Low-energy Reaction in EFT

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Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart
Institute for Nuclear Theory, Seattle, Mar 8, 2017

Outline

- Background and motivation
- Weakly bound systems at low energy
- Adiabatic projection method - proof of concept
- neutron capture
- proton-proton fusion
- Pinhole algorithm
- Efimov physics

Motivations

Astrophysics

- Low energy reactions dominate
- Need accurate cross sections but hard to measure experimentally
- Model-independent theoretical calculations important

Theoretical

- Weakly bound systems — opportunities
- First principle calculation

Use Effective Field Theory

EFT: THE LONG AND SHORT OF IT

- Identify degrees of freedom

$$\mathcal{L} = c_0\, \mathrm{O}^{(0)} + c_1\, \mathrm{O}^{(1)} + c_2\, \mathrm{O}^{(2)} + \cdots \quad \text{expansion in } \frac{Q}{\Lambda}$$

Hide UV ignorance
- short distance

IR explicit
- long distance

- Determine c_n from data (elastic, inelastic)

- EFT : ERE + currents + relativity

Not just Ward-Takahashi identity

Key ingredient: power counting

Weakly Bound Systems

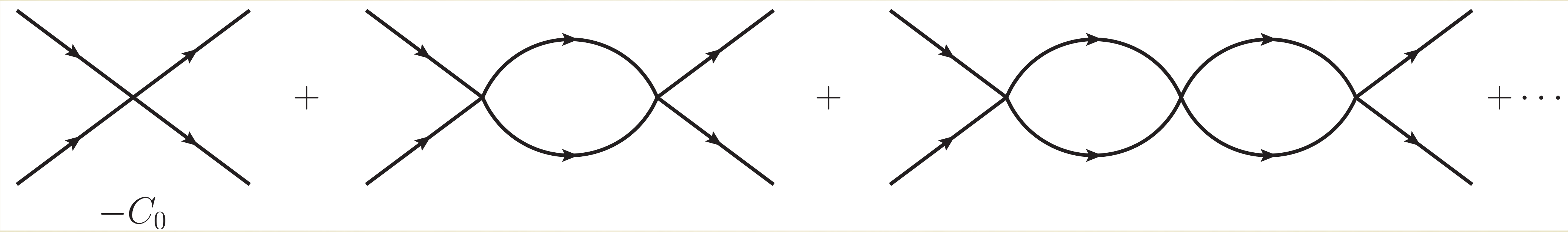
$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

--- Natural case $1/a, 1/r \sim \Lambda \gg p$, expand in small p

--- Large scattering length $a >> 1/\Lambda$

$$i\mathcal{A}(p) \approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{1}{2}\frac{rp^2}{1/a + ip} + \cdots\right]$$

EFT non-perturbative



$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 \sim \frac{2\pi a}{\mu}$$

large coupling

$1/a \sim p \sim Q \ll \Lambda$

Weinberg '90
Bedaque, van Kolck '97
Kaplan, Savage, Wise '98

P-WAVE BOUND STATE

$$i\mathcal{A}(p) = \frac{2\pi}{\mu} \frac{ip^2}{-\frac{1}{a_V} + \frac{r_1}{2}p^2 + \cdots - ip^3}$$

Shallow states

2 fine tuning Bertulani, Hammer, van Kolck (2002)

$$1/(a_V)^{1/3} \sim p \sim r_1 \sim Q \ll \Lambda$$

1 fine tuning Bedaque, Hammer, van Kolck (2003)

$$1/a_V \sim Q^2\Lambda,\ p \sim Q \ll r_1 \sim \Lambda$$

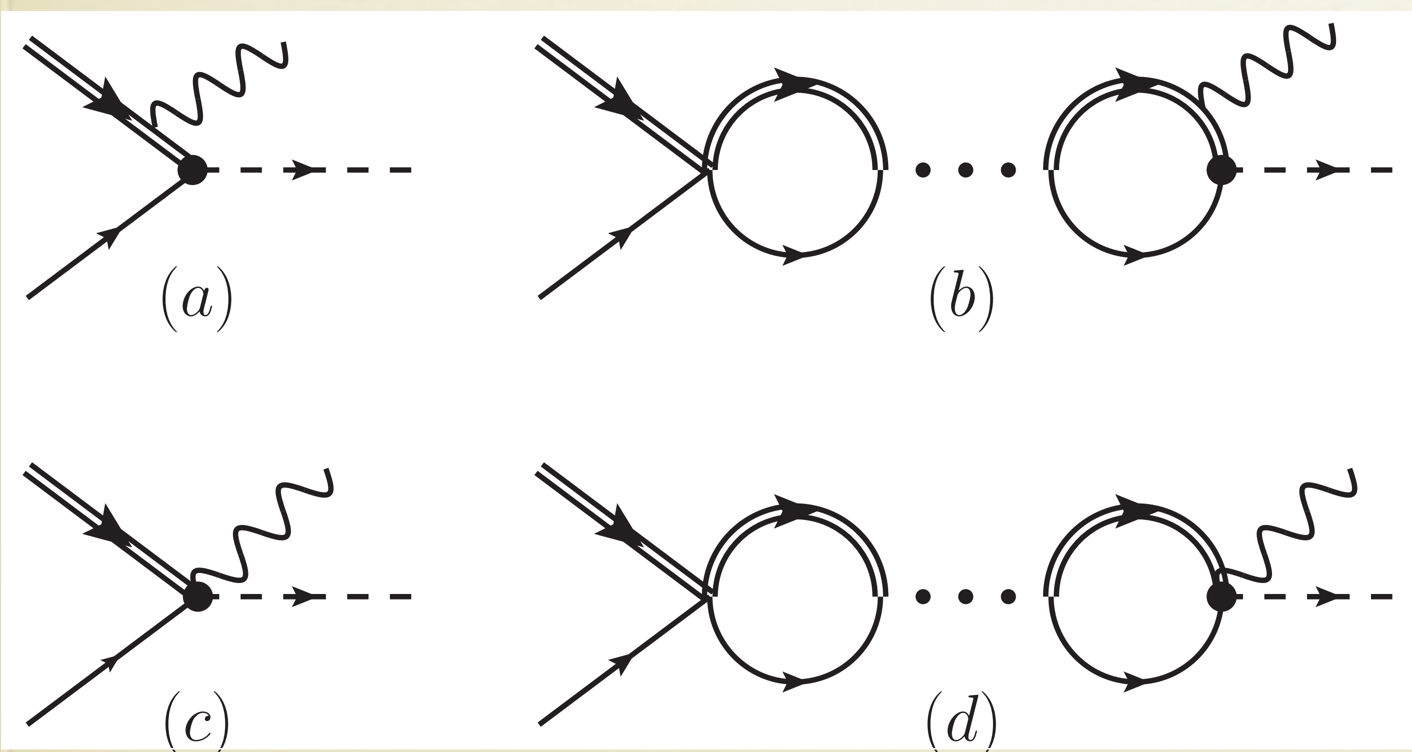
Either way requires two operators at LO

A low energy example

$^7\text{Li}(n,\gamma)^8\text{Li}$

- Isospin mirror systems $^7\text{Li}(n,\gamma)^8\text{Li} \leftrightarrow {}^7\text{Be}(p,\gamma)^8\text{B}$
- Inhomogeneous BBN

Whats the theoretical error?

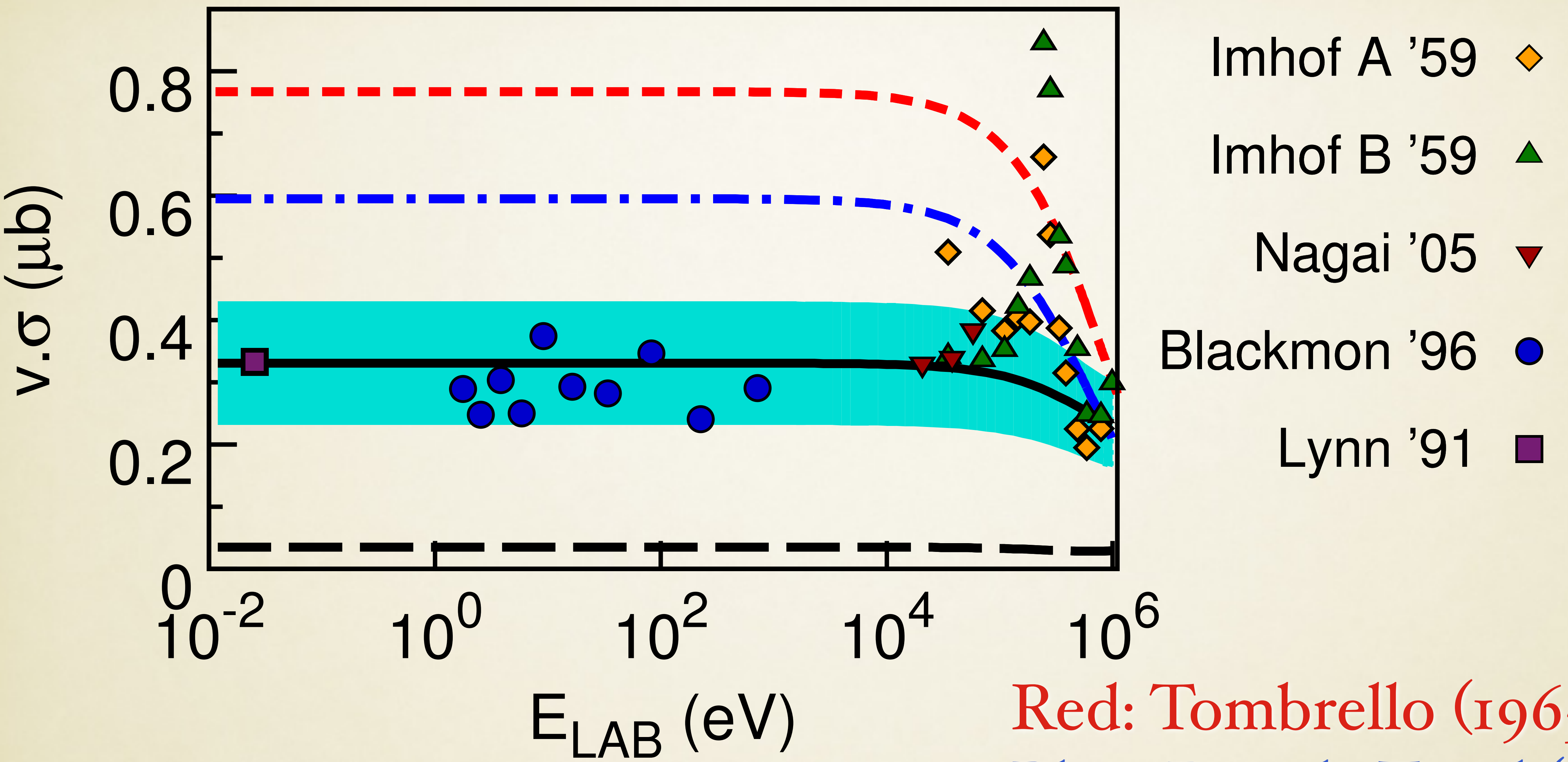


Asymptotic normalization

$$\sqrt{Z} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{1+3\gamma/r_1}}$$

Need effective range r1 and binding energy at leading order

Two EFT operators



Red: Tombrello (1965)

Blue: Davids-Typel (2003)

Black: EFT

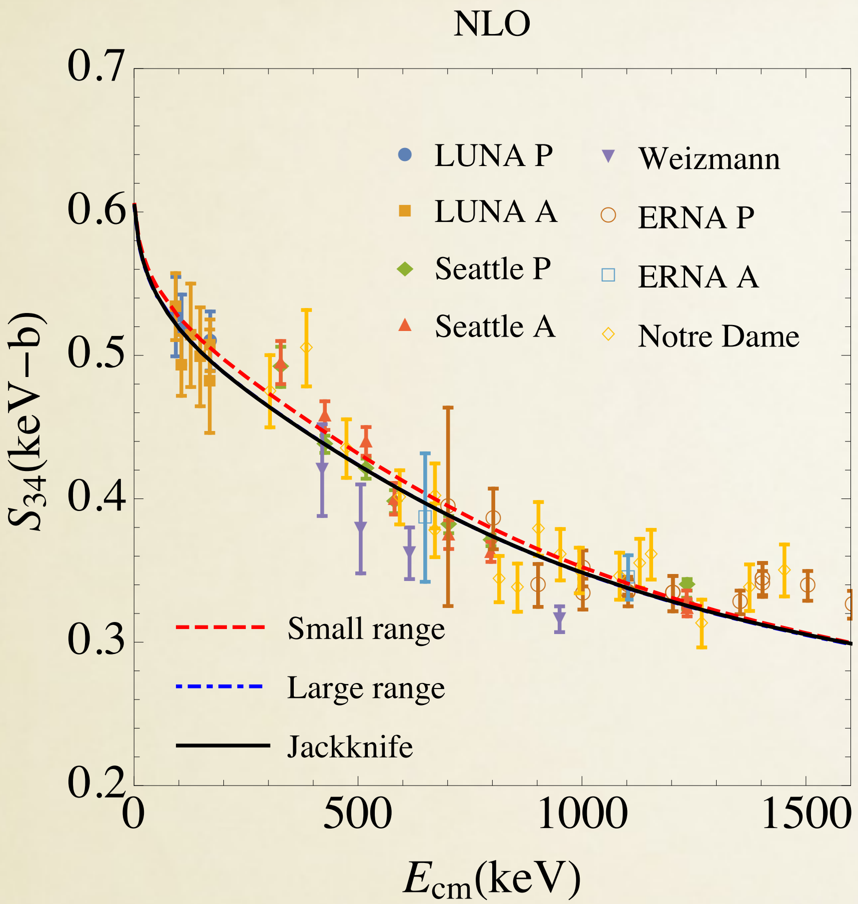
Rupak, Higa; PRL 106, 222501 (2011)

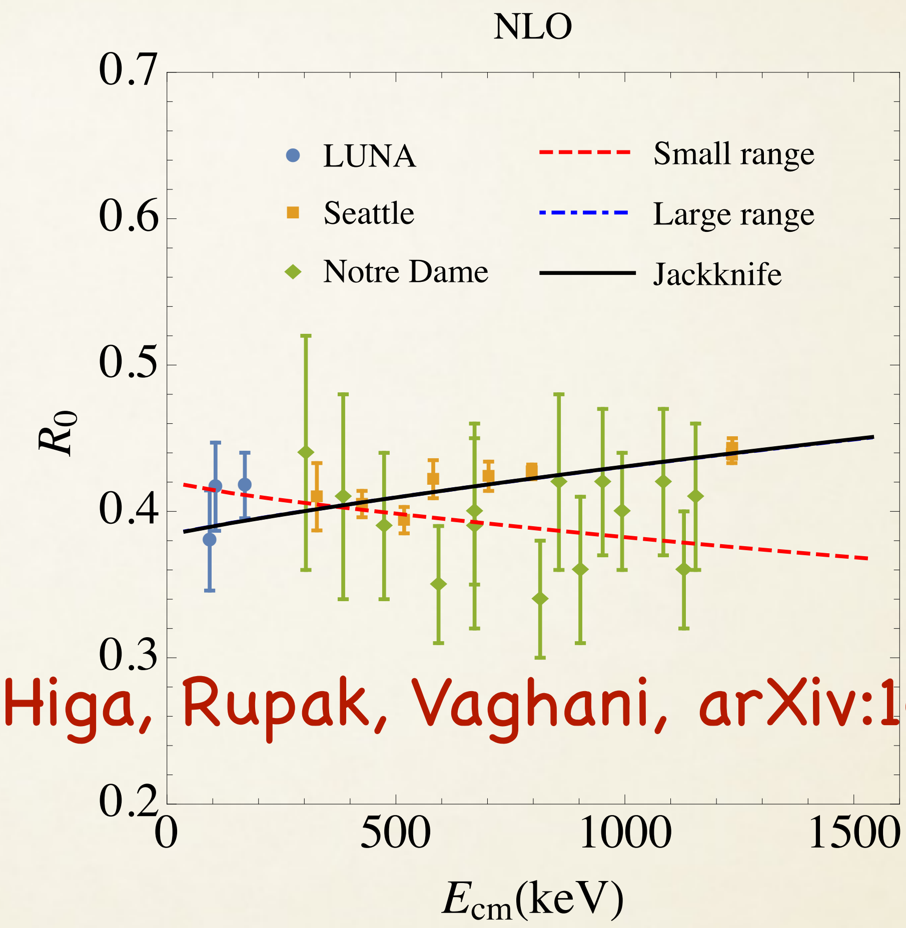
Fernando, Higa, Rupak; EPJA 48, 24 (2012)

