The Gamow Shell Model

Towards the unified theory of nuclear structure and reactions

Marek Ploszajczak
GANIL, Caen

Content:

- Introduction and recent advances in nuclear theory
- Continuum Shell Model and Gamow Shell Model
- Interplay between Hermitian and anti-Hermitian couplings
  - exceptional points, segregation of time scales, one-nucleon spectroscopic factors involving weakly- and strongly-bound nucleons
- Coupled channel formulation of the Gamow Shell Model
- Unified description of structure and reactions: (p,p'), (p/n,g), (d,p)
- Outlook

INT Program "Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart"
February 27 – March 31, 2017
and workshop
"Nuclear Reactions: A Symbiosis between Experiment, Theory, and Applications"
March 13 – 16, 2017

Collaborators

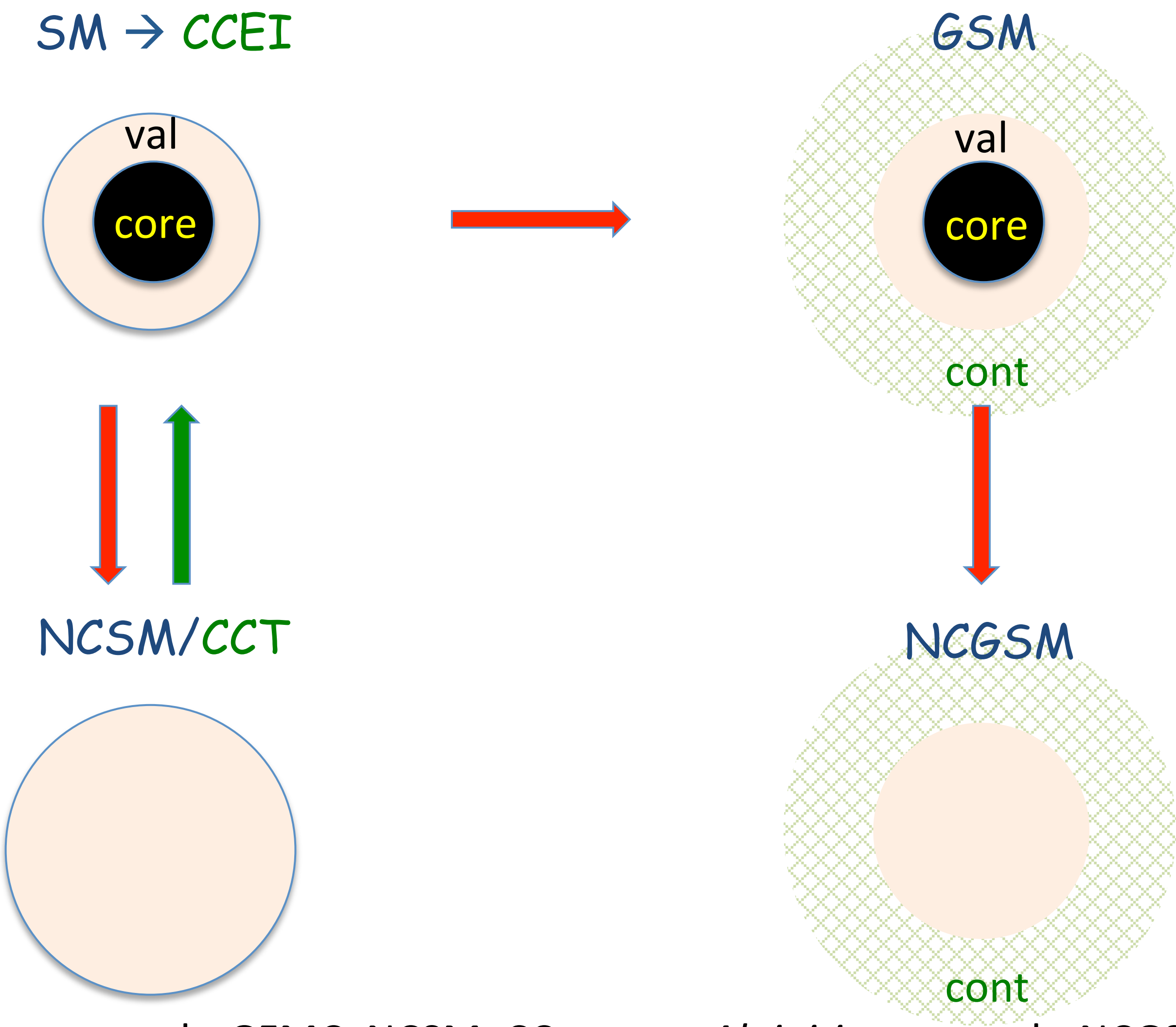
W. Nazarewicz - NSCL-FRIB MSU, East Lansing and Univ. of Warsaw
K. Fossez, Y. Jaganathen, N. Michel, J. Rotureau - NSCL-FRIB MSU, East Lansing
J. Okolowicz - Institute of Nuclear Physics, Krakow
G. Dong, A. Mercenne – GANIL
B. Barrett – Univ. of Arizona, Tucson
G. Papadimitriou – LLNL

Evolution of paradigms

- In medium nucleon-nucleon interaction from basic principles;
  3-body interactions

- *Ab initio* many-body theories for structure and reactions
  (GFMC, NCSM/RGM, NCGSM, CC,...)

- Nuclear shell model for open quantum systems;
  structure and reactions in the low-energy continuum



Ab initio approach: GFMC, NCSM, CC

Ab initio approach: NCGSM, CC(B)

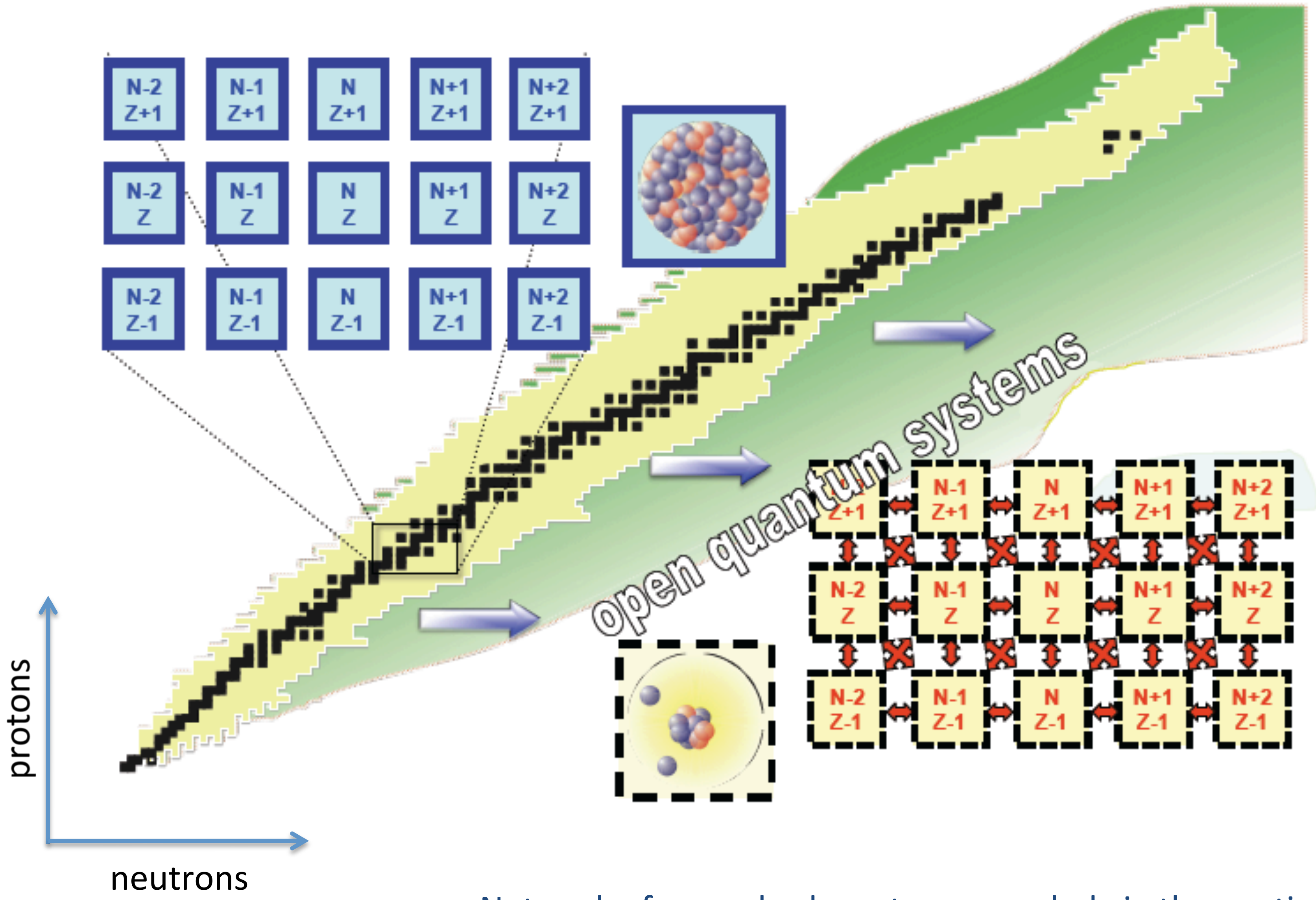
Challenges for the theory

- How to reconcile shell model with reaction models?

