Transport model for heavy quark / jet and its possible future development

INT Workshop, 05/02/2017

Shanshan Cao

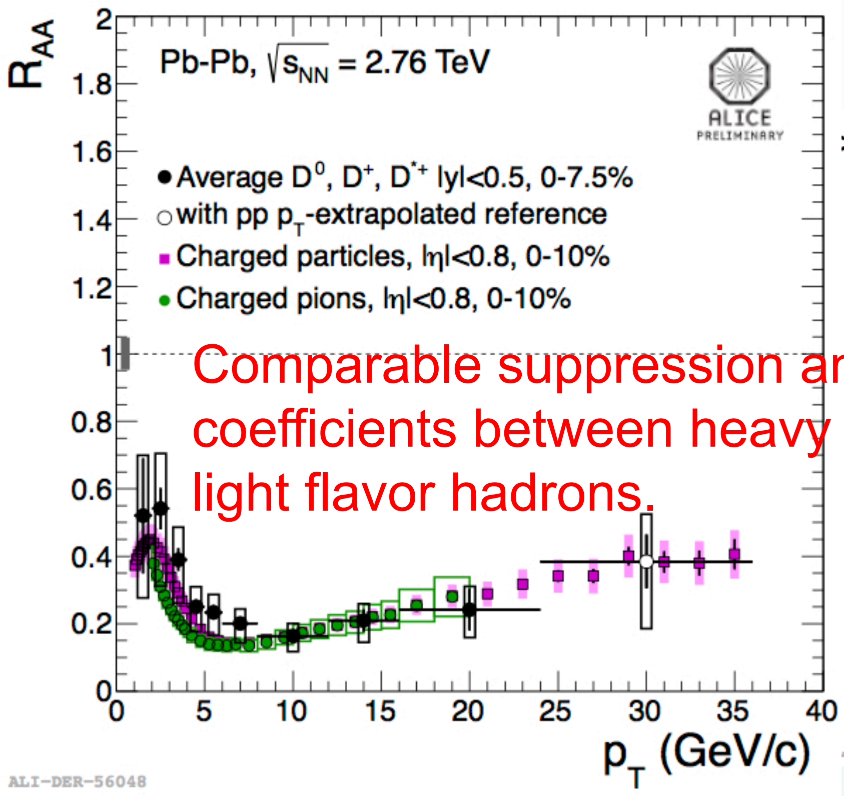
Wayne State University

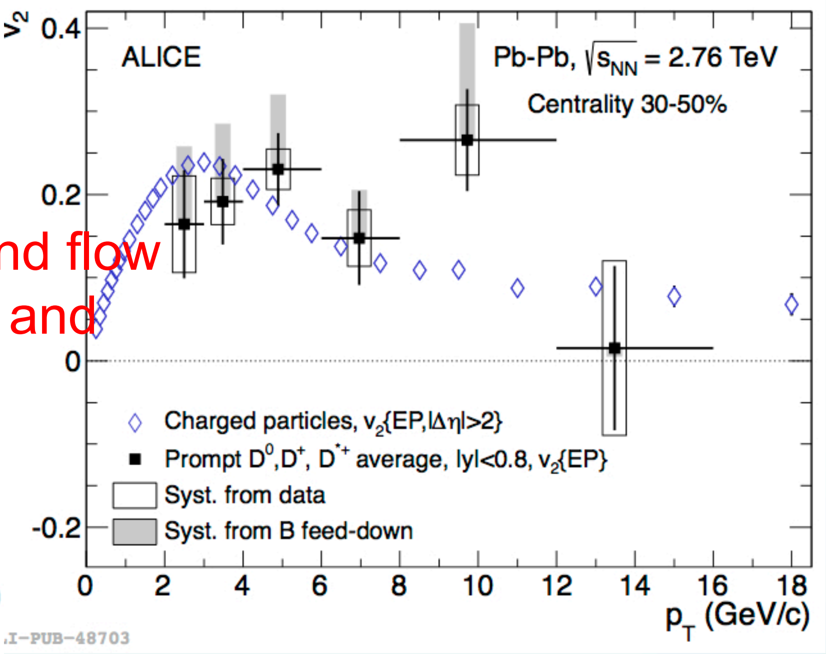
Outline

- Introduction
- A Linear Boltzmann Transport Model (LBT) for parton energy loss in QGP
- Heavy vs. light hadron suppression and anisotropic flow coefficients at RHIC and the LHC
- Medium modification of *D*-hadron correlation
- Future development: combining theories at high and low virtuality

Motivation

Hard partons: produced early and probe the full QGP history





Comparable suppression and flow coefficients between heavy and light flavor hadrons.

"Heavy vs. light flavor puzzle": is $\Delta E_g > \Delta E_q > \Delta E_c > \Delta E_b$ still right?

"R_{AA} vs. v_2 puzzle": can we describe R_{AA} and v_2 simultaneously?

Goal: fully understand heavy and light parton dynamics within a unified theoretical/numerical framework

A Linear Boltzmann Transport Model

Boltzmann equation for parton "1" distribution:

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1 C\left[f_1\right]$$

The collision term:

transition rate from p_1 to p_1-k

$$C\left[f_1\right] \equiv \int d^3k \left[w(\vec{p}_1 + \vec{k}, \vec{k}) f_1(\vec{p}_1 + \vec{k}) - w(\vec{p}_1, \vec{k}) f_1(\vec{p}_1) \right]$$

Elastic Scattering (2->2 process)

$$w(\vec{p}_1, \vec{k}) \equiv \sum_{2,3,4} w_{12\to34}(\vec{p}_1, \vec{k})$$

$$w_{12\to34}(\vec{p}_1, \vec{k}) = \gamma_2 \int \frac{d^3p_2}{(2\pi)^3} f_2(\vec{p}_2) \left[1 \pm f_3(\vec{p}_1 - \vec{k})\right] \left[1 \pm f_4(\vec{p}_2 + \vec{k})\right]$$
$$\times\, v_{\mathrm{rel}}\, d\sigma_{12\to34}(\vec{p}_1, \vec{p}_2 \to \vec{p}_1 - \vec{k}, \vec{p}_2 + \vec{k})$$

microscopic cross section of 12->34

A Linearized Boltzmann Transport Model

Scattering rate:

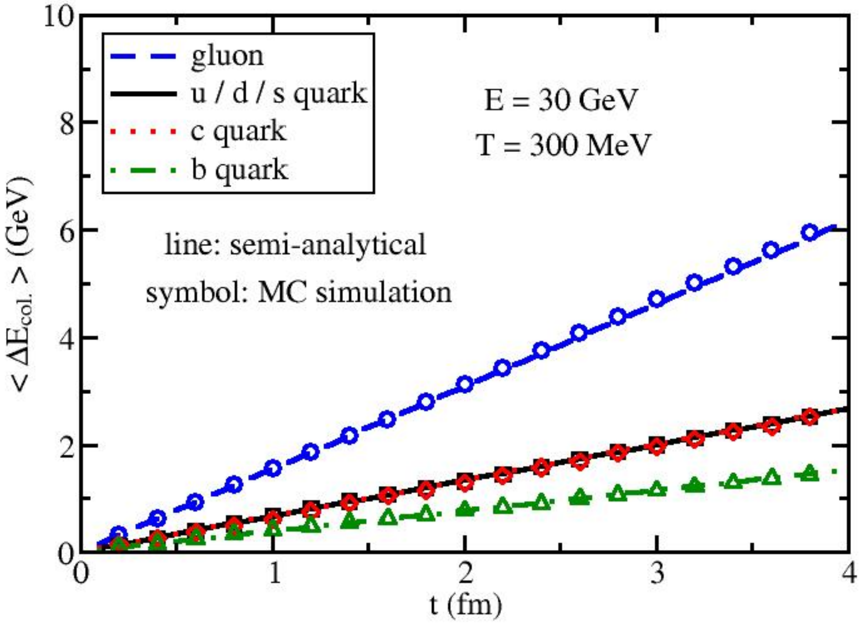
$$\Gamma_{12\to 34}(\vec{p}_1) = \int d^3k\, w_{12\to 34}(\vec{p}_1,\vec{k}) = \frac{\gamma_2}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\int \frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)\left[1\pm f_3(\vec{p}_1-\vec{k})\right]\left[1\pm f_4(\vec{p}_2+\vec{k})\right] S_2(s,t,u)$$
$$\times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12\to 34}|^2$$

In model calculation:

1. Use total rate $\Gamma = \sum_i \Gamma_i$ to determine the probability of elastic scattering $P_{\rm el} = \Gamma\Delta t$

2. Use branching ratios Γ_i/Γ to determine the scattering channel

3. Use the differential rate to sample the *p* space of the two outgoing partons



$\Delta E_{\rm col.}$ from our MC simulation agrees with the semi-analytical result.

