$^{7}\text{Be}(p,\gamma)^{8}\text{B}$: how EFT and Bayesian analysis can improve a reaction calculation

Daniel Phillips

Work done in collaboration with: K. Nollett (SDSU), X. Zhang (UW)

Phys. Rev. C 89, 024613 (2014), ibid. 051602, Phys. Lett. B751, 535 (2015), EPJ Web Conf. 113 , 06001 (2016)

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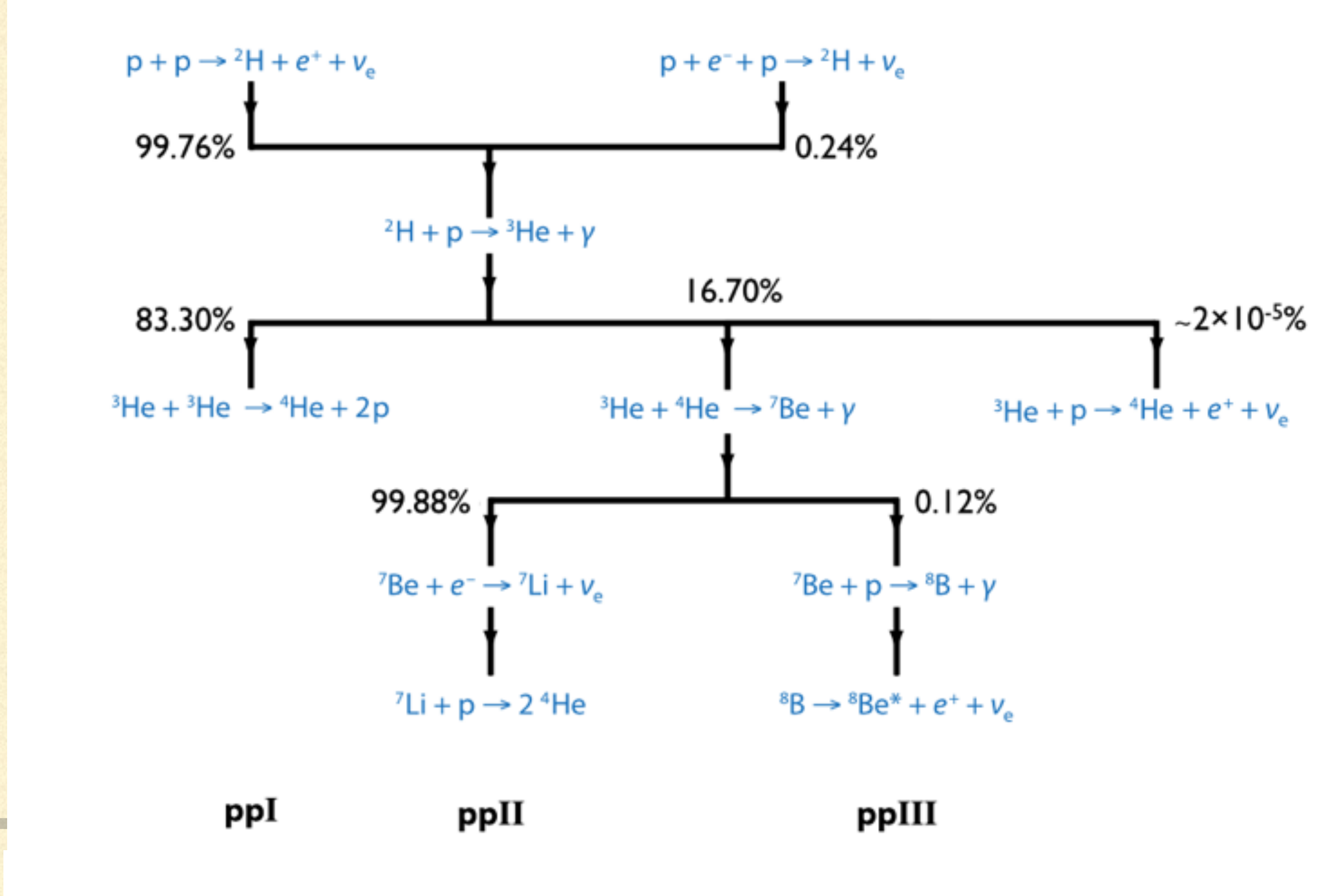
Research supported by the US Department of Energy

Why is $^7Be(p,\gamma)$ important?

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)



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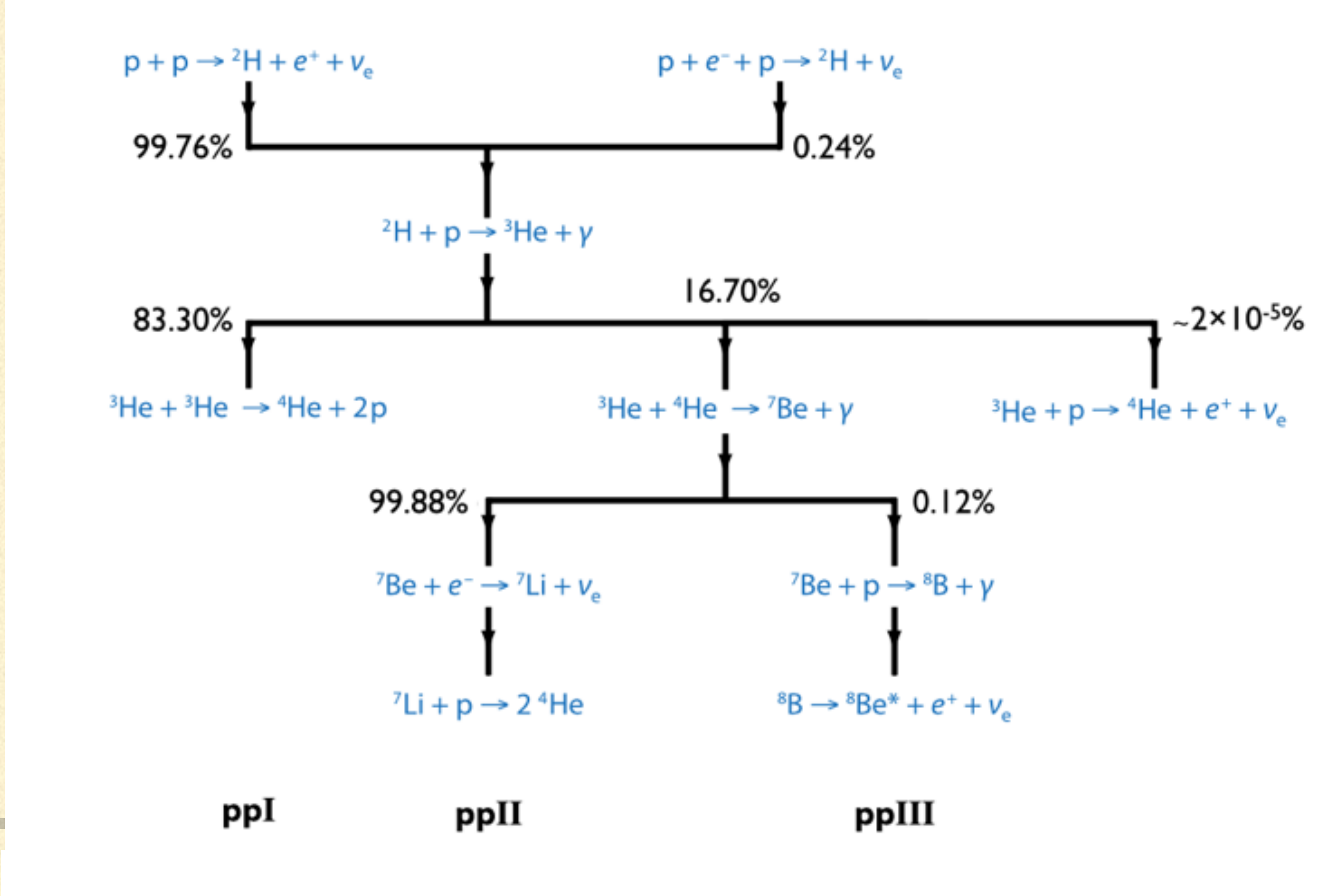

$p + p \rightarrow {}^2H + e^+ + \nu_e$

$p + e^- + p \rightarrow {}^2H + \nu_e$

99.76%

0.24%

${}^2H + p \rightarrow {}^3He + \gamma$

83.30%

16.70%

~2×10^{-5}%

${}^3He + {}^3He \rightarrow {}^4He + 2p$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$

99.88%

0.12%

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

${}^7Be + p \rightarrow {}^8B + \gamma$

${}^7Li + p \rightarrow 2\,{}^4He$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

ppI

ppII

ppIII

Why is ^{7}Be(p,γ) important?