- Weakly bound systems; role of continuum…
  How to deal with non-localities due to the coupling to decay channels and the antisymmetrization

- How to handle multi-configuration effects in reaction theory

- How to understand (optical) potentials from microscopic interactions
- …

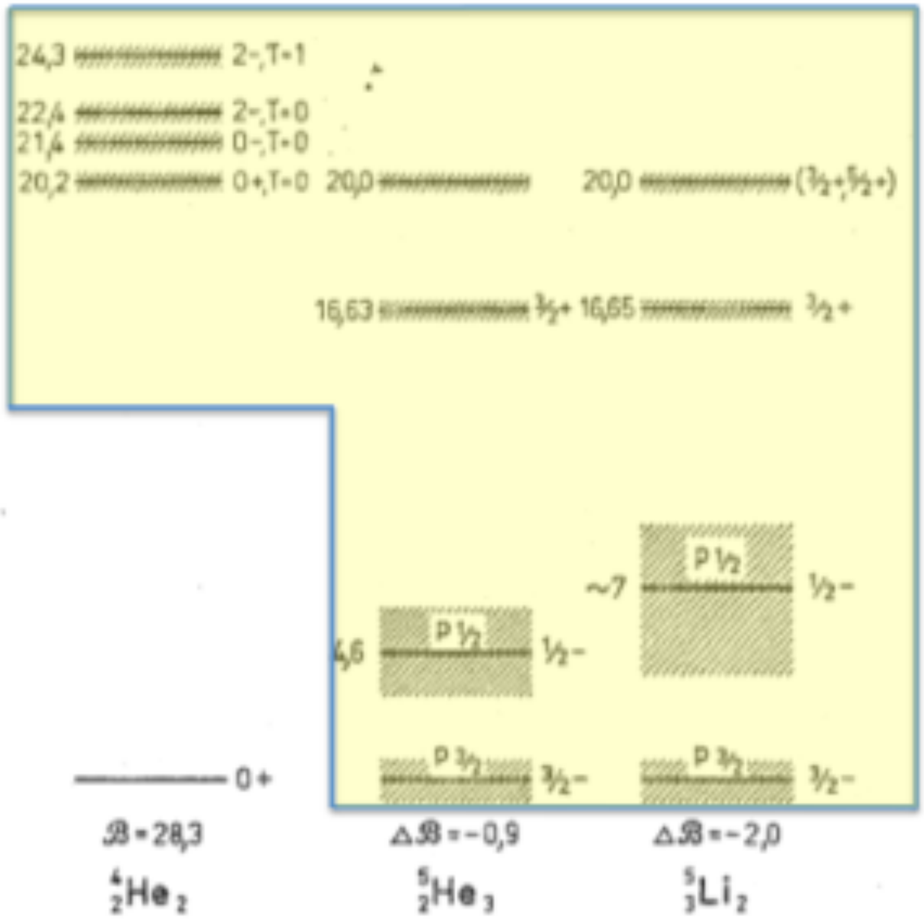


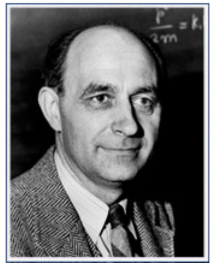
- Network of many-body systems coupled via the continuum

- Emergence of new scale(s) related to the threshold(s)

How it all begun…?

Shell model describes atomic nuclei as the closed quantum system i.e. coupling to decay channels are neglected





Enrico Fermi

Maria Goeppert-Mayer

J. Hans D. Jensen

Role of boundary conditions in universal properties of reaction cross-sections at the threshold

E.P. Wigner (1948)

To what extent the change in boundary conditions at the nuclear surface due to Coulomb wave function distortion in the external region can explain relative displacement of states in mirror nuclei?

J.B. Ehrman (1950)

The exact coincidence of the energies of different configurations makes the ordinary perturbation theory inadequate, so that special procedures are required…

U. Fano (1961)

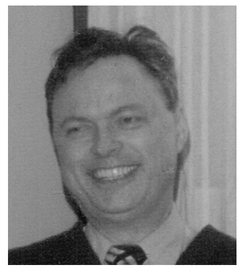
The resolution of these incosistencies took ~40 years and required:

- New mathematical concepts: Rigged Hilbert Space (≥1964),…
- Generalized completeness relation including s.p. bound states, resonances, and scattering states (~1968)
- Gamow Shell Model (~2002)

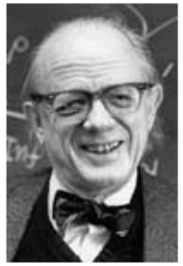
Eugene P. Wigner

U. Fano

I.M. Gelfand



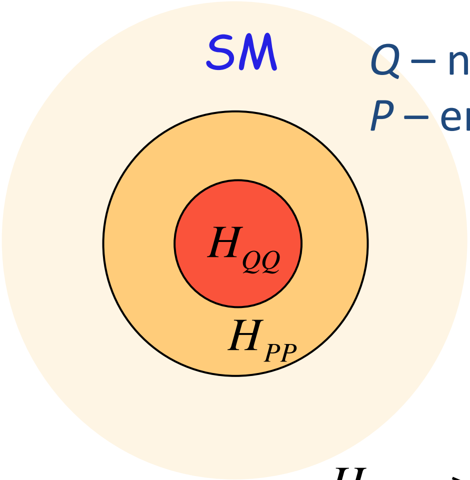
T. Berggren



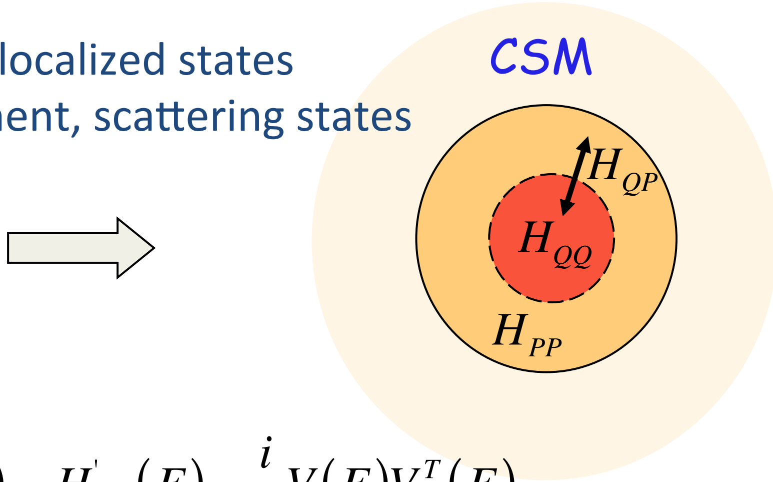
H. Feshbach

Continuum shell model with the real-energy continuum

Shell Model Embedded in the Continuum (SMEC)



Q – nucleus, localized states
P – environment, scattering states



$$H_{QQ} \rightarrow \mathcal{H}_{QQ}^{eff}(E) = H'_{QQ}(E) - \frac{i}{2}V(E)V^T(E)$$

closed quantum system — hermitian — anti-hermitian — open quantum system

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
R.J. Philpott, Nucl. Phys. A289 (1977) 109
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13

For bound states: $\mathcal{E}_\alpha(E)$ is real and $\mathcal{E}_\alpha(E) = E$
For unbound states: physical resonances = poles of S-matrix

Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations and determines the structure of many-body states

Quasi-stationary extension of the shell model in the complex k-plane

$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \qquad \Phi(r,t) = \tau(t)\Psi(r)$$

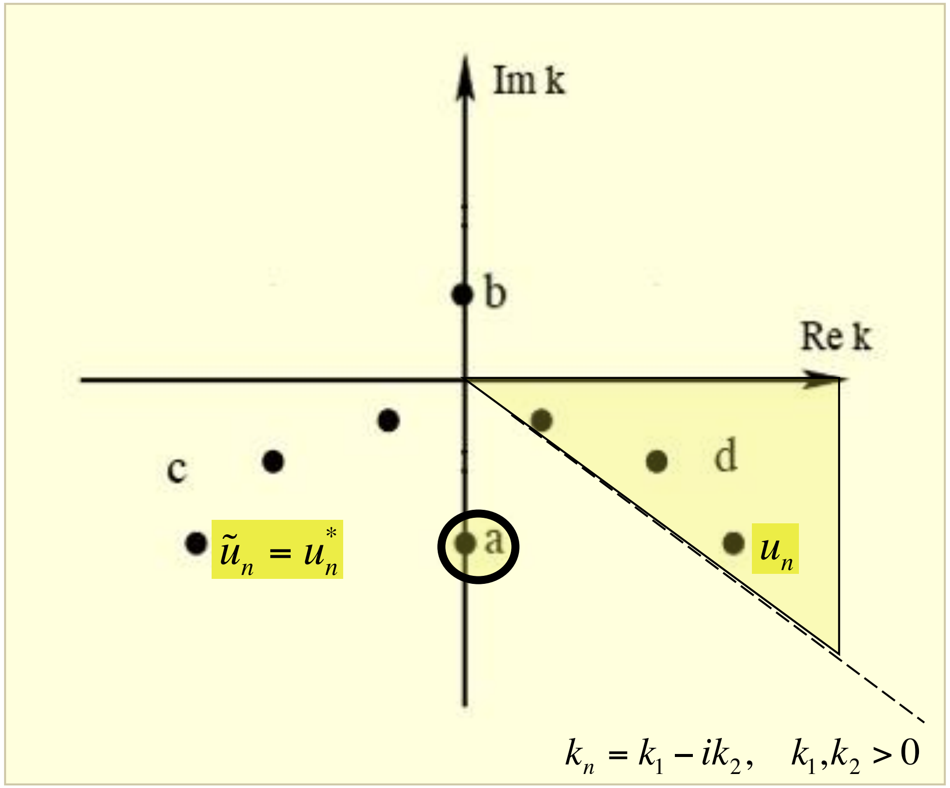
$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \rightarrow \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

$$\Psi(0,k) = 0\ ,\ \ \Psi(\vec{r},k) \underset{r\to\infty}{\rightarrow} O_l(kr)$$

$$k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$$

Poles of the S-matrix:
Bound and antibound states, resonances

Only bound states are integrable!