A Linearized Boltzmann Transport Model

Inelastic Scattering (2->2+n process)

Average gluon number in *Δt*:

$$\langle N_g\rangle(E,T,t,\Delta t) = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2\, dt}$$

Spectrum of medium-induced gluon (higher-twist formalism):

$$\frac{dN_g}{dx dk_\perp^2\, dt} = \frac{2\alpha_s C_A P(x)}{\pi k_\perp^4} \hat{q} \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4 \sin^2\left(\frac{t-t_i}{2\tau_f}\right)$$

[Guo and Wang (2000), Majumder (2012); Zhang, Wang and Wang (2004)]

$\hat{q}$: $dp_\perp^2/dt$ of quark/gluon due to 2->2 scatterings

Splitting time of radiated gluon: $\tau_f = 2Ex(1-x)/(k_\perp^2 + x^2M^2)$

Splitting functions:

$$P_{q\to qg} = \frac{(1-x)(2-2x+x^2)}{x},$$

$$P_{g\to gg} = \frac{2(1-x+x^2)^3}{x(1-x)}.$$

g->q$\bar{q}$ not included – slight effect on single HM PRC 93 (2016), 024912

A Linearized Boltzmann Transport Model

Number *n* of radiated gluons during *Δt* – Poisson distribution:

$$P(n) = \frac{\langle N_g \rangle^n}{n!} e^{-\langle N_g \rangle}$$

Probability of inelastic scattering during *Δt*: $P_{\rm inel} = 1 - e^{-\langle N_g \rangle}$

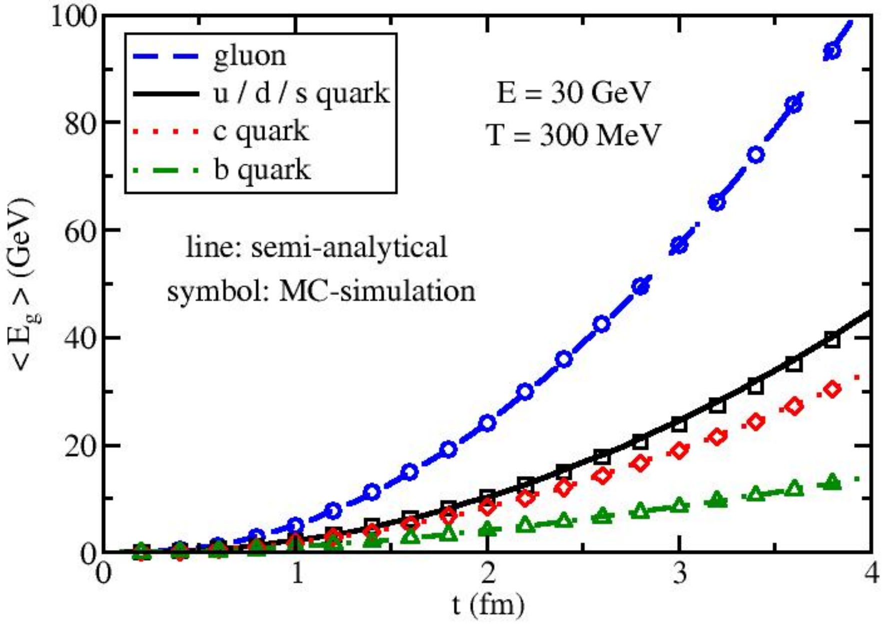
In model calculation:

1.Calculate $\langle N_g \rangle$ and thus $P_{\rm inel}$

2.If gluon radiation happens, sample *n* from $P(n)$

3.Sample *E* and *p* of gluons using the differential spectrum

4.Assume 2->2 first and adjust *E* and *p* of the 2+n final partons together to guarantee *E-p* conservation of 2->2+n process



$<E_g>$ from our MC simulation agrees with the semi-analytical result.

Elastic vs. Inelastic Energy Loss

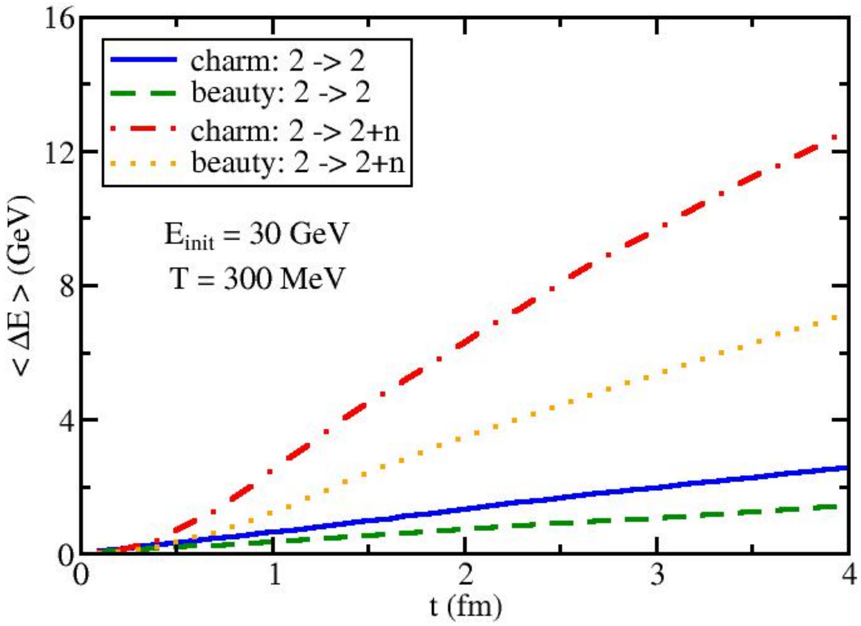
Divide scattering probability of jet parton into two regions:

1. Pure elastic scattering without radiated gluons: $P_{\mathrm{el}}(1-P_{\mathrm{inel}})$
2. Inelastic scattering: P_{inel}

Total probability: $P_{\mathrm{tot}} = P_{\mathrm{el}} + P_{\mathrm{inel}} - P_{\mathrm{el}}P_{\mathrm{inel}}$

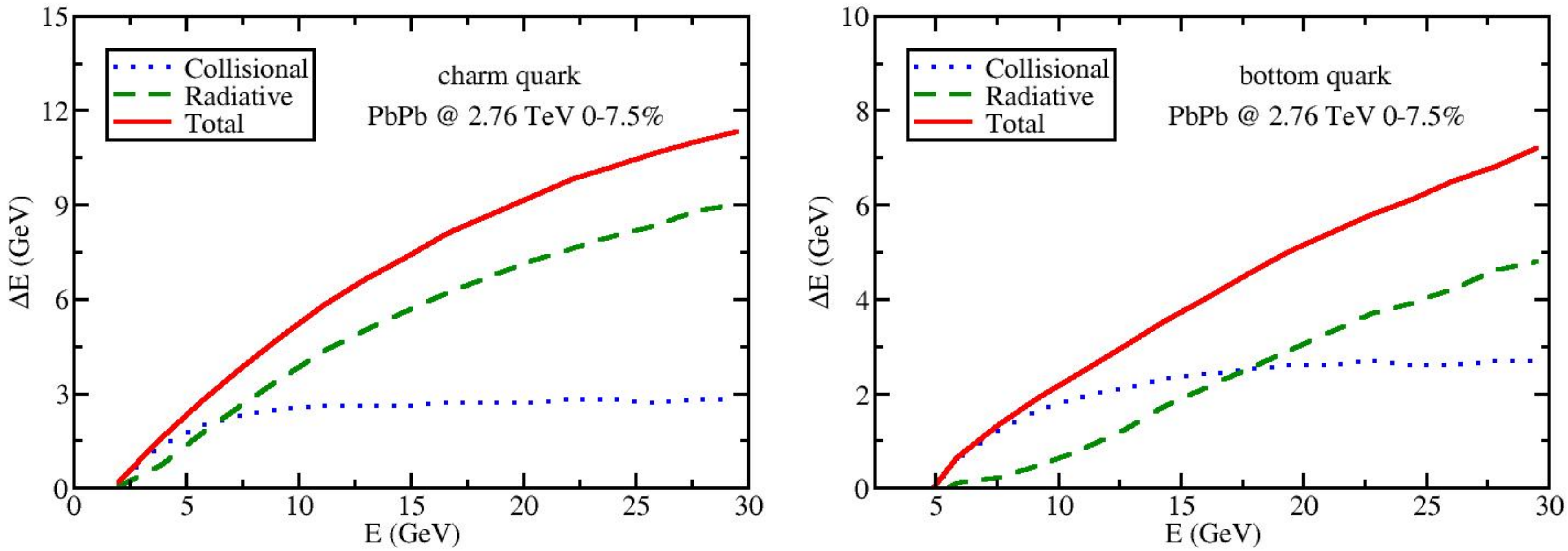
In model calculation:

1. Use P_{tot} to determine whether the jet parton scatter with the thermal medium
2. If so, we then determine whether this scattering is pure elastic or inelastic
3. Simulate the 2->2 or 2->2+n process



HQ energy loss due to elastic and inelastic processes are comparable at early time, but is dominated by the inelastic process at large *t*.

Heavy Quark Energy Loss



- Collisional energy loss dominates low energy region, while radiative dominates high energy region.
- Crossing point: 7 GeV for *c* and 18 GeV for *b* quark.
- ➔ Collisional energy loss alone may work well to describe previous RHIC data but is insufficient for LHC.

