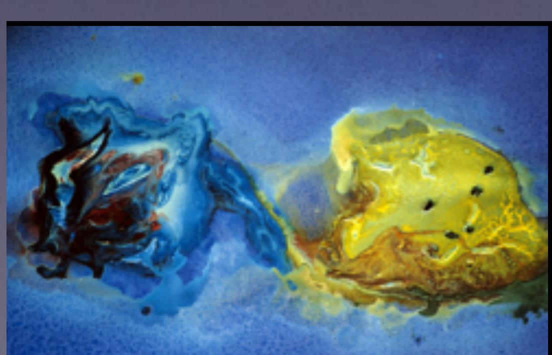
Quarkonium in the Fireball

Nora Brambilla



Physik Department TUM
T30f

- Quarkonium suppression and R_AA as a probe of QGP: original idea of Matsui and Satz based on screening

- Quarkonium works as a good probe because is a multiscale system well understood at T=0 on the basis of EFT and lattice

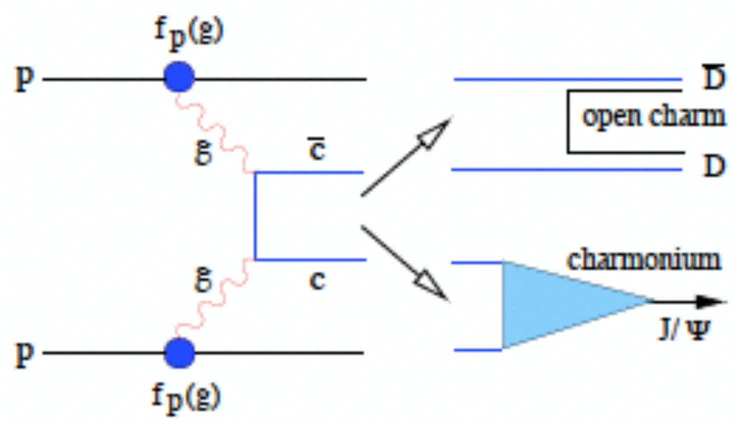
- Using EFTs to determine quarkonium interaction at finite T we get a change of paradigm: suppression driven not by screening but by imaginary parts of the potentials

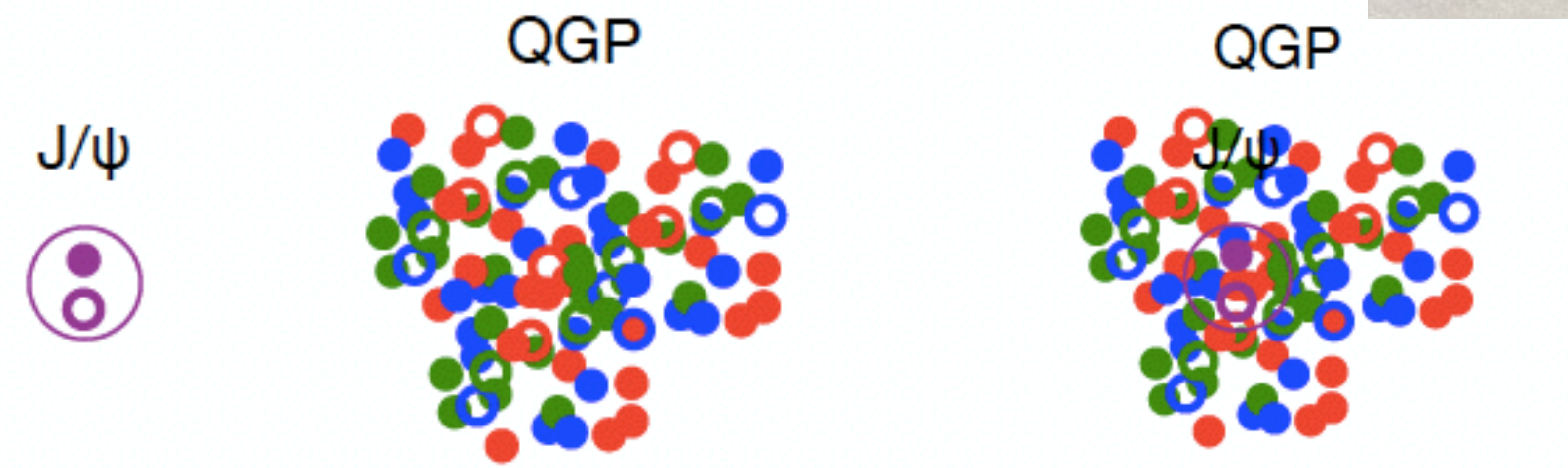
- Physical interpretation of the imaginary parts: Landau damping/inelastic parton dissociation, singlet octet transition/gluodissociation

- How to describe quarkonium evolving in the fireball: open quantum description based on the EFT. It is fully quantum, non abelian and conserve total number of heavy quarks. It contains dissociation and recombination, gives R_AA

Quarkonium as Probe of the QGP

- In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe
  - Heavy quarks are formed early in heavy-ion collisions: 1/M ~ 0.1 fm < 0.6 fm
  - Heavy quarkonium formation will be sensitive to the medium
  - Originally: Debye screening leads to binding energy dependent dissociation



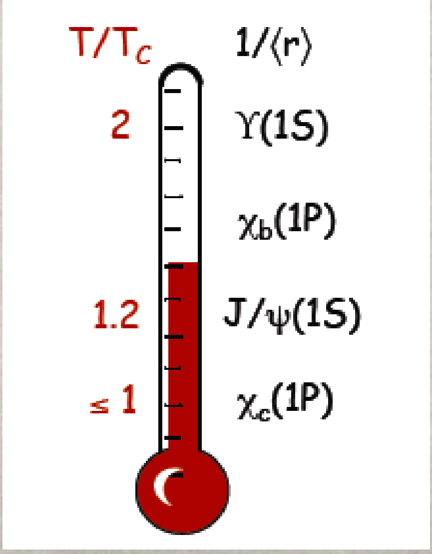


Debye charge screening

$$m_D \sim gT$$

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \longrightarrow \text{Bound state dissolves}$$



From this:

-quarkonia should dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

-R_AA should be smaller than 1

But, what is the quarkonium interaction potential in a hot medium?

Potential models

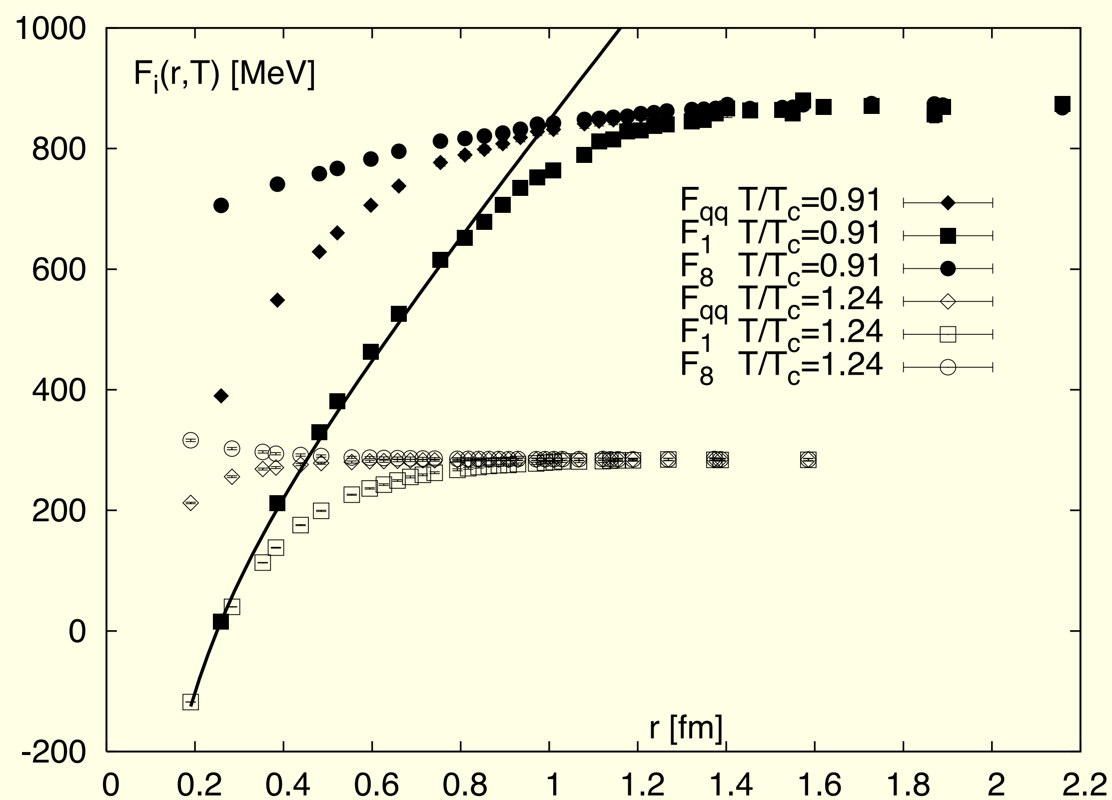
Digal, Petreczky, Satz 01
Wong 05-07
Mannarelli, Rapp 05
Mocsy, Petreczky 05-08
Alberico, Beraudo et al 05-08
Cabrera, Rapp 2007
Wong, Crater 07
Dumitru, Guo, Mocsy, Strickland 09
Rapp, Riek 10

- Either phenomenological potentials have been used so far or the free energy calculated on the lattice.

Singlet, octet and average free energy

- The free energy is not the static potential: the average free energy ($\sim \langle \mathrm{Tr}\, L^\dagger(r) \mathrm{Tr}\, L(0)\rangle$) is an overlap of singlet and octet quark-antiquark states, what is called the singlet ($\sim \langle \mathrm{Tr}\, L^\dagger(r)\, L(0)\rangle$) and the octet ($\sim \langle \mathrm{Tr}\, L^\dagger(r) \mathrm{Tr}\, L(0)\rangle$ $-1/3\, \langle \mathrm{Tr}\, L^\dagger(r)\, L(0)\rangle$) free energy are gauge dependent;

Owe Philipsen 08



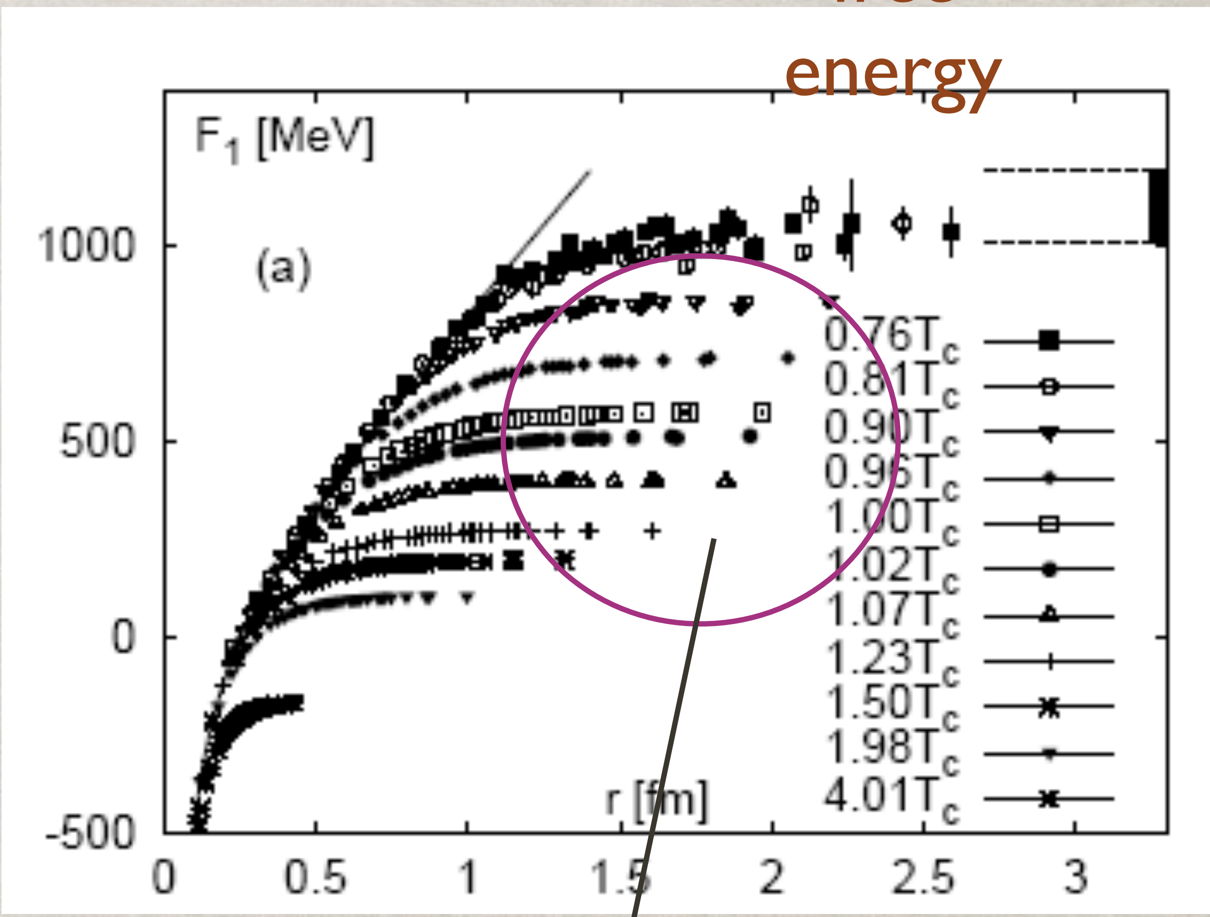
2 flavor QCD ○ Kaczmarek Zantow PRD 71 (2005) 114510

But, what is the quarkonium interaction potential in a hot medium?

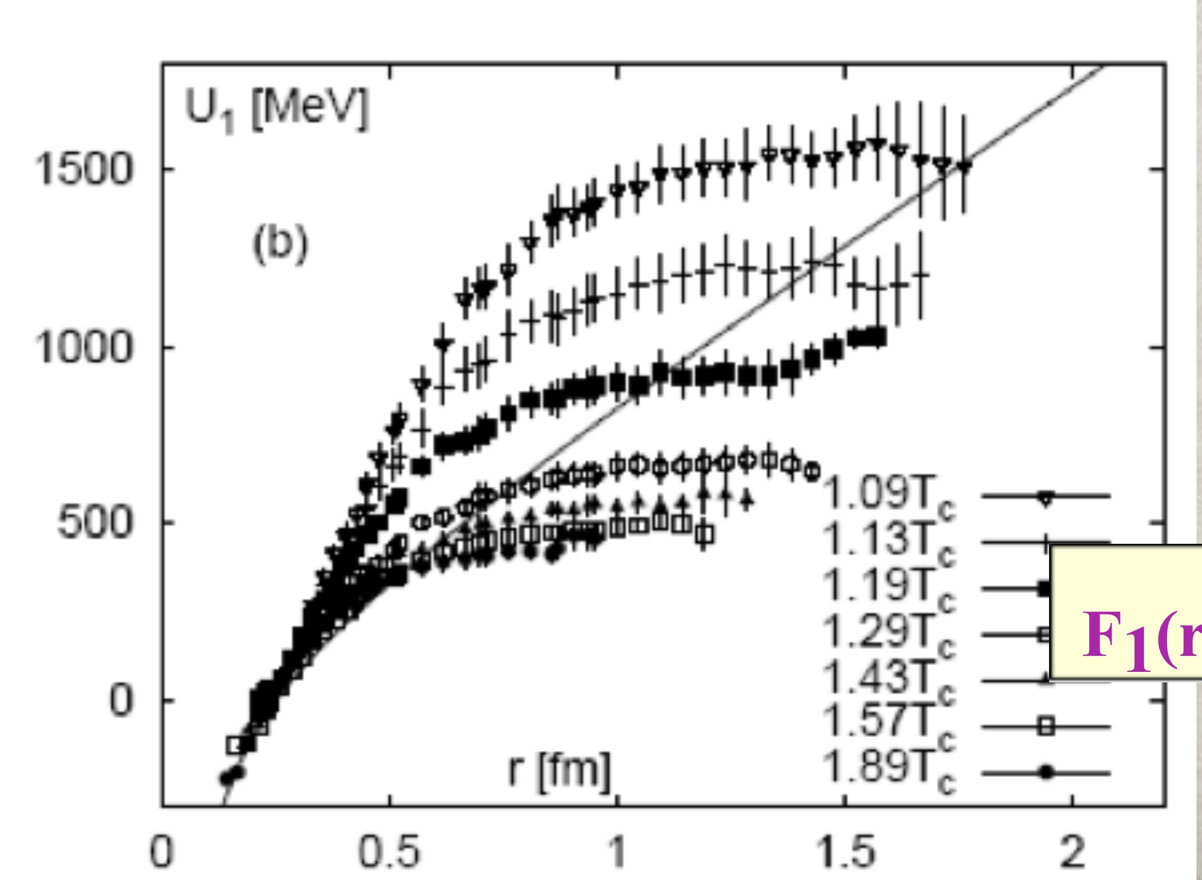
Potential models

Which of these is the QCD potential?
Are all effects incorporated?
can we devise a method to define and calculate the potential from QCD?