RADCAP: Bertulani

CDXS+: Typel

Another Example: $\alpha(^3\mathrm{He},\gamma)^7\mathrm{Be}$





Higa, Rupak, Vaghani, arXiv:1612.08959

Initial state: s-wave $a_0 \sim \Lambda/Q^2,\ r_0 \sim 1/\Lambda$ ← LO contribution

Final state: shallow p-wave (ground and excited) two operators

Need for low-energy phase shift, especially p-wave

TRIUMF

Elastic scattering of $^3\mathrm{He} + \alpha$ with SONIK

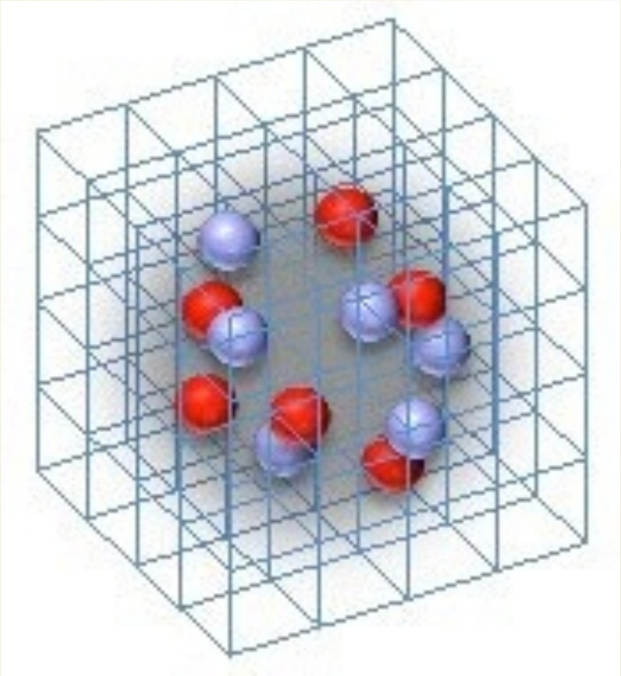
Energy range about 500 keV to 3 MeV

Spokespersons: Connolly, Davids, Greife

Reactions in lattice EFT

- Consider: $a(b,\gamma)c$; $a(b,c)d$
- Need effective "cluster" Hamiltonian -- acts in cluster coordinates, spins,etc.
- Calculate reaction with cluster Hamiltonian. Many possibilities --- traditional methods, continuum halo/cluster EFT, lattice method

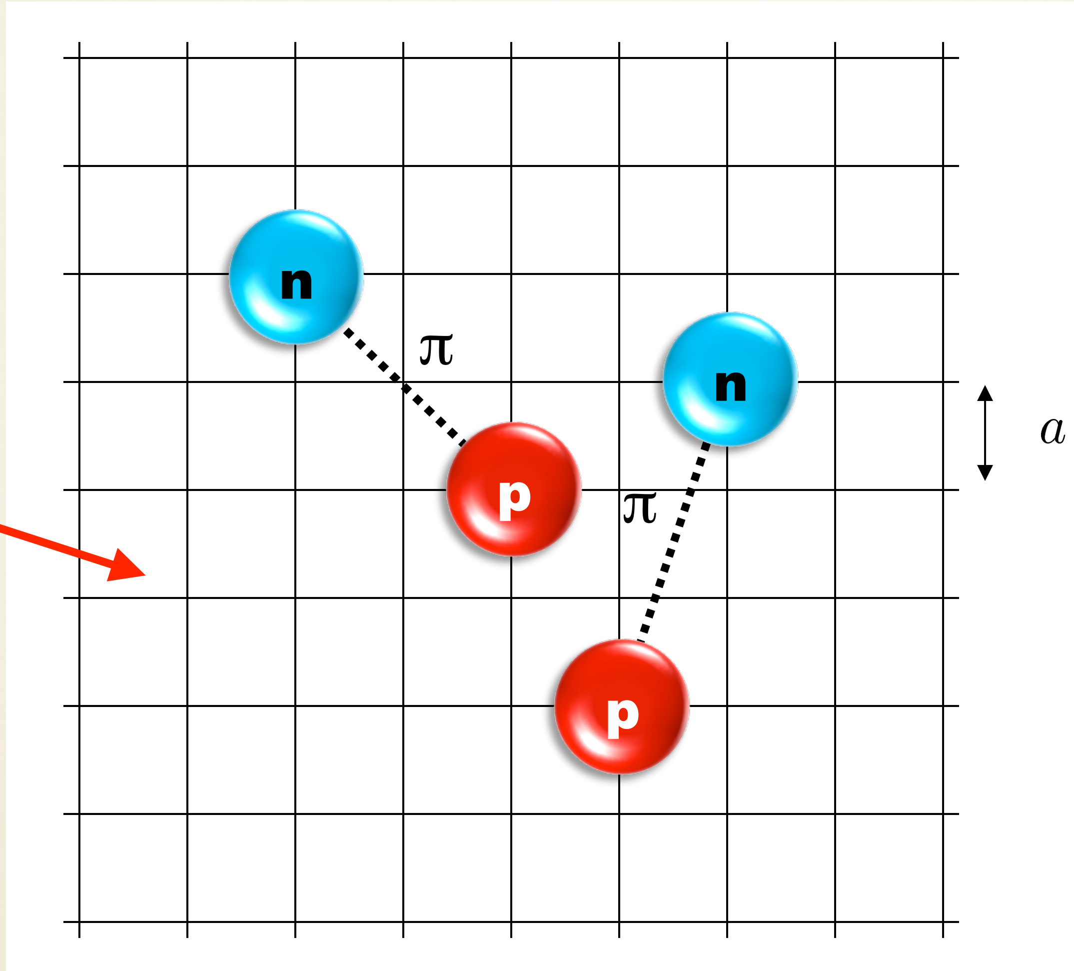
Nuclear Lattice EFT Collaboration



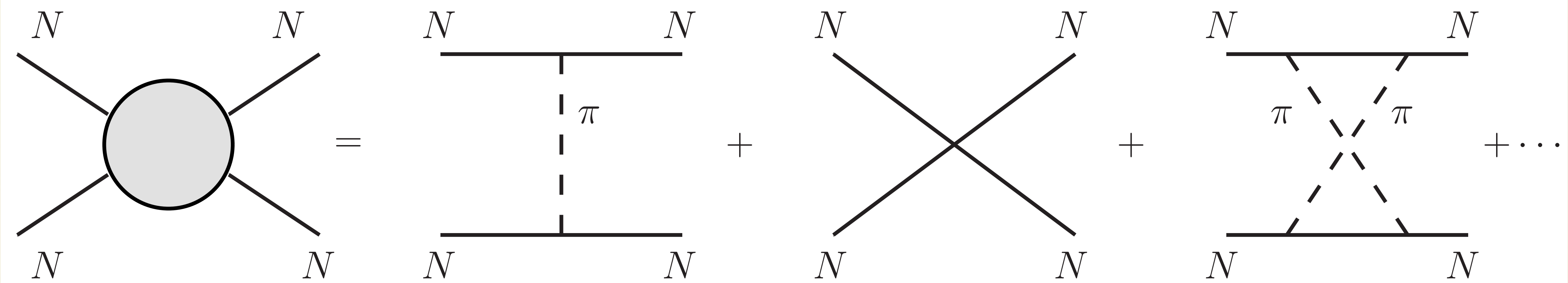
Evgeny Epelbaum (Bochum)
Hermann Krebs (Bochum)
Timo Lähde (Jülich)
Dean Lee (NCSU)
Thomas Luu (Jülich)
Ulf-G. Meißner (Bonn)

Lattice EFT

lattice



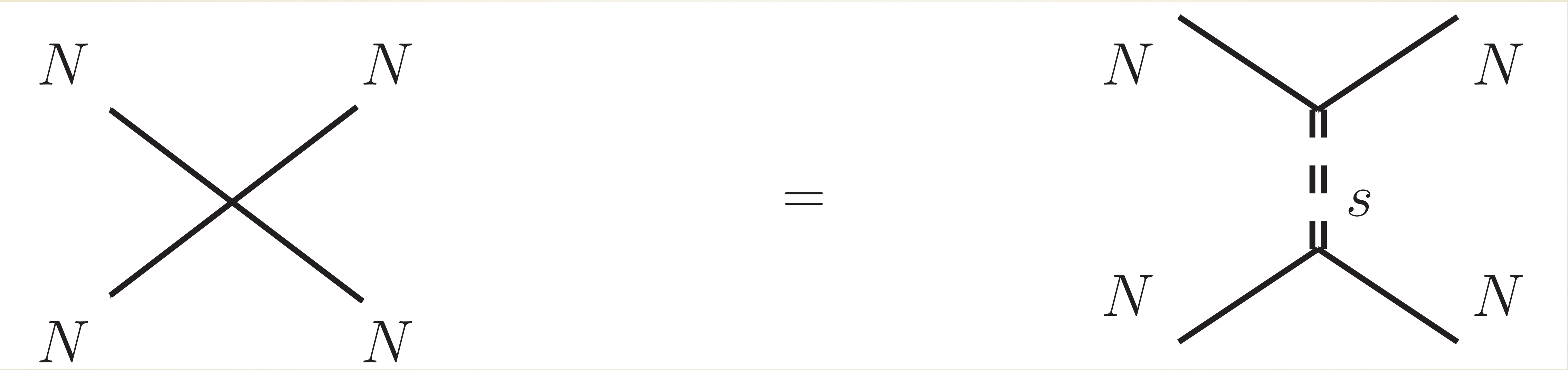
Effective Field Theory



Courtesy D. Lee

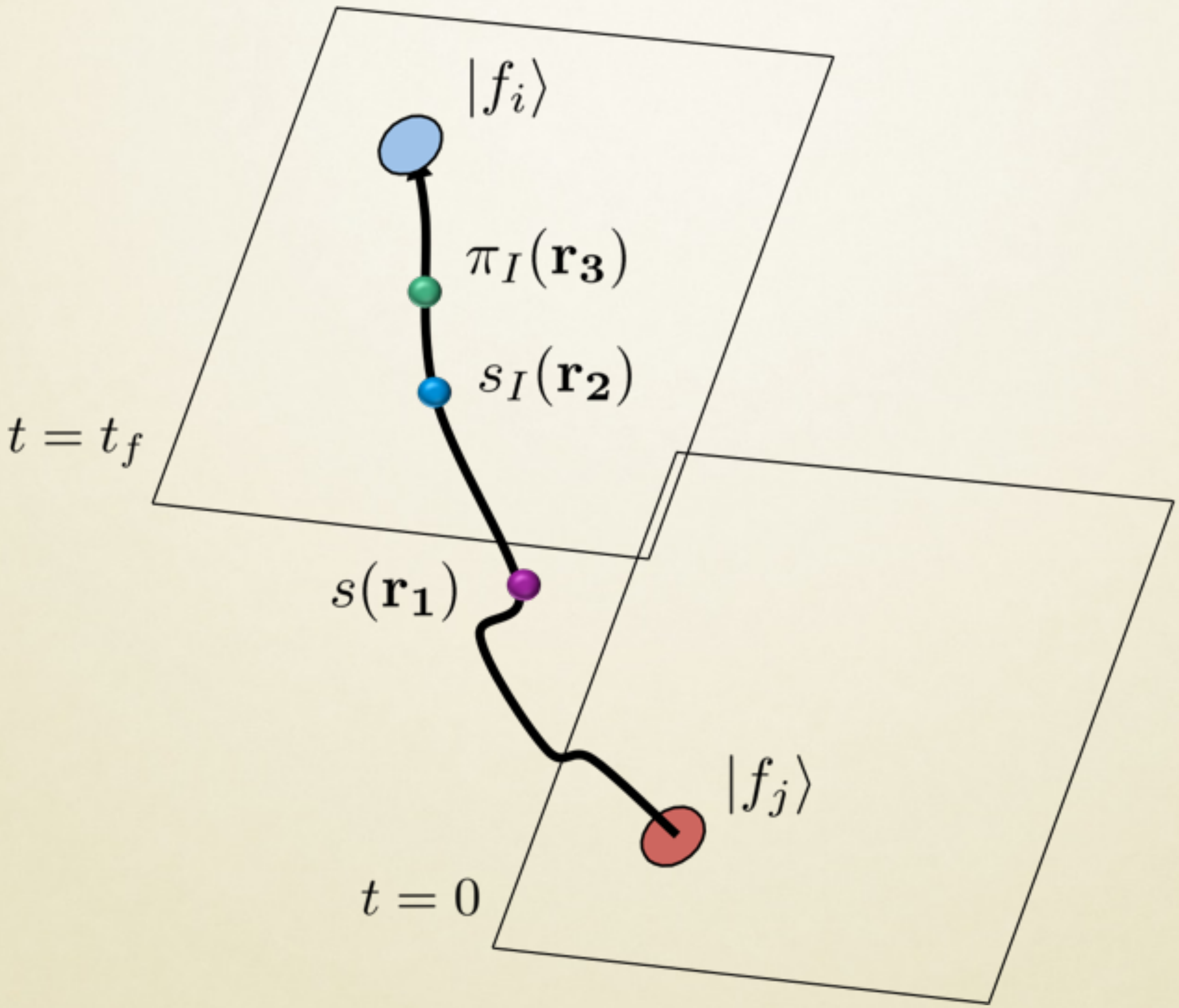
Auxiliary Field method

Replace contact-interaction by interaction of each nucleon with a background auxiliary field using a Hubbard-Stratonovich transformation



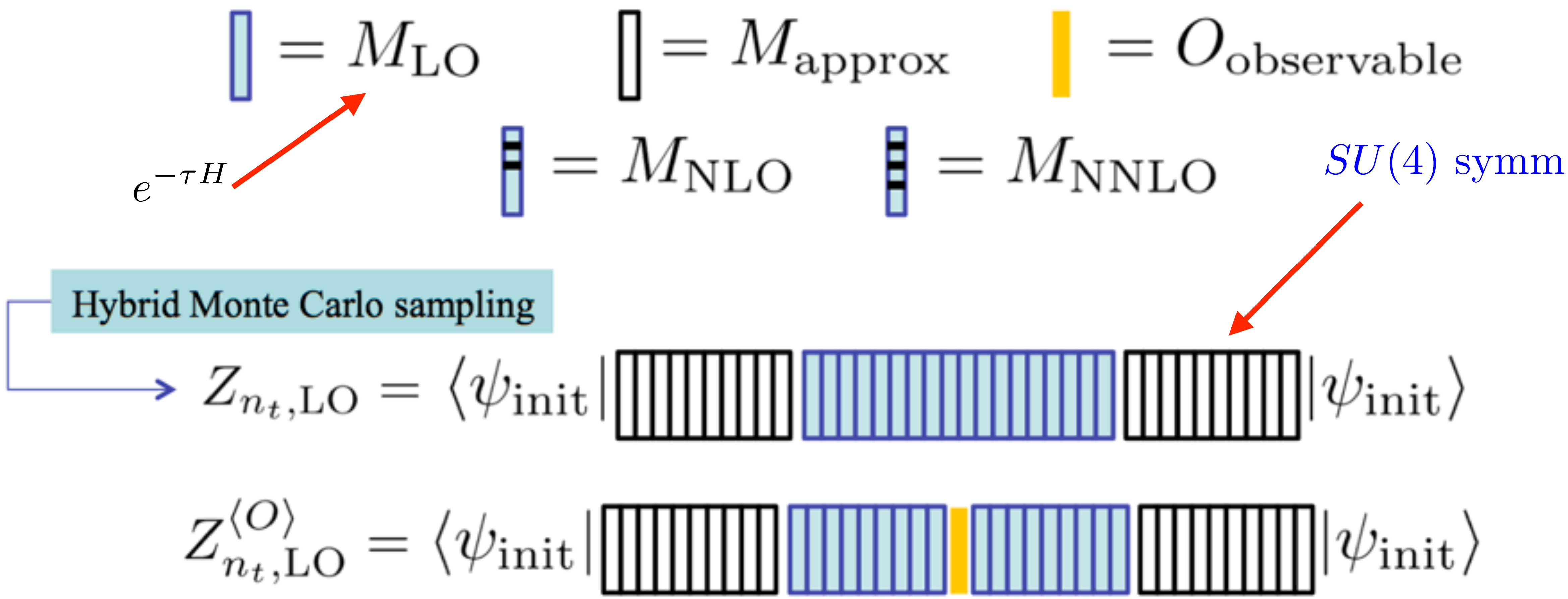
$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} ds\ \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}s\ (N^\dagger N)\right]$$

