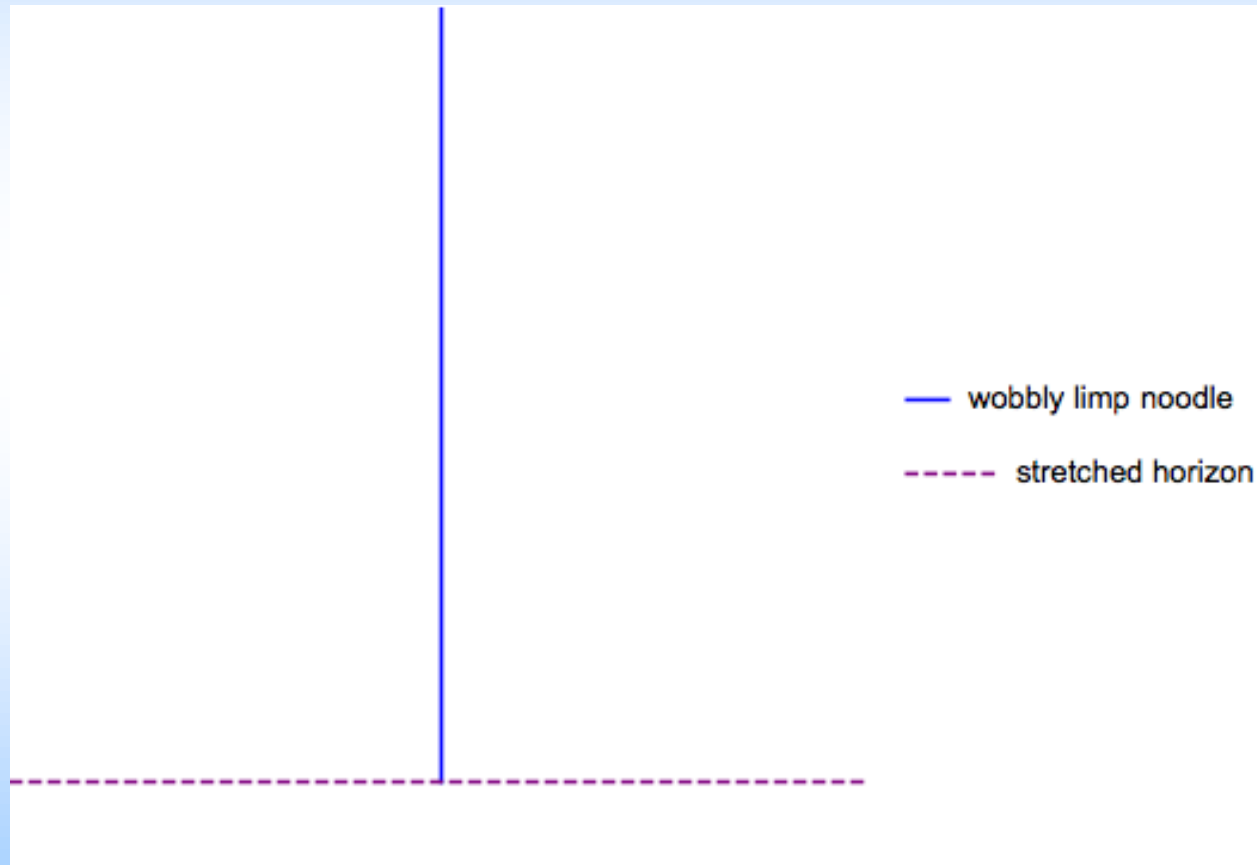
Outline of Solution:

- Derive classical solution
- Quantize perp dir's of motion given classical sol'n
- Populate quanta according to Bose statistics (semi-classical approx.)
- Compute correlators
 - For details, see slides at end or [arXiv:1605.09285](https://arxiv.org/abs/1605.09285)



Non-equilibrium Thermodynamics

- Extend to massless sol'n:



Analytic Brownian Motion

- Average distance squared travelled as a fcn of time t

$$\begin{aligned} s^2(t; a) &:= \langle : (\hat{X}_{\text{End}}(t; a) - \hat{X}_{\text{End}}(0; a))^2 : \rangle \\ &= \frac{\beta^2}{4\pi^2 \sqrt{\lambda}} \int_0^\infty \frac{d\omega}{\omega} \frac{1}{e^{\beta\omega} - 1} |f_\omega(\sigma_f - at) - f_\omega(\sigma_f) e^{i\omega t}|^2 \end{aligned}$$

– a is the speed at which the endpoint falls

- Allows interpolation btwn known HQ and new light quark results
- Will also allow for $q_{\text{hat}}(t)$



Relate $D(a, d)$ to $\hat{q}(t)$

- At late times $t \gg \beta$, $s^2 = 2 D(a, d) t$, with

$$D(a, d) := \frac{(d-1)^2 \beta}{8\pi\sqrt{\lambda}} \left(1 - \frac{a}{2}\right)$$

- Conjecture connection to moving setup:

$$\hat{q} = \frac{\langle p_T^2 \rangle}{\lambda_{mfp}} = \frac{2\kappa_T}{v} = \frac{4T^2}{vD} = \frac{32\pi\sqrt{\lambda}T^3}{(d-1)^2(1-a/2)v}$$

- Time dependence in $a(t)$ and $v(t)$
 - Rate that endpoint falls in “5th” dimension



Heavy Quark qhat

- For heavy quarks $a = 0$

$$\hat{q}_{MH} = \frac{2\pi\sqrt{\lambda}T^3}{v} \quad \Leftarrow \text{This work}$$

$$\hat{q}_{Gubser} = \frac{2\pi\sqrt{\lambda}T^3}{v} \sqrt{\gamma} \quad \Leftarrow \text{Gubser, NPB790 (2008)}$$

- MH result behaves sensibly as $v \Rightarrow 1$
- No speed limit



Light Quark q_{hat}

- For light quarks

$$\hat{q}_{MH} = \frac{2\pi}{(1 - a/2)v} \sqrt{\lambda} T^3 \simeq 2\pi \sqrt{\lambda} T^3$$

<= This work, $t = 0$

$$\hat{q}_{LRW} = \frac{\pi^{3/2} \Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3 \simeq 7.5 \sqrt{\lambda} T^3$$

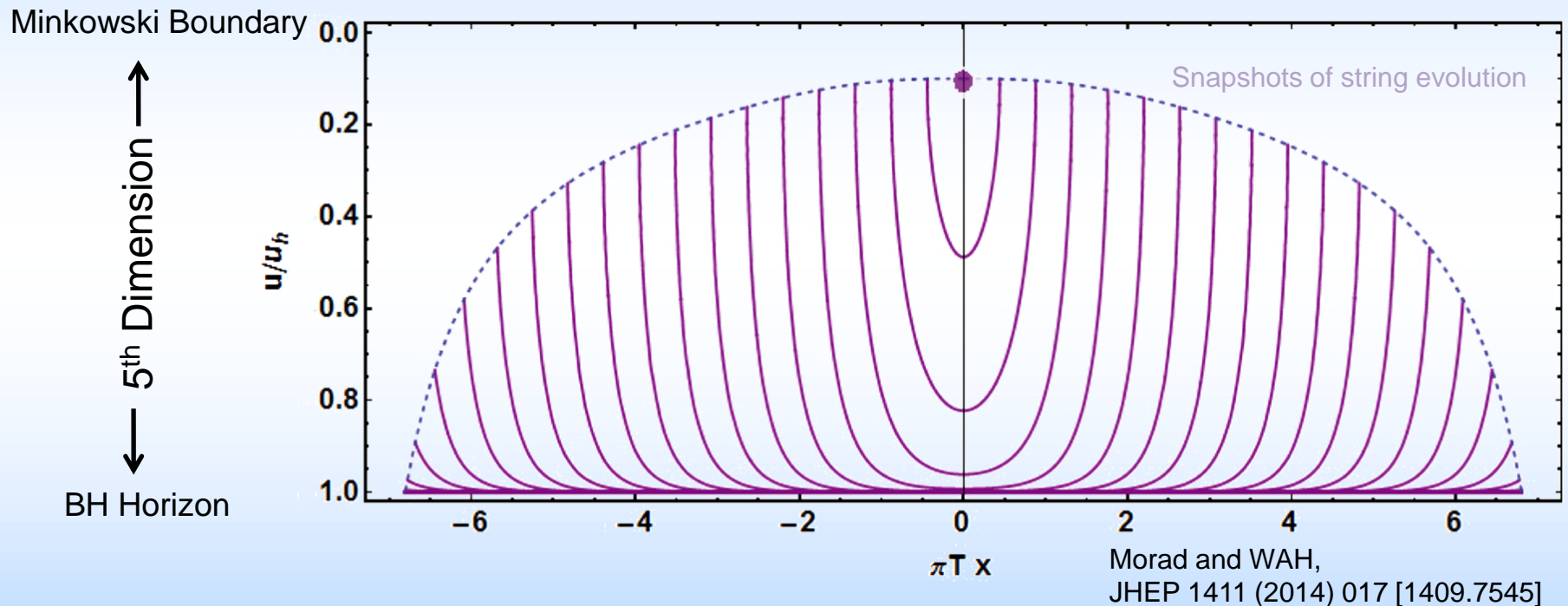
<= Liu, Rajagopal,
Wiedemann, PRL97
(2006) 182301