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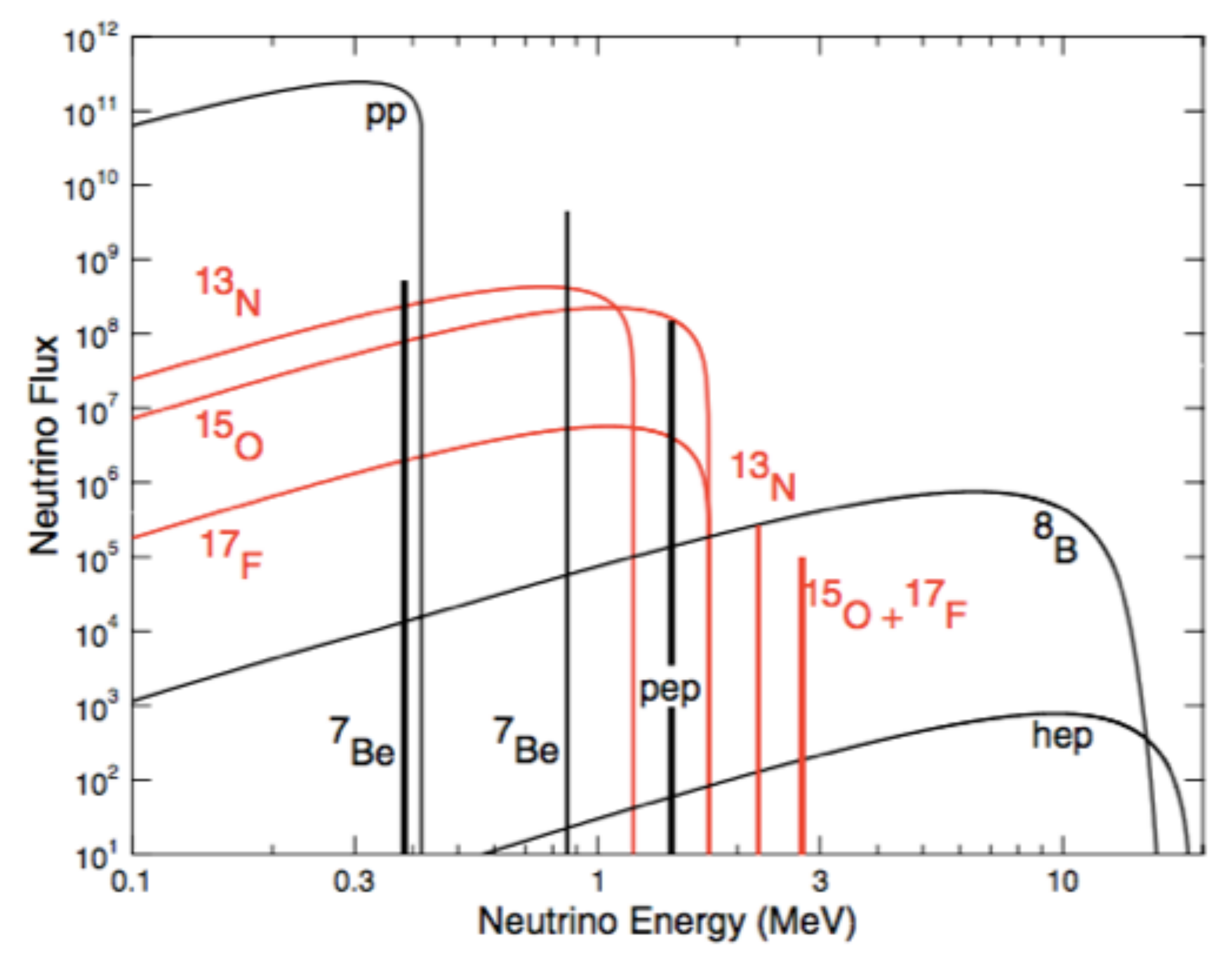

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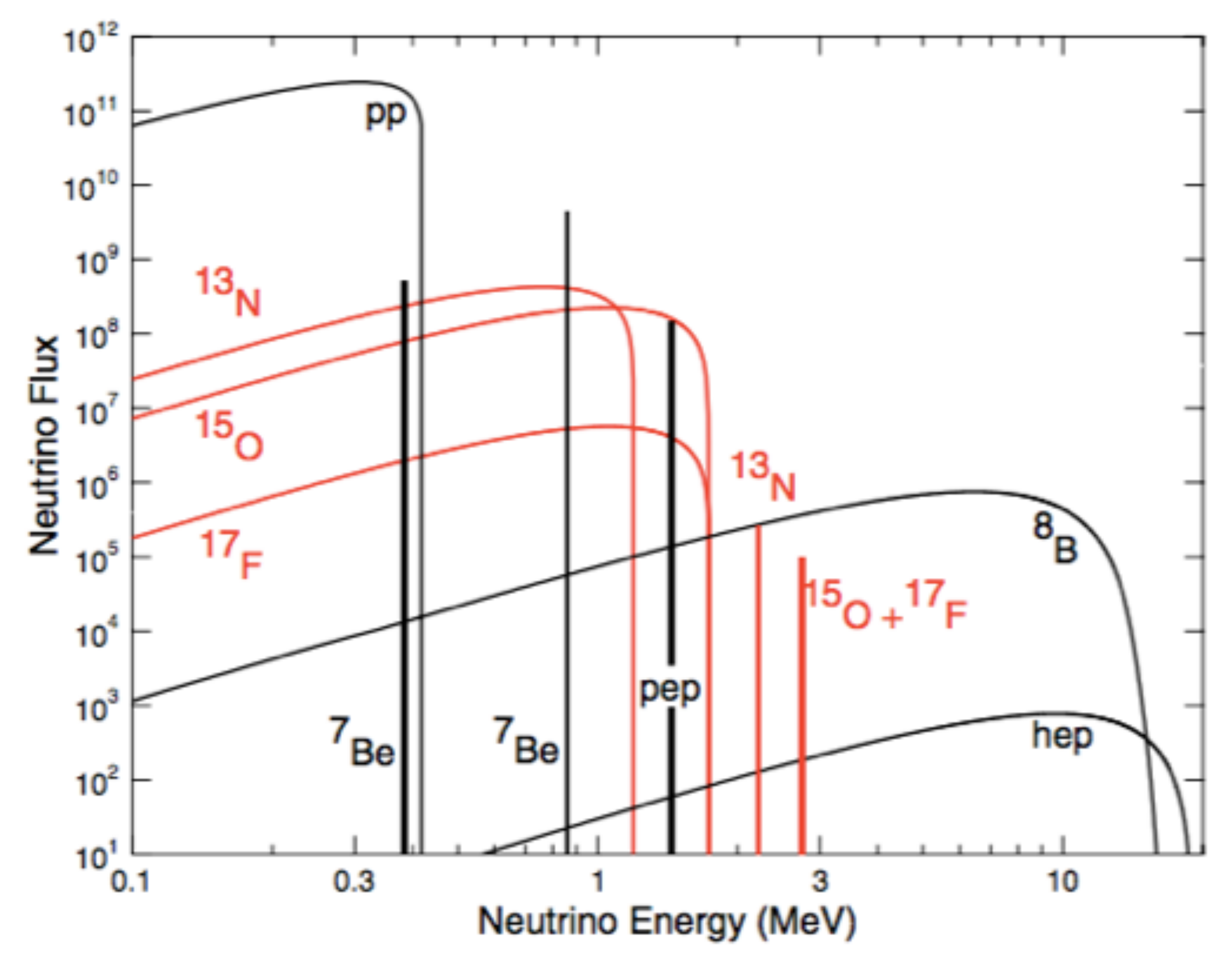
- Accurate knowledge of ^{7}Be(p,γ) needed for inferences from solar-neutrino flux regarding solar composition →solar-system formation history



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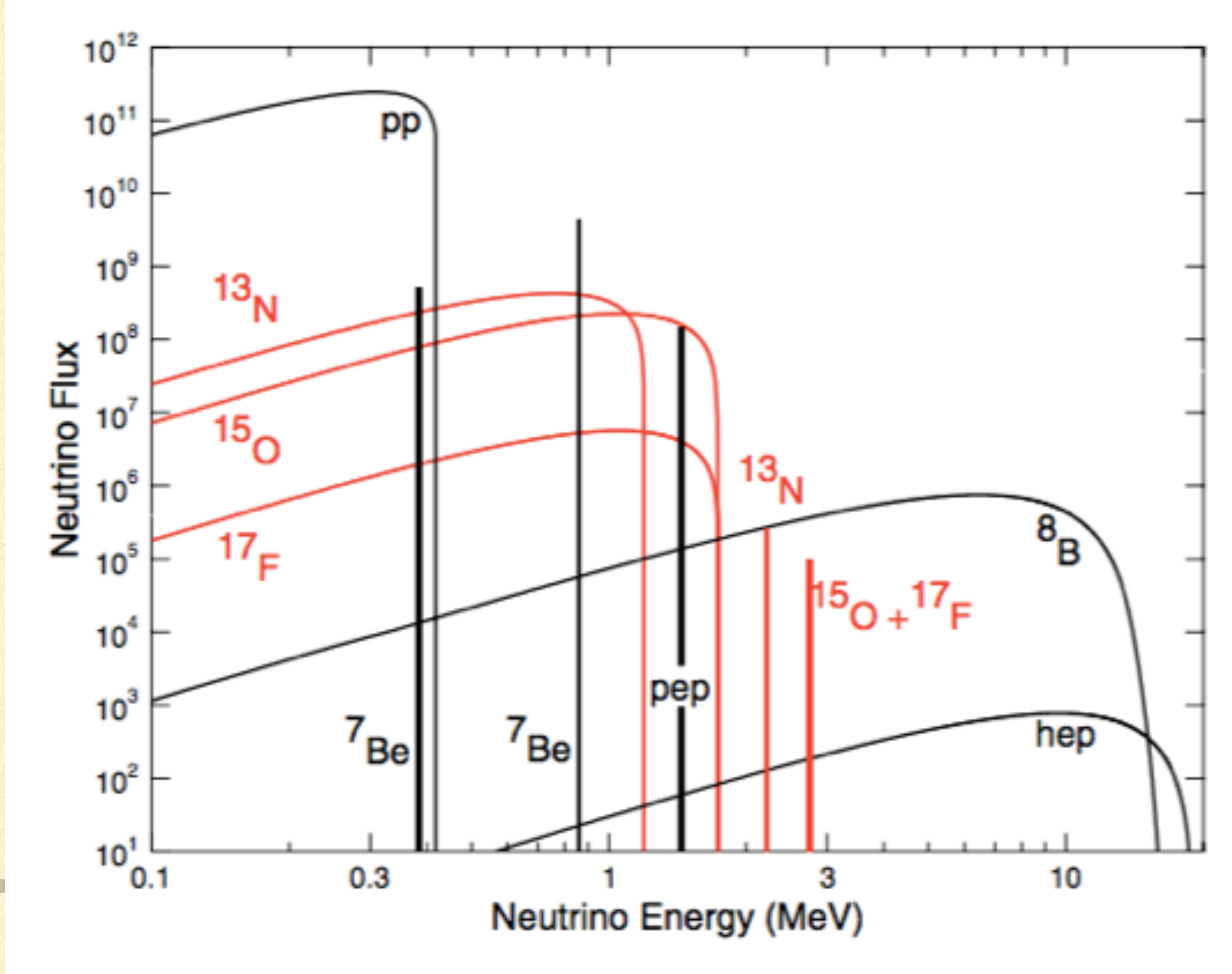