Auxiliary Field method



Courtesy D. Lee

Schematic of lattice Monte Carlo calculation

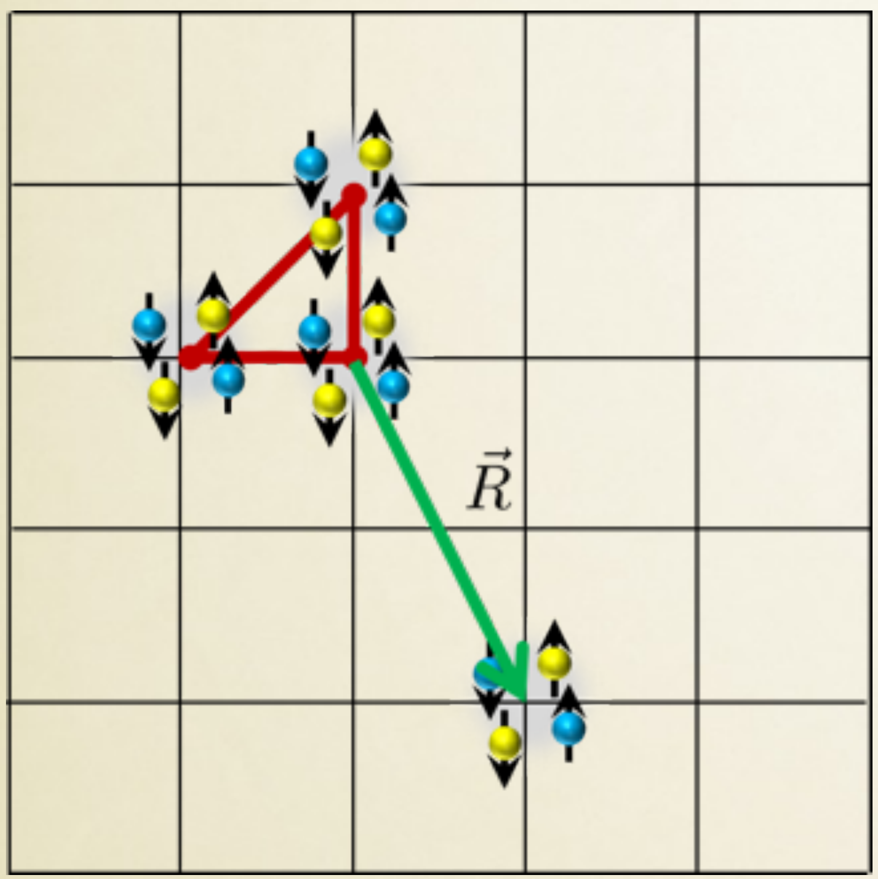


$$e^{-E_{0,\text{LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,\text{LO}} / Z_{n_t,\text{LO}}$$

$$\langle O \rangle_{0,\text{LO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\text{LO}} / Z_{n_t,\text{LO}}$$

Courtesy D. Lee

Adiabatic Projection Method



Initial state $|\vec{R}\rangle$

Evolved state $|\vec{R}\rangle_\tau = e^{-\tau H}|\vec{R}\rangle$

$${}_\tau\langle \vec{R}'|H|\vec{R}\rangle_\tau$$

Energy measurements in cluster basis.
Divide by the norm matrix as these are not orthogonal basis $[N_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle \vec{R}|\vec{R}'\rangle_\tau$

Microscopic Hamiltonian $L^{3(A-1)}$
Cluster Hamiltonian L^3 ← smaller matrices in practice!!
-- acts on the cluster CM and spins

Proof of Concept

- n-d scattering in the quartet channel
  - Low energy EFT is known, only two body
  - Shallow deuteron (large coupling)

only two-body interaction

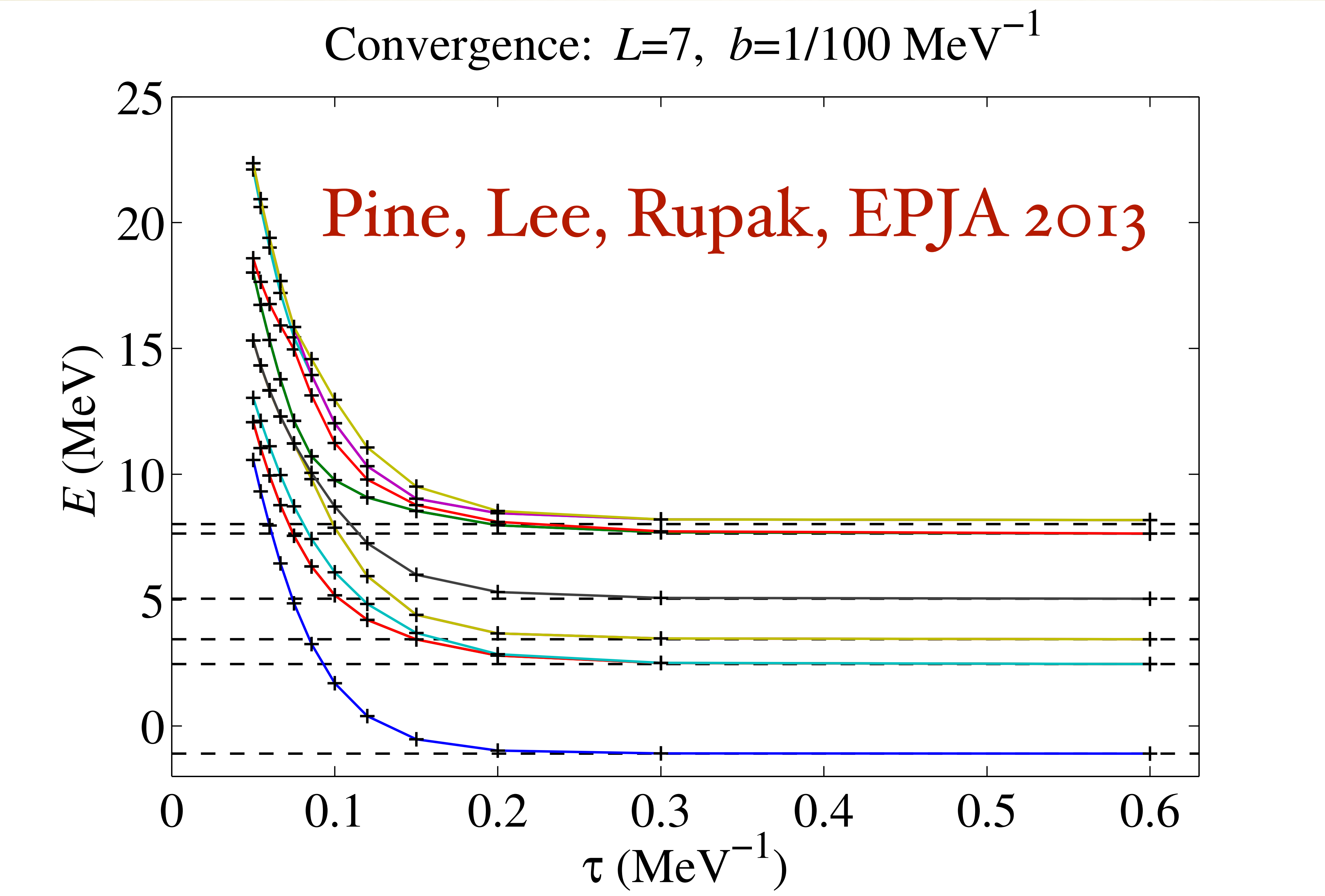
$$\mathcal{L}_I = -g(\psi_\uparrow^\dagger \psi_\uparrow)(\psi_\downarrow^\dagger \psi_\downarrow)$$

3-body Faddeev

$$T(p) = h(p) + \int dq K(p,q) T(q)$$

$$T(p) = \frac{2\pi}{\mu} \frac{1}{p \cot \delta - ip}$$

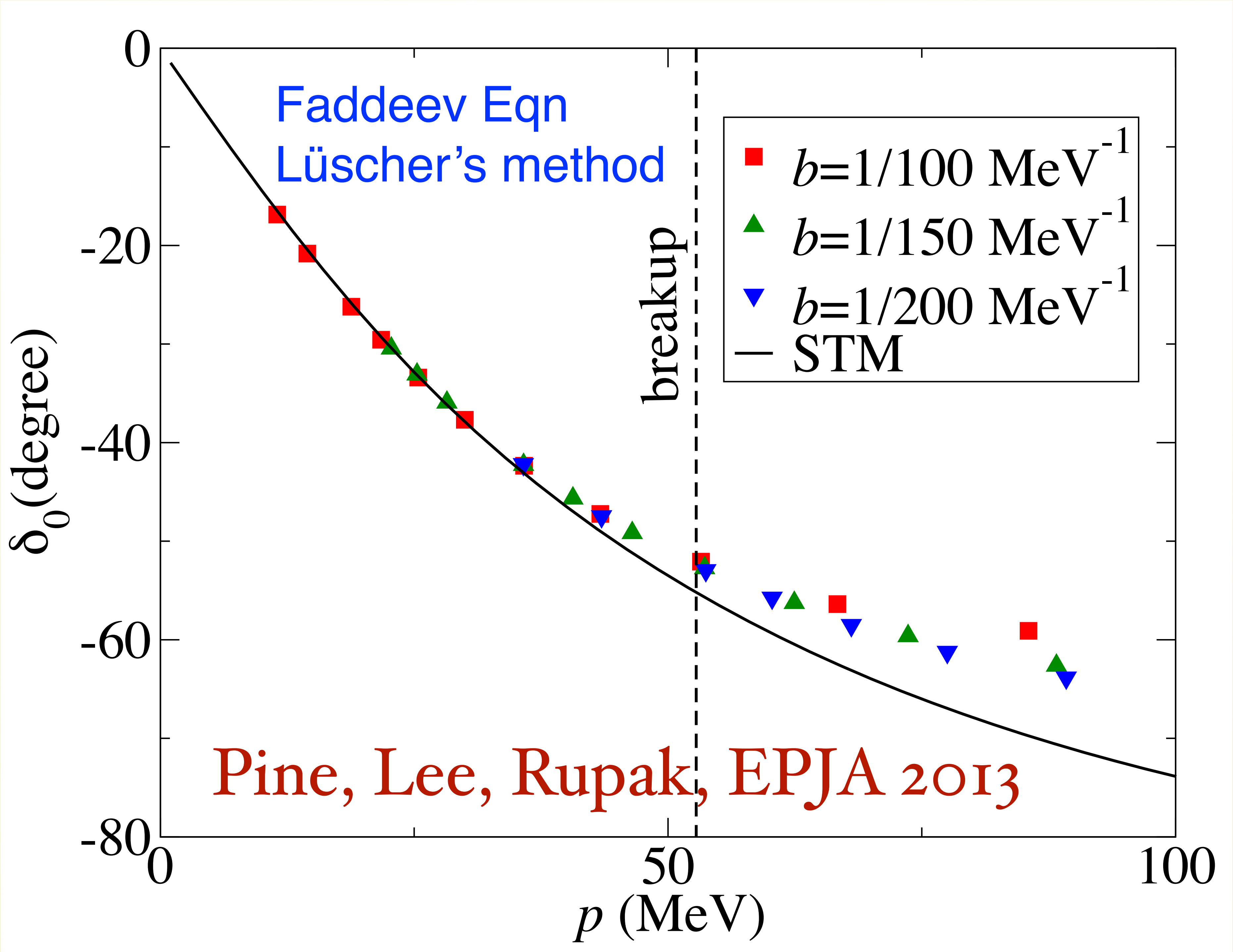
Neutron-Deuteron System



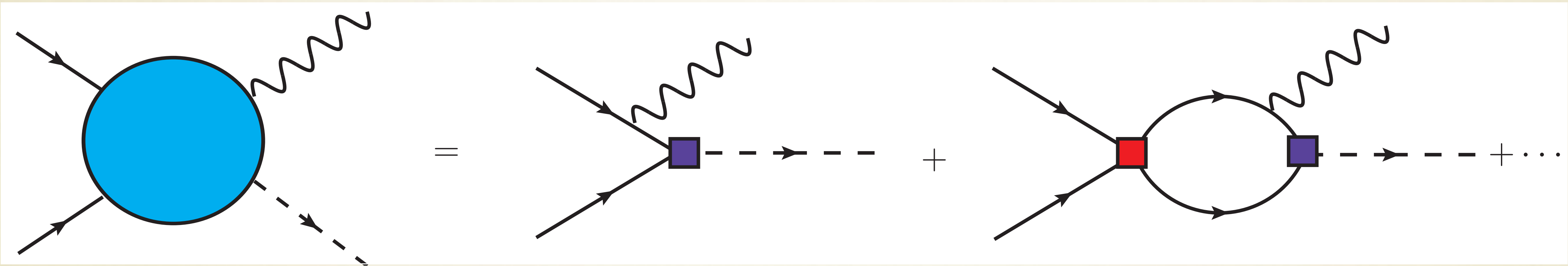
- grouping R found efficient, more later