- Similar results to LRW;
completely different method



Relate $D(a, d = 5)$ to $q_{\text{hat}}(t)$

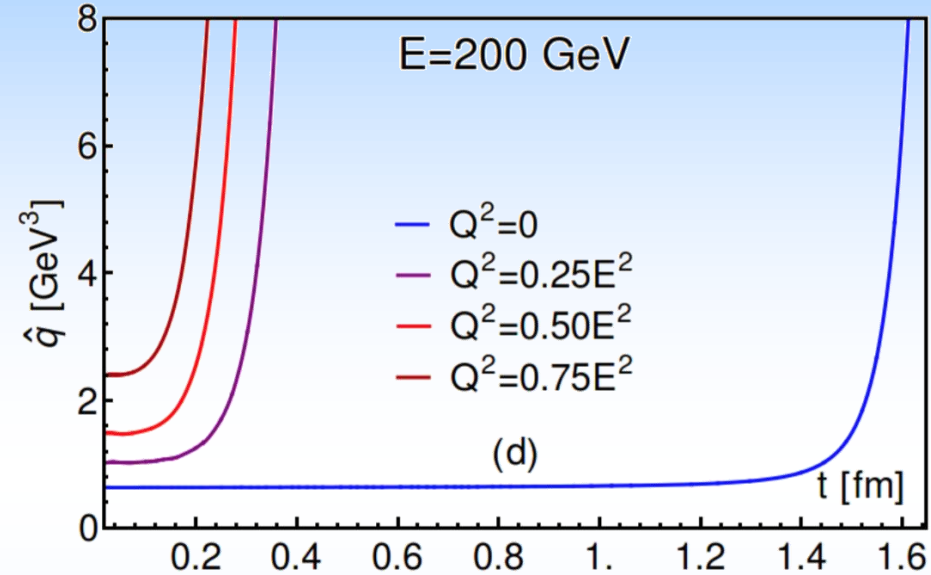
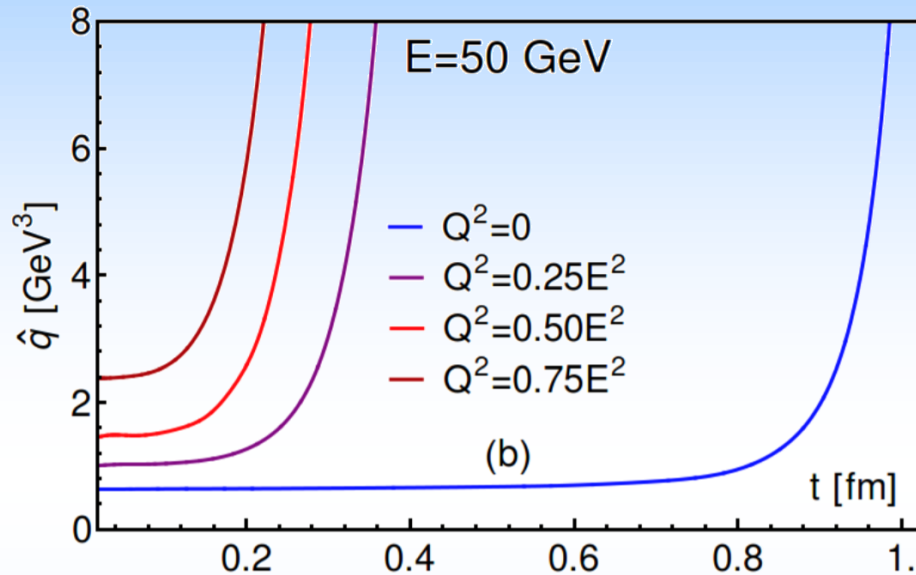
- We have numerical solutions for leading order falling strings in AdS_5



- Our numerical sol'n gives us the rate at which the endpoint is falling, the a in the previous!!



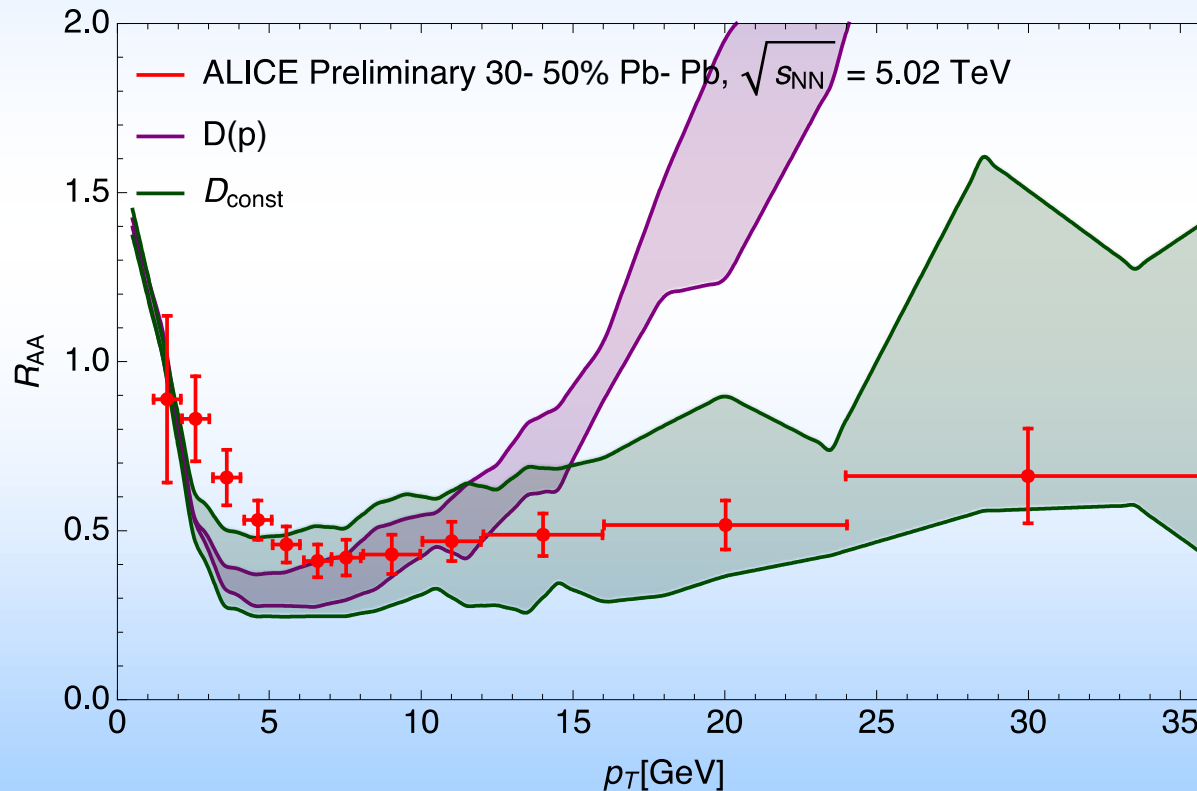
t Dependent \hat{q}



- $\hat{q}(t=0) \sim 3 - 10 \text{ GeV}^2/\text{fm}$, then increases
– $T = 350 \text{ MeV}$
- $\hat{q}(t \Rightarrow \infty) \Rightarrow \infty$ trivially as $v \Rightarrow 0$

