NUCLEAR STRUCTURE AND REACTIONS FROM LATTICE QCD

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Outline

- QCD at Low Energies and the Lattice
- Nuclear Effective Field Theories
- EFT for Lattice Nuclei
- Outlook and Conclusion

Two goals of nuclear physics

- Nucleus as a laboratory:
  properties of the Standard Model and beyond
  - nuclear matrix elements for symmetry tests
  - reaction rates for nucleosynthesis
  - equation of state for stellar structure
  - variation of parameters for cosmology
  - …

"*ab initio*" methods, phenomenology, *etc.*

- Nucleus as the simplest complex system:
  quarks and gluons interacting strongly,
  yet exhibiting many regularities
  - QCD at large distances an unsolved part of the SM
  - tools for non-perturbative quantum (field) theories,
    e.g. cold atoms

TODAY

QCD

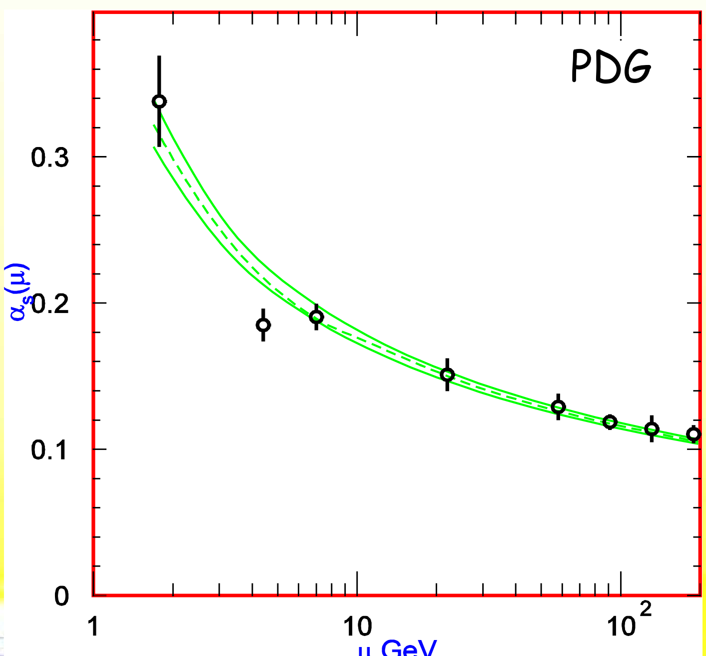
d.o.f.s quarks: $q=\begin{pmatrix}u\\d\end{pmatrix}$ gluons: G_μ^a (photon: A_μ)

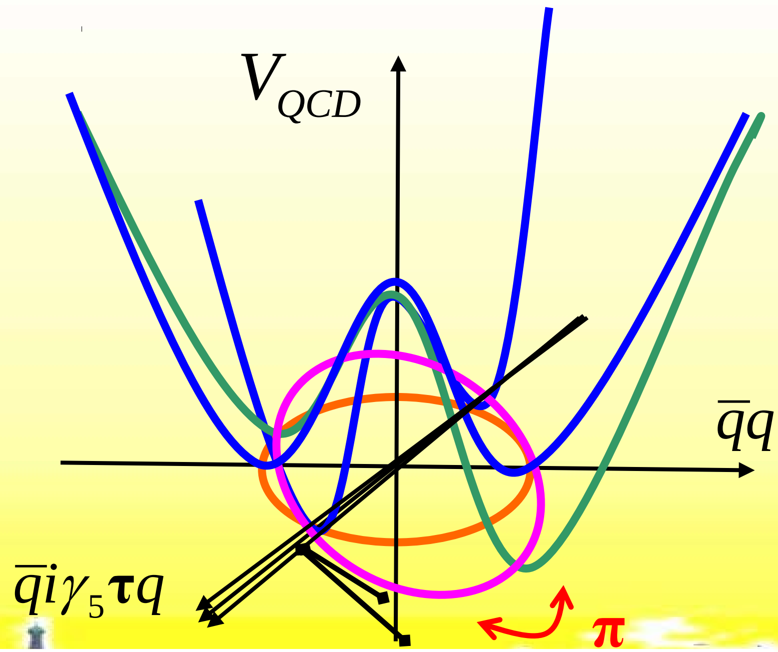
symmetries $SO(3,1)$ global, $SU_c(3)$ gauge $(+U_{em}(1)$ gauge$)$

$$\mathcal{L}_{QCD}=\bar{q}\left(i\partial\!\!\!/+g_s G\!\!\!/\right)q-\frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu}\quad+\ \bar{m}\,\bar{q}\left(1-\varepsilon\tau_3\right)q\ +\ldots$$

Basic mass scales

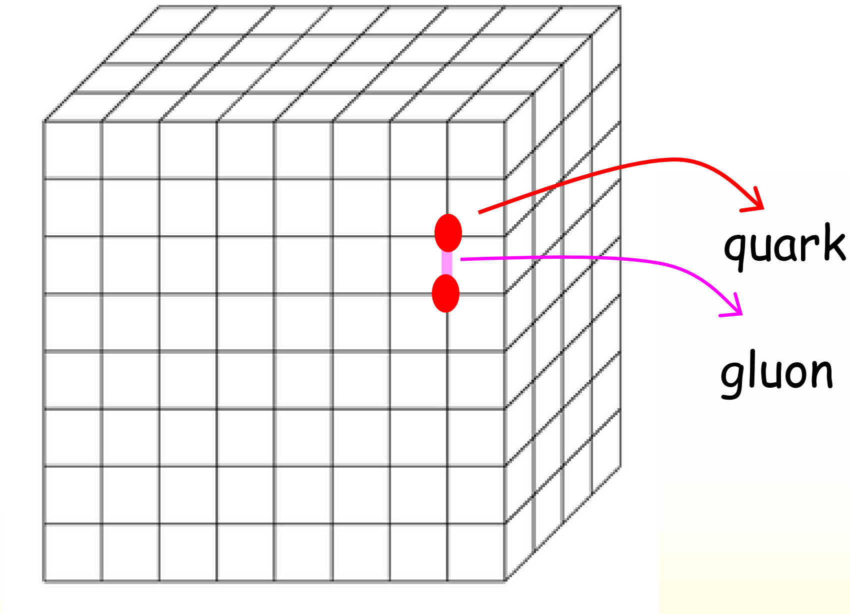
$M_{QCD}\sim m_N, m_\rho, 4\pi f_\pi, \ldots \sim 1\ \mathrm{GeV}$ $\qquad m_\pi\sim\sqrt{\bar{m}M_{QCD}}\simeq 140\ \mathrm{MeV}$





$f_\pi\sim M_{QCD}/4\pi+\mathcal{O}(\bar{m})\simeq 100\ \mathrm{MeV}$

Lattice QCD



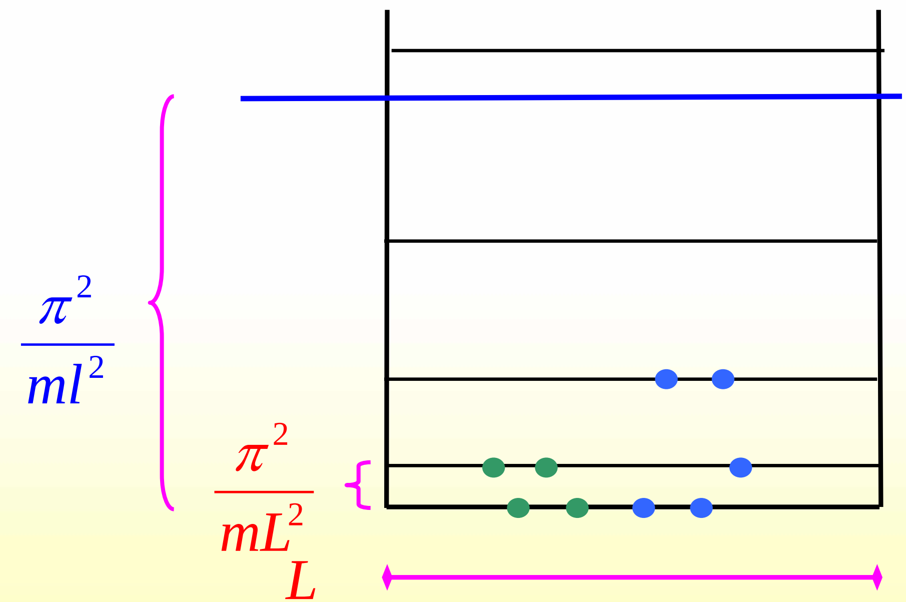
l

$L \gg l$

$\Lambda \sim 1/l$

$\lambda \sim 1/L$

lattice "model space"



path integral solved with Monte Carlo methods, typically for unrealistically large quark masses

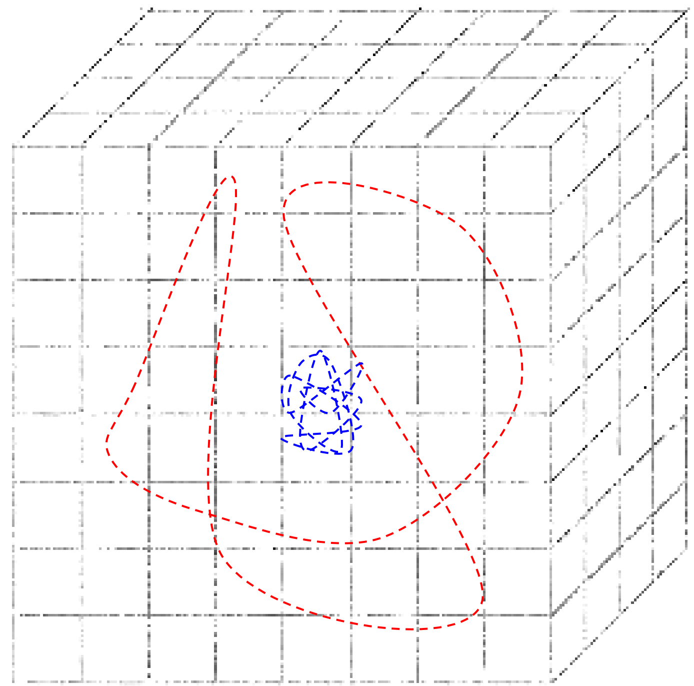
$$\cot\delta(E) = \frac{4}{\sqrt{mE}\,L}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - mEL^2} - \frac{L}{l}\right]$$