Hadronization

Heavy Flavor: fragmentation + HQ-thermal recombination

- Most high momentum heavy quarks fragment into heavy mesons: use PYTHIA 6.4
- Most low momentum heavy quarks hadronize to heavy mesons via recombination (coalescence) mechanism

[SC, Luo, Qin and Wang, Phys. Rev. C94 (2016) 014909]

Light flavor: jet fragmentation + jet-jet recombination

- Contribution from the bulk matter and jet-thermal recombination will be included in our future effort

[Han, Fries and Ko, Phys. Rev. C93 (2016) 045207]

Hadronization of Heavy Quarks

Two-particle recombination:

$$\frac{dN_M}{d^3p_M} = \int d^3p_1 d^3p_2 \frac{dN_1}{d^3p_1}\frac{dN_2}{d^3p_2} f_M^W(\vec{p}_1,\vec{p}_2)\delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

$\frac{dN_i}{d^3p_i}$ Distribution of the *i* th kind of particle

Light parton: thermal in the l.r.f of the hydro cell
Heavy quark: the distribution at T_c after LBT evolution

$f_M^W(\vec{p}_1,\vec{p}_2)$ Probability for two particles to combine

$$f_M^W(\vec{r},\vec{q}) \equiv g_M \int d^3r' e^{-i\vec{q}\cdot\vec{r}'} \phi_M(\vec{r}+\frac{\vec{r}'}{2})\phi_M^*(\vec{r}-\frac{\vec{r}'}{2})$$

$$\vec{r} = \vec{r}_1' - \vec{r}_2'$$

$$\vec{q} = \frac{1}{E_1' + E_2'}(E_2'\vec{p}_1' - E_1'\vec{p}_2')$$

→ Variables on the R.H.S. are defined in the c.m. frame of the two-particle system.

Hadronization of Heavy Quarks

Wigner function: $f_M^W(\vec{r},\vec{q}) \equiv g_M \int d^3r' e^{-i\vec{q}\cdot\vec{r}'} \phi_M(\vec{r}+\frac{\vec{r}'}{2})\phi_M^*(\vec{r}-\frac{\vec{r}'}{2})$

$$\vec{r} = \vec{r}'_1 - \vec{r}'_2 \qquad \vec{q} = \frac{1}{E'_1+E'_2}(E'_2\vec{p}'_1 - E'_1\vec{p}'_2)$$

defined in the rest frame
of the produced meson

g_M: color-spin degeneracy of the produced meson

Φ_M: meson wave function – approximated by S.H.O.

Averaging over the position space leads to

$$f_M^W(q^2) = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-q^2\sigma^2} \qquad \sigma = 1/\sqrt{\mu\omega}$$

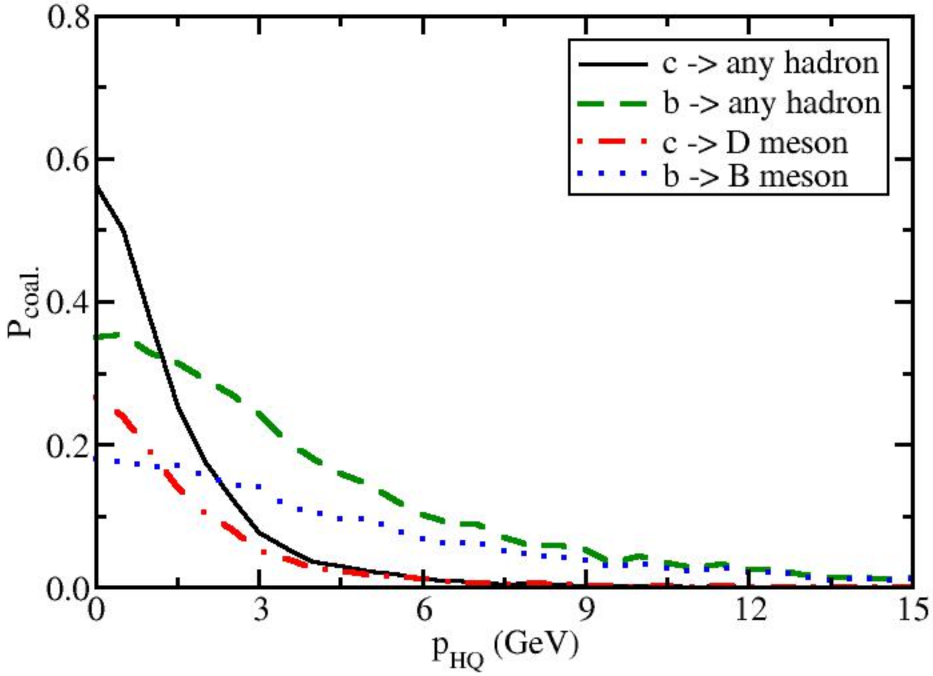
μ: reduced mass of the 2-particle system

ω: S.H.O frequency – related meson charge radius (parameter free)

$$\langle r_M^2 \rangle_{\rm ch} = \frac{3}{2\omega}\frac{1}{(m_1+m_2)(Q_1+Q_2)}$$

Can be generalized to 3-particle recombination (baryon)

Hadronization of Heavy Quarks

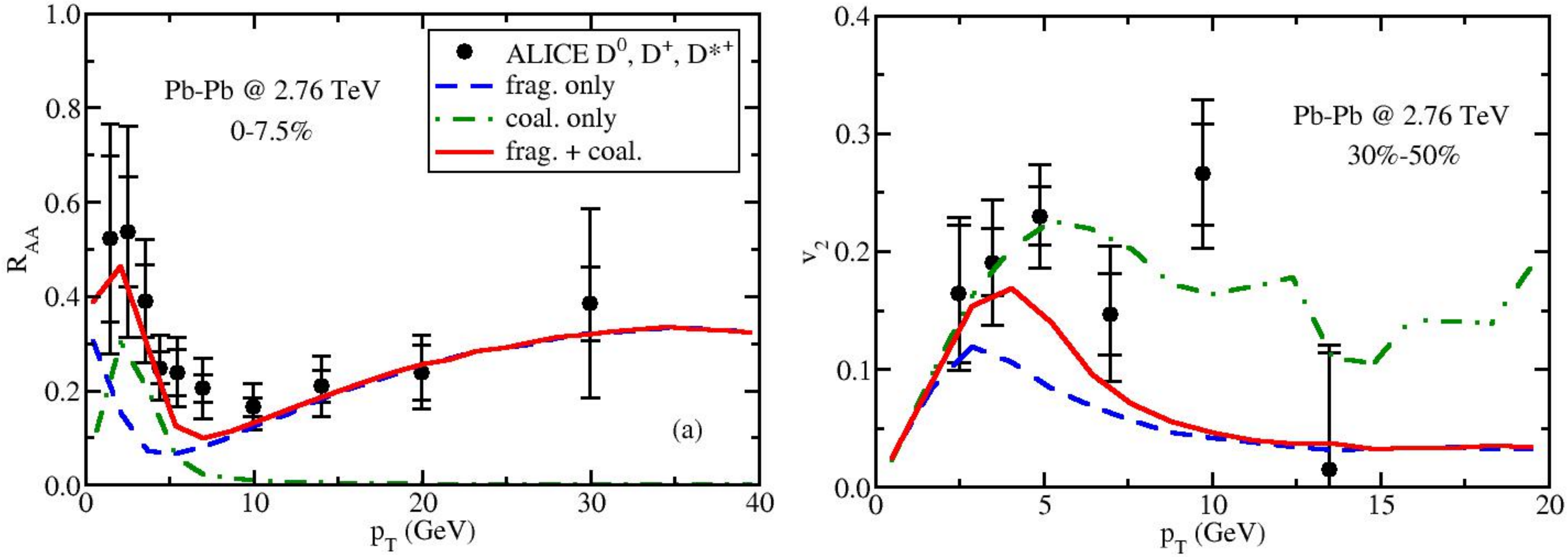


Use f^{W} to calculate $P_{coal.}(p_{HQ})$ for all channels (D/B Λ Σ Ξ Ω) at T_c

Three regions: recombination to *D*/*B* mesons, recombination to other hadrons, and fragmentation

In model calculation: in the l.r.f of the freeze-out hypersurface, determine which region each HQ belongs to, and then use either recombination model or Pythia simulation to obtain *D*/*B* mesons

Effect of Hadronization Process

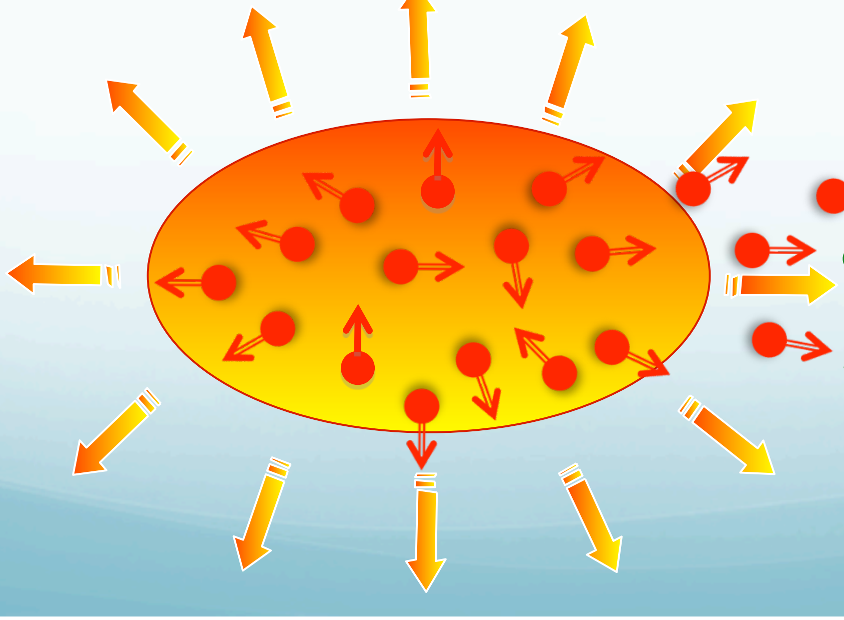


- Fragmentation dominates *D* meson production at high p_T; Coalescence enhances heavy meson production at medium p_T and forms a bump structure in R_{AA}
- Coalescence enhances heavy meson v_2 since it adds the *p*-space anisotropy of light partons from the bulk matter to heavy quarks