Apply New $D = \text{const}$ to HF

- Take $D = 2/\pi T \lambda^{1/2}$ as fundamental
 - Longitudinal fluc & drag by fluc-diss

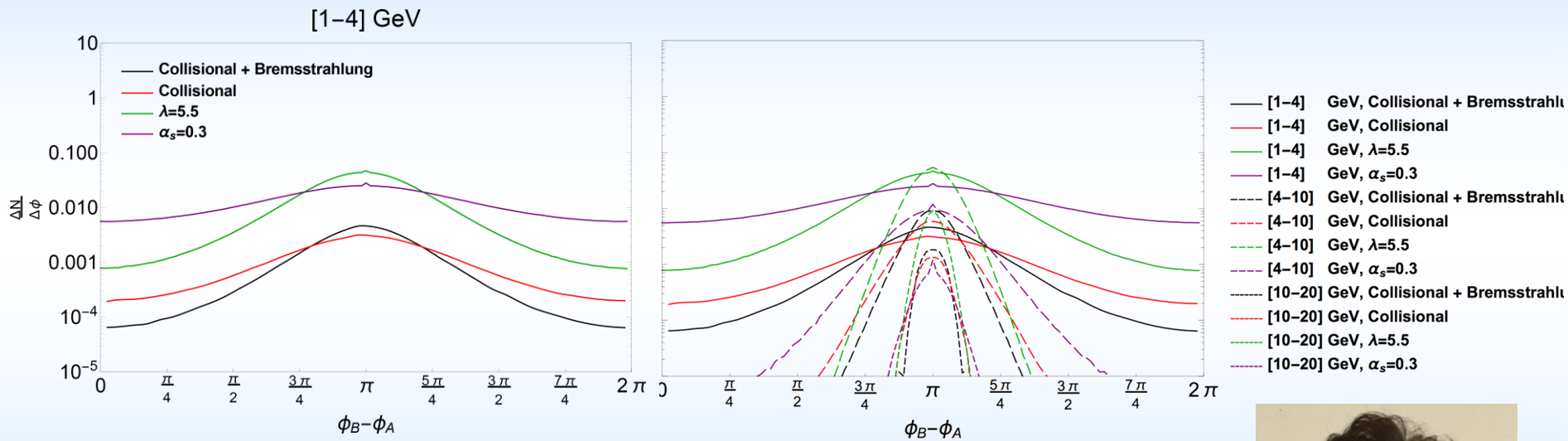


R Hambrock and WAH, *in prep*



AdS vs pQCD HF Correlations

- Attempt to differentiate between pQCD and AdS with correlations



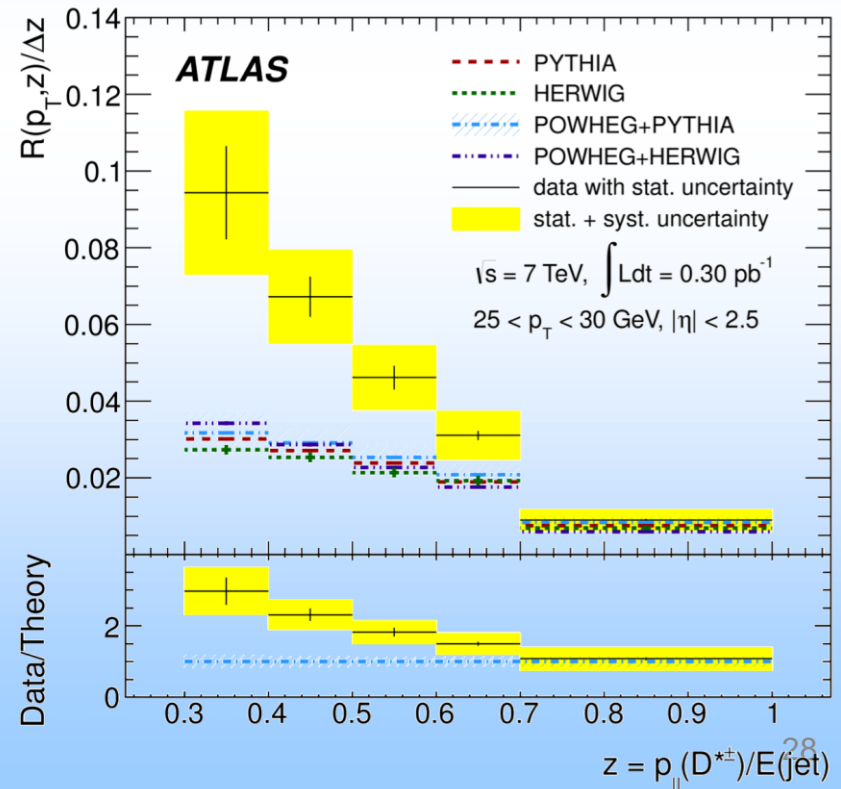
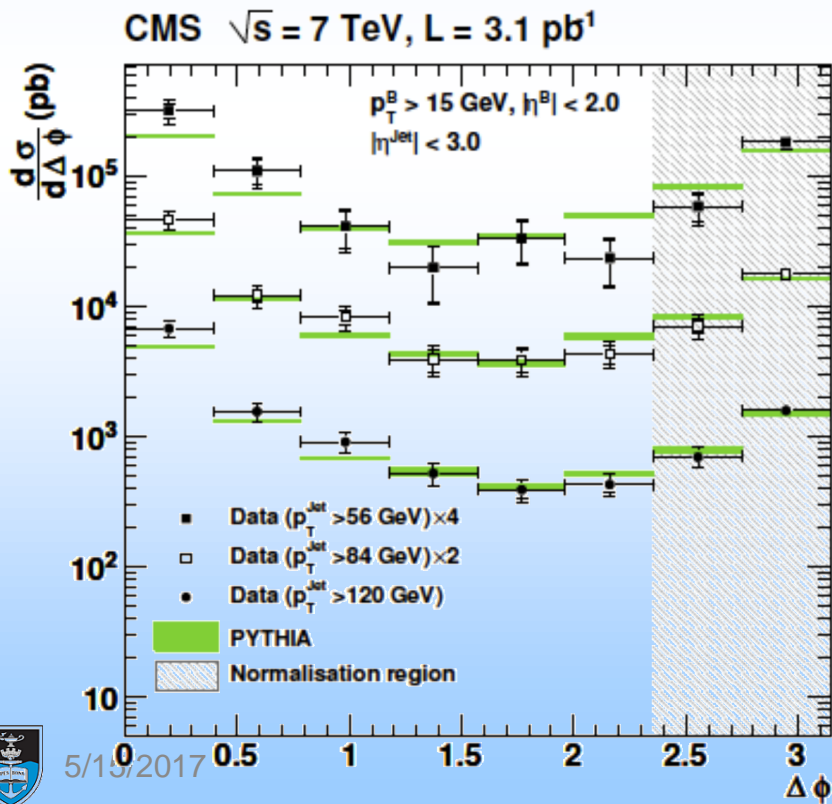
R Hambrock and WAH, *in prep*
 pQCD from Narhgang et al., PRC90 (2014)

- Difficult to differentiate with $dN/d\Delta\phi$
- Factor 10 difference in $dN/d\Delta p_T$



Plea for HF pp

- Require good theoretical control over pp baseline
 - No exclusive NLO + NLL calculation exists
 - New tools: need importance sampling
- Not much pp HF correlations to compare to