Lüscher '91

nucleon

$l \ll 1/M_{QCD}$

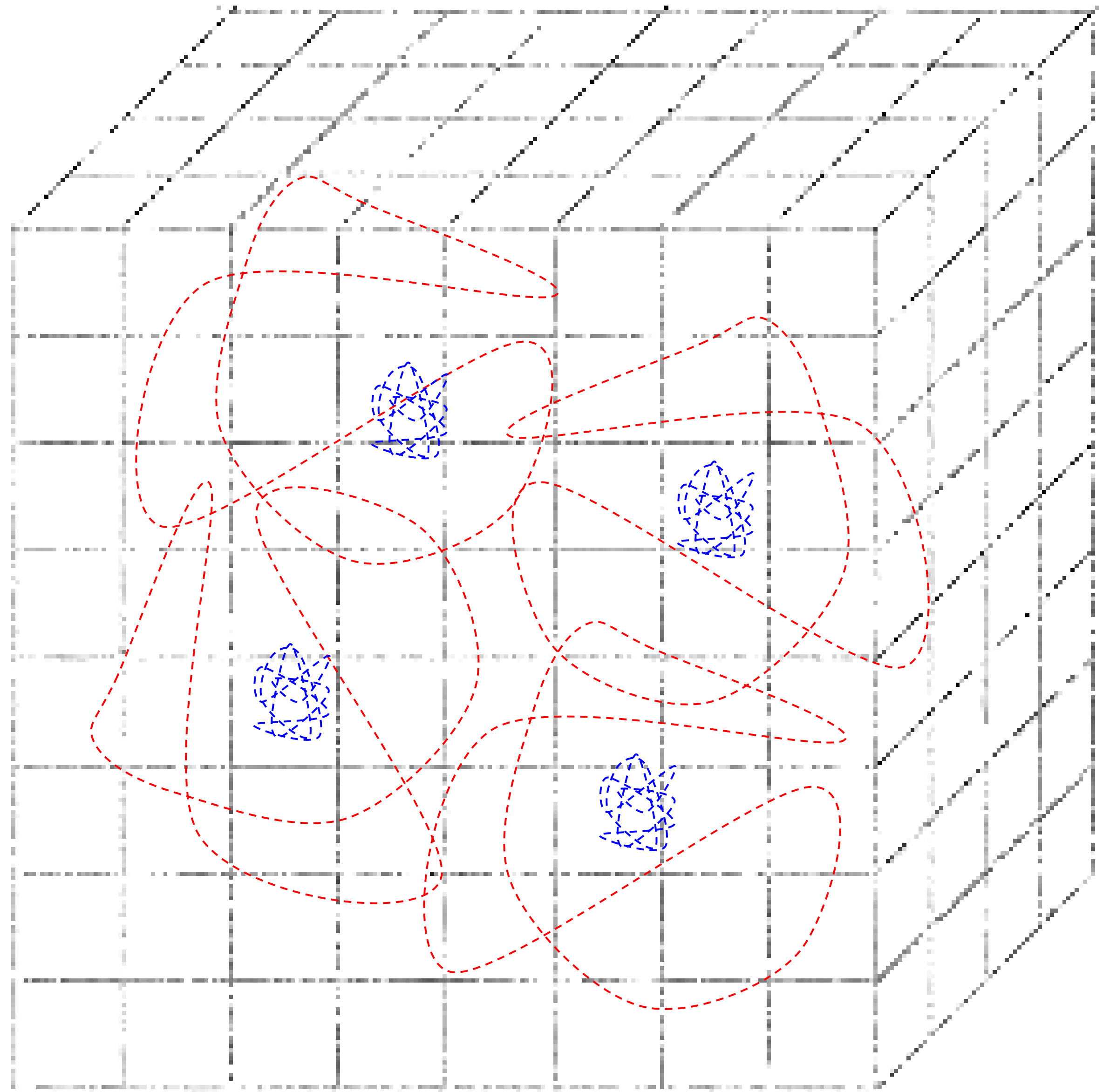
$1/M_{QCD} \approx 0.3$ fm



$1/m_\pi \cong 1.4$ fm

$L \gg 1/m_\pi$

nucleus



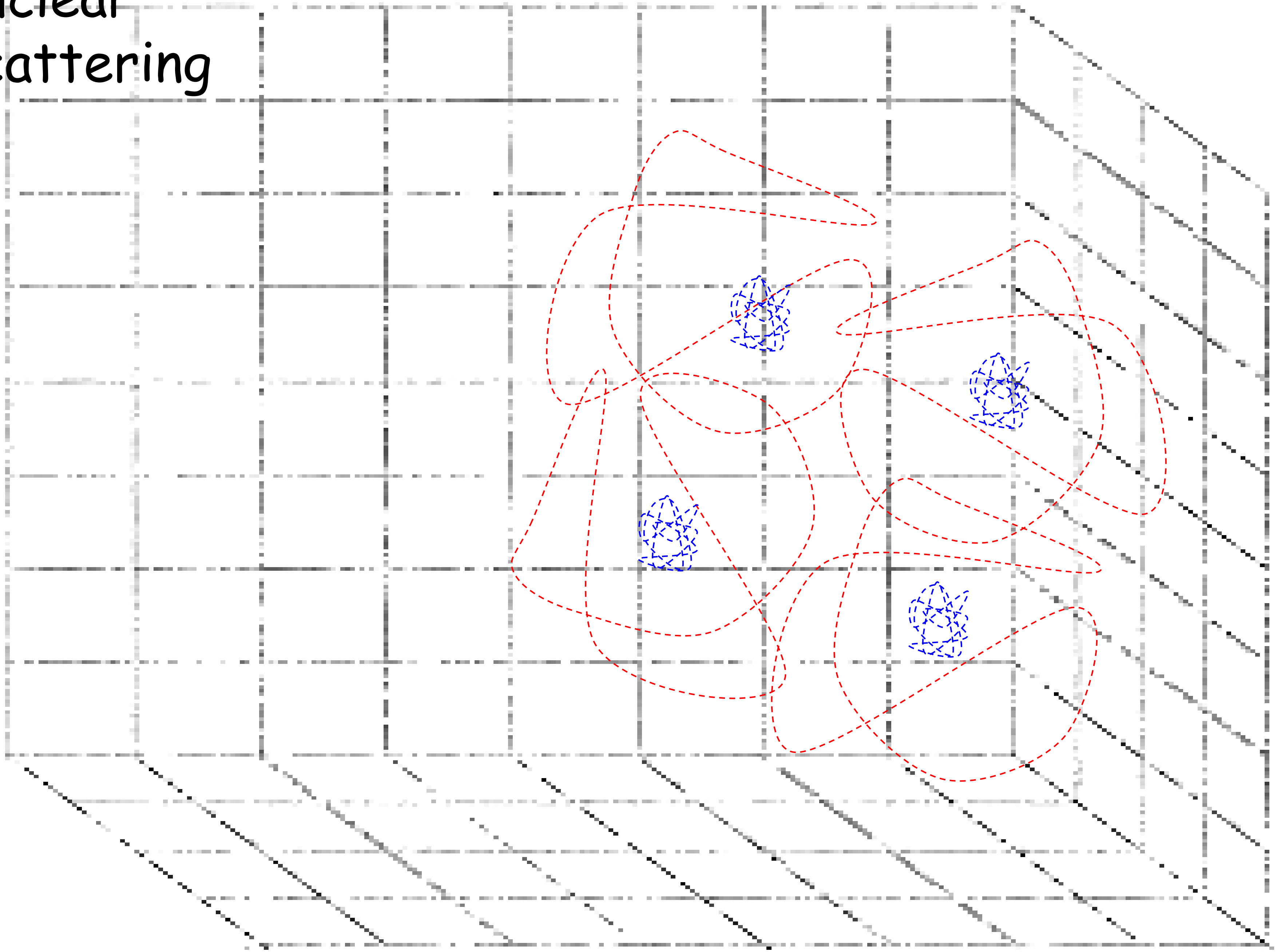
$l \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3\ \text{fm}$

$R \sim \rho\left(m_\pi / f_\pi\right) A^{1/3}/m_\pi \approx 1.2\ A^{1/3}\text{fm}$

$L \gg \rho(m_\pi / f_\pi)\, A^{1/3}/m_\pi$

nuclear scattering



How? Lattice QCD + Effective Field Theory ©

Most general *S* matrix with given symmetries

mass scales

M — Λ UV regulator

m — Q

λ IR regulator

order, expansion parameter, normalization

non-analytic functions, from solution of dynamical equation

$$S^{(\bar{\nu})}(Q \sim m \ll M) = 1 + \mathcal{N}(M) \sum_{\nu=\nu_{\min}}^{\bar{\nu}} \left[\frac{Q}{M}\right]^{\nu} F_{\nu}\left(\frac{Q}{m}, \frac{Q}{\Lambda}, \frac{\lambda}{Q}; \gamma_i\left(\frac{m}{\Lambda}, \lambda m\right)\right)$$

$$\times \left\{1 + \mathcal{O}\left(\frac{Q^{\bar{\nu}+1}}{M^{\bar{\nu}+1}}, \frac{Q^{\bar{\nu}+1}}{M^{\bar{\nu}}\Lambda}, \frac{\lambda Q^{\bar{\nu}}}{M^{\bar{\nu}+1}}\right)\right\}$$

"low-energy constants"

$\mathrm{N}^{\bar{\nu}-\nu_{\min}}\mathrm{LO}$

(unfortunately not the usage by nuclear potential modelers)

CONTROLLED UNCERTAINTY

Want large "model space"

renormalization-group invariance

$$\frac{\Lambda}{S^{(\bar{\nu})}} \frac{\partial S^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q^{\bar{\nu}+1}}{M^{\bar{\nu}}\Lambda}\right)$$

$$\frac{\lambda}{S^{(\bar{\nu})}} \frac{\partial S^{(\bar{\nu})}}{\partial \lambda} = \mathcal{O}\left(\frac{Q^{\bar{\nu}}\lambda}{M^{\bar{\nu}+1}}\right)$$

MODEL INDEPENDENCE (insensitivity to high-mom details)

$\Lambda \gtrsim M$

$\lambda \lesssim Q$

two-step strategy

ab initio primo

II) solve EFT for

fit LECs for

$A \lesssim 3, 4$

any A

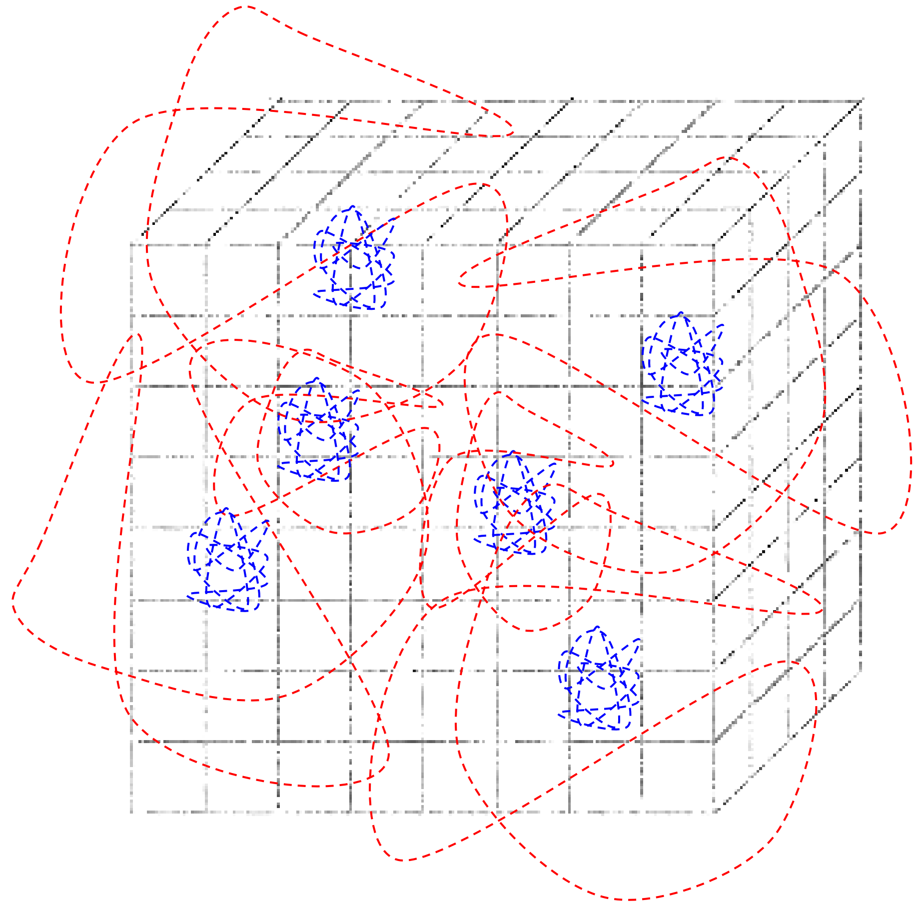
$m_\pi \geq M_\pi \sim 300, 400$ MeV

any m_π

$l \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3$ fm

Also for reactions



$1/m_\pi$ $1/M_\pi$

$\rho(M_\pi/f_\pi)\, a^{1/3}/M_\pi$

$\rho(m_\pi/f_\pi)\, A^{1/3}/m_\pi$

$L \gg \rho(M_\pi/f_\pi)\, a^{1/3}/M_\pi$

Experimental and LQCD data

+ Inoue *et al.* '12 ...

