Heavy quark bound states in a quark-gluon plasma

Precision Spectroscopy of QGP Properties with Jets and Heavy Quarks

ITP, Seattle

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A very nice idea….

The charmonium as a « non relativistic » system

$$H = 2m_c + \frac{p_1^2}{2m_c} + \frac{p_2^2}{2m_c} + V(r)$$

$$V(r) = -\frac{\alpha}{r} + \sigma r$$

Screening of binding forces in a quark-gluon plasma

Screened potential

$$V(r) = -\frac{\alpha}{r} e^{-r/r_D(T)}$$

Bound state exists for

$$r_D(T) > r_D^{min}$$

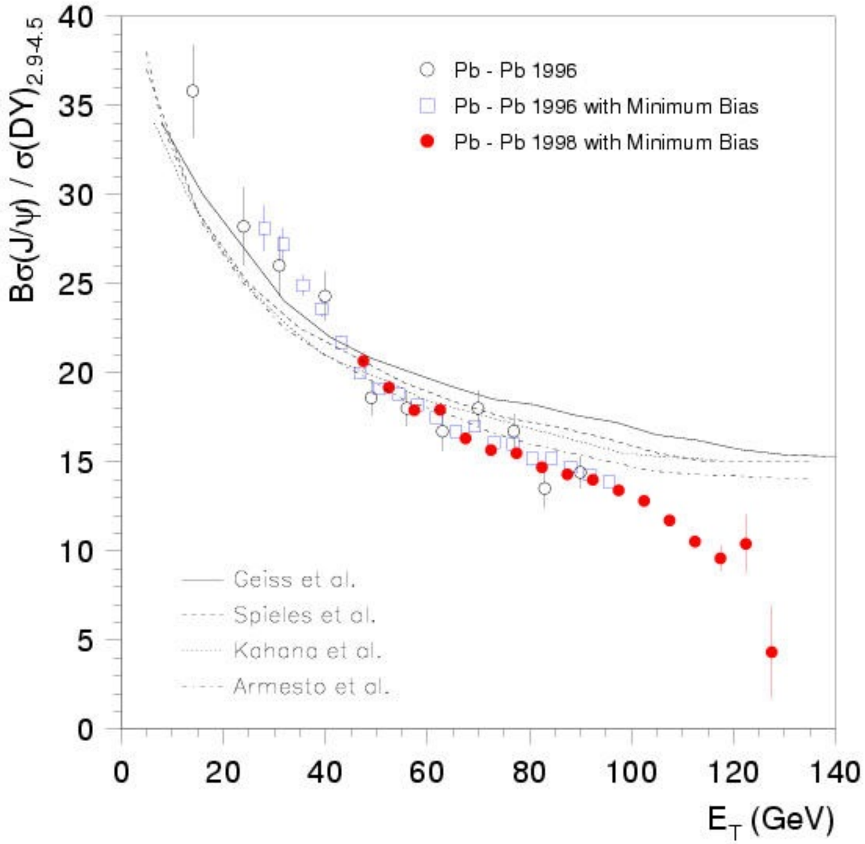
that is, for

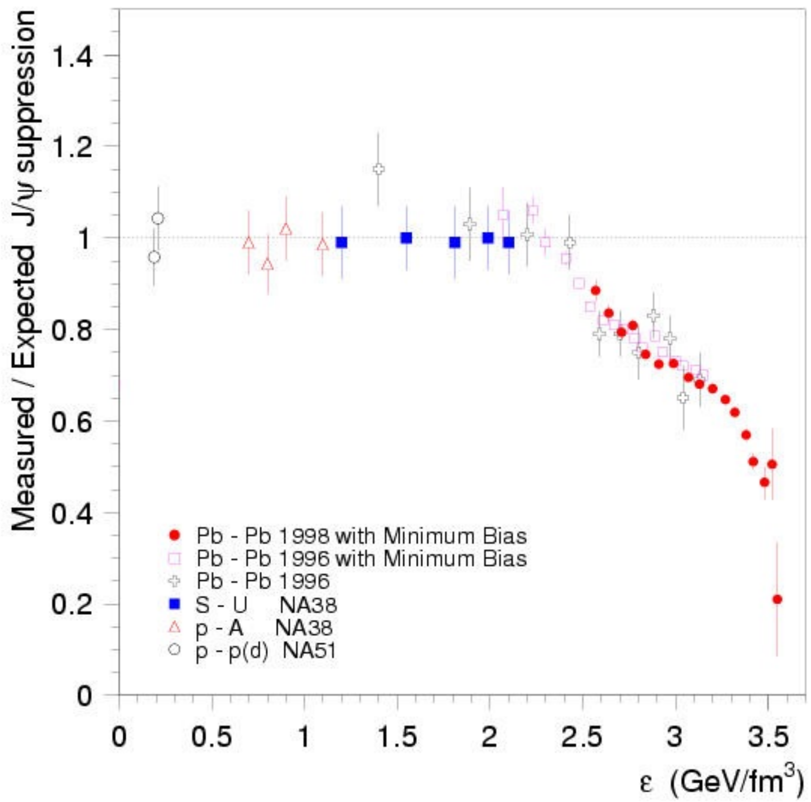
$$T < T_D$$

A considerable experimental effort ....

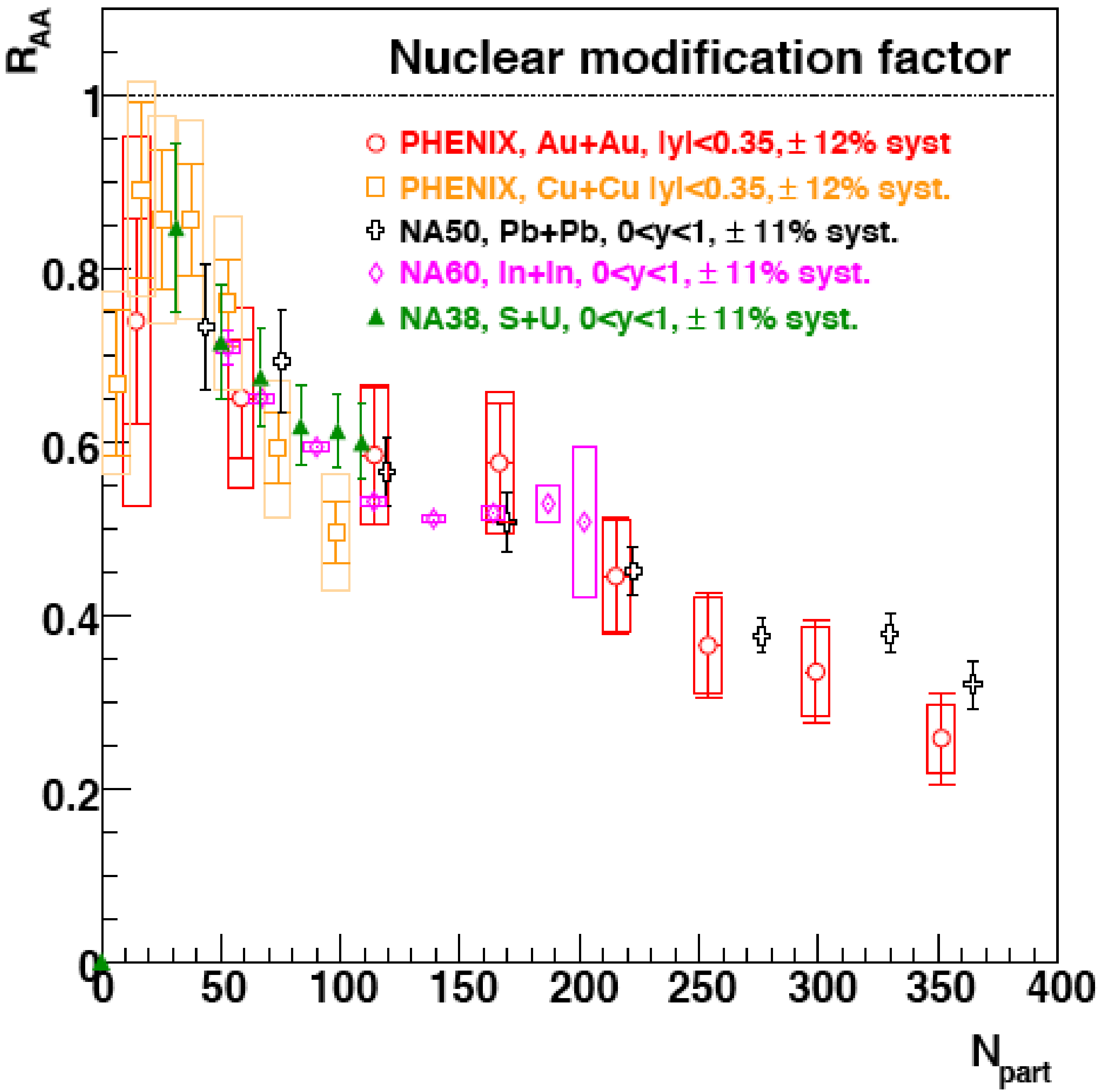
Summary of early measurements (NA38,NA50)

(CERN, 2000)