$\sim 30 \times 30$

Neutron-Deuteron Phase Shift



Lattice $p(n,\gamma)d$



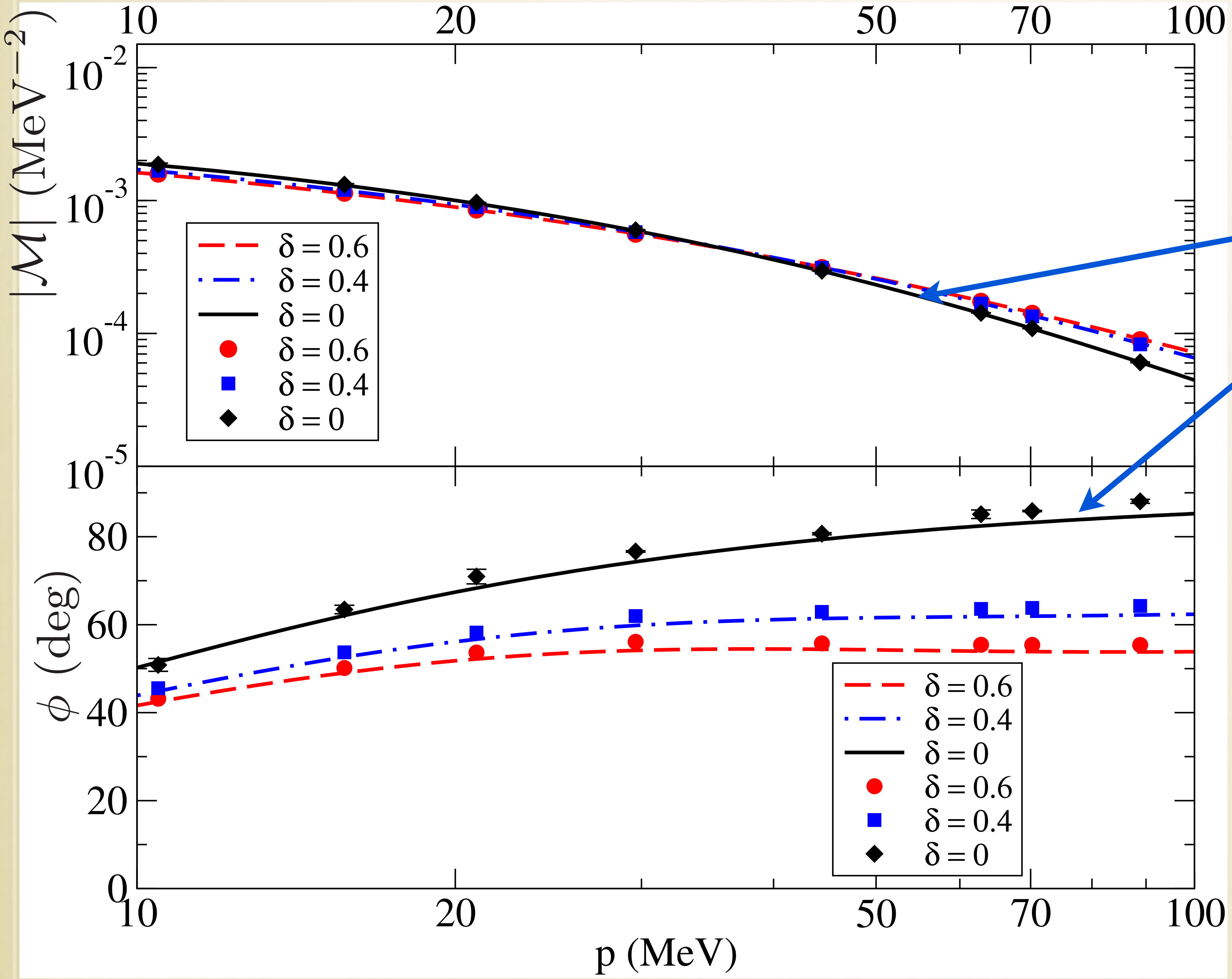
Write $\langle\psi_B|O_{\rm EM}|\psi_i\rangle$ using retarded Green's function

$$\mathcal{M}(\epsilon) = \left(\frac{p^2}{M} - E - i\epsilon\right)\sum_{\boldsymbol{x},\boldsymbol{y}} \psi_B^*(\boldsymbol{y})\langle\boldsymbol{y}|\frac{1}{E-\hat{H}_s+i\epsilon}|\boldsymbol{x}\rangle e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

cluster Hamiltonian goes here

LSZ reduction in QM

Capture Amplitude

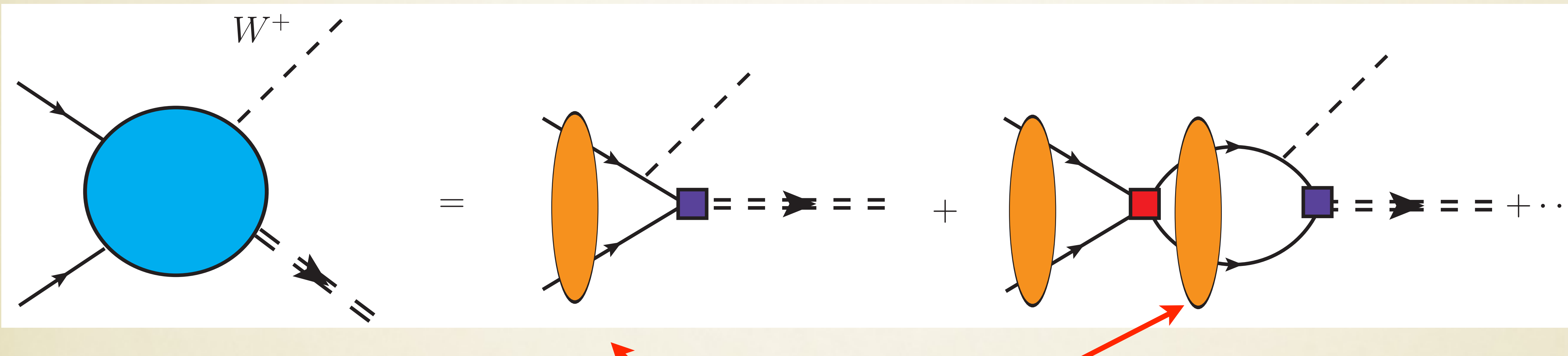


continuum results

$$\delta = \epsilon M/p^2$$

Rupak & Lee, PRL 2013

Proton fusion in continuum EFT



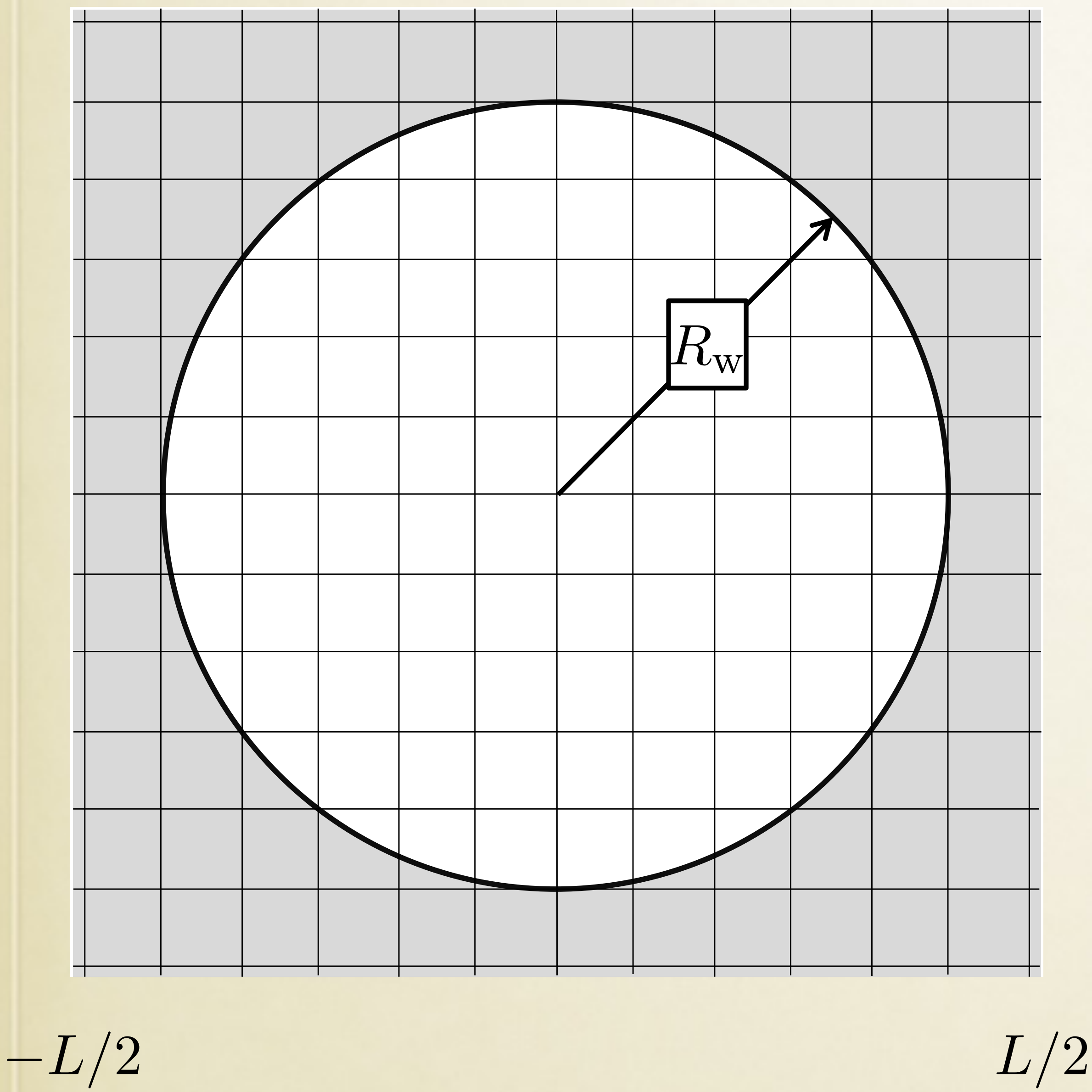
Kong, Ravndal 1999

Coulomb ladder

Peripheral scattering but how to calculate on the lattice?

Consider elastic proton-proton scattering as a warmup

Spherical-wall method



$$\psi_{\text{short}}(r) \propto j_0(kr)\cot\delta_s - n_0(kr),$$

$$\psi_{\text{Coulomb}}(r) \propto F_0(kr)\cot\delta_{sc} + G_0(kr)$$

Adjust from free theory:

$$j_0(k_0 R_w) = 0$$

IR scale setting

Hard spherical wall boundary conditions, Borasoy et al. 2007

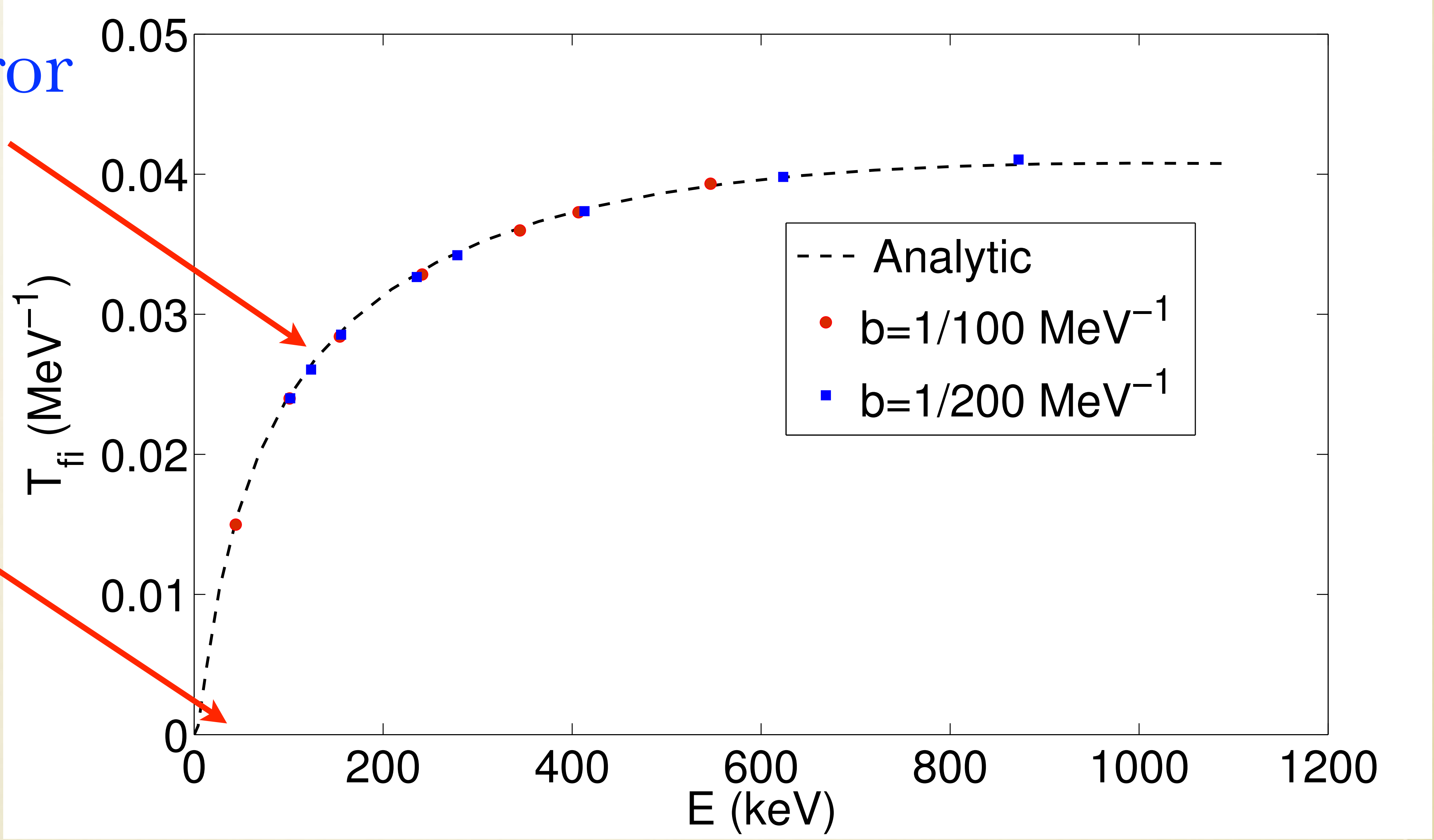
Carlson et al. 1984

Even older ?