(MeV)

m_π Nucleus	140 [nature]	300 [10]	510 [7]	805 [8]
n	939.6	1053	1320	1634
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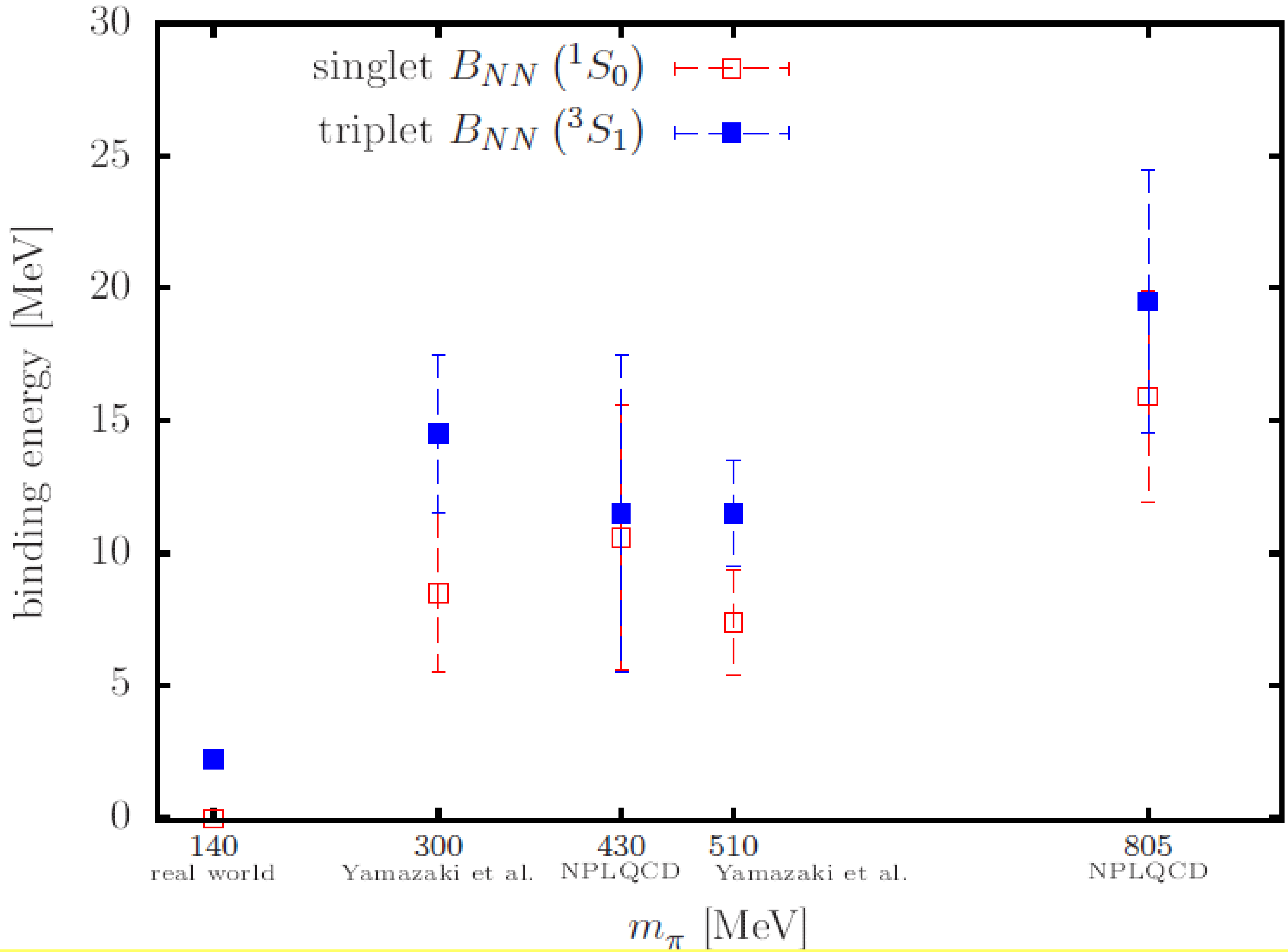
[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12

Beane *et al.* '13

$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \quad , \quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad , \quad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

+ Berkowitz *et al.* 16
+ Beane *et al.* –
  in progress



singlet B_{NN} $(^1S_0)$
triplet B_{NN} $(^3S_1)$
binding energy [MeV]
30
25
20
15
10
5
0
140
real world
300
Yamazaki et al.
430
NPLQCD
510
Yamazaki et al.
805
NPLQCD
m_π [MeV]

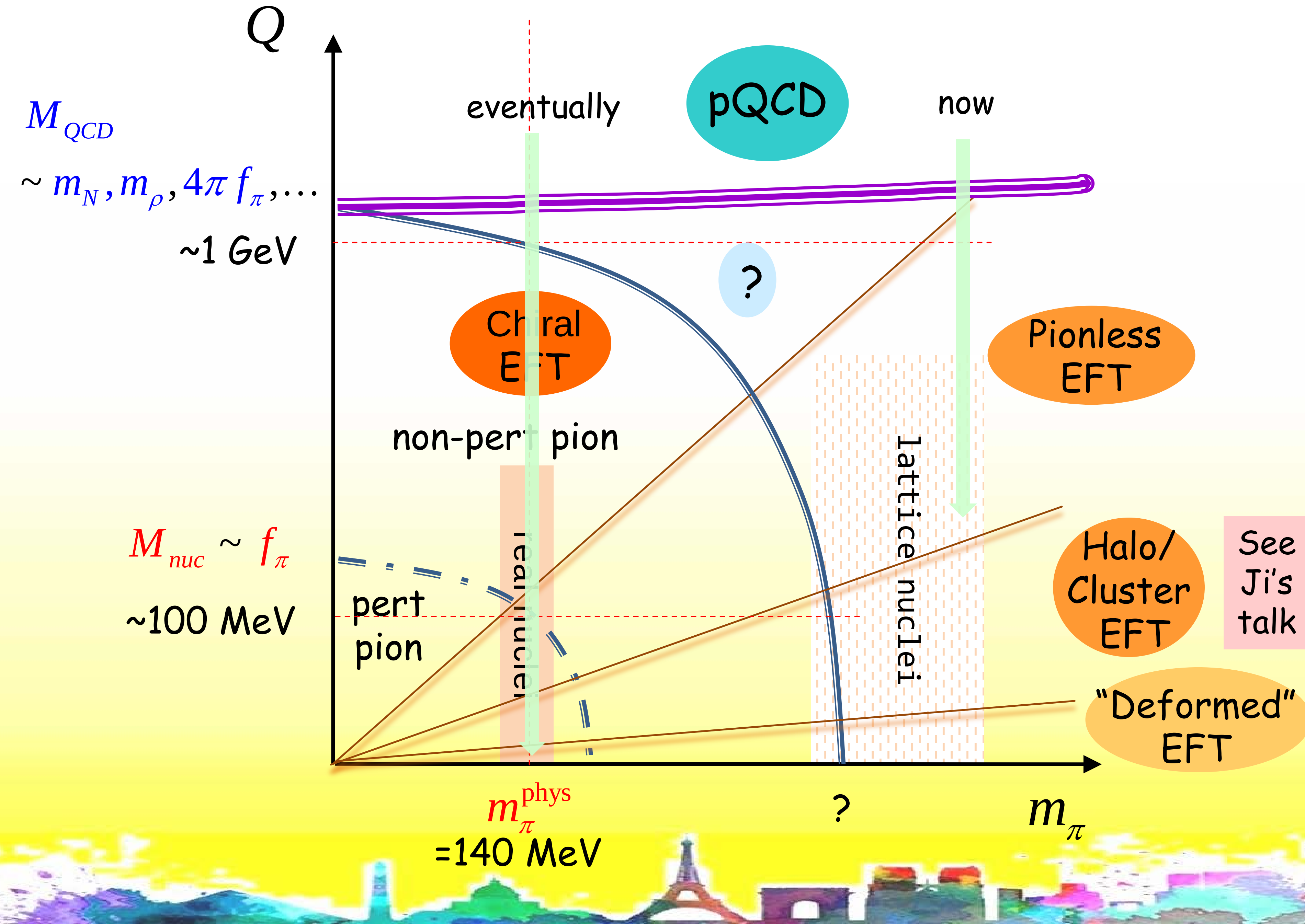
J. Kirscher

Scales (MeV)

m_N	940	1050	1320	1630
$\sqrt{2m_N(m_\Delta - m_N)}$	750	800	900	800
m_π	140	300	500	800
$\sqrt{2m_N B_A/A}\ (A = 2 \mapsto 4)$	$45 \mapsto 110$	$100 \mapsto 150$	$130 \mapsto 170$	$185 \mapsto 300$

$$Q \sim \aleph \equiv \sqrt{m_N B_2} < m_\pi \lesssim M_{QCD}$$

The Nuclear EFT Landscape



Extrapolation in pion mass

Pionful (Chiral) EFT

$$Q \sim m_\pi \ll M_{QCD}$$

Another talk…

Pionless EFT

$$Q \sim \aleph \sim \sqrt{m_N B_2} \ll m_\pi \sim M$$

- degrees of freedom: nucleons
- symmetries: Lorentz, ~~P~~, ~~T~~

$$\mathcal{L}_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N - \frac{C_0}{2} N^+ N\, N^+ N - \frac{D_0}{6} N^+ N\, N^+ N\, N^+ N$$

$$+ N^+ \frac{\nabla^4}{8m_N^3} N - \frac{C_2}{4} N^+ N\, \nabla^2 N^+ N + \ldots$$

(omitting spin, isospin)

- expansion in: $\frac{Q}{M} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$

Universality: first orders apply also to neutral atoms

$$m_\pi \to 1/l_{vdW} \quad \textit{where} \quad V(r) = -\frac{l_{vdW}^4}{2m_{at} r^6} + \ldots$$

Bedaque, Hammer + v.K. '99 '00
Bedaque, Braaten + Hammer '01
...

$$V_{ij} = \sum_{s=0}^{1} \left\{ \underbrace{C_{0(s)}\,\delta\left(\vec{r}_i - \vec{r}_j\right)}_{\text{LO}} + \underbrace{C_{2(s)}\left[\nabla^2\delta\left(\vec{r}_i - \vec{r}_j\right) + \ldots\right]}_{\text{NLO}} + \underbrace{\ldots}_{\text{NNLO and higher}} \right\}$$

v.K. '97
Kaplan, Savage + Wise '98

a_2 LO a_2, r_2 NLO NNLO and higher

$$+ \underbrace{\frac{\alpha}{\left|\vec{r}_i - \vec{r}_j\right|}}_{\text{NLO}} + \underbrace{\ldots}_{\text{NNLO and higher}}$$

...
König, Grießhammer, Hammer + v.K. '15
...

NLO NNLO and higher

$$V_{ijk} = \underbrace{D_0\,\delta\left(\vec{r}_i - \vec{r}_j\right)\delta\left(\vec{r}_j - \vec{r}_k\right)}_{\text{LO}} + \underbrace{\ldots}_{\text{NNLO and higher}}$$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '00
...

a_3 LO NNLO and higher

$$V_{ijkl} = \underbrace{E_0\,\delta\left(\vec{r}_i - \vec{r}_j\right)\delta\left(\vec{r}_j - \vec{r}_k\right)\delta\left(\vec{r}_k - \vec{r}_l\right)}_{\text{not LO}} + \ldots$$

Platter, Hammer + Meißner '04 '05
Hammer + Platter '07
...

a_4 not LO

etc.

- **Regularization** (arbitrary) $\delta\left(\vec{r}_i - \vec{r}_j\right) \to \delta_\Lambda\left(\vec{r}_i - \vec{r}_j\right)$

  e.g. local Gaussian regulator $\delta_\Lambda\left(\vec{r}\right) = \frac{\Lambda^3}{8\pi^{3/2}} \exp\left(-\Lambda^2 \vec{r}^{\,2}/4\right)$

- **Solution**

  LO $H_A^{(0)}\left|\Psi_A^{(0)}\right\rangle = E_A^{(0)}\left|\Psi_A^{(0)}\right\rangle$

  NLO $E_A^{(1)} = \left\langle\Psi_A^{(0)}\left|H_A^{(1)}\right|\Psi_A^{(0)}\right\rangle$

  etc.

