One-nucleon Transfers to Resonances

INT, Seattle.

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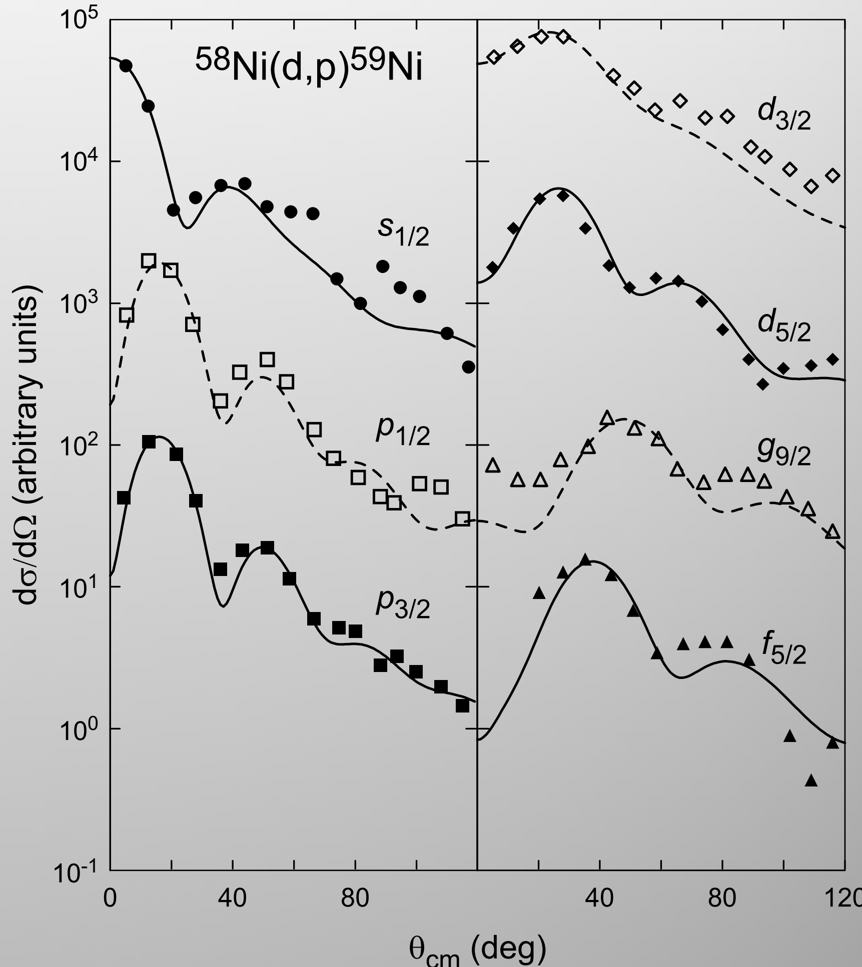
Traditional Role of (d,p) reactions

- Transfer neutron to *lj* bound state $\phi_{nlj}(r_n)$
  - usually large momentum transfer
  - Shape of proton $\sigma_p(\theta)$ depends on *l*
  - Analyzing powers depend on *j*
  - Magnitude of depends on spectroscopic factor S_{nlj}
- Higher-order corrections calculable (CRC, CCBA)



Measuring resonances with (d,p)

- Do not probe spectroscopic factors
- Do probe partial widths and resonance energies
  - These parameters come from R-matrix energies and reduced width amplitudes
- Desirable to have (d,p) calculations from R-matrix parameters
  - Is this possible?
  - Can then predict n+target scattering not otherwise measurable
  - But R-matrix values are surface and external features!
- Note: *L*-dependence of $\sigma_p(\theta)$ decreases for less bound neutrons, and hence for continuum neutrons
  - Reduced magnitude of momentum transfers

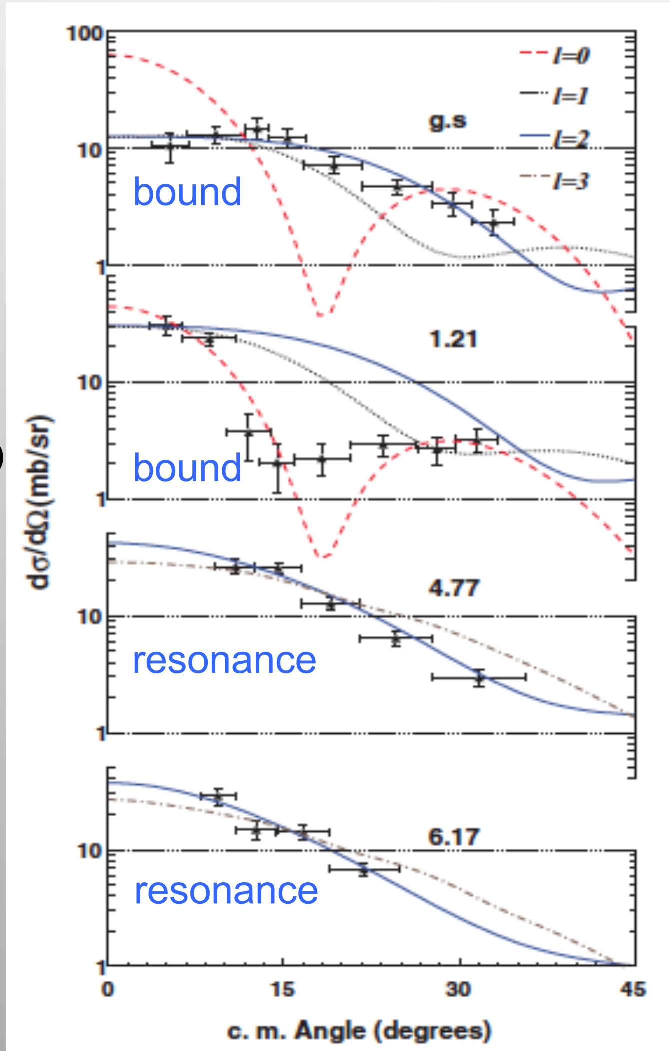
Propose to use: Surface Transfer Operator

- Work of TORUS collaboration (www.reactiontheory.org)
- Proposed: Mukhamedzhanov (PRC **84**, 044616, 2011)
- Developed for 1-step transfers:
  - Escher et al., (PRC. **89**, 054605, 2013)
  - Escher et al., (J. Phys.: Conf. Ser. **403** 012026, 2012)
- Now applied to transfers from entrance deuteron channels including breakup in CDCC basis.