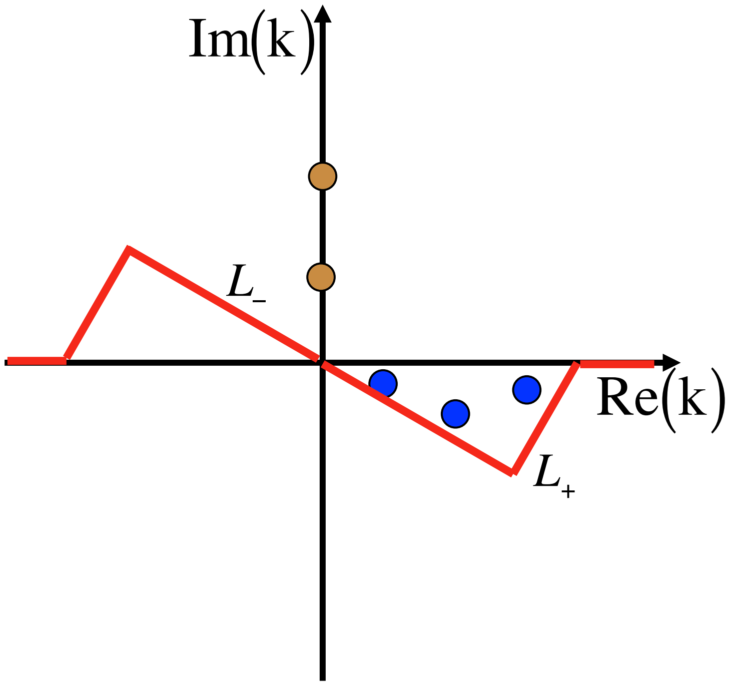
Euclidean inner product

$$\langle u_n | u_n \rangle = \int_0^\infty dr u_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2k_2 r}$$

Rigged Hilbert space inner product

$$\langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr \tilde{u}_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2ir(k_1 - ik_2)}$$

Gamow Shell Model: Continuum shell model in the complex k-plane



$$H \to [H]_{ij} = [H]_{ji}$$

Complex-symmetric eigenvalue problem for hermitian Hamiltonian

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \; ; \; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109 (1968) 265
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)

bound states resonances

non-resonant continuum

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \;\to\; \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311

Ab initio no-core GSM studies of resonances in A<8 systems

G. Papadimitriou et al, PRC 88 (2013) 044318
K. Fossez et al., arXiv:1612.01483

(see the talk of Bruce Barrett)

Interplay between hermitian and anti-hermitian couplings is a source of new phenomena

- Coalescence of eigenfunctions/eigenvalues (exceptional points)
  Example: doublet of 3+ resonances in ^{8}Be: 19.07(0.27) MeV and 19.23(0.23) MeV

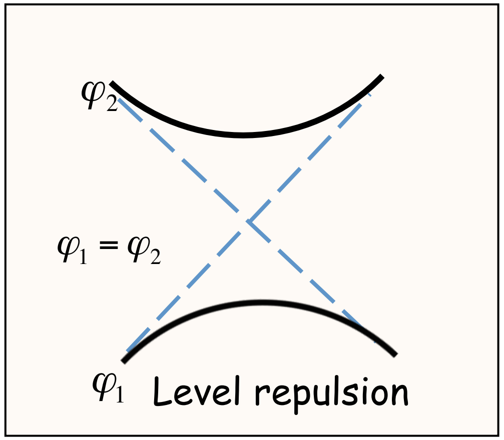
- Segregation of time scales

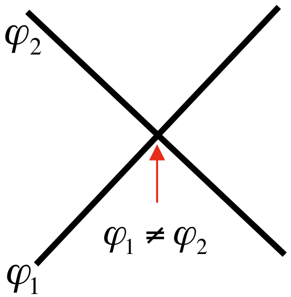
- Collective phenomena:
    - (Multi)channel effects in reaction cross-section and shell occupancies
    - Instability of SM eigenstates at the channel threshold; near-threshold clustering
    - …

- Violation of the orthogonal invariance and channel equivalence
- …

Coalescence of eigenfunctions

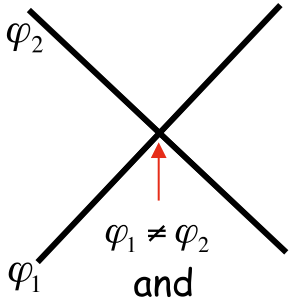
Bound states (hermitian problem)





Level crossing

Resonances (non-hermitian problem)



and

$\varphi_1 = \varphi_2$

$\varphi_1 = \varphi_1^*$

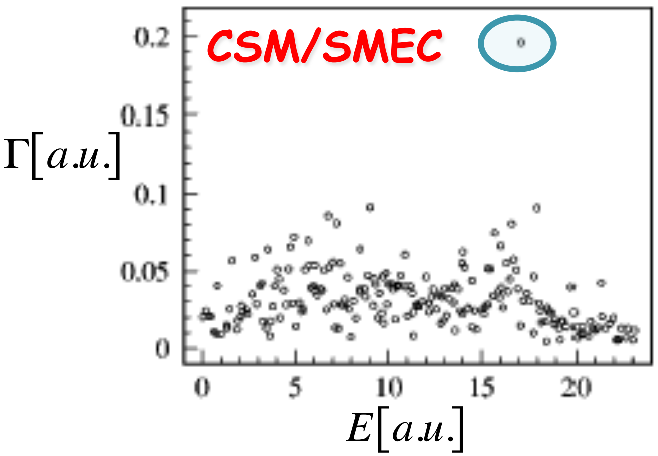
Resonances coalesce as a result of the interplay between hermitian and non-hermitian components of the residual interaction

M.R. Zirnbauer et al., Nucl. Phys. A411 (1983) 161
C. Dembowski et al., PRL 86 (2001) 787
PRL 90 (2003) 034101