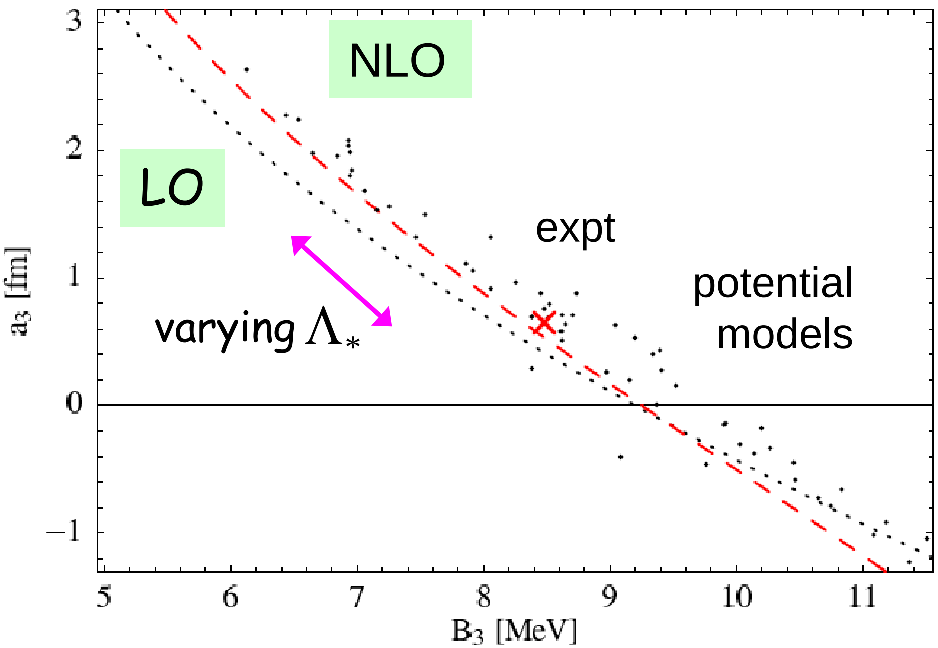
- **Renormalization** (essential)
  - $C_{0(0,1)}(\Lambda)$ fitted to two two-body data, *e.g.* $a_{NN(0,1)}$
  - $D_0(\Lambda)$ fitted to one three-body datum, *e.g.* $a_{nd(1/2)}$
  - *etc.*

$A = 2$ equivalent to (v.K. '99)
- effective-range expansion — Bethe '49
- pseudopotential — Fermi '37
- boundary condition at origin — Bethe + Peierls '35

$A \geq 3$ not *just* the effective-range expansion: includes many-body forces!

Three-Body Force at LO

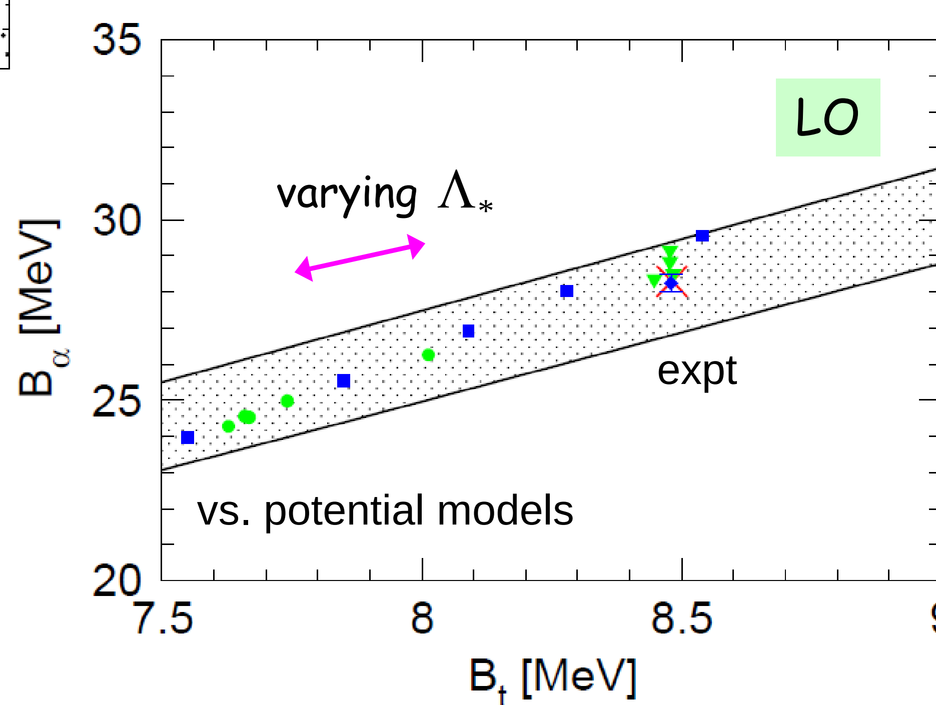
Bedaque, Hammer + v.K. '99, '00
Bedaque, Grießhammer, Hammer + Rupak '02
...
Hammer, Meißner + Platter '05
...



single $SU(4)_W$-symmetric parameter Λ_*

Phillips line

Tjon line



Pionless EFT

- ✓ reproduces effective range expansion
- ✓ explains Thomas collapse from improper renormalization
- ✓ explains correlations (Phillips, Tjon lines) from proper renormalization
- ✓ generates Efimov states and its descendants as consequence of (approximate) discrete scale invariance
- ✓ gives approximate Wigner SU(4) invariance

but

- o applies only at momenta below pion mass
- o has unknown reach in terms of nucleon number

Extrapolation in nucleon number

$m_\pi \ll M_{QCD}$ — Pionful EFT, Pionless EFT

$m_\pi \sim M_{QCD}$ — Pionless EFT

\+ <u>any</u> "exact" *"ab initio"* method

That is,

1) truncate EFT expansion at desired order
2) solve Schrödinger equation for low *A* at fixed cutoff (exactly for LO, subLOs in perturbation theory)
3) fit LECs to selected *lattice* input
4) solve Schrödinger equation for larger *A*
5) repeat steps 2-4 at other cutoffs
6) obtain observables at large cutoffs

Experimental and LQCD data

LO pionless fit:

$m_N, C_{0(0)},$

$C_{0(1)}, D_0$

Stetcu, Barrett + v.K. '06

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[10] Yamazaki *et al.* '15
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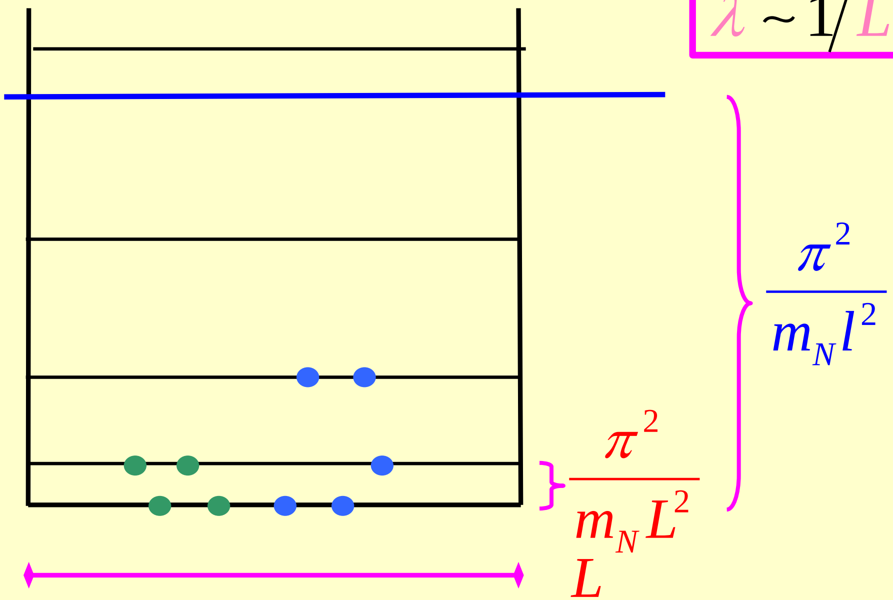
$A \gtrsim 4$

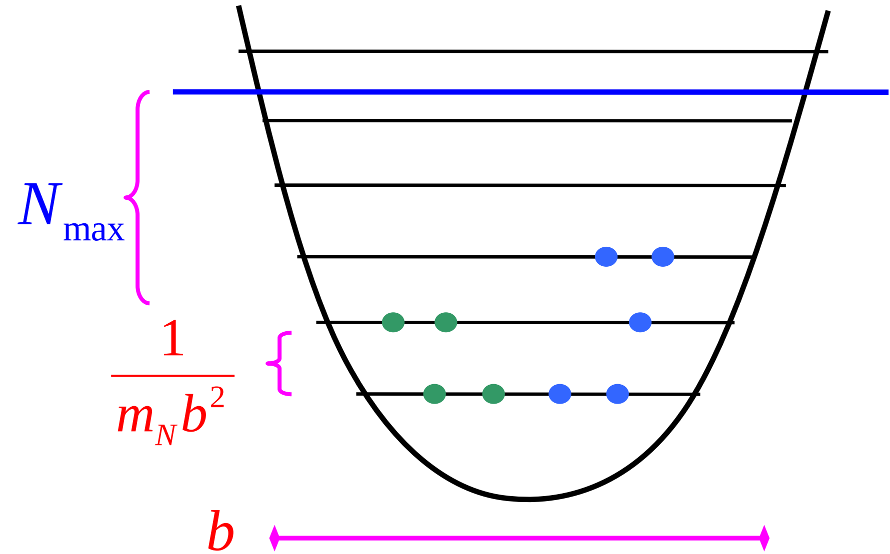
As A grows, given computational power limits number of accessible one-nucleon states ⟹ IR cutoff

Lattice	Harmonic oscillator
$\Lambda \sim 1/l$	$\Lambda \sim \sqrt{N_{max}}/b$
$\lambda \sim 1/L$	$\lambda \sim 1/\sqrt{N_{max}}\,b$

Lattice Box



Harmonic Oscillator "No-Core Shell Model"



nuclear matter — Mueller *et al.* '99

few nucleons — Lee *et al.* '05

…

finite nuclei — Stetcu *et al.* '06

…

$$\cot\delta(E) = \frac{4}{\sqrt{m_N E L}}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - m_N E L^2} - \frac{L}{l}\right]$$

Lüscher '91

…

$$\cot\delta(E) = -\frac{2}{\sqrt{m_N E b}}\,\frac{\Gamma\left(\frac{3}{4} - \frac{m_N E b^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{m_N E b^2}{2}\right)}$$