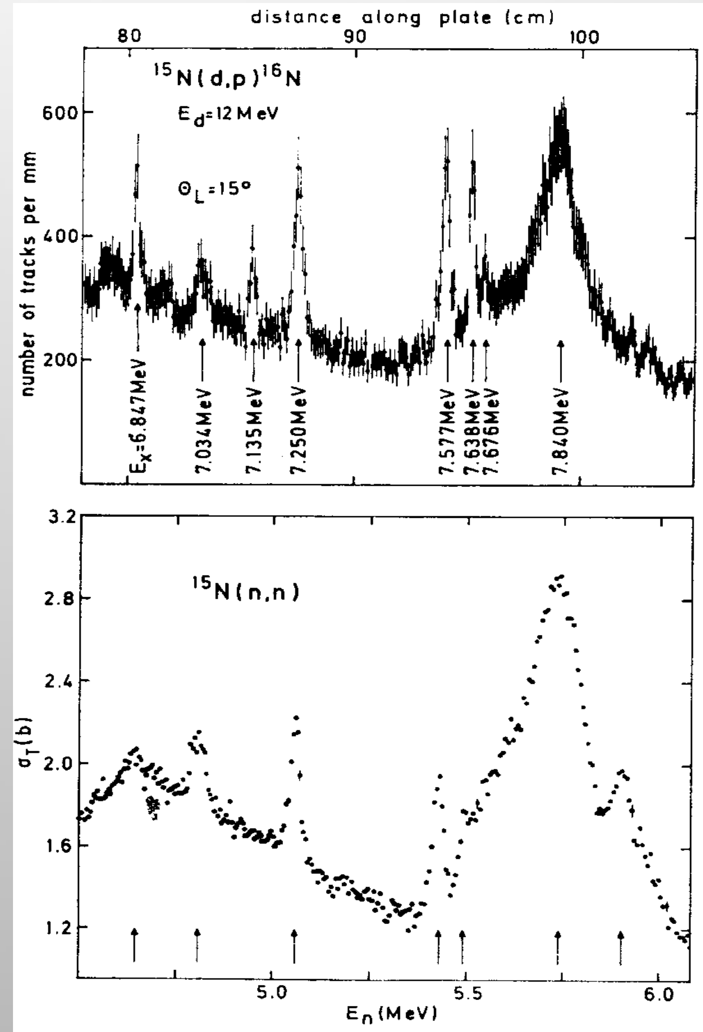
Applicable Examples of Resonances

- Near single-particle resonances

$^{20}O(d,p)^{21}O$ inverse-kinematics experiment



- Structured p-shell resonances

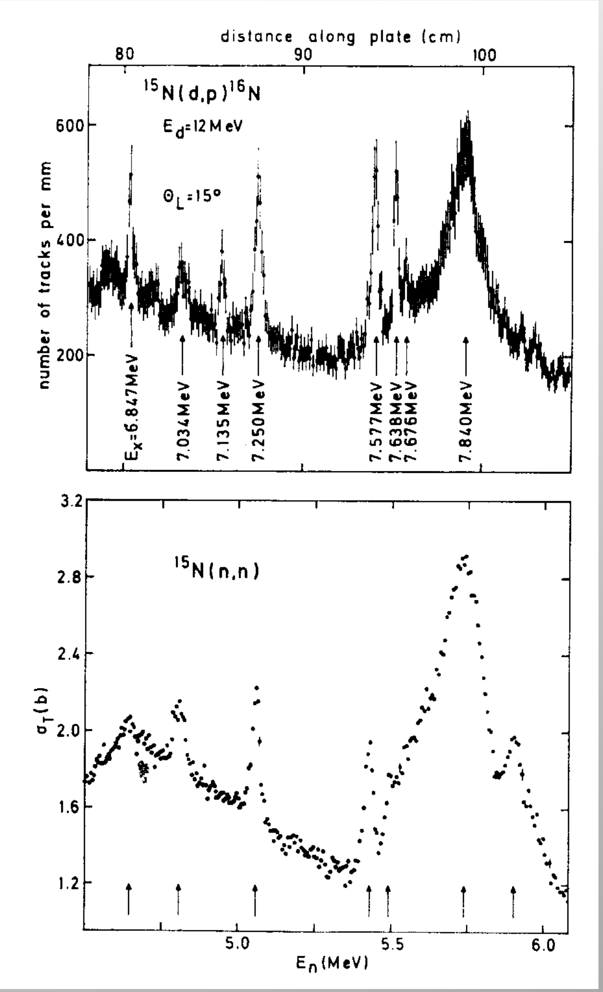


$^{15}N(d,p)^{16}N$

$^{15}N(n,n)$

Purpose of my transfer calculations

- Aim is to fit resonances in (d,p) cross sections in a region of the continuum.
- We see many wide and narrow resonances, often overlapping.
- Want to find neutron pole energies and partial widths, in entrance channel for (n,γ)



Post-prior equivalence in 1^{st} order

- Post matrix element in 1^{st} order

  $$M_{dp}^{(post)} = < \phi_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_p \mid \varphi_d \chi_{dA}^{(+)} >$$

- Prior matrix element in 1^{st} order

  $$M_{dp}^{(prior)} = < \phi_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_d \mid \varphi_d \chi_{dA}^{(+)} >$$

overlap $\phi_A^F(r_n) = < \varphi_A \mid \varphi_F >$

- Equivalent: $M_{dp}^{(post)} = M_{dp}^{(prior)}$ because
  - KE operators satisfy $<\phi_A^F(r_n) \mid T_n + T_p \mid \varphi_d(r)> = 0$
  - *Because* $\phi_A^F(r_n) \rightarrow 0$ at $r_n=0$ and $r_n \rightarrow \infty$
- If the wave functions not zero, then get surface terms.

Splitting the Transfer Matrix Element

- Define $M_{post}(a,b)$ & $M_{prior}(a,b)$ with $a < r_n < b$ limits
- General result:

$$M_{post}(a,b) = M_{surf}(a) + M_{prior}(a,b) - M_{surf}(b)$$

where $M_{surf}(\rho) = \langle f_p^{(-)}\, \phi_n \,|\, [\overleftarrow{T} - \overrightarrow{T}] \,|\, \phi_d\, f_d^{(+)} \rangle_{(r<\rho)}$

- Previous slide used $M_{surf}(0) = M_{surf}(\infty) = 0$

Mukhamedzhanov (2011):

- So, for any surface ρ:

  from: $M = M_{post}(0,\rho) + M_{post}(\rho,\infty)$

  hence: $M = M_{post}(0,\rho) + M_{surf}(\rho) + M_{prior}(\rho,\infty)$

Evaluating the Surface Matrix Element

With $M_{surf}(\rho) = \langle f_p^{(-)} \phi_n | [\overleftarrow{T} - \overrightarrow{T}] | \phi_d f_d^{(+)} \rangle_{(r<\rho)}$

- Need to calculate matrix elements like:

$$\int_{r\leqslant R} \mathrm{d}\mathbf{r}\, f(\mathbf{r})[\overleftarrow{T} - \overrightarrow{T}]g(\mathbf{r})$$
$$= -\frac{1}{2\mu}\oint_{r=R} \mathrm{d}\mathbf{S}[g(\mathbf{r})\nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r})\nabla_{\mathbf{r}} g(\mathbf{r})]$$
$$= -\frac{1}{2\mu}R^2\int \mathrm{d}\Omega_{\mathbf{r}}\left[g(\mathbf{r})\frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r})\frac{\partial g(\mathbf{r})}{\partial r}\right]_{r=R}$$

- That is, functions & derivatives on the surface $r_n=\rho$
- Do this for partial waves, in reaction code FRESCO

Preliminary Estimates of Magnitudes

- In DWBA (1st order), find surface term as:

$$M_{surf}(\rho) = M_{post}(0,\rho) - M_{prior}(0,\rho)$$

- Look at bound states and resonances.
  - See if convergence to breakup states is easier?
- Calculate all terms of

$$M = M_{post}(0,\rho) + M_{surf}(\rho) + M_{prior}(\rho,\infty)$$

Internal, surface, external contributions – $^{90}Zr(d,p)$ at E_d=11 MeV

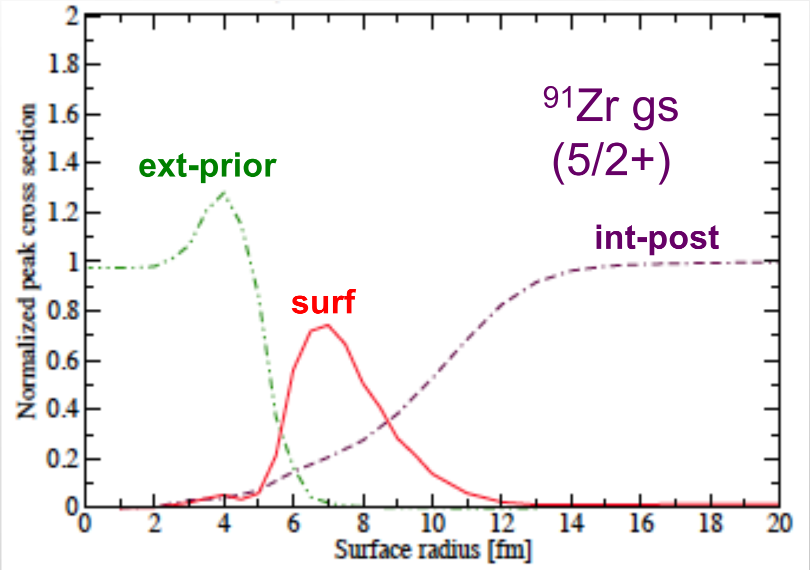
$$M = \underbrace{M^{(post)}(0,a)}_{\text{model dependence}} + \underbrace{M_{(surf)}(a) + M^{(prior)}(a,\infty)}_{\text{asymptotic quantities}}$$