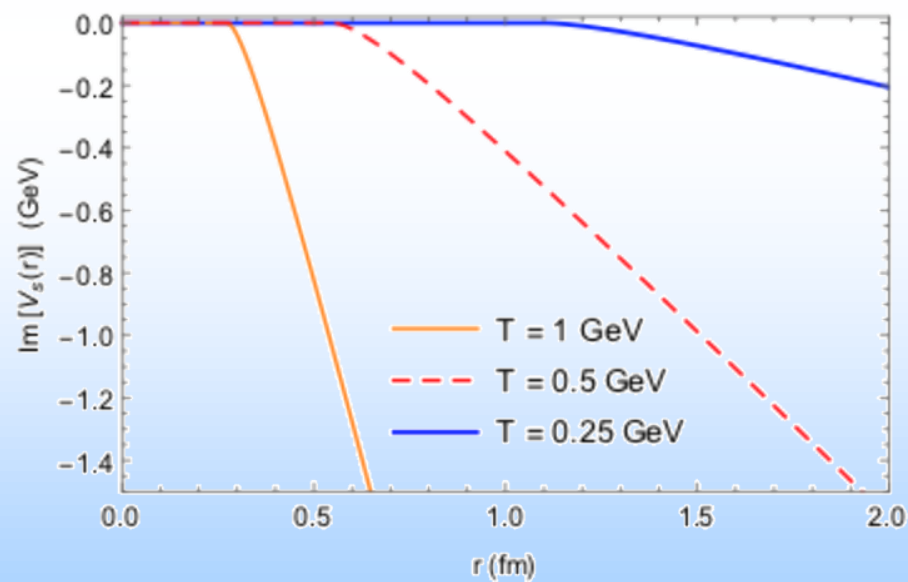
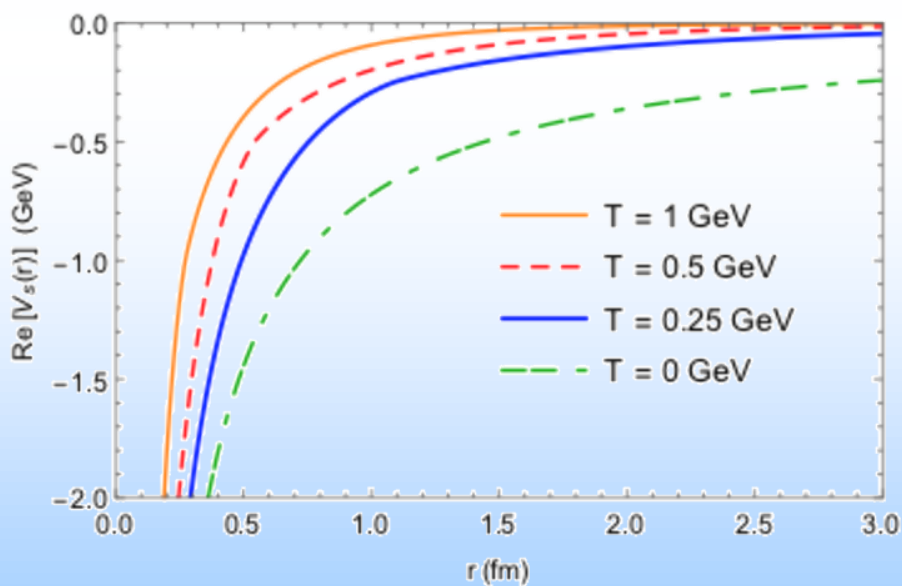


Apply AdS/CFT to Quarkonia

- Inspired originally by LRW $L \sim (1-v)^{1/4}$
- Use Albacete, Kovchegov, Taliotis $V(r)$
 - No v dependence (yet)