Busch *et al.* '99

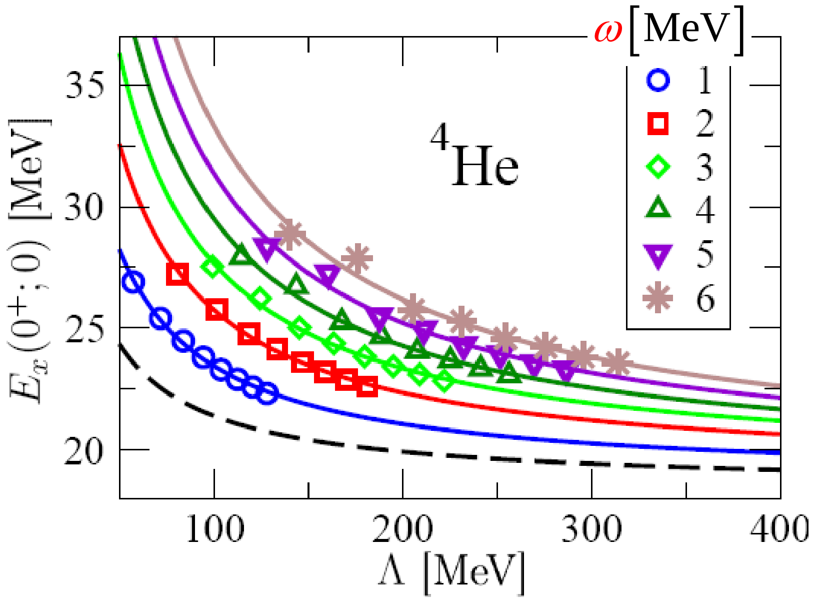
…

Stetcu, Barrett + v.K. '06

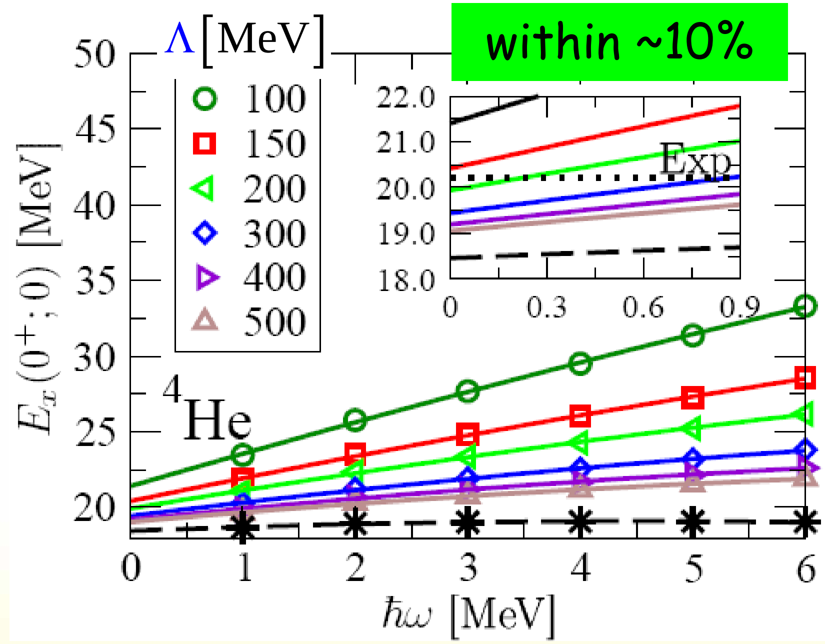
Pionless EFT: LO

(parameters fitted to d, t, α ground-state binding energies)

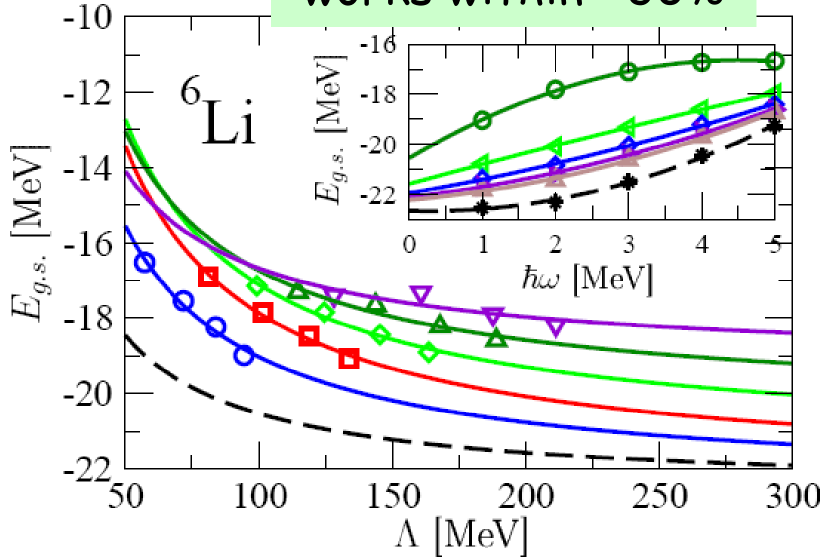
$N_{max} \leq 16$



within ~10%



works within ~30%

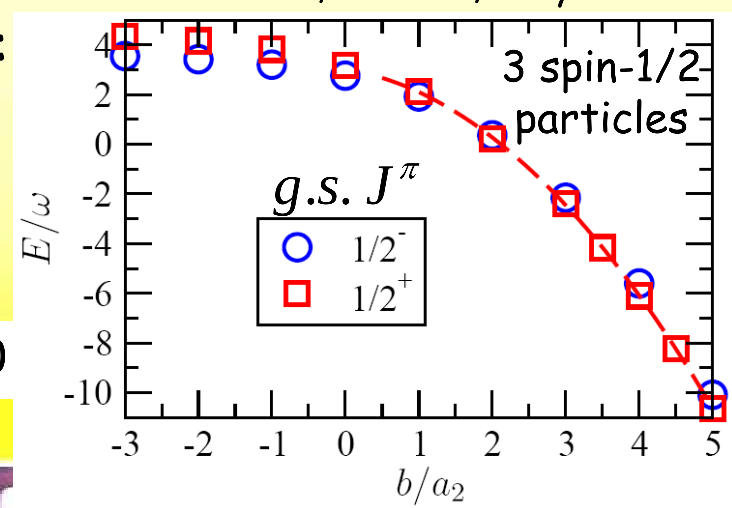


$N_{max} \leq 8$

$N_{max} \lesssim 30$

Bonus:

Stetcu, Barrett, Vary + v.K. '08



Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher, Barnea, Gazit, Pederiva + v.K. '15
Contessi, Lovato, Pederiva,
Roggero, Kirscher + v.K. '17

Experimental and LQCD data

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LO pionless fit:

m_N, $C_{0(0)}$, $C_{0(1)}$, D_0

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Ab initio methods employed

Effective-Interaction Hyperspherical Harmonics (EIHH)

Barnea *et al.* '00' 01

- ✓ hyperspherical coordinates: hyperradius + 3*A*-4 hyperangles
- ✓ model space: hyperangular momentum $K \leq K_{max}$
- ✓ wavefunction: expanded in antisymmetrized spin/isospin states
- ✓ effective interaction: Lee-Suzuki projection to subspace "in medium"
- ✓ extrapolation: $K_{max} \rightarrow \infty$

Refined Resonating Group Method (RRGM)

Hoffmann '86

- ✓ wavefunction: expanded in overcomplete basis of Gaussians in all cluster channels
- ✓ Kohn-Hulthen variational approach minimizing reactance matrix
- ✓ convergence (heavier channels, higher partial waves, Gaussian set) tested

Auxiliary-Field Diffusion Monte Carlo (AFDMC)

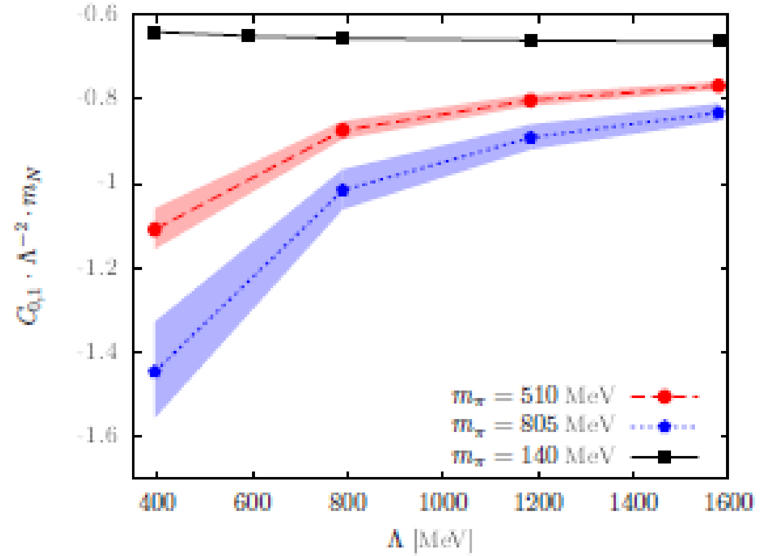
Schmidt + Fantoni '99

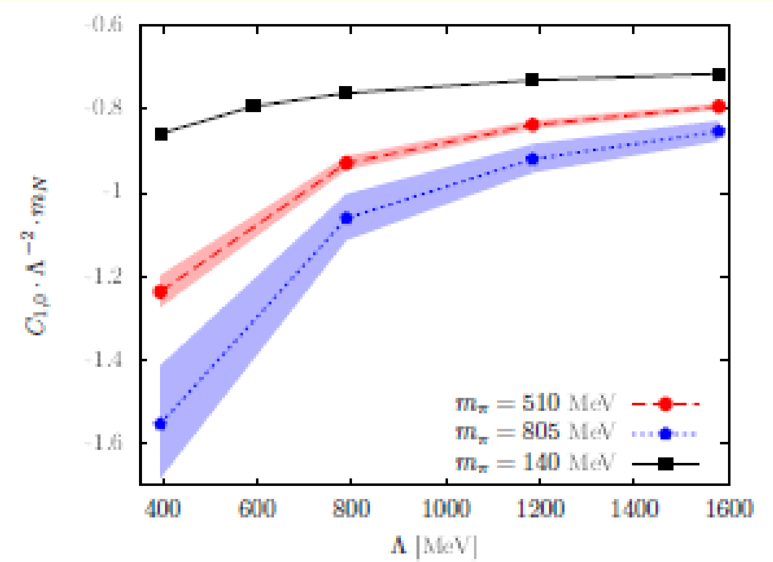
- ✓ integral equation for evolution of wavefunction in imaginary time τ in terms of Green's function (diffusion)
- ✓ two- and more-body operators linearized by auxiliary fields (Hubbard-Stratonovich transformation)
- ✓ trial wavefunction probed stochastically with weight given by the Green's function
- ✓ lowest-energy state with symmetries projected onto as τ --> ∞

Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher, Barnea, Gazit, Pederiva + v.K. '15

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4} + \sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$





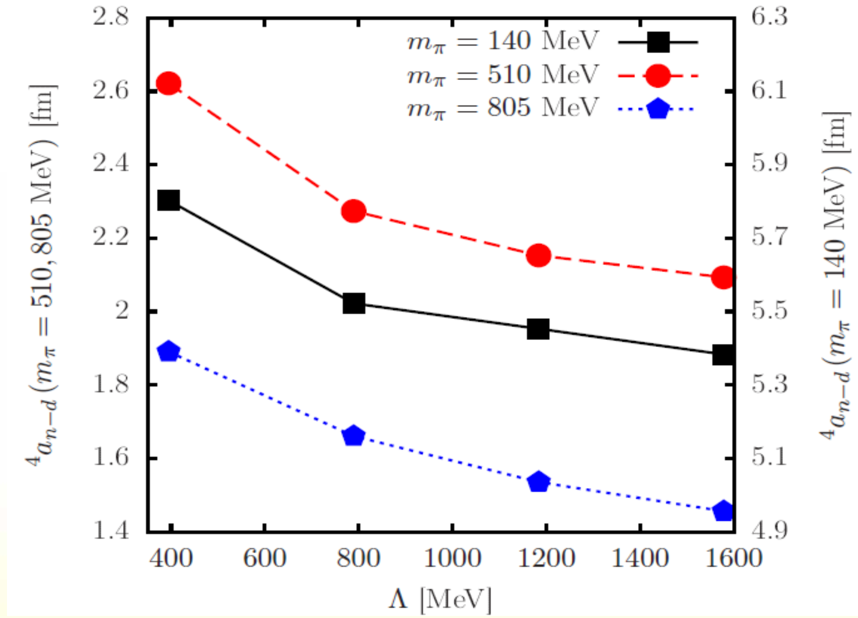
$$\frac{m_N}{\Lambda^2}C_{SI}(\Lambda) = \theta_0 + \frac{\theta_1}{a_0^{(s)}\Lambda} + O\left(\left(a_0^{(s)}\Lambda\right)^{-2}\right)$$