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- SFI: S(0)=19^{+4}_{-2} eV b

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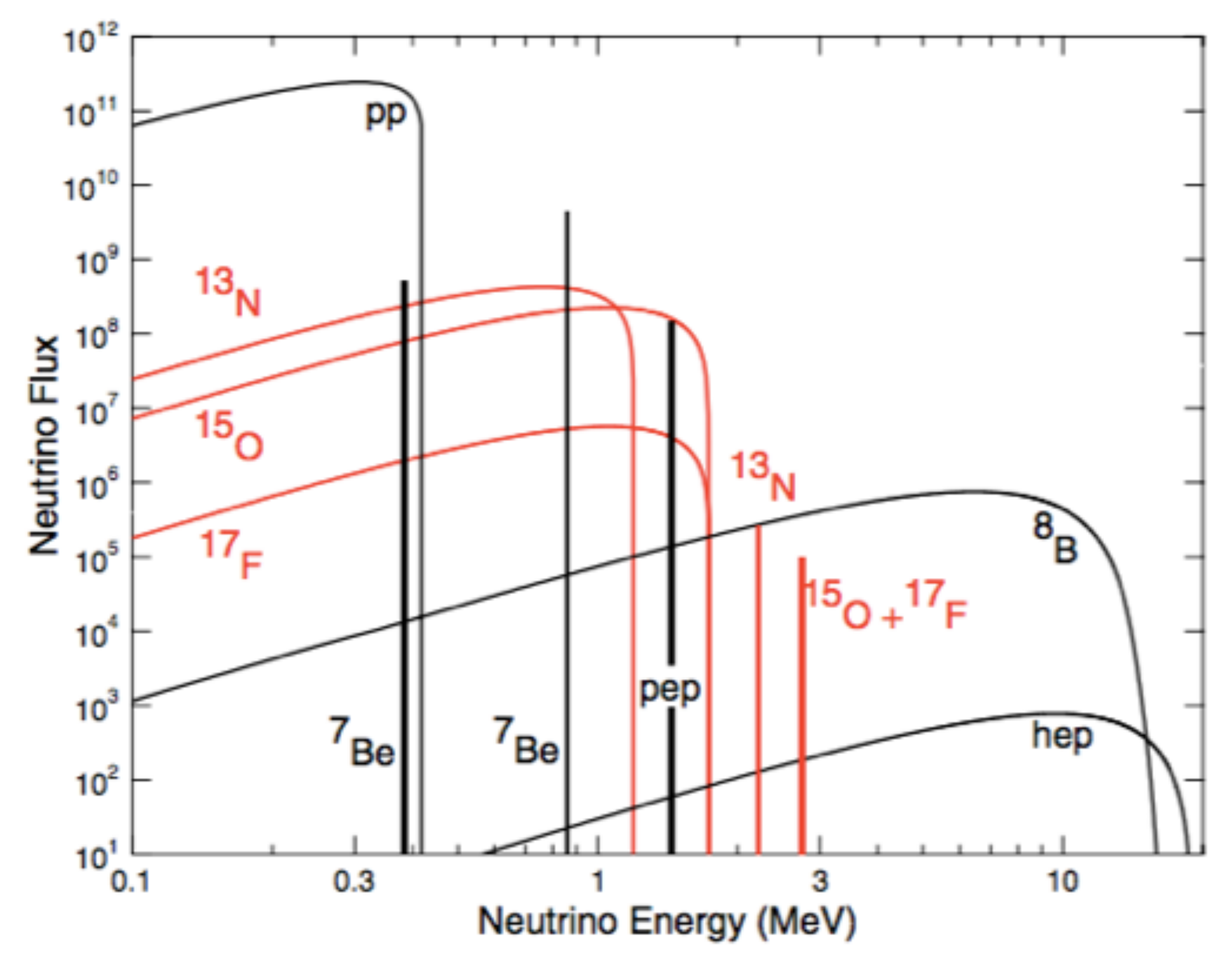
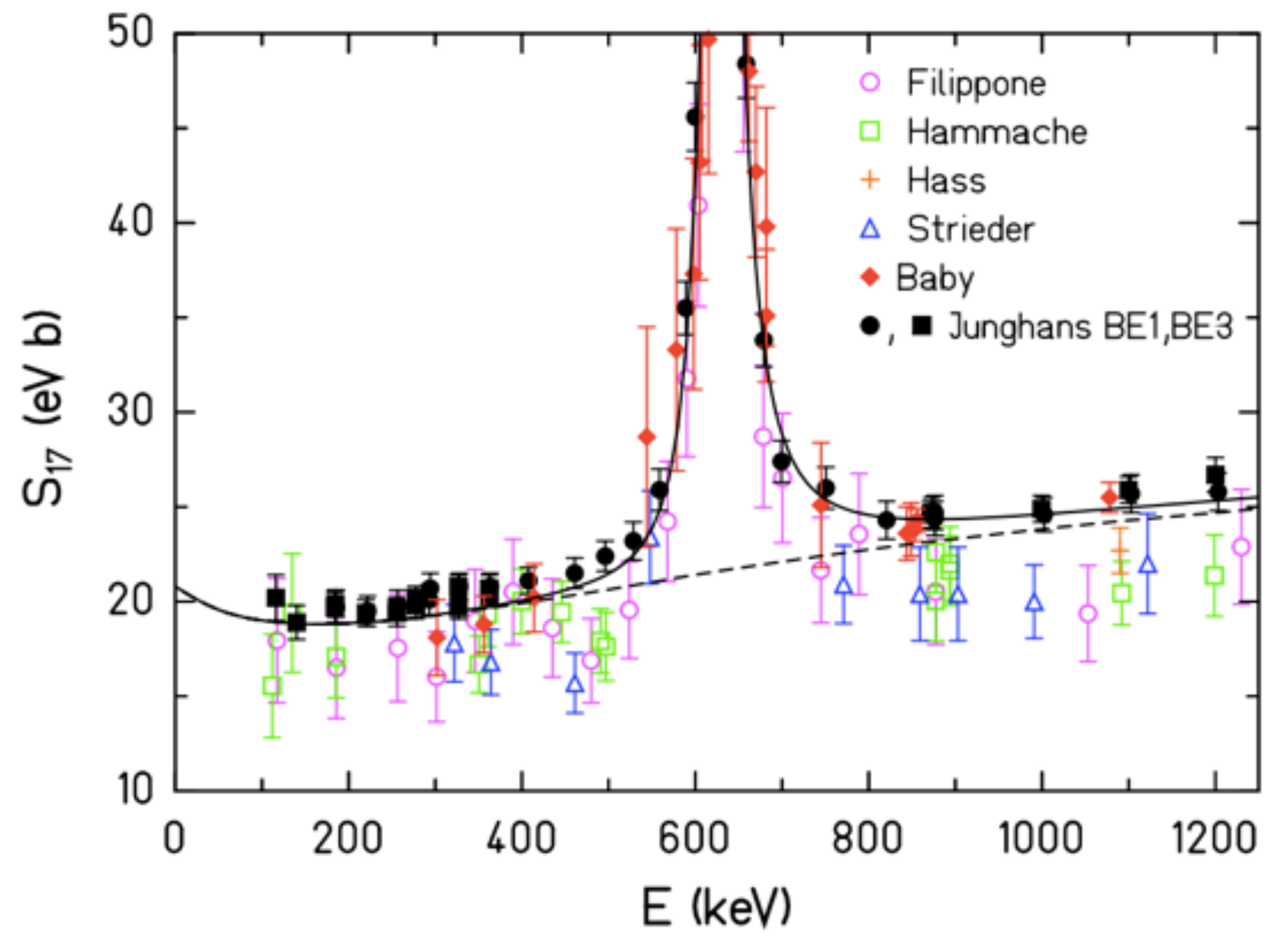


Status as in “Solar fusion II”

- Energies of relevance≈20 keV
- There dominated by ^{7}Be-p separations ~10s of fm
- Below narrow 1^+ resonance proceeds via s- and d-wave direct capture
- Energy dependence due to interplay of bound-state properties, Coulomb, strong ISI



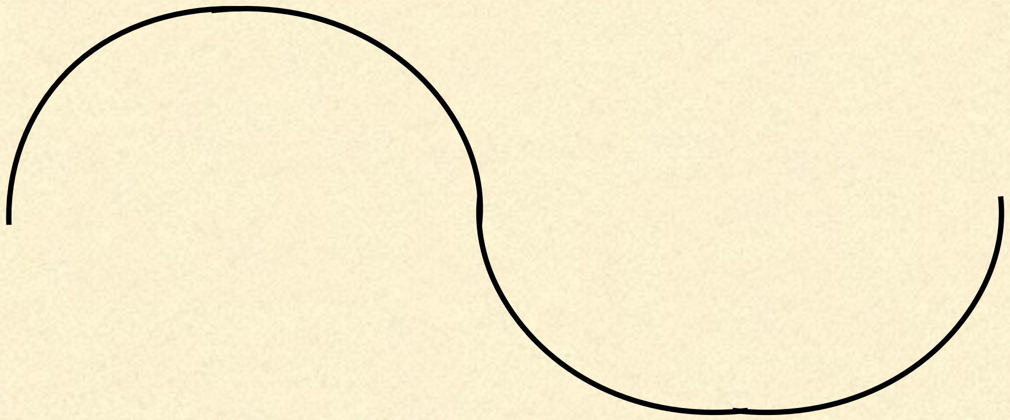
- SF II central value used energy-dependence from Descouvemont’s *ab initio* eight-body calculation. Errors from consideration of energy-dependence in a variety of “reasonable models”