Framework Overview

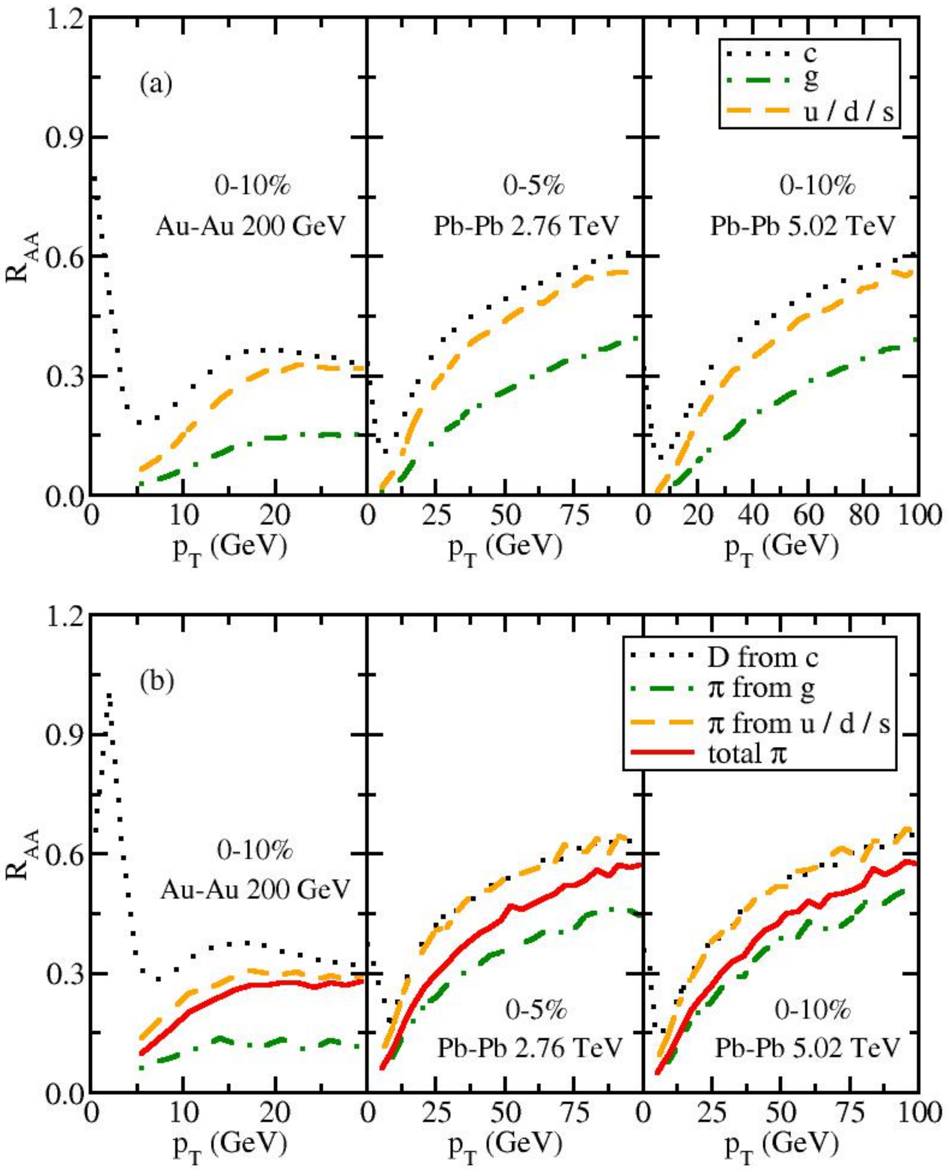
(Parton Evolution inside the QGP)

- Generation of QGP medium: viscous hydro from OSU (2+1 D) or LBL-CCNU (3+1 D) group
- Initialization of hard partons: MC-Glauber for position space and pQCD calculation for momentum space (PDF: CTEQ5+EPS09)
- Simulation of parton evolution: the Boltzmann transport model in the local rest frame of the medium
- Hadronization: fragmentation + recombination model
- Hadronic rescattering: not included [ref: Phys. Rev. C92 (2015)]



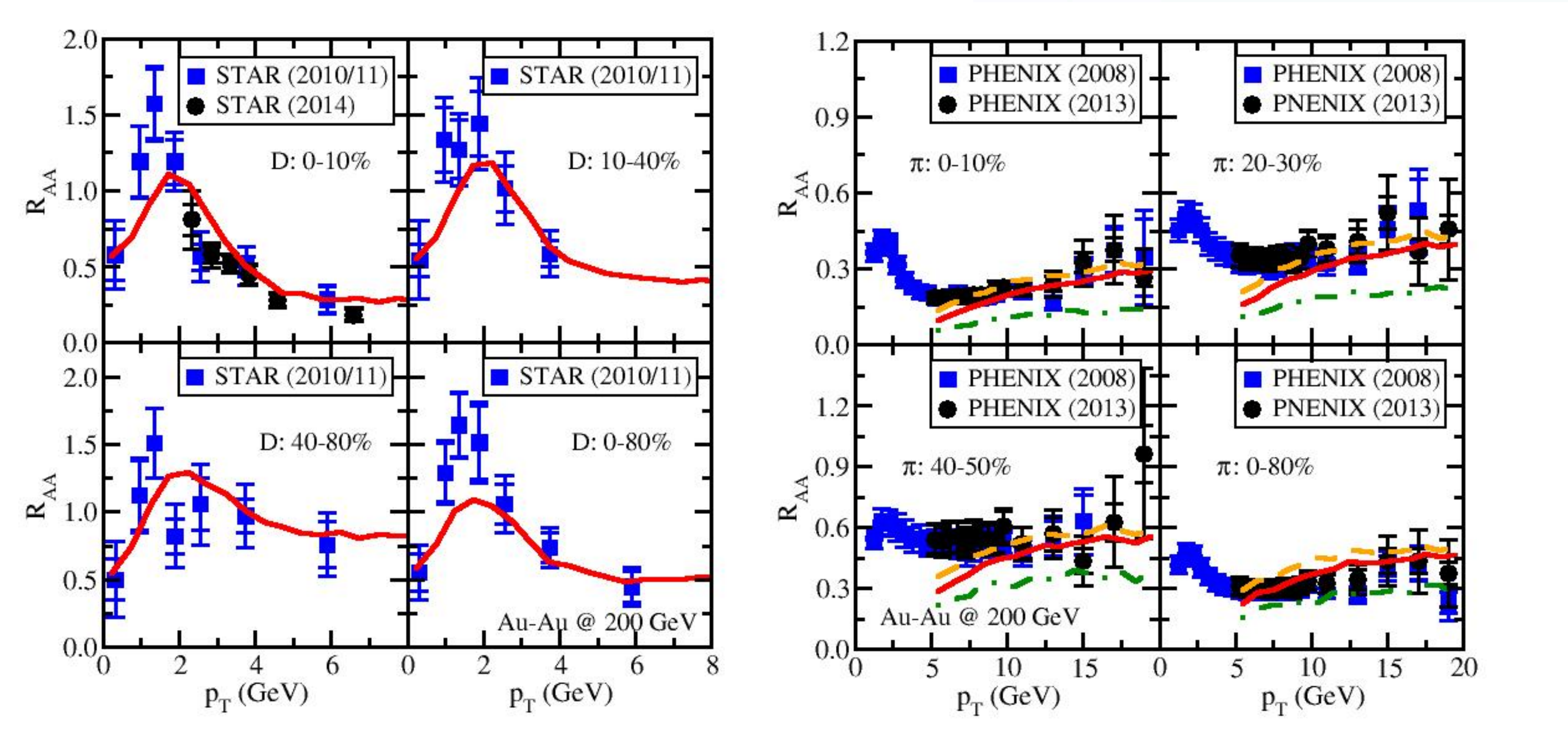
outside the medium (below T_c), converted into hadrons