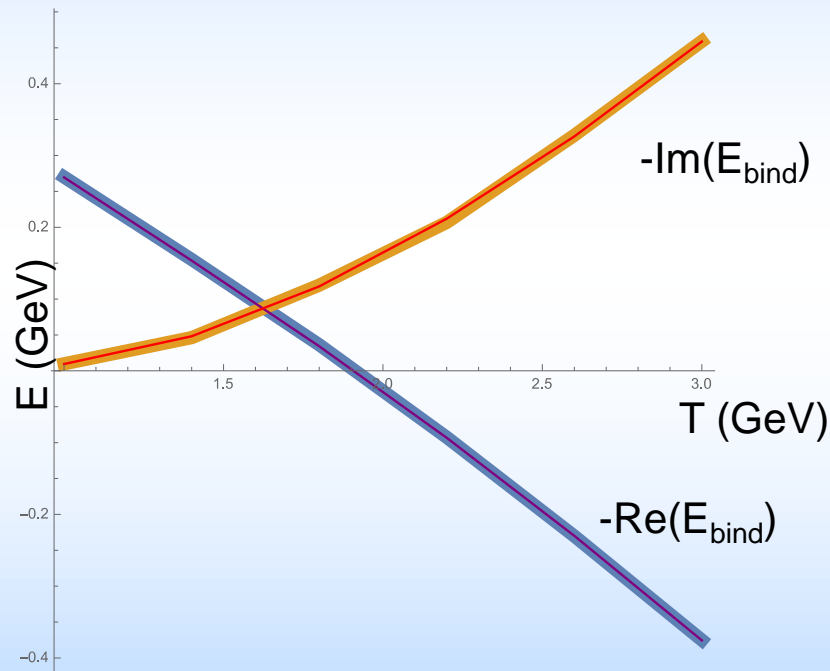


Nadia Barnard



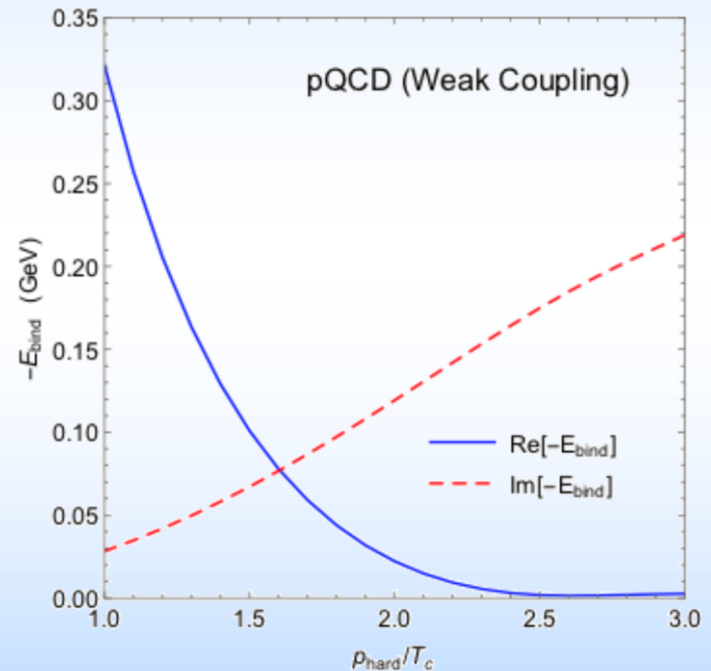
From Binding Energy to R_{AA}

- Binding Energy

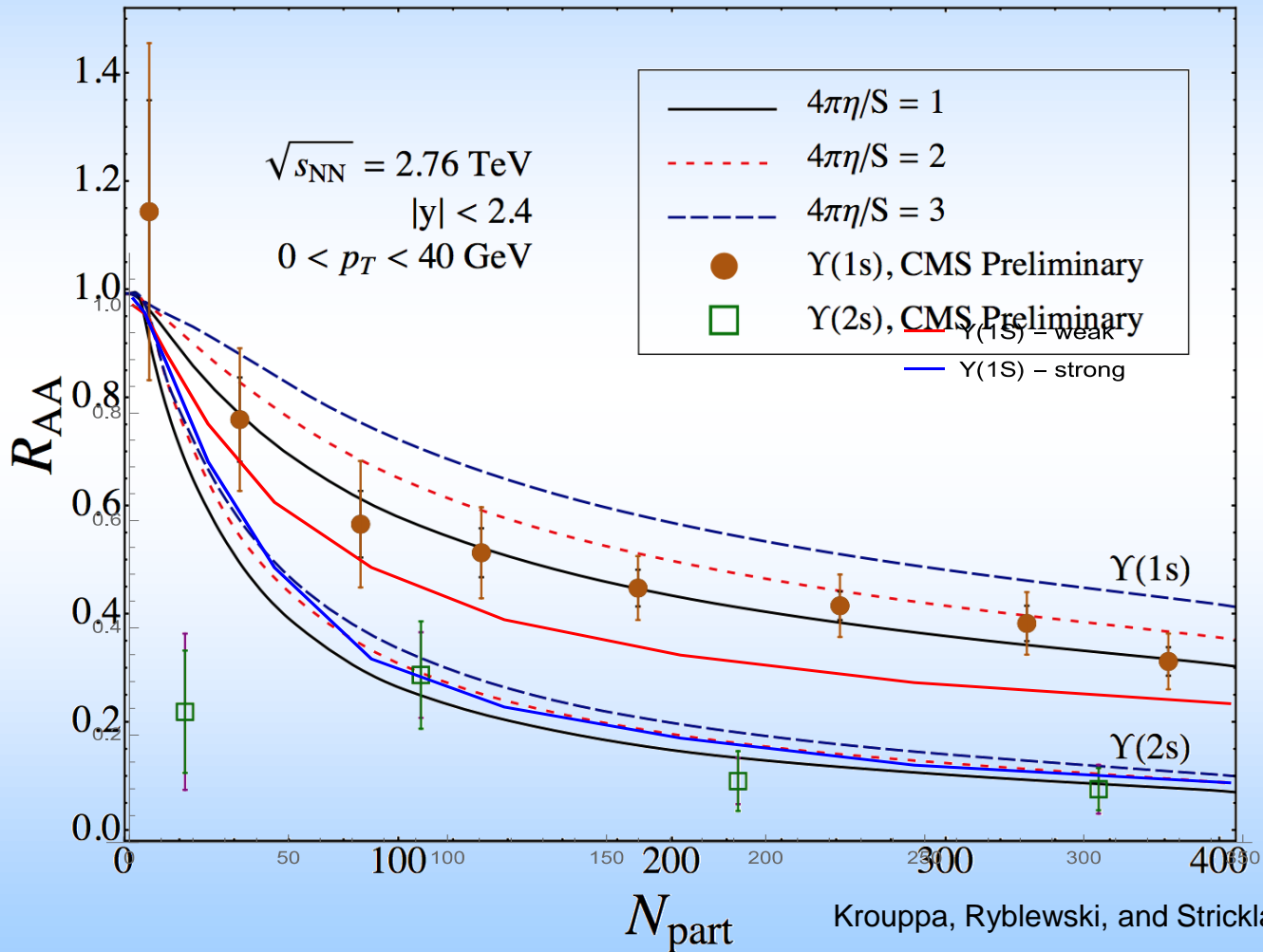


N Barnard and WAH, *in prep*

- Cf pQCD Binding E



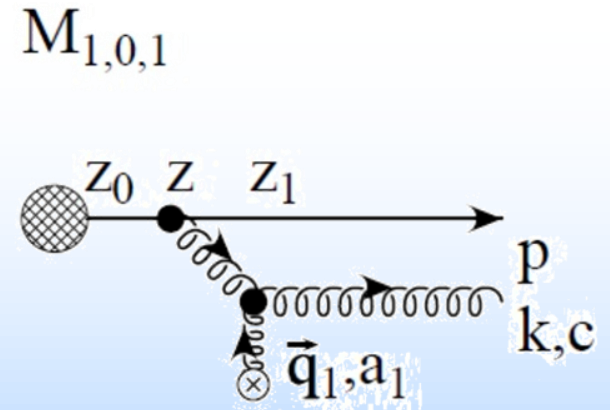
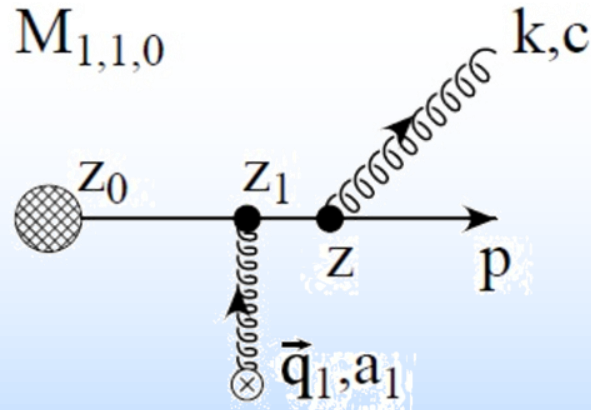
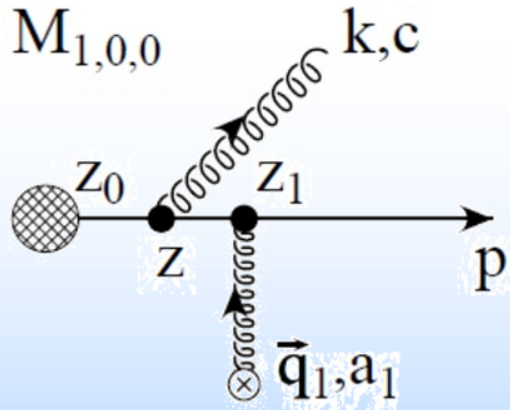
Compare to Data



Krouppa, Ryblewski, and Strickland, PRC92 (2015)



pQCD



Rummy Says We Have Some

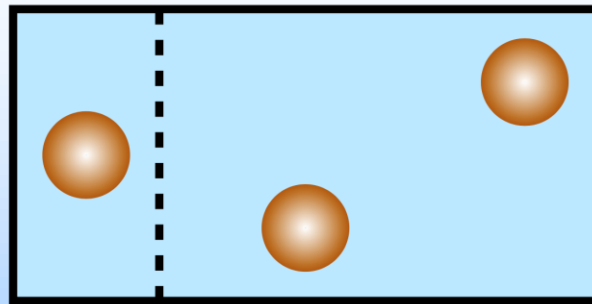
- **Known knowns:**
 - LO, all orders in opacity radiative energy loss off static scattering centers
 - Small system correction to LO and LO in opacity
 - ...
- **Known unknowns:**
 - Wide angle radiation
 - Multiple gluon emission
 - NLO, running coupling
 - Early time evolution
 - ...
- **Unknown unknowns: ???**



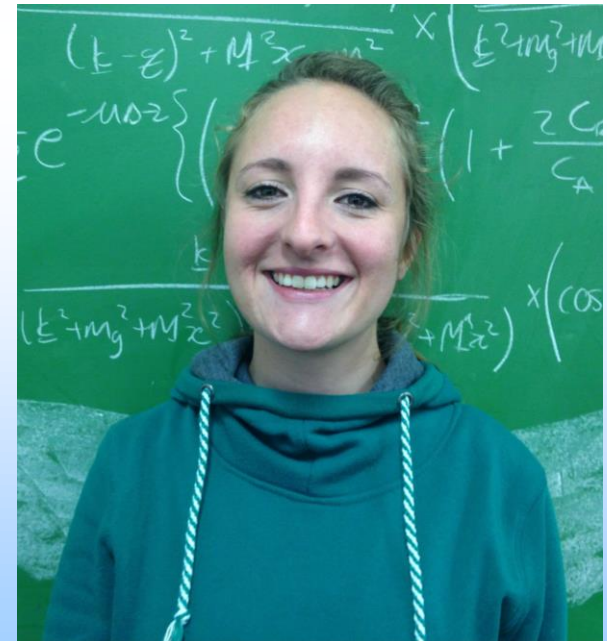
pQCD E-Loss in pA

- Take seriously potential E-loss in small systems
 - **ALL** current E-loss models *assume* large system size
- Wish to apply DGLV
 - DGLV assumes ordering of scales

