Inclusive deuteron–induced reactions

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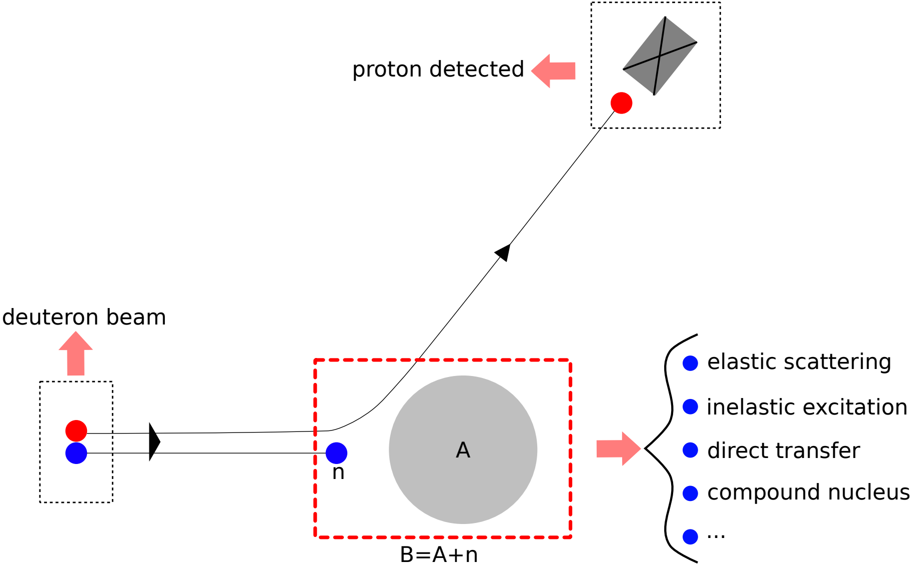
Seattle, March 28th 2017

(d, p) reactions: a probe for neutron–nucleus interactions



General framework for inclusive experiments

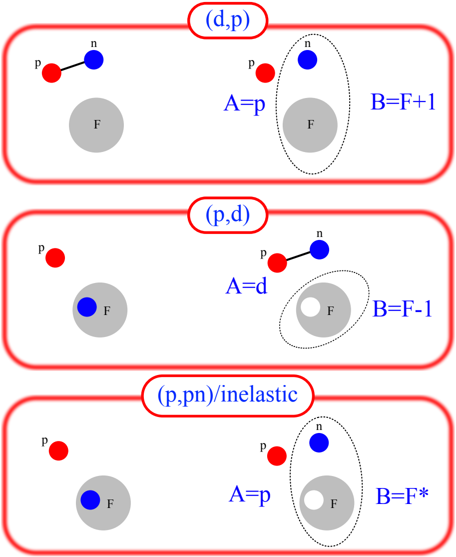
this is detected ➡ A   B ⬅ summed over states of this

$$
\begin{aligned}
\frac{d\sigma(E)}{dE_A} &\sim \sum_n |\langle \phi_A \phi_B^n | V | \Psi \rangle|^2 \\
&= \sum_n \langle \Psi | V^\dagger | \phi_A \phi_B^n \rangle \delta(E - E_A - E_B^n) \, \langle \phi_B^n \phi_A | V | \Psi \rangle \\
&= \lim_{\epsilon \to 0} \mathrm{Im} \langle \Psi | V^\dagger | \phi_A \rangle \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B + i\epsilon} \langle \phi_A | V | \Psi \rangle
\end{aligned}
$$

Main challenges

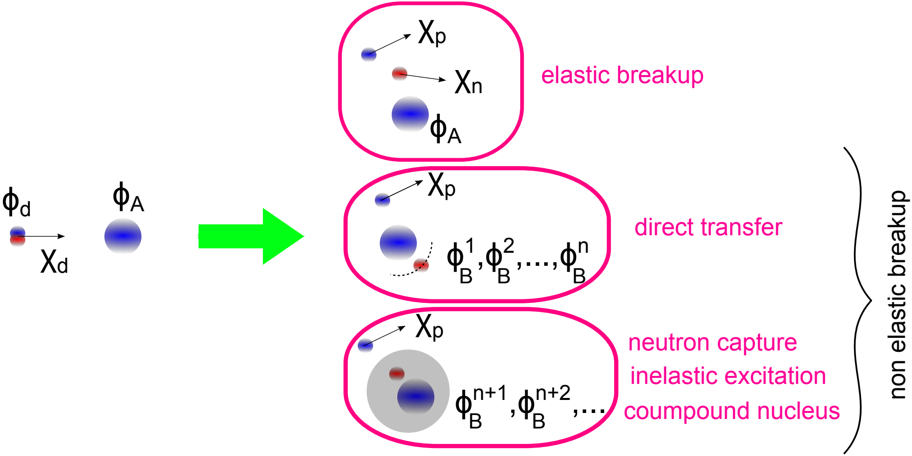
- describe propagator $G_B = \lim_{\epsilon\to 0} \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B + i\epsilon}$
- describe wave function Ψ

Some applications



Implemented so far: Inclusive (d, p) reaction

let's concentrate in the reaction A+d→ B(=A+n)+p



we are interested in the inclusive cross section, *i.e.*, we will sum over all final states ϕ_B^c.

Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final ϕ_B^c

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\left|\left\langle \chi_p\phi_B^c|V\,|\,\Psi^{(+)}\right\rangle\right|^2 .$$

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d\phi_d\phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d}\rho(E_p)$$
$$\times \sum_c \langle\, \chi_d\phi_d\phi_A|\; V \;|\chi_p\phi_B^c\rangle\, \delta(E - E_p - E_B^c)\, \langle\phi_B^c\chi_p|\; V \;|\,\phi_A\chi_d\phi_d\rangle$$

χ_d → deuteron incoming wave, ϕ_d → deuteron wavefunction,
χ_p → proton outgoing wave ϕ_A → target core ground state.

Sum over final states

the imaginary part of the Green's function G is an operator representation of the δ–function,

$$\pi\delta(E - E_p - E_B^c) = \lim_{\epsilon\to 0} \Im \sum_c \frac{|\phi_B^c\rangle\langle\phi_B^c|}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d}\rho(E_p)\Im\langle\chi_d\phi_d\phi_A|\, V\, |\chi_p\rangle\, G\, \langle\chi_p|\, V\, |\phi_A\chi_d\phi_d\rangle$$

- We got rid of the (infinite) sum over final states,
- but G is an extremely complex object!
- We still need to deal with that.

Optical reduction of G

If the interaction V do not act on ϕ_A

$$\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle = \langle \chi_d \phi_d | V | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | V | \chi_d \phi_d \rangle = \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle ,$$

where G_{opt} is the optical reduction of G

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus G_{opt} are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations

Capture and elastic breakup cross sections

the imaginary part of G_{opt} splits in two terms

$$\Im G_{opt} = \overbrace{-\pi \sum_{k_n} |\chi_n\rangle \delta\left(E - E_p - \frac{k_n^2}{2m_n}\right) \langle\chi_n|}^{\text{elastic breakup}} + \overbrace{{G_{opt}}^{\dagger} W_{An}\, G_{opt}}^{\text{non elastic breakup}},$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle\chi_p| V |\chi_d\phi_d\rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{NEB} = -\frac{2}{\hbar v_d}\rho(E_p) \langle\psi_n| W_{An} |\psi_n\rangle \,,$$

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{EB} = -\frac{2}{\hbar v_d}\rho(E_p)\rho(E_n) \left|\langle\chi_n\chi_p| V |\chi_d\phi_d\rangle\right|^2 ,$$

neutron transfer limit (isolated–resonance, first–order approximation)

Let's consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy E such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An}\to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An}\to 0} \langle\, \chi_d\phi_d|\, V\, |\chi_p\rangle$$

$$\times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle\chi_p|\, V\, |\, \chi_d\phi_d\rangle\,,$$