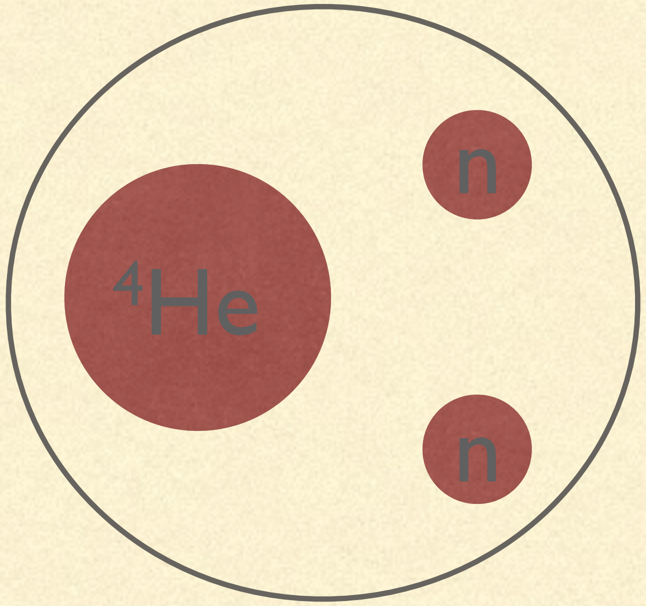
Outline

- Why all the fuss about ^{7}Be(p,γ)^{8}B?
- Halo EFT: generalities
- p-wave bound states, radiative capture, and the example of ^{7}Li(n,γ)^{8}Li
- ^{7}Be(p,γ)^{8}B in Halo EFT
- Halo EFT + data + a Bayesian analysis→a better extrapolation
- Outlook

Halo EFT

$\lambda \gg R_{core}$; $\lambda \lesssim R_{halo}$





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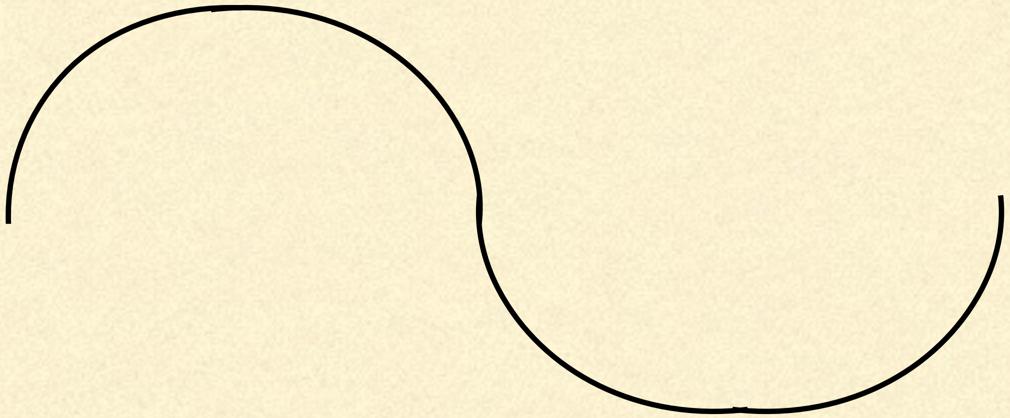
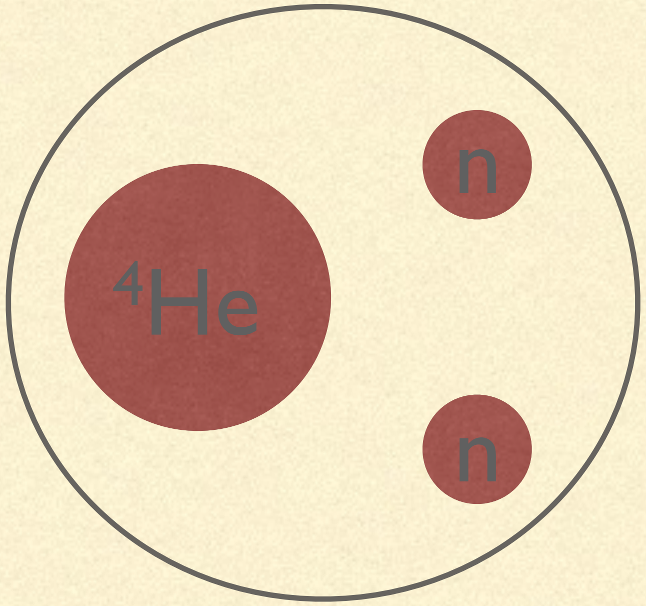
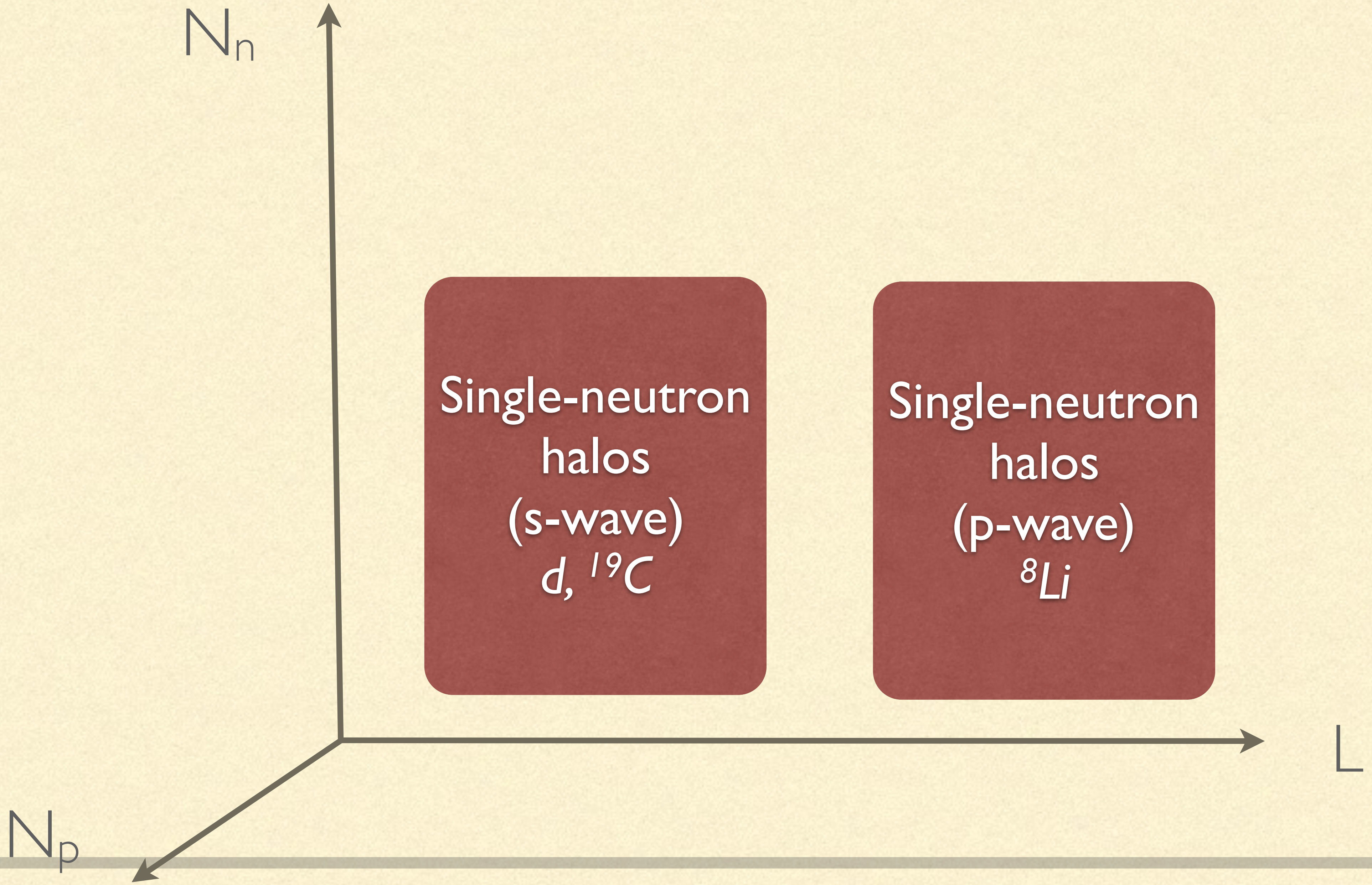




- Define $R_{halo}=\langle r^2 \rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2 \rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