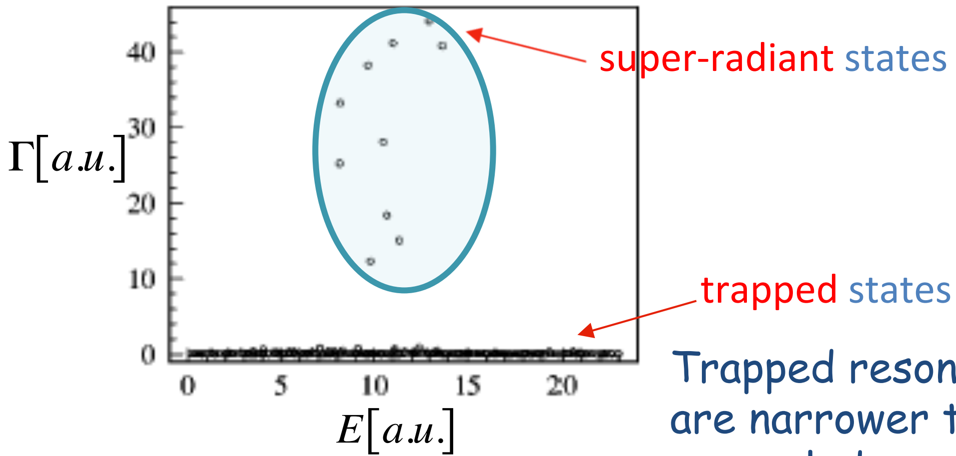
Segregation of time scales

J=0$^+$, T=0 states in ^{24}Mg, 10 channels

S. Drozdz et al, PRC 62, 24313 (2000)



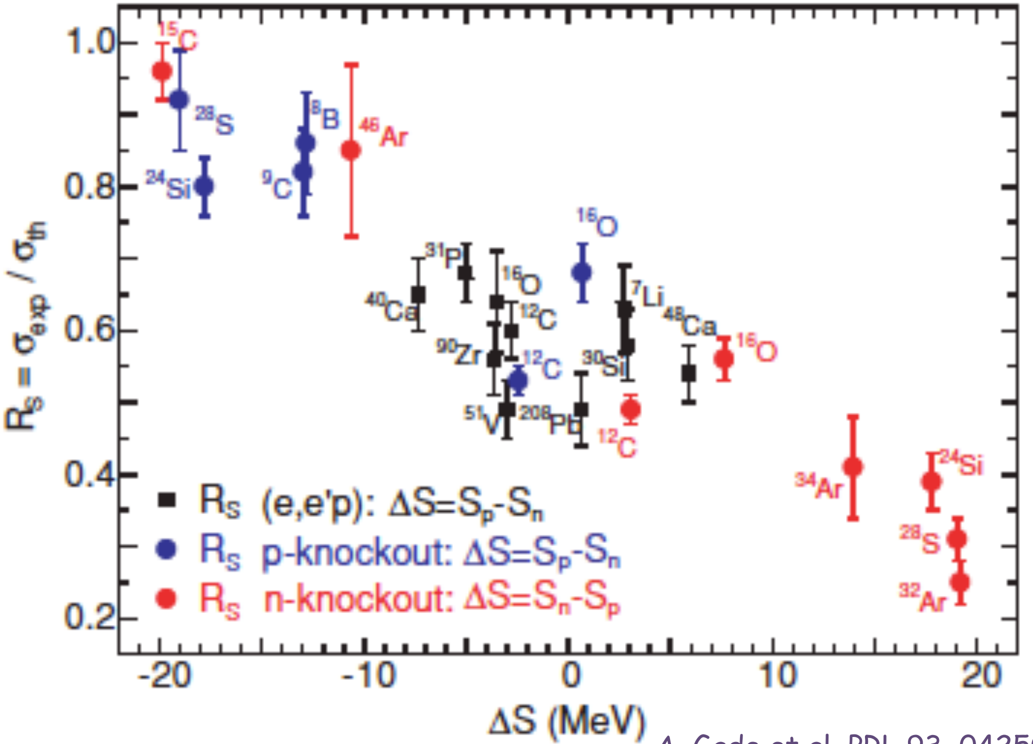
intermediate coupling



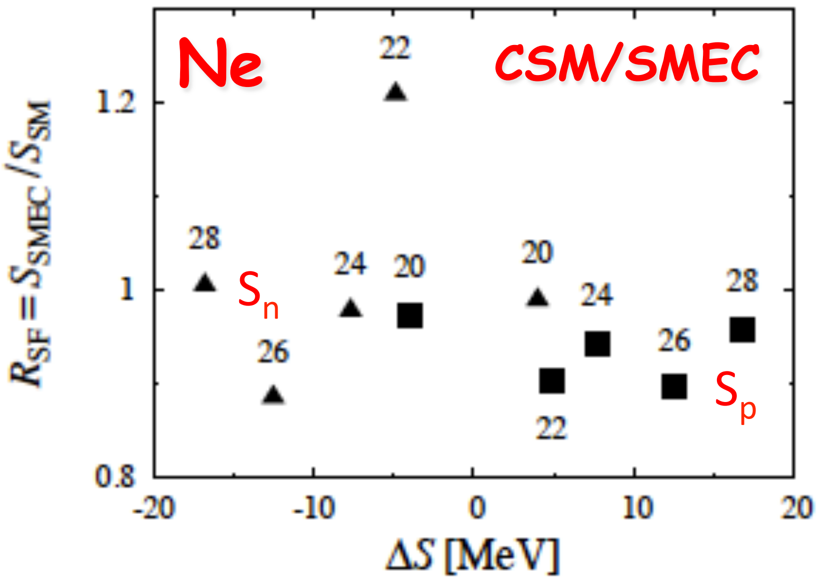
strong coupling

Trapped resonances are narrower than uncoupled resonances

One-nucleon spectroscopic factors involving weakly- and strongly-bound nucleons



A. Gade et al, PRL 93, 042501



J. Okolowicz et al, PLB 757, 303 (2016)

- Less than 15% of the ground state spectroscopic strength shifted to higher excitations
- One-nucleon spectroscopic factors are not correlated with the asymmetry of S_n and S_p separation energies

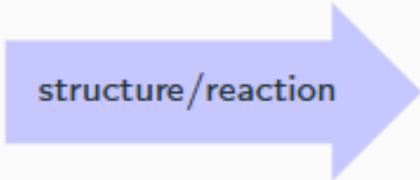
Coupled channel formulation of the Gamow shell model

GSM

$$|\Psi\rangle = \sum_n C_n \left| \text{SD} \right\rangle$$

↓

SD with s.p. states of the Berggren ensemble



GSM-CC :

$$|\Psi\rangle = \sum_c \int_0^\infty dr \frac{u_c(r)}{r} r^2 \hat{\mathcal{A}} \left| \vec{r} \right\rangle_c$$

GSM channel states

- Generalization of the SM to continuum states
- Representation of $|\Psi\rangle$ in terms of Slater determinants

Channel basis : $\{c\} = \{(A-a), J_{\mathrm{T}}, a, \ell, J_{\mathrm{int}}, J_{\mathrm{p}}\}$

$$\hat{\mathcal{A}} \left| \vec{r} \right\rangle_c \equiv |(c,r)\rangle = \hat{\mathcal{A}}\left[|\Psi_T^{J_{\mathrm{T}}}\rangle \otimes |r, \ell, J_{\mathrm{int}}, J_{\mathrm{p}}\rangle \right]^{J_{\mathrm{A}}}_{M_{\mathrm{A}}}$$

In practice : $|(c,r)\rangle = \sum_{n \in \mathrm{Berggren}} \frac{w_n(r)}{r} |(c,n)\rangle$

- Representation of $|\Psi\rangle$ in terms of the reaction channels
- Entrance and exit channels defined

- No identification of the reaction channels

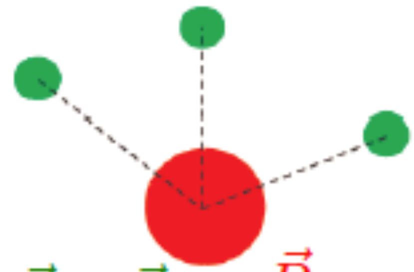
- Wave functions of projectile and target nuclei are calculated in GSM

- Center of mass treatment: *Cluster Orbital Shell Model relative coordinates*

Y. Suzuki, K. Ikeda, PRC 38 (1998) 410

$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2\mu} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{p_i p_j}{A_c}\right)$$

"Recoil" term coming from the expression of H in the COSM coordinates. No spurious states



$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

- Scattering wave functions $\left|\Psi_{\mathrm{GSM}}(A-p)\otimes\Phi_{\mathrm{proj}}(p)\right\rangle$ are the many-body states

- Antisymmetry exactly handled

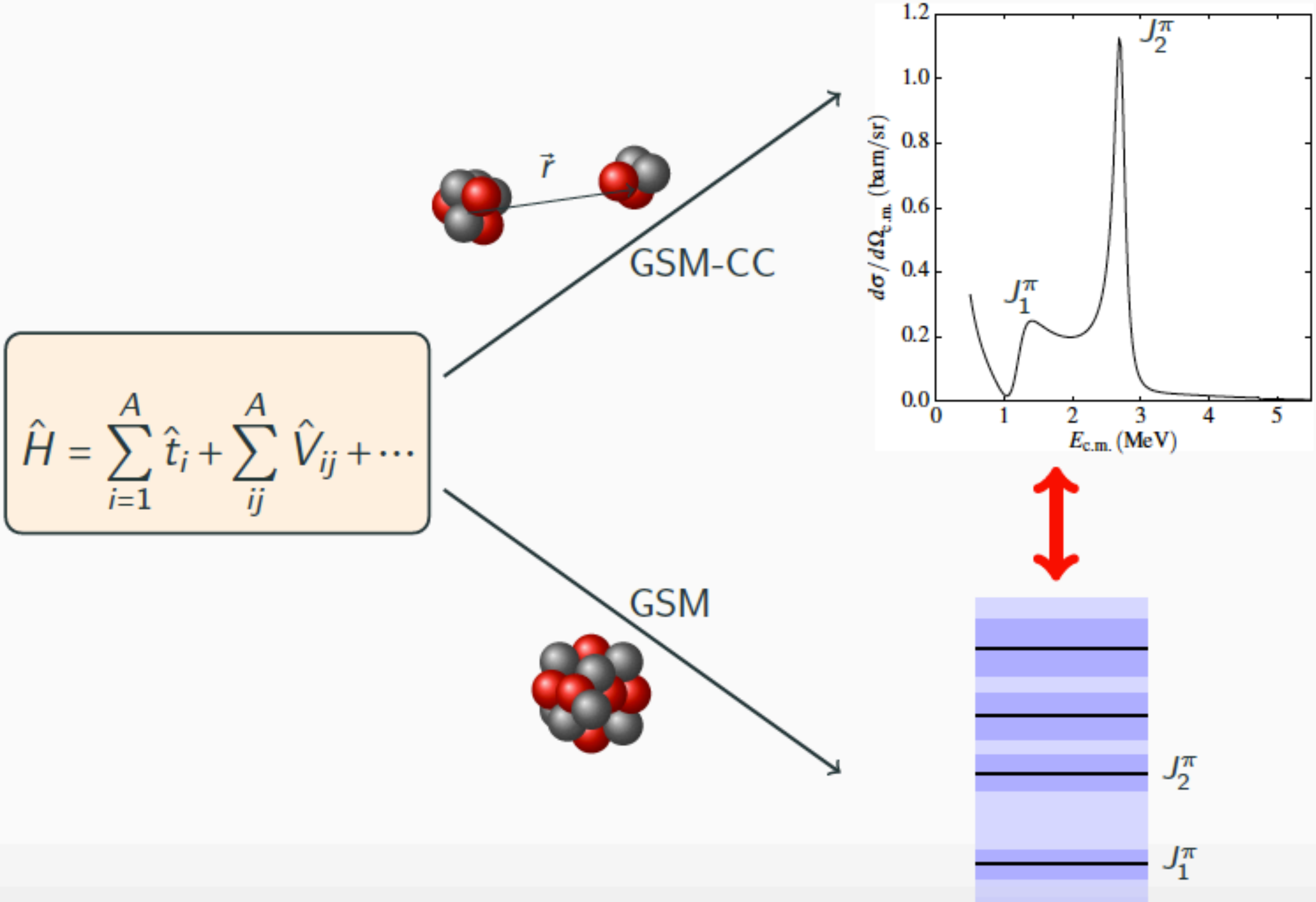
- Core arbitrary

- Resonating Group Method for A-body matrix elements:

$$\left\langle \Psi_{\mathrm{GSM;f}}(A-p)\otimes\Phi_{\mathrm{proj;f}}(p)\middle| H \middle| \Psi_{\mathrm{GSM;i}}(A-p)\otimes\Phi_{\mathrm{proj;i}}(p)\right\rangle$$