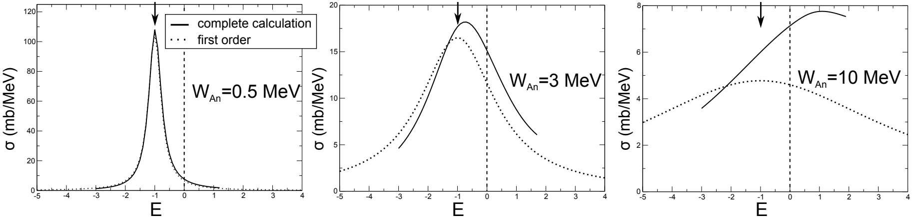
we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\,\langle\chi_p\phi_n|\, V\, |\, \chi_d\phi_d\rangle\,|^2 \delta(E - E_p - E_n)$$

Validity of first order approximation

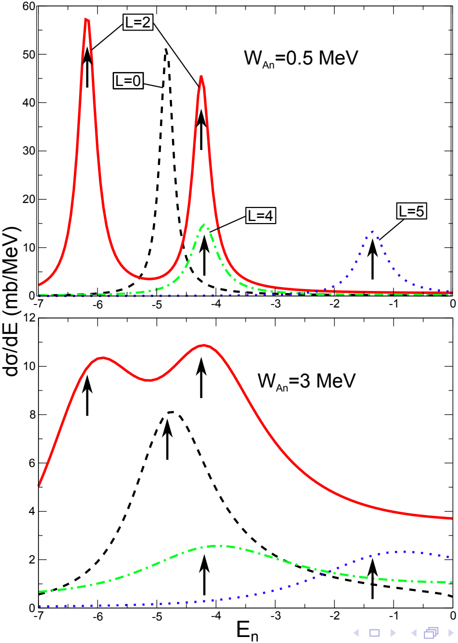
For W_{An} small, we can apply first order perturbation theory,

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}(E,\Omega)\right]^{NEB} \approx \frac{1}{\pi}\frac{\langle\phi_n|W_{An}|\phi_n\rangle}{(E_n-E)^2+\langle\phi_n|W_{An}|\phi_n\rangle^2}\left.\frac{d\sigma_n}{d\Omega}(\Omega)\right]^{transfer}$$



we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV

Spectral function and absorption cross section



Choosing the potential/propagator

Konig–Delaroche (KD)

- Phenomenological fit of elastic scattering. Local.
- Available for a wide range of stable nuclei and energies.
- Not defined for negative energies. Good reproduction of scattering observables.

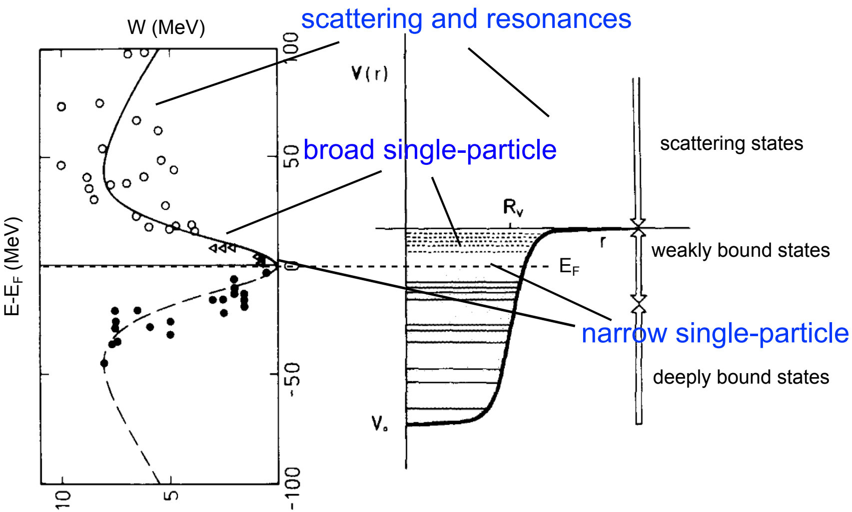
Dispersive optical model (DOM)

- Elastic scattering fit → dispersion → negative energies. Local and non–local versions.
- Limited availability. Good reproduction of scattering and low–energy observables. Satisfy sum rules (dispersive).

Coupled cluster (CC)

- Ab–initio calculation → predictive power. Non–local.
- Accuracy of reproduction of observables ?.

Neutron states in nuclei

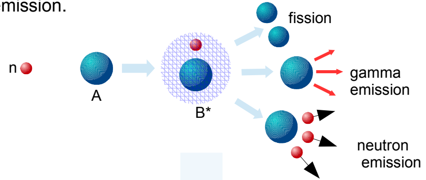


Imaginary part of optical potential neutron states

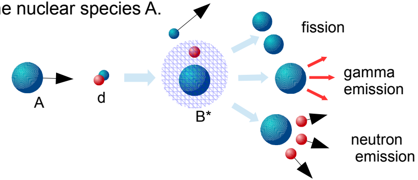
Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1

Surrogate for neutron capture

* Desired reaction: neutron induced fission, gamma emission and neutron emission.



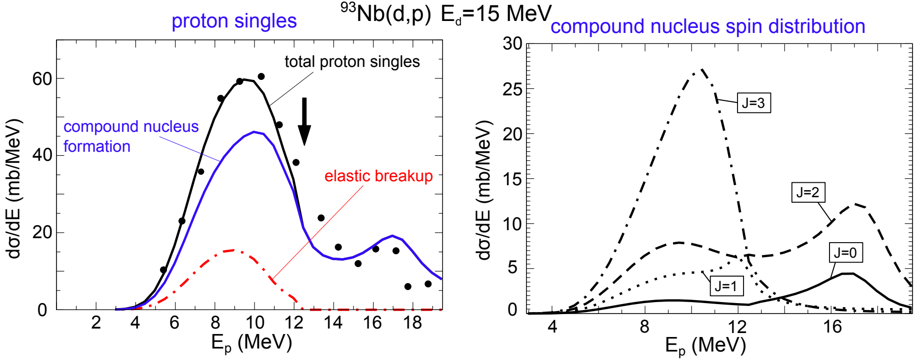
* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.



* **A theoretical reaction formalism that describes the production of all open channels B* is needed.**

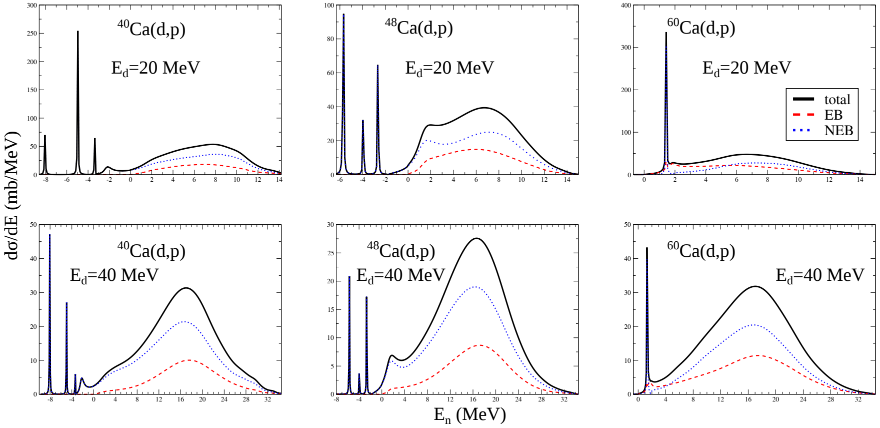
Disentangling elastic and non elastic breakup

^{93}Nb(d, p) (Mastroleo *et al.*, Phys. Rev. C **42** (1990) 683)

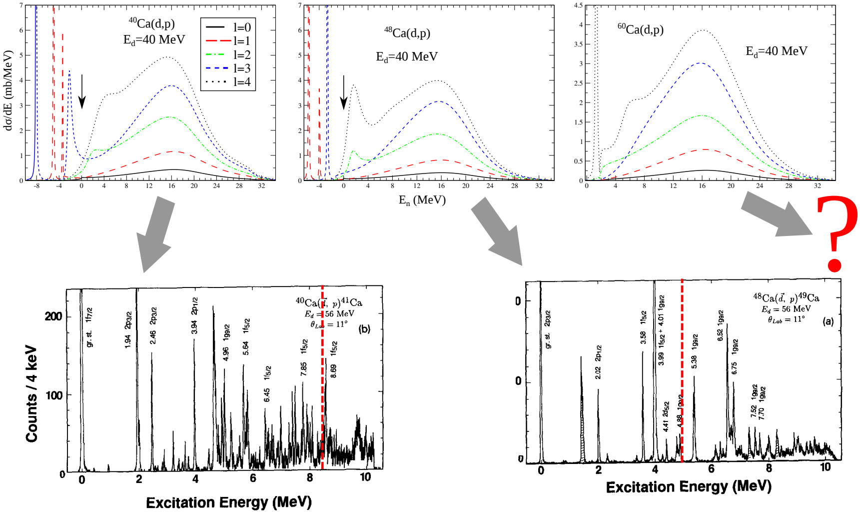


- We obtain spin–parity distributions for the compound nucleus.
- Contributions from elastic and non elastic breakup disentangled.