$$\frac{1}{\mu_D} \ll \Delta z \sim \lambda_{mfp} \ll L$$



$$\frac{1}{\mu_D} \ll \lambda_{mfp}$$



Isobel Kolbé 34



Summed, Squared Result

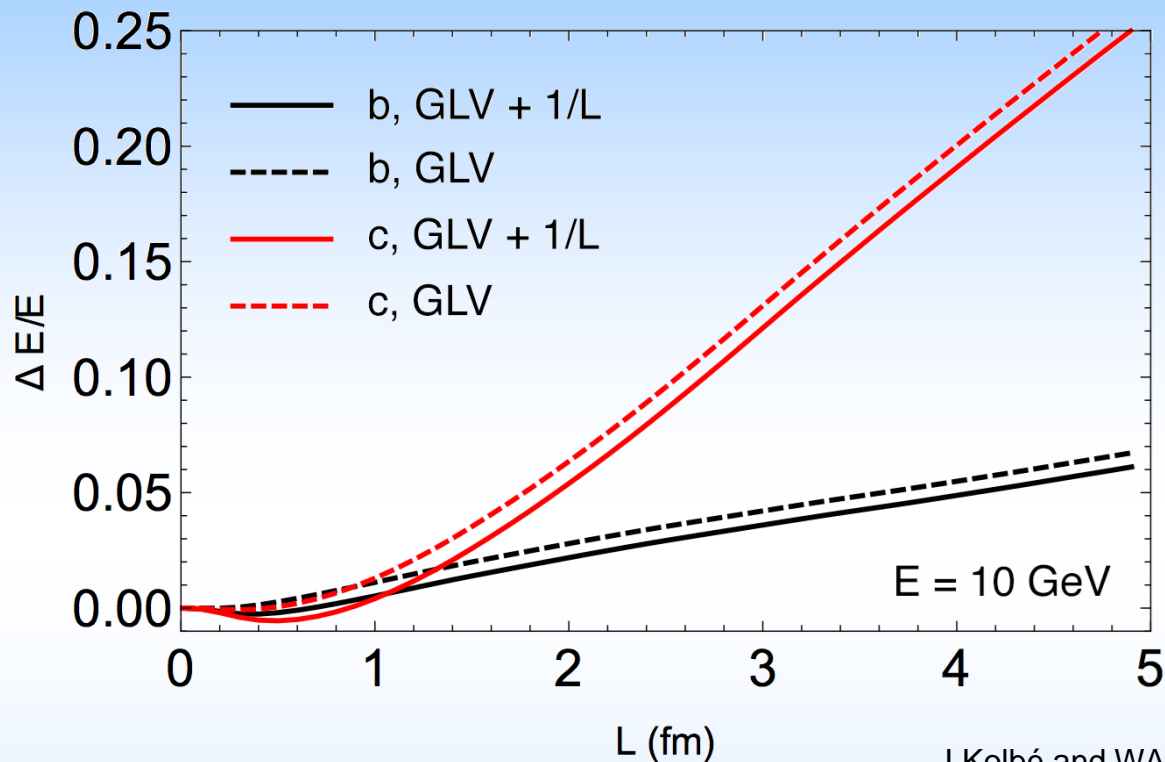
$$\begin{aligned}
 \Delta E_{ind}^{(1)} = & \frac{C_R \alpha_s L E}{\pi \lambda} \int \frac{d^2 \mathbf{q}_1}{\pi} \frac{\mu^2}{(\mu^2 + \mathbf{q}_1^2)^2} \frac{d^2 \mathbf{k}}{4\pi} \int d\Delta z \bar{\rho}(\Delta z) \times \\
 & \times \left[-2 \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times (1 - \cos\{(\omega_1 + \tilde{\omega}_m)\Delta z\}) \right. \\
 & \times \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} - \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \right) \\
 & + \frac{1}{2} e^{-\mu \Delta z} \left\{ \left(\frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \right)^2 \times \right. \\
 & \times \left(1 - \frac{2C_R}{C_A} \right) \left(1 - \cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\} \right) \\
 & + \frac{\mathbf{k}}{m_g^2 + \mathbf{k}^2 + x^2 M^2} \cdot \frac{(\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2 + M^2 x^2 + m_g^2} \times \\
 & \left. \left. \times \left(\cos\{(\omega_0 - \tilde{\omega}_m)\Delta z\} - \cos\{(\omega_0 - \omega_1)\Delta z\} \right) \right\} \right]
 \end{aligned}$$

DGLV

“Small L”
Modification



Numerics of the Correction

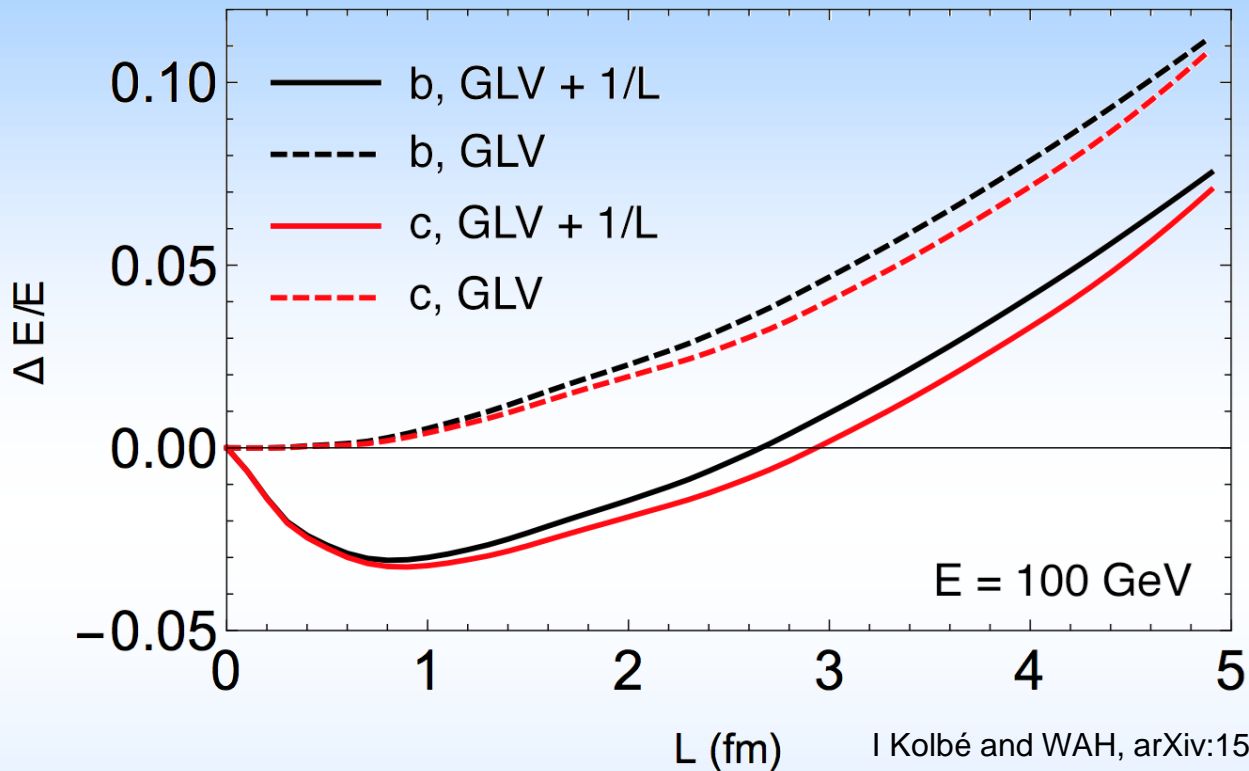


I Kolbé and WAH, arXiv:1509.06122

- Surprise 1: Correction leads to reduction in E-loss
 - LPM suppression of 0th order production radiation
- Surprise 2: Affects *all* pathlengths L
 - Due to integrating over all distances to scattering Δz in $[0, L]$



Correction at High- p_T

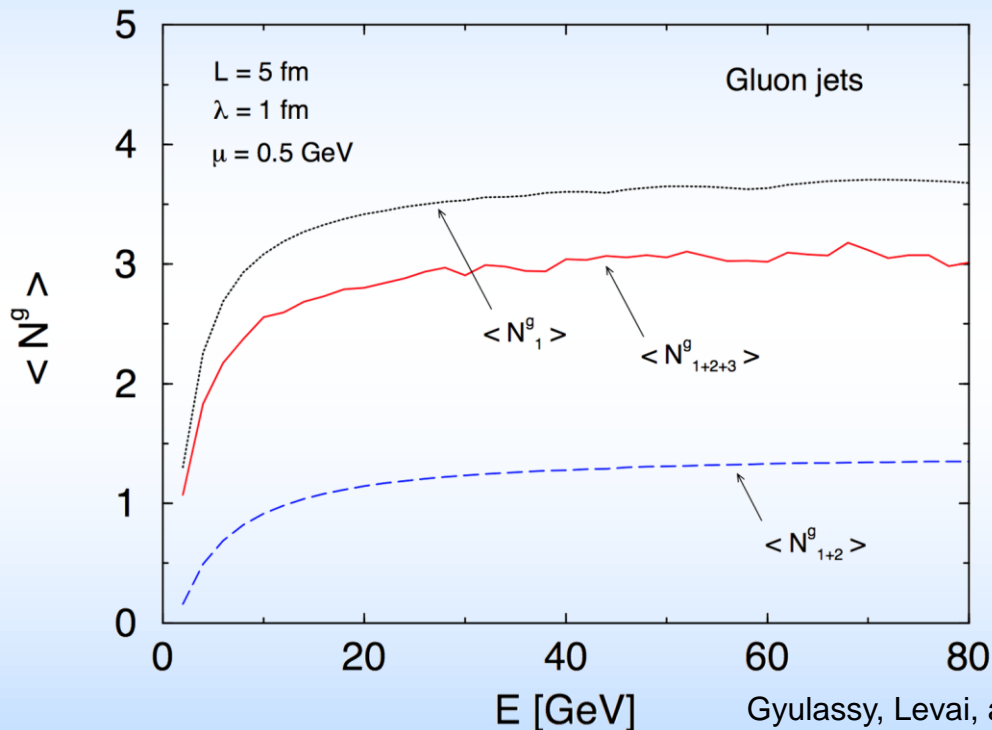


- Surprise 3: Correction grows with p_T
 - GLV $\sim L^2 \mu^2 \log E/\mu$
 - “1/L” $\sim -LE \log E/\mu$



Towards Quantitative pQCD E-Loss: Poisson Multigluon Emission?