The multi-dimensional Halo EFT space



Single-neutron halos (s-wave)
d, ^{19}C

Single-neutron halos (p-wave)
^{8}Li

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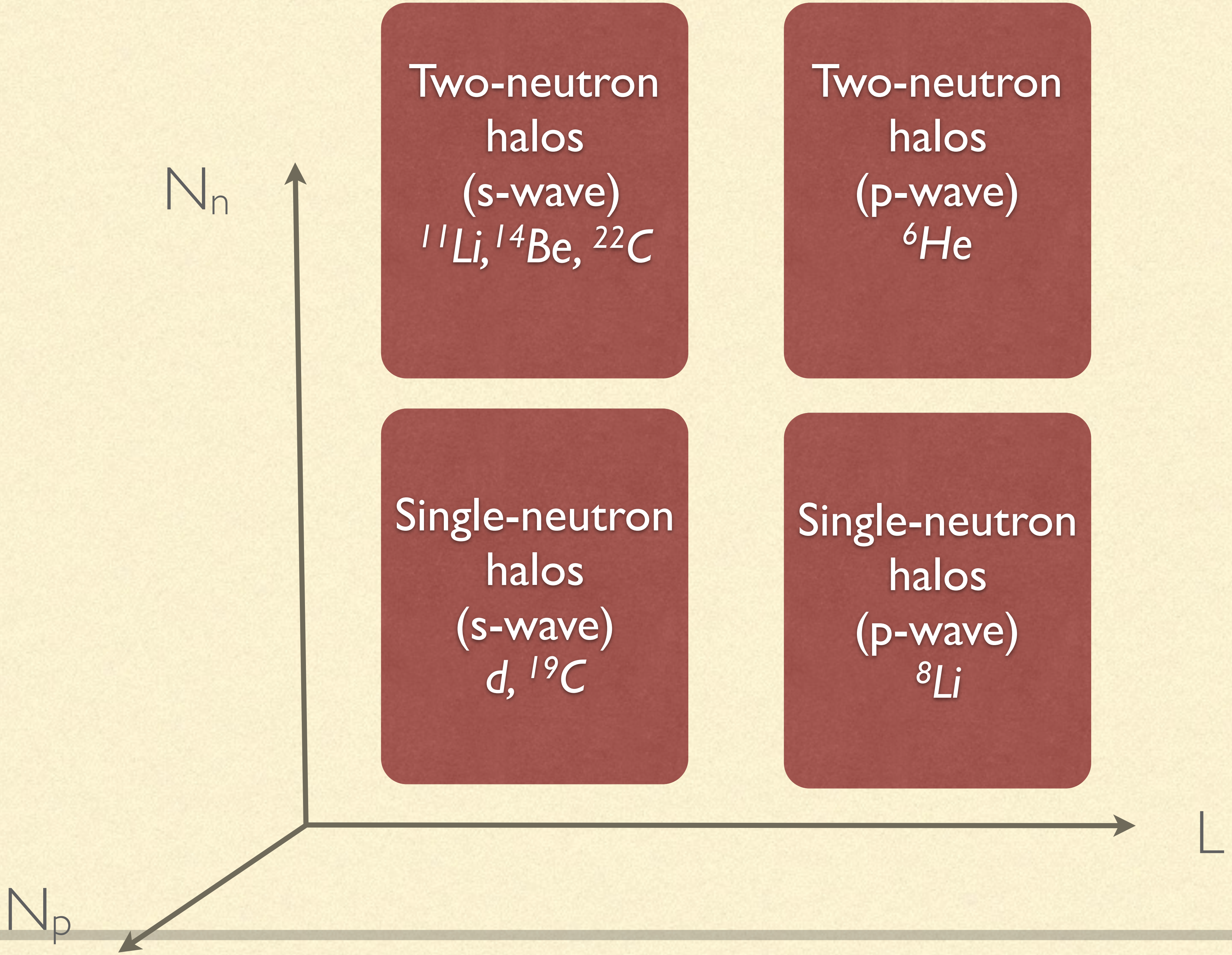

Two-neutron halos (s-wave) ^{11}Li, ^{14}Be, ^{22}C

Two-neutron halos (p-wave) ^{6}He

Single-neutron halos (s-wave) d, ^{19}C

Single-neutron halos (p-wave) ^{8}Li

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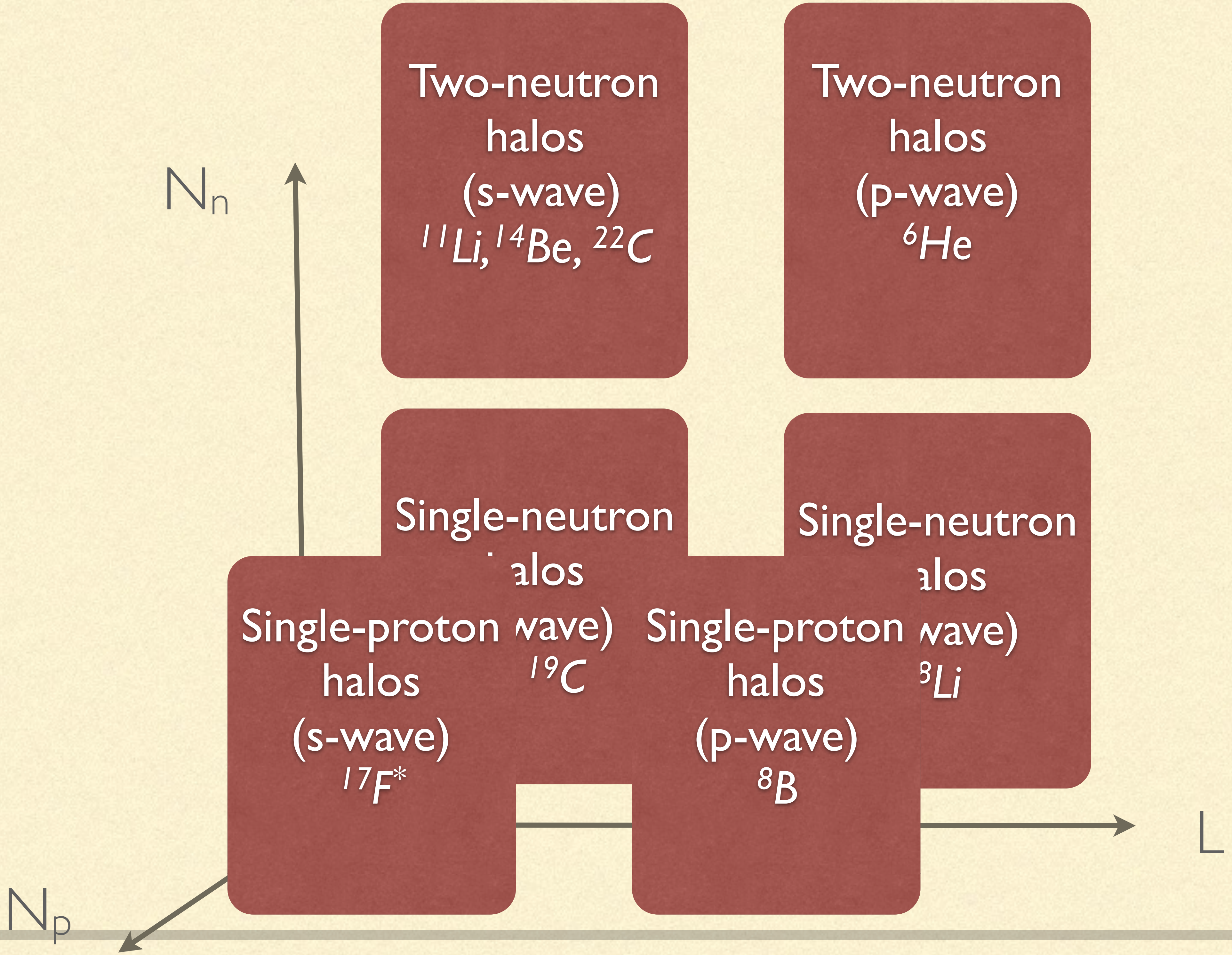

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Single-neutron
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wave)
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Single-proton halos (s-wave)
$^{17}F^*$

Single-proton halos (p-wave)
^{8}B

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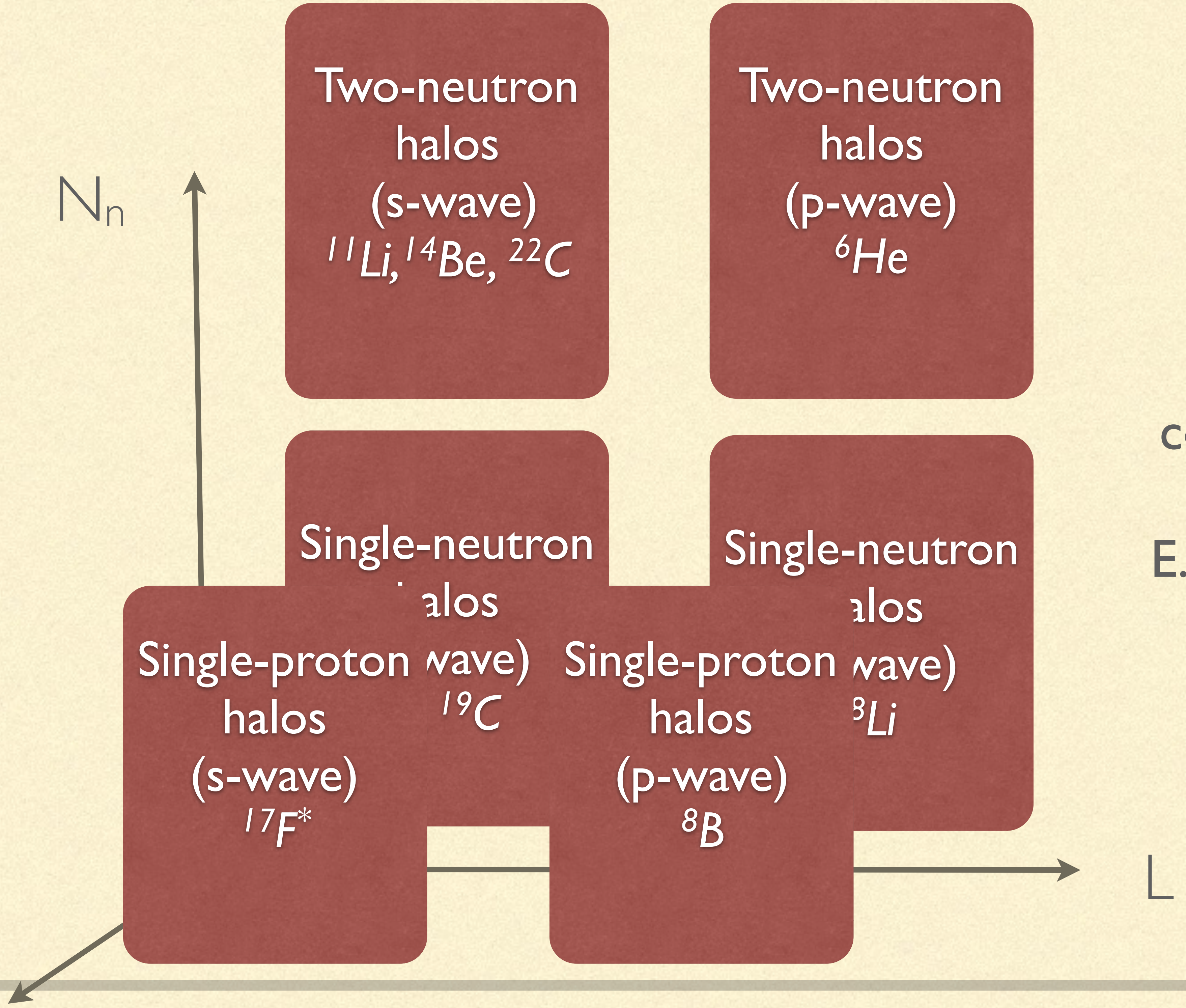

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Single-proton halos (s-wave)
$^{17}F^*$

Single-proton halos (p-wave)
^{8}B

Plus complementary direction: N_α
E.g. ^{9}Be, $^{12}C^*$, etc.