Proton-proton fusion

3% fitting error propagates

Gamow peak 6 keV

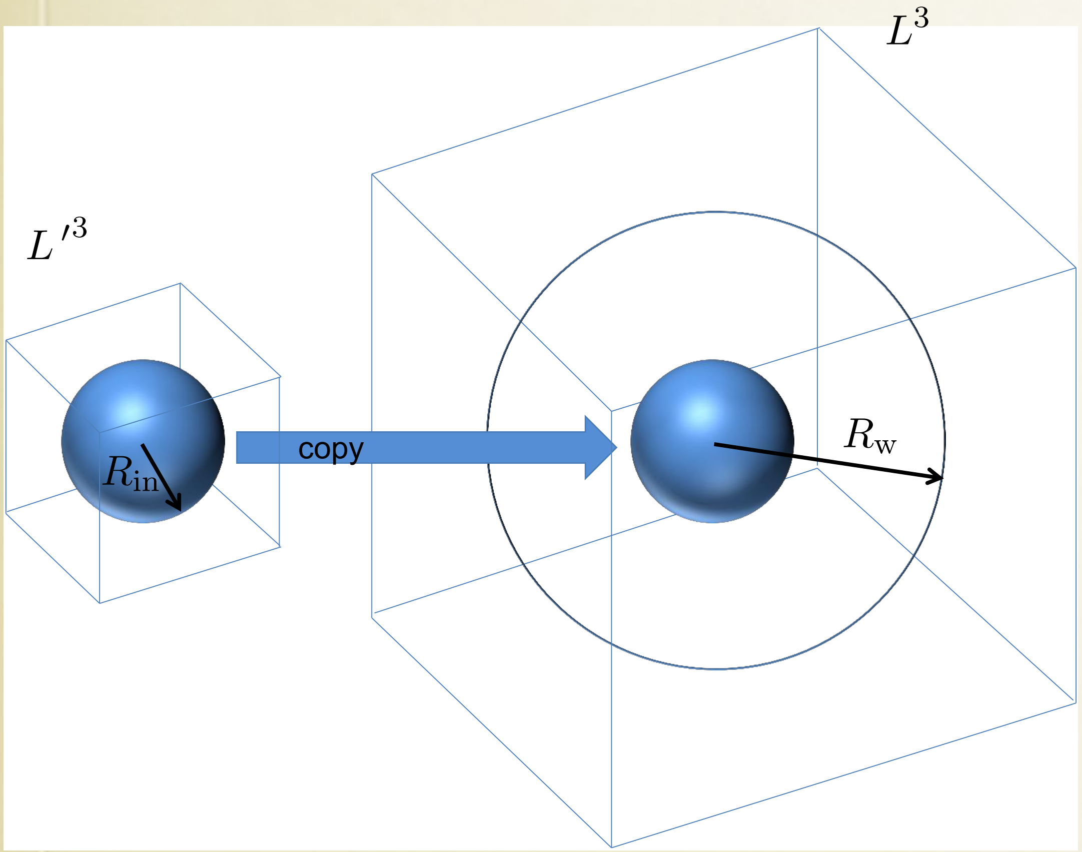


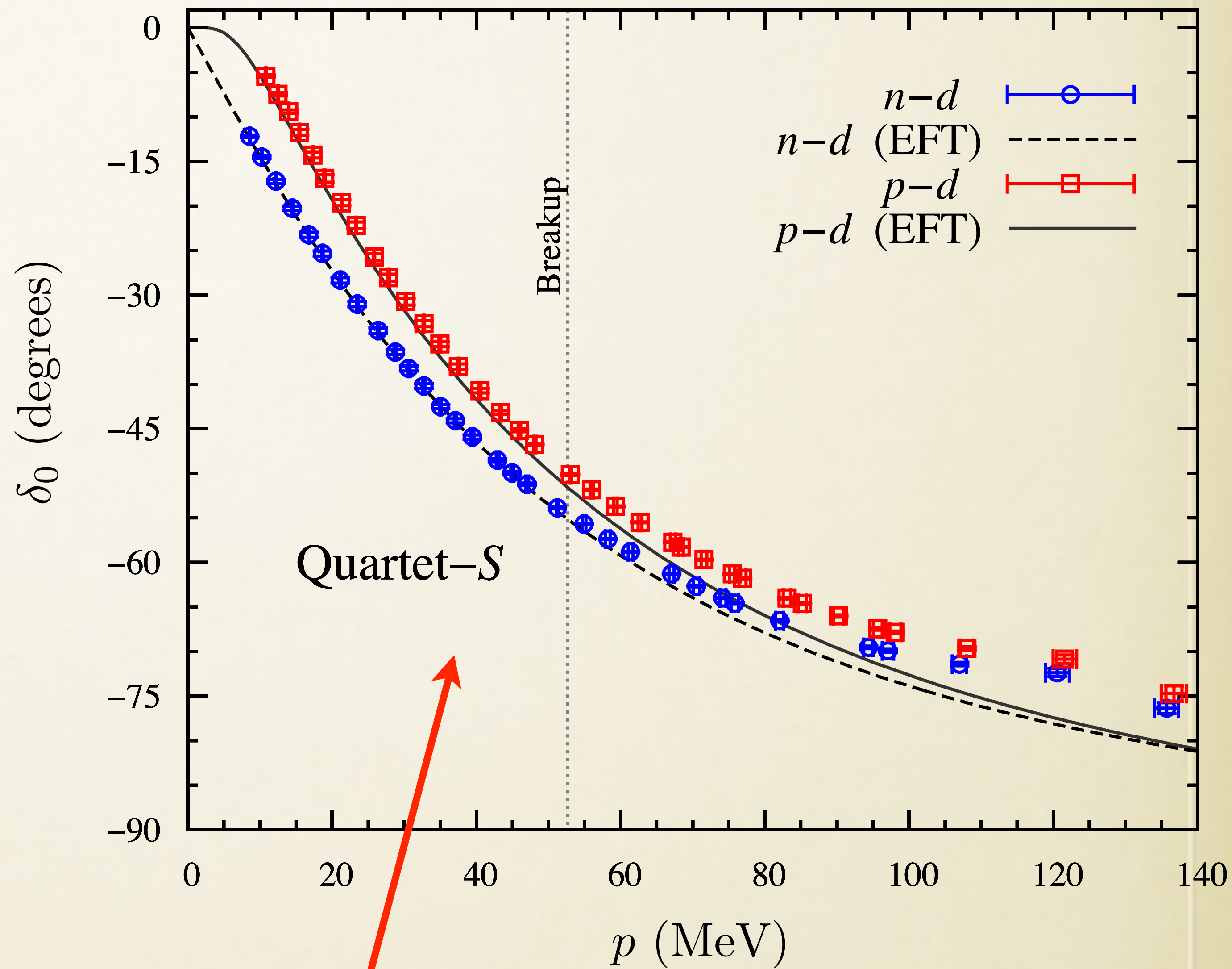
$$\Lambda(p) = \sqrt{\frac{\gamma^2}{8\pi C_\eta^2}}|T_{fi}(p)|$$

$\Lambda_{EFT}(0) \approx 2.51$ Kong, Ravndal 1999

$\text{Lattice fit} : \Lambda(0) \approx 2.49 \pm 0.02$ Rupak, Ravi PLB 2014

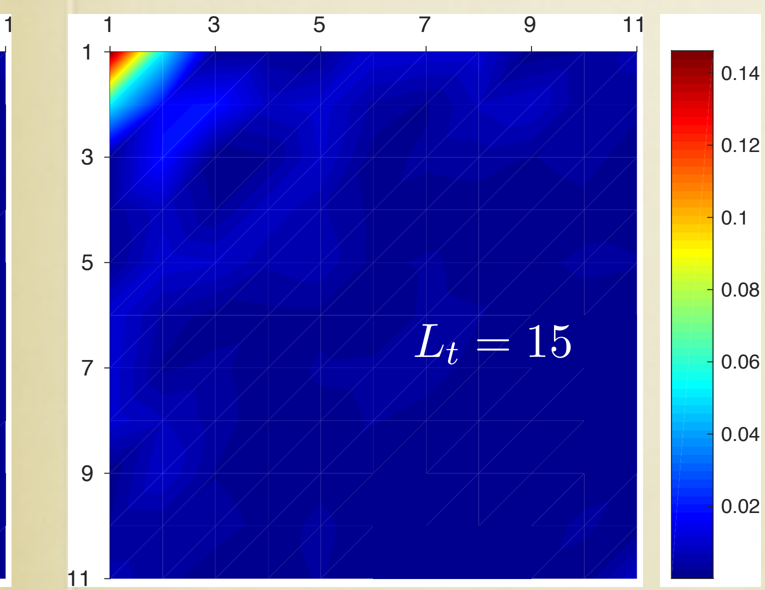
Two-cluster simulation





Matching: 15 to 82

Better lattice results than before



Fermion-dimer

Elhatisari, Lee, Meißner, Rupak, EPJA 25 (2016)

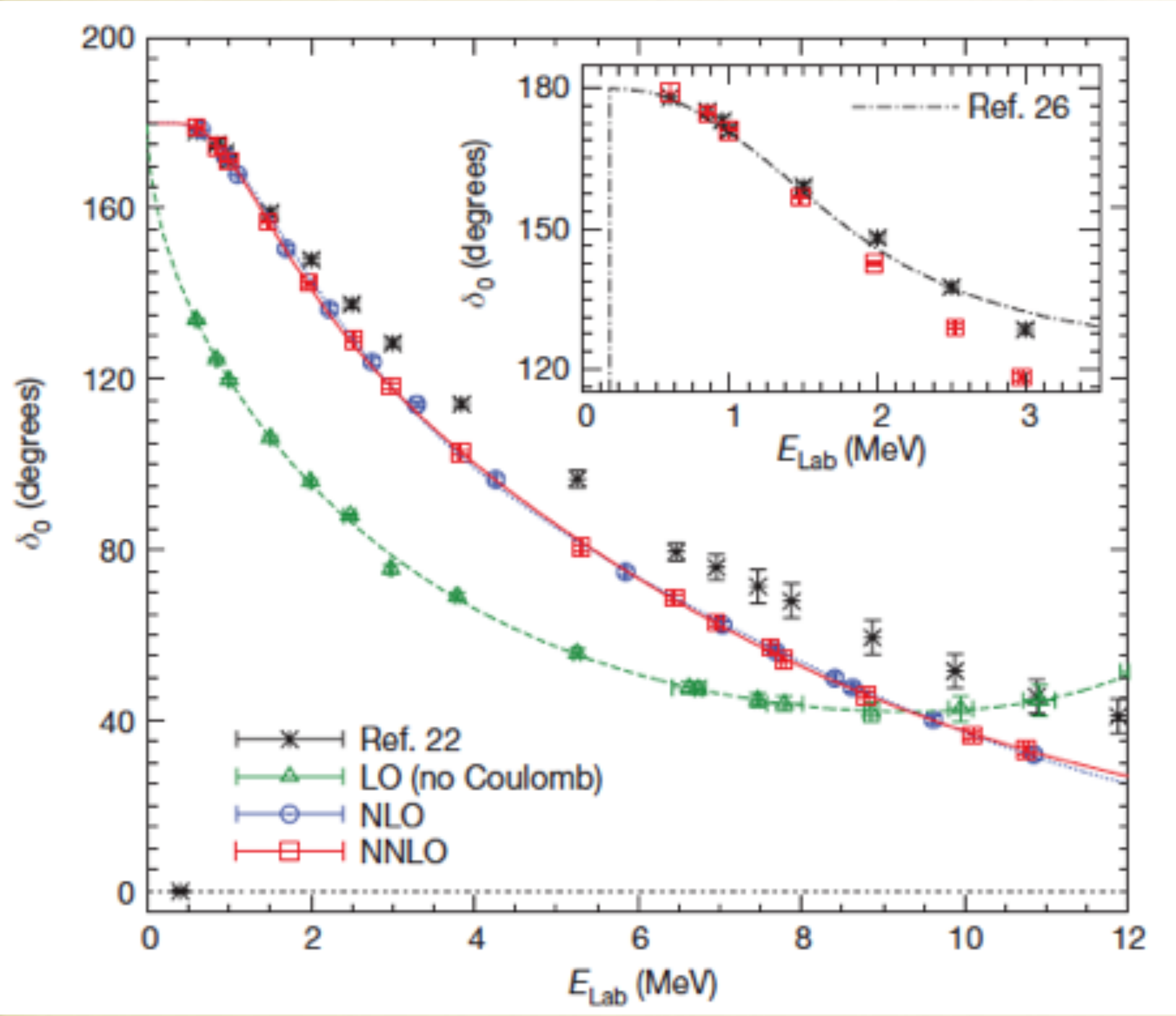
ALPHA-ALHPA SCATTERING

We have calculated alpha-alpha scattering up to NNLO using the adiabatic projection method.

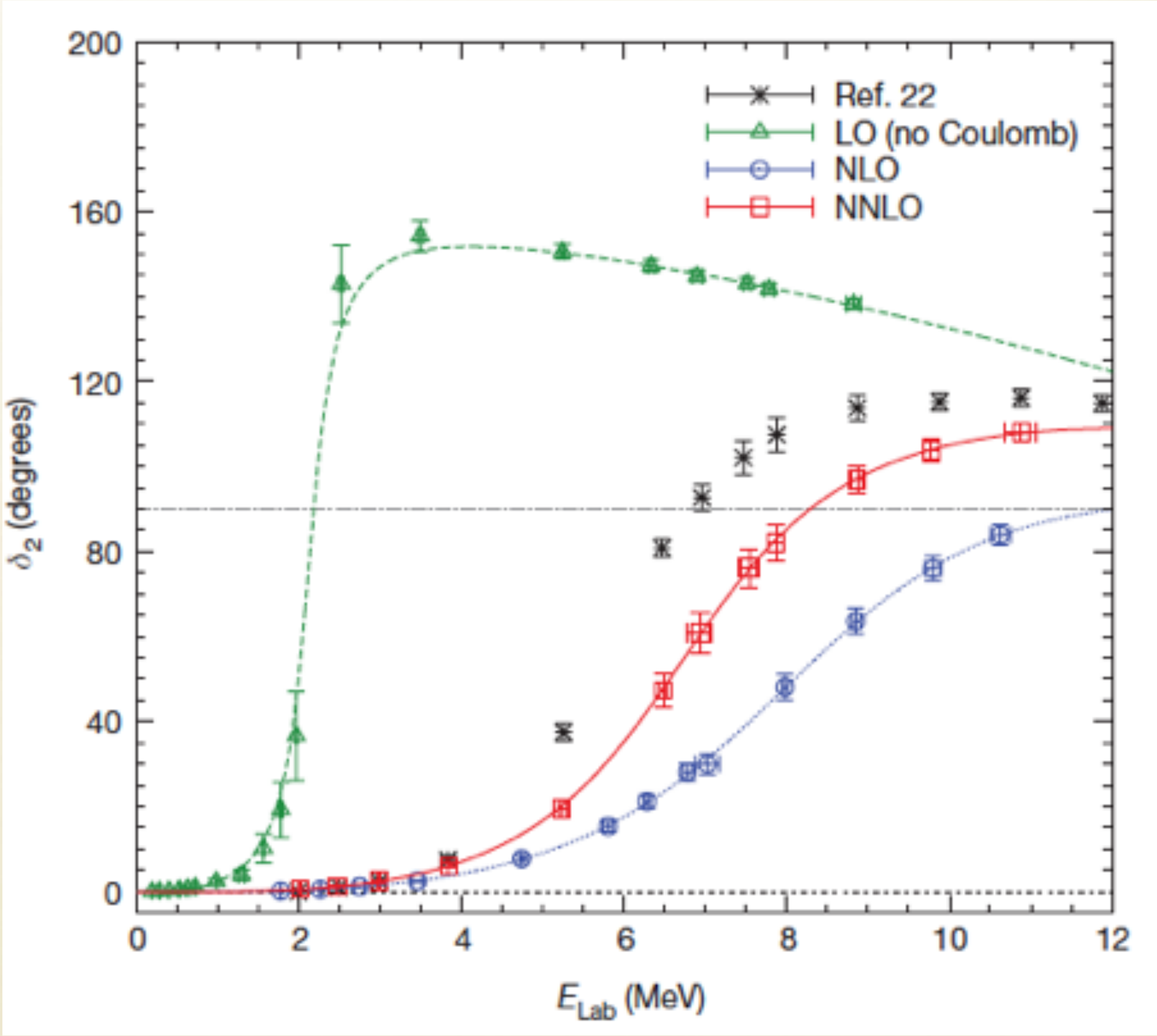
s- and d-wave phase shift were calculated using the spherical wall method in the presence of long-range Coulomb force.

Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)

S-wave



D-wave



Ref. 22: Afzal et al. (1969)
Ref. 26: Higa et. al. (2008)