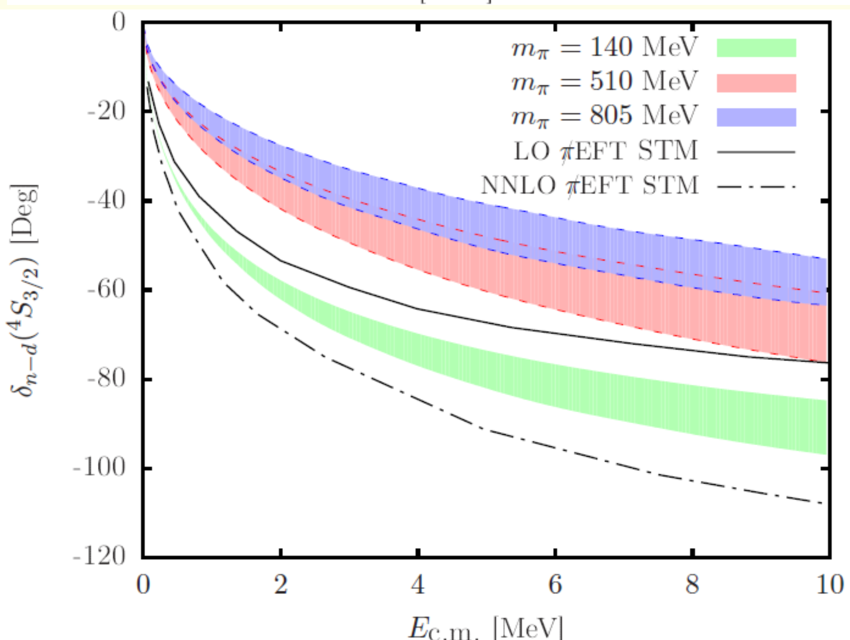
$$\theta_0 = -0.7$$

$$a^{(^3S_1)} = (1.2 \pm 0.5)\ \text{fm}$$

cutoff variation 2 to 14 fm^{-1}

Neutron-deuteron scattering: quartet





	$\not\pi$EFT	$\not\pi$EFT	
m_π [MeV]	140	510	805
$^4a_{nD}$ [fm]	5.5 ± 1.3	2.3 ± 1.3	1.6 ± 1.3
	experiment [79]	LQCD	
$^4a_{nD}$ [fm]	6.4 ± 0.020	?	?

for all masses $\quad {}^4a_{nD} = \mathcal{O}\left(1/k_{pn}\right)$

$$k_{pn} = \sqrt{4m_N B_D/3}$$

deuteron break-up threshold

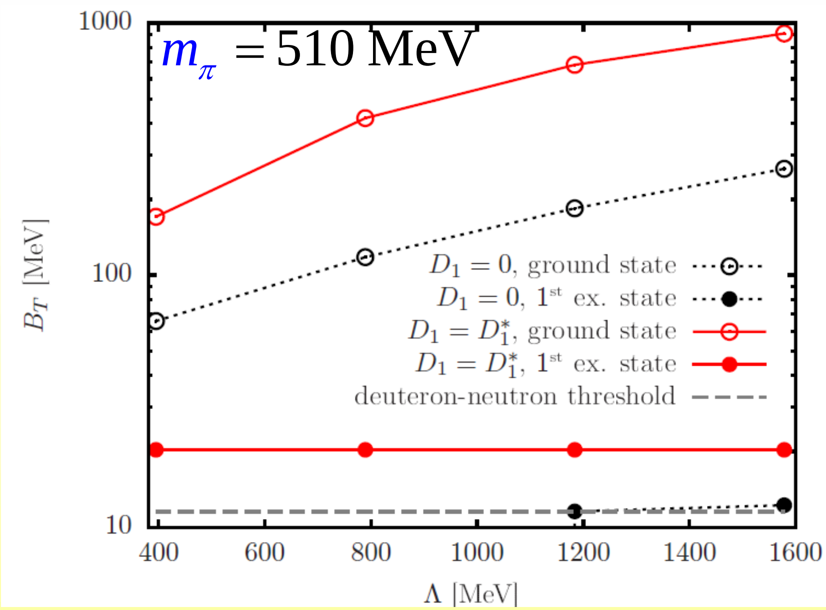
Kirscher, Barnea, Gazit, Pederiva + v.K. '15

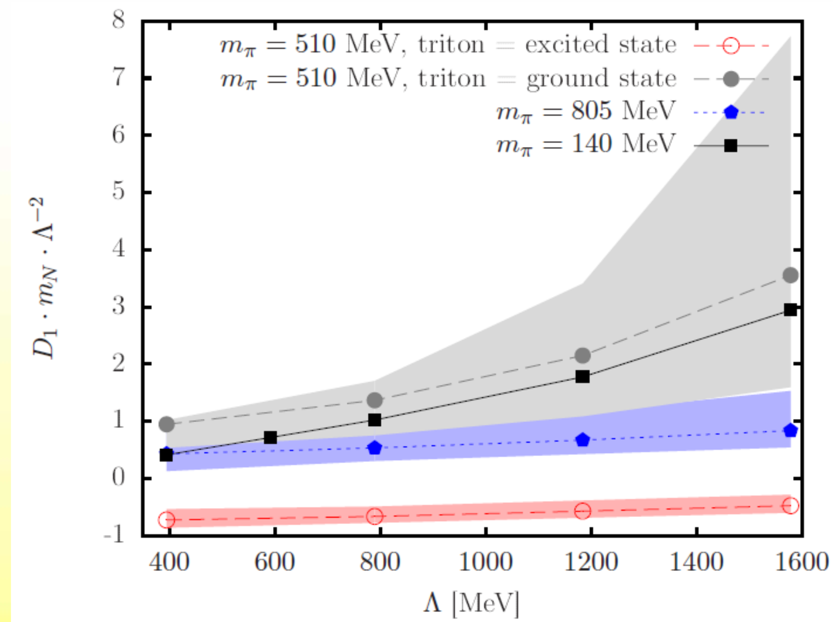
Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher, Barnea, Gazit, Pederiva + v.K. '15

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4} + \sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

$$\frac{m_N}{\Lambda^2}D_1(\Lambda) = F\left(\Lambda/\Lambda_*\right)$$

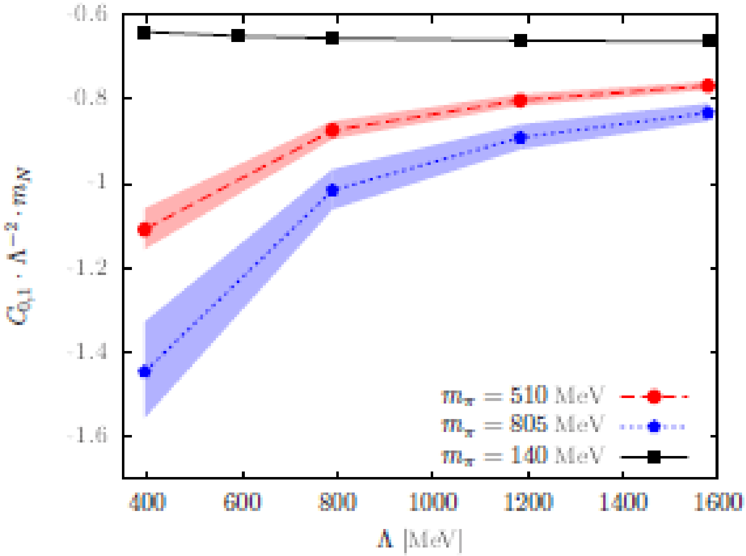


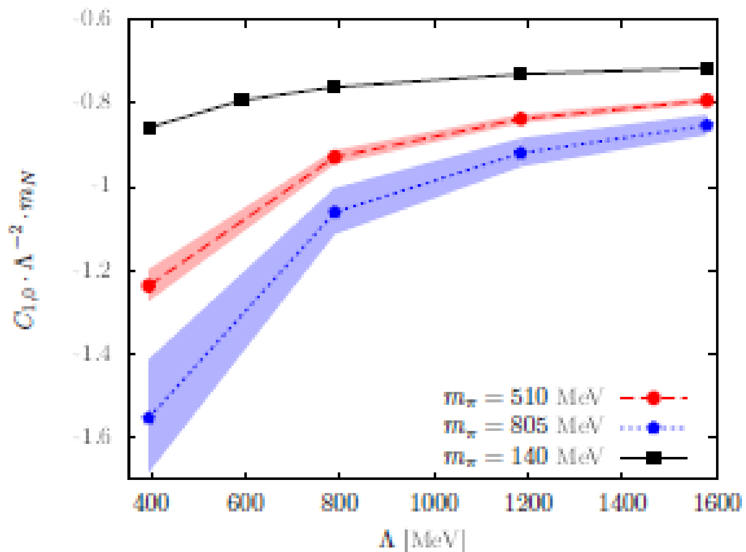


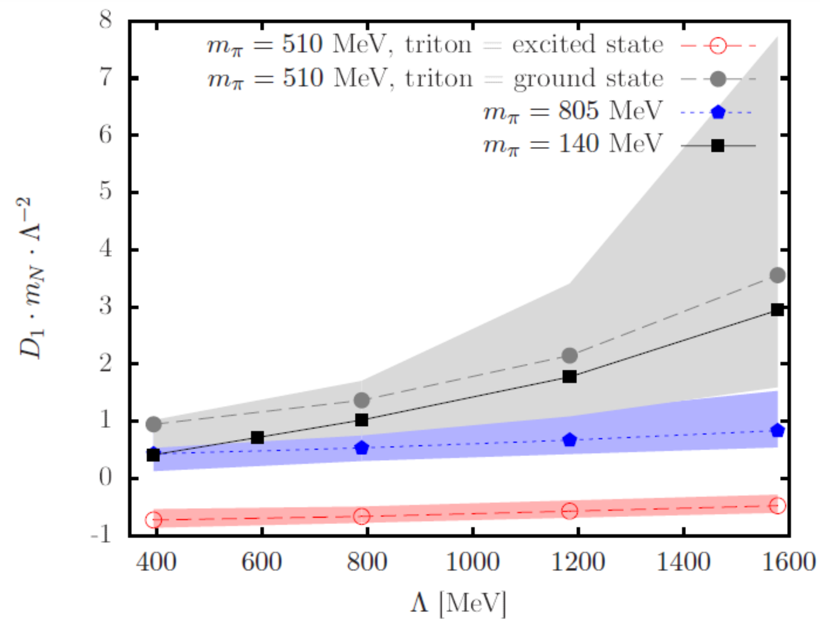
Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher, Barnea, Gazit, Pederiva + v.K. '15

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4} + \sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$



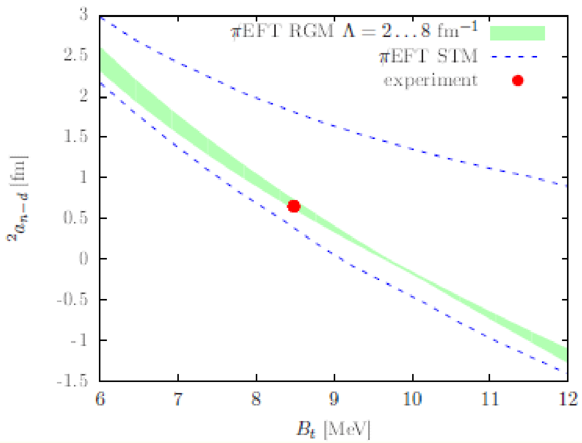


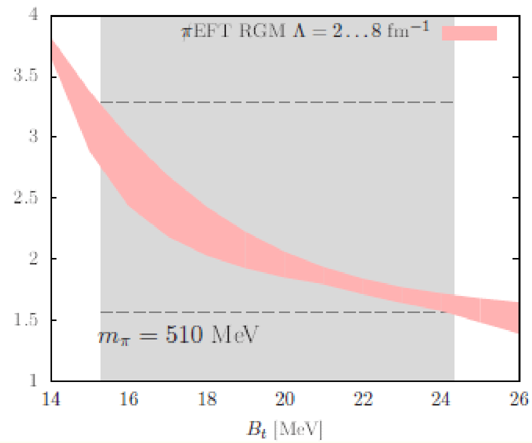


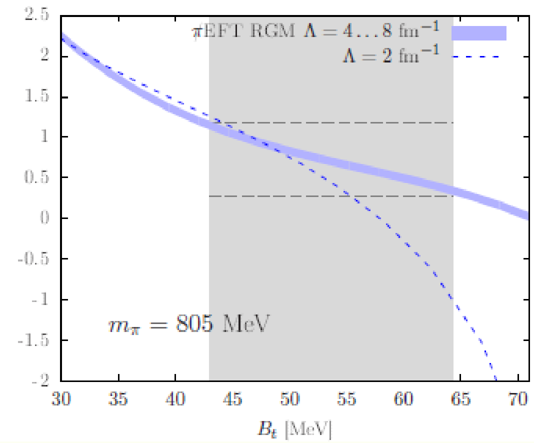
Kirscher, Barnea, Gazit, Pederiva + v.K. '15