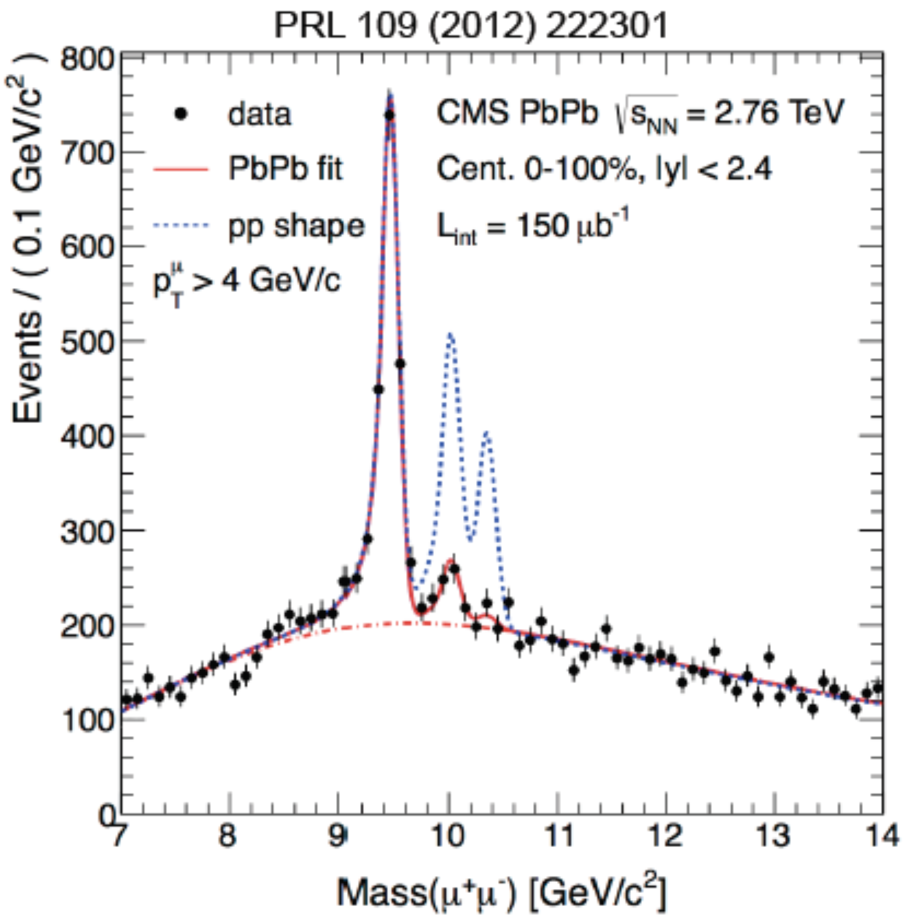


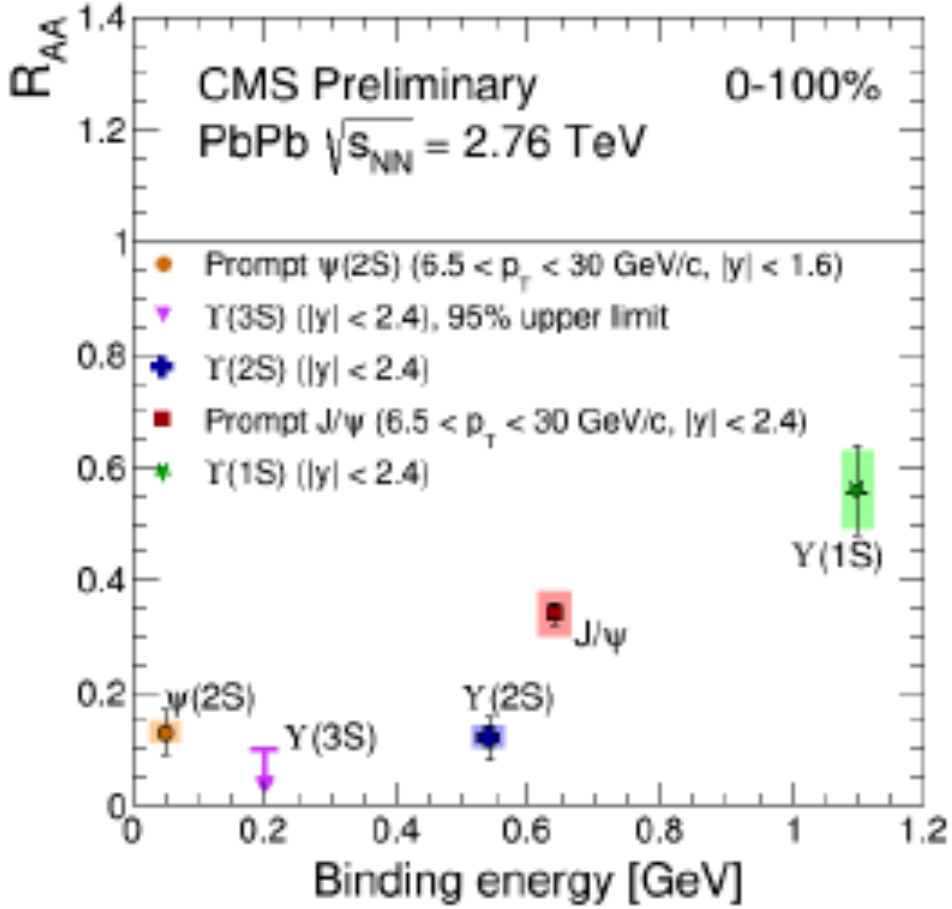


What about RHIC ?



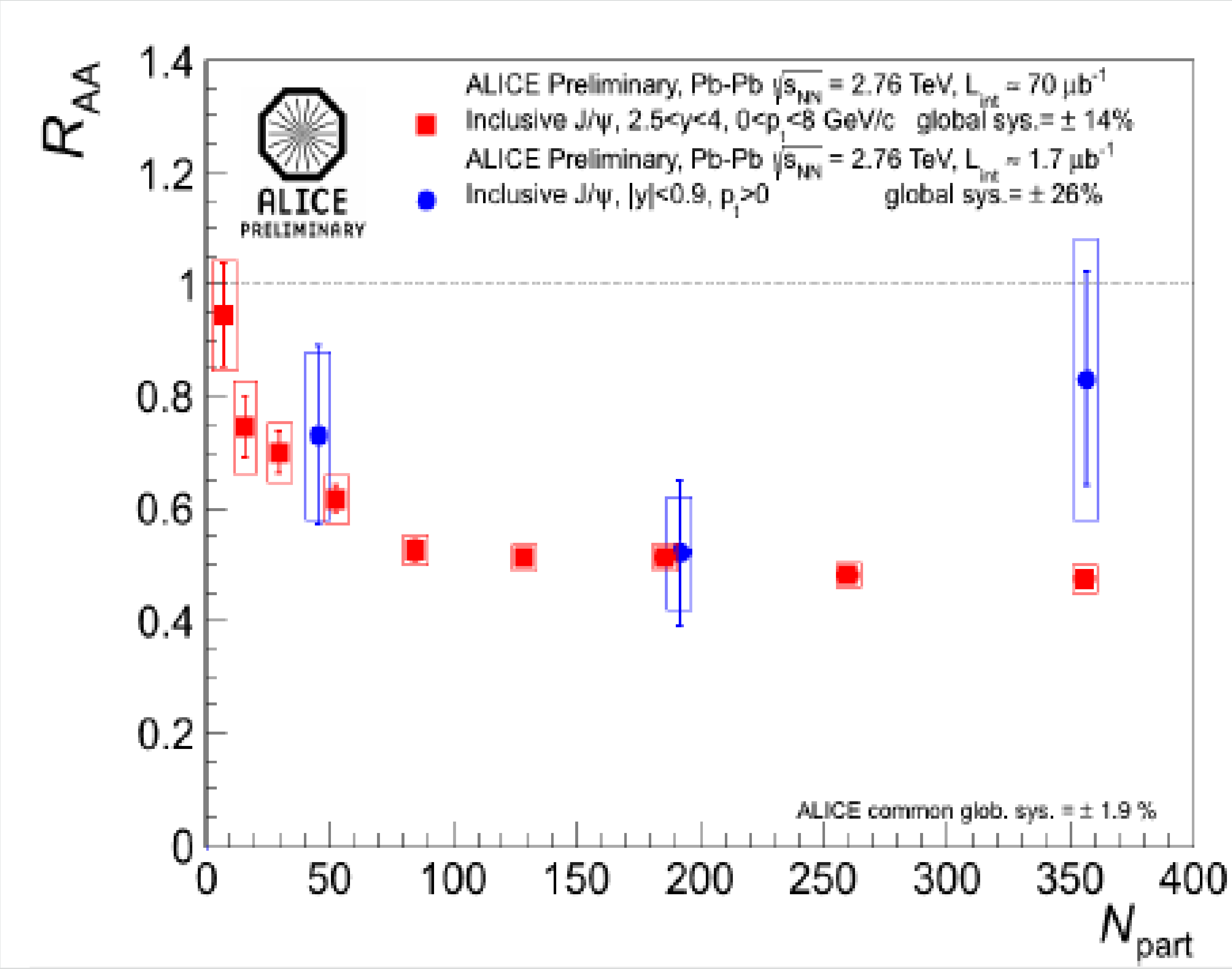
Υ suppression





excited states are more 'fragile'....

findings in line with expectations....


ALICE Preliminary, Pb-Pb √sNN = 2.76 TeV, Lint = 70 μb-1
Inclusive J/ψ, 2.5<y<4, 0<pt<8 GeV/c global sys.= ± 14%
ALICE Preliminary, Pb-Pb √sNN = 2.76 TeV, Lint ≈ 1.7 μb-1
Inclusive J/ψ, |y|<0.9, pt>0 global sys.= ± 26%
ALICE
PRELIMINARY
ALICE common glob. sys. = ± 1.9 %
RAA
Npart
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
50
100
150
200
250
300
350
400

A very nice idea....

a considerable experimental effort

but a very difficult many-body problem !

a large variety of theoretical approaches

- -potential models
- -spectral functions
- -Euclidean correlators (lattice), maximum entropy techniques
- -coupled channels
- -path integrals
- -open quantum systems
- -effective field theory, non relativistic heavy quark effective theory
- -strong coupling techniques
- -etc

Which problem do we need to solve ?

- -full dynamics, including plasma expansion
- -dynamics of bound state formation (stationary states are not enough)
- -dynamics of dissociation and recombination

WORK IN PROGRESS !

Results presented are based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]

A. Beraudo, JPB, P. Faccioli and G. Garberoglio [arXiv: 1005.1245]

JPB, D. de Boni, P. Faccioli and G. Garberoglio, Nucl.Phys. A946 (2016) 49-88 [arXiv: 15003.03857]