Singlet free energy



flattening is due to screening



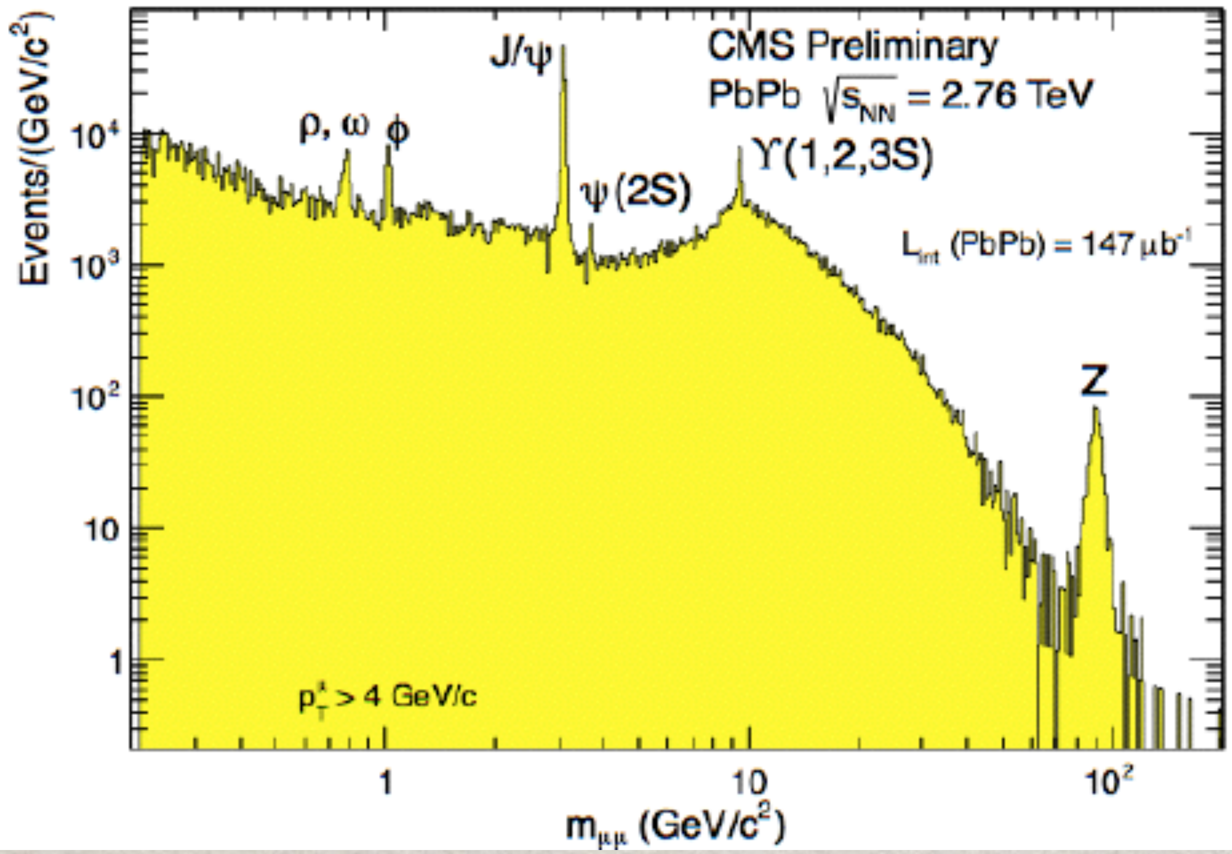
Internal energy

$$F_1(r,T) = U_1(r,T) - T\,S_1(r,T)$$

Kaczmarek Zantow'05
2 flavor QCD

Heavy Quarkonium is a special probe of QGP

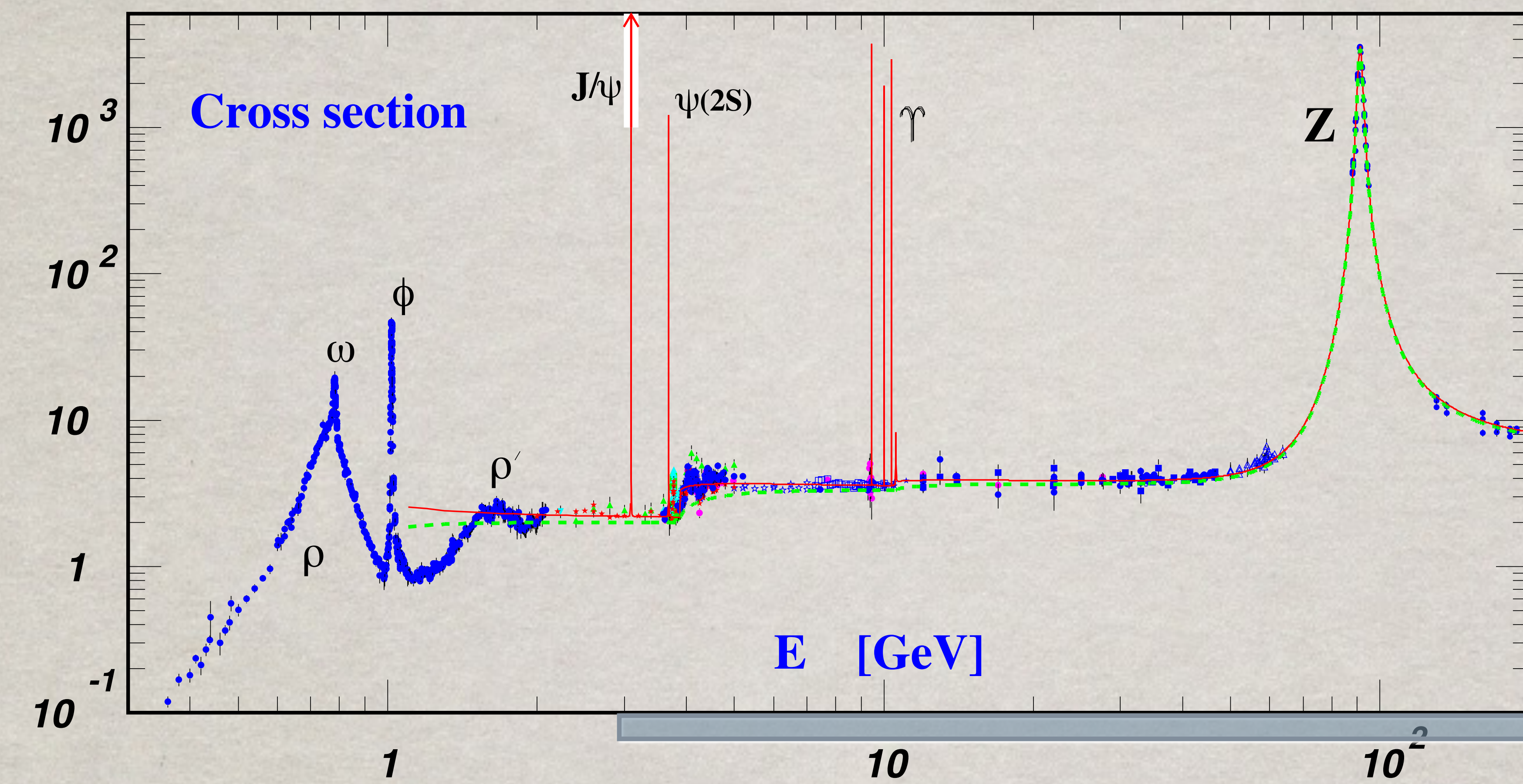
- Experimentally:
  - Heavy quarkonia decay in vacuum: dilepton signal makes quarkonia a clean experimental probe

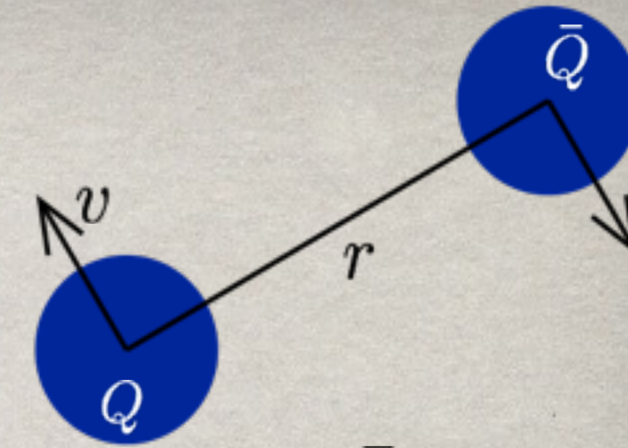


- Theoretically
  - The heavy-quark mass introduces one or more large scales, whose contributions may be factorised and computed in perturbation theory
  - Low-energy contributions may be accessible via lattice calculations

Low-energy scales are sensitive to the temperature.

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

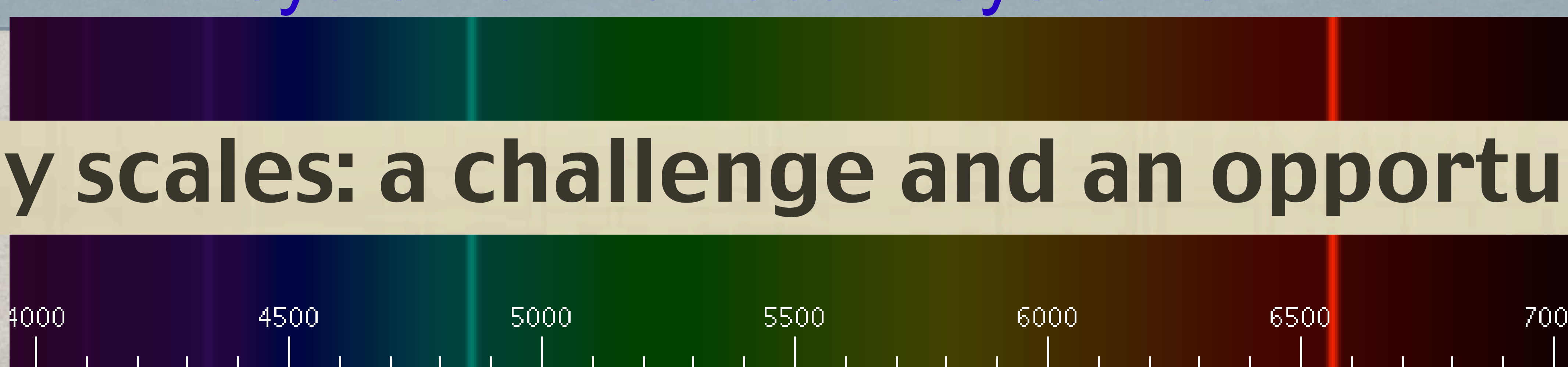
$m_c \sim 1.5$GeV

$m_b \sim 5$GeV

$m_t \sim 170$GeV

A large scale $m_Q \gg \Lambda_{QCD}$ $\alpha_s(m_Q) \ll 1$

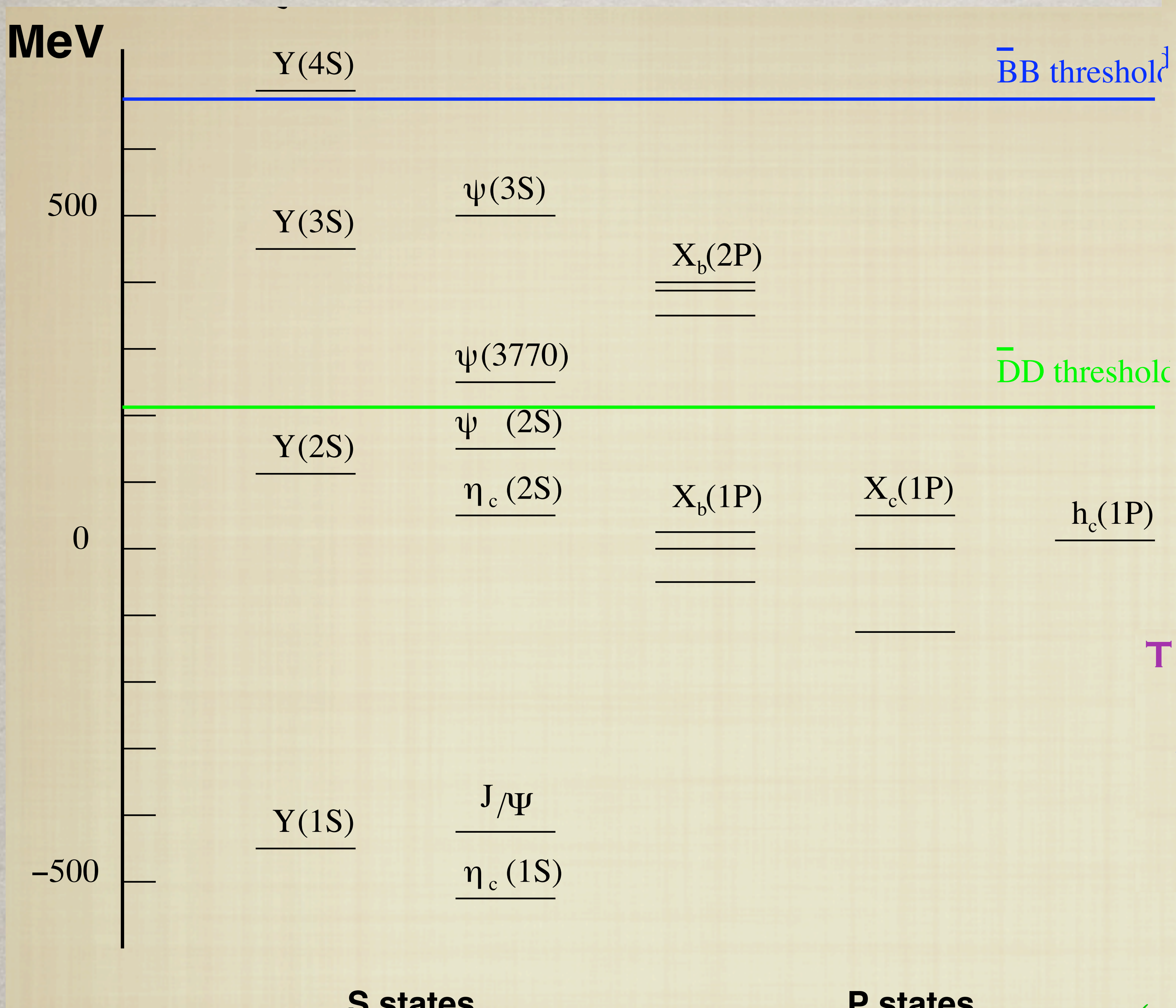
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



many scales: a challenge and an opportunity

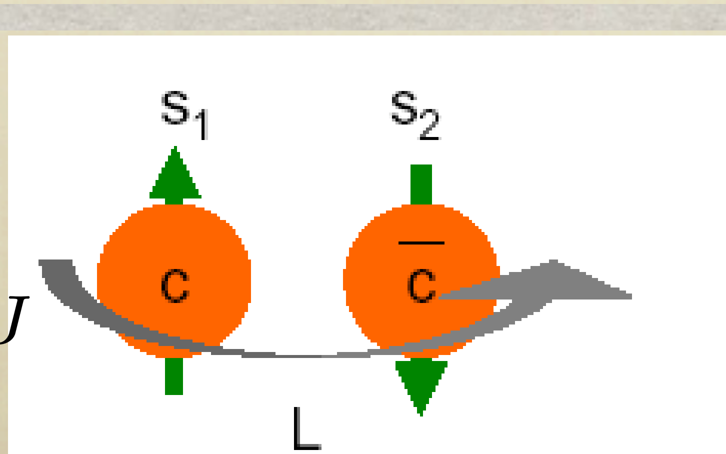
Electromagnetic bound states: atoms, molecules,