Using the DOM: Calcium isotopes

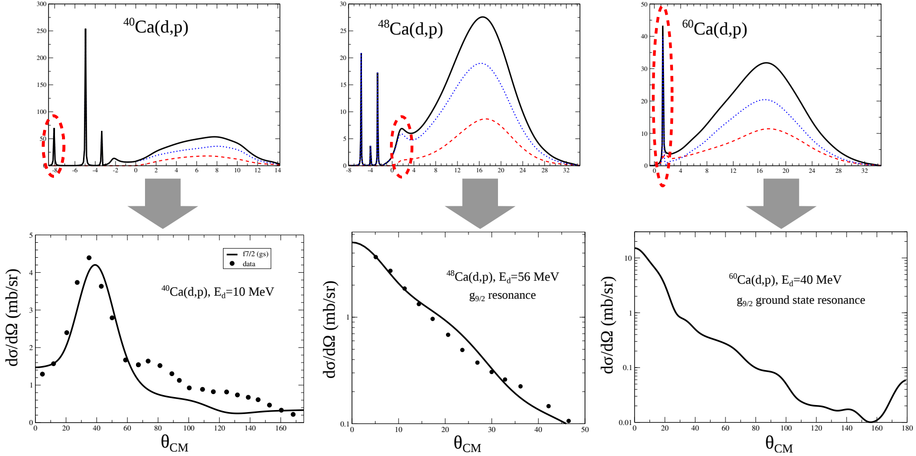


Using the DOM: angular momenta



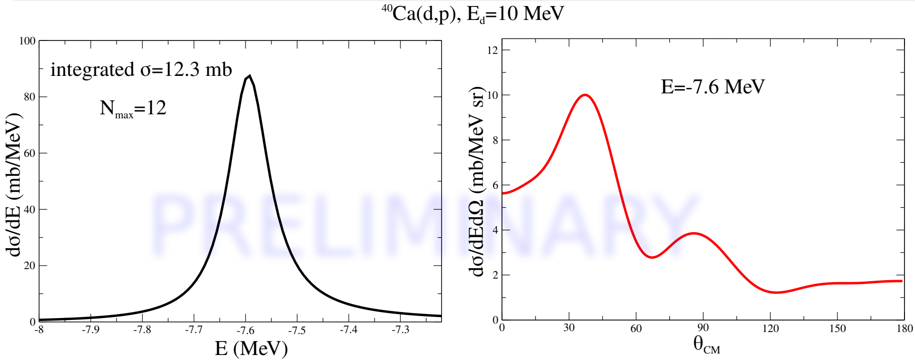
Uozimi *et al.*, NPA **576** (1994) 123

Using the DOM: comparing with data



CC: preliminary results (with J. Rotureau, MSU and G. Hagen, ORNL)

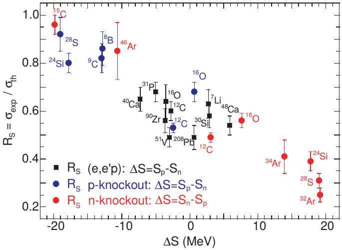
first results using a propagator and self–energy generated within the coupled–cluster formalism (J. Rotureau, MSU and G. Hagen, ORNL)



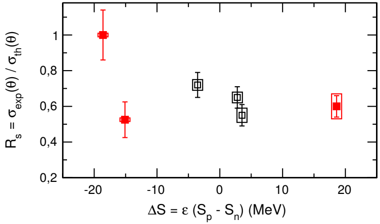
Open questions (personal selection)

- Separate different channels in the NEB (possibly with M. Dupuis and G. Blanchon ?).
- Better description of deuteron channel (adiabatic approximation? CDCC? Coupled cluster?)
- Implement (p, pn). Could shed light on the problem of spectroscopic factors as a function of asymmetry. Two–particle Green's function? Final neutron as a spectator?

Gade *et al.* PRC **77**, 044306 (2008)



Flavigny *et al.* PRL **110**, 122503 (2013)



Inclusive cross section

Single–particle Green's function of B system:

$$G_B = \lim_{\eta \to 0} (E - T - U_B + i\eta)^{-1}$$

Strength at given $E, \mathbf{r}_n, \mathbf{r}'_n, J, \pi \to \mathrm{Im}\, G_B(E, \mathbf{r}_n, \mathbf{r}'_n, J, \pi)$
inclusive cross section $\to$ folding of strength with "breakup probability density" $|\rho_{bu}(\mathbf{r}_n, \mathbf{k}_i, \mathbf{k}_f)|^2$:

$$\rho_{bu}(\mathbf{r}_n, \mathbf{k}_i, \mathbf{k}_f) = (\chi_p(\mathbf{r}_p, \mathbf{k}_f)|V|\chi_d(\mathbf{r}_d, \mathbf{k}_i)\phi_d(\mathbf{r}_{pn})\rangle$$

$$\frac{d\sigma}{dEd\Omega} \sim \int \rho^*_{bu}(\mathbf{r}_n, \mathbf{k}_i, \mathbf{k}_f)\mathrm{Im}\, G_B(E, \mathbf{r}_n, \mathbf{r}'_n, J, \Pi)\rho_{bu}(\mathbf{r}'_n, \mathbf{k}_i, \mathbf{k}_f) d\mathbf{r}_n d\mathbf{r}'_n$$

$$\mathrm{Im}\, G_B = \text{elastic breakup spectrum} + G_B^\dagger \mathrm{Im}\, U_B G_B$$

How to write $U_B = U_{compound} + U_{rest}$?

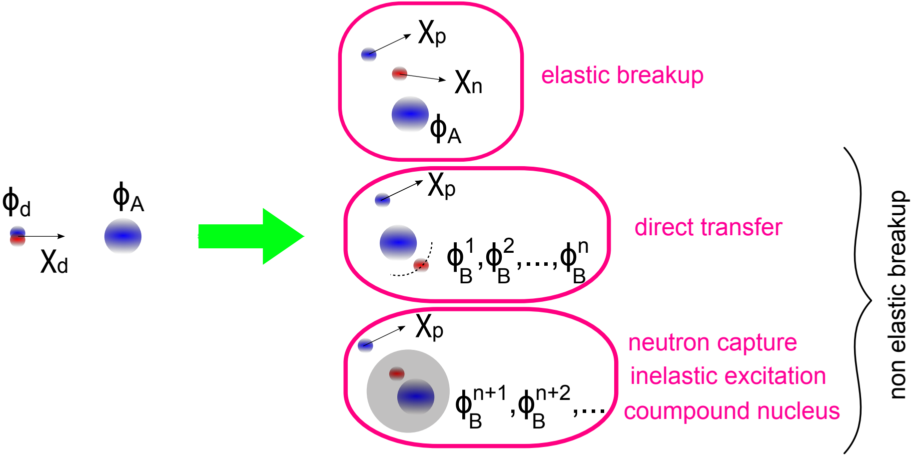
Inclusive deuteron–induced reactions

Grégory Potel Aguilar (NSCL/FRIB)
Filomena Nunes (NSCL)
Ian Thompson (LLNL)