JPB, M. Escobedo-Espinosa, in preparation

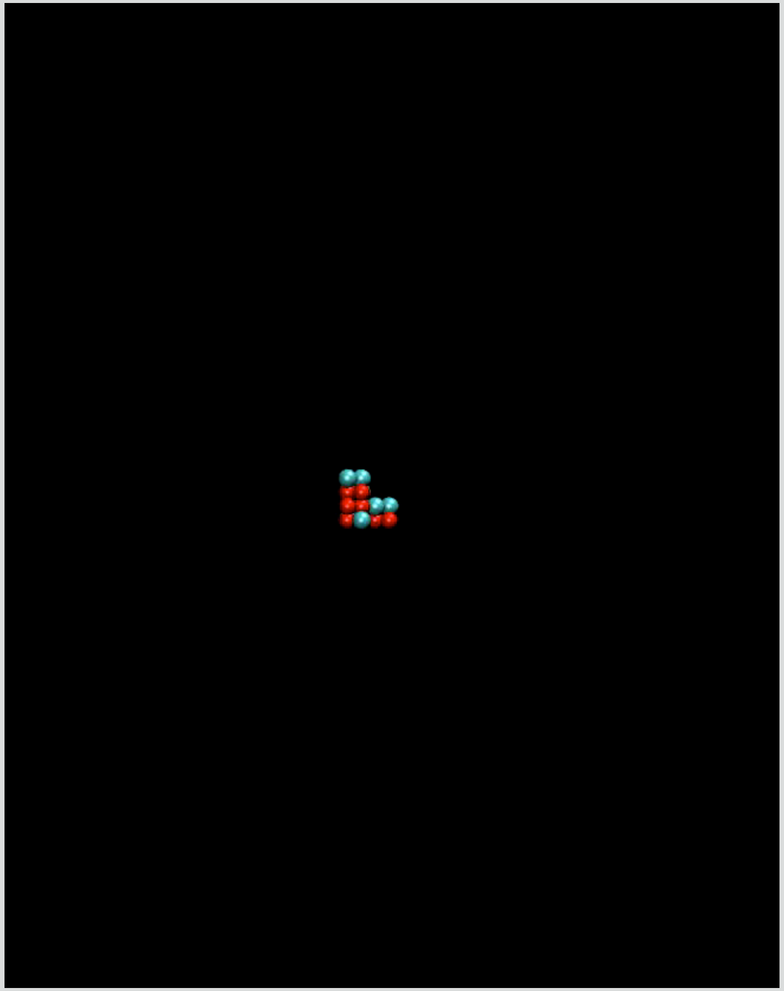
Outline

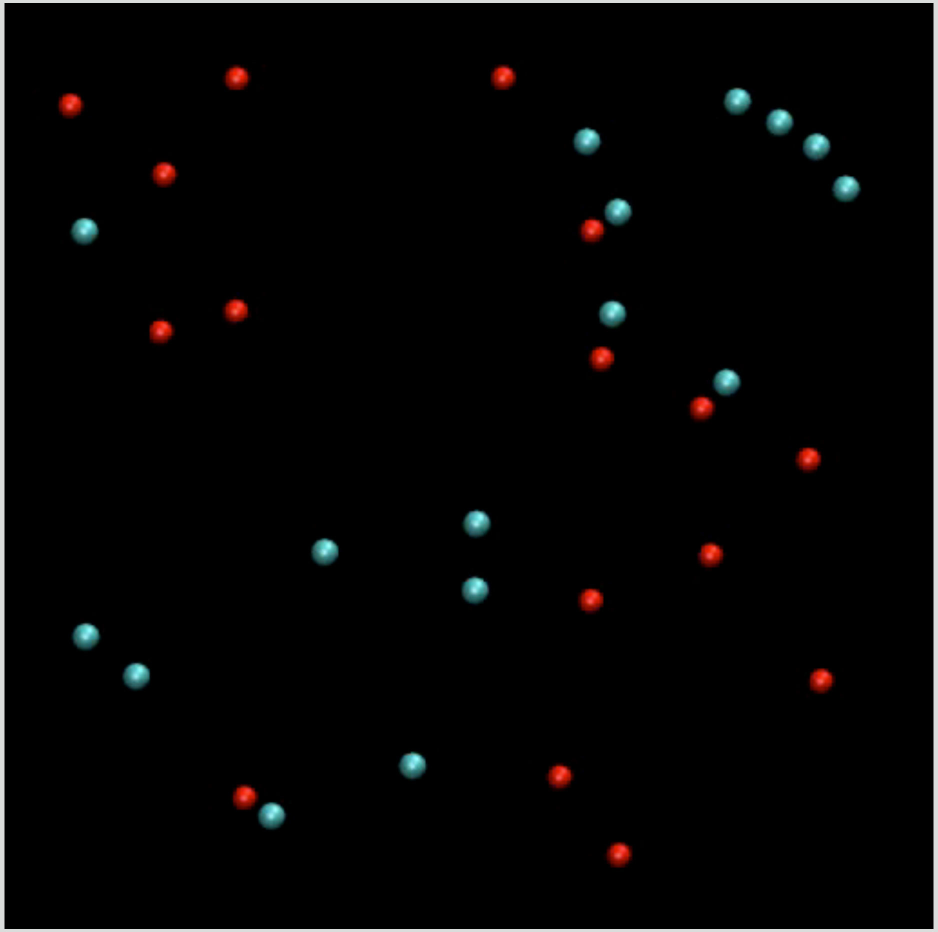
Basic concepts

influence functional,
complex potential,
etc, (QED)

Some numerical results (QED)

QCD: new features





Dynamics

$$H = H_Q + H_{med} + H_{int}$$

Heavy quark

$$H_Q = M \int d^3\boldsymbol{r}\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}) + \int d^3\boldsymbol{r}\, \psi^\dagger(\boldsymbol{r}) \left(-\frac{\nabla^2}{2M}\right) \psi(\boldsymbol{r})$$

linearly coupled to gauge field

$$H_{int} = g \int d^3\boldsymbol{r}\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}) A_0(\boldsymbol{r})$$

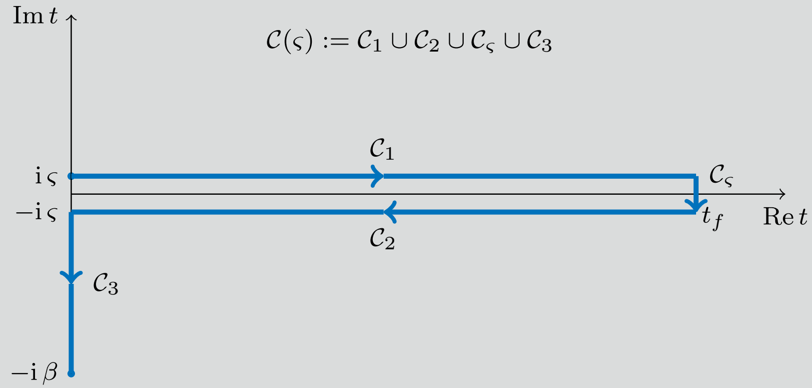
The hot plasma

$$H_{med} = \int d^3r\, \xi^\dagger(\boldsymbol{r}) h_0\, \xi(\boldsymbol{r}) + \frac{1}{2} \int d^3r d^3r' \hat{\rho}(\boldsymbol{r}) \frac{g^2}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|} \hat{\rho}(\boldsymbol{r}')$$

Path integral formulation

$$(Q_f, t_f | Q_i t_i) = \int_{x(t_i)=Q_i}^{x(t_f)=Q_f} [\mathcal{D}x(t)] \exp\left[i \int_{t_i}^{t_f} dt \left(\frac{1}{2} M \dot{x}^2 - V(x) \right) \right]$$

$$P(\boldsymbol{Q}_f, t_f | \boldsymbol{Q}_i, t_i) = \left| (\boldsymbol{Q}_f, t_f | \boldsymbol{Q}_i, t_i) \right|^2$$



$$P(Q_f, t_f | Q_i t_i) = \int_C [\mathcal{D}x(t)] \exp\left[i \int_C dt_C \left(\frac{1}{2} M \dot{x}^2 - V(x) \right) \right]$$

$$V(x) = g A_0(x)$$

Path integral and influence functional

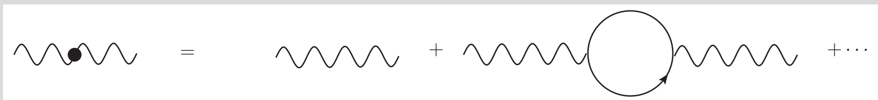
$$P(Q_f, t_f | Q_i, t_i) = \int_C DQ \, e^{iS_0[Q]} \, e^{i\Phi[Q]}$$

$$e^{i\Phi[\boldsymbol{Q}]} = \int \boldsymbol{D} A_0 \, e^{-i \int_C d^4x \, g\rho(x) A_0(x)} e^{iS_2[A_0]}$$

$$\rho(x) = \sum_{j=1}^{N} (\delta(\boldsymbol{x} - \boldsymbol{q}_j(t) - \delta(\boldsymbol{x} - \bar{\boldsymbol{q}}_j(t))$$

'Integrate out' the light particles and keep the quadratic part of the resulting action (HTL approximation)

$$S_2[A_0] \;=\; -\frac{1}{2} \int_c dx \, (\, A_0(x) \nabla^2 A_0(x) \,) - i \; \mathrm{Tr} \; \ln \left[\, i\gamma^\mu \partial_\mu - m - e\gamma^0 A_0(x) \, \right]$$



$$\Phi[\boldsymbol{Q}] = \frac{g^2}{2} \iint_C d^4x d^4y \; \rho(x) \Delta_c(x - y) \rho(y)$$

$$\Delta(x - y) \equiv i \langle \, T_C \left[A_0(x) A_0(y) \, \right] \rangle$$

Infinite mass limit

(single heavy quark)

$$G^{>}(t,\boldsymbol{r}) = \delta(\boldsymbol{r})\,\mathrm{e}^{-iMt}\,\mathrm{e}^{iF(t)} \qquad F(t) = \frac{g^2}{2}\int_0^t dt' \int_0^t dt'' D(t'-t'',0)$$

long time limit is determined by static response of plasma

$$F(t) \simeq \frac{g^2}{2} t D(\omega=0, \boldsymbol{r}=0) \equiv -tV_{opt}$$

'Optical potential'

$$\begin{aligned} V_{\rm opt} &\equiv -\frac{g^2}{2}\int \frac{d\boldsymbol{q}}{(2\pi)^3} D(\omega=0,\boldsymbol{q}) \\ &= \frac{g^2}{2}\int \frac{d\boldsymbol{q}}{(2\pi)^3}\Big[\frac{1}{\boldsymbol{q}^2+m_D^2} - \frac{1}{\boldsymbol{q}^2} - i\frac{\pi m_D^2 T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}\Big] \\ &= -\frac{\alpha}{2} m_D - i\frac{\alpha T}{2}, \end{aligned}$$

Quark antiquark pair

Large time behaviour $(t\, m_D \gg 1)$

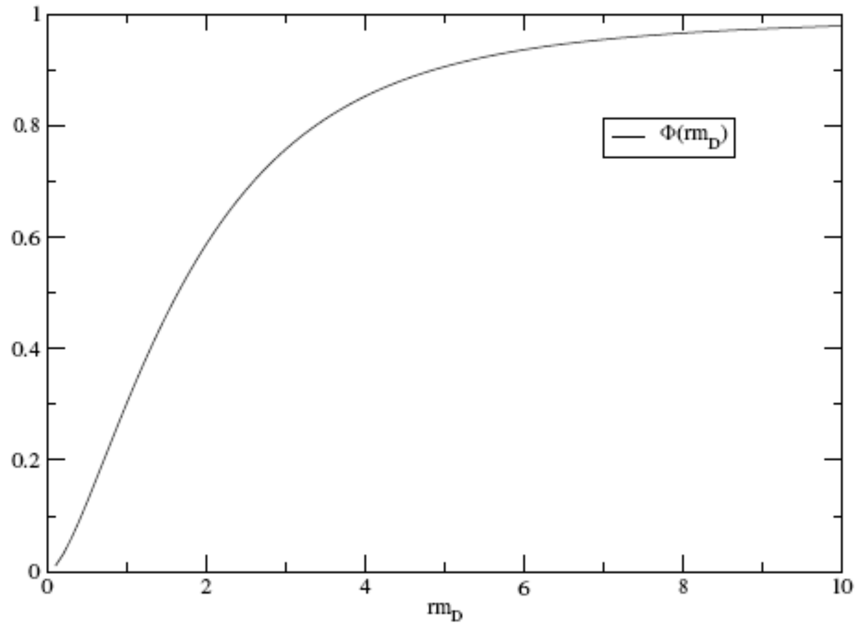
$$\overline{G}(t, r_1 - r_2) \underset{t\to\infty}{\sim} \exp[-iV_{\text{eff}}(r_1 - r_2)t]$$

V_{eff} has real and imaginary part (*)

$$\begin{aligned} V_{\text{eff}}(r_1 - r_2) &\equiv g^2 \int \frac{dq}{(2\pi)^3} \left(1 - e^{iq\cdot(r_1 - r_2)}\right) D_{00}(\omega = 0, q) \\ &= g^2 \int \frac{dq}{(2\pi)^3} \left(1 - e^{iq\cdot(r_1 - r_2)}\right) \left[\frac{1}{q^2 + m_D^2} - i\frac{\pi m_D^2 T}{|q|(q^2 + m_D^2)^2}\right] \\ &= -\frac{g^2}{4\pi}\left[m_D + \frac{e^{-m_D r}}{r}\right] - i\frac{g^2 T}{4\pi}\phi(m_D r) \end{aligned}$$

