One-nucleon Transfers to Resonances

INT, Seattle.

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Traditional Role of (d,p) reactions

- **Transfer neutron to // bound** state $\phi_{nlj}(r_n)$
	- usually large momentum transfer
	- Shape of proton $\sigma_{p}(\theta)$ depends on *l*
	- Analyzing powers depend on *j*
	- Magnitude of depends on spectroscopic factor S_{nli}
- **Higher-order corrections** calculable (CRC, CCBA)

Measuring resonances with (d,p)

- Do not probe spectroscopic factors
- Do probe partial widths and resonance energies
	- These parameters come from R-matrix energies and reduced width amplitudes
- Desirable to have (d,p) calculations from R-matrix parameters
	- Is this possible?
	- Can then predict n+target scattering not otherwise measurable
	- But R-matrix values are surface and external features!
- Note: *L*-dependence of $\sigma_p(\theta)$ decreases for less bound neutrons, and hence for continuum neutrons
	- Reduced magnitude of momentum transfers

Propose to use: Surface Transfer Operator

- Work of TORUS collaboration (www.reactiontheory.org)
- § Proposed: Mukhamedzhanov (PRC **84**, 044616, 2011)
- Developed for 1-step transfers:
	- Escher et al., (PRC. **89**, 054605, 2013)
	- Escher et al., (J. Phys.: Conf. Ser. **403** 012026, 2012)
- Now applied to transfers from entrance deuteron channels including breakup in CDCC basis.

Applicable Examples of Resonances

- Near single-particle resonances Structured p-shell resonances
-

Purpose of my transfer calculations

- Aim is to fit resonances in (d,p) cross sections in a region of the continuum. - 1 \mathbf{F}
- § We see many wide and narrow resonances, often overlapping. $\overline{}$ \mathbf{v} **1 o 1 o 1 o 1 o 1 o 1 o 1 o 1 o 1**
- Want to find neutron pole energies and partial widths, in entrance channel for (n, γ) ali vi i noje 5.0~, <u>10. IUL IUL III.</u> En(MeVI

a uniform error of 10 % attributed to the scanning procedure. The 0L = 15 ° data were taken from ref. 7). Note that the energy scales of (n, n) and (d, p) do not coincide exactly.

 \overline{a} z.

©

Post-prior equivalence in 1st order

- Post matrix element in 1st order $M_{dp}^{(post)} = \langle \phi_A^F \chi_{pF}^{(-)} | \Delta \underline{V}_p | \phi_d \chi_{dA}^{(+)} \rangle$
- Prior matrix element in 1st order $M_{dp}^{(prior)} = \langle \phi_A^F \chi_{pF}^{(-)} | \Delta \underline{V}_d | \phi_d \chi_{dA}^{(+)} \rangle$

$$
\begin{aligned}\n\text{overlap} \\
\phi_{A}{}^{F}(r_{n}) &= \\
<\phi_{A} | \phi_{F} >
$$

- **Equivalent:** $M_{dp}^{(post)} = M_{dp}^{(prior)}$ because
	- KE operators satisfy $\langle \phi_A^F(r_n) | T_n + T_p | \phi_d(r) \rangle = 0$
	- *Because* $\phi_A^F(r_n) \rightarrow 0$ at $r_n=0$ and $r_n \rightarrow \infty$
- If the wave functions not zero, then get surface terms.