East Lansing, July 19th 2016

Inclusive (d, p) reaction

let's concentrate in the reaction A+d→ B(=A+n)+p

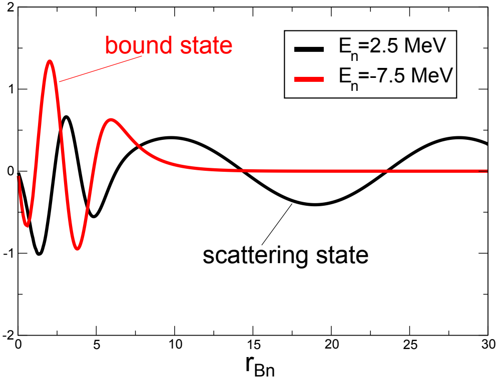


we are interested in the inclusive cross section, *i.e.*, we will sum over all final states ϕ_B^c.

the neutron wavefunctions

$$|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$$

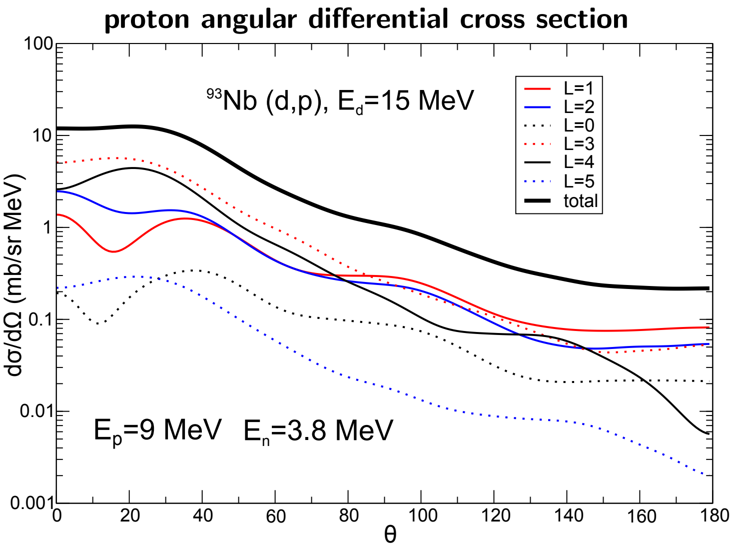
can be computed for **ANY** neutron energy, positive or negative



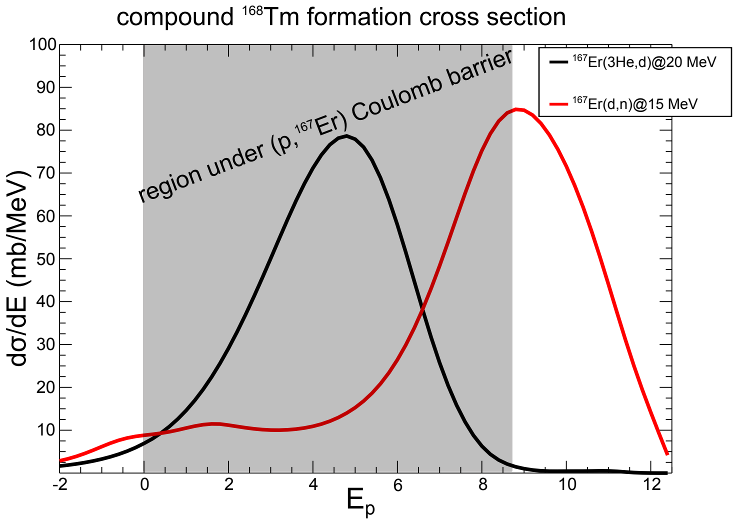
transfer to resonant and non-resonant continuum well described

$|\psi_n\rangle$ are the solutions of an inhomogeneous Schrödinger equation
$(H_{An} - E_{An}) |\psi_n\rangle = \langle \chi_p | V | \chi_d \phi_d \rangle$

Breakup above neutron–emission threshold



Dropping a proton



We can also transfer charged clusters

Austern (post)–Udagawa (prior) formalisms

The interaction V can be taken either in the *prior* or the *post* representation,

- Austern (post)$\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, from Sevilla and Carlson from São Paulo)
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$ (used in calculations showed here)

in the prior representation, V can act on ϕ_A $\rightarrow$ the optical reduction gives rise to new terms:

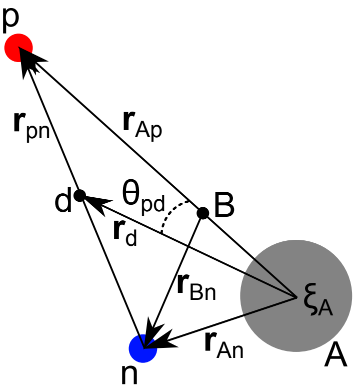
$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{post} = -\frac{2}{\hbar v_d}\rho(E_p)\left[\Im\left\langle \psi_n^{prior}\right| W_{An} \left|\psi_n^{prior}\right\rangle + 2\Re\left\langle \psi_n^{NON}\right| W_{An} \left|\psi_n^{prior}\right\rangle + \left\langle \psi_n^{NON}\right| W_{An} \left|\psi_n^{NON}\right\rangle\right],$$

where $\psi_n^{NON} = \langle \chi_p | \; \chi_d \phi_d \rangle$.

The nature of the 2–step process depends on the representation

Summary, conclusions and some prospectives

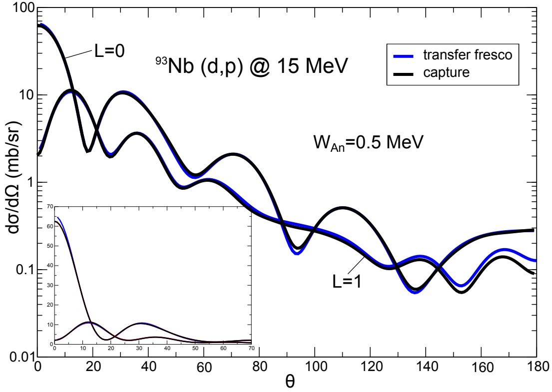
- We have presented a reaction formalism for inclusive deuteron–induced reactions.
- Valid for final neutron states from Fermi energy $\rightarrow$ to scattering states
- Disentangles elastic and non elastic breakup contributions to the proton singles.
- Probe of nuclear structure in the continuum.
- Provides spin–parity distributions.
- Useful for surrogate reactions.
- Need for optical potentials.
- Need to address non–locality.
- Can be generalized to other three–body problems.
- Can be extended for (p, d) reactions (hole states).



From H to H_{3B}

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$

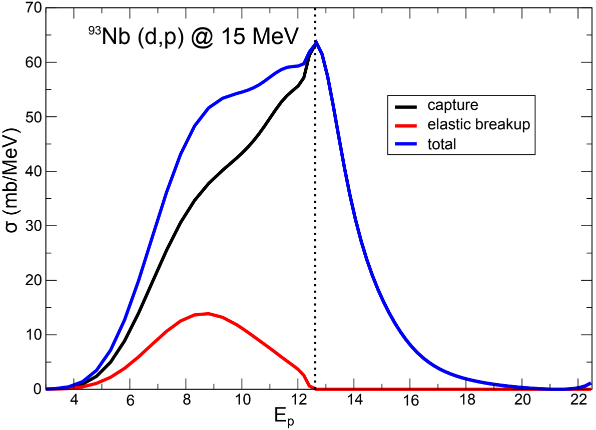
Observables: angular differential cross sections (neutron bound states)



- capture at resonant energies compared with
- direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

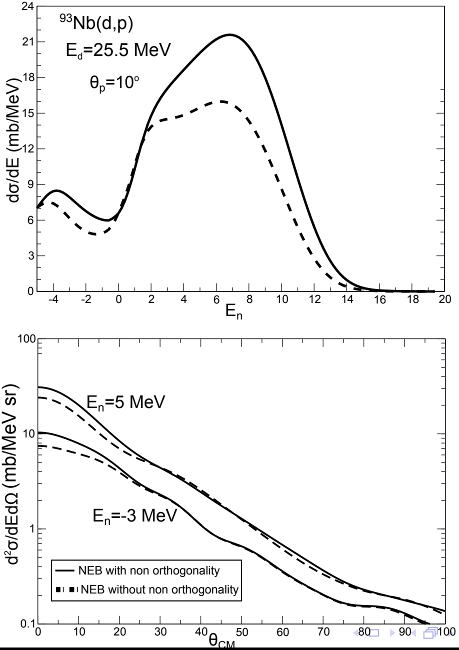
double proton differential cross section

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y^{l_p}_{-m}(\theta_p) \right|^2 W(r_{An})\ dr_{Bn}.$$

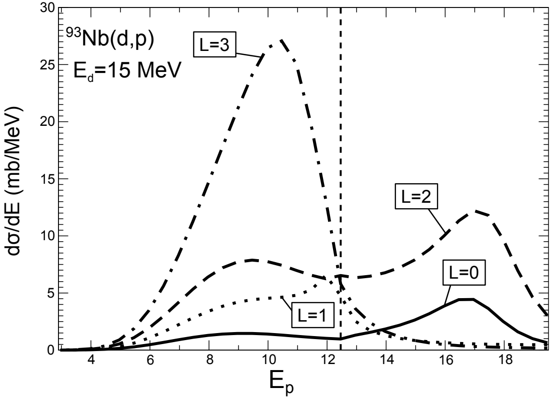


elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the U_{An} interaction (Koning and Delaroche, Nucl. Phys. A **713** (2003) 231).