leads to coupled-channel equations with microscopic potentials

Unified desciption of structure and reactions in the Gamow Shell Model



p+^{18}Ne excitation function

Y. Jaganathen, N. Michel, M.P., PRC 89 (2014) 034624

18Ne	EXP	GSM	GSM-CC
0+	0.00	0.00	
2+	1.89	1.56	

S_p=3.921 MeV
S_n=19.237 MeV

19Na	EXP	GSM	GSM-CC
5/2+	0.32	0.28	0.29
3/2+	0.44	0.25	0.27
1/2+	1.07	1.08	1.13

S_p=-0.32 MeV
S_n=20.18 MeV

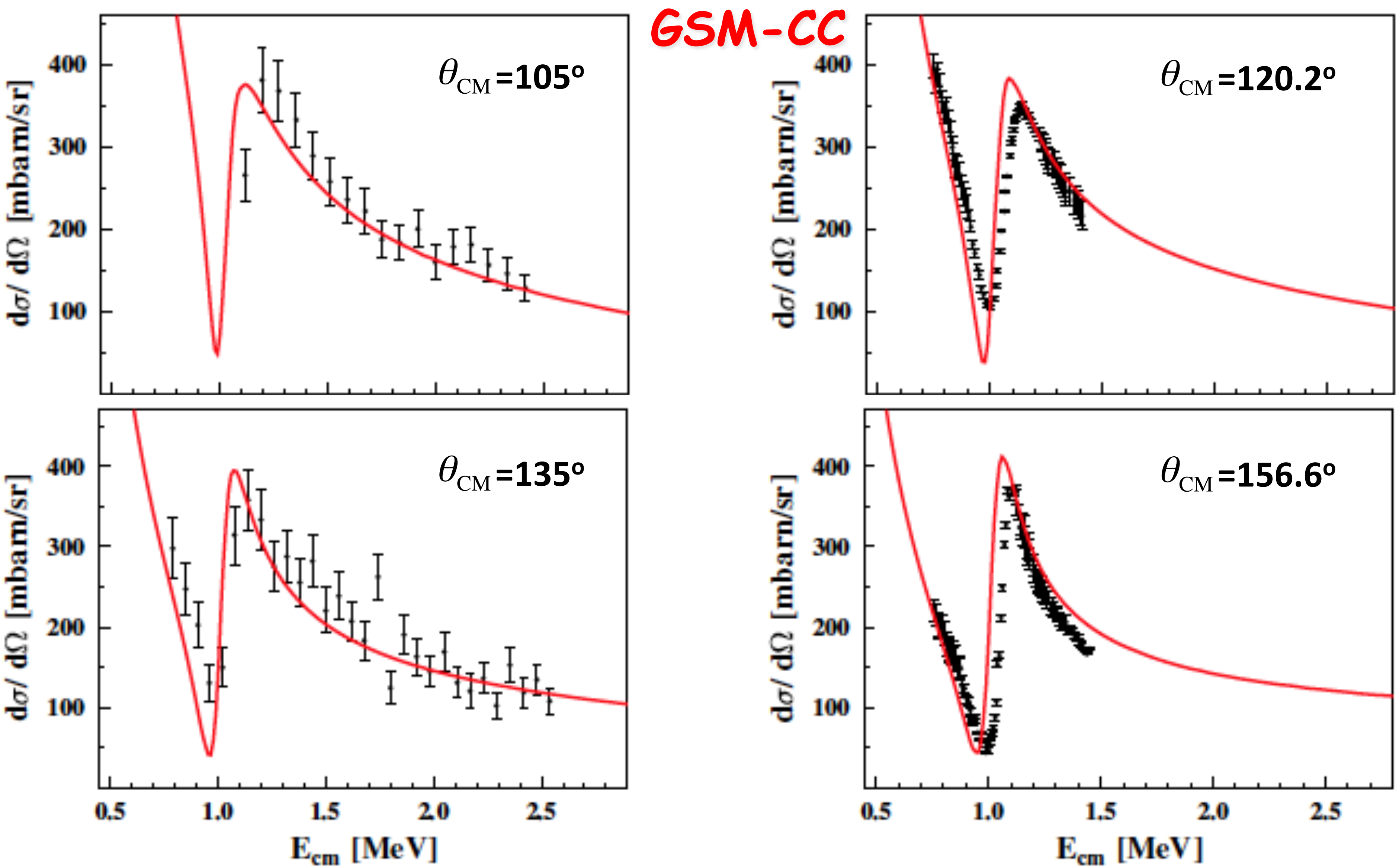
Model space: $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $1p_{3/2}$, $1p_{1/2}$

Interaction: FHT finite-range interaction: $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

H. Furutani, H. Horiuchi, R. Tamagaki, PTP 60 (1978) 307; 62 (1979) 981

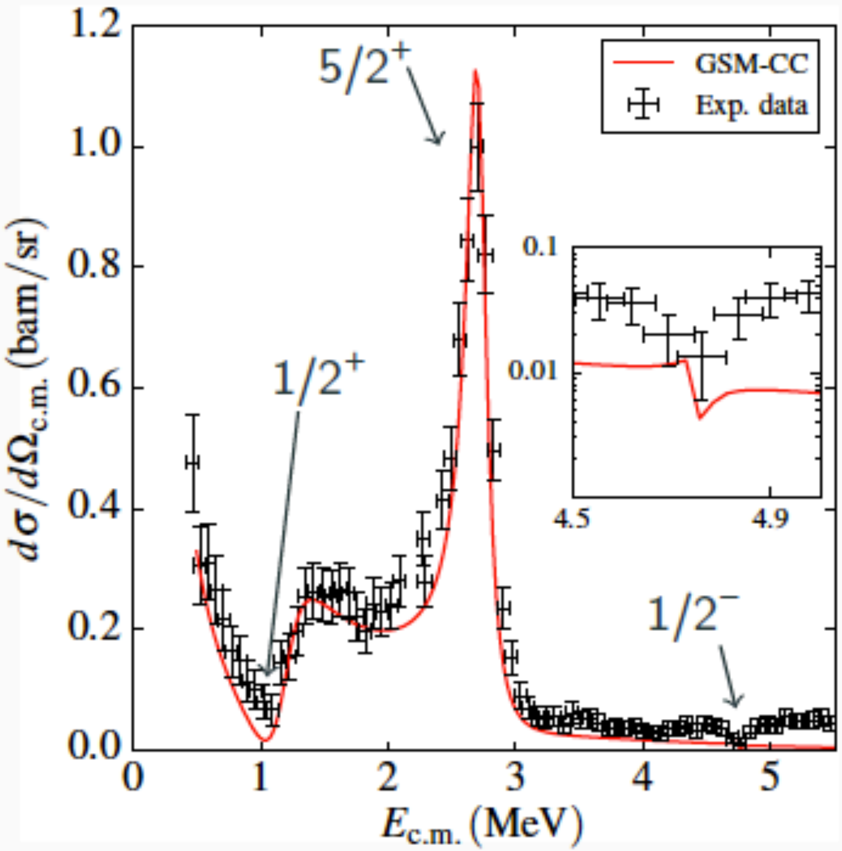
GSM and GSM-CC results (almost) identical → Scattering states $J=0^+,1^+,2^+$,... and higher lying (bound) states in ^{18}Ne are unimportant.

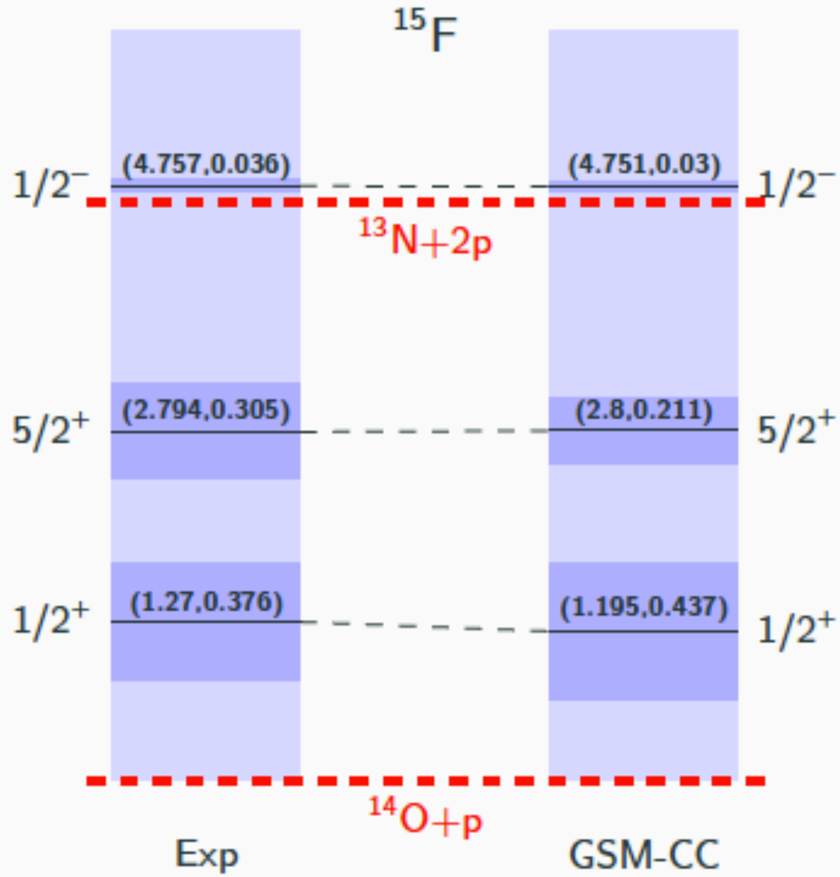
p+^{18}Ne excitation function at different angles