Splitting the Transfer Matrix Element % **x Element**

- Define M_{post} (a,b) & M_{prior} (a,b) with $a < r_n < b$ limits μ, ω we approximation to the volume volume volume value of μ
- General result: $M_{\text{post}}(a,b) = M_{\text{surf}}(a) + M_{\text{prior}}(a,b) - M_{\text{surf}}(b)$ where $M_{\text{surf}}(\rho) = \langle f_p^{(-)} \phi_n | [\tilde{T} - \vec{T}] | \phi_d f_d^{(+)} \rangle_{(r \le \rho)}$ $\overline{2}$ $\mathsf{null}:$ into the surface one, which encircles the inner volume $\mathsf{null}:$ $\overline{}$ ϕ _n | [\overleftarrow{T} − \overrightarrow{T}]| ϕ _d 1
	- Previous slide used $M_{surf}(0) = M_{surf}(\infty) = 0$ r!R usea ivi_{si} d**S**[g(**r**)∇**r**f (**r**) − f (**r**)∇**r**g(**r**)]
- is is So, for any surface *ρ*: from: $M = M_{\text{post}}(0, \rho) + M_{\text{post}}(\rho, \infty)$ $hence \t M = M_{post}(0, \rho) + M_{surf}(\rho) + M_{prior}(\rho, \infty)$ $\overline{}$ radius $\overline{}$ $frac$ e p \bigcap * g(**r**) amedzł **Production** $\left(\frac{1}{2011} \right)$ the unit vector **rˆ** is the normal vector to the sphere directed Mukhamedzhanov (2011):

h(1)

 $\overline{}$

% %

It is worth mentioning that, owing to the presence of the

∂g(**r**)

r=R

Evaluating the Surface Matrix Element % <mark>ce Matrix Elem</mark> age the Currence Motrix Element over the coordinate **r**:

With $M_{\text{surf}}(\rho) = \langle f_p^{(-)} \phi_n | [\overline{T} - \overrightarrow{T}] | \phi_d f_d^{(+)} \rangle_{(r \le \rho)}$ lna Wit h l \overline{a} ϕ _n | [\overleftarrow{T} − \overrightarrow{T}]| ϕ _d.

• Need to calculate matrix elements like: over the coordinate **r**: r ile

Calculate that k elements are:

\n
$$
\int_{r \le R} dr f(\mathbf{r}) \left[\overline{T} - \overline{T} \right] g(\mathbf{r})
$$
\n
$$
= -\frac{1}{2\mu} \oint_{r=R} dS[g(\mathbf{r}) \nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r}) \nabla_{\mathbf{r}} g(\mathbf{r})]
$$
\n
$$
= -\frac{1}{2\mu} R^2 \int d\Omega_{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right]_{r=R}
$$
\nTherefore, $\Omega_{\mathbf{r}} = 0$, where $\Omega_{\mathbf{r}} = 0$ and $\Omega_{\mathbf{r}} = 0$, and $\Omega_{\mathbf{r}} = 0$.

(**k**pF , **^k**dA) ⁺ ^MDW

SRnA

That is, functions & derivatives on the surface r_n= ρ becomes crucial with increasing binding energy. We use also Functions α derivatives on the surface t_r inctions α derivatives on the surface $v_n - p$

SRnA

Do this for partial waves, in reaction code FRESCO for partial waves, in reaction coue in Eq. (24) over **r**nA is taken over the external volume restricted Do this for partial waves, in reaction code by two spherical surfaces: the inner surface with the radius RnA

and the external surface with the radius R′

h(1)

 $\frac{1}{2}$ \mathbb{R}^n \mathbb{R}^n %

 $\overline{}$

It is worth mentioning that, owing to the presence of the

It is worth mentioning that, owing to the presence of the

channel radius RnA, the reduced width, in contrast to the ANC,

is model-dependent. The dependence on the channel radius

becomes crucial with increasing binding energy. We use also

the boundary condition, which is the logarithmic derivative of

 $\mathbb{R}^{\mathbb{N}}$

is also small. In the external region, rnA > RnA, the nuclear

channel radius RnA, the reduced width, in contrast to the ANC,

is model-dependent. The dependence on the channel radius

lnA (iκnARnA)

ext can be

r=R

nA → ∞; that is,

Preliminary Estimates of Magnitudes

• In DWBA (1st order), find surface term as: $M_{\text{surf}}(\rho) = M_{\text{post}}(0,\rho) - M_{\text{prior}}(0,\rho)$

- Look at bound states and resonances.
	- See if convergence to breakup states is easier?
- § Calculate all terms of $M = M_{\text{post}}(0,\rho) + M_{\text{surf}}(\rho) + M_{\text{prior}}(\rho,\infty)$

Internal, surface, external contributions – $^{90}Zr(d,p)$ **at E_d=11 MeV**

$$
M = M^{(post)}(0, a) + M_{(surf)}(a) + M^{(prior)}(a, \infty)
$$

model dependence asymptotic quantities

Observations

- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

Escher, Thompson, Mukhamedzhanov, JPCS (2012).