Non–orthogonality term



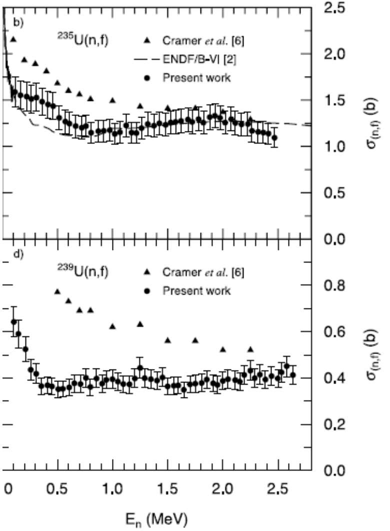
Obtaining spin distributions



spin distribution of compound nucleus

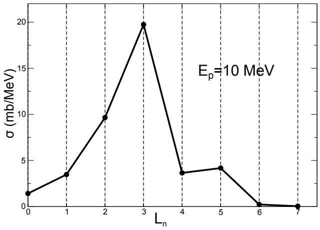
$$\frac{d\sigma_I}{dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\sum_{l_p,m}\int \left|\varphi_{Iml_p}(r_{Bn};k_p)\right|^2 W(r_{An})\, dr_{Bn}.$$

Getting rid of Weisskopf–Ewing approximation



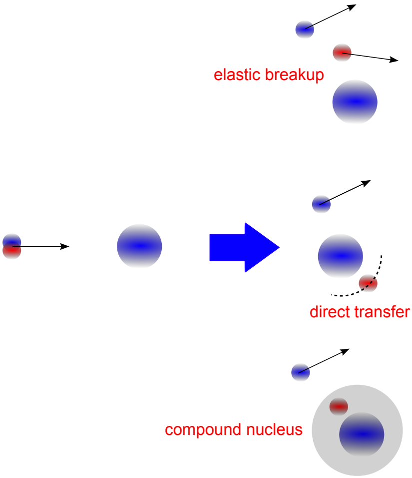
Younes and Britt, PRC **68**(2003)034610

- Weisskopf–Ewing approximation: $P(d,nx) = \sigma(E)G(E,x)$
- inaccurate for $x = \gamma$ and for $x = f$ in the low–energy regime
- can be replaced by $P(d,nx) = \sum_{J,\pi} \sigma(E,J,\pi)G(E,J,\pi,x)$ if $\sigma(E,J,\pi)$ can be predicted.



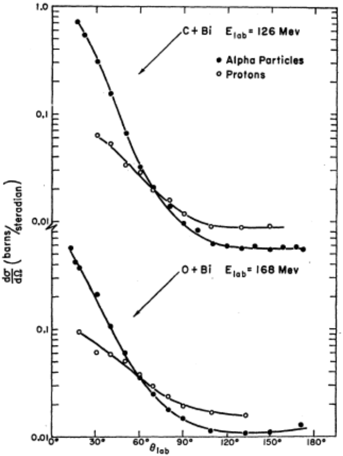
Introduction

We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:



- Direct neutron transfer: should be compatible with existing theories.
- Elastic deuteron breakup: **"transfer" to continuum states**.
- Non elastic breakup (direct transfer, inelastic excitation and compound nucleus formation): **absorption above and below neutron emission threshold**.
- Important application in surrogate reactions: **obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation**.

breakup-fusion reactions

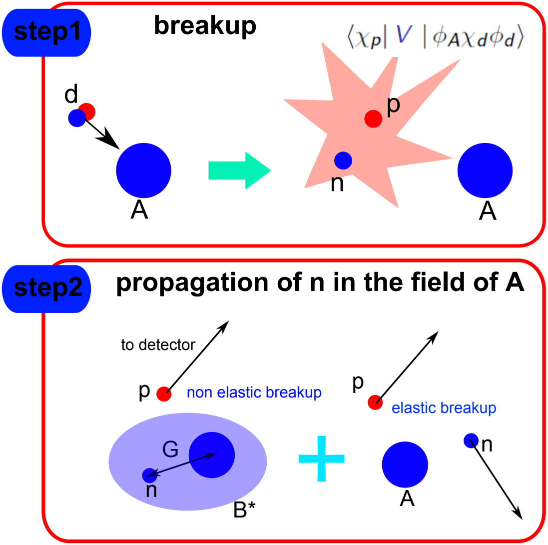


Britt and Quinton, Phys. Rev. **124** (1961) 877

protons and α yields bombarding ^{209}Bi with ^{12}C and ^{16}O

- Kerman and McVoy, Ann. Phys. **122** (1979)197
- Austern and Vincent, Phys. Rev. C**23** (1981) 1847
- Udagawa and Tamura, Phys. Rev. C**24**(1981) 1348
- Last paper: Mastroleo, Udagawa, Mustafa Phys. Rev. C**42** (1990) 683
- Controversy between Udagawa and Austern formalism left somehow unresolved.

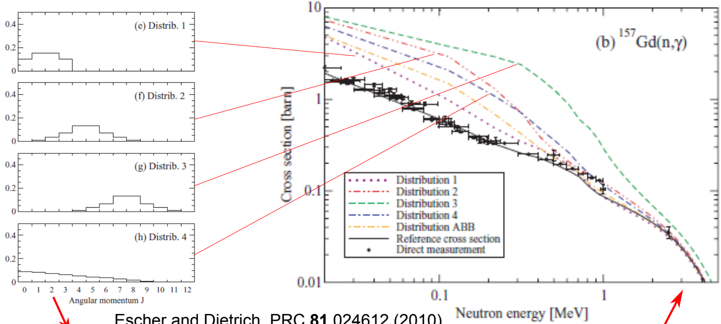
2–step process (post representation)



Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E_{ex}, J, \pi) G_\chi^{CN}(E_{ex}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{WE}(E_a) = \sigma_\alpha^{CN}(E_{ex}) \mathcal{G}_\chi^{CN}(E_{ex})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



Escher and Dietrich, PRC **81** 024612 (2010)

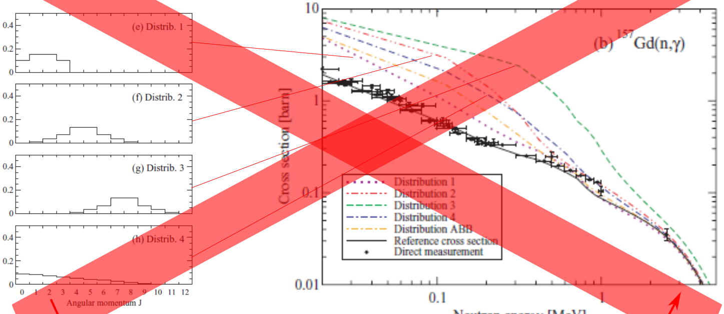
Different J,π → Different cross section for γ emission

Weisskopf–Ewing is inaccurate for (n, γ)

Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E_{ex}, J, \pi) G_\chi^{CN}(E_{ex}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{WE}(E_a) = \sigma_\alpha^{CN}(E_{ex}) \mathcal{G}_\chi^{CN}(E_{ex})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