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



NR BOUND STATES HAVE AT LEAST 3 SCALES

$$m \gg mv \gg mv^2 \qquad v \ll 1$$

$$mv \sim r^{-1}$$

and Λ_{QCD}

THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

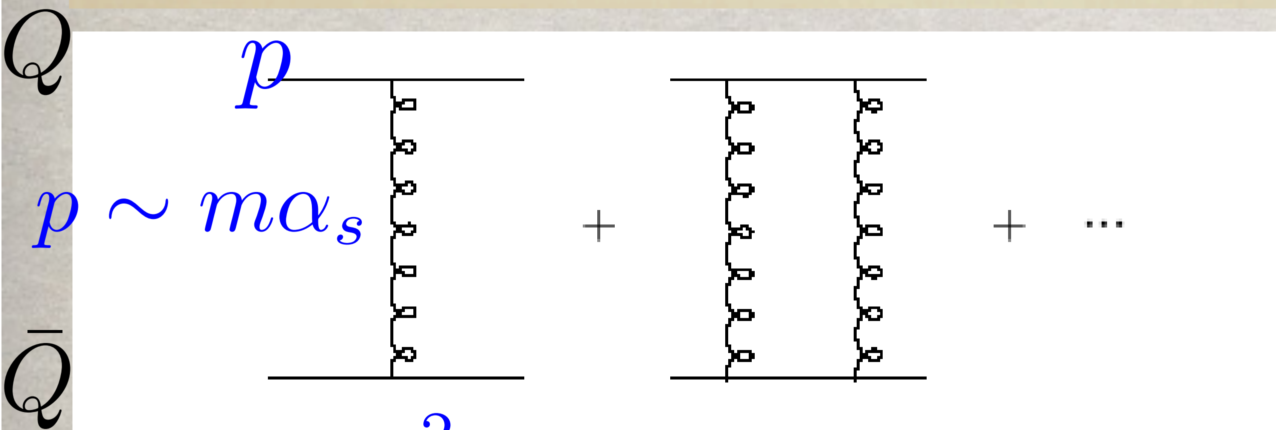
THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{QCD}$$

$$m_b \simeq 5\,\text{GeV}; m_c \simeq 1.5\,\text{GeV}$$

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

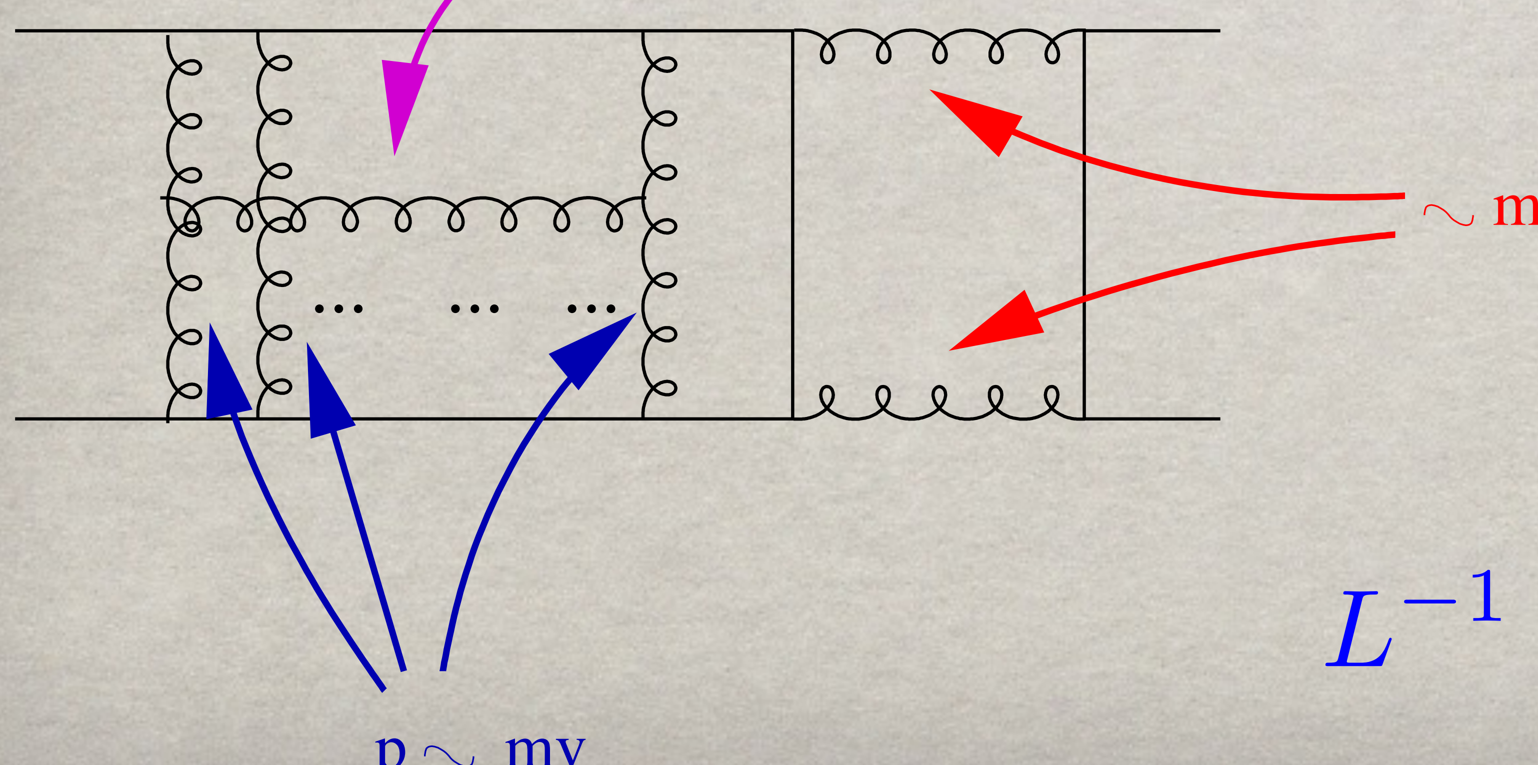


$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

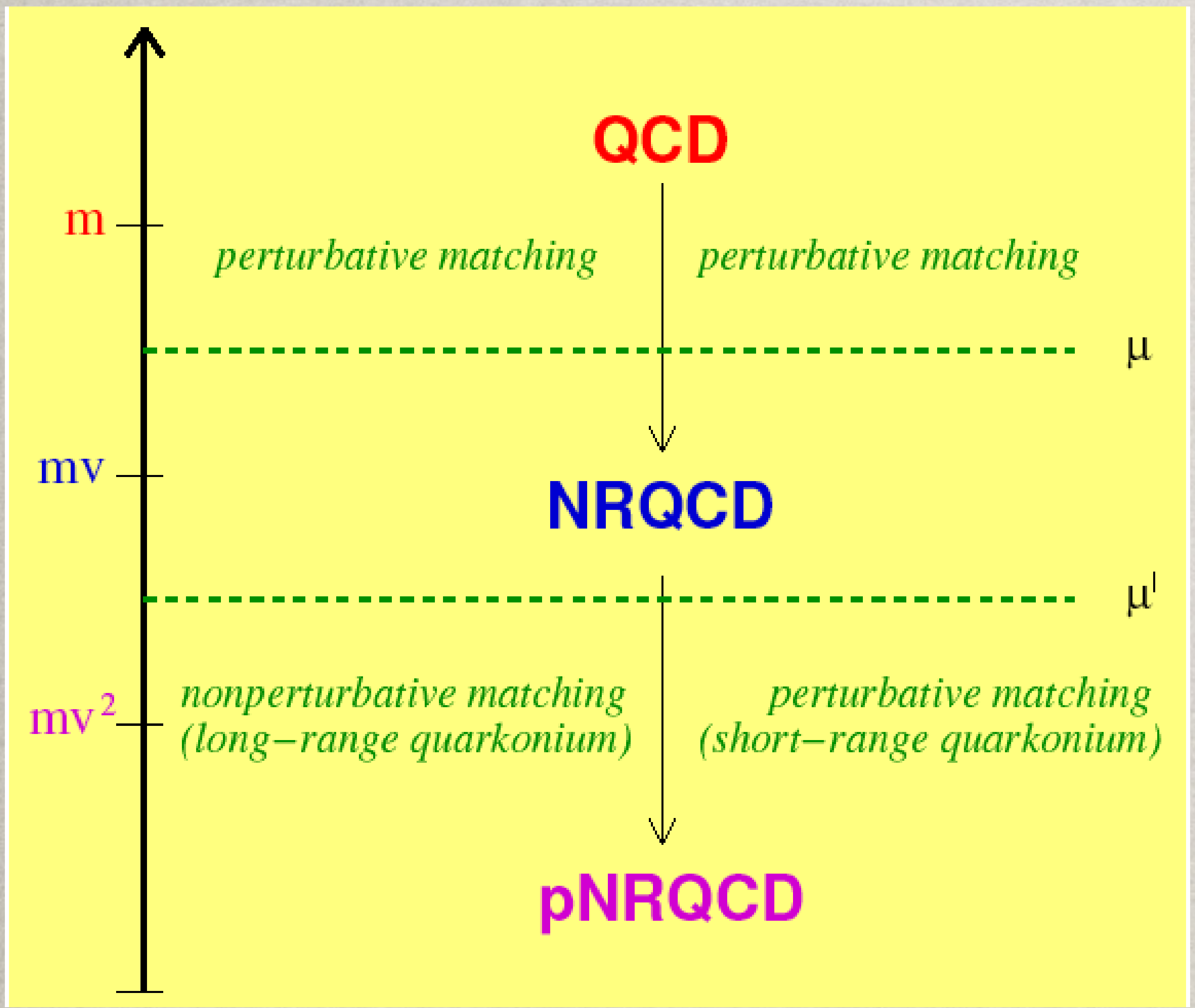


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with NR EFT

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft (relative momentum)

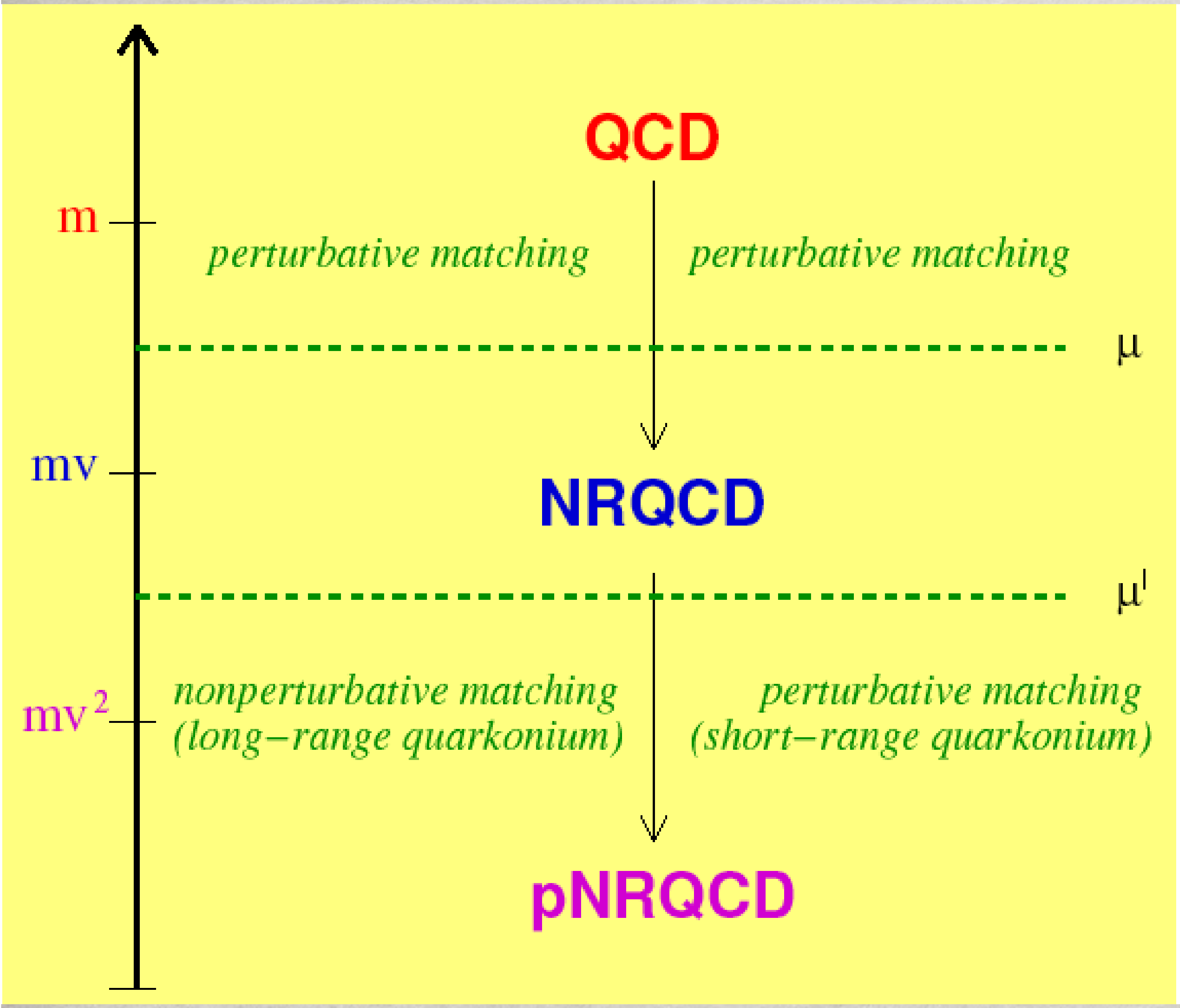
$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

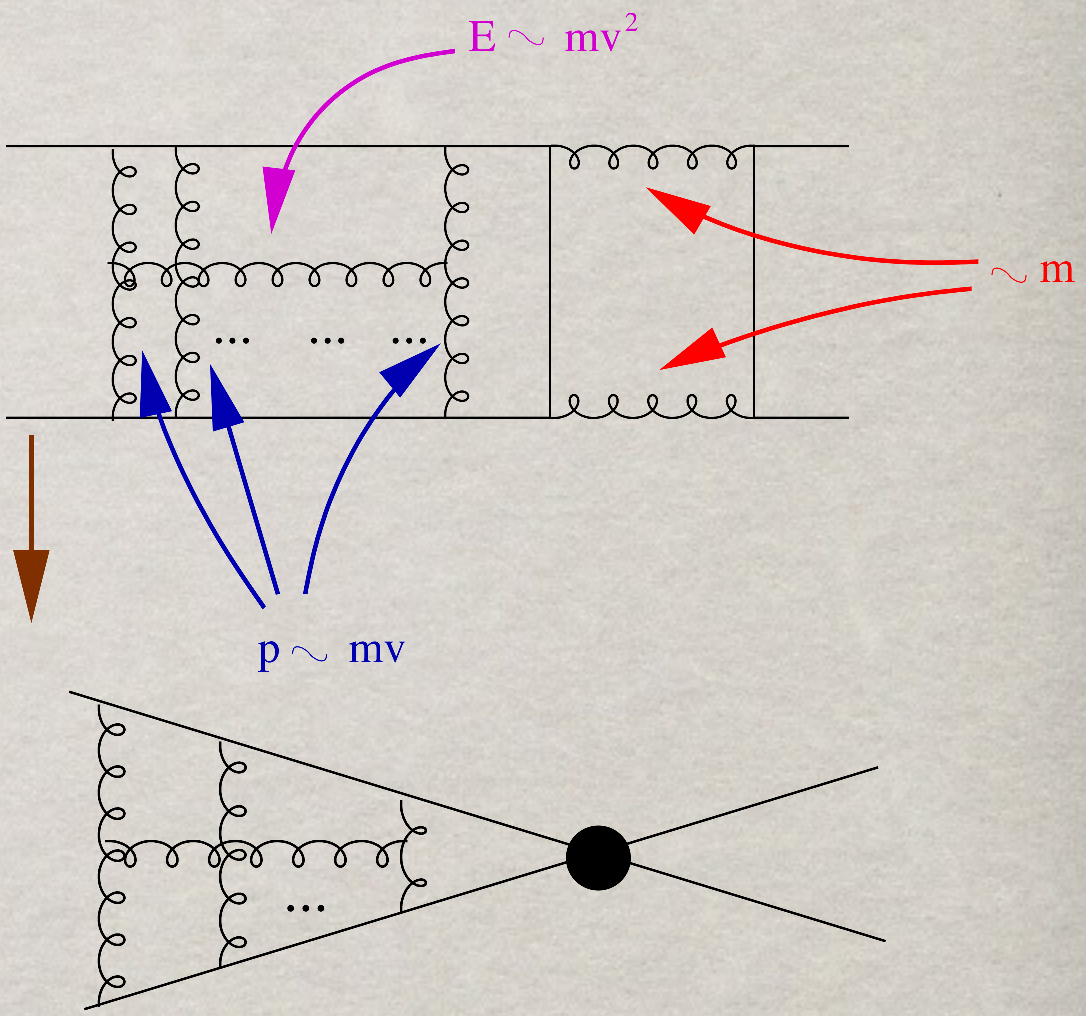
Ultrasoft (binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

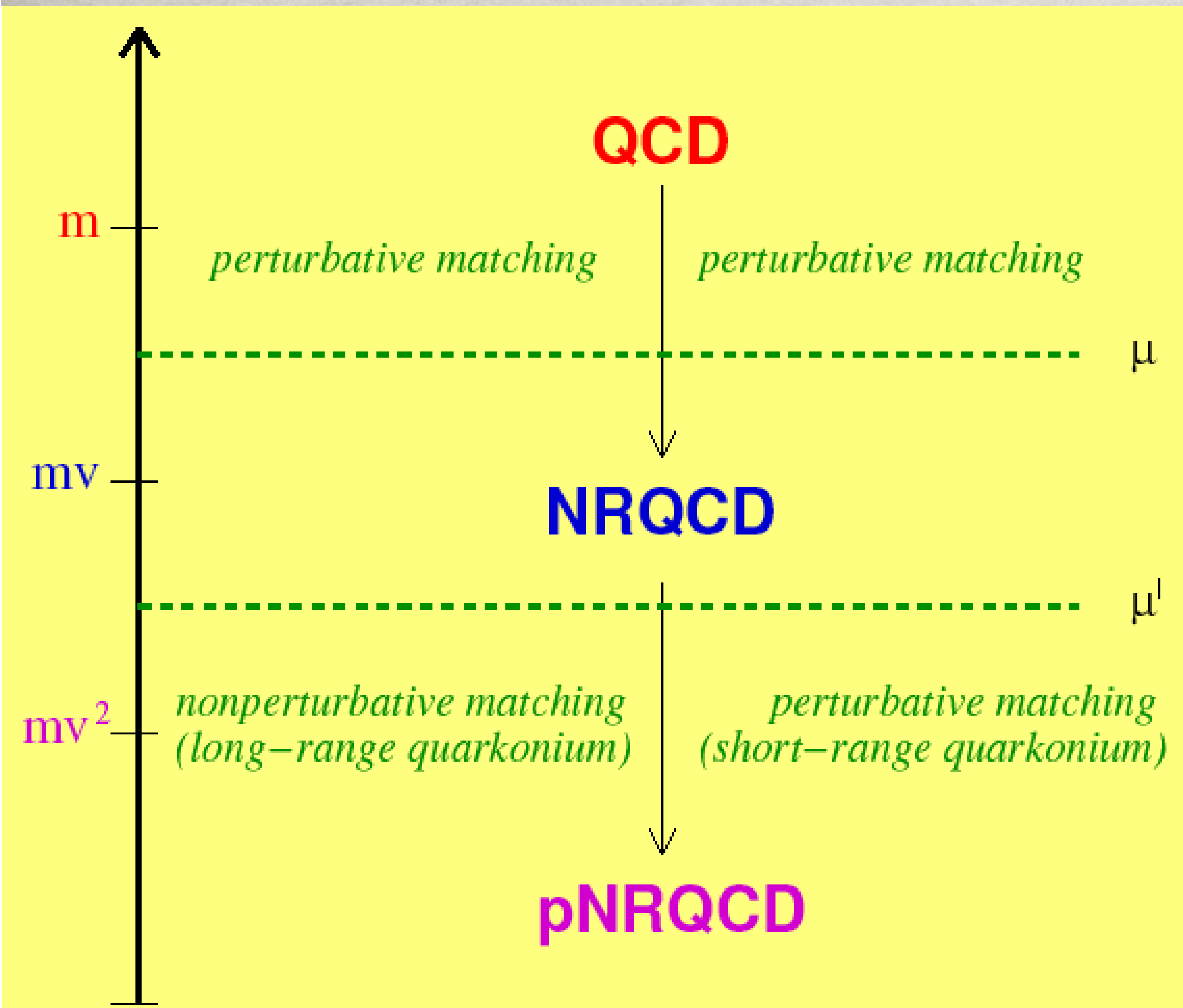
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