Peak cross section relative to full calculation

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Numerical tests of the formalism (DWBA) 48 Ca(d,p) at $E_d=13$, 19.3, 56 MeV

Surface formalism for DWBA – resonance states

Total post matrix element for $b + B \neq n + A$ example:

$$
M^{\text{DW}(post)}(P, \mathbf{k}_{dA}) = 2\pi \sqrt{\frac{1}{\mu_{bB}k_{bB}}} \sum_{J_{F}M_{F}s'l'm_{\pi'}m_{\mu'}M_{\alpha}} i^{l} \langle s m_{s}l m_{l} | J_{F}M_{F} \rangle \langle s' m_{s'}l' m_{l'} | J_{F}M_{F} \rangle \langle J_{n}M_{n} J_{A}M_{A} | s' m_{s'} \rangle \langle J_{n}M_{n} J_{p}M_{p} | J_{d}M_{d} \rangle
$$

\n
$$
\times e^{-i\delta_{bB}^{kt}} Y_{lm_{l}}^{*}(-\hat{\mathbf{k}}_{bB}) \sum_{v,\tau=1}^{N} [\Gamma_{v b B s l J_{F}}(E_{bB})]^{1/2} [\mathbf{A}^{-1}]_{v\tau} \left\{ \langle \chi_{pF}^{(-)} I_{A s' l' J_{F}}^{F} | \Delta \overline{V}_{pF} | \varphi_{d} \chi_{dA}^{(+)} \rangle \right\}_{r_{aA} \leq R_{aA}}
$$

\n
$$
+ \sqrt{\frac{2\mu_{nA}}{R_{nA}} \gamma_{\tau n A s' l' J_{F}} \left\langle \chi_{pF}^{(-)} \frac{O_{l}^{*}(k_{nA}, r_{nA})}{r_{nA}} \frac{R_{nA}}{O_{l}^{*}(k_{nA}, R_{nA})} Y_{l'm_{l'}}^{*}(\hat{\mathbf{r}}_{nA}) \right\} \Delta \overline{V}_{dA} \left| \varphi_{d} \chi_{dA}^{(+)} \right\rangle \right\}_{r_{aA} > R_{aA}} + \sqrt{\frac{R_{nA}}{2\mu_{nA}} \gamma_{\tau n A s' l' J_{F}}}
$$

\n
$$
\times \int d\mathbf{r}_{pF} \chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{aA}} Y_{l'm_{l'}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_{d}(\mathbf{r}_{p n}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})(B_{nA} - 1) - R_{nA} \frac{\partial \varphi_{d}(\mathbf{r}_{p n}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \
$$

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The oxygen case - ²⁰O at E_d=21 MeV Angular cross section – Surface term only Peak cross section relative to full calculation $50₁$ (b) $d_{2/2}$, E=4.77 MeV **21O @ 4.77 MeV (3/2+)** 1.8 40 $a=5.0$ fm 1.6 Cross section [mb/sr] 1.4 **int-post** 1.2 **Surface at ~5 fm** $\begin{array}{c} 1 \\ 1 \end{array}$ **R**_x $\begin{array}{c} 1 \\ 1 \end{array}$ **ext-prior surf** approximately 20 0.8 reproduces 0.6 measurement. 0.4 10 0.2 $\bf{0}$ 10 12 16 18 20 50 10 20 30 40 60 Angle [deg] 2 **21O @ 6.17 MeV (3/2+)** 1.8 (d) d_{30} , E=6.17 MeV 1.6 $a=5.5$ fm 40 1.4 $Cross section [mb/sr]$
 $\frac{\omega}{\omega}$ experiment 1.2 $\mathbf{R}_\mathbf{X}$ surface -1 $surface + priorExt$ 0.8 exact 0.6 0.4 10 0.2 0 $\bf{0}$ 10 12 14 16 18 20 $a [fm]$ 50 60 10 20 30 40 $\frac{10}{10}$ 20 $\frac{30}{15}$ 40 $\frac{50}{10}$ 60 Escher et al, PRC 89, 054605 (2014) LLNL-PRES-673281