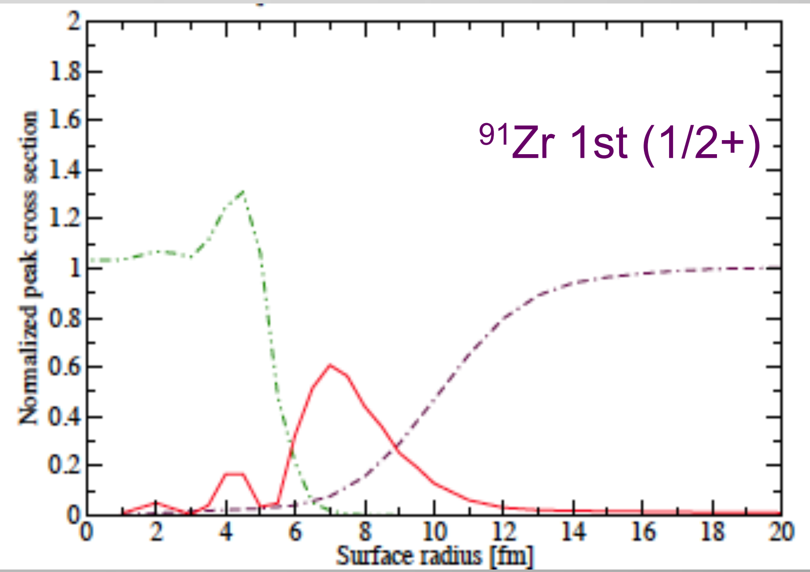
Observations

- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

Peak cross section relative to full calculation

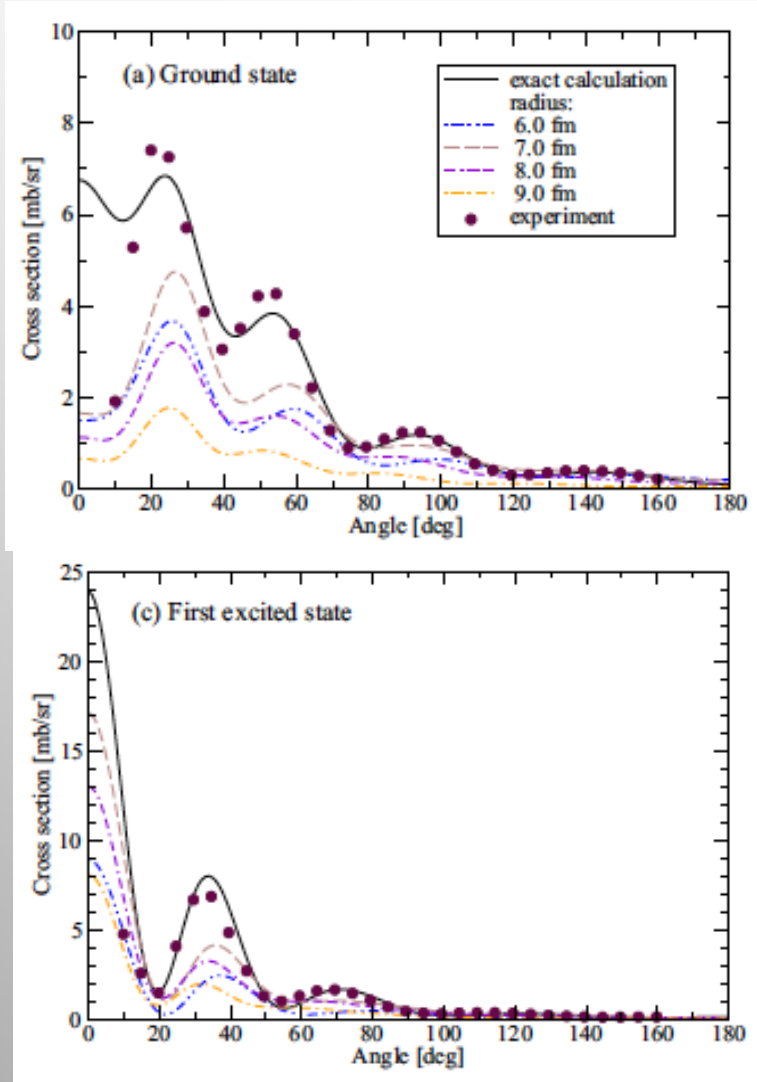




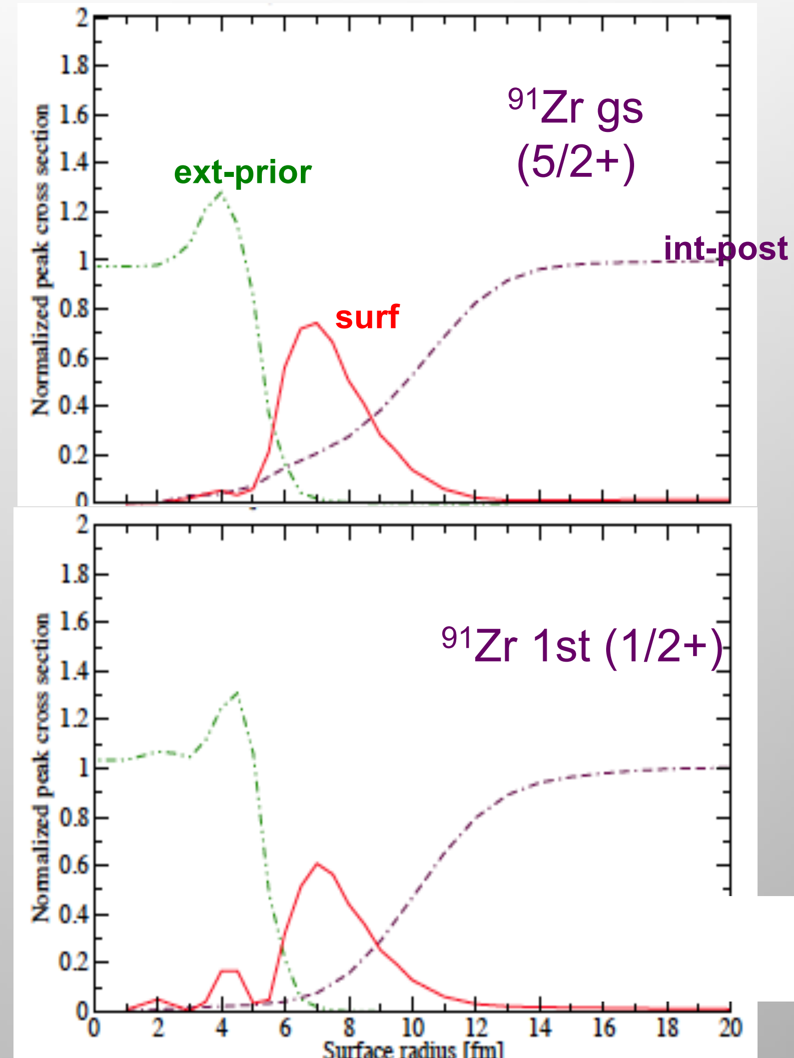
Escher, Thompson, Mukhamedzhanov, JPCS (2012).