We need theory to predict J, π distributions

Inclusive three–body cross sections

January 31, 2017

Introduction

General description of experiments in which we measure the energy of final product A and do not measure the energy of final product B:

$$\frac{d\sigma}{dE_A}(E) \sim \sum_n |\langle \phi_A \phi_B^n | V | \Psi \rangle|^2$$

$$= \sum_n \langle \Psi | V^\dagger | \phi_A \phi_B^n \rangle \delta(E - E_A - E_B^n) \langle \phi_B^n \phi_A | V | \Psi \rangle$$

$$= \Im \lim_{\epsilon \to 0} \langle \Psi | V^\dagger | \phi_A \rangle \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B} \langle \phi_A | V | \Psi \rangle$$

Main challenge: get the B–system propagator $G_B = \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B}$

ICNT Workshop "Deuteron–induced reactions and beyond: Inclusive breakup fragment cross sections" (July 2016)

structure

W. Dickhoff (St. Louis)
J. Rotureau (MSU)
J. Escher (LLNL)

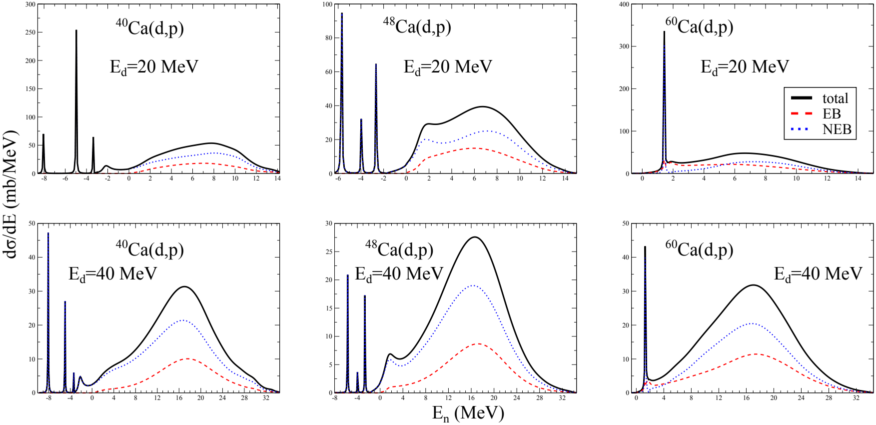
experiment

G. Perdikakis (CMU, NSCL)
A. Macchiavelli (LBNL)
S. Pain (ORNL)

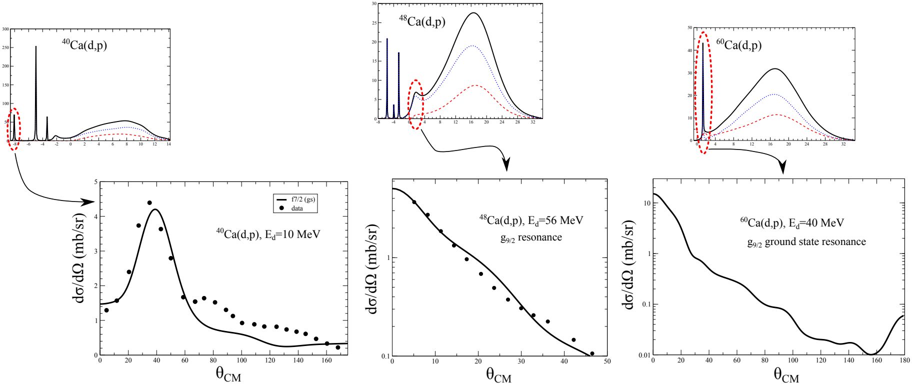
reactions

B. Carlson (Sao Paulo)
A. Moro (Sevilla)
F. Nunes (MSU)
M. Husein (Sao Paulo)
P. Capel (Bruxelles)
G. Potel (MSU)

Recent results (I): Ca isotopes



Recent results (II): Ca isotopes



- Implement (p, d) (with DOM optical potential from Wim Dickhoff and Mack Atkinson).
- Include non–locality in the propagator (with Weichuan).
- Use Coupled–Clusters propagator (with Jimmy).

Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final ϕ_B^c

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\left|\left\langle \chi_p \phi_B^c | V \,|\, \Psi^{(+)} \right\rangle\right|^2 .$$

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d}\rho(E_p)$$
$$\times \sum_c \langle \chi_d \phi_d \phi_A | \, V \, | \chi_p \phi_B^c \rangle \, \delta(E - E_p - E_B^c) \, \langle \phi_B^c \chi_p | \, V \, | \phi_A \chi_d \phi_d \rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,
$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.

Sum over final states

the imaginary part of the Green's function G is an operator representation of the δ–function,

$$\pi\delta(E - E_p - E_B^c) = \lim_{\epsilon \to 0} \Im \sum_c \frac{|\phi_B^c\rangle \langle\phi_B^c|}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d}\rho(E_p)\Im \langle \chi_d\phi_d\phi_A| V |\chi_p\rangle G \langle\chi_p| V |\phi_A\chi_d\phi_d\rangle$$

- We got rid of the (infinite) sum over final states,
- but G is an extremely complex object!
- We still need to deal with that.

Optical reduction of G

If the interaction V do not act on ϕ_A

$$\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle = \langle \chi_d \phi_d | V | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | V | \chi_d \phi_d \rangle = \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle ,$$

where G_{opt} is the optical reduction of G

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus G_{opt} are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations

Capture and elastic breakup cross sections

the imaginary part of G_{opt} splits in two terms

$$\Im G_{opt} = \overbrace{-\pi \sum_{k_n} |\chi_n\rangle \delta\left(E - E_p - \frac{k_n^2}{2m_n}\right) \langle \chi_n|}^{\text{elastic breakup}} + \overbrace{G_{opt}{}^{\dagger} W_{An}\, G_{opt}}^{\text{non elastic breakup}},$$

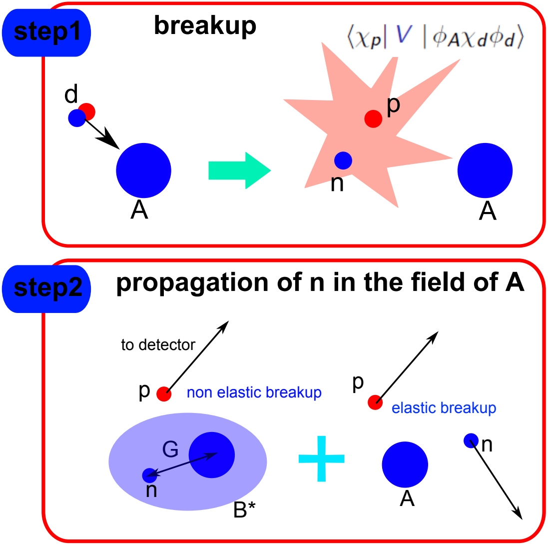
we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{NEB} = -\frac{2}{\hbar v_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n \rangle ,$$

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{EB} = -\frac{2}{\hbar v_d} \rho(E_p) \rho(E_n) \left|\langle \chi_n \chi_p | V | \chi_d \phi_d \rangle\right|^2 ,$$

2–step process (post representation)



Austern (post)–Udagawa (prior) controversy

The interaction V can be taken either in the *prior* or the *post* representation,

- Austern (post)$\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, University of Sevilla)
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$

in the prior representation, V can act on ϕ_A $\rightarrow$ the optical reduction gives rise to new terms:

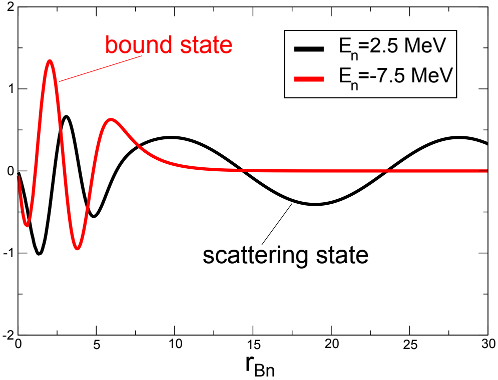
$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{post} = -\frac{2}{\hbar v_d}\rho(E_p)\left[\Im\left\langle \psi_n^{prior}\right|W_{An}\left|\psi_n^{prior}\right\rangle + 2\Re\left\langle \psi_n^{NON}\middle|W_{An}\middle|\psi_n^{prior}\right\rangle + \left\langle \psi_n^{NON}\right|W_{An}\left|\psi_n^{NON}\right\rangle\right],$$

where $\psi_n^{NON} = \langle \chi_p | \ \chi_d \phi_d \rangle$.