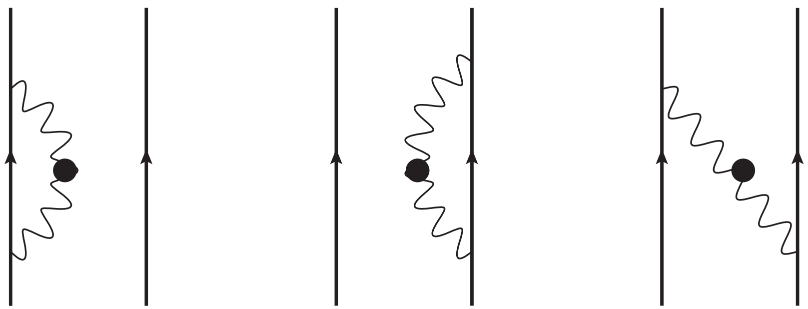
(*first observed by M. Laine et al hep-ph/ 0611300)

The imaginary part of the effective potential



At large distance the imaginary part is twice the damping rate of the heavy quark

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



Physical content of the influence functional

$$\Phi[\boldsymbol{Q}] = \frac{g^2}{2} \iint_{\mathcal{C}} \mathrm{d}^4x \mathrm{d}^4y \; \rho(x) \Delta_{\mathcal{C}}(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i \langle \, T_C \, [A_0(x) A_0(y) \,] \rangle$$

$$V(x) \sim \Delta_{11}(\omega = 0, x)$$

Heavy quark potential (complex)

$$D(x) \sim \Delta_{12}(\omega = 0, x) \sim \mathrm{Im} V(x)$$

dissipation

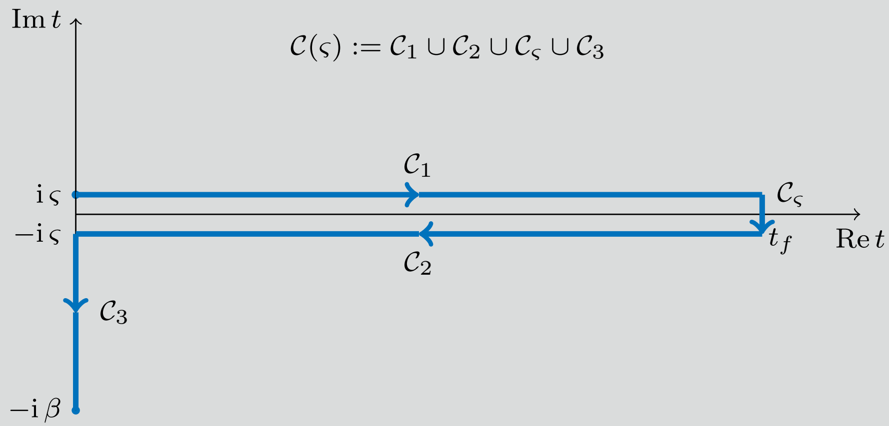
$$\frac{g^2}{2MT} \left. \frac{\partial^2 D}{\partial x_i \partial x_j} \right|_{x=0} = \delta_{ij} \gamma$$

γ friction coefficient

Low frequency expansion

$$r_i = \frac{1}{2}(q_{i,1} + q_{i,2})$$

$$y_i = q_{i,1} - q_{i,2}$$



$$P(R_f, t_f | R_i, t_i) = \int_{R_i}^{R_f} DR \int_0^0 DY \, \mathrm{e}^{\int_{t_i}^{t_f} \mathrm{d}t \mathcal{L}(R,Y)}$$

$$\mathcal{L}(R, Y) = -i\, Y \left(M\ddot{R} + \frac{\beta}{2}\, \mathcal{H}(R)\dot{R} - \mathbf{F}(R) \right) - \frac{1}{2}\, Y\mathcal{H}(R)Y$$

$$\mathbf{F}(R) \sim \nabla \mathrm{Re} V(R) \qquad \mathcal{H}_{ij} \sim \left. \frac{\partial^2 D}{\partial x_i \partial x_j} \right|_{x=0}$$

Equivalent langevin equation

$$M\,\ddot{R} = -\frac{\beta}{2}\,\mathcal{H}(R)\,\dot{R} + \mathbf{F}(R) + \Psi(R,t)$$

$$\mathcal{H}_{ij} \sim \left.\frac{\partial^2 D}{\partial x_i \partial x_j}\right|_{x=0} \qquad \mathbf{F}(R) \sim \nabla \mathrm{Re} V(R)$$

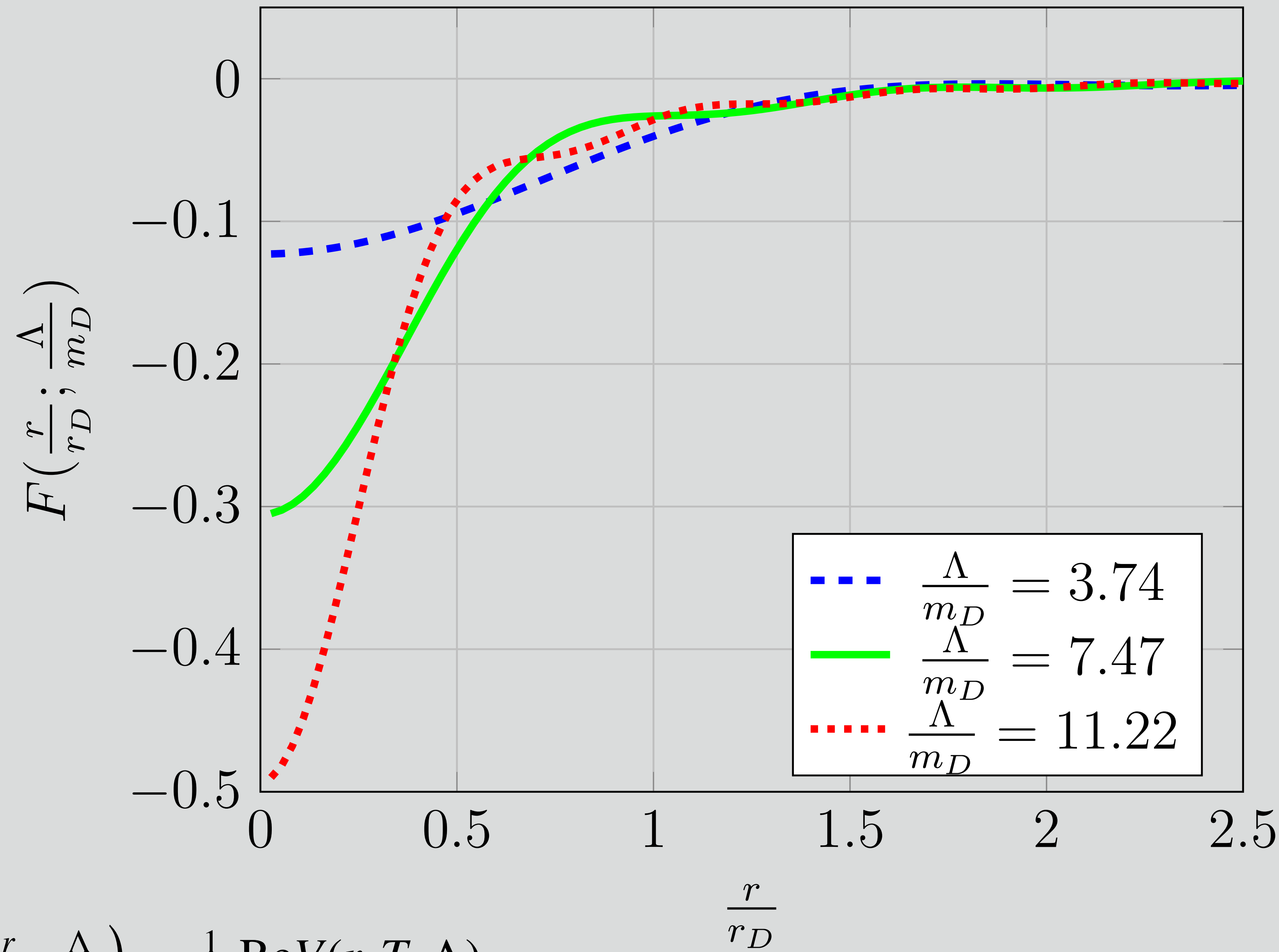
$$\langle\, \Psi(R,t)\, \rangle = 0$$

$$\langle\, \Psi_k(R,t)\, \Psi_m(R,t')\, \rangle = \mathcal{H}_{km}(R)\delta(t-t')$$