The surface contribution – $^{90}Zr(d,p)$ at E_d=11 MeV

Angular cross section – Surface term only



Peak cross section relative to full calculation

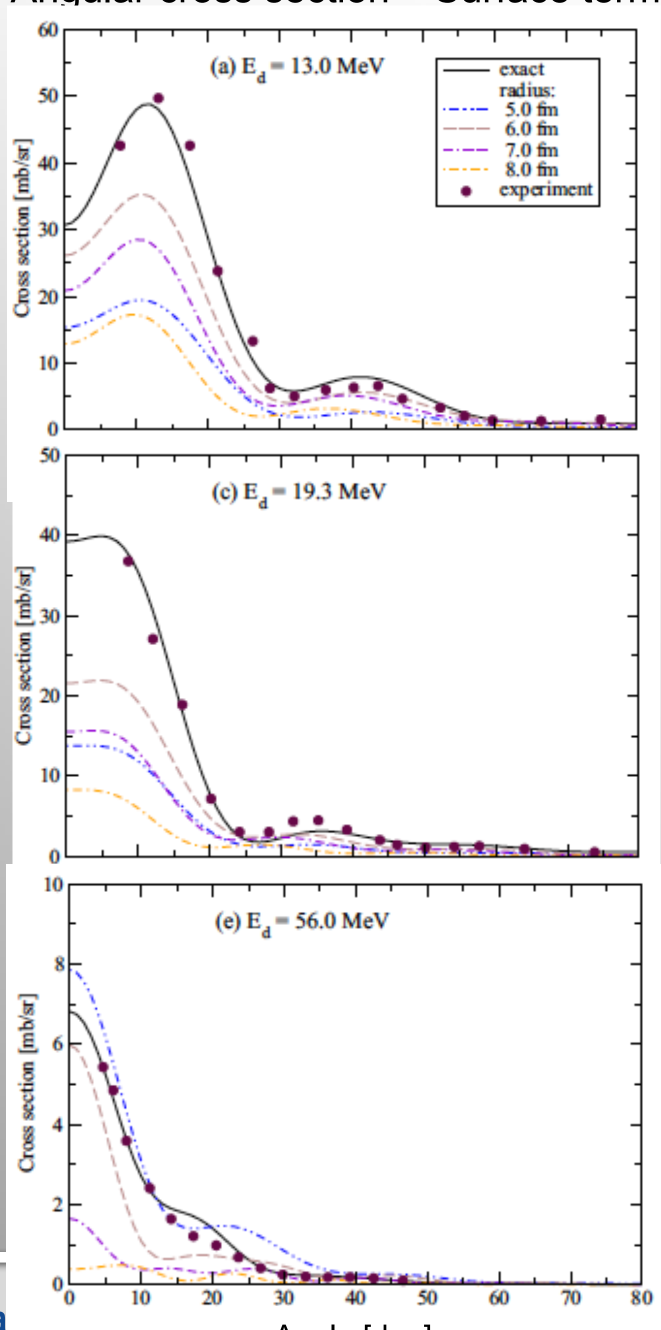


Escher et al, PRC 89, 054605 (2014)

- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain

Numerical tests of the formalism (DWBA) – ^{48}Ca(d,p) at E_d=13, 19.3, 56 MeV

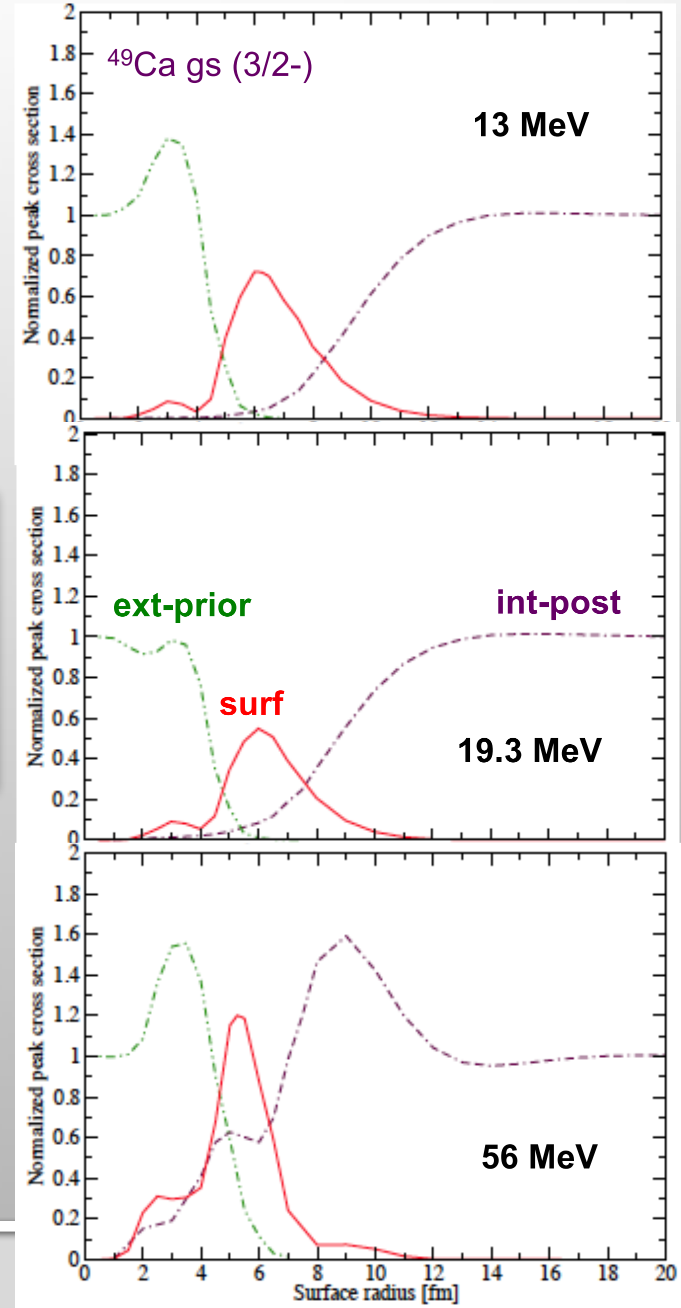
Angular cross section – Surface term only



Surface term approximation improves with decreasing energy

Calculations for ^{49}Ca 1st (1/2-) give similar results

Peak cross section relative to full calculation



Escher et al, PRC (2014)

Surface formalism for DWBA – resonance states

d+A → p + (n+A)

↓

b+B

$$M^{\mathrm{DW(post)}}(P,\mathbf{k}_{dA}) = \langle \chi_{pF}^{(-)} \Psi_{bB}^{(\mathrm{int})(-)} | \Delta \overline{V}_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle, \qquad (94)$$