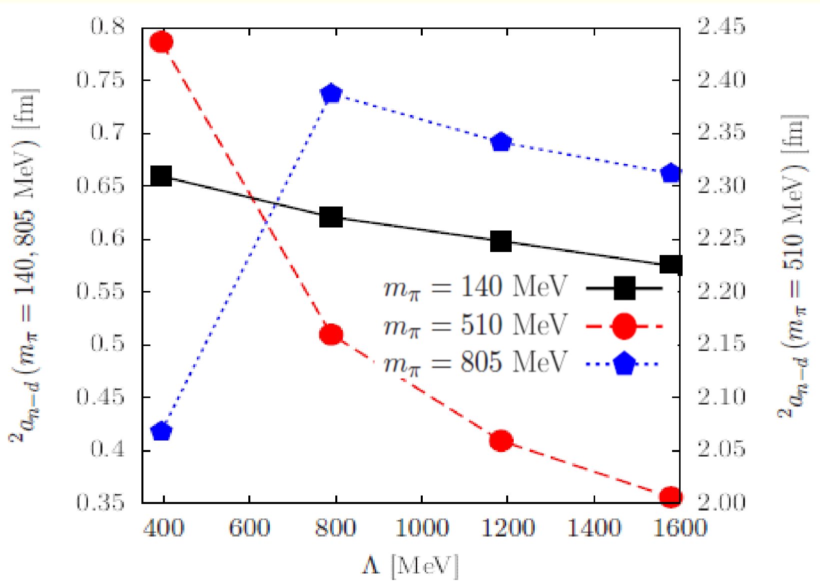
Neutron-deuteron scattering: doublet

Phillips correlations at various pion masses









	$\not\pi$EFT	$\not\pi$EFT	
m_π [MeV]	140	510	805
$^2a_{\mathrm{nD}}$ [fm]	0.61 ± 0.50	2.2 ± 2.1	0.62 ± 1.0
	experiment [79]	LQCD	
$^2a_{\mathrm{nD}}$ [fm]	0.65 ± 0.040	?	?

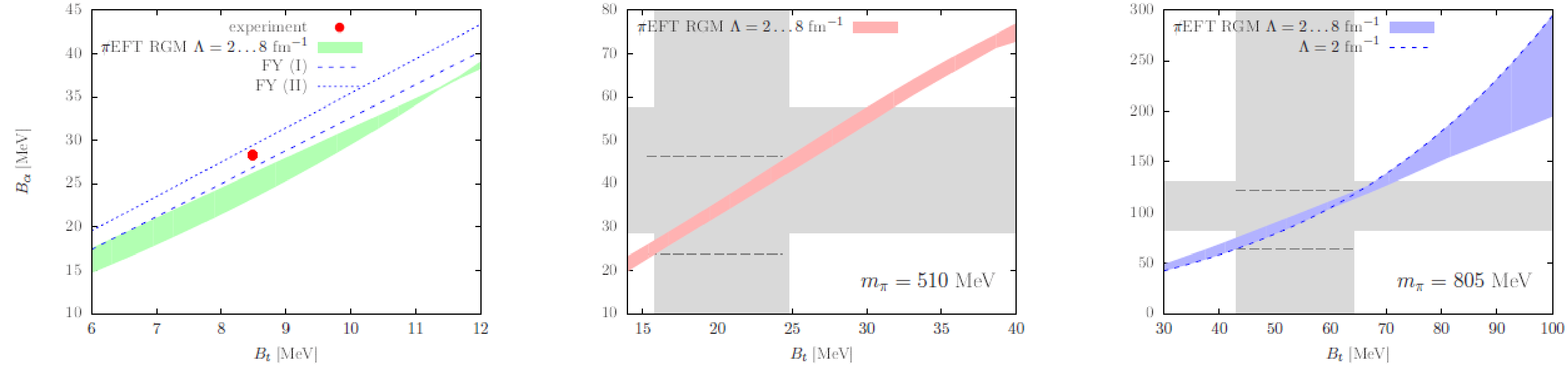
zero of T
close to threshold

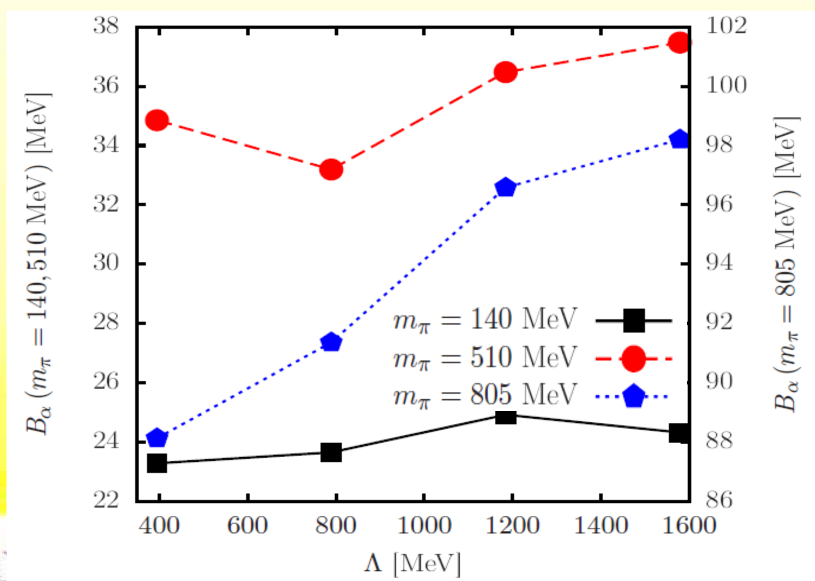
$$^4a_{nD} = \mathcal{O}\left(1/k_{pn}\right)$$

Kirscher, Barnea, Gazit, Pederiva + v.K. '15

Alpha Particle

Tjon correlations at various pion masses





m_π [MeV]	140	510	805
	$\pi\!\!\!/$EFT	$\pi\!\!\!/$EFT	
B_α [MeV]	24.9 ± 4.3	35 ± 22	94 ± 45
B_α/B_T	2.9 ± 0.51	1.7 ± 1.1	1.8 ± 0.9
	experiment	LQCD	
B_α [MeV]	28.3	43.0 ± 14.4	107.0 ± 24.2
B_α/B_T	3.34	2.1 ± 0.85	2.0 ± 0.6

Barnea, Contessi, Gazit, Pederiva + v.K. '13

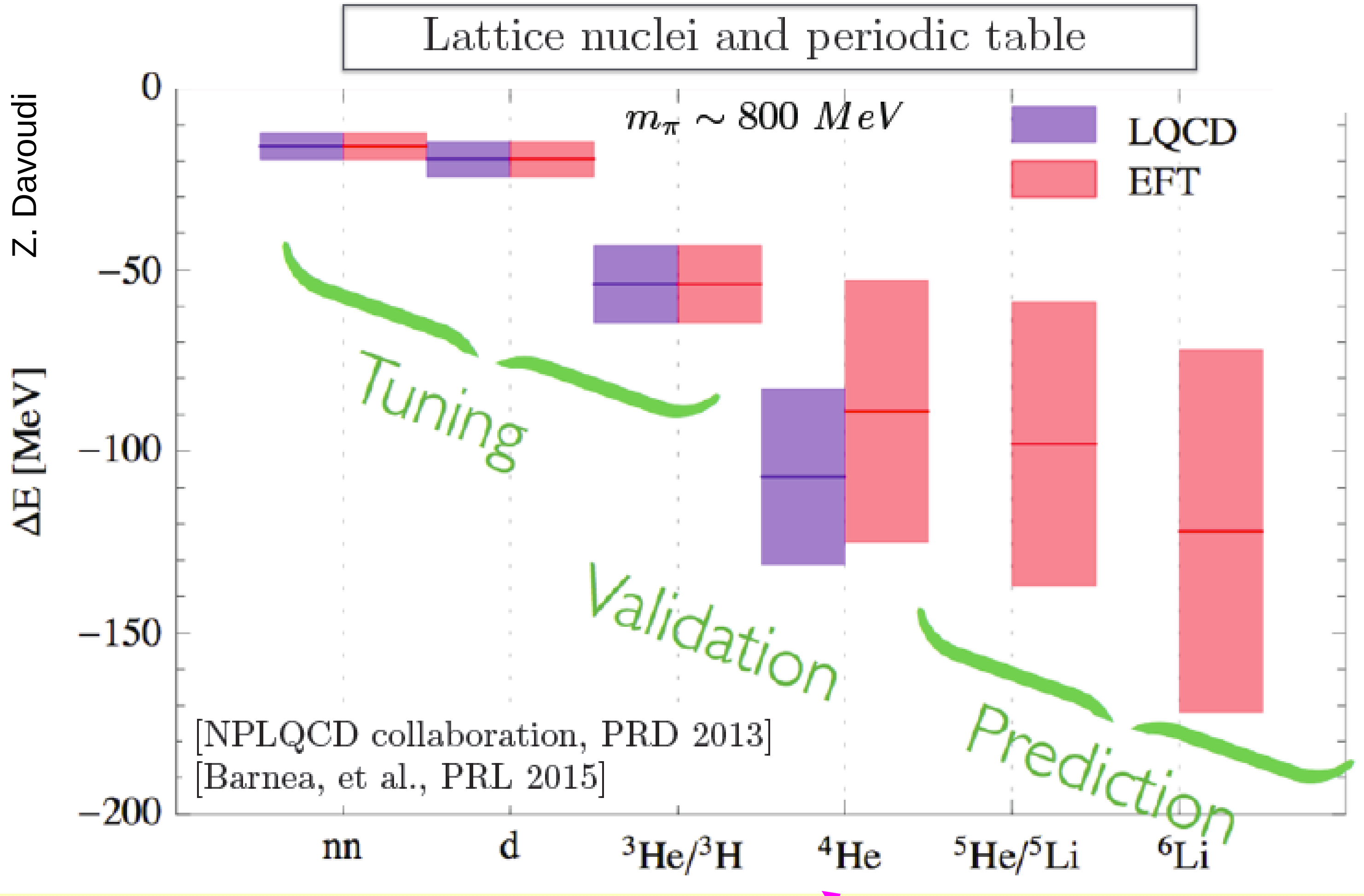
- no excited states for *A* =2,3,4
- no ^{3}n droplet

m_π Nucleus	140 [nature]	140 [23]	300 [10]	510 [7]	805 [8]	805 [4]
n	939.6	939.0 $*$	1053	1320	1634	1634 $*$
p	938.3	939.0	1053	1320	1634	1634
^{2}n	—	—	$8.5 \pm 0.7\ ^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8	15.9 ± 3.8 $*$
^{2}H	2.224	2.224 $*$	$14.5 \pm 0.7\ ^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8	19.5 ± 4.8 $*$
^{3}n	—					< 12.1
^{3}H	8.482	8.482 $*$	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7 $*$	53.9 ± 10.7 $*$
^{3}He	7.718		$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^{4}He	28.30	28.30 $*$	$47 \pm 7\ ^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
^{4}He*	8.09	10 ± 3				< 43.2
^{5}He	27.50					98 ± 39
^{5}Li	26.61					98 ± 39
^{6}Li	32.00	23 ± 7				122 ± 50

[23] Stetcu *et al.* '06
[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12
[4] Barnea *et al.* '13

predictions

Barnea, Contessi, Gazit, Pederiva + v.K. '13



consistency between EFT and LQCD, including plateau id

$B_5 \approx B_4$

A=5 gap persists!?

$$\frac{B_6}{6} \approx \frac{B_4}{4}$$

nuclear saturation survives!?

Maybe pions play less of a role than we are used to think?

A Scene from Program INT-16-1

...

--- Effective field theorist: (nonchalant)
Pionless EFT works pretty well for light nuclei.

--- Esteemed nuclear theorist: (doubtful)
But has it been applied to a *real* nucleus?