- $GLV \Rightarrow N_g^{\text{emitted}} \sim 3$



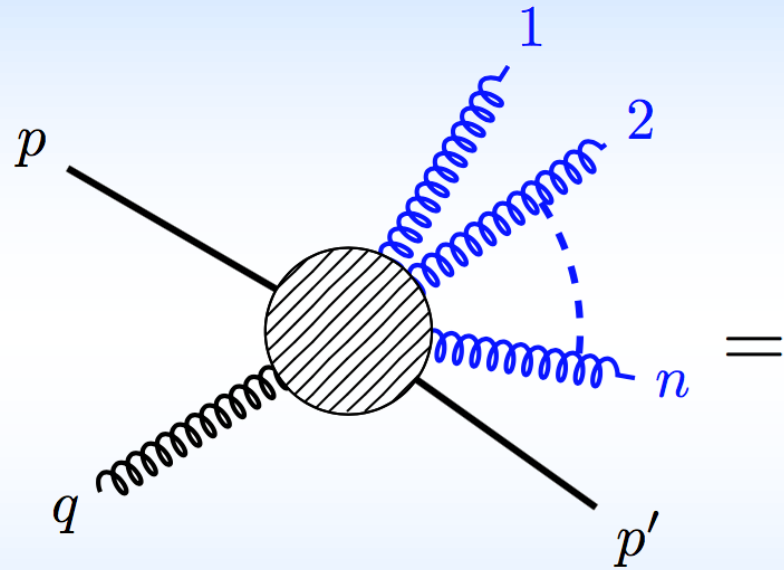
– Assume multi-g emission follows Poisson

- Reasonable approx based on QED



Multigluon Emission in QCD

- Use MHV Techniques
 - PhD project for Andri Rasoanaivo



$$g^{n-1} \frac{\langle pq \rangle^3 \langle qp' \rangle}{\langle pp' \rangle \langle p'q \rangle \langle q1 \rangle \langle 12 \rangle \cdots \langle np \rangle}$$

Deviations from Poisson

- 2 gluon emission

$$|J_2(k_1, k_2)|^2 = \frac{1}{4}[3 + F(k_1, k_2)] \times \prod_{i=1}^2 |J_1(k_i)|^2$$

- For gluons in the same plane

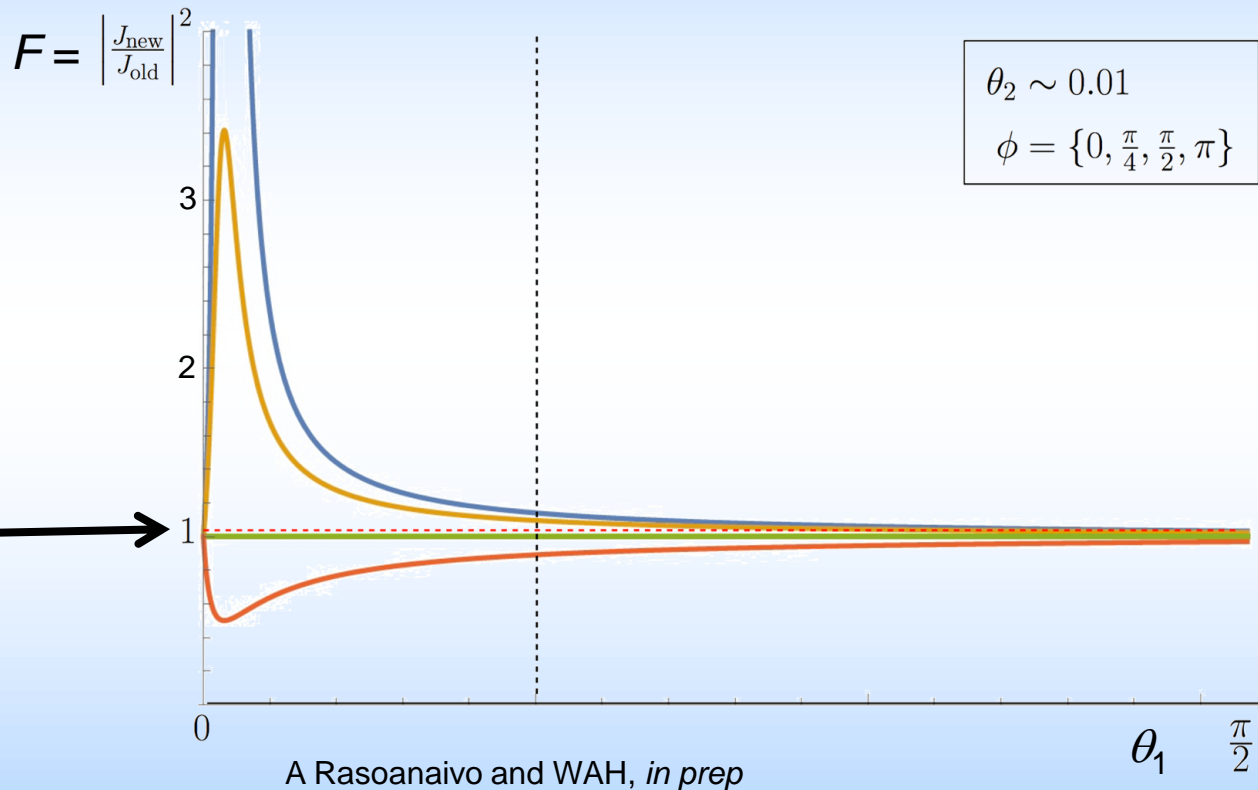
$$F(\theta_1, \theta_2) = \frac{1 - \cos \theta_1 \cos \theta_2}{1 - \cos(\theta_1 - \theta_2)}$$