<u>Surface formulation</u>

$M = M^{(\mathrm{post})}(0,a)$

$+ M_{\mathrm{surf}}(a)$

$+ M^{(\mathrm{prior})}(a,\infty)$

$f(\Gamma^{1/2}, [A^{-1}], I_A^F)$:
contribution hopefully small

b + B = n + A
b + B ≠ n + A

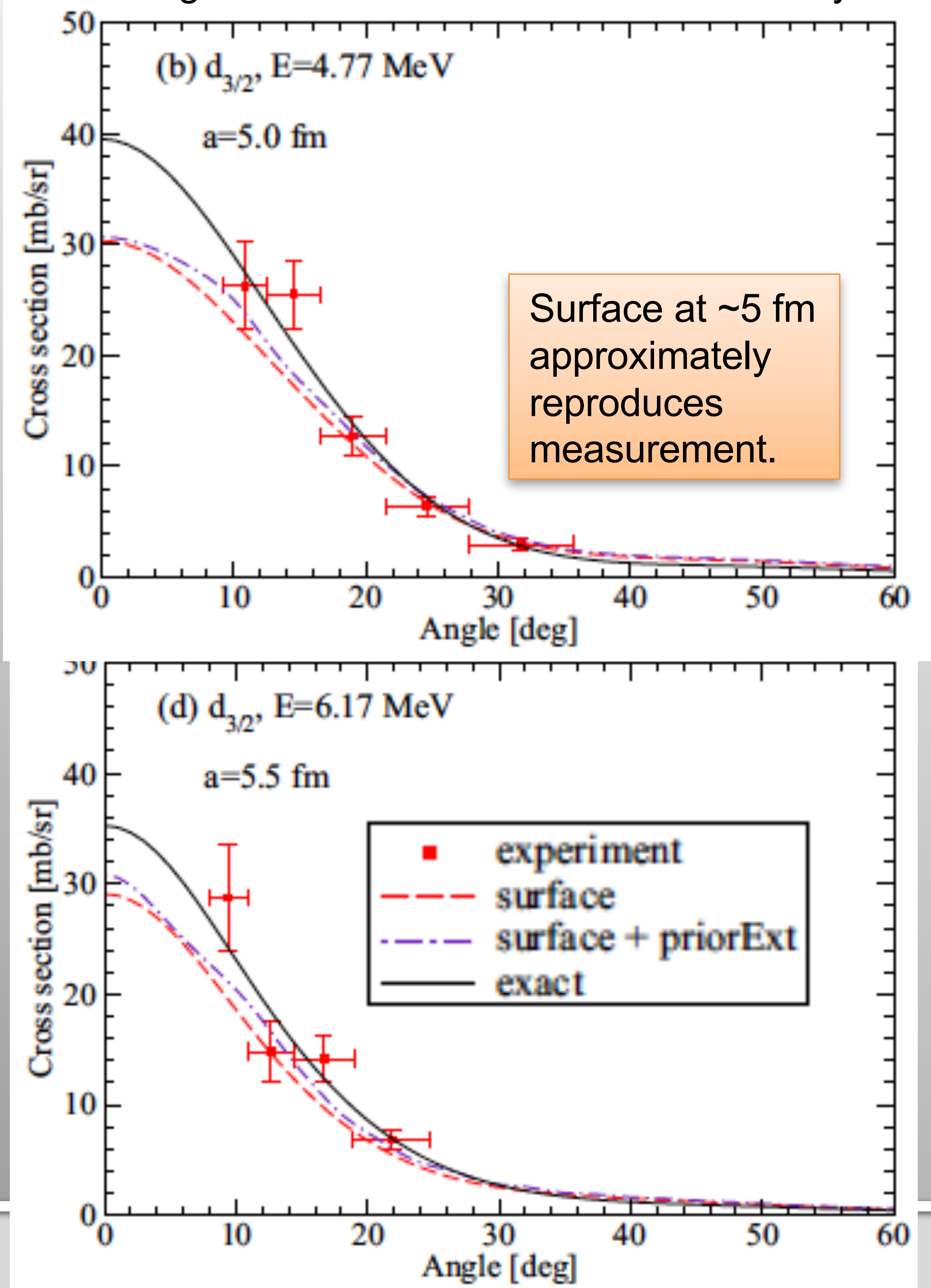
b + B = n + A
b + B ≠ n + A

Total post matrix element for b + B ≠ n + A example:

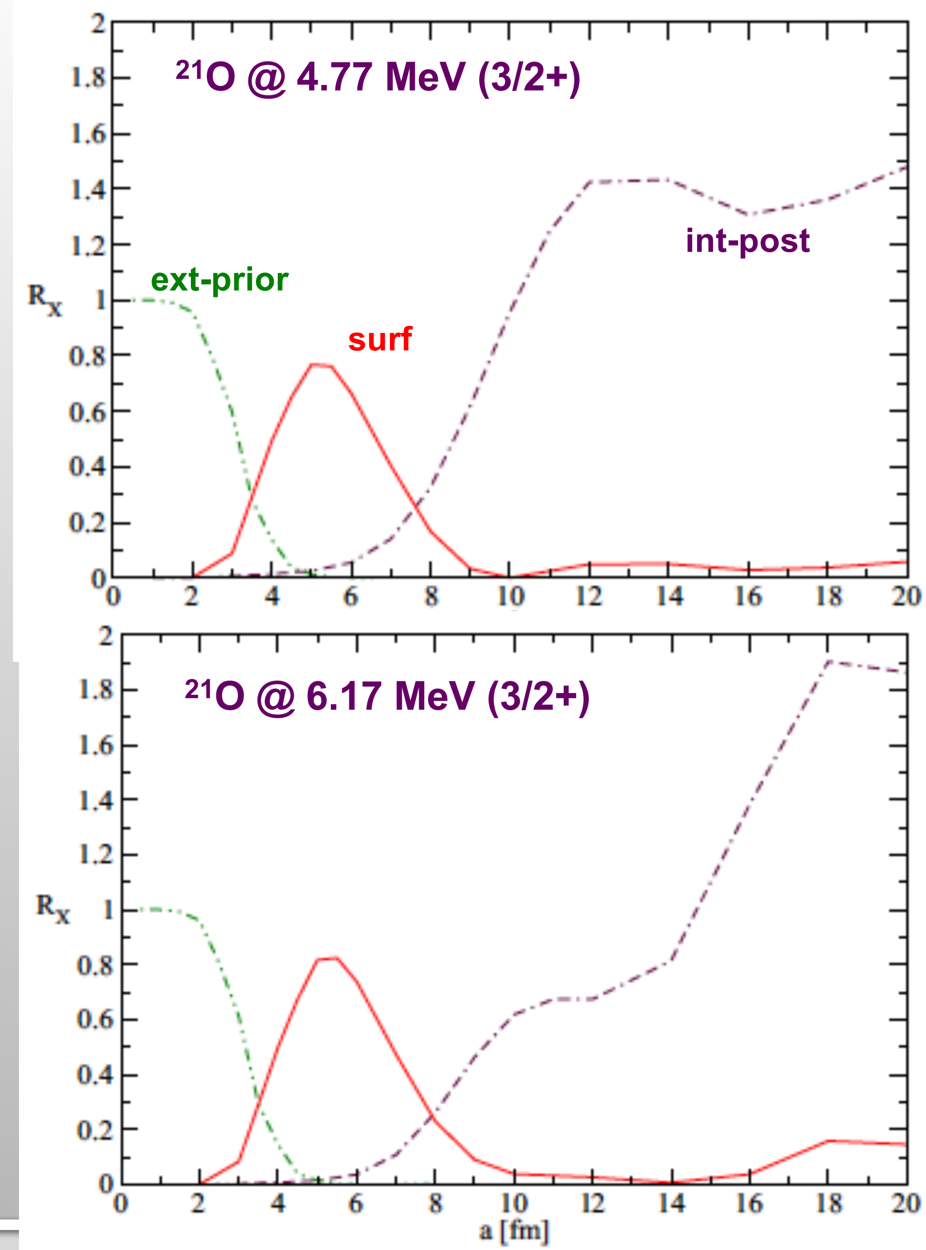
$$M^{\mathrm{DW(post)}}(P,\mathbf{k}_{dA}) = 2\pi\sqrt{\frac{1}{\mu_{bB}k_{bB}}} \sum_{J_F M_F s' l l' m_{s'} m_l m_{l'} M_n} i^l \langle s m_s l m_l | J_F M_F \rangle \langle s' m_{s'} l' m_{l'} | J_F M_F \rangle \langle J_n M_n J_A M_A | s' m_{s'} \rangle \langle J_n M_n J_p M_p | J_d M_d \rangle$$
$$\times e^{-i\delta_{bBl}^{hs}} Y_{lm_l}^{*}(-\hat{\mathbf{k}}_{bB}) \sum_{\nu,\tau=1}^{N} [\Gamma_{\nu bBslJ_F}(E_{bB})]^{1/2} [\mathbf{A}^{-1}]_{\nu\tau} \Bigg\{ \langle \chi_{pF}^{(-)} I_{As'l'J_F}^{F} | \Delta\overline{V}_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle \Big|_{r_{nA} \leqslant R_{nA}}$$
$$+ \sqrt{\frac{2\mu_{nA}}{R_{nA}}} \gamma_{\tau nAs'l'J_F} \left\langle \chi_{pF}^{(-)} \frac{O_{l'}^{*}(k_{nA}, r_{nA})}{r_{nA}} \frac{R_{nA}}{O_{l'}^{*}(k_{nA}, R_{nA})} Y_{l'm_{l'}}^{*}(\hat{\mathbf{r}}_{nA}) \right| \Delta\overline{V}_{dA} \left| \varphi_d \chi_{dA}^{(+)} \right\rangle \Bigg|_{r_{nA} > R_{nA}} + \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \gamma_{\tau nAs'l'J_F}$$
$$\times \int \mathrm{d}\mathbf{r}_{pF} \chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF}) \int \mathrm{d}\Omega_{\mathbf{r}_{nA}} Y_{l'm_{l'}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA} = R_{nA}} \Bigg\}. \qquad (117)$$