Heavy vs. Light Hadron Suppression

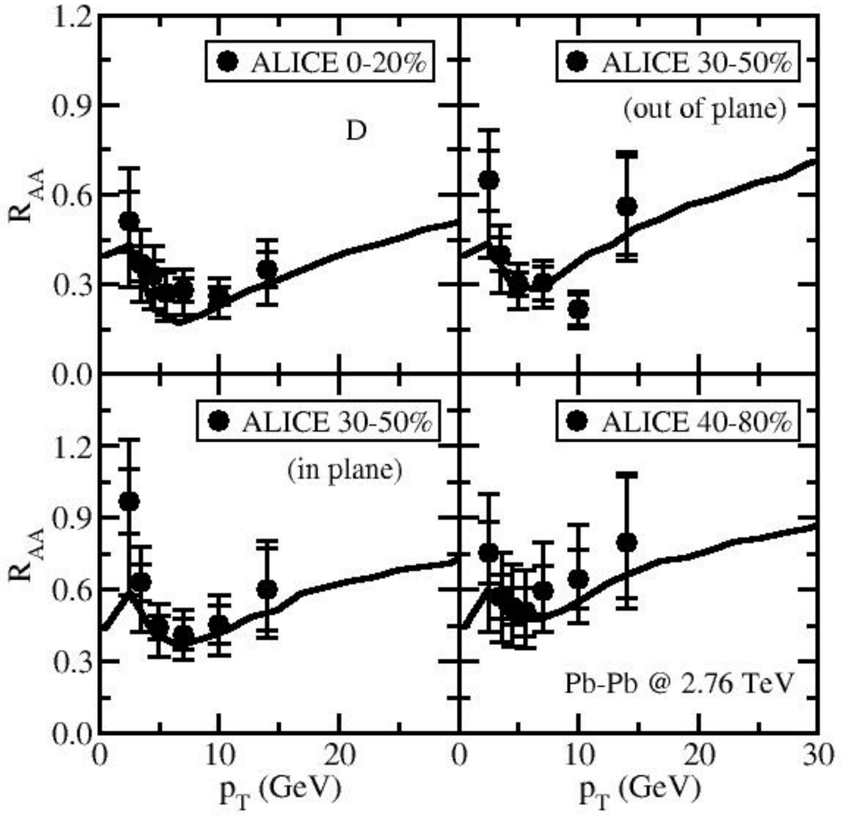


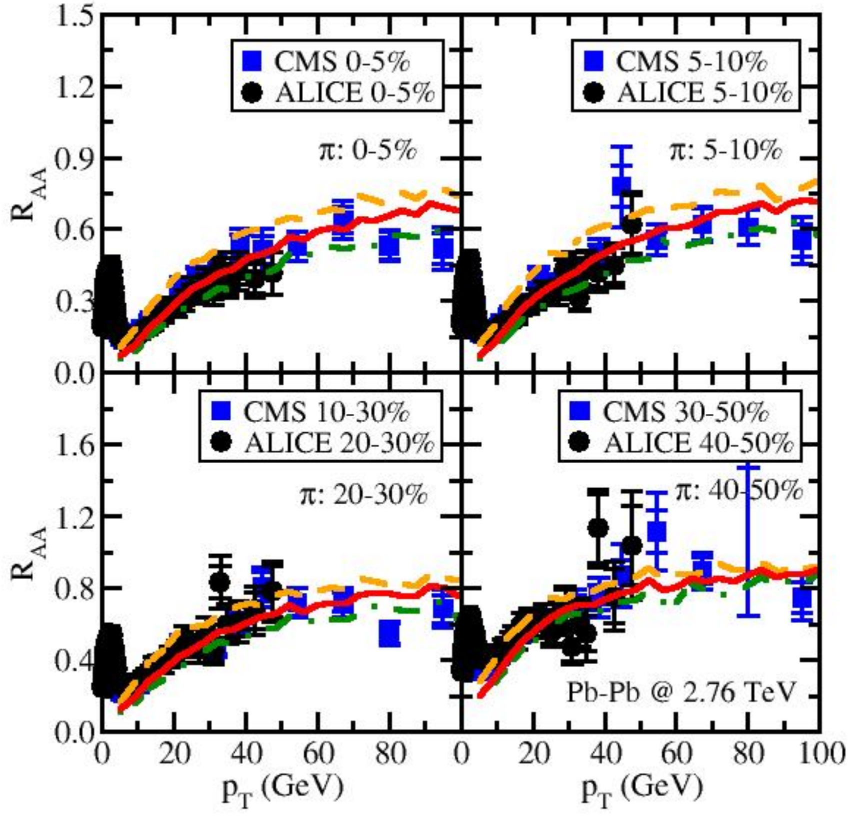
- *u/d/s* are slightly more suppressed than *c* quark, *g* is significantly more suppressed
- Due to different fragmentation function (harder for *c* than for *u/d/s*), *π* from light quark has similar R_{AA} *to D*, *π* from gluon is still more suppressed
- Final *π* is dominated by contribution from quark jet at small $\sqrt{s_{NN}}$, but is dominated by gluon jet at large $\sqrt{s_{NN}}$