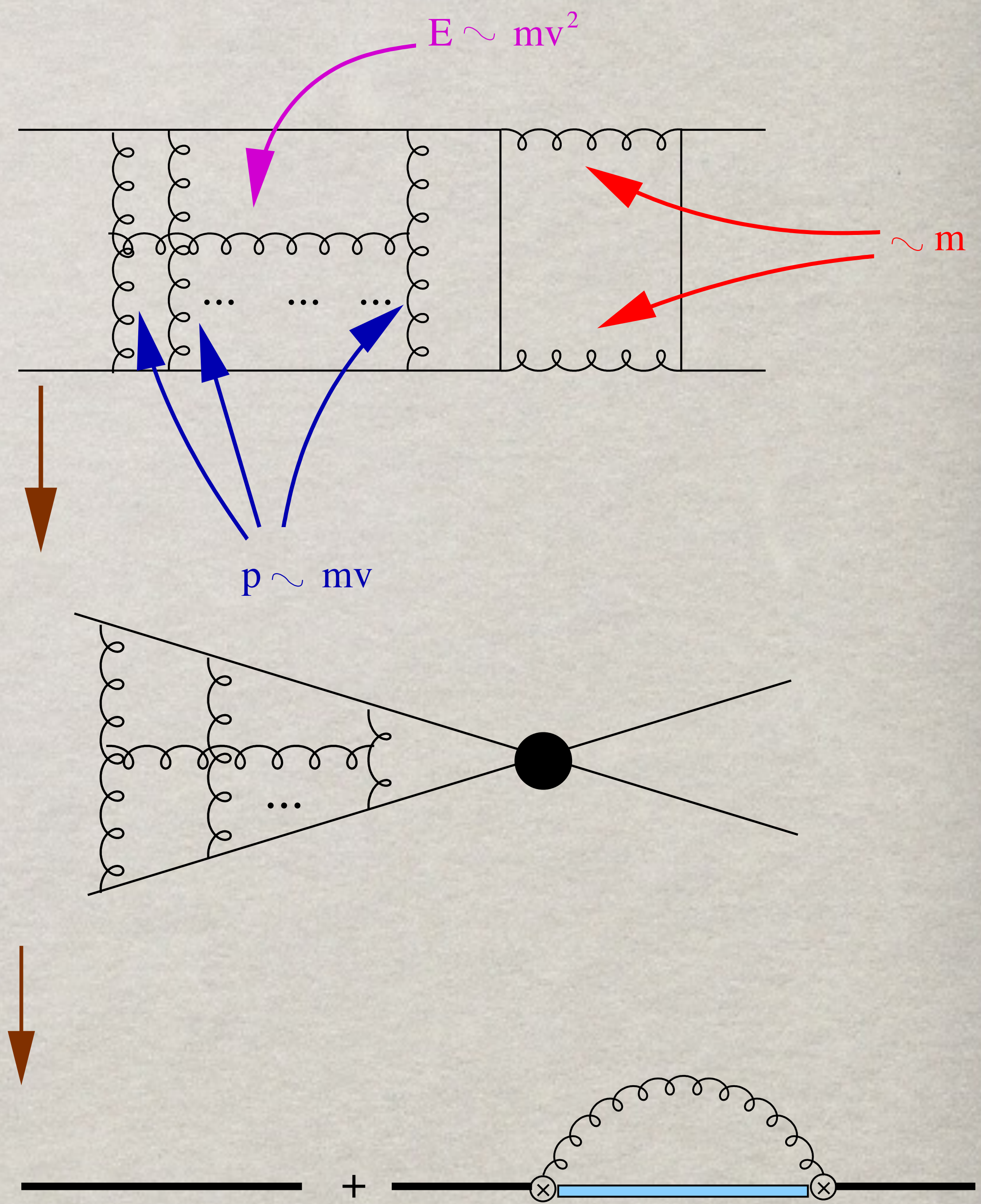




$$\mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

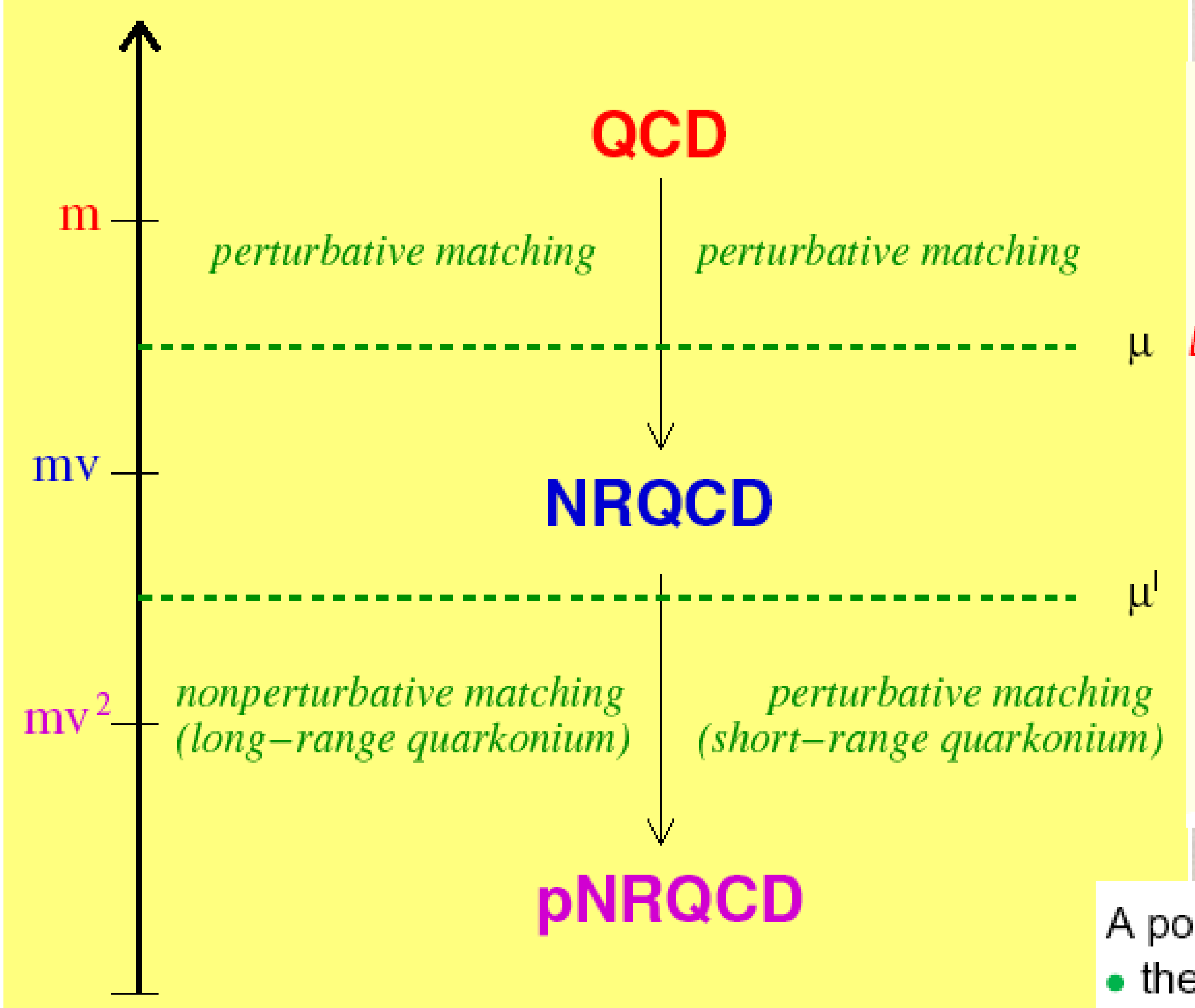
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)

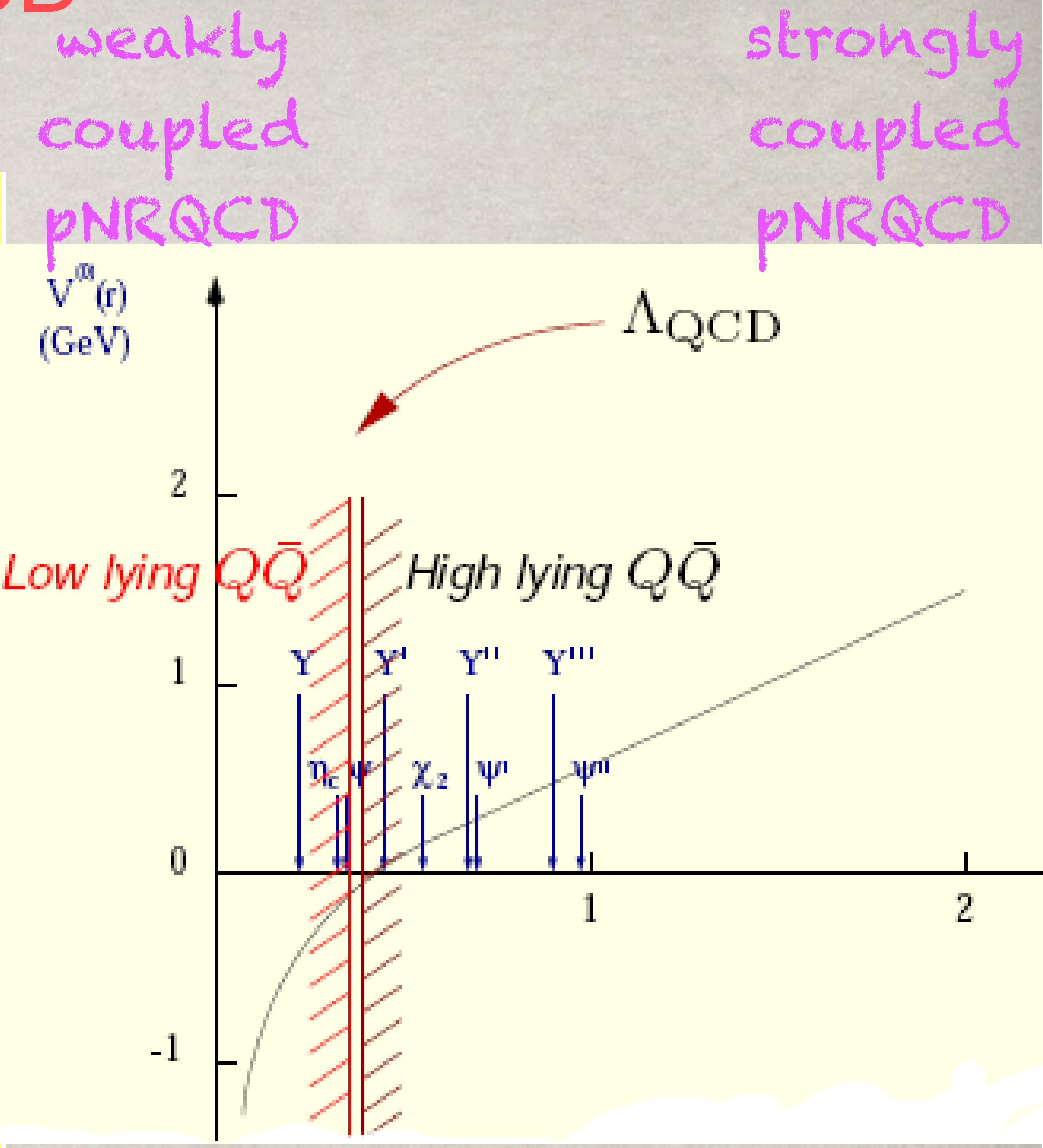




$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: pNRQCD





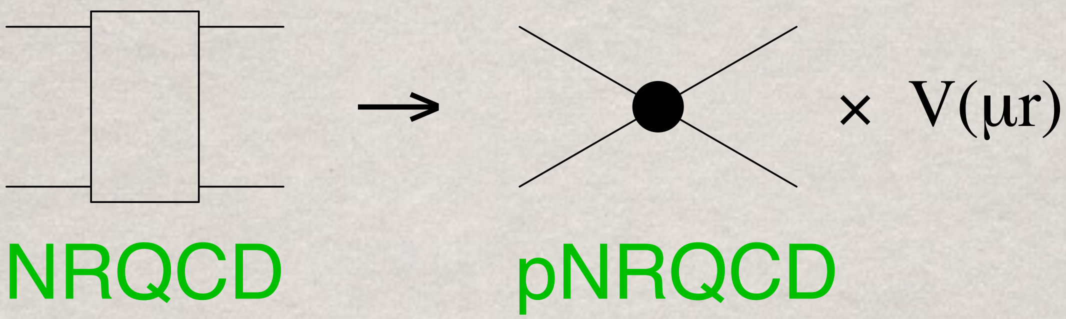
A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

In QCD another scale is relevant Λ_{QCD}

pNRQCD for quarkonia with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

Degrees of freedom that scale like mv are integrated out:



- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet S (ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Definite power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:
  $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$

  Non-analytic behaviour in $r \to$ matching coefficients V

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

Octet static potential

$$+V_A {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} \right\}$$
$$+\frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O}\, \mathbf{r}\cdot g\mathbf{E} \right\}$$
$$+\cdots$$

NLO in r

S singlet field

O octet field

singlet propagator

octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

pNRQCD is nowadays the EFT used at T=0 to describe quarkonium properties

but what about finite T?

with the EFT one can define what is the quarkonium potential at finite T

The potential V(r,T) dictates throught the Schroedinger equation the real time evolution of the QQbar pair in the medium-> use the EFT to define and calculate it

more scales $m \gg mv \gg mv^2$

?

and $\Lambda_{\rm QCD}$

$$\pi T \gg gT \gg g^2T \ldots$$

$m_D \sim gT$
Debye mass
Screening Scale

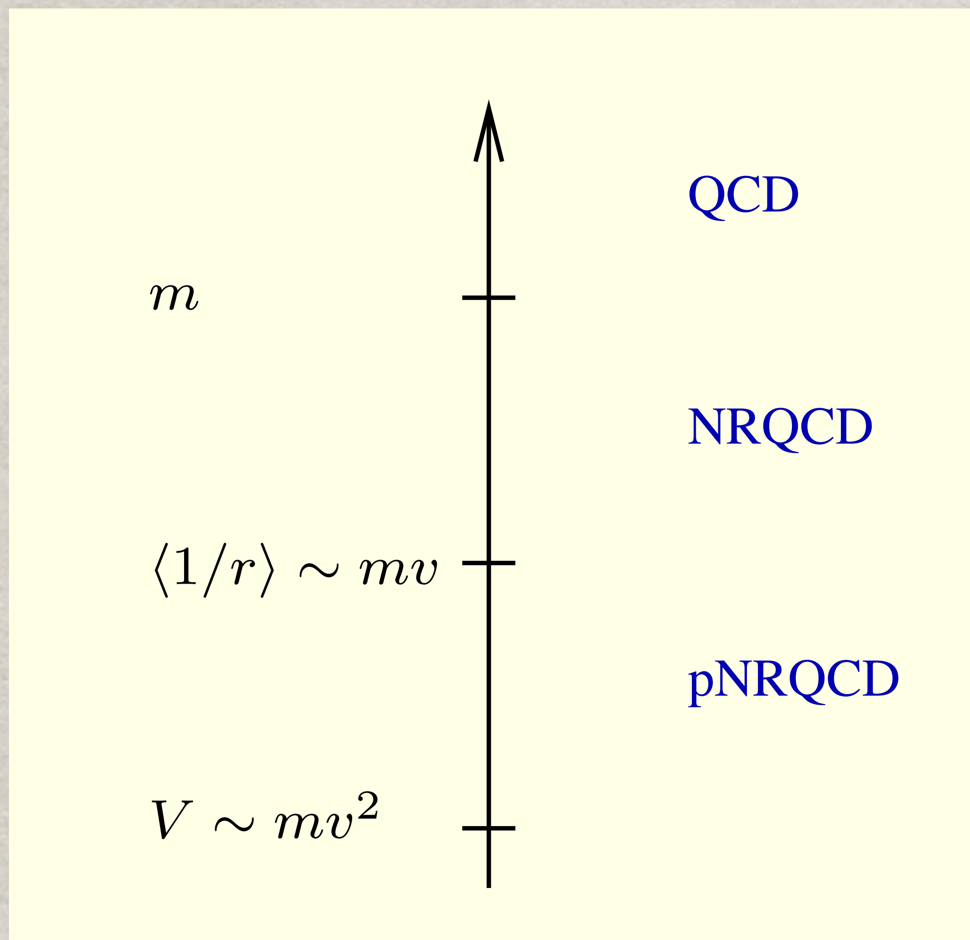
Without heavy quarks an EFT already exists that comes from integrating out hard gluon of p \sim T:
Hard Thermal Loop EFT

Braaten Pisarski 90

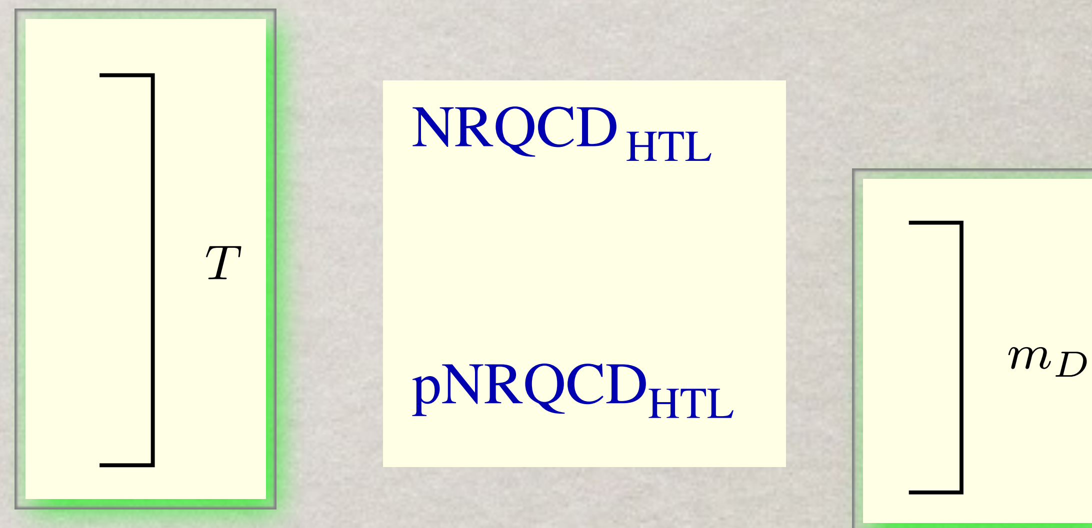
-> obtain pNRQCD at finite T

Quarkonium at finite T with pNRQCD

N. B., Ghiglieri,Petreczky, Vairo 08



pNRQCD at finite T allows us to define the static QQbar potential in the medium in real time



We work under the conditions:

We assume that bound states exist for

- $T \ll m$
- $\langle 1/r \rangle \sim mv \gtrsim m_D$

We neglect smaller thermodynamical scales.