Non trivial noise

Selected results

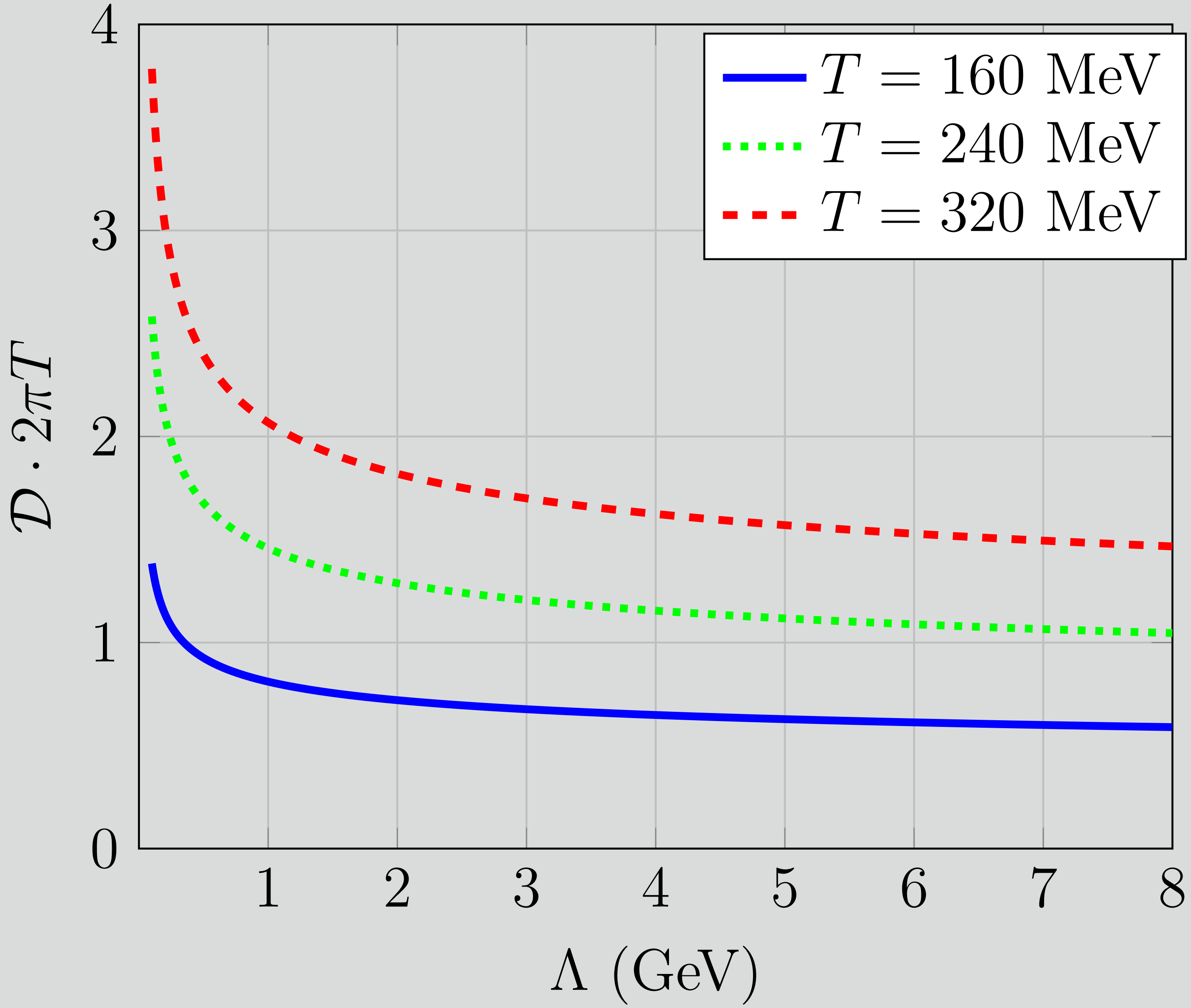
Regularized Coulomb potential



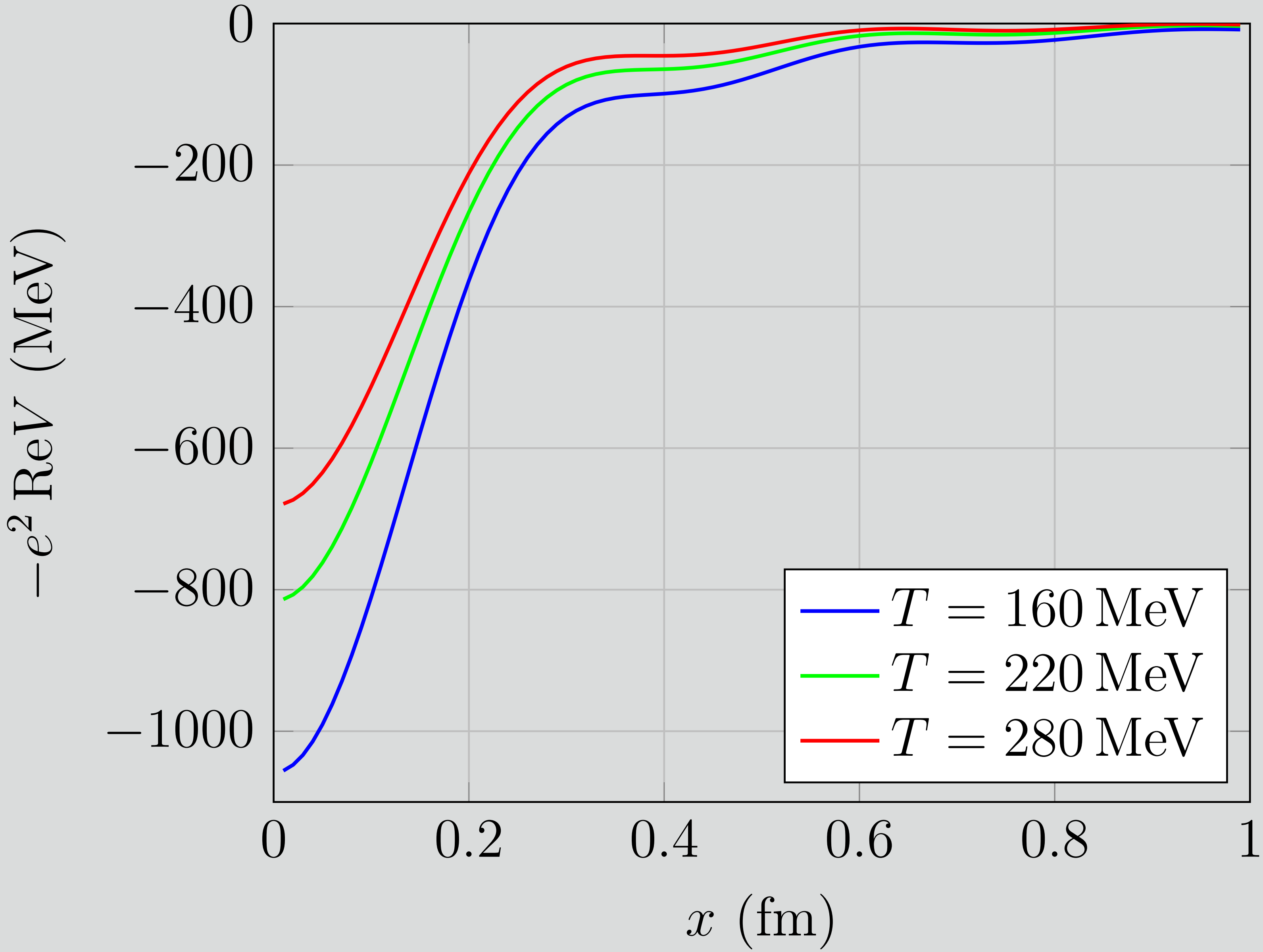
$$F\left(\frac{r}{r_D}, \frac{\Lambda}{m_D}\right) = \frac{1}{m_D} \mathrm{Re} V(r, T, \Lambda)$$