Resonances – ⁹⁰Zr at E_d=11 MeV

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The surface formalism: can we save it? ²⁰O at E_d=21 MeV

Extension of the formalism to include breakup

DWBA matrix element

CDCC (Continum-discretized coupled channels)

- Approximate treatment of 3-body problem
- Describes breakup of deuteron

- Successfully used for describing data
- Currently revisited via comparison with Fadeev

CDCC matrix element

CDCC extension of R-matrix formalism

- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant

Derivatives of products of 2 wfns 23 Dr products of 2 Wills

$$
\frac{\partial}{\partial r'} \langle \hat{R}, \hat{r} | \alpha \rangle \frac{1}{rR} \varphi_{\alpha}(r) u_{\alpha}(R) = \frac{1}{rR} \sum_{M_L m_\ell} C_{\alpha}^{M_L m_\ell : M}
$$
\n
$$
\left(Y_L^{M_L}(\hat{\mathbf{R}}) u_{\alpha}(R) \frac{p}{r} \left\{ \sqrt{\frac{4\pi \ell (2\ell+1)}{3}} \sum_{\lambda=-1}^1 \langle \ell-1 | m-\lambda, 1\lambda | \ell m \rangle Y_{\ell-1}^{m-\lambda}(\hat{\mathbf{r}}) Y_1^{\lambda}(\hat{\mathbf{r}}') \frac{\varphi_{\alpha}(r)}{r} + Y_\ell^m(\hat{\mathbf{r}}) \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' \left[\varphi_{\alpha}'(r) - \frac{\ell+1}{r} \varphi_{\alpha}(r) \right] \right\}
$$
\n
$$
+ Y_l^{m_\ell}(\hat{\mathbf{r}}) \varphi_{\alpha}(r) \frac{P}{R} \left\{ \sqrt{\frac{4\pi L (2L+1)}{3}} \sum_{\Lambda=-1}^1 \langle L-1 | M_L - \Lambda, 1\Lambda | L M_L \rangle Y_{L-1}^{M_L - \Lambda}(\hat{\mathbf{R}}) Y_1^{\Lambda}(\hat{\mathbf{r}}') \frac{u_{\alpha}(R)}{R} + Y_L^{M_L}(\hat{\mathbf{R}}) \hat{\mathbf{R}} \cdot \hat{\mathbf{r}}' \left[u_{\alpha}'(R) - \frac{L+1}{R} u_{\alpha}(R) \right] \right\} \right)
$$
\n
$$
\mathbf{r} = p \mathbf{r}' + q \mathbf{R}'
$$
\n
$$
\mathbf{R} = P \mathbf{r}' + Q \mathbf{R}'
$$

*[|]*h*R,* ^ˆ *^r*ˆ*|*↵ⁱ ¹

`:*M*⇤ ^h*^Y ^M*⁰

*^L*⁰ (Rˆ⁰

) *Y ^m*⁰

 \sim

)*|* @

*Lm*⁰

*, r*ˆ0

*F ^M*⁰

 \sim

Source term complete for r,R wfns

$$
S_{\beta\alpha}^{\text{surf}}(R') = -\frac{\hbar^2 \rho^2}{2\mu_n} \sum_{M'_L m'_\ell M_L m_\ell} F_{\beta}^{M'_L m'_\ell : M*} C_{\alpha}^{M_L m_\ell : M} \langle Y_{L'}^{M'_L}(\hat{\mathbf{R}}) Y_{\ell'}^{m'_\ell}(\hat{\mathbf{r}}')|_{r'=\rho} \frac{1}{rR}
$$