In the weak coupling regime:

- $v \sim \alpha_s \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ, ...
- $T \gg gT \sim m_D$.

Effects due to the scale Λ_{QCD} will not be considered.

pNRQCD supply the potential (weak coupling regime T>>gT)

- The thermal part of the potential has a real and an imaginary part

ReVs (r,T)

ImVs (r,T)

thermal width of $Q\bar{Q}$

New effect, specific of QCD
dominates for E/m_D>>1

Known from QED
dominates for m_D/E>>1

Singlet-to-octet
N.B Ghiglieri, Petreczky, Vairo 2008

(gluo dissociation)
N. B. Escobedo, Ghiglieri , Vairo 2011

Landau damping
Laine et al 07, Escobedo Soto 07

(inelastic parton scattering)
N. B. Escobedo, Ghiglieri , Vairo 2013

The singlet static potential and the static energy (pNRQCD)

- Temperature effects can be other than screening

$T > 1/r$ and $1/r \sim m_D \sim gT$

exponential screening but $\mathrm{Im}V \gg \mathrm{Re}V$

$T > 1/r$ and $1/r > m_D \sim gT$

no exponential screening but
power-like T corrections

$T < E_{bin}$

no corrections to the potential,
corrections to the energy

imaginary parts in the potential have subsequently been found also for a strongly coupled plasma on the lattice (A. Rothkopf et al, Petreczky, Weber..) and in strings calculations

Screening vs Landau damping

For temperatures such that $Mv^2 \ll m_D$ and $Mv \gg \pi T$

- $\mathrm{Re}\, V_s(r) \sim \alpha_s/r$, quarkonium is a Coulombic bound state.

○ Escobedo Soto arXiv:0804.069

- Quarkonium dissociates at a temperature such that $\mathrm{Im}\, V_s(r) \sim \mathrm{Re}\, V_s(r) \sim \alpha_s/r$:

$$\pi T_{\text{dissociation}} \sim mg^{4/3}$$

Laine arXiv:0810.1112

- The interaction is screened when $\langle 1/r \rangle \sim m_D$, hence

$$\pi T_{\text{screening}} \sim mg \gg \pi T_{\text{dissociation}}$$

offer a systematic framework to do the calculations of the energy and width in a hot medium

The bottomonium ground state , which is a weakly coupled non-relativistic bound state: $mv \sim m\alpha_s,\, mv^2 \sim m\alpha_s^2 \gtrsim \Lambda_{QCD}$, produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy

$$m \approx 5 \text{ GeV } > m\alpha_s \approx 1.5 \text{ GeV } > \pi T \approx 1 \text{ GeV } > m\alpha_s^2 \approx 0.5 \text{ GeV } \gtrsim m_D, \Lambda_{QCD}$$

T_dissociation in the $\Upsilon(1S)$ case is about 450 MeV.

$M = m =$ mass heavy quark

bottomonium 1S below the melting temperature T_d

The complete mass and width up to $\mathcal{O}(m\alpha_s^5)$

$$\delta E_{1S}^{(\mathrm{thermal})} = \frac{34\pi}{27}\alpha_s^2\,T^2 a_0 + \frac{7225}{324}\frac{E_1\alpha_s^3}{\pi}\left[\ln\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right] + \frac{128 E_1\alpha_s^3}{81\pi}L_{1,0} - 3a_0^2\left\{\left[\frac{6}{\pi}\zeta(3)+\frac{4\pi}{3}\right]\alpha_s\,T\,m_D^2 - \frac{8}{3}\zeta(3)\,\alpha_s^2\,T^3\right\}$$

$$\Gamma_{1S}^{(\mathrm{thermal})} = \frac{1156}{81}\alpha_s^3 T + \frac{7225}{162}E_1\alpha_s^3 + \frac{32}{9}\alpha_s\,T m_D^2\,a_0^2\,I_{1,0} - \left[\frac{4}{3}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \ln 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{32\pi}{3}\ln 2\,\alpha_s^2\,T^3\right]a_0^2$$

where $E_1 = -\frac{4m\alpha_s^2}{9}$, $a_0 = \frac{3}{2m\alpha_s}$ and $L_{1,0}$ (similar $I_{1,0}$) is the Bethe logarithm.

- Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038

Consistent with lattice calculations of spectral functions

- Aarts Allton Kim Lombardo Oktay Ryan Sinclair Skullerud JHEP 1111 (2011) 103

The imaginary parts in the potentials give origin to a quarkonium thermal dissociation width and they correspond to two process

Two distinct dissociation mechanisms may be identified at leading order:

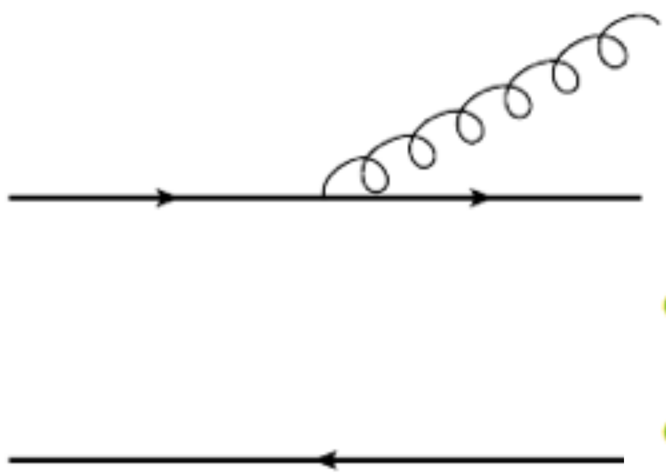
- gluodissociation,
  which is the dominant mechanism for $Mv^2 \gg m_D$;
- dissociation by inelastic parton scattering,
  which is the dominant mechanism for $Mv^2 \ll m_D$.

Beyond leading order the two mechanisms are intertwined and distinguishing between them becomes unphysical, whereas the physical quantity is the total decay width.

- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
- Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

Gluodissociation

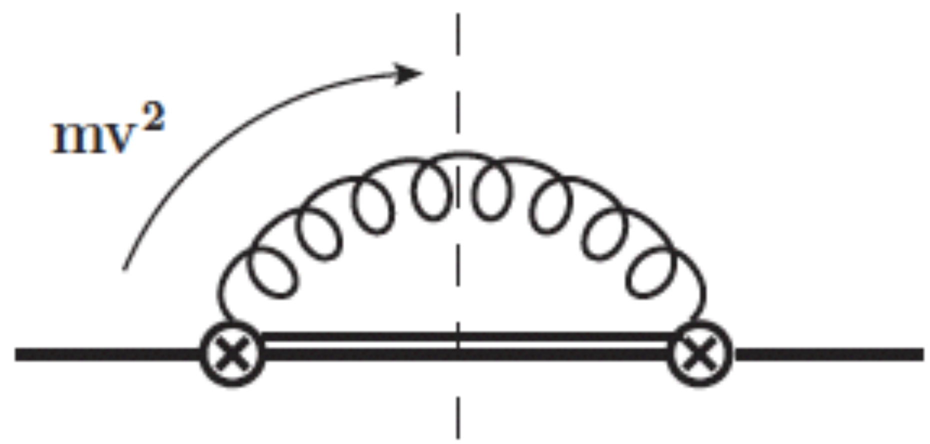
Gluodissociation is the dissociation of quarkonium by absorption of a gluon from the medium.



- Kharzeev Satz PLB 334 (1994) 155
  Xu Kharzeev Satz Wang PRC 53 (1996) 3051

- The exchanged gluon is lightlike or timelike.
- The process happens when the gluon has an energy of order Mv^2.

From the optical theorem, the gluodissociation width follows from cutting the gluon propagator in the following pNRQCD diagram

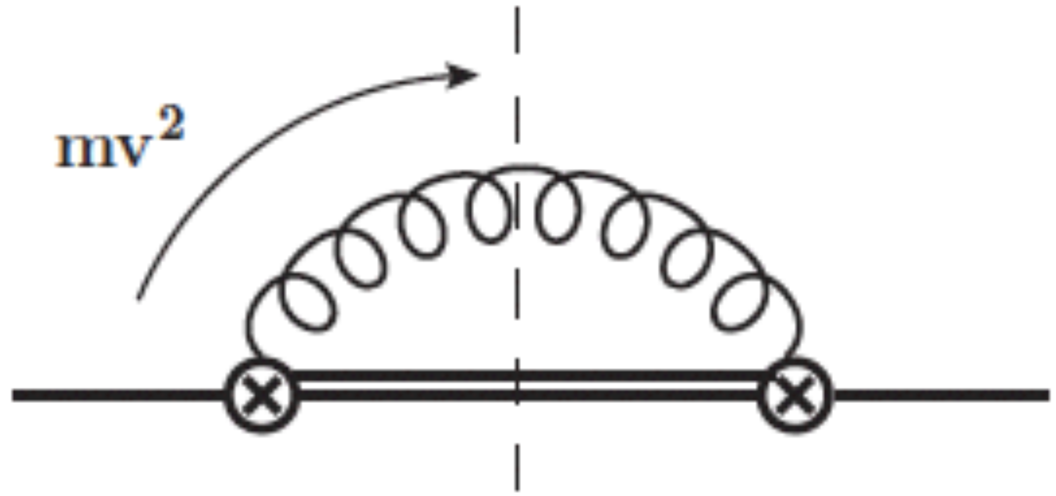


For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma_{nl} = \int_{q_{\rm min}} \frac{d^3q}{(2\pi)^3}\, n_{\rm B}(q)\, \sigma^{nl}_{\rm gluo}(q)\,.$$

- $\sigma^{nl}_{\rm gluo}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} + g \to Q + \overline{Q}$.
- Gluodissociation is also known as singlet-to-octet break up.

$1S$ gluodissociation at LO



The LO gluodissociation cross section for $1S$ Coulombic states is

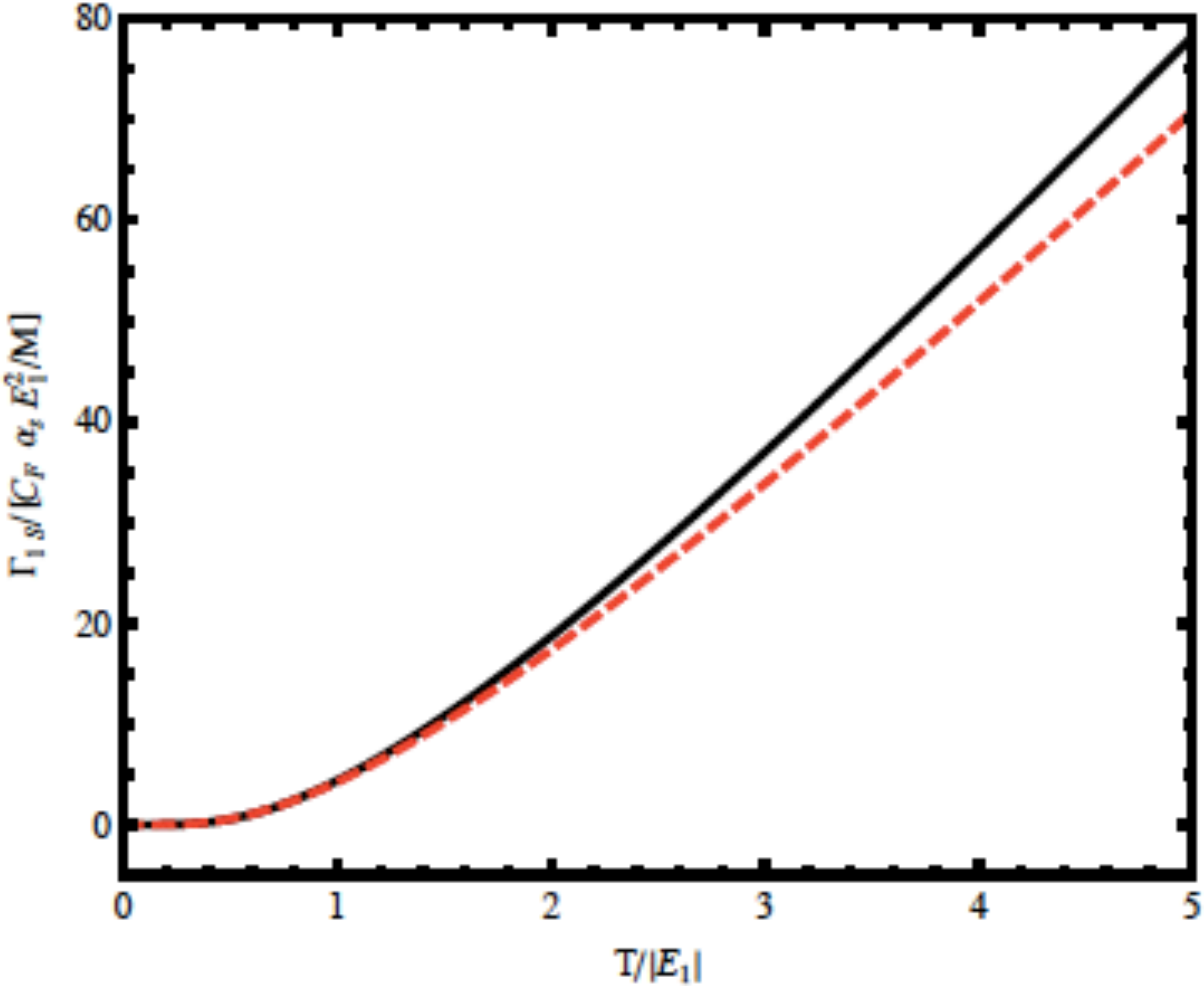
$$\sigma^{1S}_{\rm gluo\,LO}(q) = \frac{\alpha_{\rm s} C_F}{3} 2^{10}\pi^2 \rho(\rho+2)^2 \frac{E_1^4}{M q^5}\left(t(q)^2+\rho^2\right)\frac{\exp\left(\frac{4\rho}{t(q)}\arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}}-1}$$

where $\rho \equiv 1/(N_c^2-1)$, $t(q) \equiv \sqrt{q/|E_1|-1}$ and $E_1 = -MC_F^2\alpha_{\rm s}^2/4$.

- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
  Brezinski Wolschin PLB 707 (2012) 534

The Bhanot–Peskin approximation corresponds to the large N_c limit, i.e. to neglecting final state interactions (the rescattering of a $Q\overline{Q}$ pair in a color octet configuration).

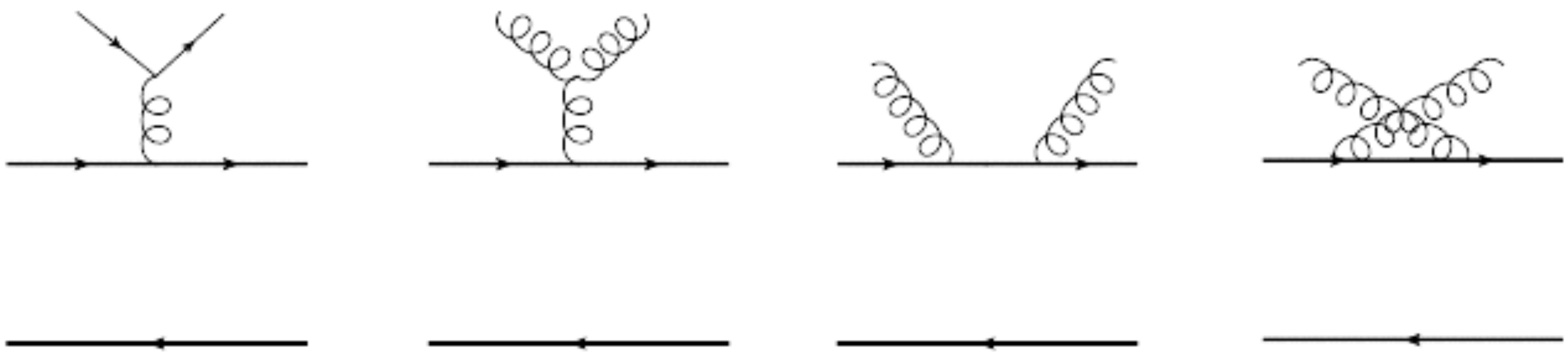
- Peskin NPB 156 (1979) 365, Bhanot Peskin NPB 156 (1979) 391



80
60
40
20
0
0
1
2
3
4
5
$\Gamma_{1S}/[C_F\ \alpha_s\ E_1^2/M]$
$T/|E_1|$

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.