--- EFTst: (stunned)
What the hell is a *real* nucleus? When we did deuteron we were told triton was more like a real nucleus. When we did triton we were told it's the alpha particle that's a real nucleus. Now you tell me the alpha particle is *not* a real nucleus?

--- ENTst: (didactic)
No, you need to show saturation for a real nucleus like ^{16}O.

--- EFTst: (outraged)
But we have shown saturation for triton and alpha particle! It's due to the three-body force, so ^{16}O will saturate.

--- ENTst: (jaded)
I believe it when I see it.

...

Contessi, Lovato, Pederiva, Roggero, Kirscher + v.K. '17

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

$$+\sum_{i<j}\left[C_1(\Lambda)+C_2(\Lambda)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4} + \sum_{i<j<k}\sum_{cyc} D_0(\Lambda)\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

^{4}He

Λ	m_π = 140 MeV	m_π = 510 MeV	m_π = 805 MeV
2 fm^{-1}	-23.17 ± 0.02	-31.15 ± 0.02	-88.09 ± 0.01
4 fm^{-1}	-23.63 ± 0.03	-34.88 ± 0.03	-91.40 ± 0.03
6 fm^{-1}	-25.06 ± 0.02	-36.89 ± 0.02	-96.97 ± 0.01
8 fm^{-1}	-26.04 ± 0.05	-37.65 ± 0.03	-101.72 ± 0.03
$\to \infty$	$-30^{\pm 0.3\,(\mathrm{sys})}_{\pm 2\,(\mathrm{stat})}$	$-39^{\pm 1\,(\mathrm{sys})}_{\pm 2\,(\mathrm{stat})}$	$-124^{\pm 3\,(\mathrm{sys})}_{\pm 1\,(\mathrm{stat})}$
Exp.	-28.30	–	–
LQCD	–	-43.0 ± 14.4	-107.0 ± 24.2

within previous error bar

in good agreement with
- previous calculations
- experiment

^{16}O

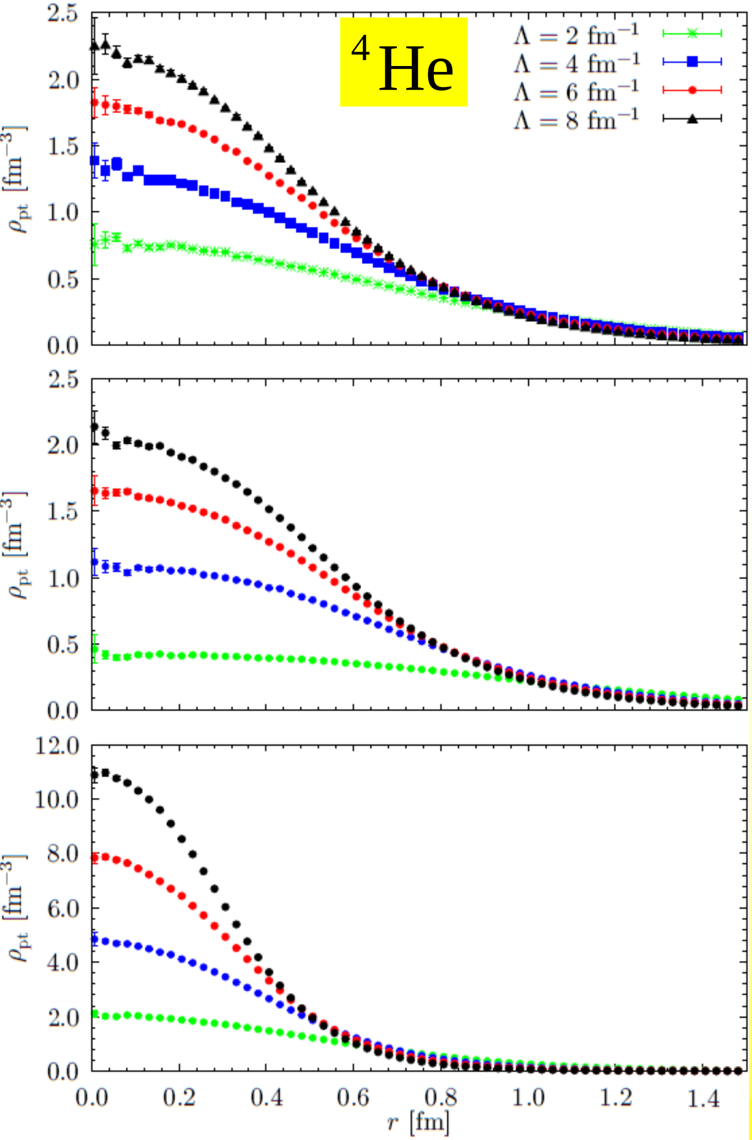
Λ	m_π = 140 MeV	m_π = 510 MeV	m_π = 805 MeV
2 fm^{-1}	-97.19 ± 0.06	-116.59 ± 0.08	-350.69 ± 0.05
4 fm^{-1}	-92.23 ± 0.14	-137.15 ± 0.15	-362.92 ± 0.07
6 fm^{-1}	-97.51 ± 0.14	-143.84 ± 0.17	-382.17 ± 0.25
8 fm^{-1}	-100.97 ± 0.20	-146.37 ± 0.27	-402.24 ± 0.39
$\to \infty$	$-115^{\pm 1\,(\mathrm{sys})}_{\pm 8\,(\mathrm{stat})}$	$-151^{\pm 2\,(\mathrm{sys})}_{\pm 10\,(\mathrm{stat})}$	$-504^{\pm 20\,(\mathrm{sys})}_{\pm 12\,(\mathrm{stat})}$
Exp.	-127.62	–	–

indistinguishable from four-alpha threshold; if anything, too much saturation…

Alpha-particle clusterization

Contessi, Lovato, Pederiva, Roggero, Kirscher + v.K. '17

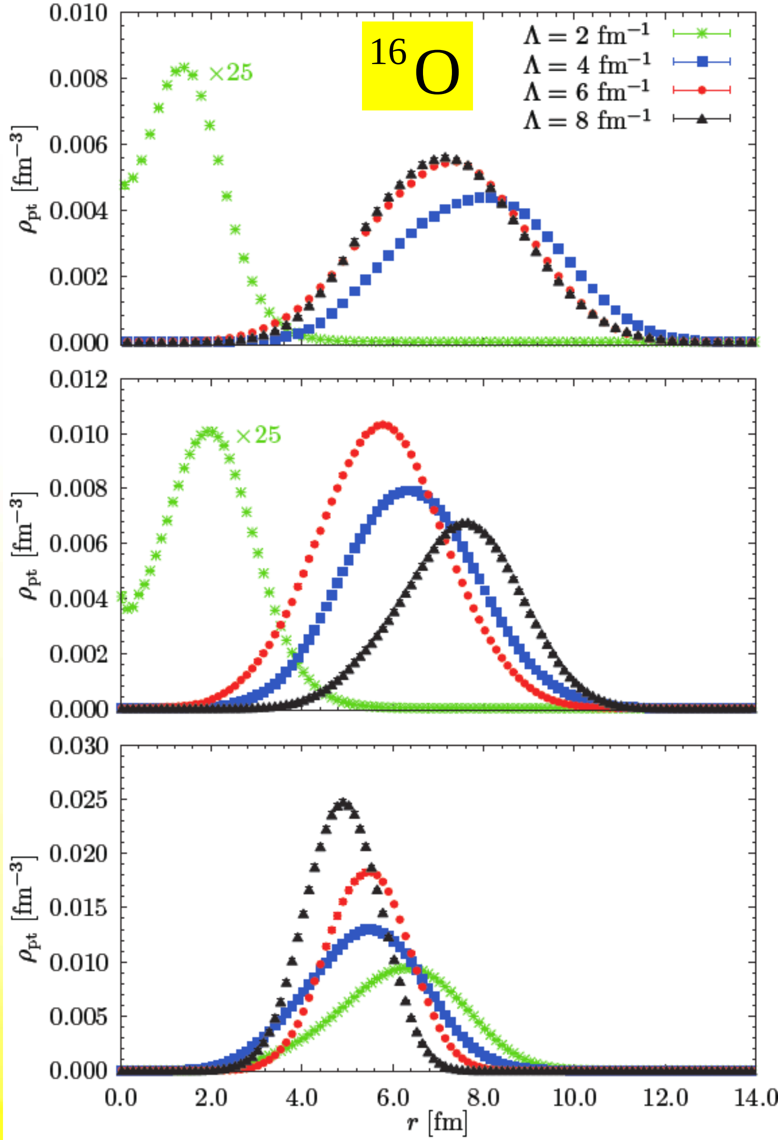
Single-nucleon point densities



$m_\pi = 140$ MeV

$m_\pi = 510$ MeV

$m_\pi = 805$ MeV



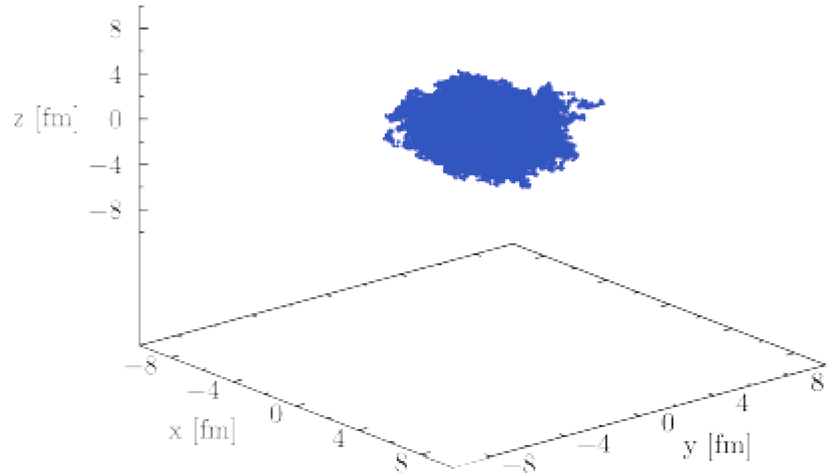
Contessi, Lovato, Pederiva, Roggero, Kirscher + v.K. '17

^{16}O

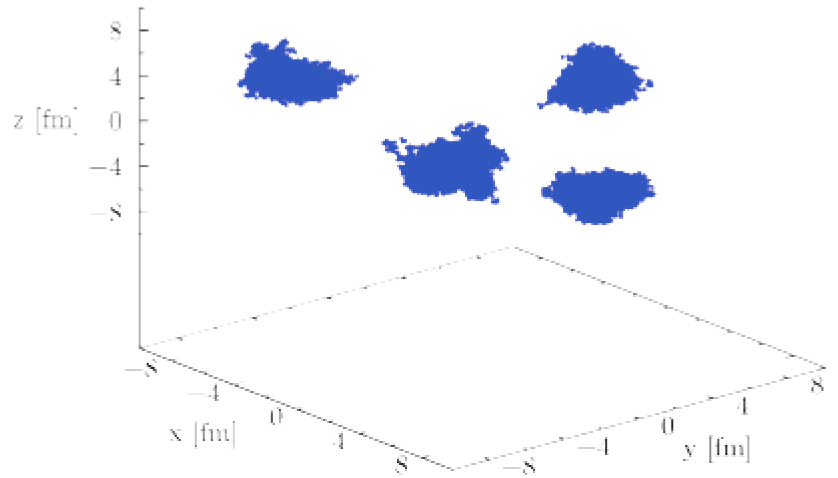
imaginary-time diffusion of a single walker

$m_\pi = 140$ MeV

$\Lambda = 2\ \mathrm{fm}^{-1}$



$\Lambda = 8\ \mathrm{fm}^{-1}$



What next?

- NLO   Bazak *et al.*, in progress
- Other *A*
- hypernuclei
- Chiral EFT at lower m_π (when available)
- ...

Conclusion

- EFT is constrained *only* by symmetries and thus can be matched onto lattice QCD

- EFT allows controlled extrapolations of lattice results in nucleon number (and pion mass), including reactions!

- First, proof-of-principle calculations carried out at $m_\pi \approx 500, 800$ MeV with Pionless EFT

- World at large pion mass *might* be just a denser version of ours