Simultaneous Description of *D* and *π* R_{AA} in 200 GeV Au-Au Collisions

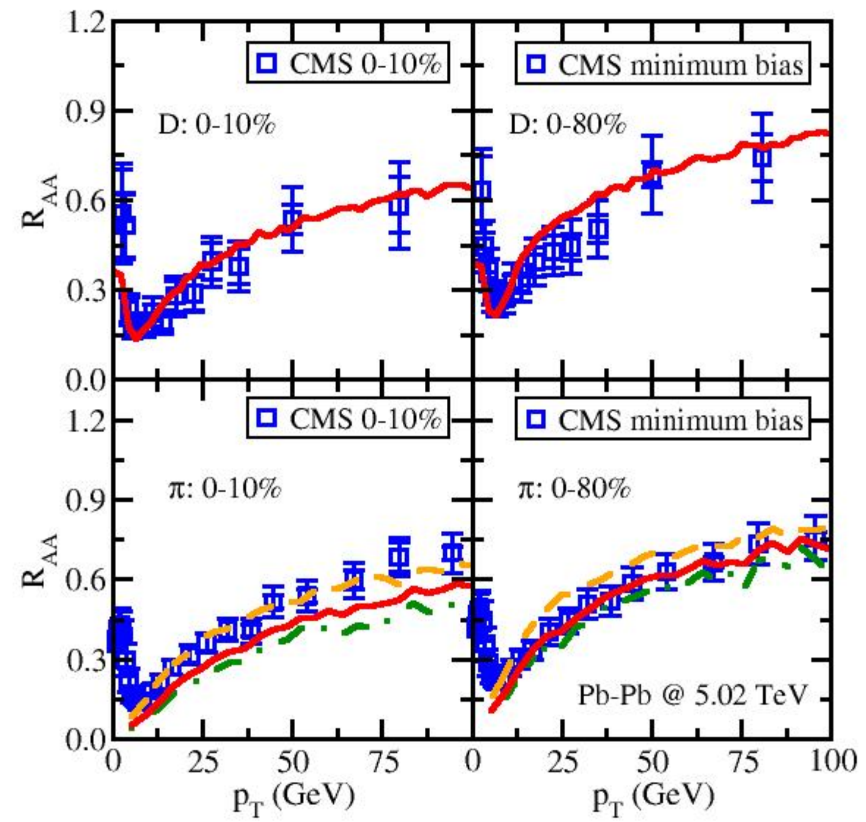


Simultaneous Description of *D* and *π* R_{AA} in 2.76 TeV Pb-Pb Collisions



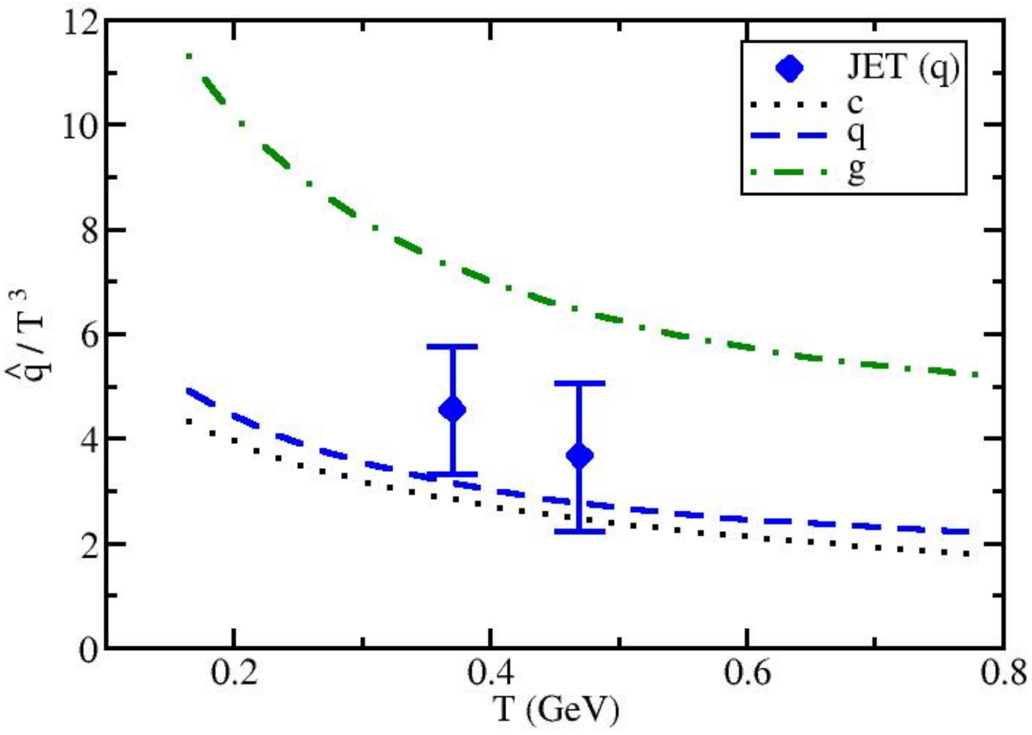


Simultaneous Description of *D* and *π* R_{AA} in 5.02 TeV Pb-Pb Collisions



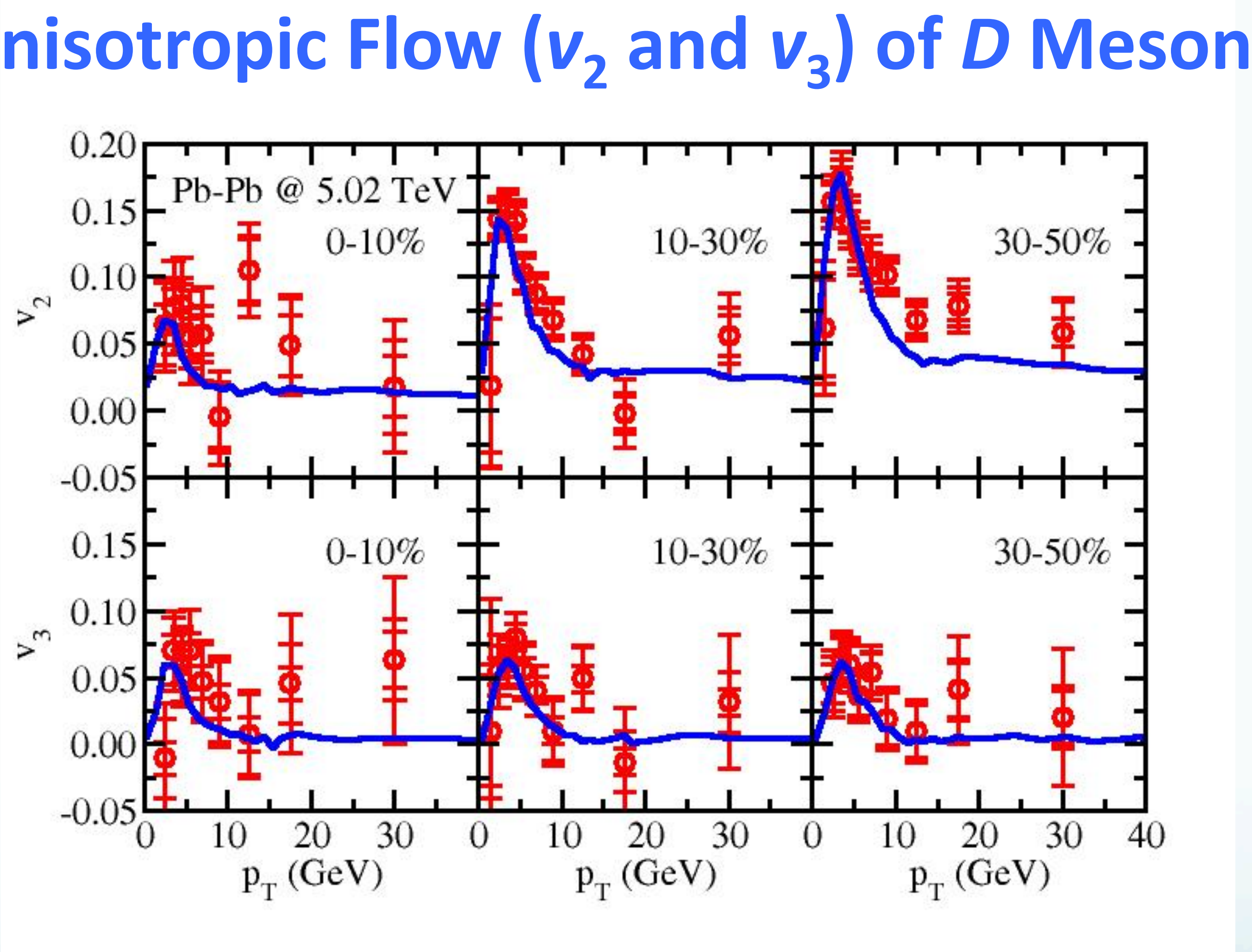
With a delicate treatment of heavy and light parton in-medium evolution and their hadronization, one may provide reasonable description of heavy and light hadron suppression simultaneously.

Quark and Gluon Transport Coefficient: $\hat{q}$



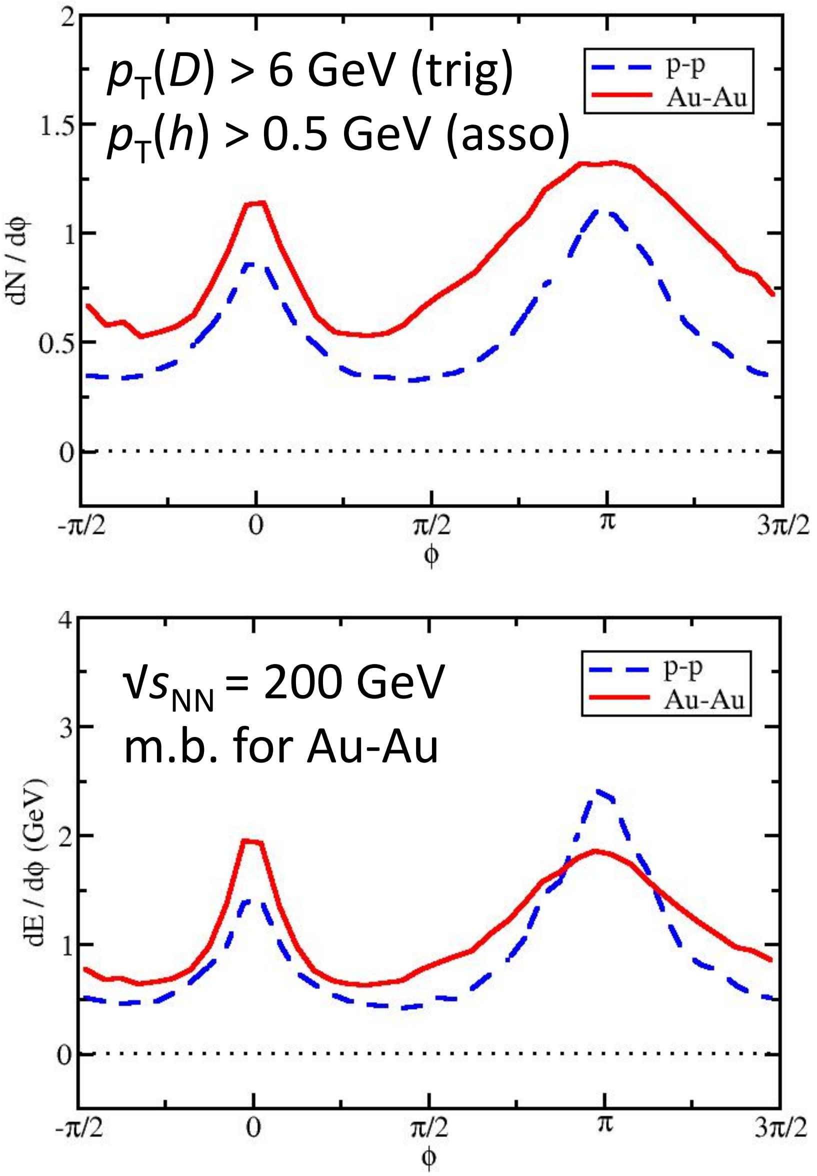
The extracted $\hat{q}$ from model to data comparison within our LBT framework is consistent with the value constrained by the earlier work by the JET Collaboration [Phys. Rev. C90, 014909 (2014)].

Anisotropic Flow (v_2 and v_3) of *D* Mesons



- Predictions of v_2 and v_3 are consistent with CMS data at 5.02 TeV.
- Strong v_2 is observed for the full p_T range.
- Strong v_3 is observed at low p_T, but it is consistent with 0 at high p_T.

D-hadron Correlation Functions



- Single hadron observables quantify the amount of parton energy loss; *D*-hadron correlation reveals how the lost energy is re-distributed.
- p-p baseline: Pythia
- Au-Au: all charged hadrons from heavy and light parton shower, recoiled parton from and back reaction to the medium (thermal hadrons emitted by QGP are not included)
- dN/dΦ is increased at all Φ due to parton shower in Au-Au
- dE/dΦ is enhanced at 0 due to *c* energy loss in Au-Au; and broadened at π due to parton shower and scattering in QGP
- Will quantify energy loss and jet broadening in upcoming work

Summary of the transport model

- Established a Linear Boltzmann Transport (LBT) Model that treats heavy and light parton evolution on the same footing and simultaneously incorporates their elastic and inelastic scattering inside QGP
- Provided reasonable descriptions of both heavy and light hadron suppression and flow at RHIC and the LHC
- Discussed *D*-hadron correlation functions for the first time: not only quantify the amount of energy loss of heavy quarks, but also reveal how the lost energy is re-distributed inside the parton shower

Future development

No virtuality ($Q^2 = p^2 - m^2$) in the current transport model.