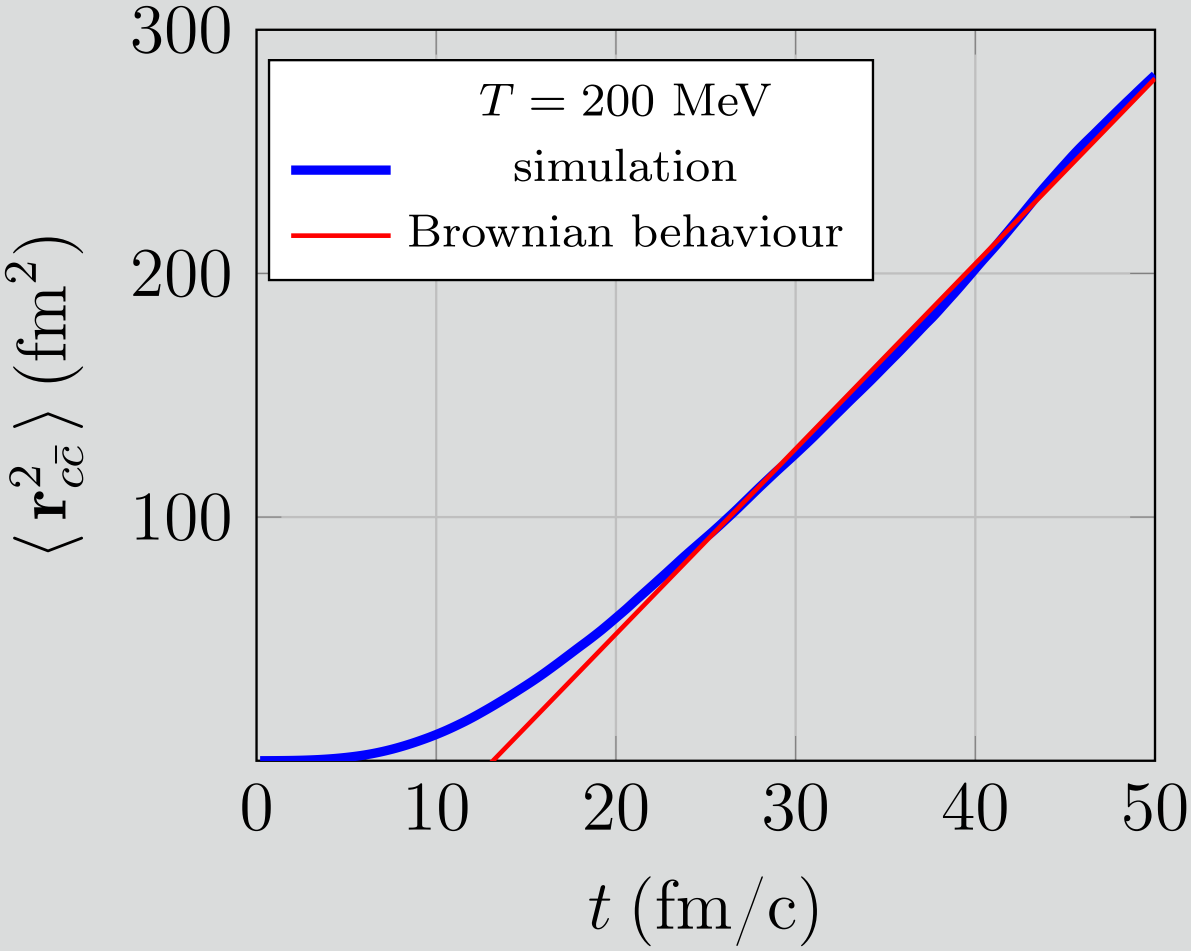
Diffusion constant

$$\mathcal{D} = \frac{T}{M\gamma}$$



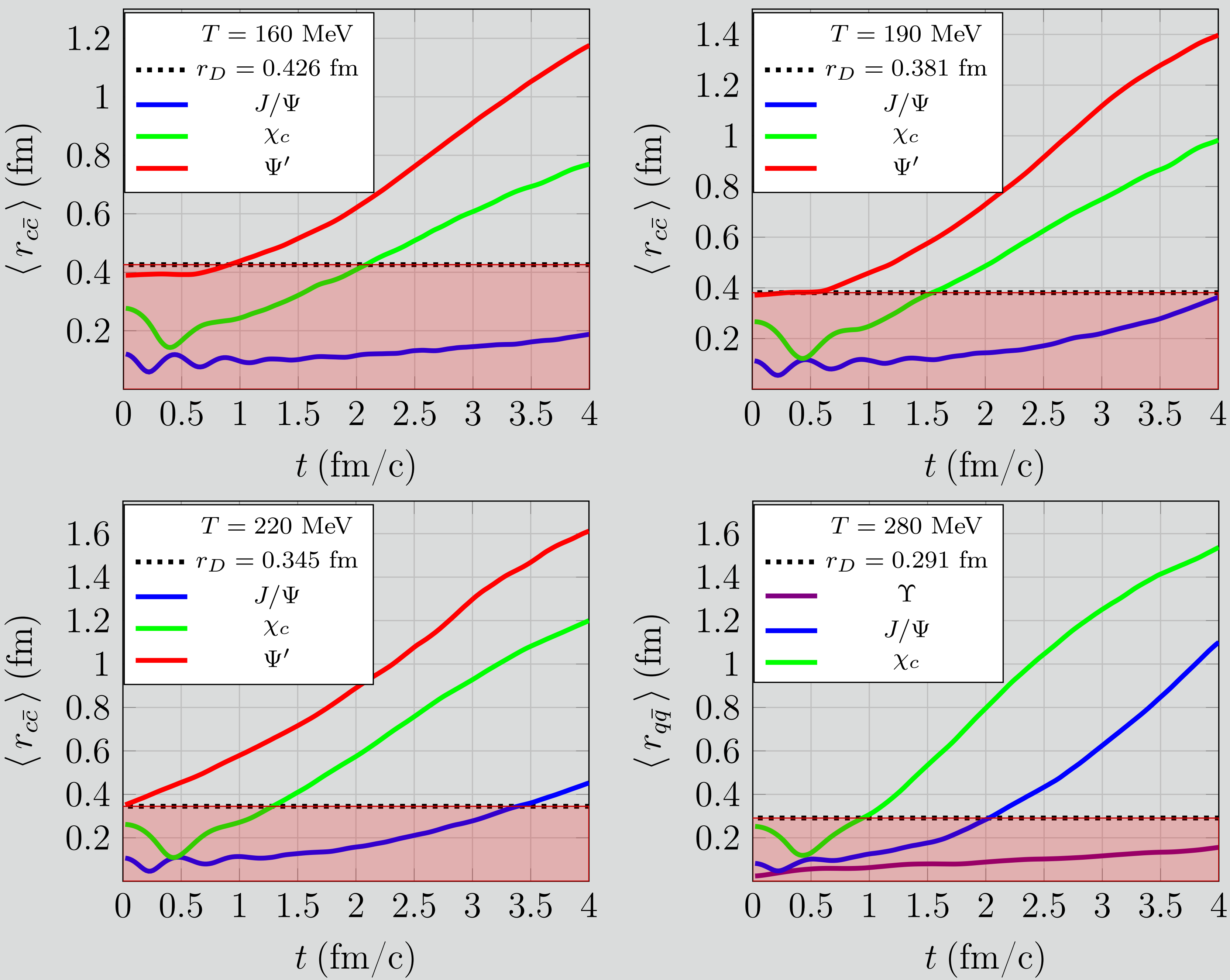
Potential (real part) - charmonium



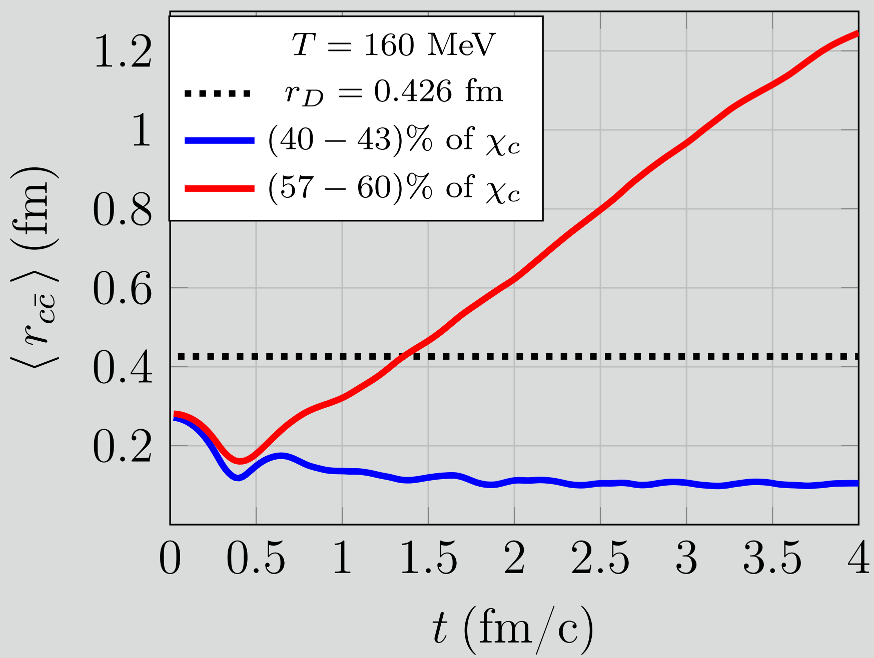


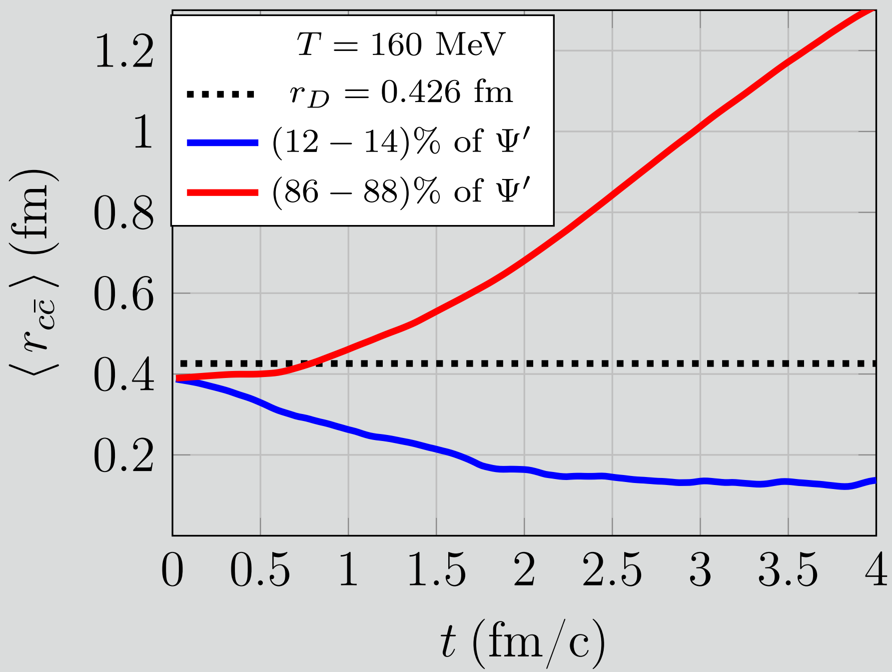
$T = 200$ MeV
simulation
Brownian behaviour
300
200
100
0
10
20
30
40
50
$\langle \mathbf{r}^2_{c\bar{c}} \rangle$ (fm^2)
t (fm/c)

Sequential suppression

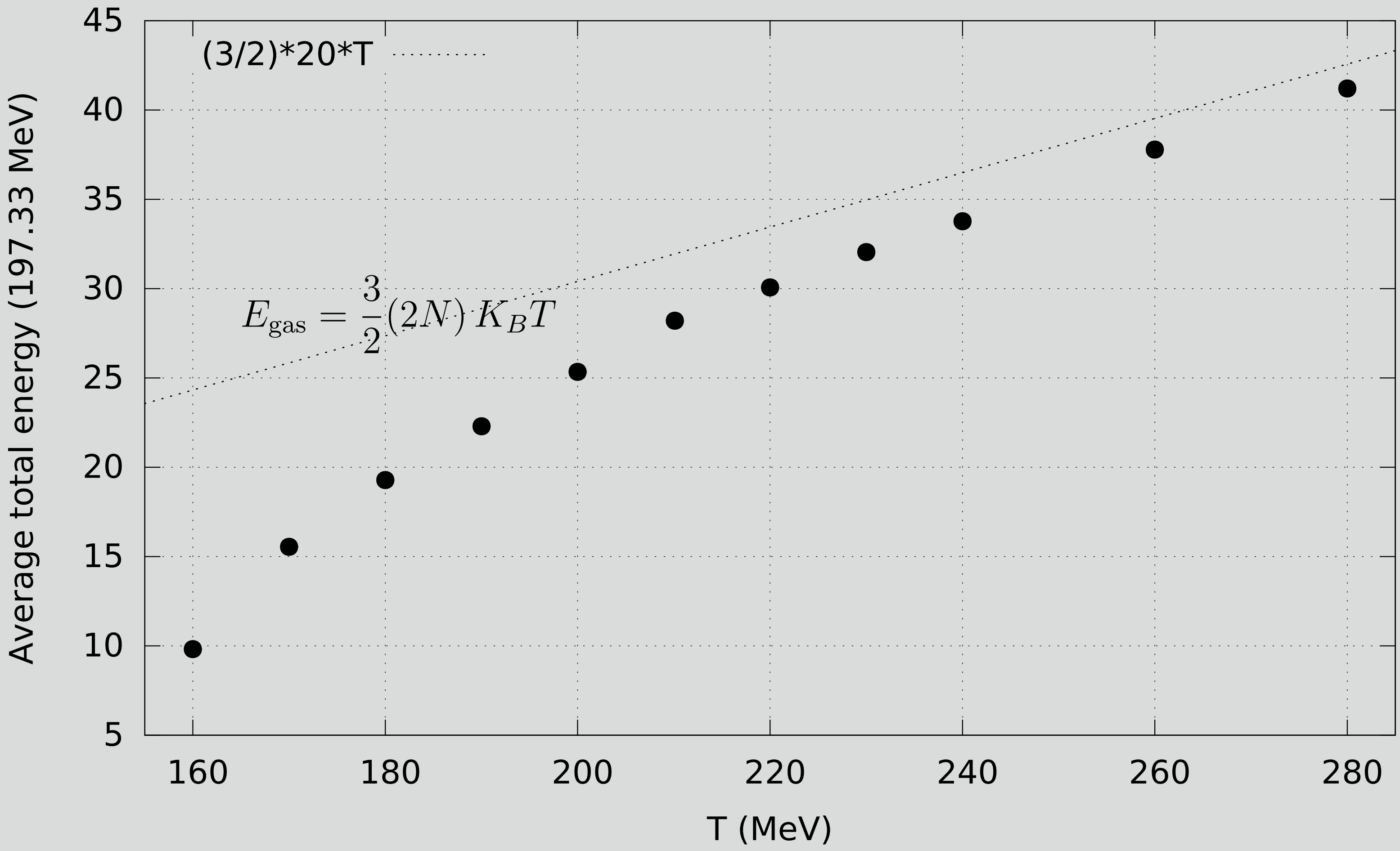


Effective feed down from excited states!