Beyond-Alpha Clusters

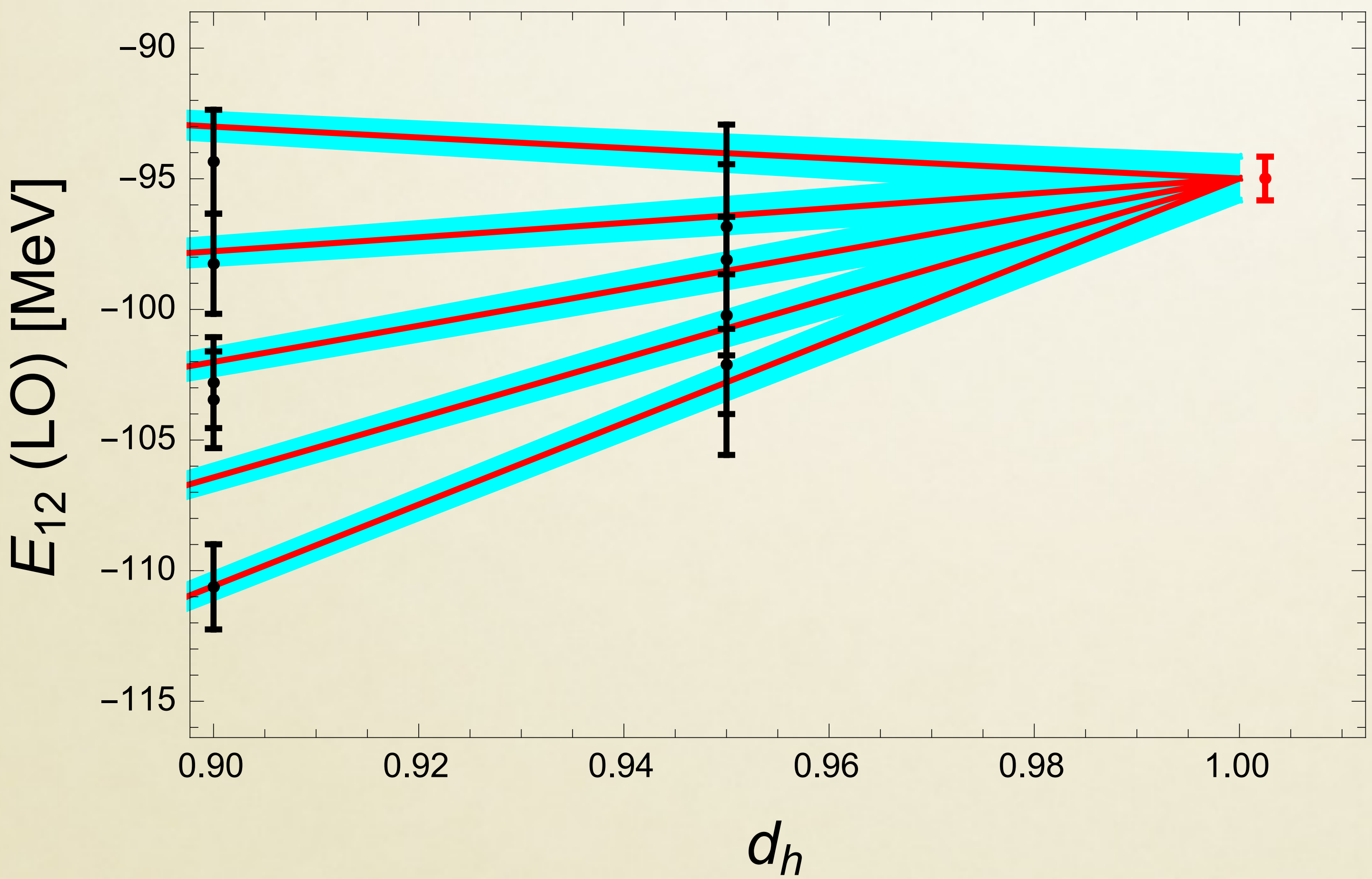
Symmetry Sign Extrapolation

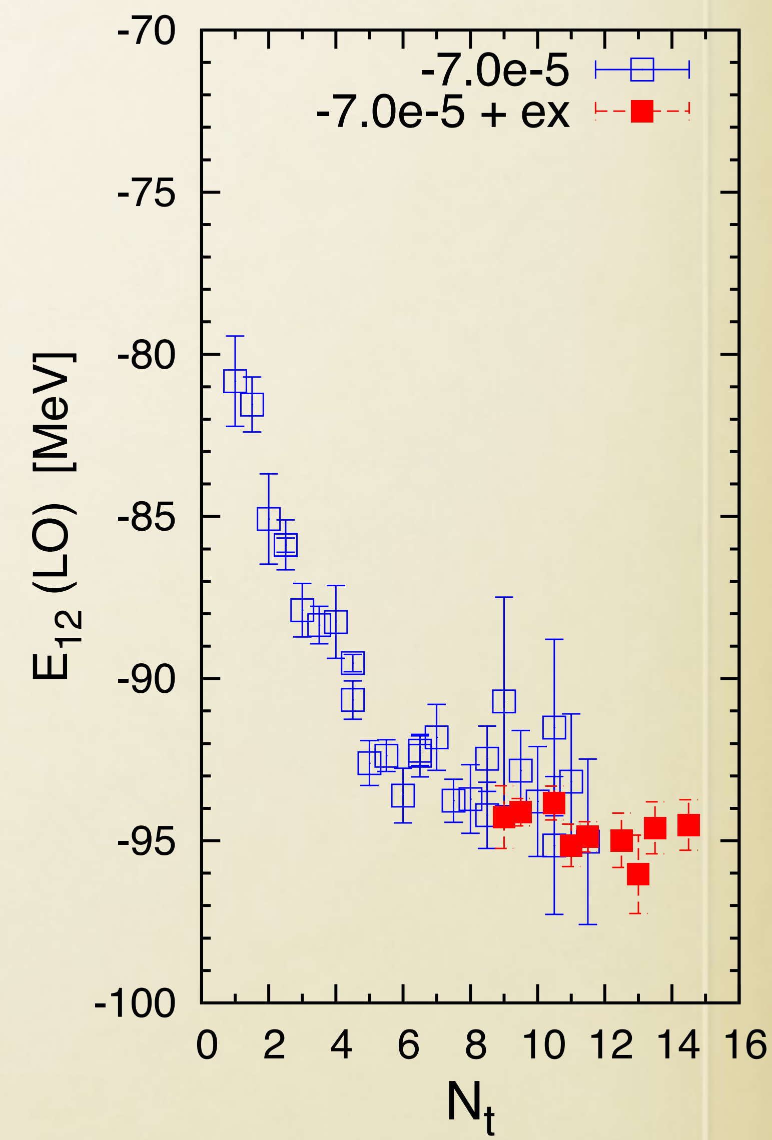
$$H \equiv d_h H_{\text{physical}} + (1 - d_h) H_{SU(4)}$$

$$H_{SU(4)} = \frac{1}{2} C_{SU(4)} (N^\dagger N)^2$$

Alhassid et. al (94), Koonin at. al (97)

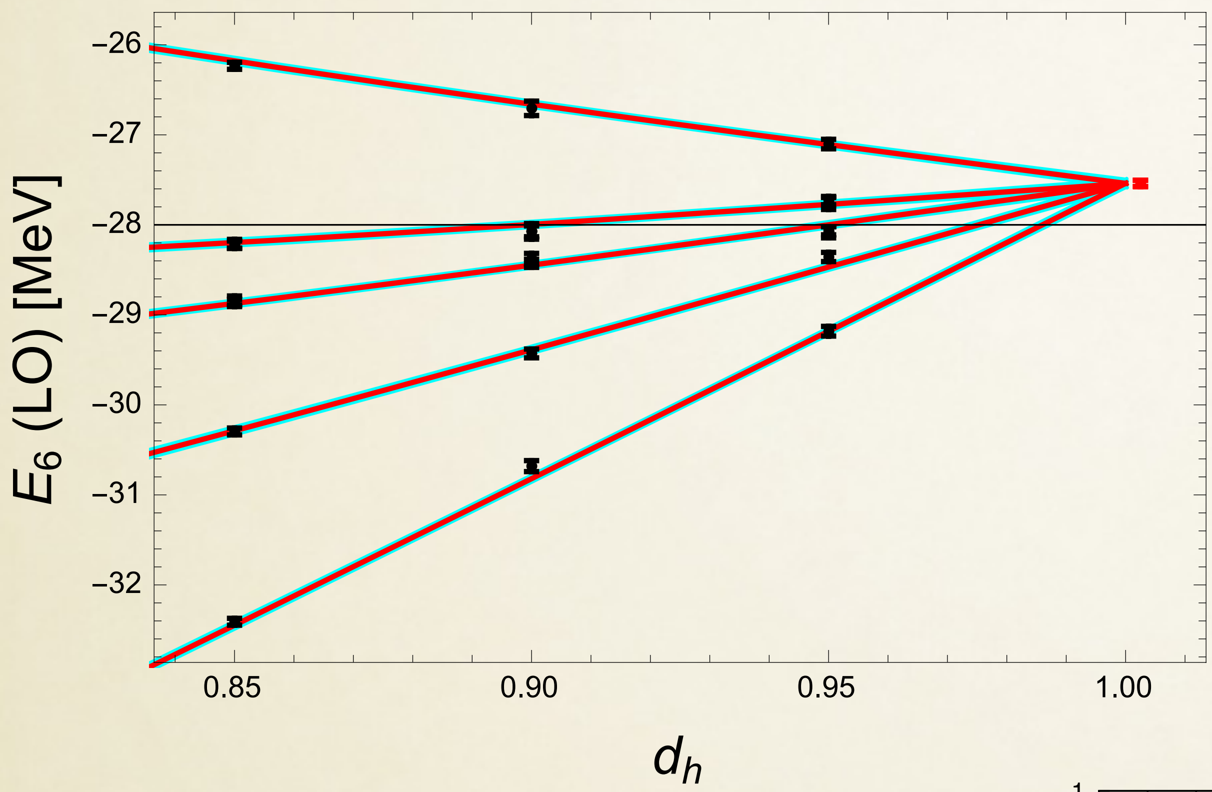
First verify in C-12



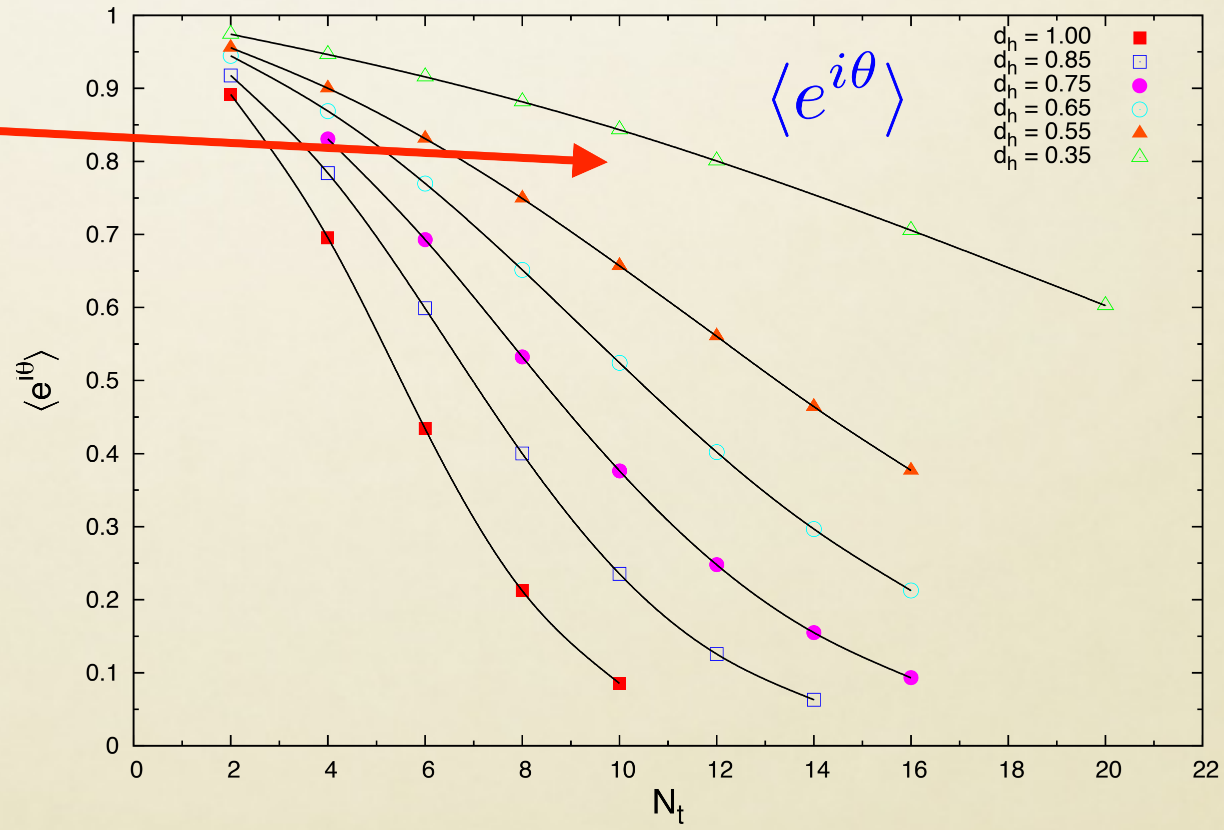


Lähde, Luu, Lee, Meißner, Epelbaum, Krebs, Rupak, EPJA 51, 92 (2015)

Results for A=6



Sign problem ameliorated
- transfer amplitude phase



Tools to explore

Explore $N \neq Z$

Pin-hole Algorithm

Calculates charge distribution in CM coordinates

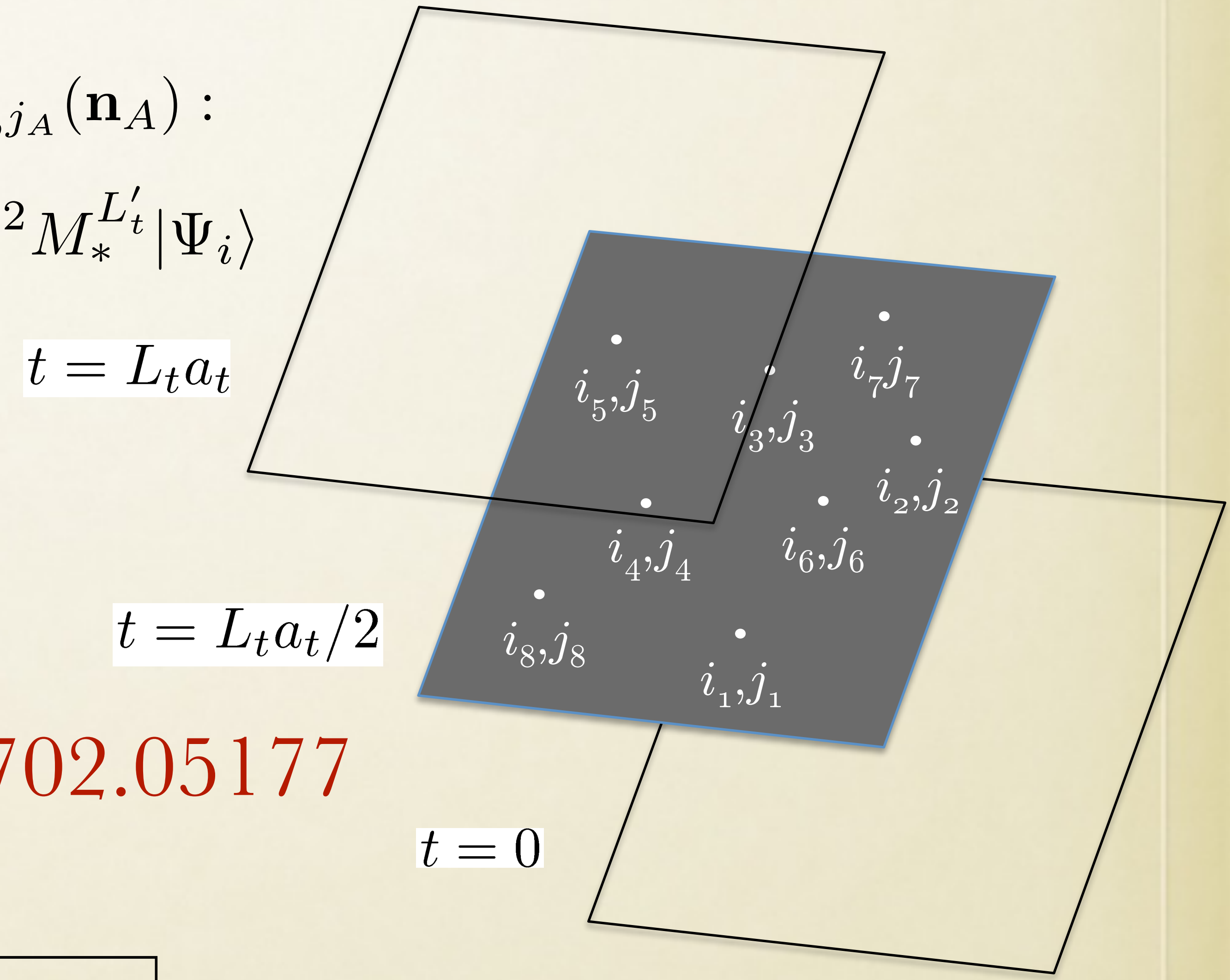
Importance sampling according to magnitude for

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A) = :\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