Exp: F. de Oliveira Santos et al., Eur. Phys. J. A24, 237 (2005)
B. Skorodumov et al., Phys. Atom. Nucl. 69, 1979 (2006)
C. Angulo et al., PRC 67, 014308 (2003)

p+^{14}O excitation function



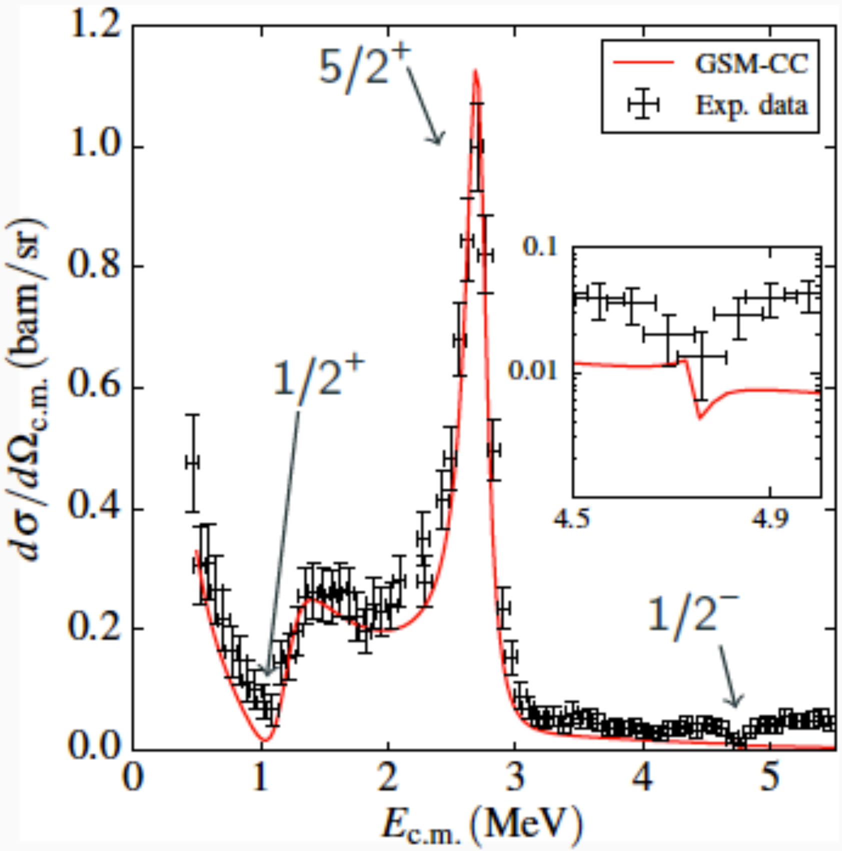


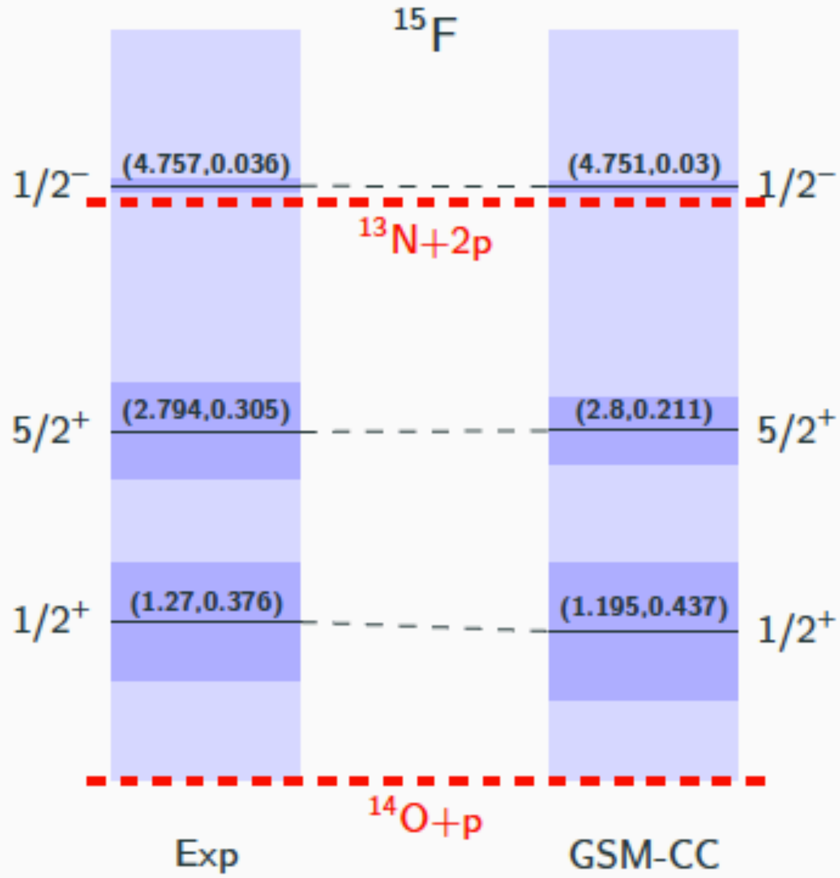
F. De Grancey, A. Mercenne, et al. PLB 758, 26 (2016)

- Model space: $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $1p_{1/2}$
- Channels: $|J^\pi(^{15}\mathrm{F})\rangle = \hat{\mathcal{A}}[|J^\pi(^{14}\mathrm{O})\rangle \otimes |\mathrm{p}\rangle]^{J^\pi(^{15}\mathrm{F})}$ $\rightarrow \{J^\pi(^{14}\mathrm{O})\} = 0_1^+,\ 1_1^-,\ 0_2^+,\ 3_1^-,\ 2_1^+,\ 0_1^-,\ 2_2^+,\ 2_1^-$

  Non-resonant channels: $[|J^\pi_{scat}(^{14}\mathrm{O})\rangle \otimes |\mathrm{p}\rangle]^{J^\pi(^{15}\mathrm{F})}$ are neglected
- FHT finite-range interaction: $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

p+^{14}O excitation function



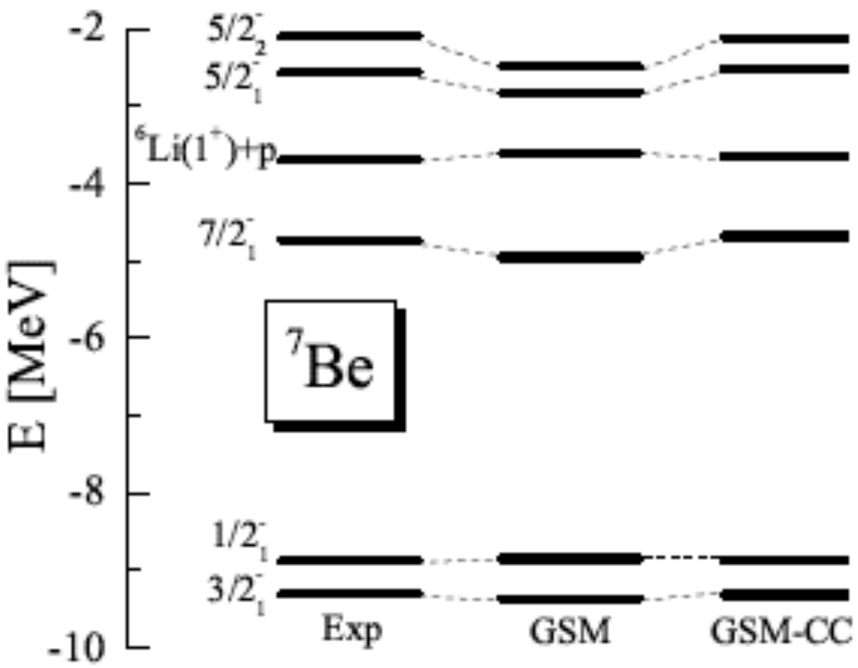


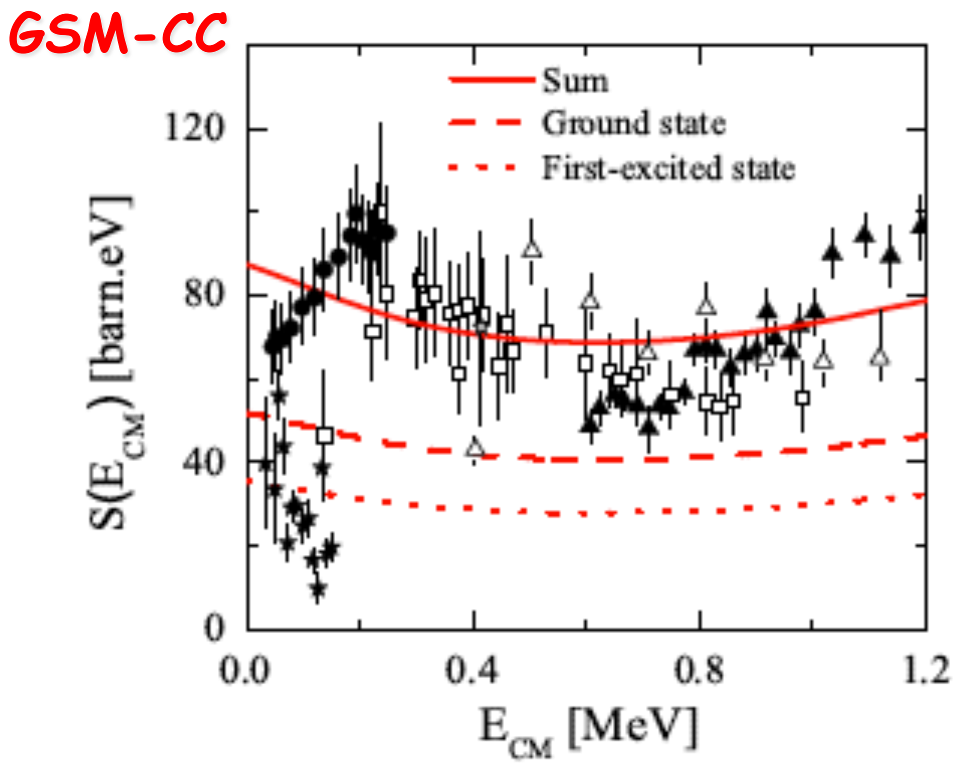
Small correction factors in the channel-channel coupling potentials to compensate for neglected non-resonant channels