The oxygen case - ^{20}O at E_d=21 MeV

Angular cross section – Surface term only



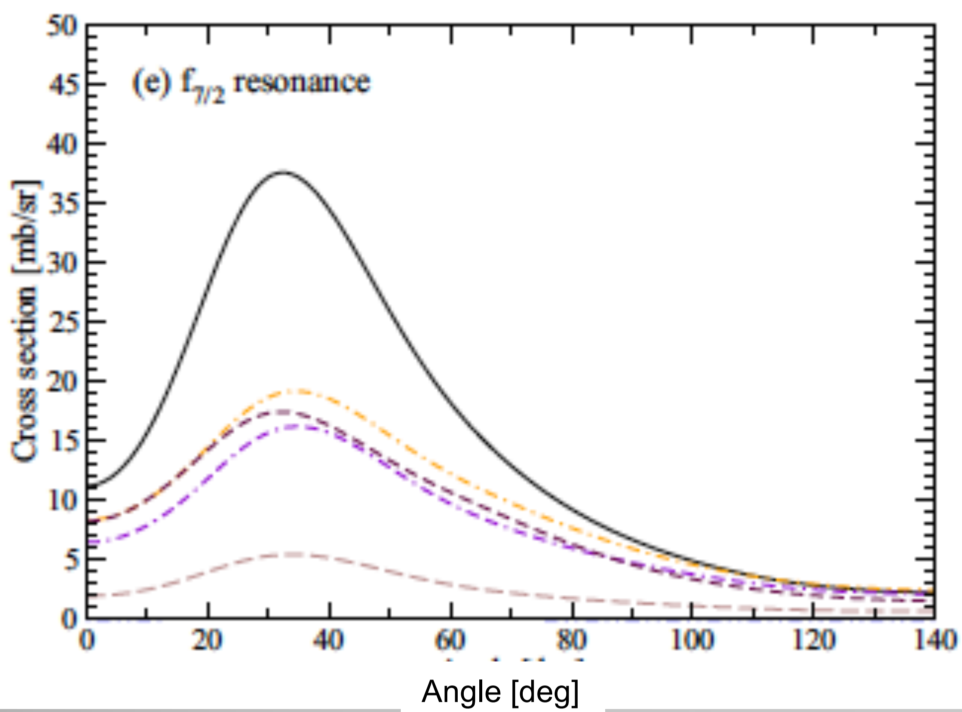
Peak cross section relative to full calculation



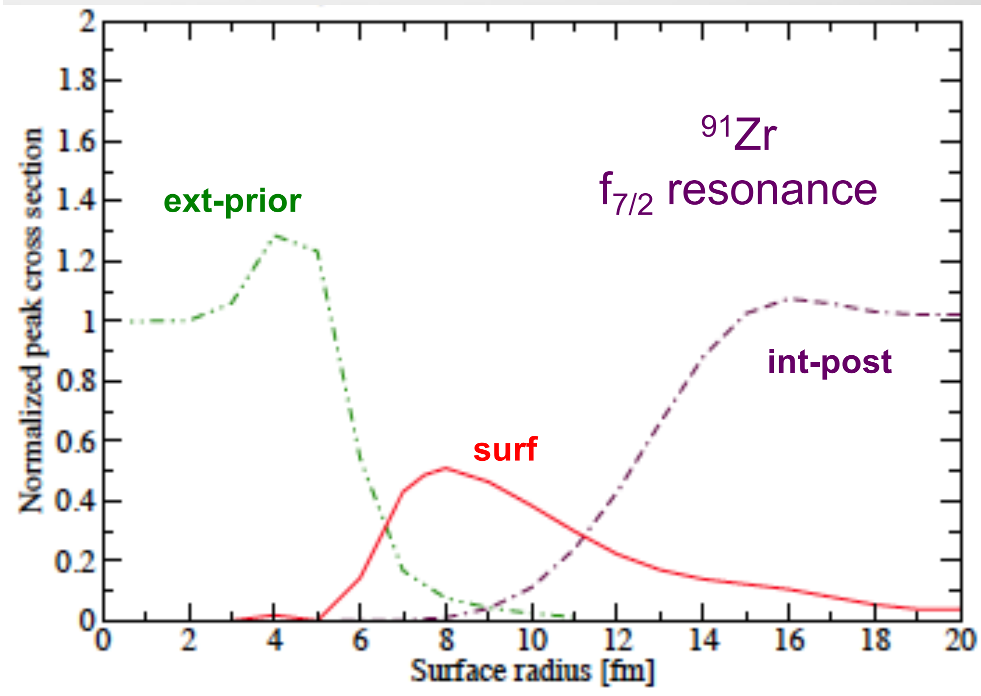
Escher et al, PRC 89, 054605 (2014)

Resonances – ^{90}Zr at E_d=11 MeV

Angular cross section – Surface term only



Peak cross section relative to full calculation



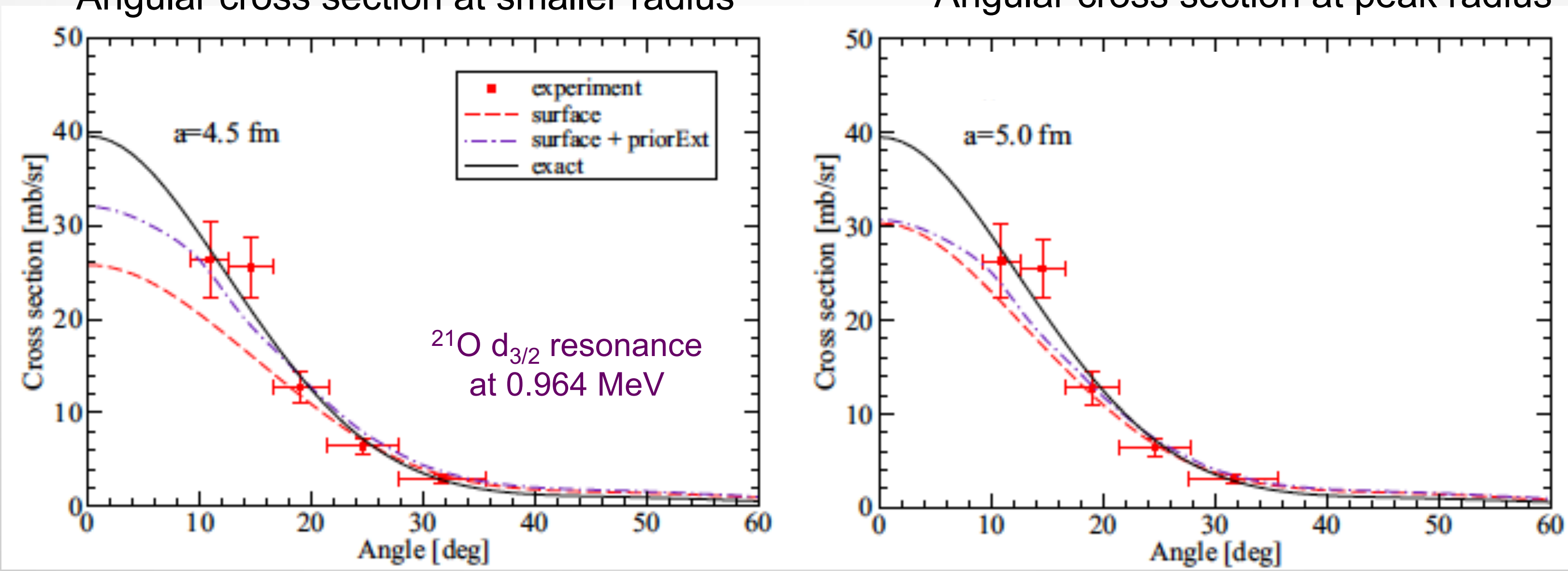
Escher et al, PRC 89, 054605 (2014)

- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute

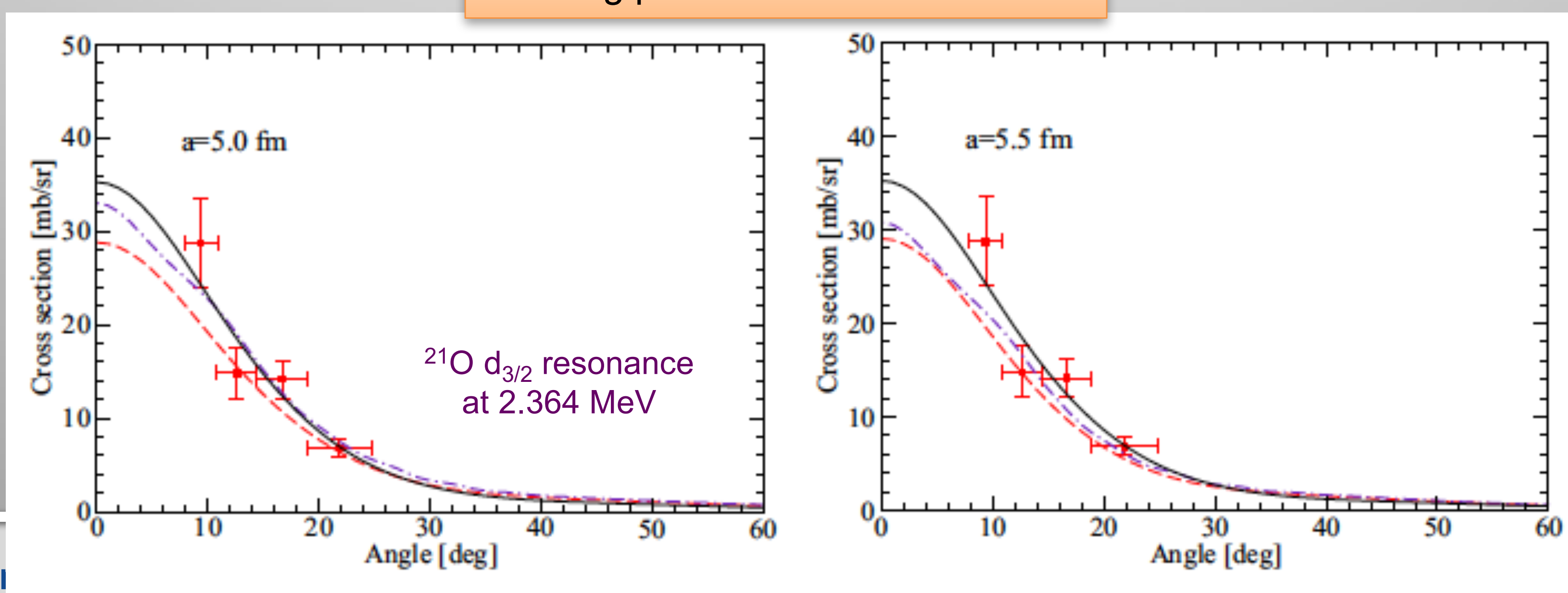
The surface formalism: can we save it? ^{20}O at E_d=21 MeV

Angular cross section at smaller radius

Angular cross section at peak radius



- reducing the surface radius
- adding prior-exterior contribution



Extension of the formalism to include breakup

DWBA matrix element

$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$

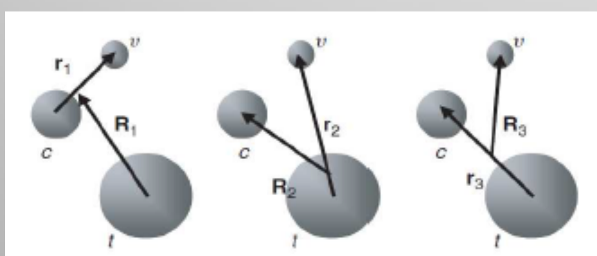
CDCC matrix element

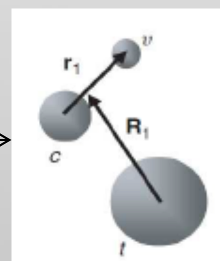
$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a)$

$M^{(prior)}(a,\infty) = 0$ (is included in breakup)

CDCC (Continum-discretized coupled channels)

- Approximate treatment of 3-body problem
- Describes breakup of deuteron





- Successfully used for describing data
- Currently revisited via comparison with Fadeev

CDCC extension of R-matrix formalism

- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant

Derivatives of products of 2 wfns

$$\frac{\partial}{\partial r'}\langle \hat{R}, \hat{r}\,|\alpha\rangle \frac{1}{rR}\,\varphi_\alpha(r)u_\alpha(R) = \frac{1}{rR}\sum_{M_L m_\ell} C_\alpha^{M_L m_\ell : M}$$

$$\Bigg(Y_L^{M_L}(\hat{\mathbf{R}})u_\alpha(R)\frac{p}{r}\Bigg\{\sqrt{\frac{4\pi\ell(2\ell+1)}{3}}\sum_{\lambda=-1}^{1}\langle \ell{-}1\ m{-}\lambda, 1\lambda|\ell m\rangle Y_{\ell-1}^{m-\lambda}(\hat{\mathbf{r}})Y_1^{\lambda}(\hat{\mathbf{r}'})\frac{\varphi_\alpha(r)}{r}$$

$$+\,Y_\ell^m(\hat{\mathbf{r}})\ \hat{\mathbf{r}}\cdot\hat{\mathbf{r}'}\left[\varphi'_\alpha(r) - \frac{\ell+1}{r}\varphi_\alpha(r)\right]\Bigg\}$$

$$+\,Y_\ell^{m_\ell}(\hat{\mathbf{r}})\varphi_\alpha(r)\frac{P}{R}\Bigg\{\sqrt{\frac{4\pi L(2L+1)}{3}}\sum_{\Lambda=-1}^{1}\langle L{-}1\ M_L{-}\Lambda, 1\Lambda|LM_L\rangle Y_{L-1}^{M_L-\Lambda}(\hat{\mathbf{R}})Y_1^{\Lambda}(\hat{\mathbf{r}'})\frac{u_\alpha(R)}{R}$$

$$+\,Y_L^{M_L}(\hat{\mathbf{R}})\ \hat{\mathbf{R}}\cdot\hat{\mathbf{r}'}\left[u'_\alpha(R) - \frac{L+1}{R}u_\alpha(R)\right]\Bigg\}\quad\Bigg)$$

$$\mathbf{r} = p\mathbf{r}' + q\mathbf{R}'$$

$$\mathbf{R} = P\mathbf{r}' + Q\mathbf{R}'$$

Source term complete for r,R wfns

$$S^{\rm surf}_{\beta\alpha}(R') = -\frac{\hbar^2\rho^2}{2\mu_n} \sum_{M'_L m'_\ell M_L m_\ell} F_\beta^{M'_L m'_\ell:M*} C_\alpha^{M_L m_\ell:M} \langle Y_{L'}^{M'_L}(\hat{\mathbf{R}}') \; Y_{\ell'}^{m'_\ell}(\hat{\mathbf{r}}')|_{r'=\rho} \frac{1}{rR}$$

$$\Bigg[\phi'_\beta(\rho) \;\; Y_\ell^{m_\ell}(\hat{\mathbf{r}}) \; Y_L^{M_L}(\hat{\mathbf{R}}) \quad \varphi_\alpha(r) \; u_\alpha(R)$$

$$- \phi_\beta(\rho) \Bigg(Y_L^{M_L}(\hat{\mathbf{R}}) u_\alpha(R) \frac{p}{r} \Bigg\{ \sqrt{\frac{4\pi\ell(2\ell{+}1)}{3}} \sum_{\lambda=-1}^{1} \langle \ell{-}1 \; m{-}\lambda, 1\lambda | \ell m \rangle Y_{\ell-1}^{m-\lambda}(\hat{\mathbf{r}}) Y_1^{\lambda}(\hat{\mathbf{r}'}) \frac{\varphi_\alpha(r)}{r}$$