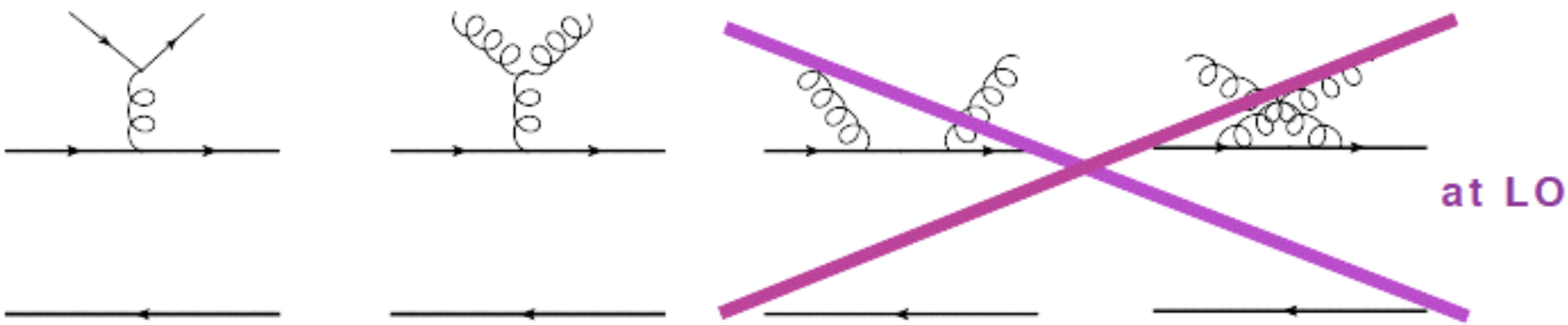


○ Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415

- The exchanged gluon is spacelike.
- External thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.

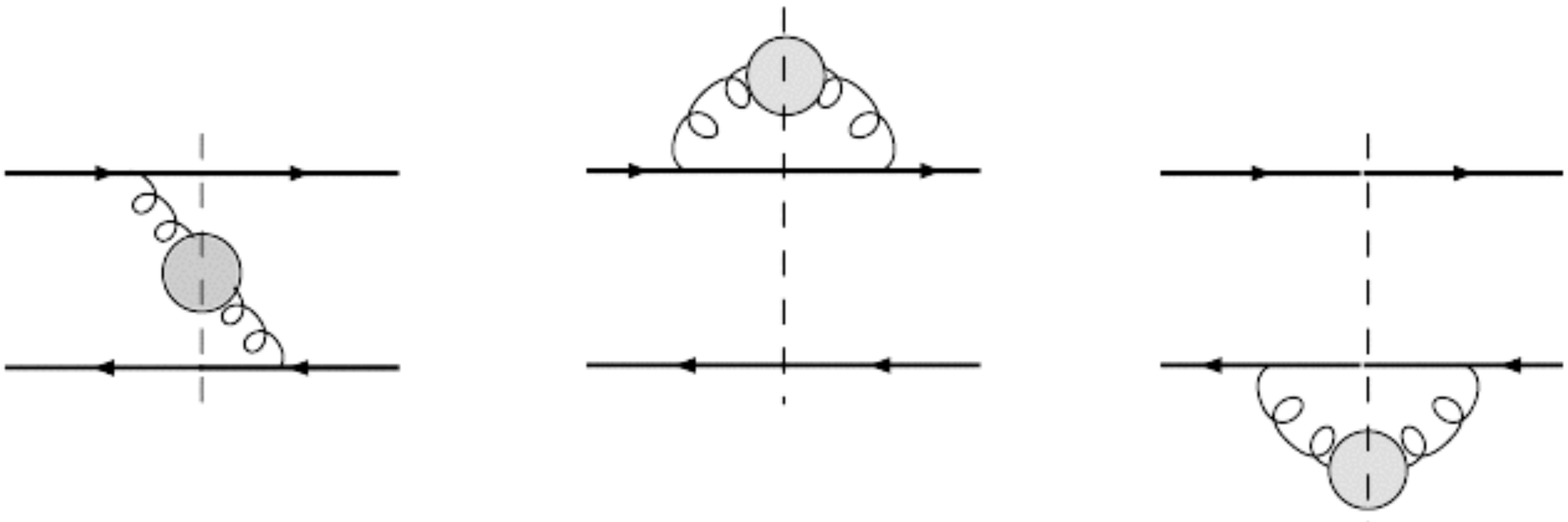


○ Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415

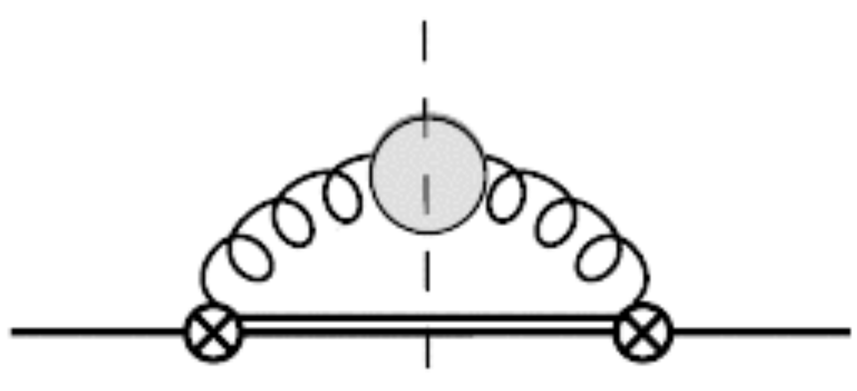
- The exchanged gluon is spacelike.
- External thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

Dissociation by inelastic parton scattering

From the optical theorem, the thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams (momentum of the gluon $\gtrsim Mv$)



and/or pNRQCD diagram (momentum of the gluon $\ll Mv$)



- Dissociation by inelastic parton scattering is also known as Landau damping.

Dissociation by inelastic parton scattering

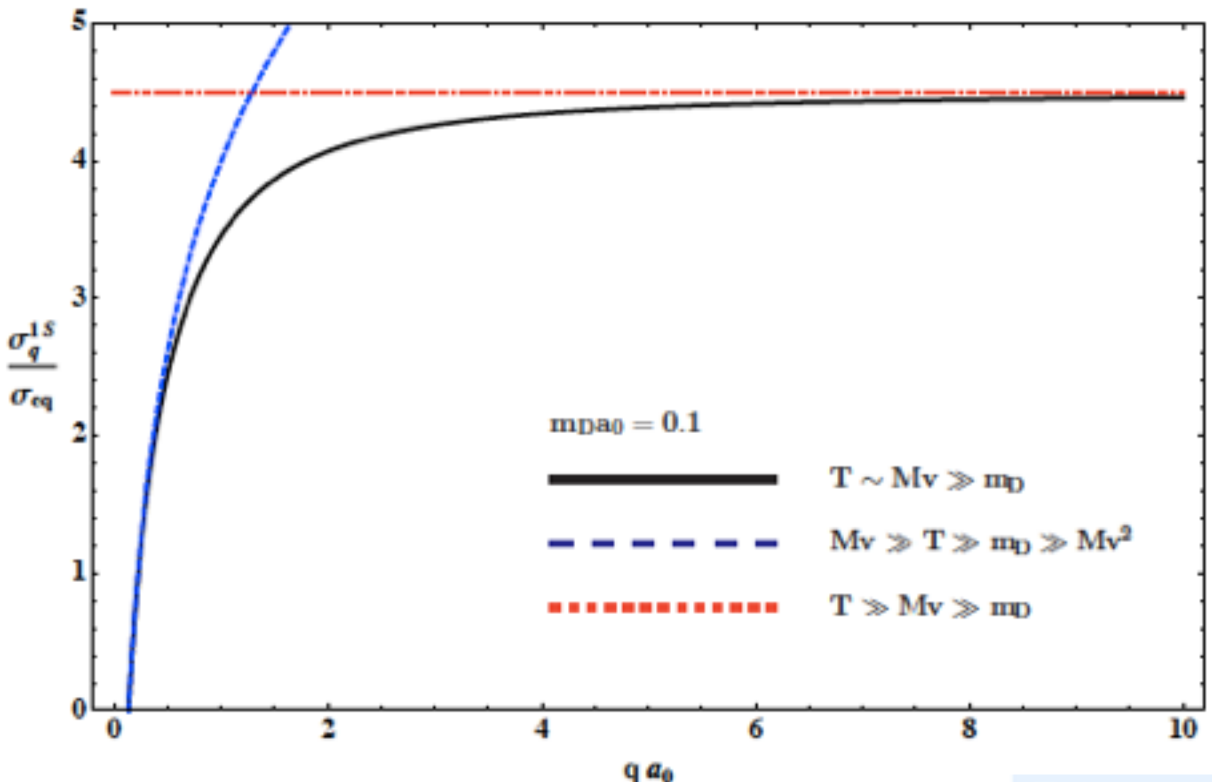
For a quarkonium at rest with respect to the medium, the thermal width has the form

$$\Gamma_{nl} = \sum_p \int_{q_{\min}} \frac{d^3q}{(2\pi)^3} f_p(q) \, [1 \pm f_p(q)] \, \sigma_p^{nl}(q)$$

where the sum runs over the different incoming light partons and $f_g = n_{\rm B}$ or $f_q = n_{\rm F}$.

- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.
- The formula differs from the one used for long in the literature, which has been inspired by the gluodissociation formula.
  - Grandchamp Rapp PLB 523 (2001)
    Park Kim Song Lee Wong PRC 76 (2007) 044907, ...

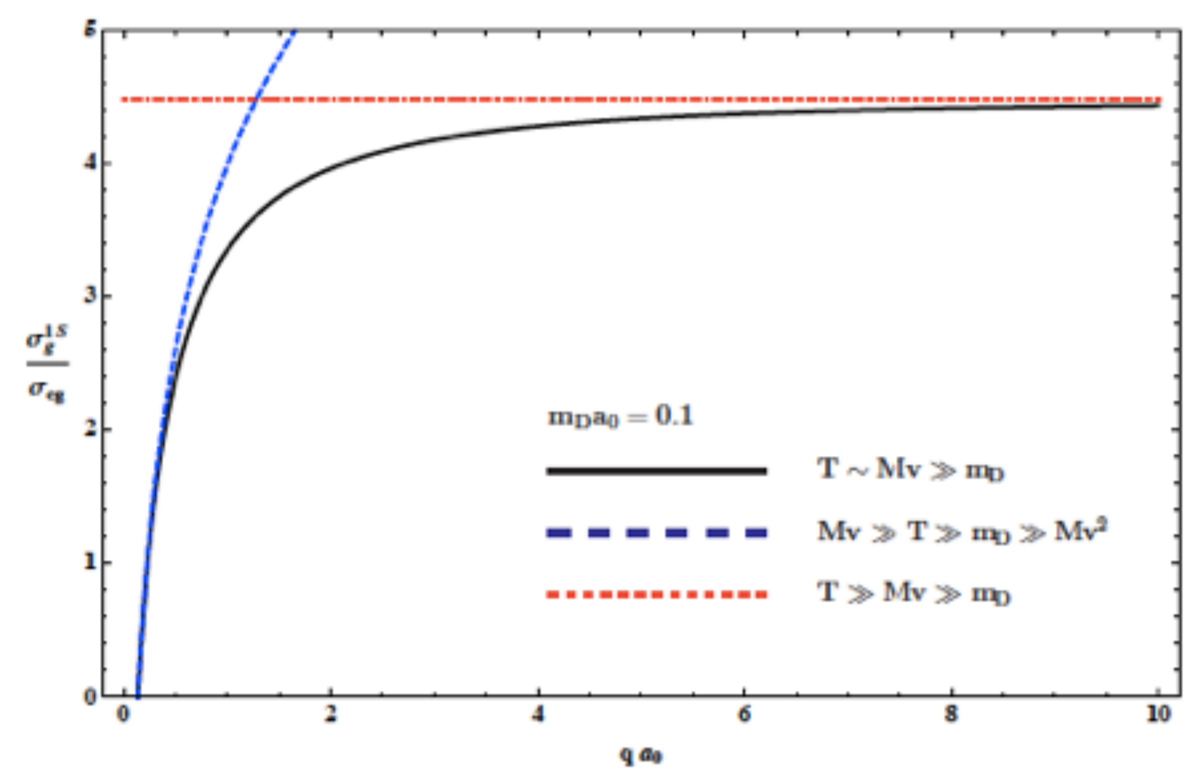
Dissociation by quark inelastic scattering



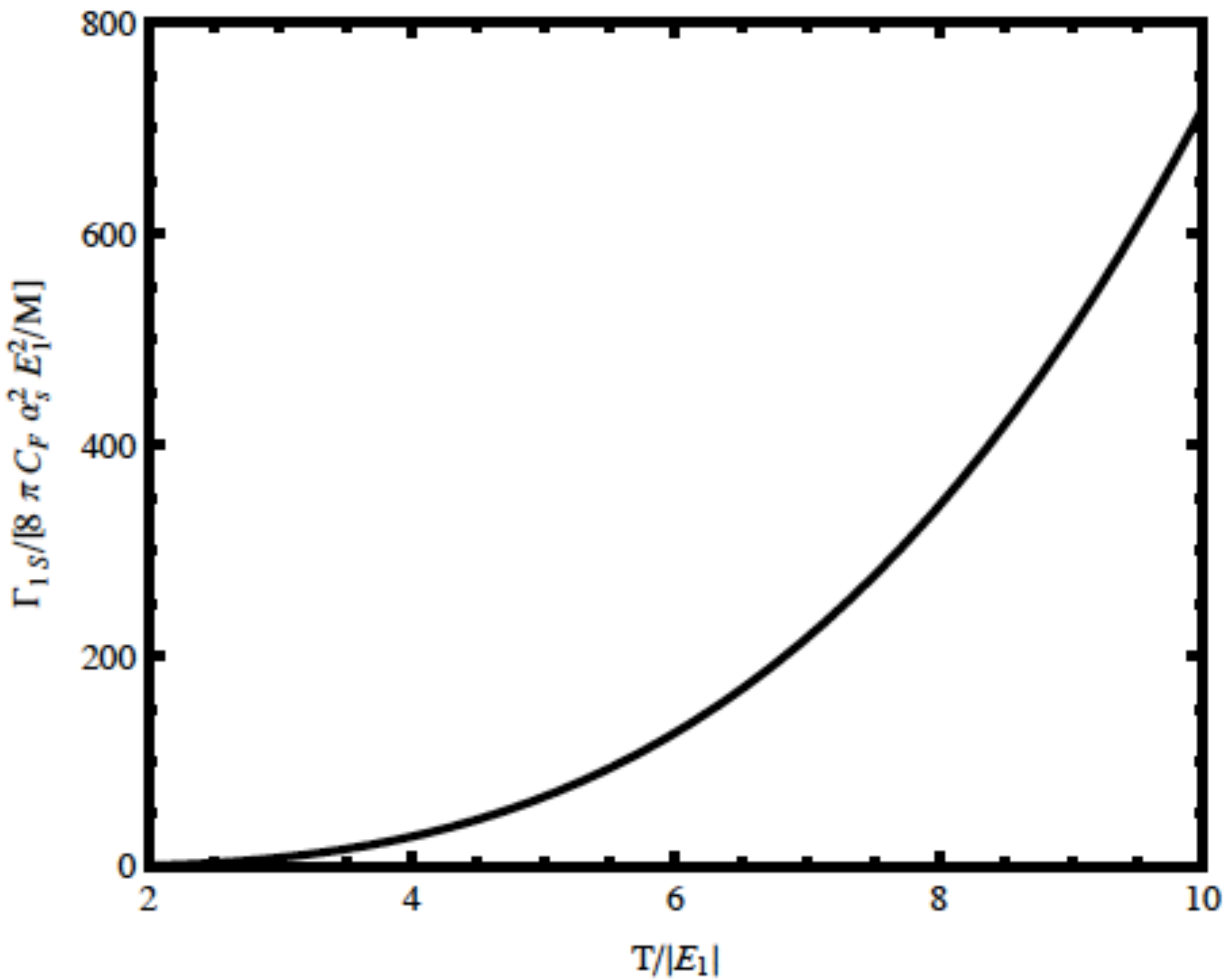
different from quasi free approximation

$a_0 = 2/(C_F M \alpha_s)$ is the Bohr radius.

Dissociation by gluon inelastic scattering



Dissociation width



$m_D a_0 = 0.5$

$|E_1|/m_D = 0.5$

$n_f = 3$

Vairo EPJ Web Conf 71 (2014) 00135

Γ1S due to inelastic parton scattering

All these are in equilibrium results

How can we describe the evolution of quarkonium in the fireball and obtain R_AA?