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It does:

- Connect structure and reactions, including in multi-nucleon halos
- Collect information from different theories/experiments in one calculation
- Treat same physics as cluster models, in a systematically improvable way
- Provide information on inter-dependencies of low-energy observables, including along the core + n, core + 2n, core + 3n, etc. chain

Lagrangian: shallow S- and P-states

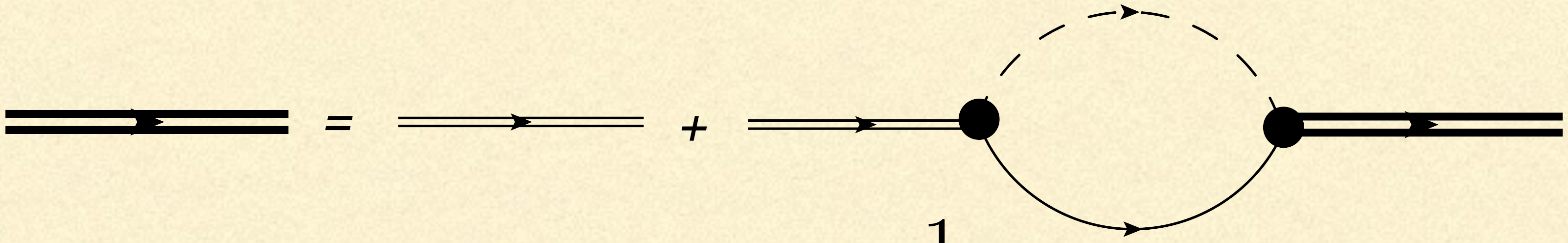
$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \; i\overleftrightarrow{\nabla}_j \; c) + (c^\dagger \; i\overleftrightarrow{\nabla}_j \; n^\dagger) \pi_j \right] \\
& - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[\pi_j^\dagger \; i\overrightarrow{\nabla}_j \; (nc) - \; i\overleftrightarrow{\nabla}_j \; (n^\dagger c^\dagger) \pi_j \right] + \ldots \, ,
\end{aligned}
$$

- c, n: “core”, “neutron” fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Dyson equation for (cn)-system propagator



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

- Reproduces ERE. But here (cf. s waves) cannot take r_1=0 at LO

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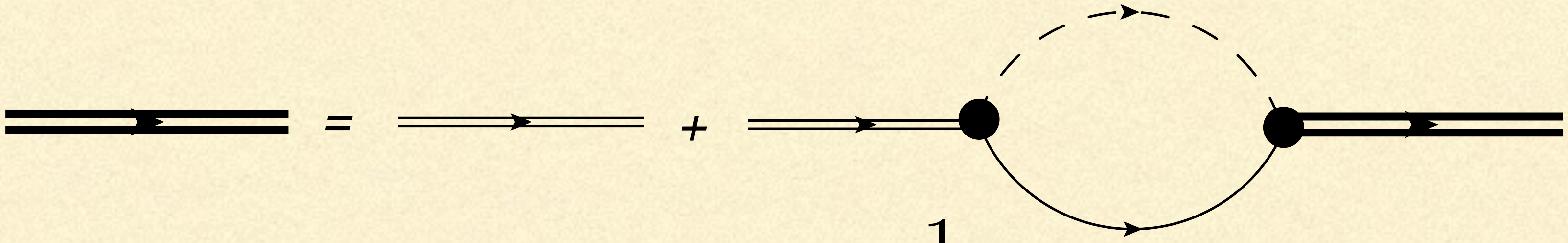

$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

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- If a_l > 0 then pole is at $k=i\gamma_l$ with $B_l=\gamma_l^2/(2m_R)$:

$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1 + 3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

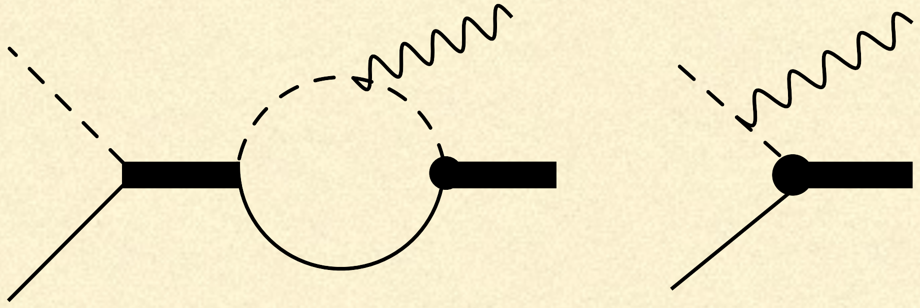
p-wave bound states and capture thereto

Hammer & DP, NPA (2011)

- At LO: p-wave 1n halo described solely by its ANC and binding energy

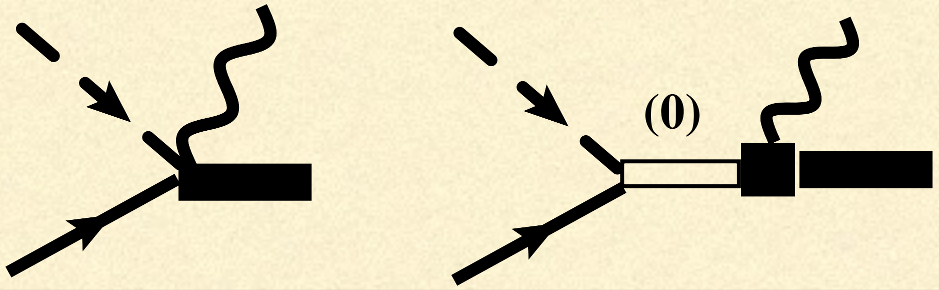
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

- Capture to the p-wave state proceeds via the one-body E1 operator: "external direct capture"



$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



Fixing ^{8}Li parameters

- ^{8}Li ground state is 2^+: both $^5P_2$ and $^3P_2$ components
- ^{8}Li first excited state: 1^+, bound by 1.05 MeV

Zhang, Nollett, Phillips, PRC (2014)

c.f. Rupak, Higa, PRL 106, 222501 (2011);
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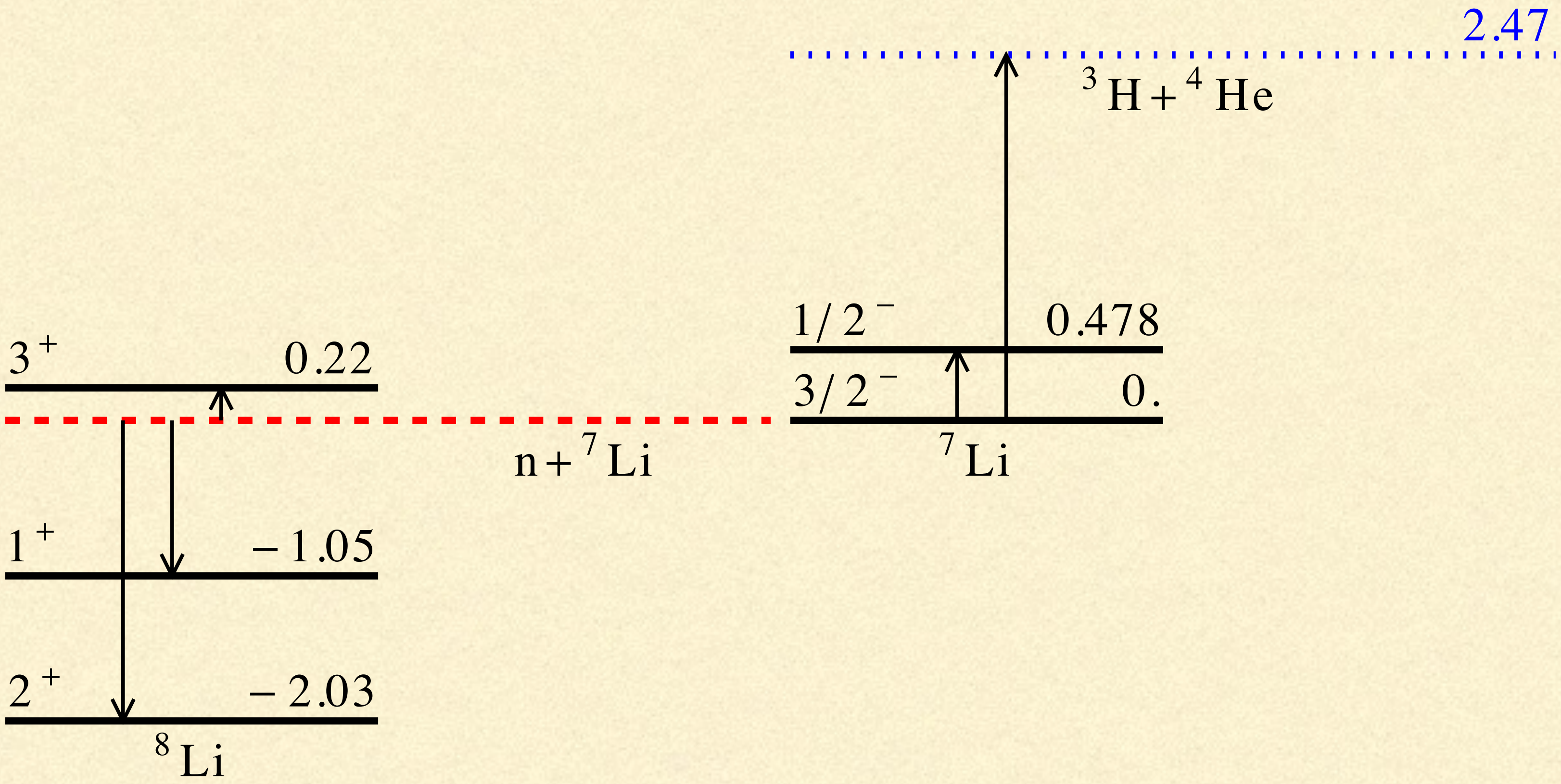