- Poisson for strong angular ordering $\theta_2 \ll \theta_1$



Non-Poisson Corrections

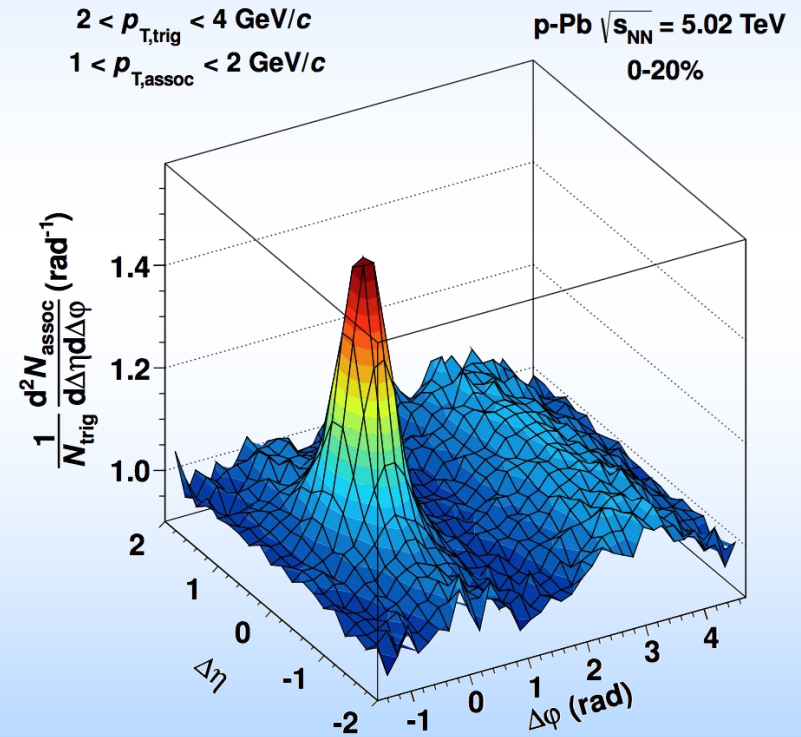
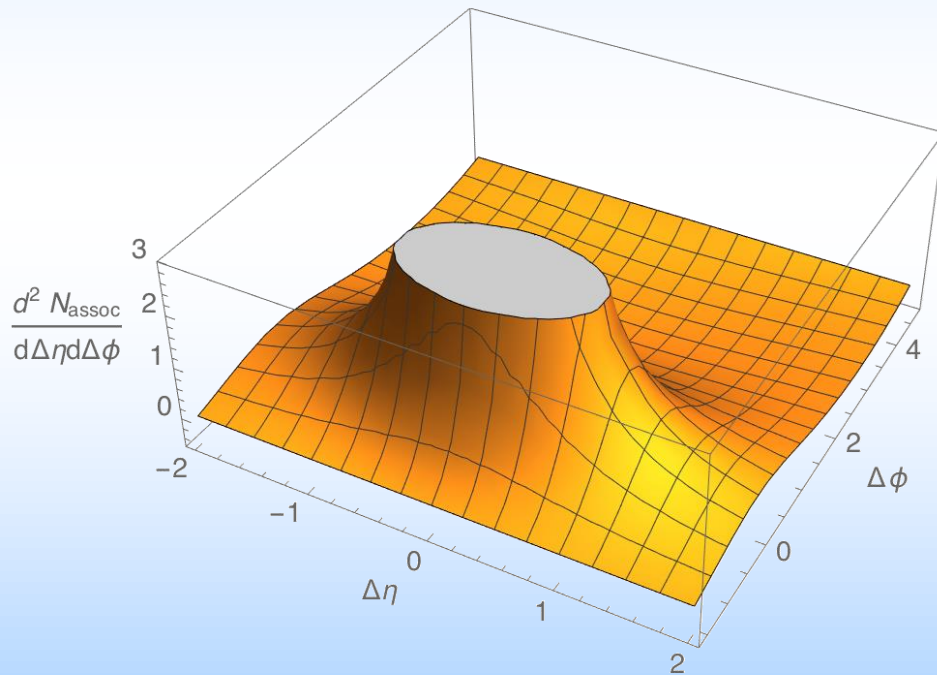
$$|J_2(k_1, k_2)|^2 = \frac{1}{4}[3 + F(k_1, k_2)] \times \prod_{i=1}^2 |J_1(k_i)|^2$$



- For general angle, potentially *large* corrections

Correlations

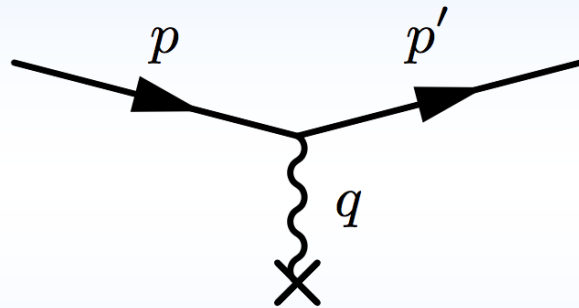
- Multiple gluon emission naturally yields suggestive correlations



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Towards NLO Energy Loss

- Start with simpler problem:
 - Rutherford scattering in QED

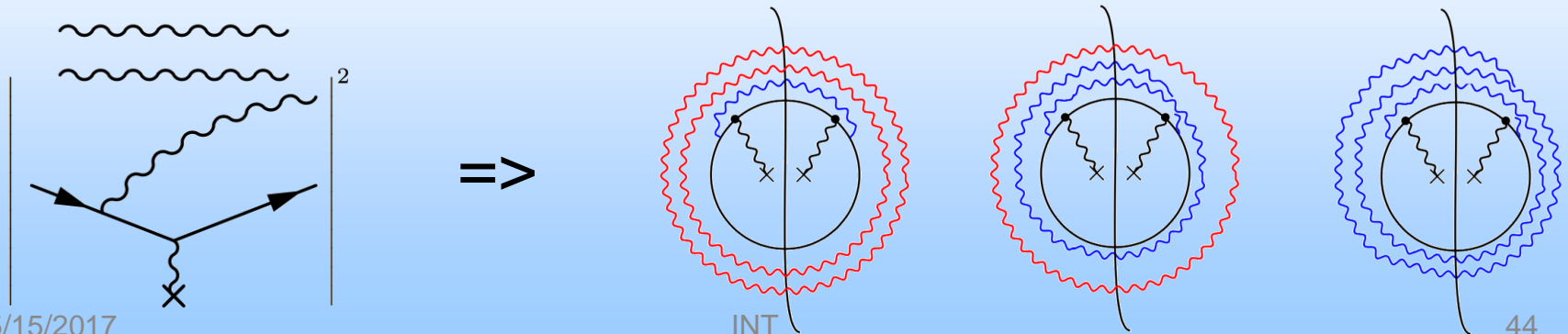
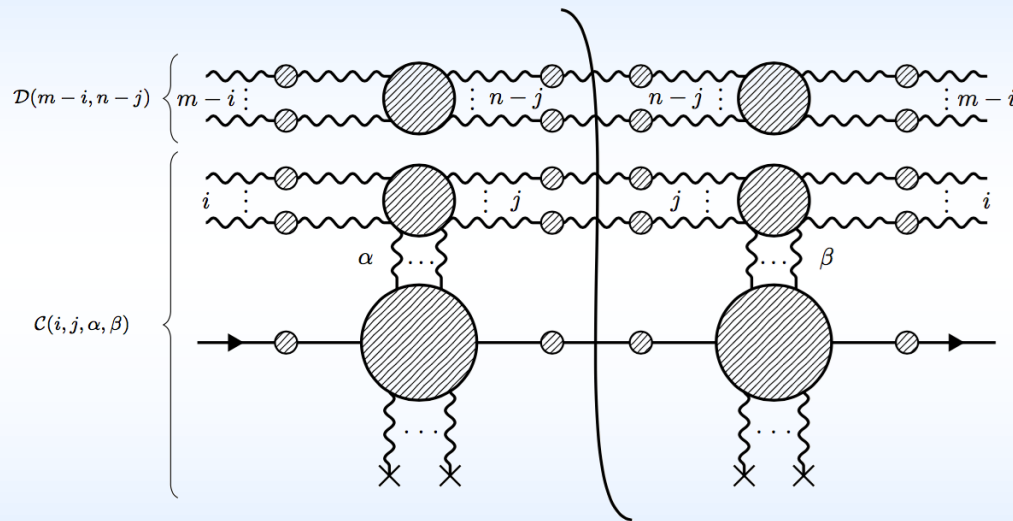


Abdullah Khalil

- Similar to GW interaction
- What is NLO QED Rutherford?
 - 50 year old, fundamental open problem
 - For solution details, see [arXiv:1701.00763](https://arxiv.org/abs/1701.00763)

Infinite Number of Soft Diagrams

- Must carefully rearrange formally divergent series to obtain finite, sensible result



Final Result

- First ever complete to $O(1)$ NLO Rutherford x-sc

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{NLO}} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left\{ 1 + \frac{\alpha}{\pi} \left[\log\left(\frac{Q^2}{\delta^2 E^2}\right) \left(\log\left(\frac{E^2}{\Delta^2}\right) + \frac{3}{2} \right) + \frac{2}{3} \log\left(\frac{Q^2}{\mu_{\text{MS}}^2}\right) - \pi^2 \left(\frac{1}{\left(\frac{2E}{Q} + 1\right)} + \frac{1}{3} \right) + \frac{5}{36} + \mathcal{O}(m^2, \delta^2) \right] \right\}$$

- δ : experimental angular resolution
- Δ : experimental energy resolution
- Non-trivial check: satisfies Callan-Symanzik



Conclusions

- Many areas of theoretical uncertainty, some addressed in this work
- AdS Heavy flavor:
 - Increased understanding of momentum fluctuations
 - Conjecture: HF diffusion coef. ind. of v
 - Smooth transition from heavy to light quarks in one picture
 - *Momentum correlations* as distinguishing observable
 - Need for more precise pp theory & exp for HF production
 - Brand new $Y R_{AA}(N_{part})$
- pQCD:
 - Small system E-loss
 - Corrections for 2 gluon emission
 - Potential ridge component
 - Partial results for n gluon emission
 - First ever full NLO Rutherford, correct implementation of KLN theorem
- Future Work:
 - Continue improving theoretical understanding, implement results into energy loss models, and compare with data