The nature of the 2–step process depends on the representation the neutron wavefunctions

$$|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$$

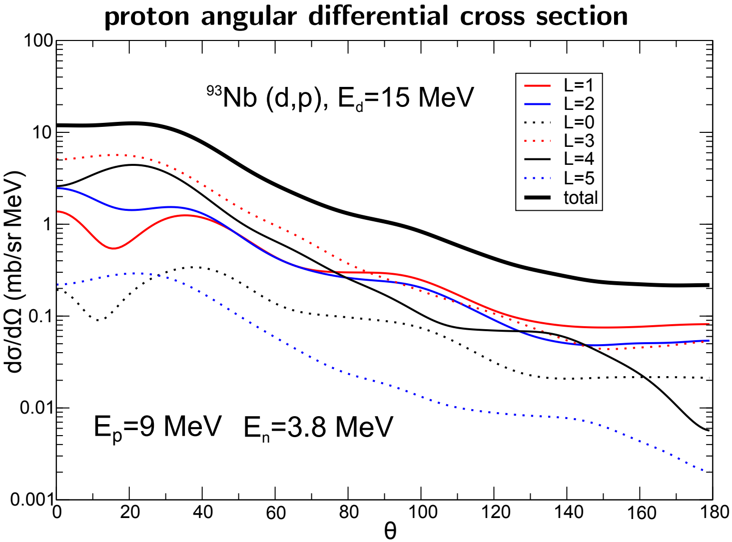
can be computed for any neutron energy



transfer to resonant and non-resonant continuum well described

these wavefunctions are not eigenfunctions of the Hamiltonian $H_{An} = T_n + \Re(U_{An})$

Breakup above neutron–emission threshold



neutron transfer limit (isolated–resonance, first–order approximation)

Let's consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy E such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An}\to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An}\to 0} \langle\, \chi_d\phi_d|\, V\, |\chi_p\rangle \times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle\chi_p|\, V\, |\,\chi_d\phi_d\rangle\,,$$

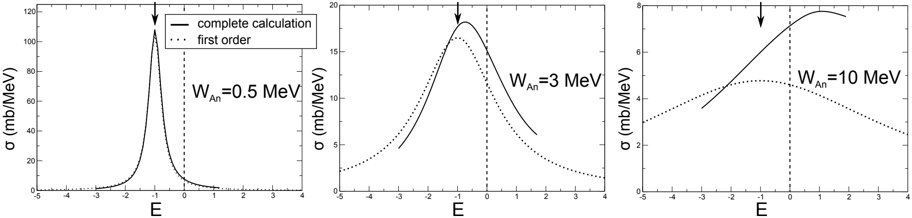
we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\,\langle\chi_p\phi_n|\, V\, |\,\chi_d\phi_d\rangle\,|^2\delta(E - E_p - E_n)$$

Validity of first order approximation

For W_{An} small, we can apply first order perturbation theory,

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}(E,\Omega)\right]^{NEB} \approx \frac{1}{\pi}\frac{\langle\phi_n|W_{An}|\phi_n\rangle}{(E_n-E)^2+\langle\phi_n|W_{An}|\phi_n\rangle^2}\left.\frac{d\sigma_n}{d\Omega}(\Omega)\right]^{transfer}$$

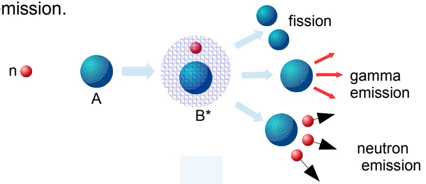


we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV

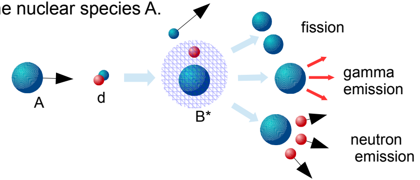
Application to surrogate reactions

Surrogate for neutron capture

* Desired reaction: neutron induced fission, gamma emission and neutron emission.



* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.

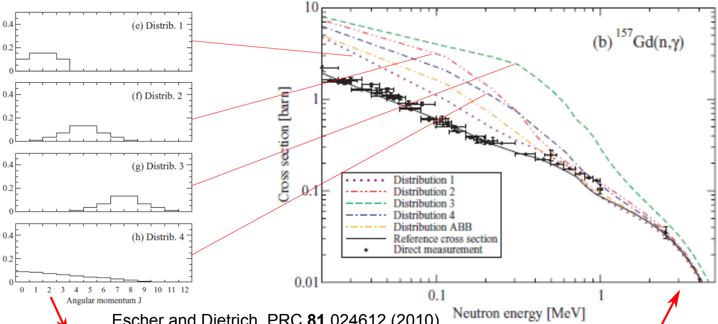


* **A theoretical reaction formalism that describes the production of all open channels B* is needed.**

Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E_{ex}, J, \pi) G_\chi^{CN}(E_{ex}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{WE}(E_a) = \sigma_\alpha^{CN}(E_{ex}) \mathcal{G}_\chi^{CN}(E_{ex})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



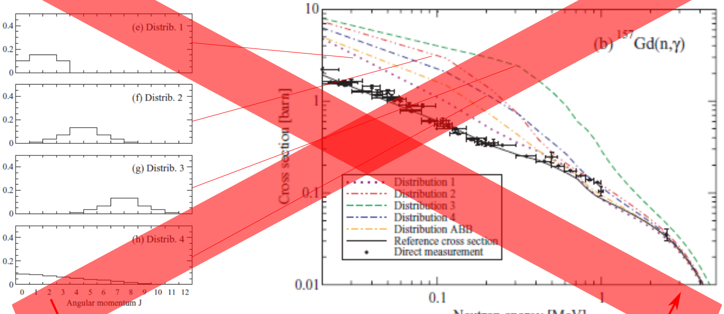
Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

Weisskopf–Ewing is inaccurate for (n, γ)

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E_{ex}, J, \pi) G_\chi^{CN}(E_{ex}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{WE}(E_a) = \sigma_\alpha^{CN}(E_{ex}) \mathcal{G}_\chi^{CN}(E_{ex})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



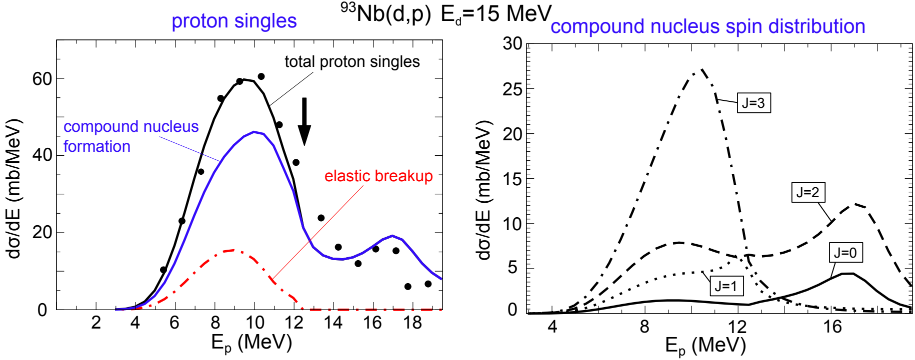
Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

We need theory to predict J, π distributions

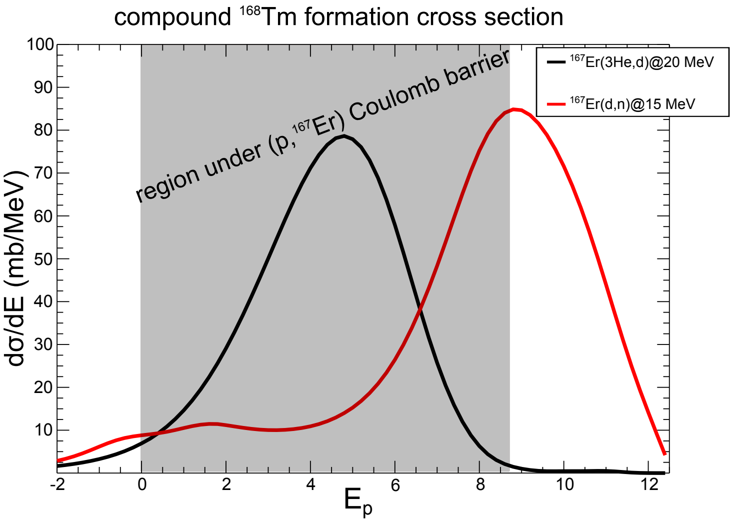
Disentangling elastic and non elastic breakup

^{93}Nb(d,p) (Mastroleo *et al.*, Phys. Rev. C **42** (1990) 683)



- We obtain spin–parity distributions for the compound nucleus.
- Contributions from elastic and non elastic breakup disentangled.