Quarkonium in a fireball

N.B., M. Escobedo, J. Soto. A. Vairo
1612.07248 and in preparation 017

- After the heavy-ion collisions, quarkonium propagates freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t it propagates in the medium.
- We assume the medium infinite, homogeneous, isotropic and in thermal equilibrium.
- The temperature T of the medium changes with time:

$$T = T_0 \left(\frac{t_0}{t}\right)^{v_s^2}, \quad t_0 = 0.6 \text{ fm}, \quad v_s^2 = \frac{1}{3} \text{ (sound velocity)}$$

○ Bjorken PRD 27 (1983) 140

The initial temperature T_0 may account for different centralities

centrality (%)	$\langle b \rangle$ (fm)	T_0 (MeV) @ LHC
$0-10$	3.4	471
$10-20$	6.0	461
$20-30$	7.8	449
$30-50$	9.9	425
$50-100$	13.6	304

- We assume the heavy quarks comoving with the medium.

Quarkonium as a Coulombic bound state

The lowest quarkonium states (1S bottomonium and charmonium, 2S bottomonium) are the most tightly bound. For these we can assume the hierarchy of energy scales

$$m \gg \frac{1}{a_0} \sim m\alpha_s \gg \pi T \sim m_D \gg \text{any other scale}\,.$$

This qualifies the bound state as Coulombic:

- quark-antiquark color singlet Hamiltonian $= h_s = \dfrac{\mathbf{p}^2}{M} - \dfrac{4}{3}\dfrac{\alpha_s}{r}$
- quark-antiquark color octet Hamiltonian $= h_o = \dfrac{\mathbf{p}^2}{M} + \dfrac{\alpha_s}{6r}$

The attractive factor $-4/3$ and repulsive factor $1/6$ come from the color quantum numbers. The octet potential describes an unbound quark-antiquark pair.

$$M = m$$

Quarkonium evolution equations

Quarkonium is not in equilibrium, as it can be created (in a color singlet state) or dissociated (in a color octet state) through emission of gluons. The singlet and octet density matrices can be defined in the close-time path formalism:

we want to compute $\mathrm{Tr}\{\rho(t_F)\, S^{\ell\,\dagger} S^{\ell}\}$.

where rho is the QCD density

the experimental fact that the number of b quark is much smaller than the number of light quarks imply

$$\int d^3r \int d^3r' \int d^3R \int d^3R'\, \mathrm{Tr}\{\rho\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t,\mathbf{r}',\mathbf{R}')\} \ll \mathrm{Tr}\{\rho\}$$
$$\int d^3r \int d^3r' \int d^3R \int d^3R'\, \mathrm{Tr}\{\rho\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^a(t,\mathbf{r}',\mathbf{R}')\} \ll \mathrm{Tr}\{\rho\}$$

We look at the heavy quarks as an open quantum system that interacts with the (slowly) evolving medium of the fireball made of quarks and gluons

$$\mathrm{Tr}\{\rho\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\} = \langle S_1(t',\mathbf{r}',\mathbf{R}') S_2^\dagger(t,\mathbf{r},\mathbf{R})\rangle \equiv \langle \mathbf{r}',\mathbf{R}'|\rho_s(t';t)|\mathbf{r},\mathbf{R}\rangle,,$$
$$\mathrm{Tr}\{\rho\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\} = \langle O_1^b(t',\mathbf{r}',\mathbf{R}') O_2^{a\dagger}(t,\mathbf{r},\mathbf{R})\rangle \equiv \langle \mathbf{r}',\mathbf{R}'|\rho_o(t';t)|\mathbf{r},\mathbf{R}\rangle \frac{\delta^{ab}}{N_c^2-1},$$

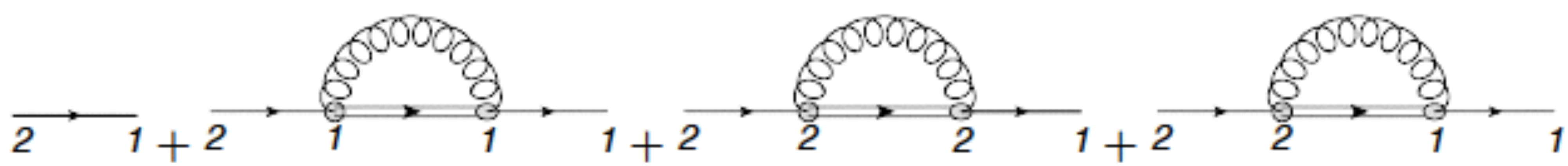
Quarkonium evolution equations

Quarkonium is not in equilibrium, as it can be created (in a color singlet state) or dissociated (in a color octet state) through emission of gluons. The singlet and octet density matrices can be defined in the close-time path formalism:

$$\mathrm{Tr}\{\rho\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\} = \langle S_1(t',\mathbf{r}',\mathbf{R}') S_2^\dagger(t,\mathbf{r},\mathbf{R})\rangle \equiv \langle \mathbf{r}',\mathbf{R}'|\rho_s(t';t)|\mathbf{r},\mathbf{R}\rangle,,$$

$$\mathrm{Tr}\{\rho\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\} = \langle O_1^b(t',\mathbf{r}',\mathbf{R}') O_2^{a\dagger}(t,\mathbf{r},\mathbf{R})\rangle \equiv \langle \mathbf{r}',\mathbf{R}'|\rho_o(t';t)|\mathbf{r},\mathbf{R}\rangle \frac{\delta^{ab}}{N_c^2-1},$$

By resumming self-energy contributions,



they satisfy the evolution equations

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s,\rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_s(\rho_o,t)\,,$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o,\rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{o1}(\rho_s,t) + \Xi_{o2}(\rho_o,t)$$

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_s(\rho_o, t)\,,$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{o1}(\rho_s, t) + \Xi_{o2}(\rho_o, t)$$

$$\Sigma_s(t) = \frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right]\,,$$

$$\Sigma_o(t) = \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right]\,,$$

$$\Xi_{so}(\rho_o, t) = \frac{1}{N_c^2-1}\, r^i\,\rho_o\, r^i\,\kappa(t)\,,$$

$$\Xi_{os}(\rho_s, t) = r^i\,\rho_s\, r^i\,\kappa(t)\,,$$

$$\Xi_{oo}(\rho_o, t) = \frac{N_c^2-4}{2(N_c^2-1)}\, r^i\,\rho_o\, r^i\,\kappa(t)\,.$$

The equations show the coupled evolution of the singlet and octet densities. Their interpretation is straightforward: the function Ξ_{so} accounts for the production (or regeneration) of singlets through the decay of octets, while the functions Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets re-

Quarkonium as an open quantum system

Quarkonium is not in equilibrium, as it can be created (in a color singlet state) or dissociated (in a color octet state) through emission of gluons. The singlet and octet density matrices are ρ_s and ρ_o. If $(1/T)\, dT/dt \ll E$ and for a strongly-coupled plasma the evolution equations can be written in the Lindblad form

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

H is an effective Hermitian Hamiltonian and C's are called collapse operators.

- Akamatsu PRD 91 (2015) 056002

STRONG-COUPLING CASE: $1/a_0 \gg T \sim m_D \gg E$

Results:
The Out of Equilibrium Evolution

Numerical solutions: QuTiP

There exist numerical toolboxes for open quantum systems. An example is QuTiP.

QuTiP
Quantum Toolbox in Python

- Johansson, Nation, Nori, CPC 183 (2012) 1760, 184 (2013) 1234

Huge matrix. We can simplify it by making an expansion in spherical armonics and cutting at some point.

- If the initial condition is diagonal in the spherical harmonics space it will remain always so.
- Including up to p-wave gives a good result if you are interested in s-wave. It does not change the results a lot to include also the d-wave.
- Similar arguments apply to f_o in color space. $f_o^{ab} \propto \delta^{ab}$.

Initial conditions

The production of singlets is α_s suppressed compared to that of octets.

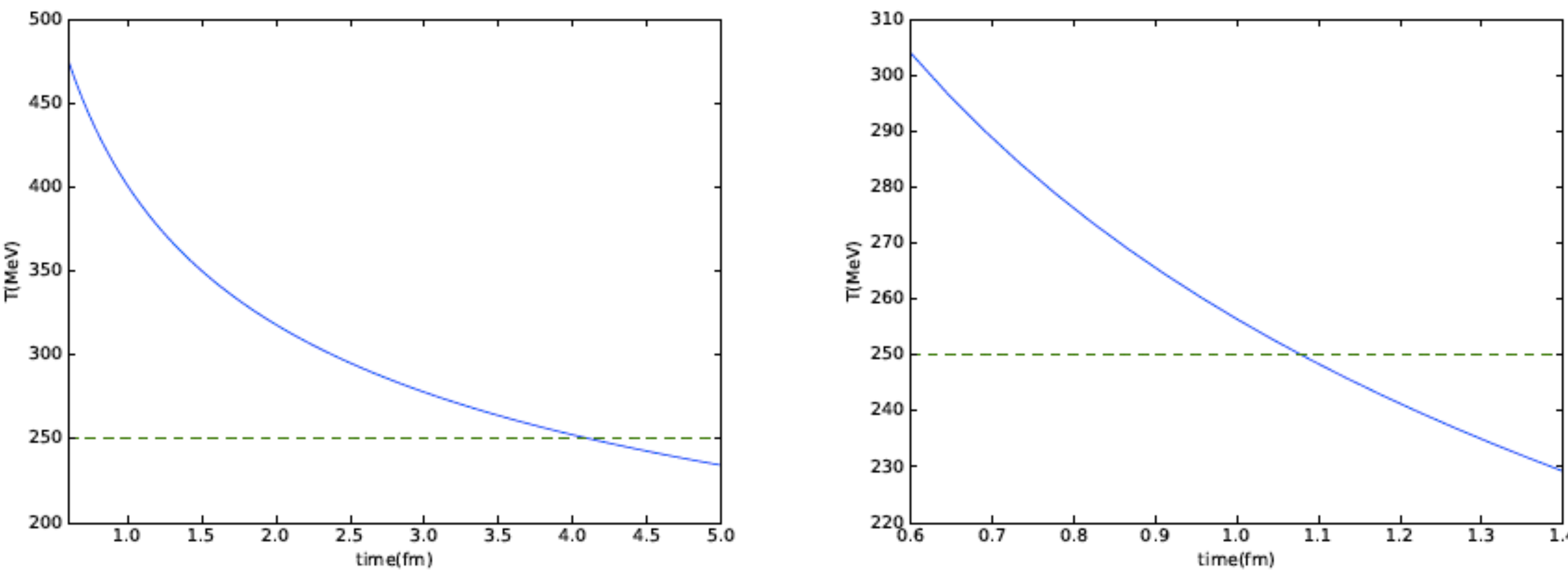
- Cho Leibovich PRD 53 (1996) 6203

Our choice at $t = 0$ is

$$\rho_s = A|\mathbf{0}\rangle\langle\mathbf{0}|, \qquad \rho_o = \frac{\delta}{\alpha_s(M_b)}\rho_s$$

A is fixed by $\mathrm{Tr}(\rho_s) + \mathrm{Tr}(\rho_o) = 1$

δ fixes the octet fraction with respect to the singlet: $\delta = 1$,

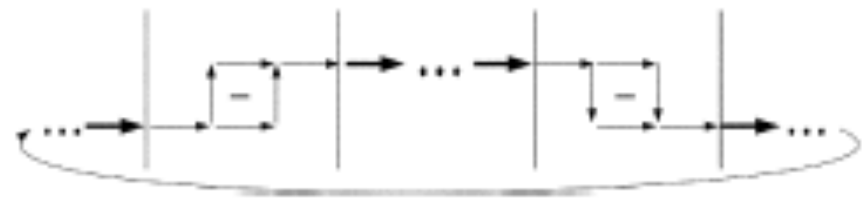


. Time evolution of the temperature according to (18) for the most central (left) and the most peripheral (right) collisions

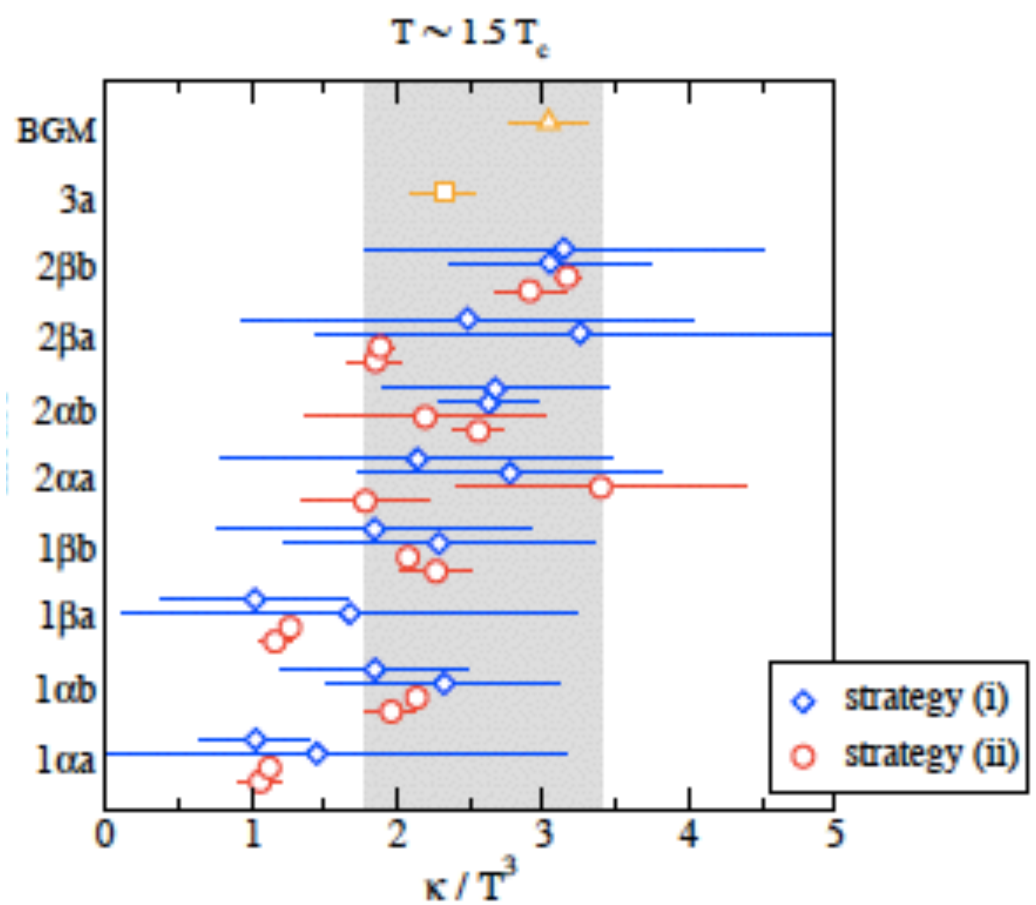
Lattice QCD and low-energy coefficients

Low energy parameters may be determined by numerical calculations in lattice QCD.
κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{6\,N_c}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\,\langle \mathrm{T}\, E^{a,i}(s,0)E^{a,i}(0,0)\rangle =$$



$$1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4\,.$$



- Francis Kaczmarek Laine Neuhaus Ohno, PRD 92 (2015) 116003

Lattice QCD and low-energy coefficients

Low energy parameters may be determined by numerical calculations in lattice QCD.

$$\gamma = \frac{g^2}{6\,N_c}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\,\langle \mathrm{T}\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

γ is known only in perturbation theory.

$$\gamma = -2\zeta(3)\,C_F\left(\frac{4}{3}N_c + n_f\right)\alpha_s^2\,T^3\,,$$

Dilepton suppression rate

We compute the dilepton suppression rate R_{AA}:

$$R_{AA} \sim \frac{\rho_S|^{AA}_{1S\,1S}}{\rho_S|^{pp}_{1S\,1S}}$$

non equilibrium modification of the equilibrium formula of L. McLerran, T. Toimela 1985

Bottomonium nuclear modification factor

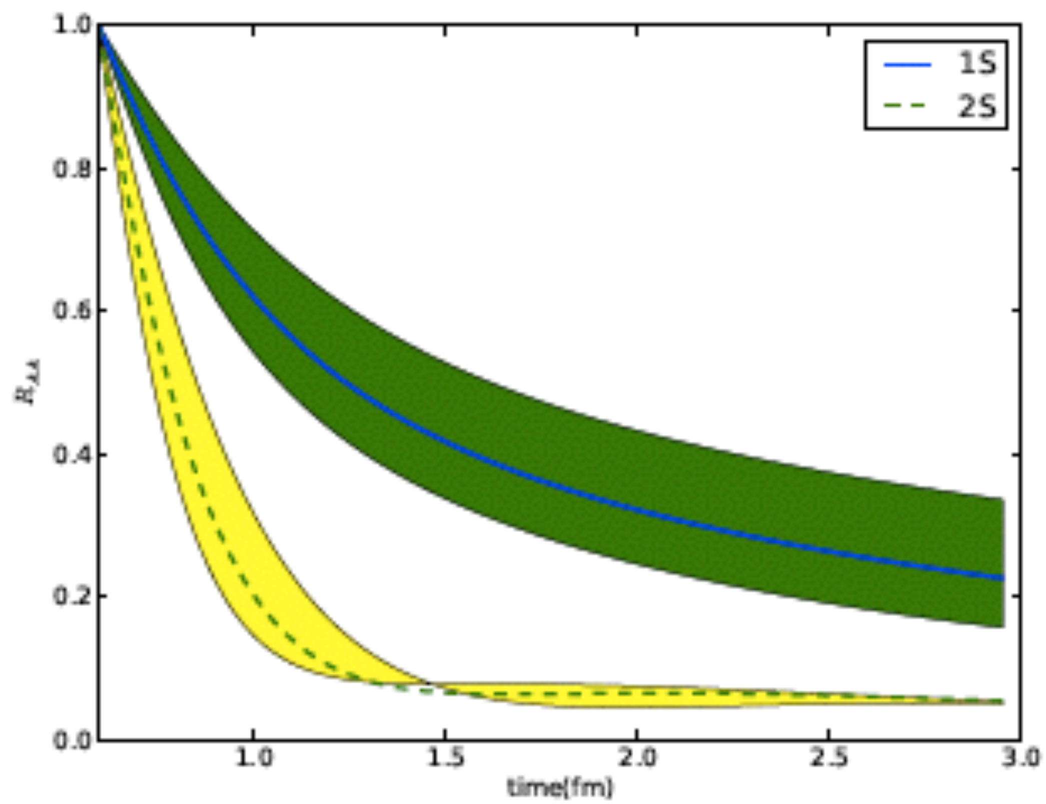
30-50% centrality		50-100% centrality	
$R_{AA}(1S)$	$\frac{R_{AA}(2S)}{R_{AA}(1S)}$	$R_{AA}(1S)$	$\frac{R_{AA}(2S)}{R_{AA}(1S)}$
$0.23^{+0.10}_{-0.07}$	0.24 ± 0.09	0.80 ± 0.05	0.59 ± 0.10