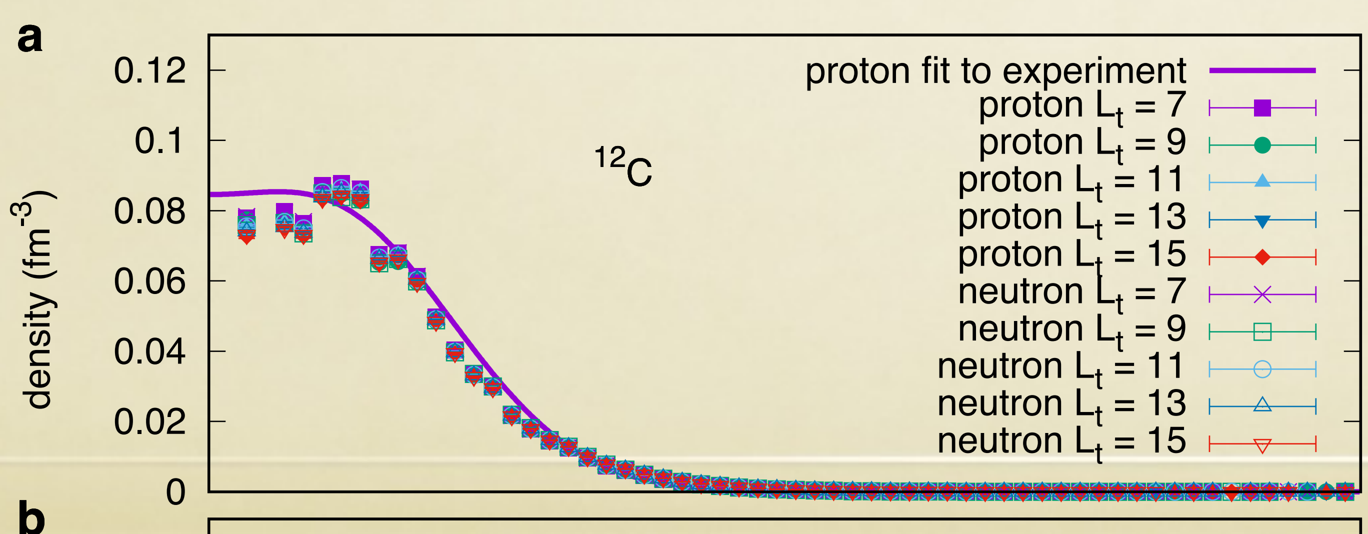
$$\langle\Psi_f|M_*^{L'_t}M^{L_t/2}\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A)M^{L_t/2}M_*^{L'_t}|\Psi_i\rangle$$

CM location minimizes the rms radius

$$\sum_i|\vec{\boldsymbol{R}}_{\rm CM}-\vec{\boldsymbol{r}}_i|^2$$

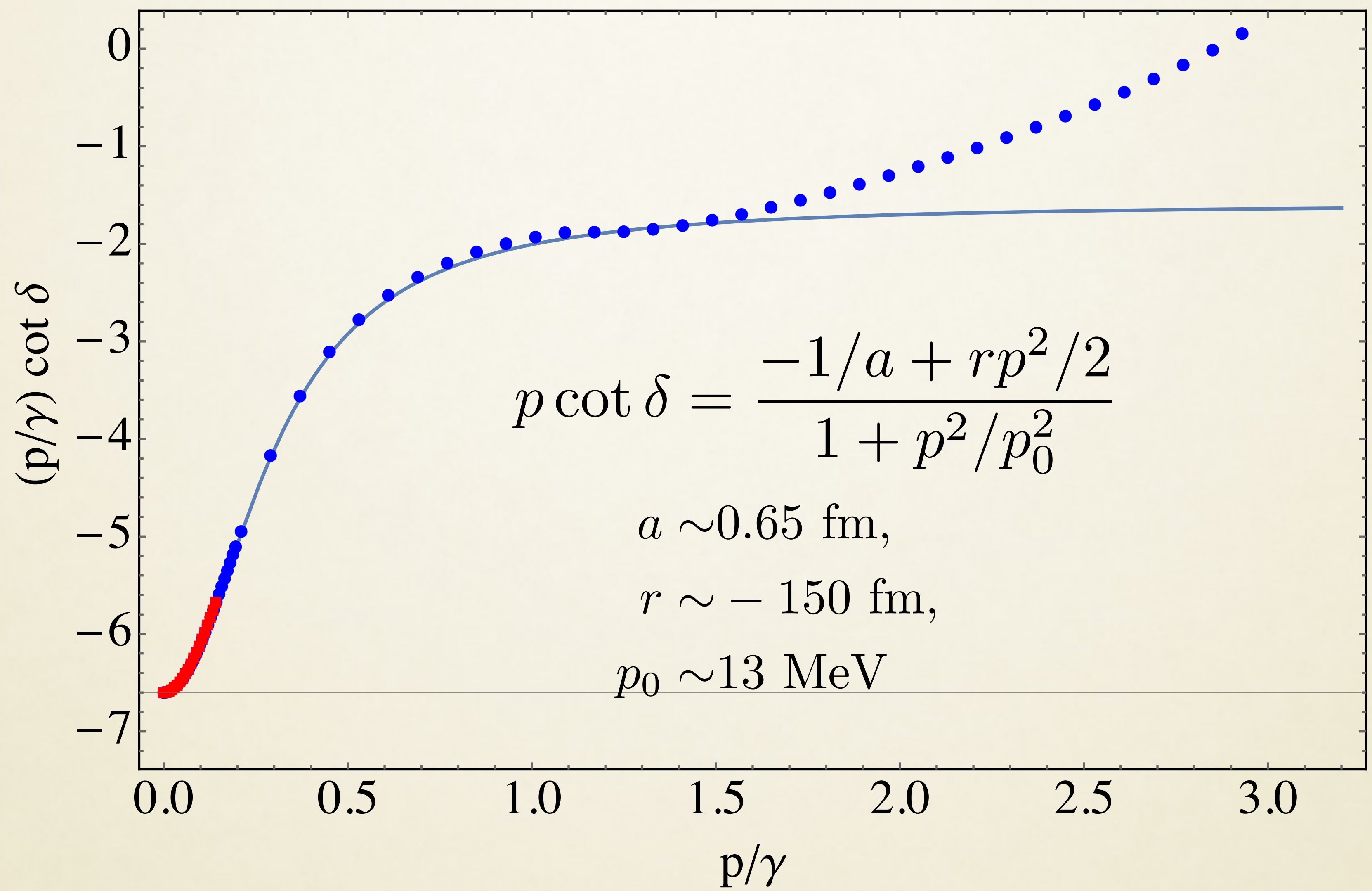


Elhatisari et al. , arXiv 1702.05177



Efimov Physics

N-D DOUBLET CHANNEL



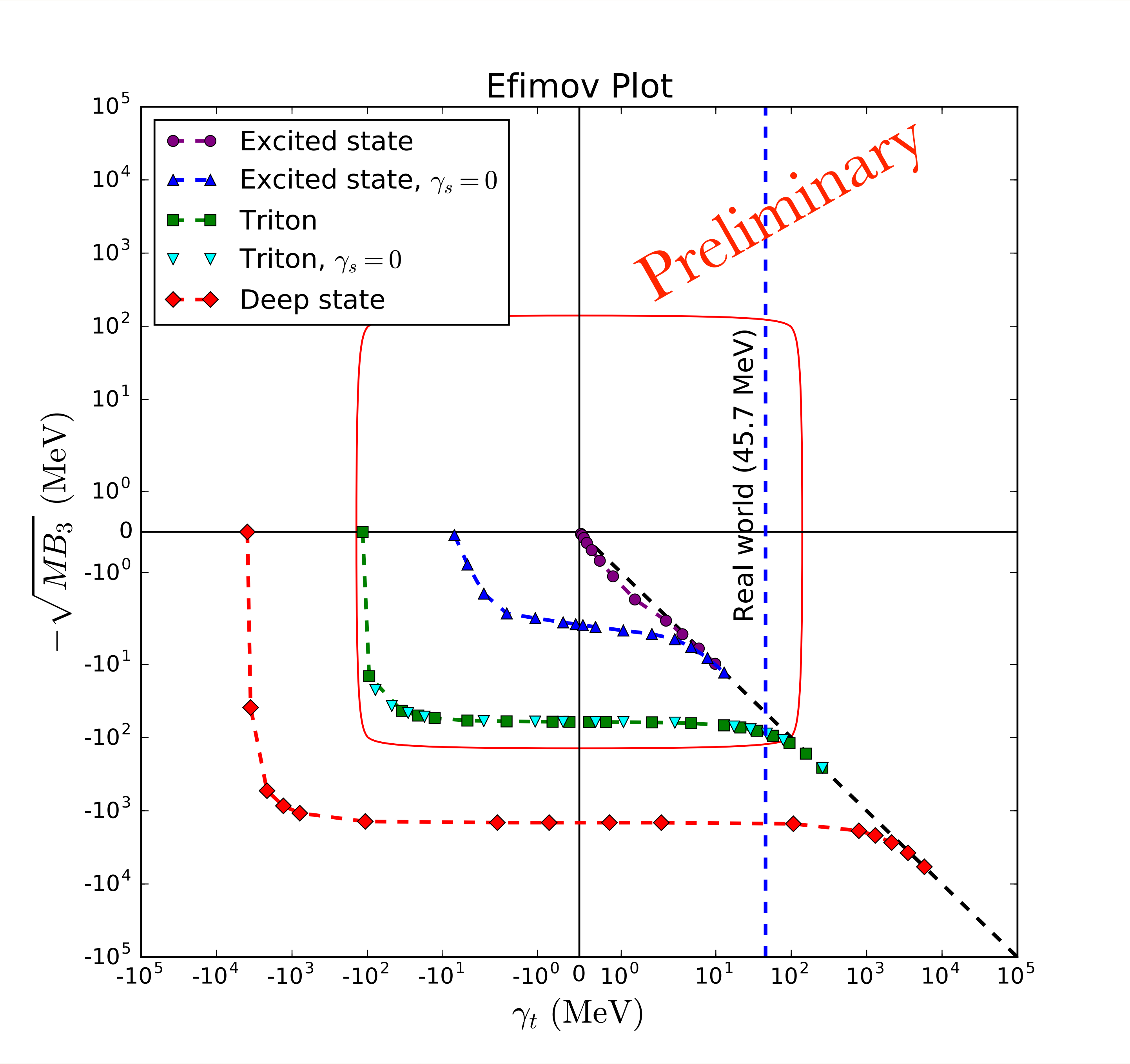
ERE form van Oers & Seagrave (1967)

-what EFT for modified ERE

Virtual state at 0.5 MeV Girard & Fuda (1979)