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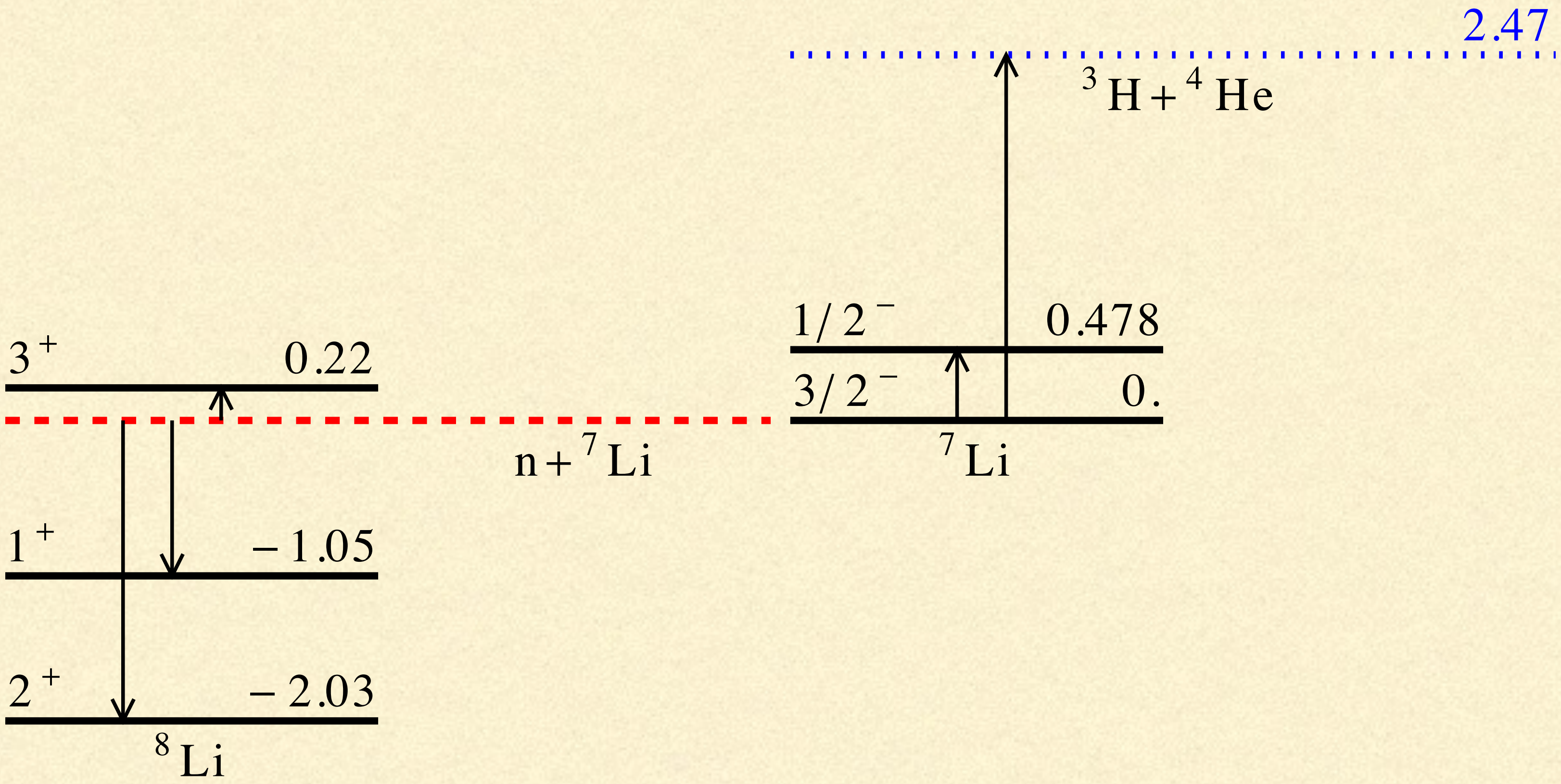

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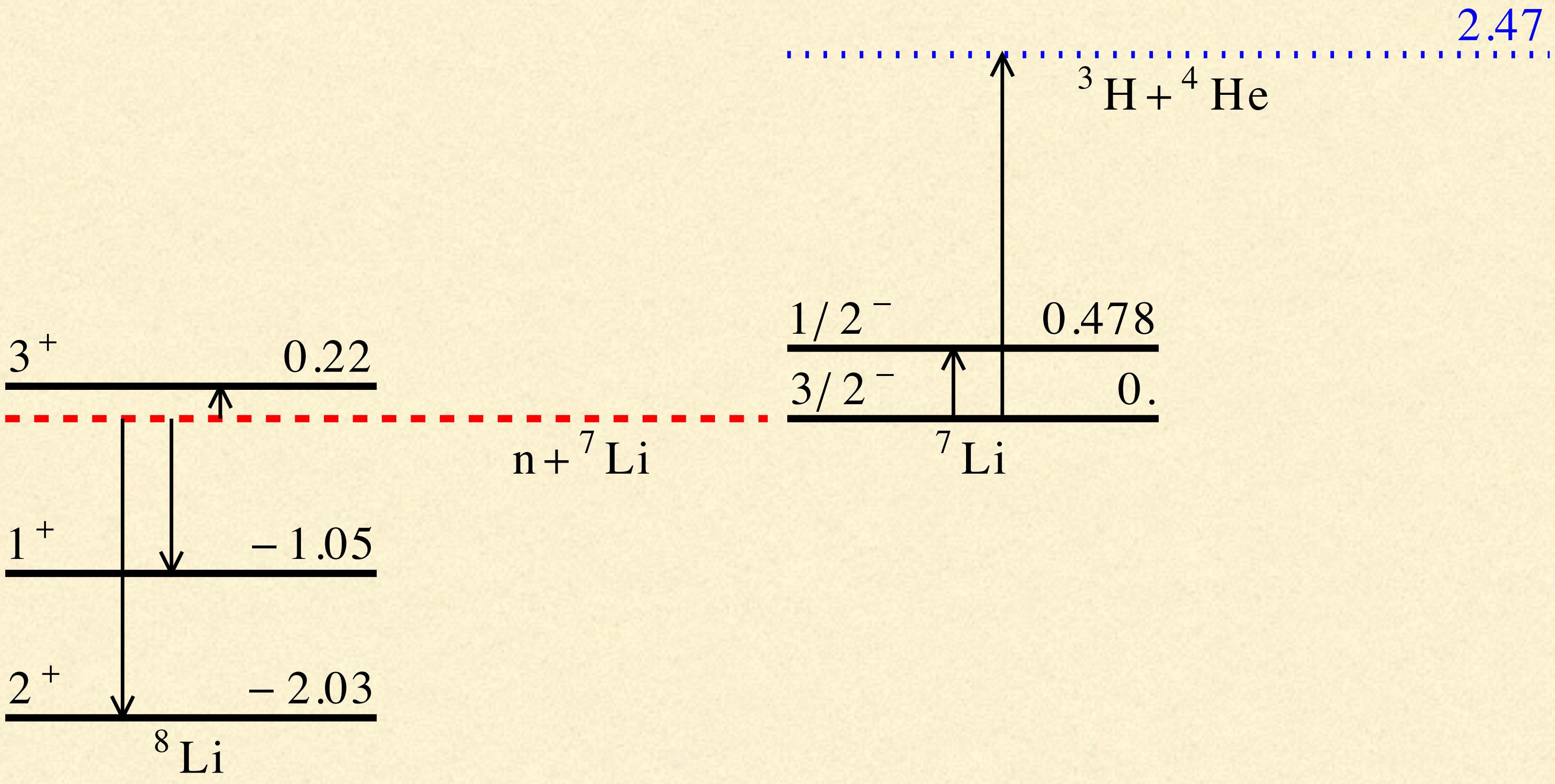

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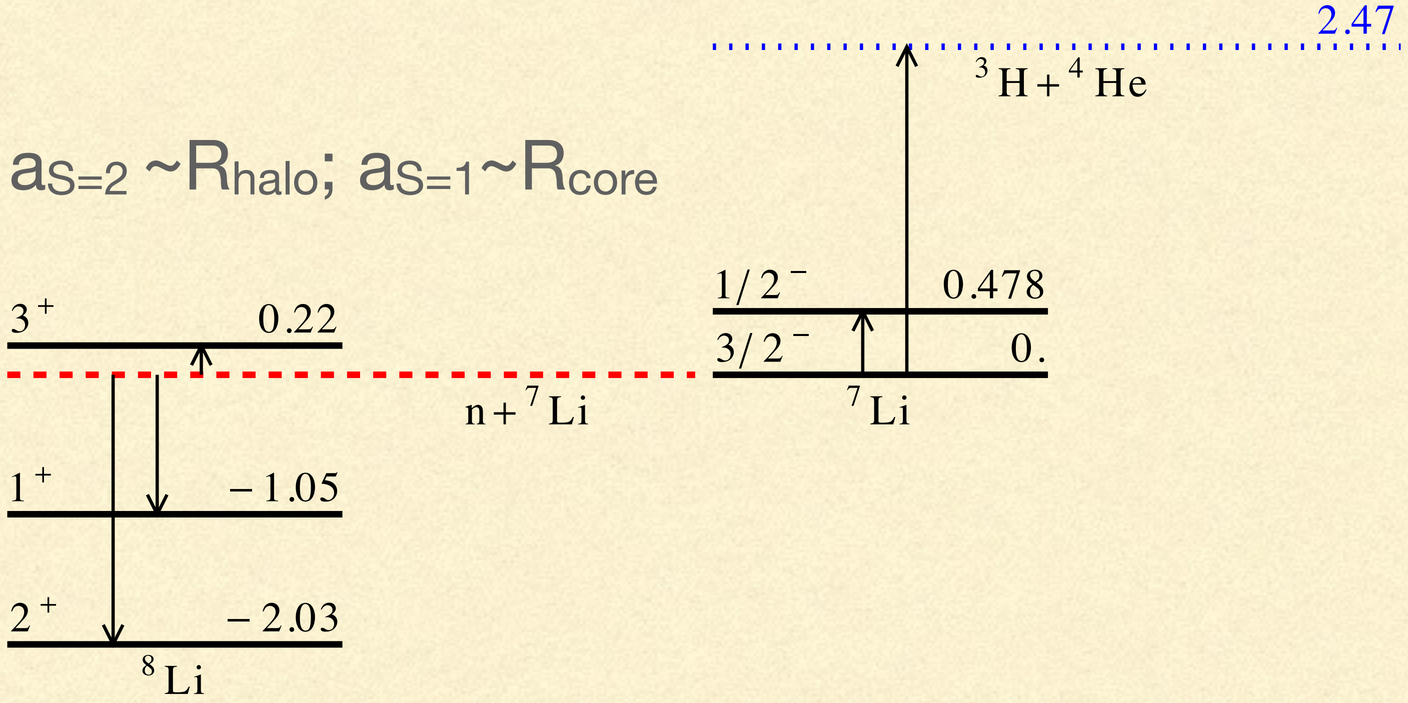
	$A_{(3P2)}$	$A_{(5P2)}$	$A_{(3P2*)}$	$A_{(3P1)}^*$	$A_{(5P1)}^*$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)	0.220(6)	0.197(5)
Trache	-0.284(23)	-0.593(23)		0.187(16)	0.217(13)