$$
\begin{bmatrix} \phi'_{\beta}(\rho) Y_{\ell}^{m_{\ell}}(\hat{\mathbf{r}}) Y_{L}^{M_L}(\hat{\mathbf{R}}) \varphi_{\alpha}(r) u_{\alpha}(R) \\ -\phi_{\beta}(\rho) \left(Y_{L}^{M_L}(\hat{\mathbf{R}}) u_{\alpha}(R) \frac{p}{r} \left\{ \sqrt{\frac{4\pi\ell(2\ell+1)}{3}} \sum_{\lambda=-1}^{1} \langle \ell-1 \ m-\lambda, 1\lambda | \ell m \rangle Y_{\ell-1}^{m-\lambda}(\hat{\mathbf{r}}) Y_{1}^{\lambda}(\hat{\mathbf{r}}') \frac{\varphi_{\alpha}(r)}{r} \right. \\ + Y_{\ell}^{m}(\hat{\mathbf{r}}) \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' \left[\varphi'_{\alpha}(r) - \frac{\ell+1}{r} \varphi_{\alpha}(r) \right] \right\}
$$

$$
+ Y_{\ell}^{m_{\ell}}(\hat{\mathbf{r}}) \varphi_{\alpha}(r) \frac{P}{R} \left\{ \sqrt{\frac{4\pi L(2L+1)}{3}} \sum_{\Lambda=-1}^{1} \langle L-1 \ M_L-\Lambda, 1\Lambda | L M_L \rangle Y_{L-1}^{M_L-\Lambda}(\hat{\mathbf{R}}) Y_1^{\Lambda}(\hat{\mathbf{r}}') \frac{u_{\alpha}(R)}{R} \right. \\ + Y_L^{M_L}(\hat{\mathbf{R}}) \hat{\mathbf{R}} \cdot \hat{\mathbf{r}}' \left[u'_{\alpha}(R) - \frac{L+1}{R} u_{\alpha}(R) \right] \right\} \Bigg) \Bigg] \ .
$$

 $\mathbf{F} = \mathbf{F}^{M'_I} m'_i \colon M *_{\mathbf{C}} M_I, m_i \colon M$ \mathbf{r}_{β} are the \mathbf{r}_{α} are that The $F_\beta^{M'_L m'_\ell : M*} C_\alpha^{M_L m_\ell : M}$ are the channel-defin *rR* The $F_\beta^{m_L m_\ell.m*\,C^{M_L m_\ell:M}_\alpha}$ are the channel-defining Clebsch-Gordon coefs.

E

↵(*R*) *^L*+1

, R)*u*↵(*R*) + ^Z

Using Eq. (66), this is

*M*0 *Lm*⁰

(⇢) *^Y ^m*`

`*MLm*`

E

Implementation ⇢) = ⁸⇡2⇢

- **As** $\mathbf{R}' \neq \mathbf{R}'$ **, transfer couplings are still non-local** $\mathbf{A} \in \mathbf{D}' \perp \mathbf{D}'$ transfor countings are
- With A, B, C as non-local operators, the transferchannel exit equation is \sim channol ovit caugtion is , <u>r</u>

 $[H_{\beta}-E_{\beta}]u_{\beta} + \phi'_{\beta}(\rho)$ $A_{\beta\alpha}u_{\alpha} + \phi_{\beta}(\rho)$ $B_{\beta\alpha}u_{\alpha} + \phi_{\beta}(\rho)$ $C_{\beta\alpha}\left[u'_{\alpha}\right]$ $\frac{L_{\alpha}+1}{R}u_{\alpha}$ $\overline{\mathsf{I}}$ $\phi_{\beta}(\rho) \,\, \mathsf{B}_{\beta\alpha} u_{\alpha} + \phi_{\beta}(\rho) \,\, \mathsf{C}_{\beta\alpha} \,\, \bigg| \, u'_{\alpha} - \frac{L_{\alpha}+1}{R} u_{\alpha} \bigg| \, = 0 \quad \, \Box$

• More complicated than standard transfers, because of derivative $\mathbf{F} = \mathbf{F} \cdot \mathbf{F$ If the final valence wave function at *r*⁰ = ⇢ has logarithmic derivative *B* such that ⁰ $u^\prime_\alpha(R)$

= 0 (87)