GSM structure of $1/2^-_1$:
$\langle\Psi|0p_{1/2}[1]1s_{1/2}[2]\rangle^2 = 0.97$
$\langle\Psi|0p_{1/2}[1]0d_{5/2}[2]\rangle^2 = 0.02$
almost pure configuration of 2 protons in $1s_{1/2}$ and $\{s_{1/2}\}$ shells

Diproton nature of $1/2^-_1$ favors the two-proton emission, but the available decay energy is very small (Q_{2p} = 129 keV) $\Rightarrow$ one-proton emission is hindered

$^{6}Li(p,g)$ radiative capture cross sections





- Model space: $0p_{3/2}$, $0p_{1/2}$, $0s_{1/2}$, $0d_{5/2}$, $0d_{3/2}$

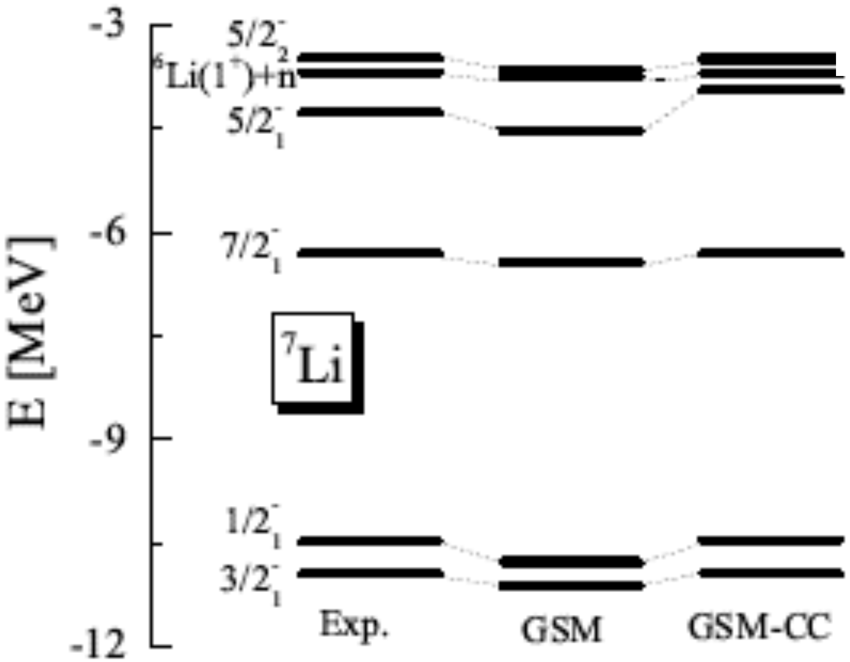
- Interaction: FHT finite-range interaction

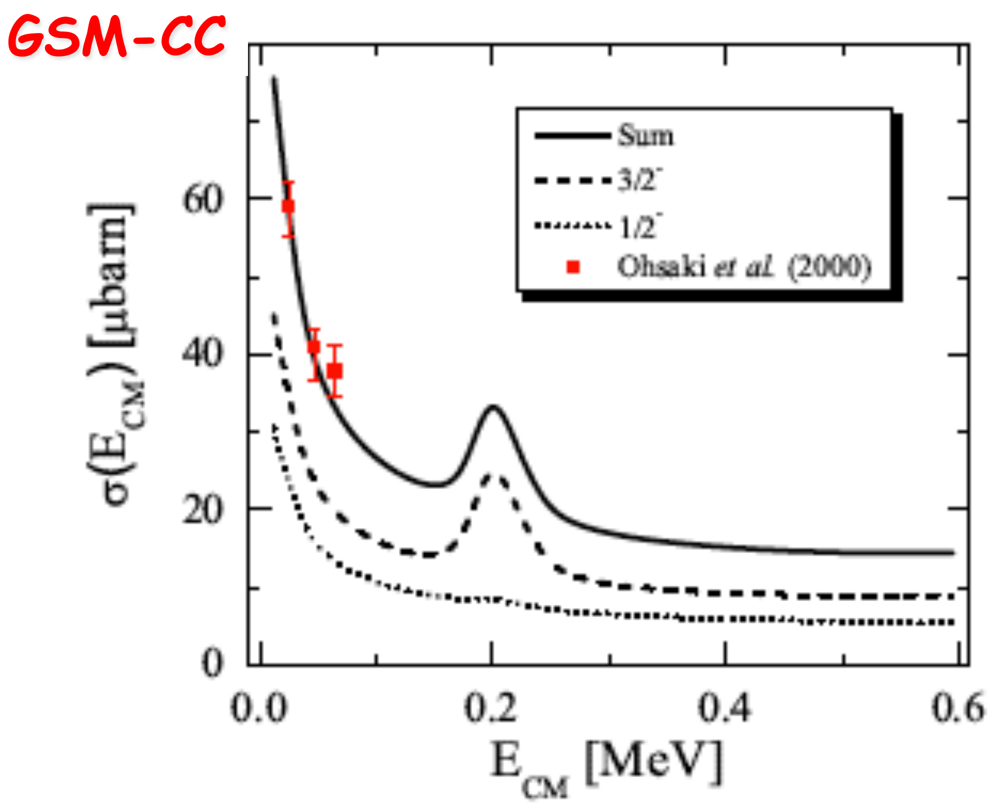
E1, E2, M1 components included

$S^{GSM\text{-}CC}(0)$=88.34 b eV
$S^{exp\text{-}acc}(0)$=79+/-18 b eV

G. Dong, et al., J. Phys. G: Nucl. Part Phys. 44. 045201 (2017)

Mirror reaction: ^{6}Li(n,g)





E1, E2, M1 components included

G. Dong, et al., J. Phys. G: Nucl. Part Phys. 44. 045201 (2017)

^{40}Ca(d,p) transfer reaction

- Channels : $|J^\pi(^{42}\mathrm{Sc})\rangle = \hat{\mathcal{A}}[|J^\pi(^{40}\mathrm{Ca})\rangle \otimes |\mathrm{d}\rangle]^{J^\pi(^{42}\mathrm{Sc})}$

$$|J^\pi(^{41}\mathrm{Ca})\rangle \otimes |\mathrm{p}\rangle \longleftarrow \{J^\pi(^{41}\mathrm{Ca})\} = 7/2^-_1,\ 3/2^-_1,\ 5/2^-_1,\ 1/2^-_1$$

$$|J^\pi(^{41}\mathrm{Sc})\rangle \otimes |\mathrm{n}\rangle \longleftarrow \{J^\pi(^{41}\mathrm{Sc})\} = 7/2^-_1,\ 3/2^-_1,\ 5/2^-_1,\ 1/2^-_1$$

- Model space : ^{40}Ca core + valence particles
- S.p. basis :

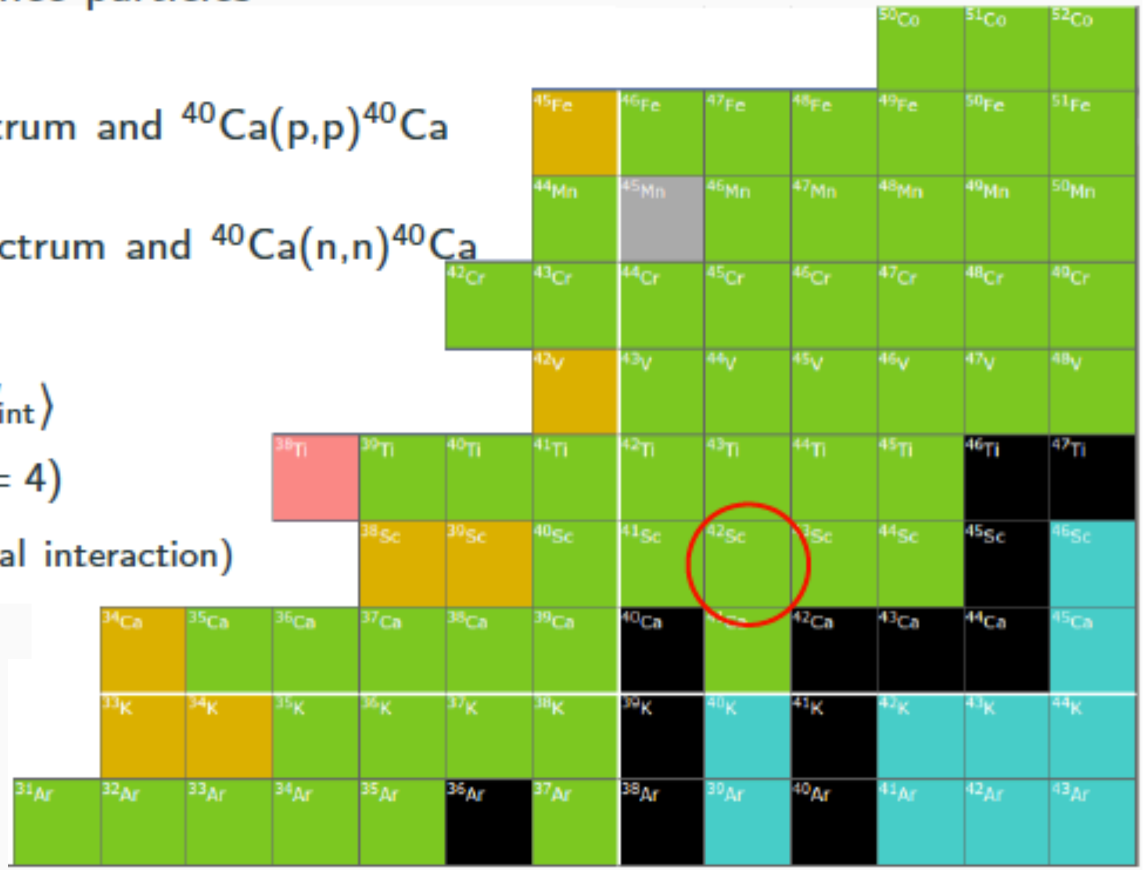
→ Proton WS potential : ^{41}Sc spectrum and ^{40}Ca(p,p)^{40}Ca cross section