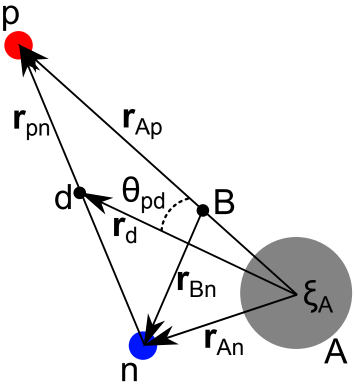
Extending the formalism



We can also transfer charged clusters

Summary, conclusions and some prospectives

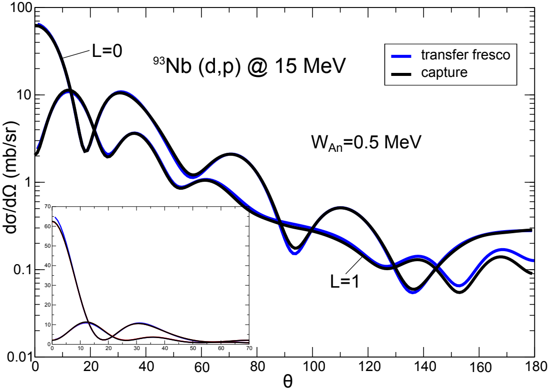
- We have presented a reaction formalism for inclusive deuteron–induced reactions.
- Valid for final neutron states from Fermi energy $\rightarrow$ to scattering states
- Disentangles elastic and non elastic breakup contributions to the proton singles.
- Probe of nuclear structure in the continuum.
- Provides spin–parity distributions.
- Useful for surrogate reactions.
- Need for optical potentials.
- Can easily be generalized to other three–body problems.
- Can be extended for (p, d) reactions (hole states).



From H to H_{3B}

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$

Observables: angular differential cross sections (neutron bound states)

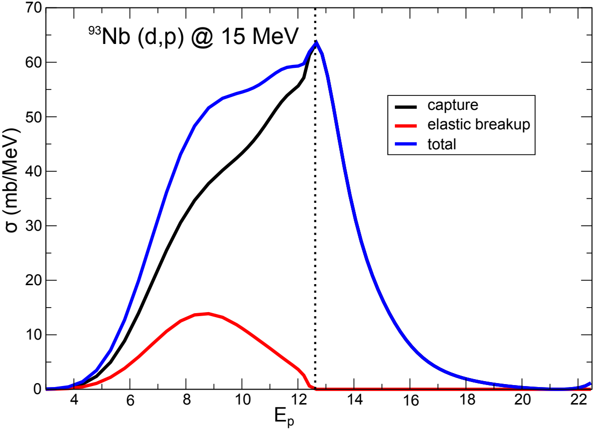


- capture at resonant energies compared with
- direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

double proton differential cross section

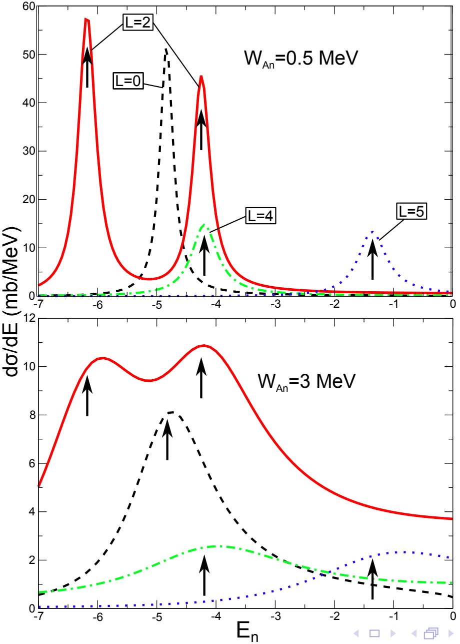
$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y^{l_p}_{-m}(\theta_p) \right|^2 W(r_{An})\, dr_{Bn}.$$

Observables: elastic breakup and capture cross sections

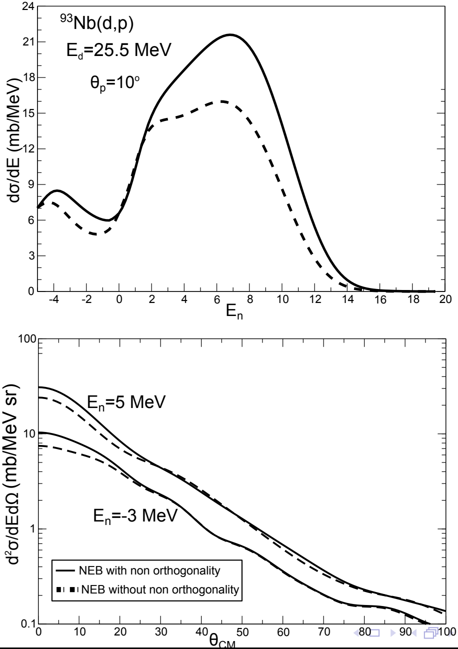


elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the U_{An} interaction (Koning and Delaroche, Nucl. Phys. A **713** (2003) 231).

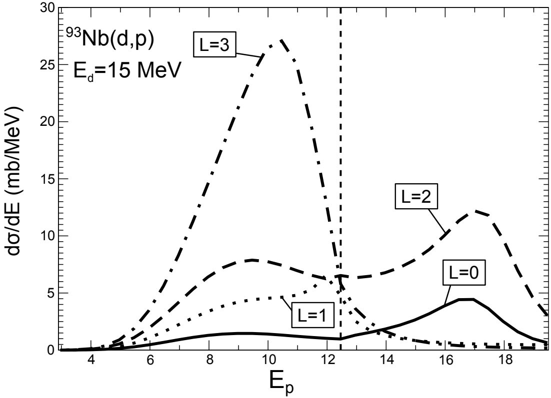
Sub–threshold capture



Non–orthogonality term



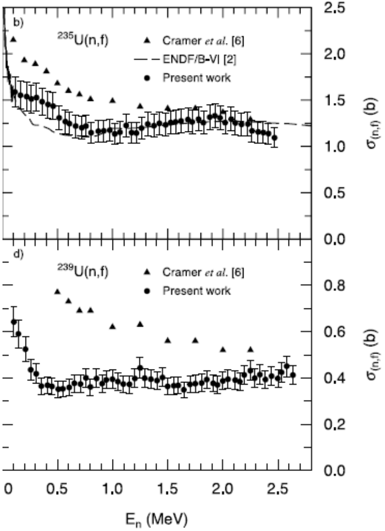
Obtaining spin distributions



spin distribution of compound nucleus

$$\frac{d\sigma_I}{dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\sum_{l_p,m}\int \left|\varphi_{Iml_p}(r_{Bn};k_p)\right|^2 W(r_{An})\ dr_{Bn}.$$

Getting rid of Weisskopf–Ewing approximation



Younes and Britt, PRC **68**(2003)034610

- Weisskopf–Ewing approximation: $P(d,nx) = \sigma(E)G(E,x)$
- inaccurate for $x=\gamma$ and for $x=f$ in the low–energy regime
- can be replaced by $P(d,nx) = \sum_{J,\pi}\sigma(E,J,\pi)G(E,J,\pi,x)$ if $\sigma(E,J,\pi)$ can be predicted.