Full evolution of partons:

high *Q* and high *E* (**DGLAP**)

-> low *Q* and high *E* -> low *Q* and low *E* (**Transport**)

Combine different theories at different scales into a unified approach. Details can be found in:

- Talk on next Thursday (05/11) 4:30 pm
- First JESTCAPE paper arXiv:1705.00050

Combining theories at high *Q* and low *Q*

DGLAP evolution for parton fragmentation function at high *Q*:

$$\frac{\partial}{\partial Q^2} D(z,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}\int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}, Q^2\right)$$

Sudakov form factor (probability of NO splitting between *Q* and $Q_{\max}$):

$$\Delta(Q_{\max}, Q) = \exp\left[-\frac{\alpha_s}{2\pi}\int_{Q^2}^{Q^2_{\max}} \frac{dQ^2}{Q^2}\int_{z_c}^{1-z_c}\frac{dy}{y}P(y)\right]$$

Splitting at (or below) scale *Q* happens if:

$$\Delta(Q_{\max}, Q_0) \le r \le \Delta(Q_{\max}, Q) = \frac{\Delta(Q_{\max}, Q_0)}{\Delta(Q, Q_0)}$$

MATTER [Phys. Rev. C88, 014909, arXiv:1702.05862)] simulation of this *virtuality-ordered* parton showers from $Q_{\max}$ down to Q_0 [$P(y)$: vacuum splitting + medium-modified (higher-twist) splitting function]

LBT simulation of *time-ordered* parton showers at (or below) Q_0 (with on-shell approximation)

Separation scale Q_0 between MATTER and LBT

Fixed Q_0 (both in vacuum and medium): 1, 2 or 3 GeV will be used and compared.

Dynamical Q_0 (virtuality gain from scattering with the medium):

$$Q_0^2 = \hat{q}\tau_f \qquad \tau_f = 2E/Q_0^2$$

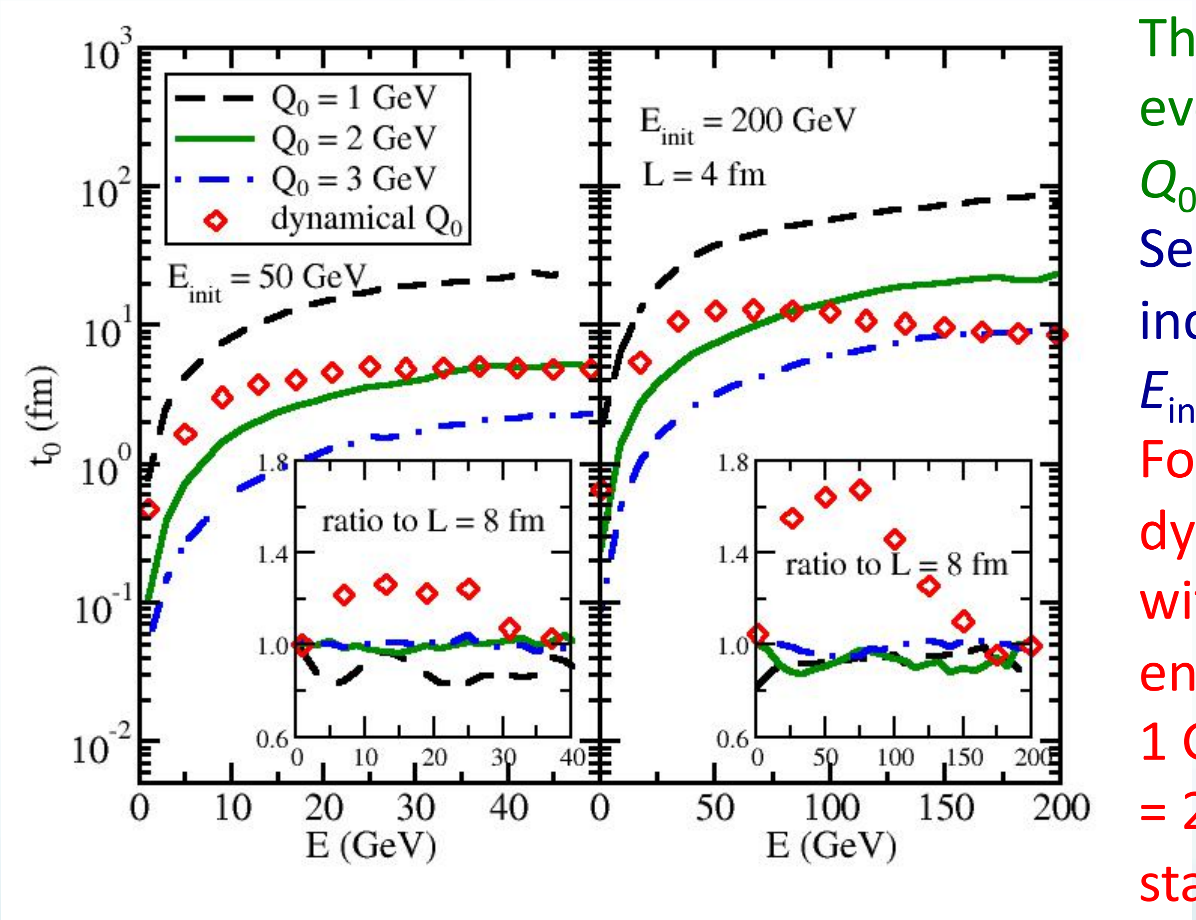
$$\Longrightarrow \qquad Q_0^2 = \sqrt{2E\hat{q}}$$

$$\hat{q} = C_R \alpha_s \mu_D^2 T \log\left(\frac{6ET}{\mu_D^2}\right) \qquad \mu_D^2 = 6\pi\alpha_s T^2$$

* Dynamical Q_0 is only meaningful in a thermal medium, in vacuum, Q_0 = 1GeV vacuum

In this work, static medium with T = 250 MeV is used. Effects of medium length L and initial parton (quark) energy E will be investigated.

Switching t_0 between MATTER and LBT

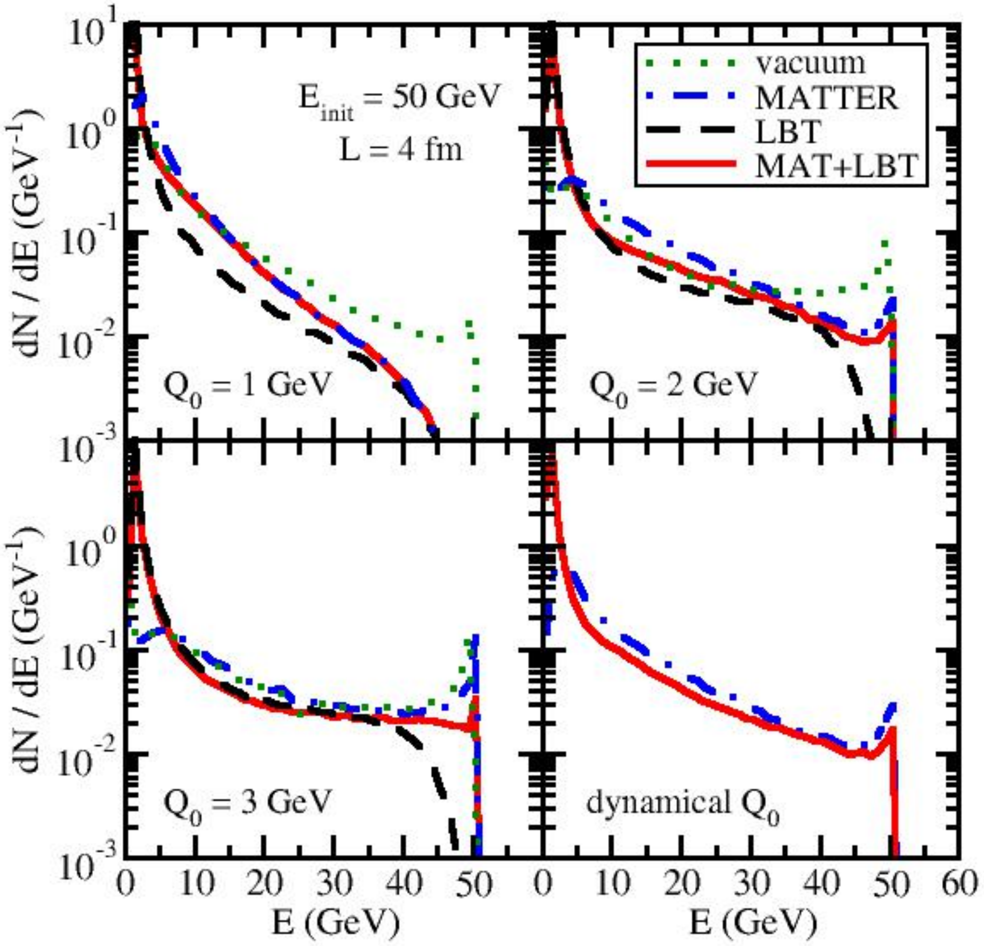


The time MATTER takes to evolve initial parton down to Q_0 is NOT small (vs. τ_0). Separation time (t_0) increases if Q_0 decreases or E_{init} increases.