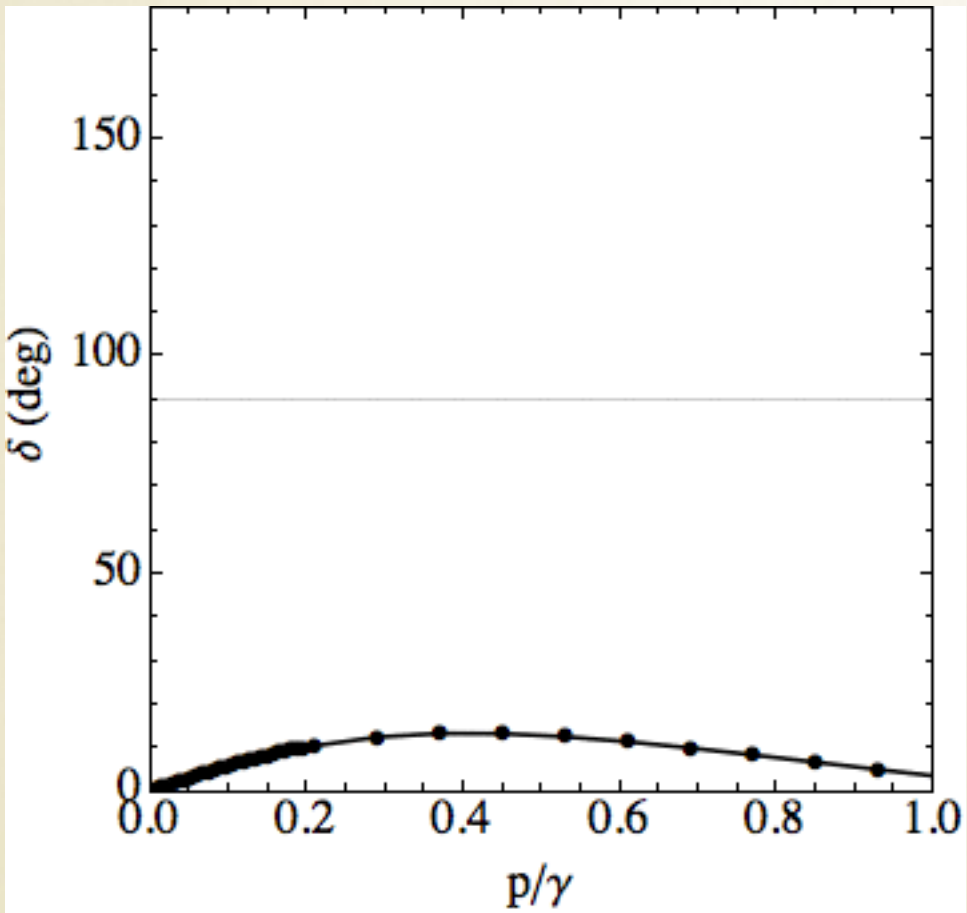
- Efimov physics

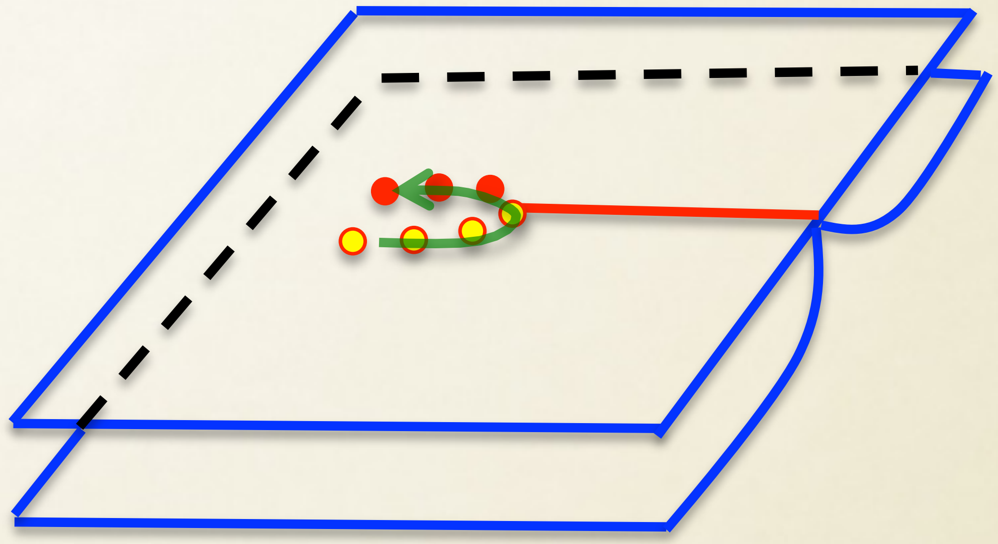
Efimov plot



Higa, Rupak, Vaghani, van Kolck

Virtual State

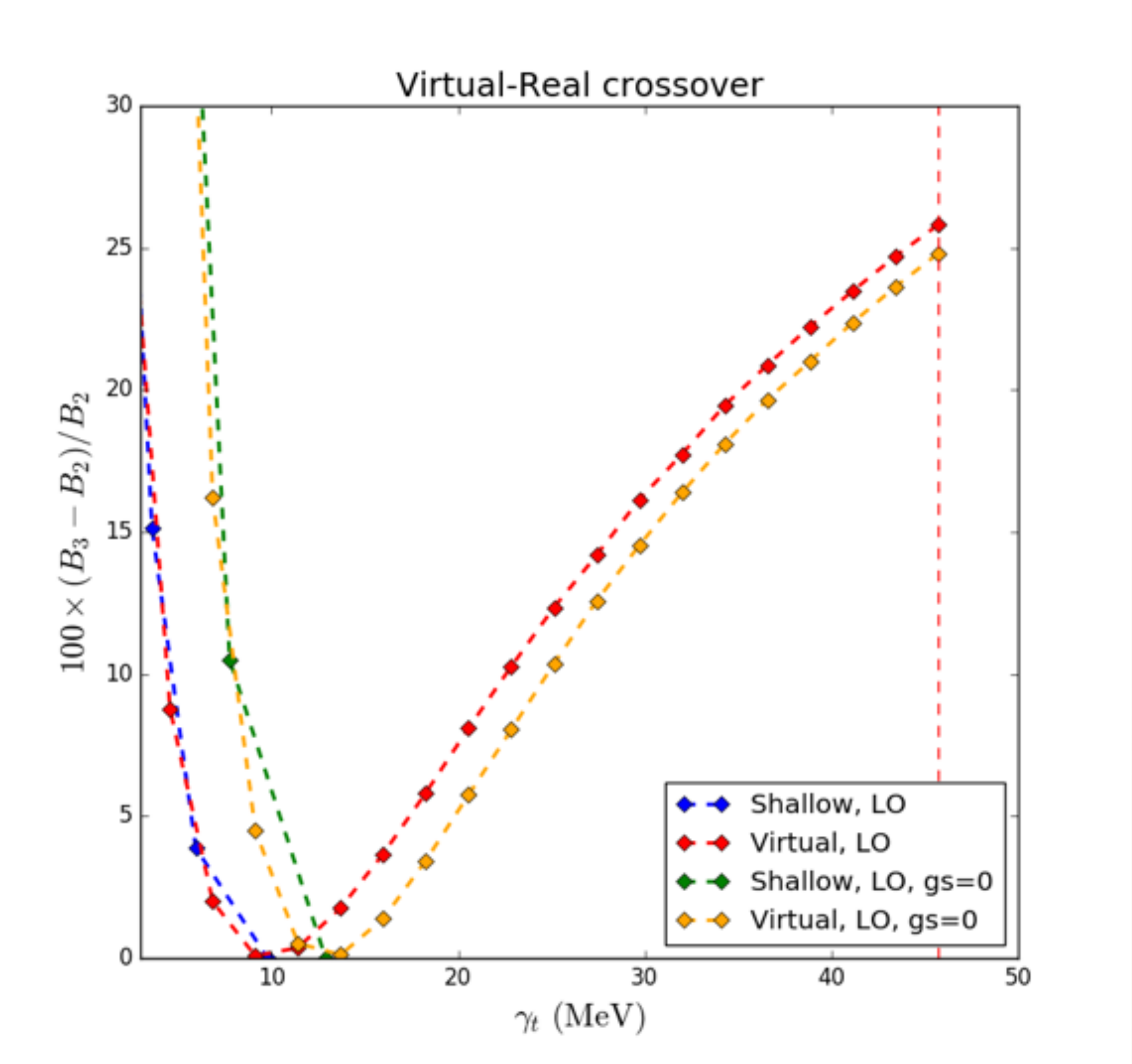




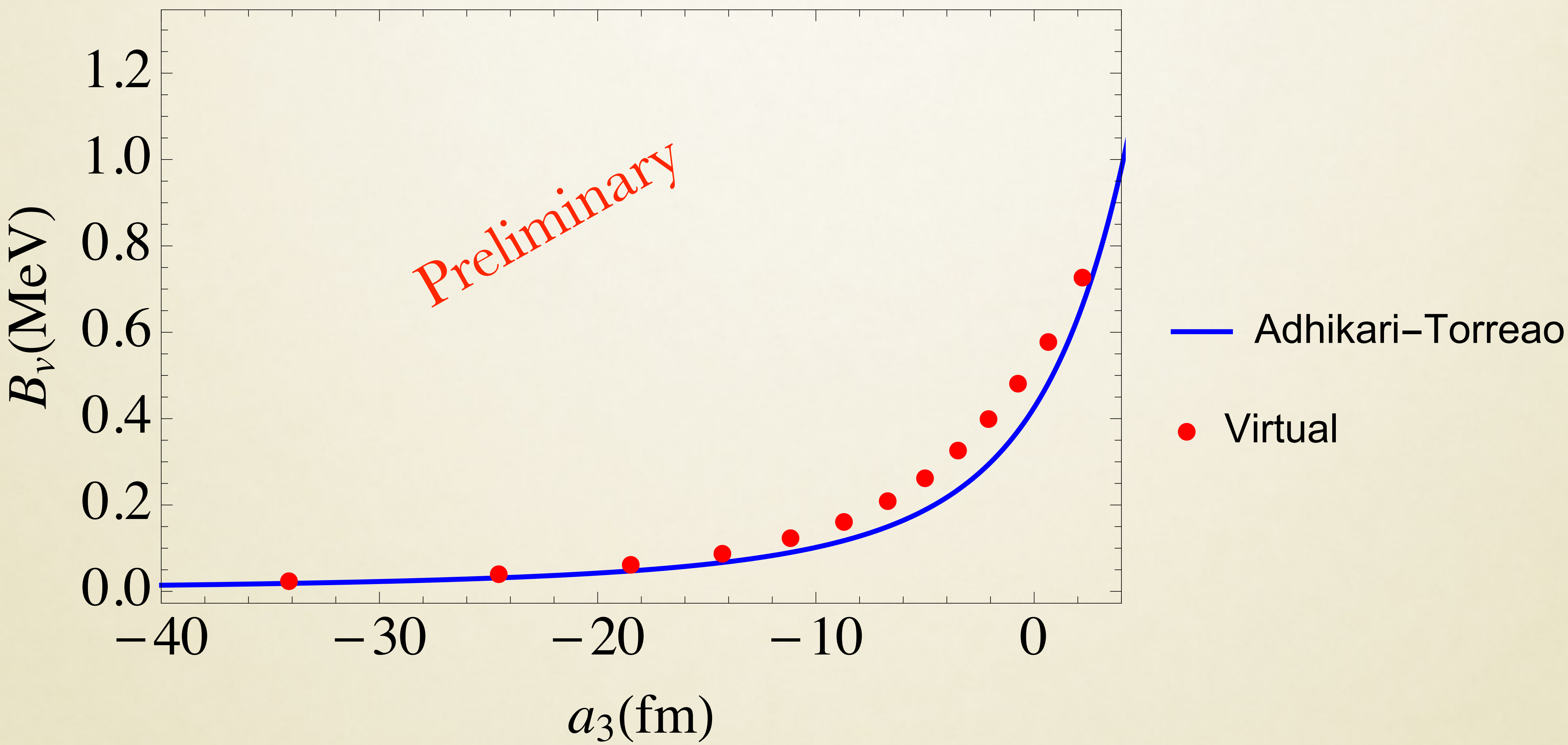
Shallow virtual to bound state

lattice QCD with B field, even with heavy pions?

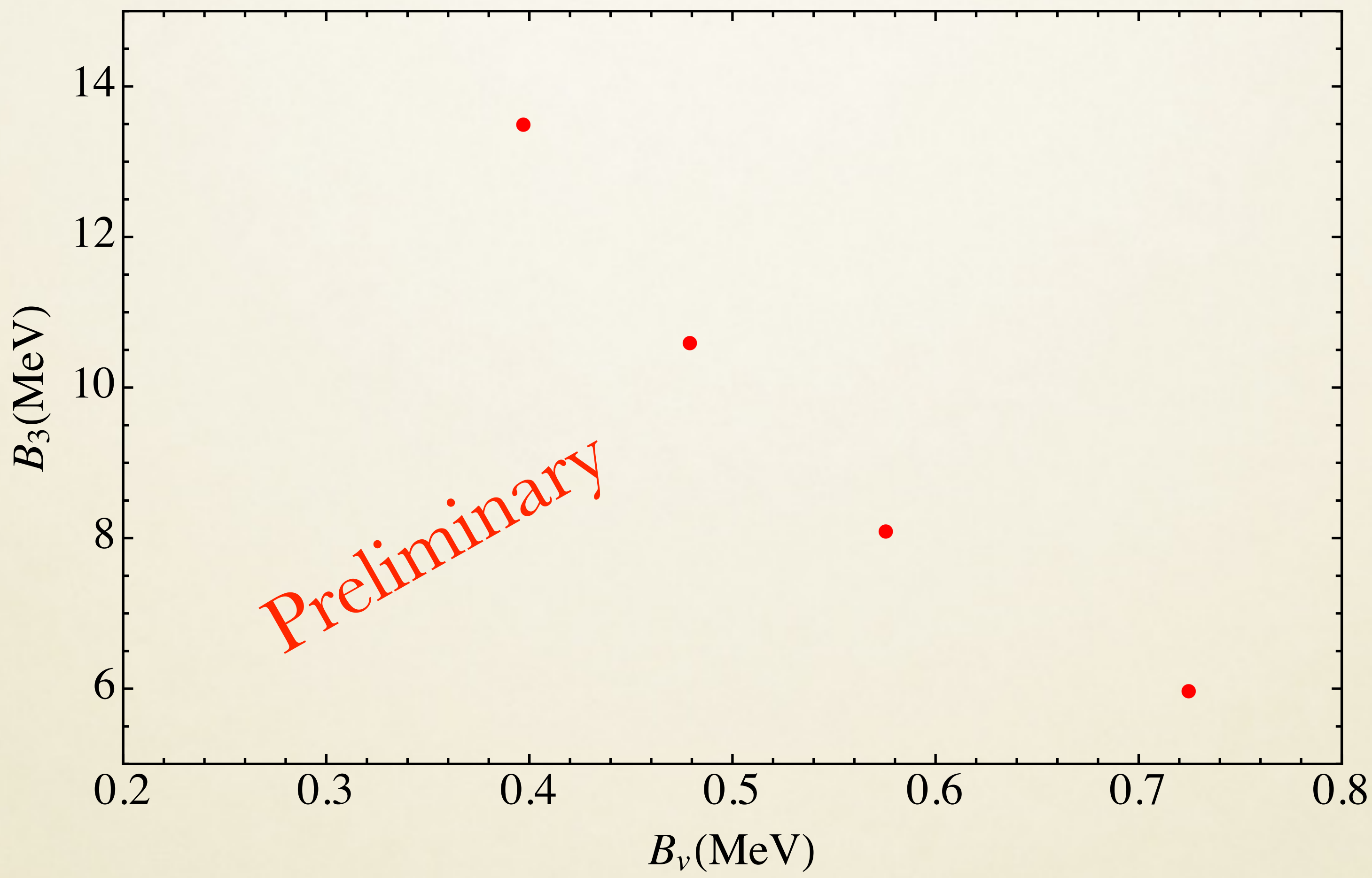
Virtual-Real Corssover



Adhikari-Torreao

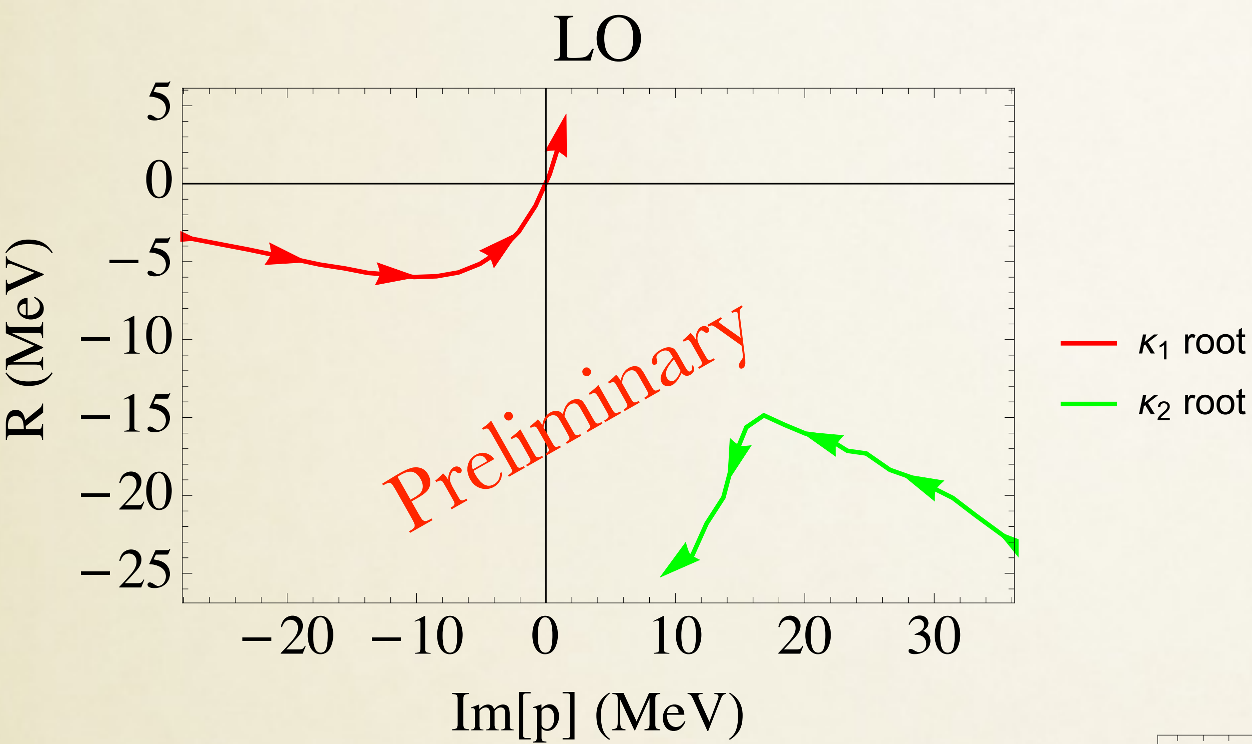


Phillips-Girard-Fuda



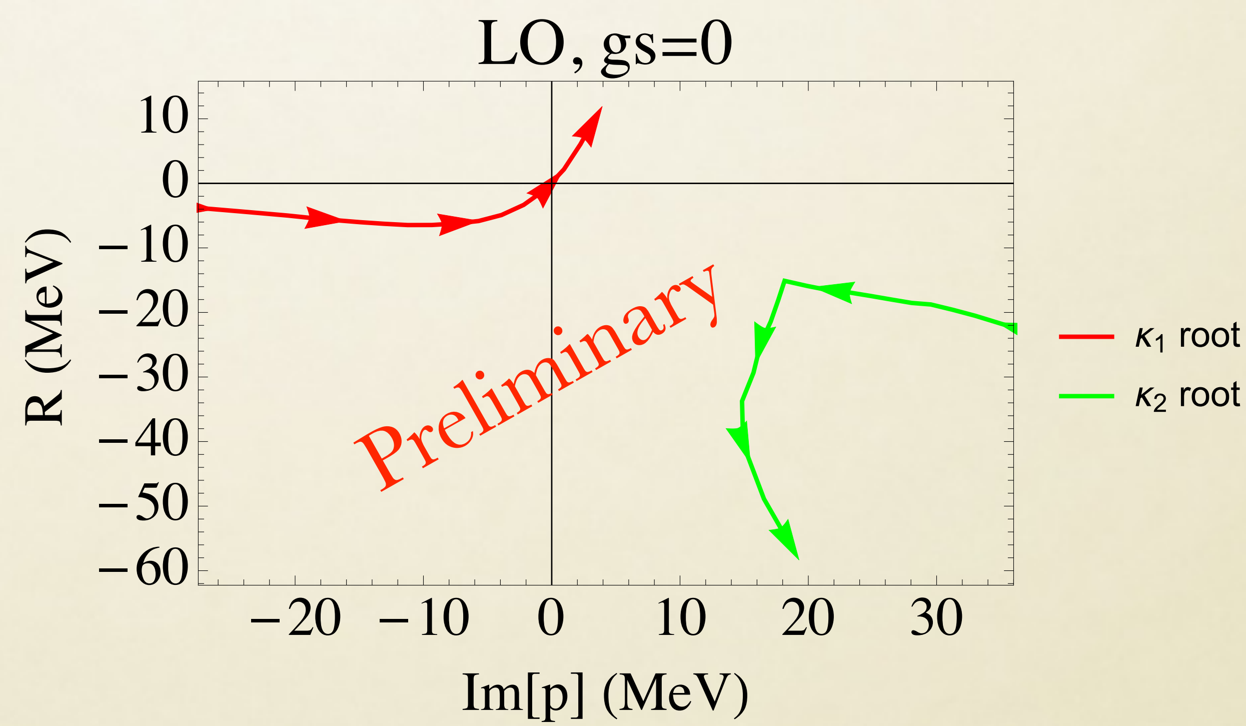
3-body correlation

Redundant Pole



Ma, Phys. Rev. (1946) and (1947)
— Exponentially decaying pot.
Negative norm states

Nelson, Rajagopal, Shastry (1971)



Conclusions and Outlook

- Halo/cluster EFT
- Adiabatic Projection Method to derive effective two-body Hamiltonian in lattice EFT
- Reaction calculations with or without Coulomb
- Pinhole algorithm
- Efimov physics

Thank you