R-matrix continuum parameterisation [*H E*]*u* + (⇢; *e*) $\frac{1}{\phi(e;e_0)}$ ameterisation terms of reduced widths ² *^p* and pole energies "*p*. The one-pole expansion, for pole *p*, is from which we obtain the *S*-matrix $\begin{array}{c} \text{R} \text{C} \text{H} \text{C} \text{C} \text{C} \text{C} \end{array}$ **unuum param** $\int d(\omega \cdot \mathbf{e}_0)$ eterisation $F_{\rm t}$ transfers to unbound states, we use scattering solutions (\sim **EX-matrix continuum parameterisation (***R ^R ^u*↵ **Here, EX-matrix continuum parameterisation**

(<u>† 1888)</u>
1900 - Paul VI, prinses politik († 1888)
1900 - Paul VI, prinses politik († 1888)

• Definition $R(e_{\beta}) = \frac{1}{2}$ ρ $\phi(\rho;e_\beta)$ $\frac{\varphi(p, e_p)}{\phi'(p; e_p)}$ _N₂ $\frac{e_{\beta}}{s}$ $\mathsf{R}(e_\beta) =$ $\frac{\partial}{\partial} \overrightarrow{\phi'(\rho; e_{\phi})}$ *^p* and pole energies "*p*. The one-pole expansion, for pole *p*, is **For the state of the local** $B(e_{\beta}) = \frac{1}{2} \frac{\phi(\rho; e_{\beta})}{\phi(\rho; e_{\beta})}$ **EXEMENDENTERIZATION INTERVISION in the asymptotic normalization coefficient (ANC),** $\overline{\rho} \overline{\phi'(\rho; e_{\beta})}$ $\frac{1}{\phi(\rho; e_{\beta})}$

in channel at relative energy *e*. We define the *R*-matrix for the wave function (*r*⁰

The A, B are can be similar to be simila

the will be parameterized in terms of the asymptotic normalization coecient (ANC), *C*, as

the will be parameterized in terms of the asymptotic normalization coecient (ANC), *C*, as

- Parameterization: $R(e_\beta) = \sum \frac{p_\beta}{p_\beta}$ (*N*-pole case) *p*−1 = 0 (89) $\frac{P \notin (P, \nabla_{\beta})}{P} \frac{N}{N} = \frac{\gamma_p^2}{N}$ (*N*-nole case) **p** and proportion $R(e_{\beta}) = \sum_{p=1}^{\infty} \frac{1}{\varepsilon_p - e_{\beta}}$ (i.e. pole case "*^p e* $\sum_{p=1}^{\infty} \varepsilon_p$ $\sum_{p=1}^{\infty} \varepsilon_p - e_{\beta}$ $p=1$ $\cup p$ \cup_p $P \notin (P, P, P)$ N γ_p^2 γ_p^2 **the Parameterization.** $R(e_\beta) = \sum_i \frac{1}{2\pi}$ (A-pole case) (⇢) = *CW*(*k*⇢). ⇢ $\rho \varphi(\rho; e_{\beta})$
: R(e_{β}) = $\sum_{ }^{\mathcal{N}} \frac{\gamma_p^2}{\rho}$ (*N*-pole case) **Paramete** *p* "*^p e* le c $p=1$ $\circ p$ $\circ p$ $R(e_{\beta}) = \sum$ *N p*=1 γ_p^2 $\varepsilon_p - e_\beta$
	- From $R(e_{\beta})$, get S-matrix $S(e_{\beta})$ and wf by usual theory, for every energy $e_{\beta} = E_{\rm tot} - E_{\beta}$ [*H E*]*u* + (⇢; *e*) $R(e_\beta)$, get S-matrix $S(e_\beta)$ and wf ϕ_β terms of reduced widths ² *p* "*^p e* $S(e_{\beta})$ and wf $\phi_{\beta}(\rho;e_{\beta})$ $\mathsf{P}_\beta \mathsf{M}(\rho;e_\beta) \ = \ F_{\text{tot}} - F_{\text{tot}} \, ,$ (i) , get S-matrix $S(e_\beta)$ and wf $\phi_\beta(\rho;e_\beta)$ [*H E*]*u* + (⇢; *e*) *R* $\frac{1}{2}$ by usual theory, for every energy $e_{\beta} = E_{\text{tot}} - E_{\beta}$ For transfers to unbound states, we use scattering solutions (⇢; *e*) for *v* + *A* scattering **• From** R(e_{β}), get S-matrix S(e_{β}) and wf $\phi_{\beta}(\rho; e_{\beta})$ $\mathcal{L}(\mathbf{u}) = \mathcal{L}(\mathbf{v})$ $\frac{1}{h}$ $\frac{1}{h}$ \mathbf{F} **r** rom $\mathbf{R}(\mathcal{E}_{\beta})$, yet onitating $\mathbf{F}(\mathcal{E}_{\beta})$ and writh $\mathcal{E}_{\beta}(\mathcal{P}, \mathcal{E}_{\beta})$ d wf ϕ_{β} (β) by usual theory, for every energy $e_\beta = E_\mathrm{tot} - E_\mathrm{tot}$ got on:
pry, for e ⇢ $(0, 0, 0, 0, 0, 0)$ $\mathsf{p} \circ \mathsf{p} = \mathsf{p} \circ \mathsf{p} \circ \mathsf{p} = \mathsf{p} \circ \mathsf{p} \circ \mathsf{p} + \mathsf{p} \circ \mathsf{p} = \mathsf{p} \circ \mathsf{p} + \mathsf{p} \circ \mathsf{p} = \mathsf{p} \circ \mathsf{p} + \mathsf{p} \circ \$ from which we obtain the *S*-matrix *H* (*k*) $e_{\beta} = E_{\text{tot}} - E_{\beta}$
- **Then exit channel eqn, for continuous** E_{β} is *^p* and pole energies "*p*. The one-pole expansion, for pole *p*, is from which we obtain the *S*-matrix T hen exit channel egn for continuous \mathbf{F}_{α} is Γ Γ **Herefore, we have a set of the exit channel eqn, for continuous** E_β **is** and then the scattering wave function