→ Neutron WS potential : ^{41}Ca spectrum and ^{40}Ca(n,n)^{40}Ca cross section

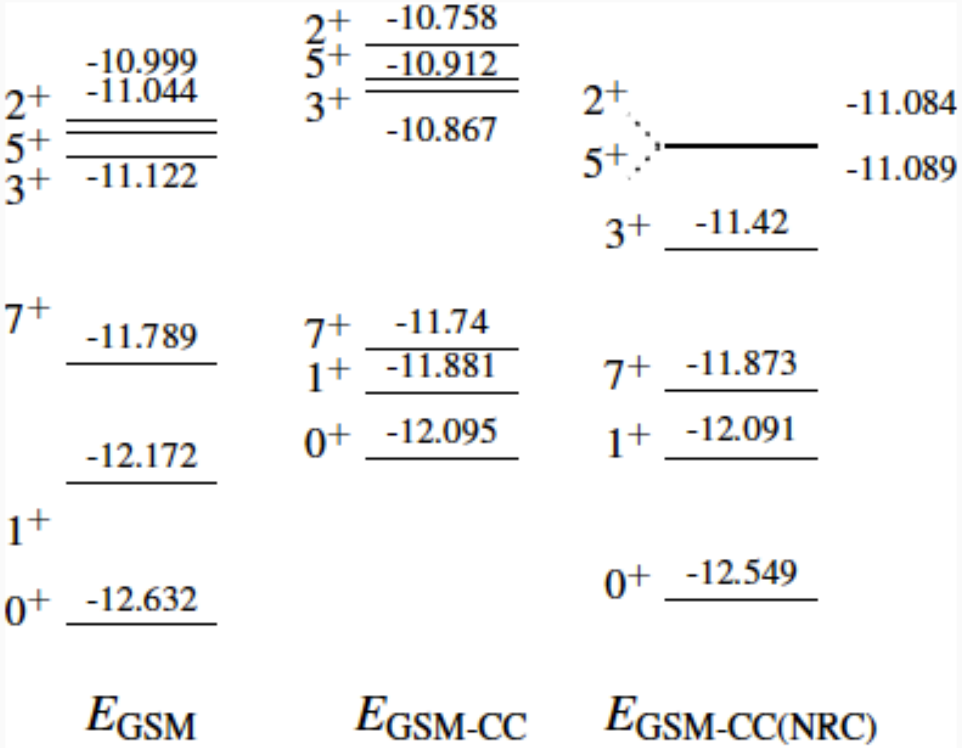
- Deuteron state : $|d\rangle = |n,\ell\rangle \otimes |J_{\mathrm{int}}\rangle$

  → $|n,\ell\rangle$: Berggren states ($\ell_{max} = 4$)

  → $|J^\pi_{\mathrm{int}}\rangle$: 1^+ (NCSM + N^{3}LO chiral interaction)



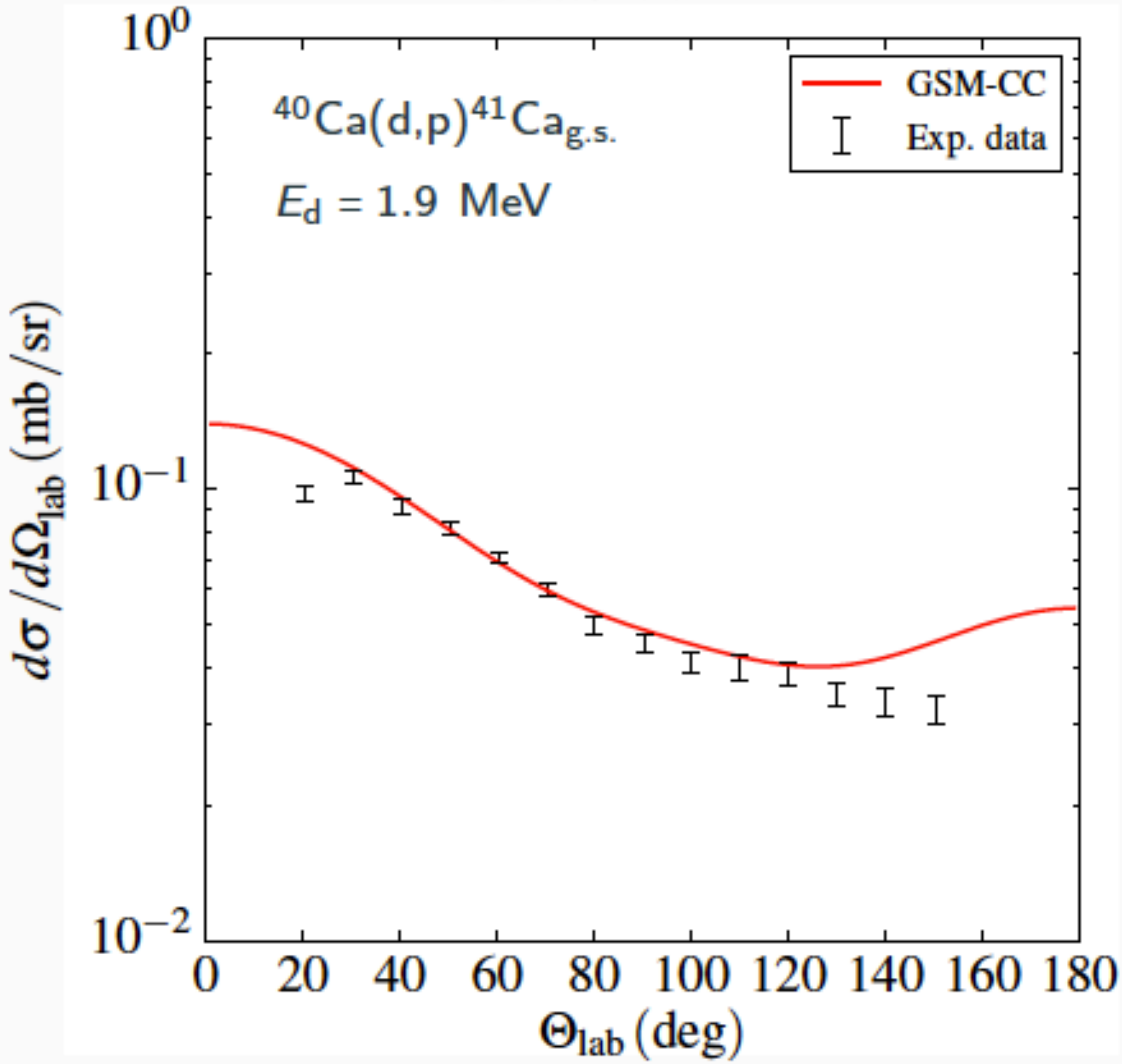
Non-resonant channels



- Channels : ^{40}Ca + d, ^{41}Ca + p, and ^{41}Sc + n
- Non-resonant channels (NRC) built of continuum states $\{1/2^+\}$, $\{3/2^+\}$, $\{5/2^+\}$, $\{7/2^+\}$, $\{9/2^+\}$, $\{1/2^-\}$, $\{3/2^-\}$, $\{5/2^-\}$, $\{7/2^-\}$ in ^{41}Sc and ^{41}Ca are considered
- $E_{\text{GSM-CC}}$: Diagonalization in the channel basis *without* the non-resonant channels
- $E_{\text{GSM-CC(NRC)}}$: Diagonalization in the complete channel basis *with* non-resonant channels

The inclusion of non-resonant channels improves the description of ^{42}Sc spectrum $\Rightarrow$ no corrective factors in channel-channel coupling potentials

GSM-CC



A. Mercenne, N. Michel, M.P., (2017) in preparation

Outlook

1. Shell model treatment of weakly bound/unbound states ➔ unification of nuclear structure and reactions
2. Collectivization of nuclear wave functions can be the result of
   - internal mixing by interactions (low-lying collective vibrations, rotational states, giant resonances,…)
   - external mixing via the decay channel(s) (coherent enhancement/ suppression of radiation, multi-channel effects in spectroscopic factors and reaction cross-sections,…)
   - interplay of internal & external mixing (near-threshold clustering, halo states, pygmy resonances, modification of spectral fluctuations, coalescence of eigenvalues, …)
3. GSM-CC approach has been extended to reactions with many-nucleon projectiles
   - important role of the non-resonant reaction channels

Thank You