For E_{init} = 50 GeV, L = 4 fm, dynamical Q_0 is consistent with 2 GeV at the high energy end, but approaches 1 GeV at low energy. For E_{init} = 200 GeV, dynamical Q_0 starts at 3 GeV in high E.

For fixed Q_0, changing from L = 4 to 8 fm increases scattering process (virtuality gain) and thus may delay t_0; for dynamical Q_0, extending L increases the range where larger Q_0 is applied and shortens t_0.

dN/dE for E_i = 50 GeV and L = 4 fm



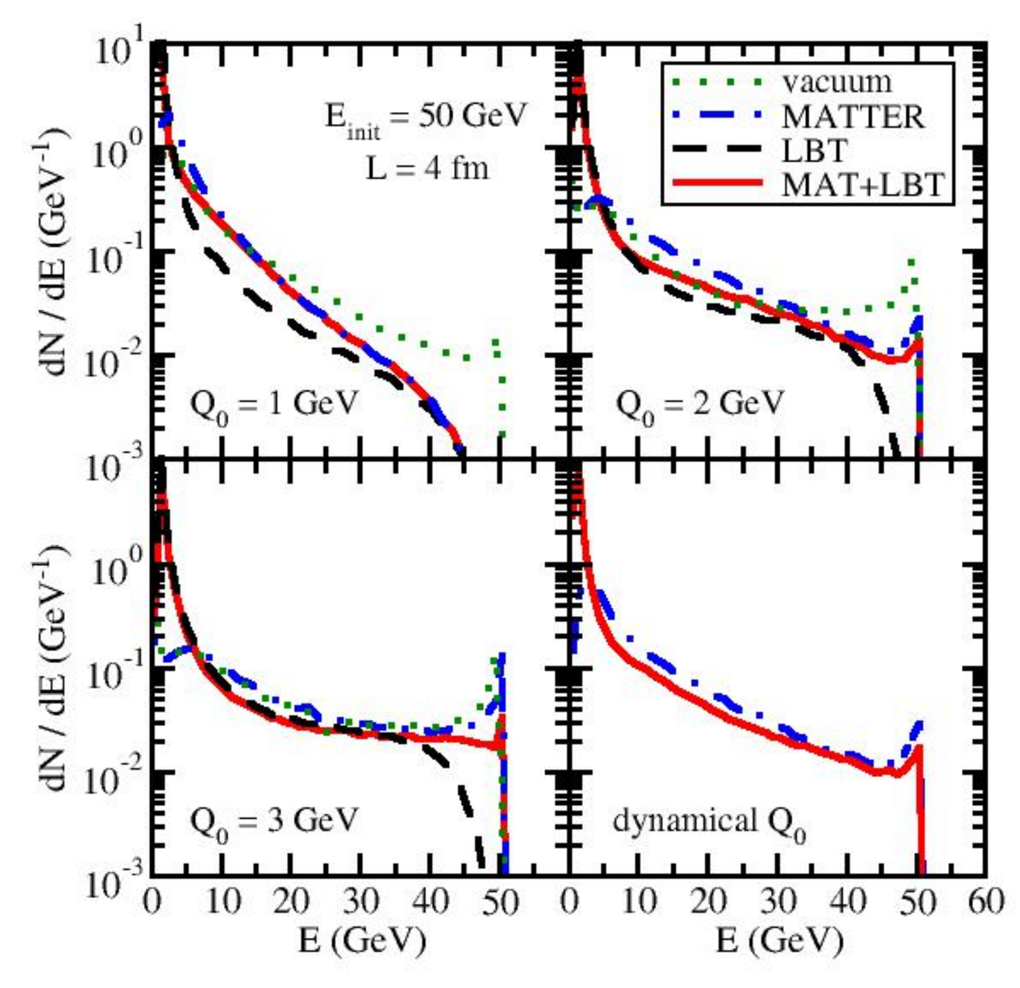
Vacuum: Sudakov type of shower with vacuum splitting function

MATTER: Sudakov type of shower with vacuum + medium modified splitting function

LBT: Partons provided vacuum shower evolve the entire 4 fm in LBT

MATTER+LBT: Combined scheme – partons evolve in MATTER up to t_0 and then in LBT up to 4 fm

dN/*dE* for E_i = 50 GeV and *L* = 4 fm



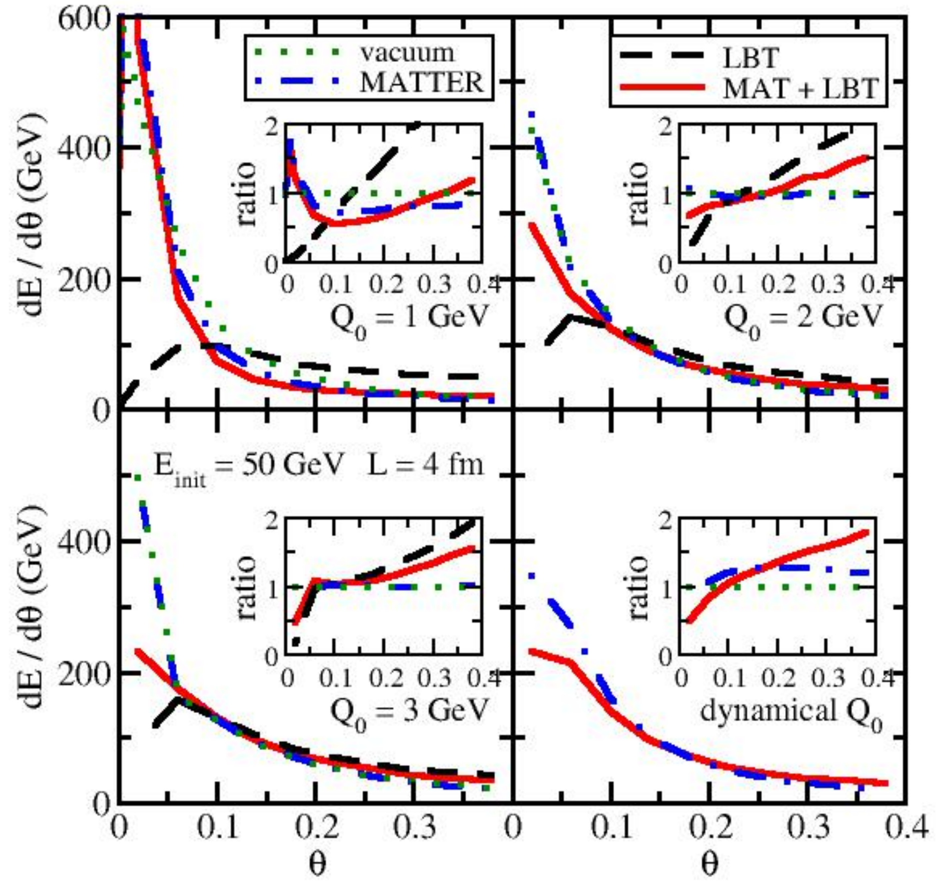
For E_{init} = 50 GeV, MATTER evolution (w.r.t vacuum shower) is weak if Q_0 > 2 GeV (scale of the medium $\hat{q}\tau_f$)

Pure LBT evolution is stronger than pure MATTER.

Effect of LBT in MATTER+LBT is strong for low energy partons; but is weaker for high energy ones since it took longer time for them to hit Q_0 in MATTER and leaves no time for LBT evolution. This is more obvious when Q_0 is smaller.

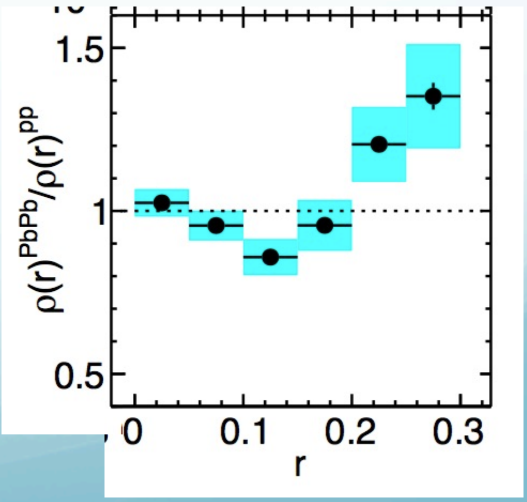
Dynamical Q_0 is close to the fixed Q_0 = 2 GeV case when E_{init} = 50 GeV.

$dE/d\vartheta$ for E_i = 50 GeV and L = 4 fm



Medium evolution changes the jet shape – depletes energy in small cone and enhance energy in large cone.

LBT is more effective than MATTER in shifting energy distribution into larger angle since elastic scattering is included in LBT.



Interesting non-monotonic behavior at Q_0 = 1 GeV -- enhanced Sudakov type splitting at very small *r* and LBT scattering at large *r*. (Never seen when single energy loss formalism is applied)

Outlook

- Implement the combined energy loss approach in realistic hydrodynamic medium
- Include more approaches into the same framework, such as the strongly coupled approach for the near thermal partons
- Apply this framework to heavy quark

Thank you!