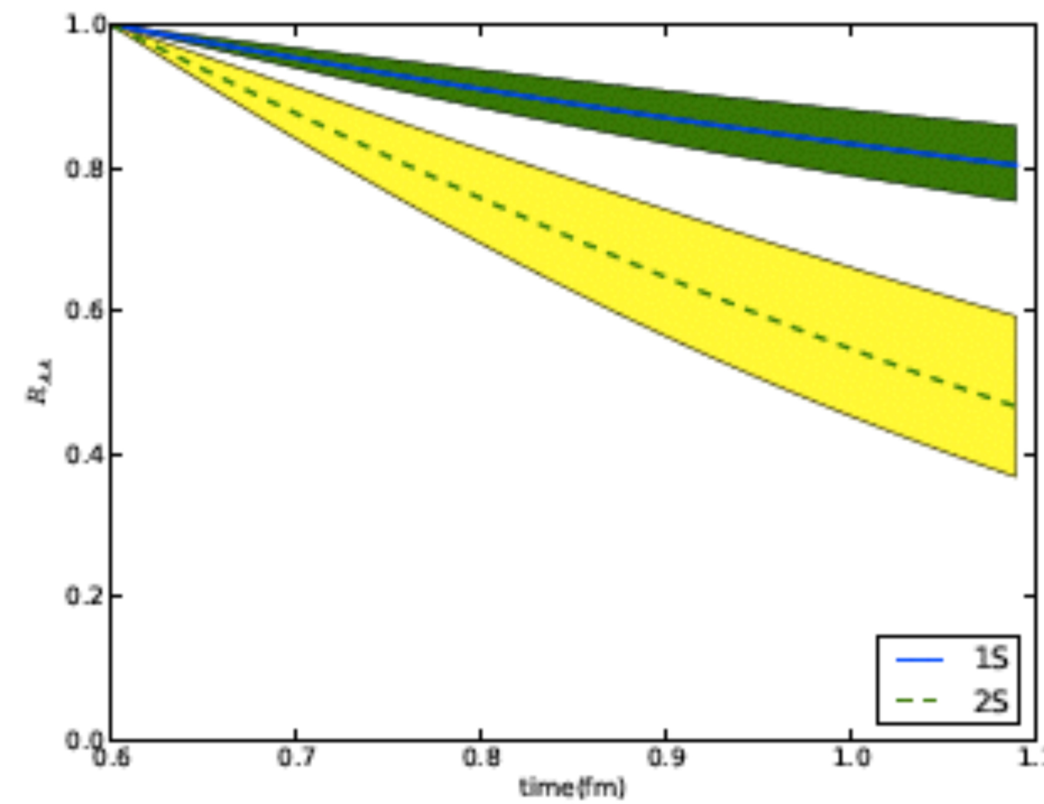
Results for $R_{AA}(1S)$ and $R_{AA}(2S)$ for κ/T^3 in

$$1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4\,.$$

$\gamma = 0$ and $\delta = 1$ in the bottomonium case.

Time evolution of R_{AA}

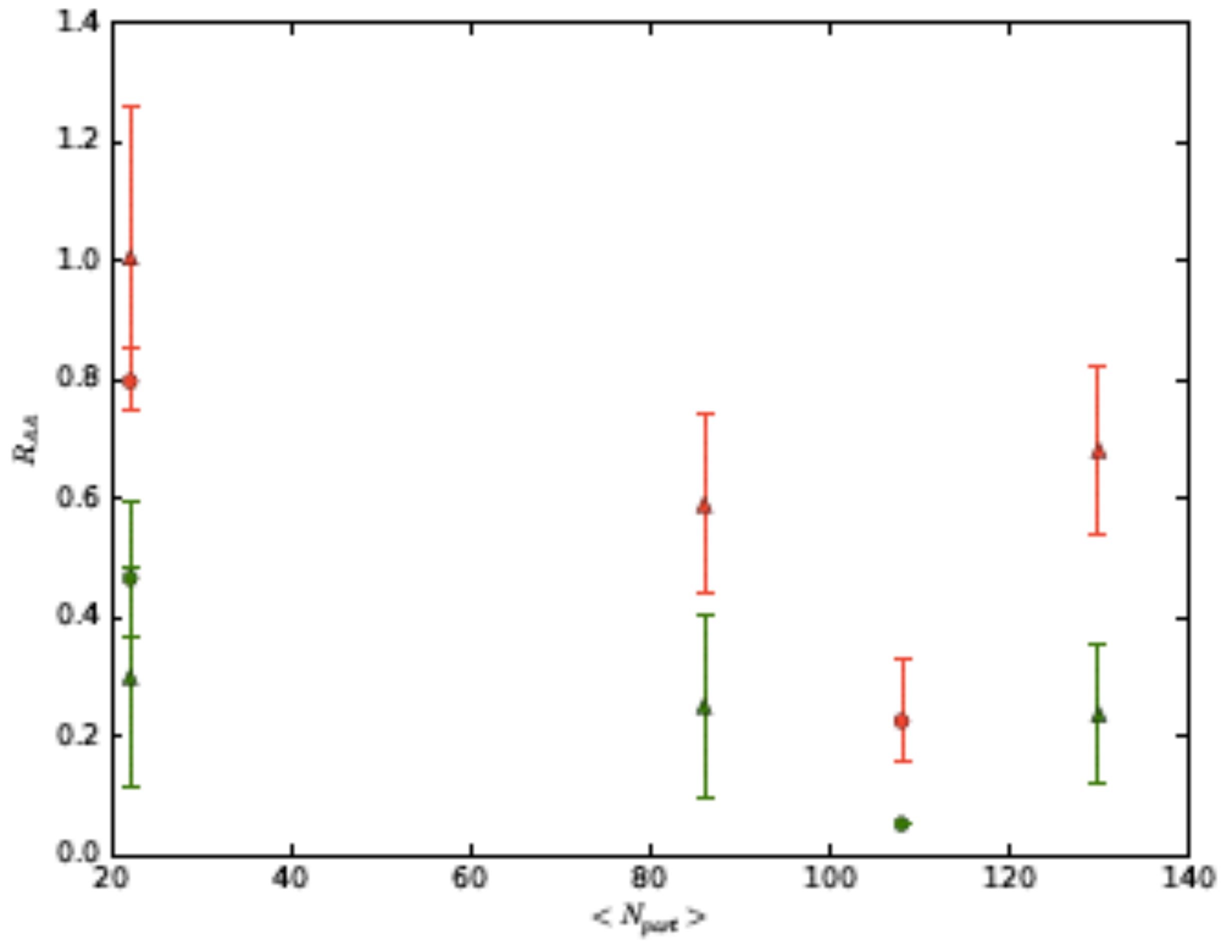




$30 - 50\%$ centrality (left) and $50 - 100\%$ centrality (right)

$\gamma = 0$ and $\delta = 1$

Bottomonium nuclear modification factor vs CMS data



○: theory results; △: CMS data; red: $\Upsilon(1S)$; green: $\Upsilon(2S)$.

○ CMS coll. PRL 109 (2012) 222301

Brambilla Escobedo Soto Vairo, arXiv:1612.07248

Conclusions

We have shown a realistic particle physics example where a complex full system made out of a multiscale subsystem (quarkonium) interacting with a rich and inherently non-perturbative environment (the quark-gluon plasma) could be studied in its out-of-equilibrium evolution with the methods of

- effective field theories, to factorize contributions coming from different energy scales. Contributions coming from high-energy scales (mass, ...) can be computed in perturbation theory.
- lattice QCD, to compute numerically on a space-time lattice low-energy non-perturbative contributions.
- open quantum systems, to compute the out-of-equilibrium evolution of the subsystem and its non-trivial interaction with the environment (production, dissociation and recombination of quarkonium).

As a result the study describes for the first time quarkonium dissociation taking into account the conservation of the total number of heavy quarks, the non-Abelian nature of QCD, without any classical approximation.

- Quarkonium suppression may be systematically studied with the use of **effective field theories** and **lattice QCD**
- **In equilibrium** properties like **dissociation width, cross section, mass shift…** have been computed as expansions in the small parameters of the system.
- **Out of equilibrium** properties, like octet recombination, can be studied by treating quarkonium as an open quantum system. Lattice input is crucial.

Outlook

—assumed that medium is infinite homogeneous and isotropic in space but changes in time: appropriate for large nuclei in central collisions, used Bjorken evolution—> can improve on this using hydrodynamics

—we have used a particular initial condition—> initial conditions can be tuned to account for pre-equilibrium states like Plasma

—we work at linear order in the density expansion—> go to next order (not relevant for bottomonium but can be for charmonium)

—more info on kappa and gamma needed

—we are now exploring other hierarchies and weakly coupled plasma

QCD and strongly coupled gauge theories: challenges and perspective

N. Brambilla*†,[1] S. Eidelman†,[2,3] P. Foka†‡,[4] S. Gardner†‡,[5] A.S. Kronfeld†,[6]
M.G. Alford‡,[7] R. Alkofer‡,[8] M. Butenschön‡,[9] T.D. Cohen‡,[10] J. Erdmenger‡,[11] L. Fabbietti‡,[12]
M. Faber‡,[13] J.L. Goity‡,[14,15] B. Ketzer‡§,[1] H.W. Lin‡,[16] F.J. Llanes-Estrada‡,[17]
H.B. Meyer‡,[18] P. Pakhlov‡,[19,20] E. Pallante‡,[21] M.I. Polikarpov‡,[19,20] H. Sazdjian‡,[22]
A. Schmitt‡,[23] W.M. Snow‡,[24] A. Vairo‡,[1] R. Vogt‡,[25,26] A. Vuorinen‡,[27] H. Wittig‡,[18]
P. Arnold,[28] P. Christakoglou,[29] P. Di Nezza,[30] Z. Fodor,[31,32,33] X. Garcia i Tormo,[34] R. Höllwieser,[13]
M.A. Janik,[35] A. Kalweit,[36] D. Keane,[37] E. Kiritsis,[38,39,40] A. Mischke,[41] R. Mizuk,[19,42]
G. Odyniec,[43] K. Papadodimas,[21] A. Pich,[44] R. Pittau,[45] J.-W. Qiu,[46,47] G. Ricciardi,[48,49]
C.A. Salgado,[50] K. Schwenzer,[7] N.G. Stefanis,[51] G.M. von Hippel,[18] and V.I. Zakharov[11,19]

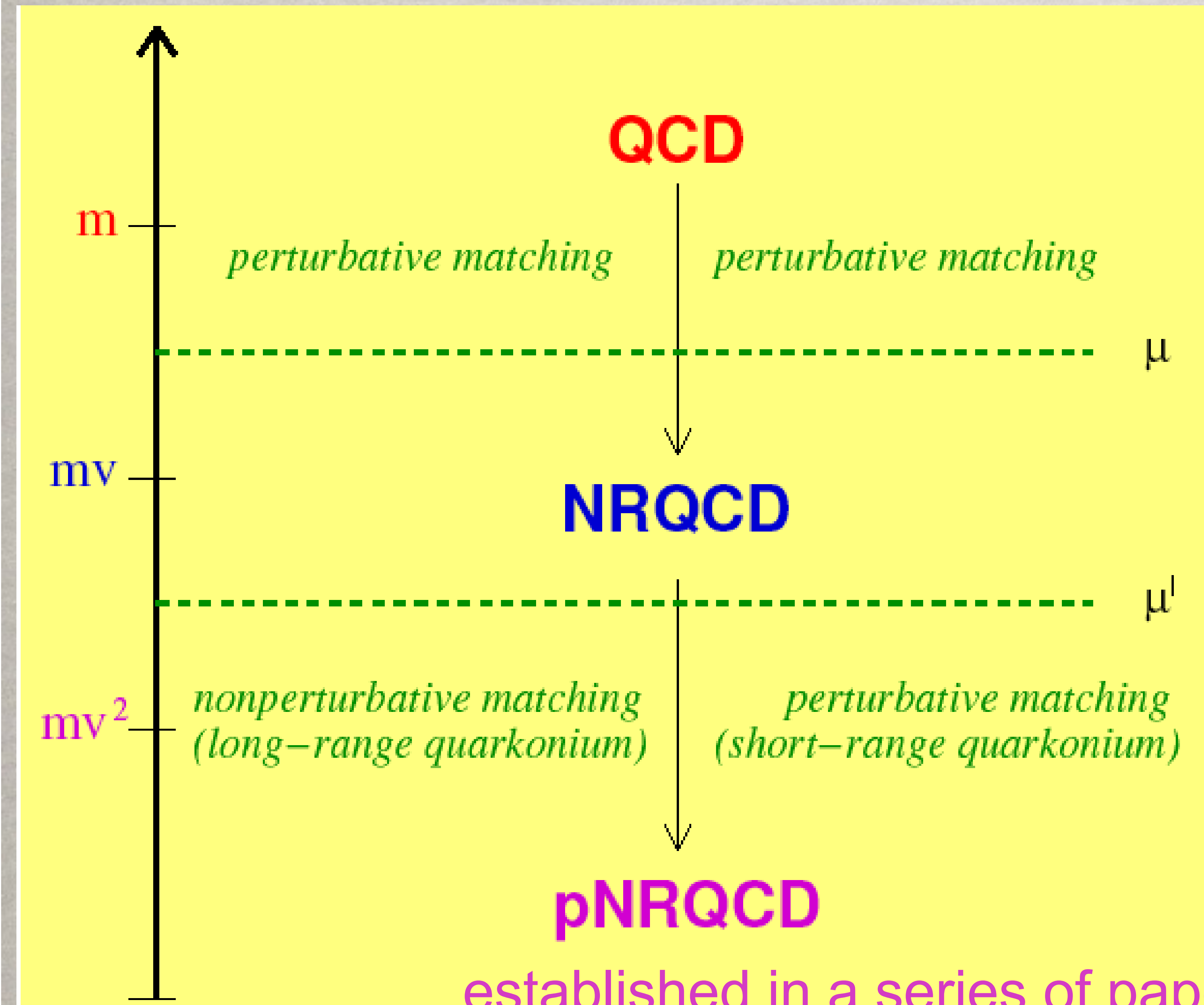
arXiv:1404.3723v1 [hep-ph] 14 Apr 2014

Heavy quarkonium: progress, puzzles, and opportunities

N. Brambilla∗†,1 S. Eidelman∗†,2, 3 B. K. Heltsley∗†‡,4 R. Vogt∗†,5, 6 G. T. Bodwin†,7 E. Eichten†,8 A. D. Frawley†,9 A. B. Meyer†,10 R. E. Mitchell†,11 V. Papadimitriou†,8 P. Petreczky†,12 A. A. Petrov†,13 P. Robbe†,14 A. Vairo†,1 A. Andronic,15 Arnaldi,16 P. Artoisenet,17 G. Bali,18 A. Bertolin,19 D. Bettoni,20 J. Brodzicka,21 G. E. Bruno,22 A. Caldwell,23 J. Catmore,2 C.-H. Chang,25, 26 K.-T. Chao,27 E. Chudakov,28 P. Cortese,16 P. Crochet,29 A. Drutskoy,30 U. Ellwanger,31 P. Faccioli,32 Gabareen Mokhtar,33 X. Garcia i Tormo,34 C. Hanhart,35 F. A. Harris,36 D. M. Kaplan,37 S. R. Klein,38 H. Kowalski,10 J.-P. Lansberg,39, 40 E. Levichev,2 V. Lombardo,41 C. Lourenc¸o,42 F. Maltoni,43 A. Mocsy,44 R. Mussa,16 F. S. Navarra,45 M. Negrini,20 M. Nielsen,45 S. L. Olsen,46 P. Pakhlov,47 G. Pakhlova,47 K. Peters,15 A. D. Polosa,48 W. Qian,49, 14 J.-W. Qiu 12, 50 G. Rong,51 M. A. Sanchis-Lozano,52 E. Scomparin,16 P. Senger,15 F. Simon,23, 53 S. Stracka,41, 54 Y. Sumino,55 M Voloshin,56 C. Weiss,28 H. K. W¨ohri,32 and C.-Z. Yuan51

arXiv.org/abs/arXiv:1010.5827

Quarkonium with EFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

established in a series of papers:
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N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423