$$
[H_{\beta} - E_{\beta}]u_{\beta} + \phi_{\beta}(\rho; e_{\beta}) \left\{ \frac{1}{\rho R_{\beta}(e_{\beta})} A_{\beta\alpha} u_{\alpha} + B_{\beta\alpha} u_{\alpha} + C_{\beta\alpha} \left[u_{\alpha}^{\prime} - \frac{L_{\alpha} + 1}{R} u_{\alpha} \right] \right\} = 0
$$

Note that A B C and *u* are independent of exit energy *E*₀

and then the scattering wave function **from Wore that A**, Here, we need to know that a set of the that A B C and u_{α} are independent of exit energy E_{α} the reduced widths μ_{α} are independent of exit energy E_{β} . Note that A, B, C and u_{α} are independent of exit energy E_{β} .

^R(*e*) = ²

terms of reduced widths ²

from which we obtain the *S*-matrix

and then the scattering wave function

from which we obtain the *S*-matrix

(⇢) = *CW*(*k*⇢).

(90)

; *e*);

^R(*e*) = ²

^S(*e*) = *^H*(*k*⇢) ⇢R(*e*)*H*0(*k*⇢)

= 0 (89)

^H+(*k*⇢) ⇢R(*e*)*H*0+(*k*⇢) *,* (91)

[*H*(*k*⇢) S(*e*)*H*0(*k*⇢)] *.* (92)

Interior-post + Surface-term

Black: interior post (depends on inside wf)

Red: surface (depends on R-matrix parameters)

The 2 terms add as matrix elements: **Coherent** sums.

The sums (**green**) should be the same for any surface position.

(should be outside the neutron potential!)

Conclusions

Surface formalism for studying resonances with (d,p):

- Uses successful R-matrix ideas to emphasize **asymptotic properties** of the wave function
- Separation into interior and exterior leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing **spectroscopic information**
- Our DWBA and CDCC studies show **surface term is dominant**; and dependence on model for nuclear interior is reduced.
- The surface term alone is **not sufficient** to describe transfer reactions, corrections are required
- Within a CDCC implementation (which includes breakup effects) the **exterior is already included**: not needed for transfer operator.