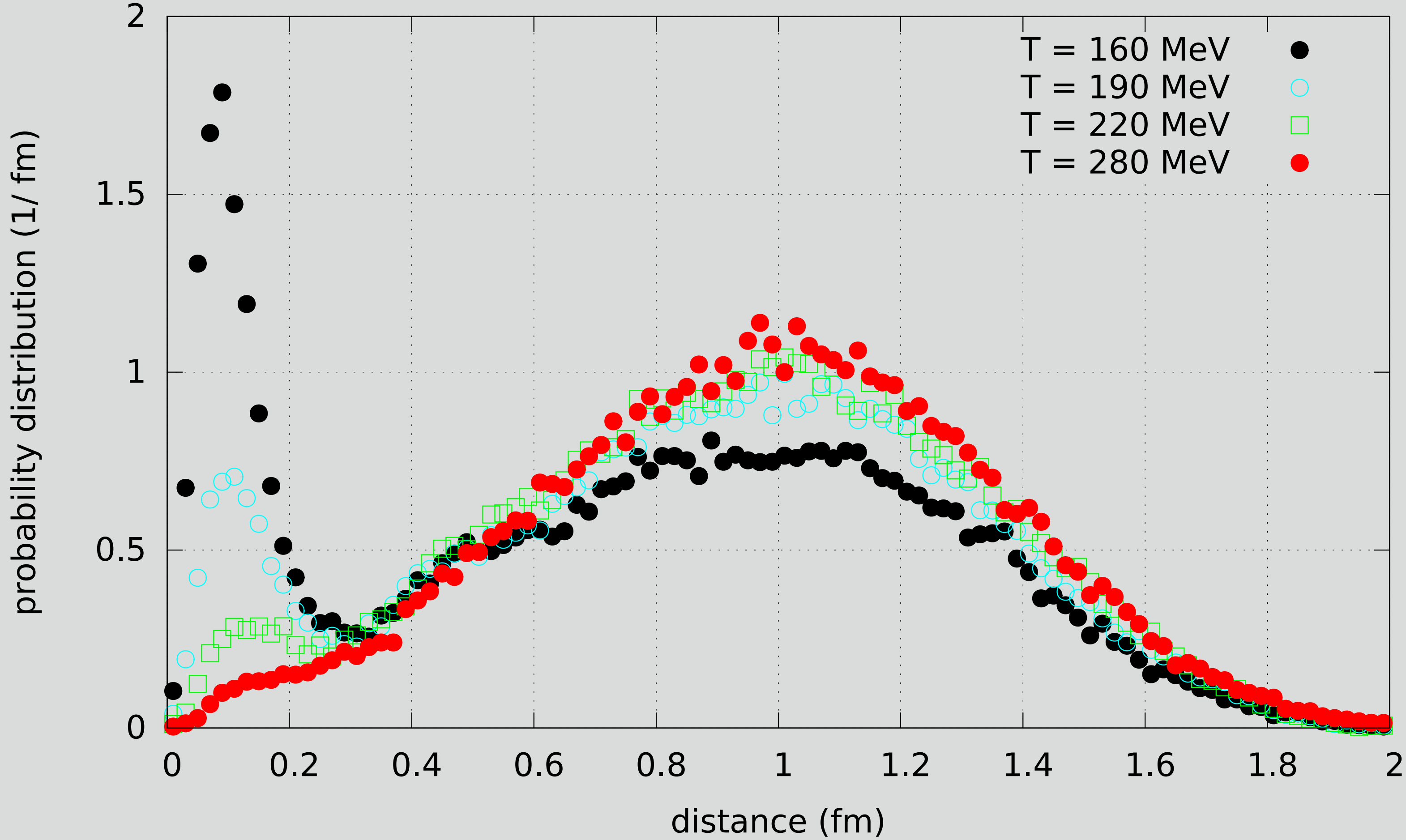




10 pairs in plasma

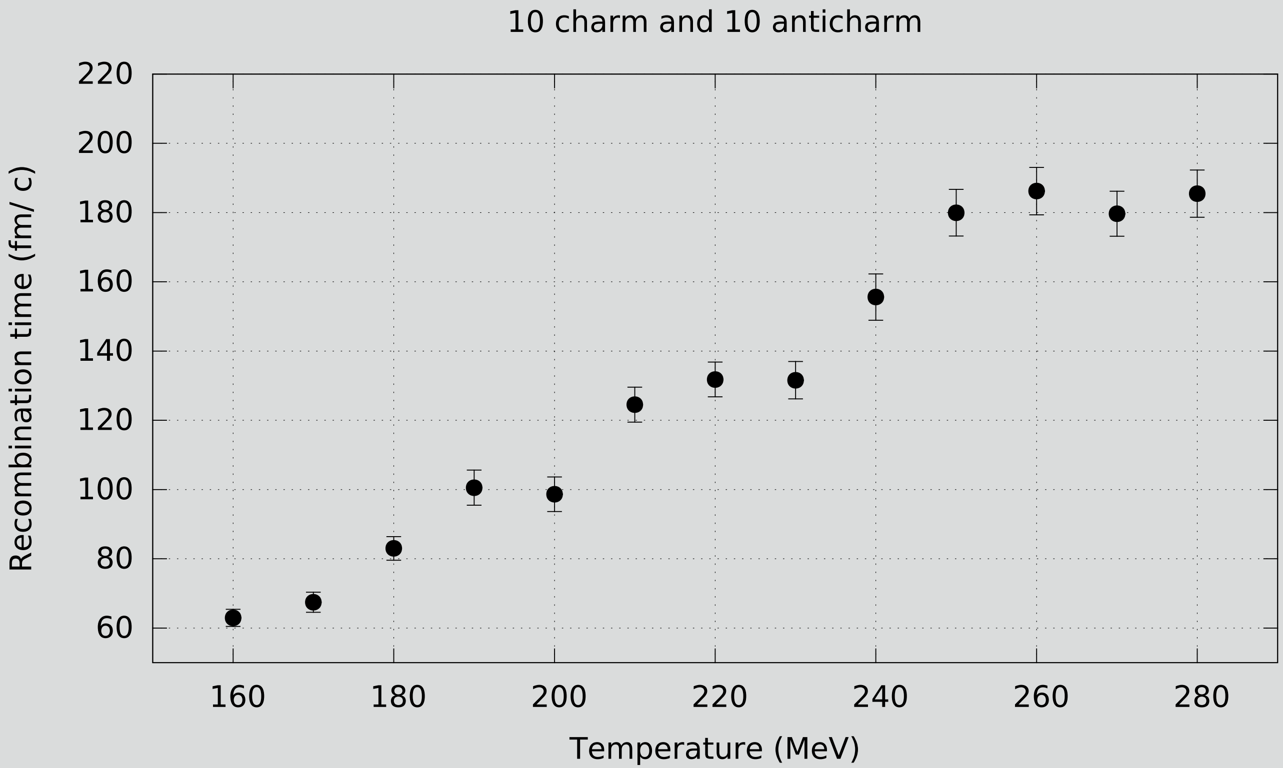


Probability distribution of distance to nearest neighbor

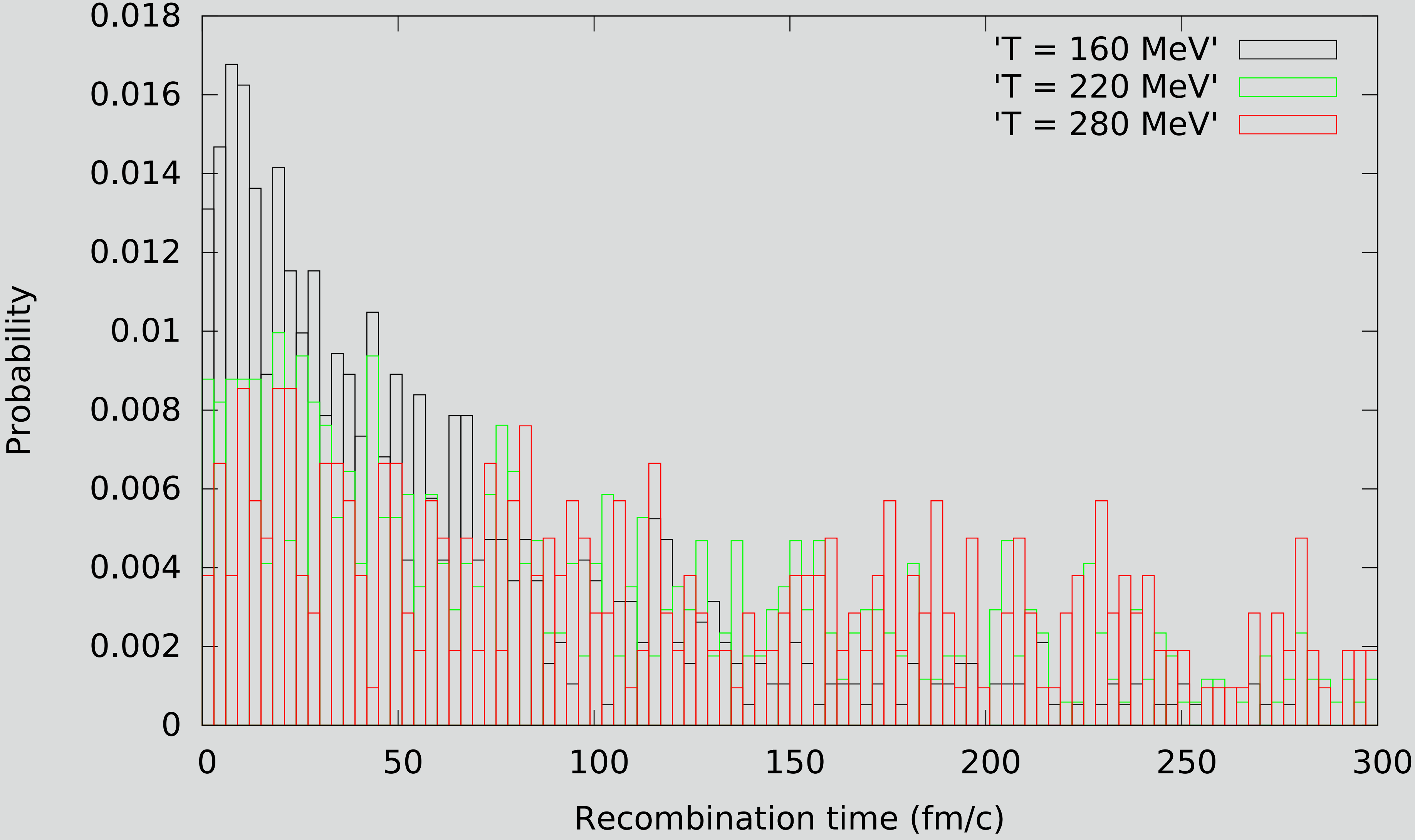


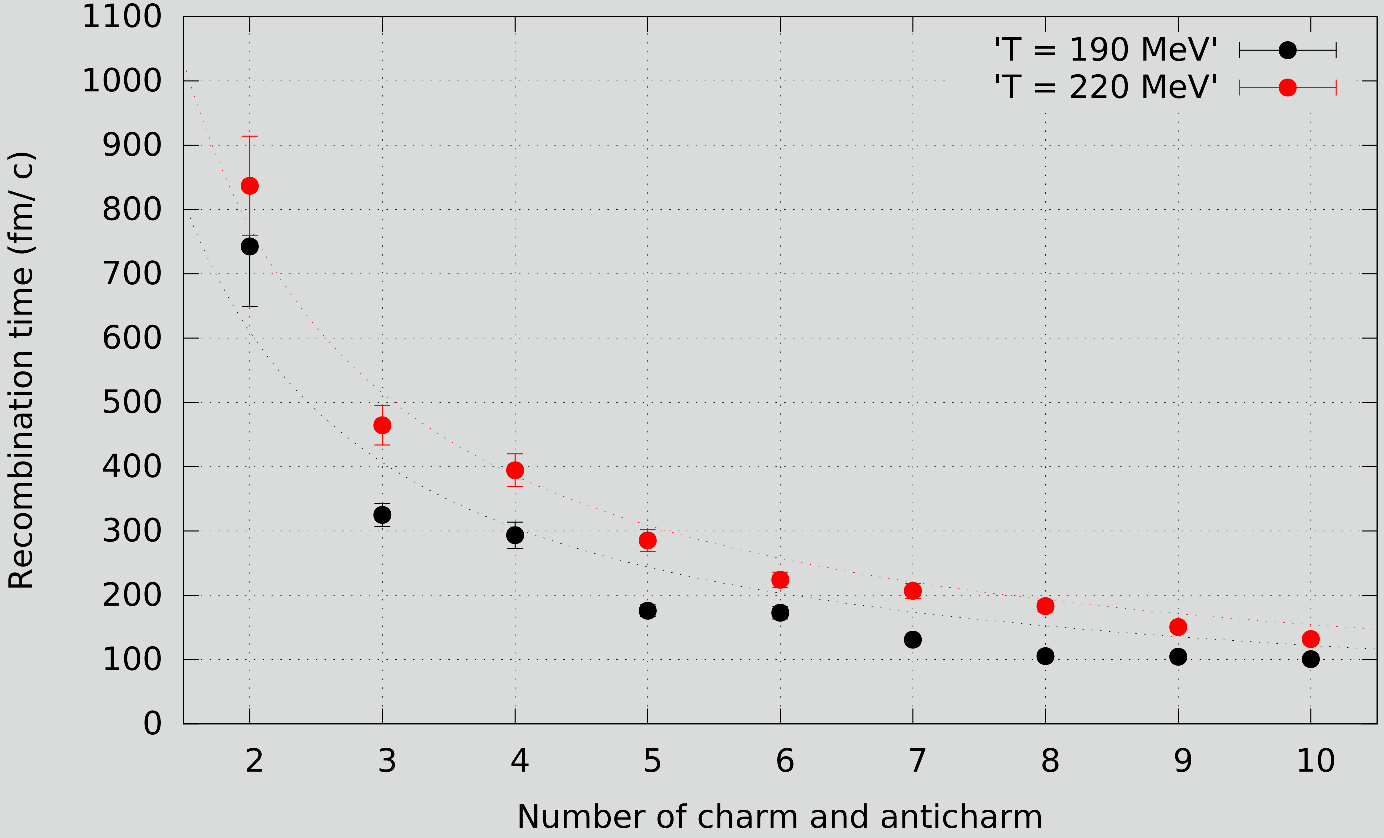
$$P_{q\bar{q}}^{\mathrm{ideal}}(r) = \frac{3}{a}\left(\frac{r}{a}\right)^2\left(1-\left(\frac{r}{a}\right)^3\frac{1}{N}\right)^{N-1} \overset{N\gg 1}{\simeq} \frac{3}{a}\left(\frac{r}{a}\right)^2 e^{-(r/a)^{\frac{1}{3}}}$$

Recombination time



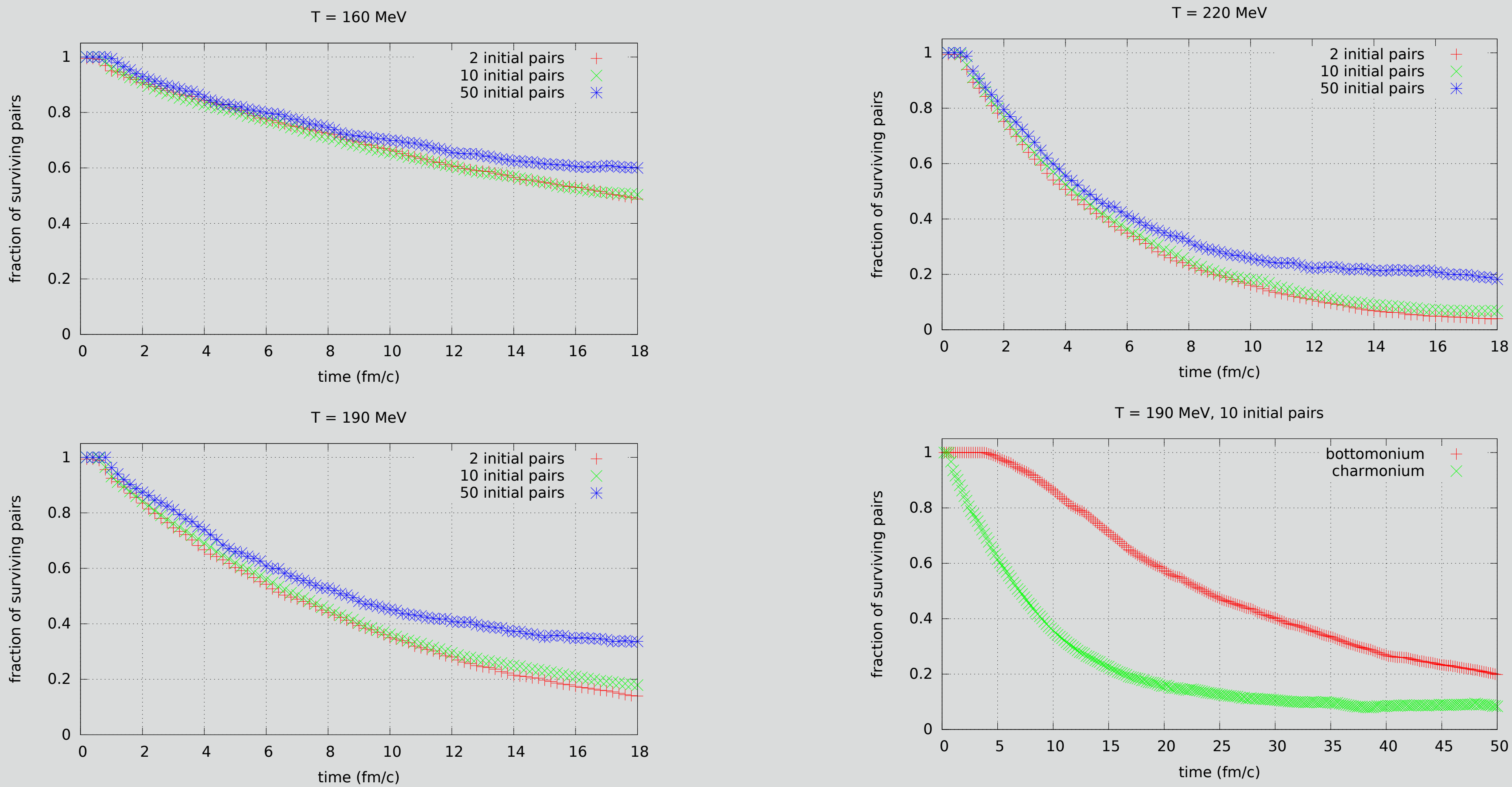
Distribution of recombination times



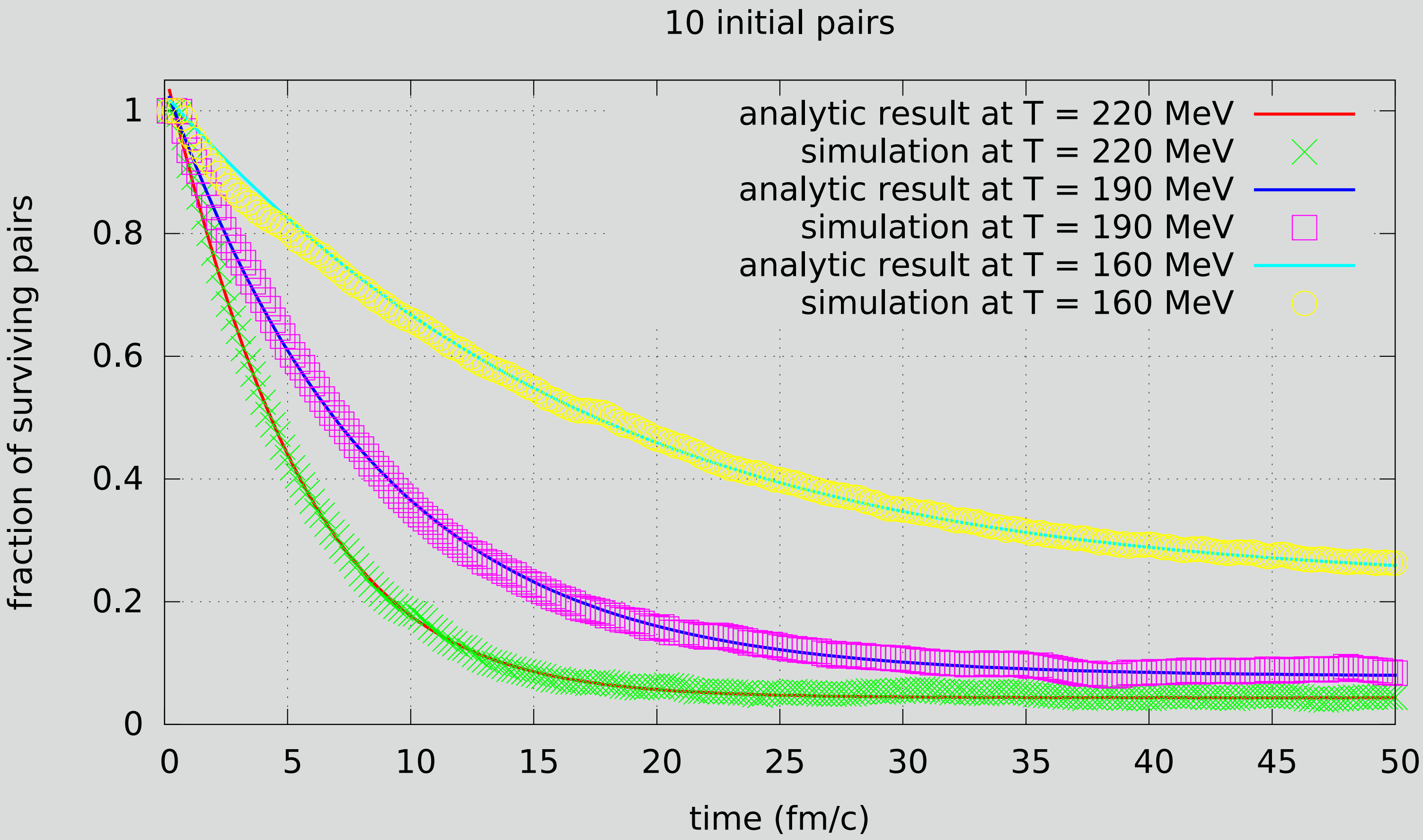


'T = 190 MeV'
'T = 220 MeV'
Recombination time (fm/ c)
Number of charm and anticharm
0
100
200
300
400
500
600
700
800
900
1000
1100
2
3
4
5
6
7
8
9
10

Dissociation/recombination



Evolution of population of bound states is well described by a simple rate equation



$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -\lambda N(t) + \zeta N_q(t) N_{\bar{q}}(t)$$

Extension to QCD

Much of the previous discussion goes through

New random force, dependent on color

Subtle interplay between color and coordinate space dynamics

'Separate' treatment of binding potential and 'imaginary part' seems required

Stay tuned : JPB, M. Escobedo-Espinosa, in preparation