$$+ Y_\ell^m(\hat{\mathbf{r}}) \; \hat{\mathbf{r}} \cdot \hat{\mathbf{r}'} \left[\varphi'_\alpha(r) - \frac{\ell{+}1}{r} \varphi_\alpha(r) \right] \Bigg\}$$

$$+ Y_\ell^{m_\ell}(\hat{\mathbf{r}}) \varphi_\alpha(r) \frac{P}{R} \Bigg\{ \sqrt{\frac{4\pi L(2L{+}1)}{3}} \sum_{\Lambda=-1}^{1} \langle L{-}1 \; M_L{-}\Lambda, 1\Lambda | L M_L \rangle Y_{L-1}^{M_L-\Lambda}(\hat{\mathbf{R}}) Y_1^{\Lambda}(\hat{\mathbf{r}'}) \frac{u_\alpha(R)}{R}$$

$$+ Y_L^{M_L}(\hat{\mathbf{R}}) \; \hat{\mathbf{R}} \cdot \hat{\mathbf{r}'} \left[u'_\alpha(R) - \frac{L{+}1}{R} u_\alpha(R) \right] \Bigg\} \quad \Bigg) \Bigg] .$$

The $F_\beta^{M'_L m'_\ell:M*} C_\alpha^{M_L m_\ell:M}$ are the channel-defining Clebsch-Gordon coefs.

Implementation

- As $\mathbf{R}' \neq \mathbf{R}'$, transfer couplings are still non-local
- With A, B, C as non-local operators, the transfer-channel exit equation is

$$[H_\beta - E_\beta]u_\beta + \phi'_\beta(\rho)\ \mathsf{A}_{\beta\alpha}u_\alpha + \phi_\beta(\rho)\ \mathsf{B}_{\beta\alpha}u_\alpha + \phi_\beta(\rho)\ \mathsf{C}_{\beta\alpha}\left[u'_\alpha - \frac{L_\alpha+1}{R}u_\alpha\right] = 0$$

- More complicated than standard transfers, because of derivative $u'_\alpha(R)$

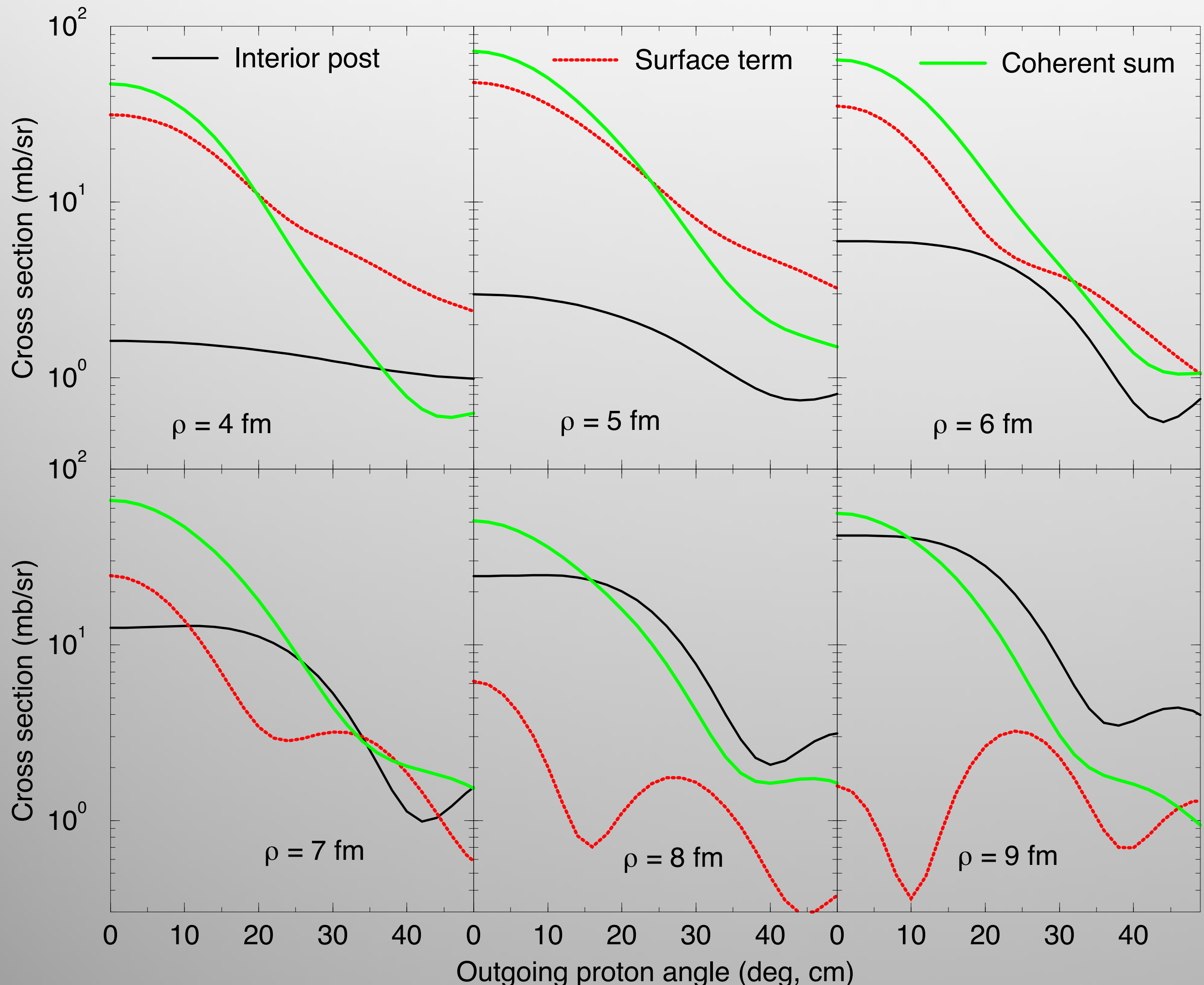
R-matrix continuum parameterisation

- Definition $\mathsf{R}(e_\beta) = \frac{1}{\rho}\frac{\phi(\rho;e_\beta)}{\phi'(\rho;e_\beta)}$
- Parameterization: $\mathsf{R}(e_\beta) = \sum_{p=1}^{N} \frac{\gamma_p^2}{\varepsilon_p - e_\beta}$ (N-pole case)
- From $\mathsf{R}(e_\beta)$, get S-matrix $\mathsf{S}(e_\beta)$ and wf $\phi_\beta(\rho;e_\beta)$ by usual theory, for every energy $e_\beta = E_{\rm tot} - E_\beta$
- Then exit channel eqn, for continuous E_β is

$$[H_\beta - E_\beta]u_\beta + \phi_\beta(\rho;e_\beta)\left\{\frac{1}{\rho\mathsf{R}_\beta(e_\beta)}\mathsf{A}_{\beta\alpha}u_\alpha + \mathsf{B}_{\beta\alpha}u_\alpha + \mathsf{C}_{\beta\alpha}\left[u'_\alpha - \frac{L_\alpha+1}{R}u_\alpha\right]\right\} = 0$$

Note that A, B, C and u_α are independent of exit energy E_β.

Interior-post + Surface-term



Black: interior post (depends on inside wf)

Red: surface (depends on R-matrix parameters)

The 2 terms add as matrix elements: **Coherent** sums.

The sums (**green**) should be the same for any surface position.

(should be outside the neutron potential!)

Conclusions

Surface formalism for studying resonances with (d,p):

- Uses successful R-matrix ideas to emphasize **asymptotic properties** of the wave function
- Separation into interior and exterior leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing **spectroscopic information**
- Our DWBA and CDCC studies show **surface term is dominant**; and dependence on model for nuclear interior is reduced.
- The surface term alone is **not sufficient** to describe transfer reactions, corrections are required
- Within a CDCC implementation (which includes breakup effects) the **exterior is already included**: not needed for transfer operator.

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