^{7}Li + n→^{8}Li + γ_{E1}

- ^{7}Li ground state is 3/2$^-$: S-wave n scattering in $^5S_2$ and $^3S_1$

$a_{S=2}$ ~R_{halo}; $a_{S=1}$~R_{core}



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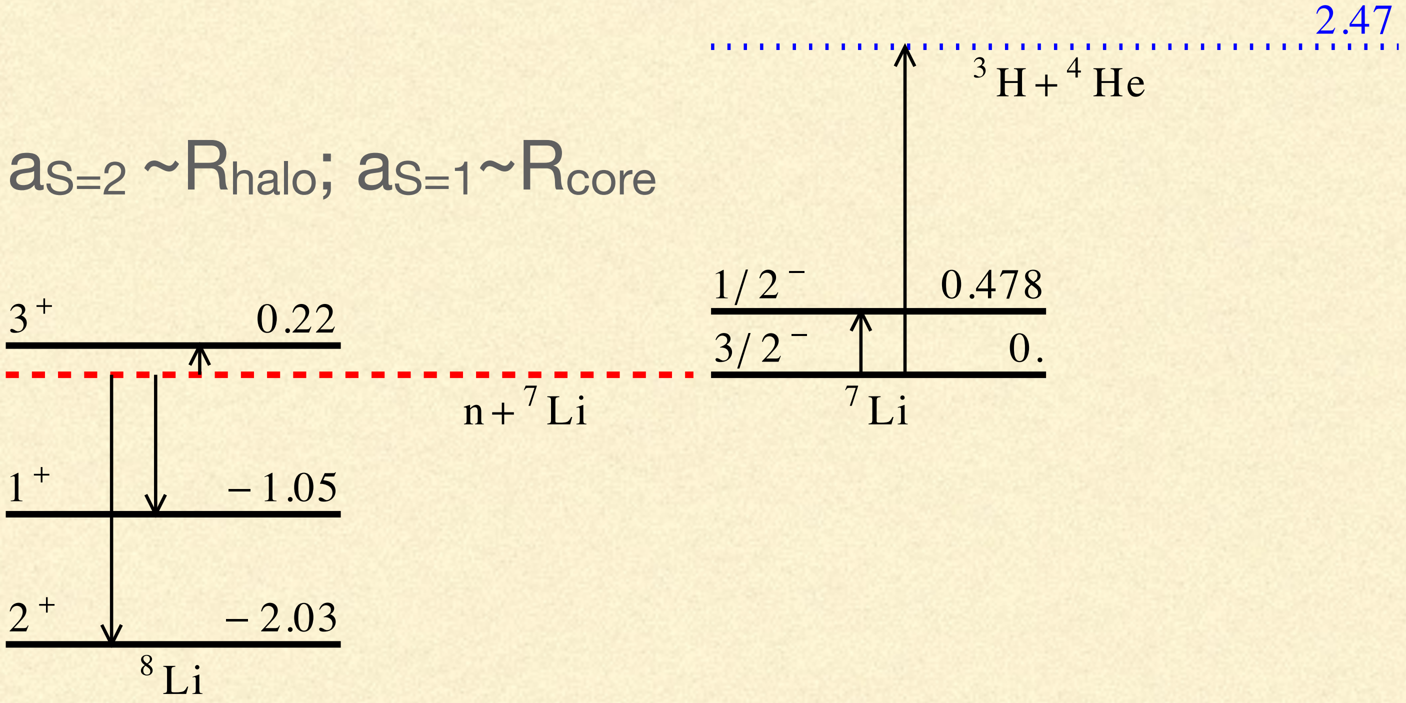

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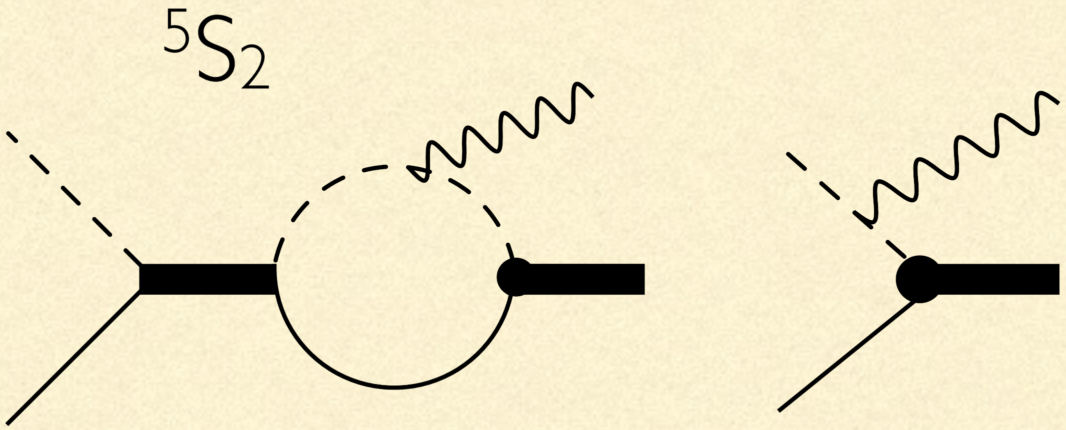

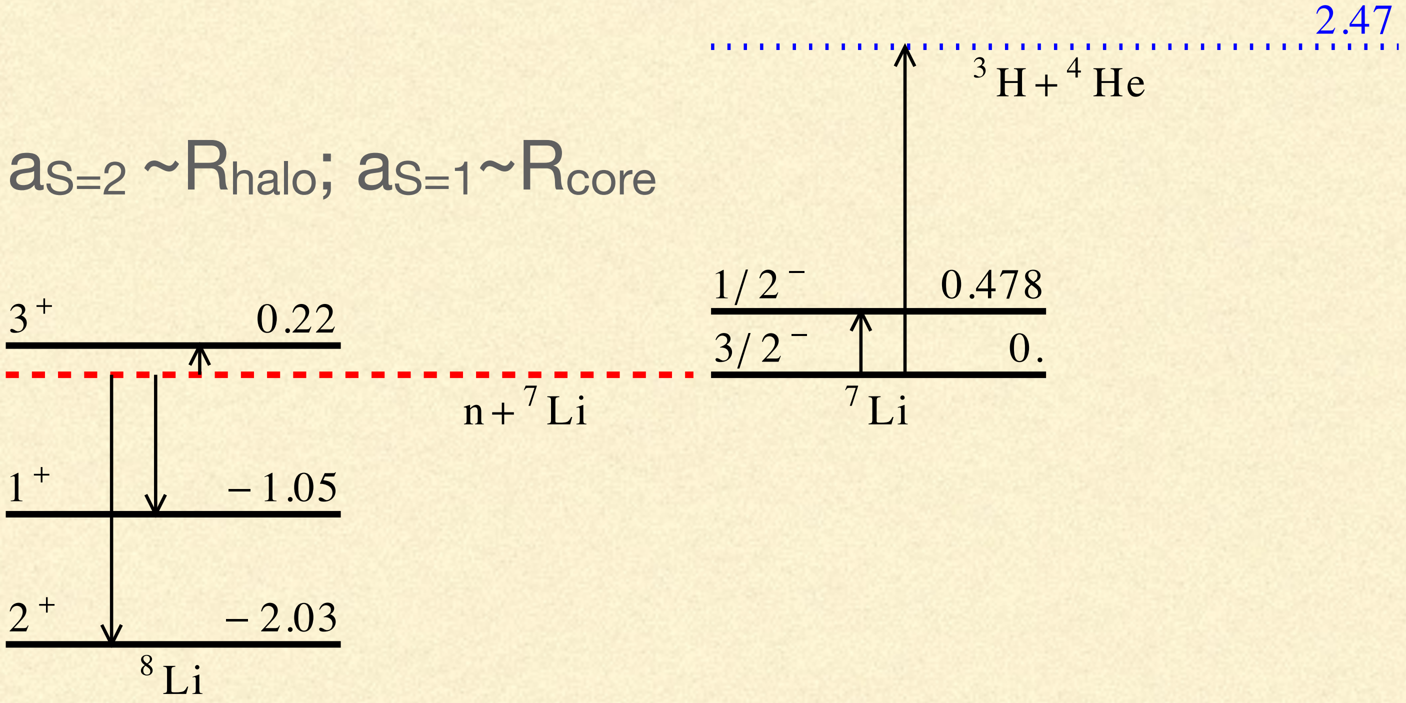

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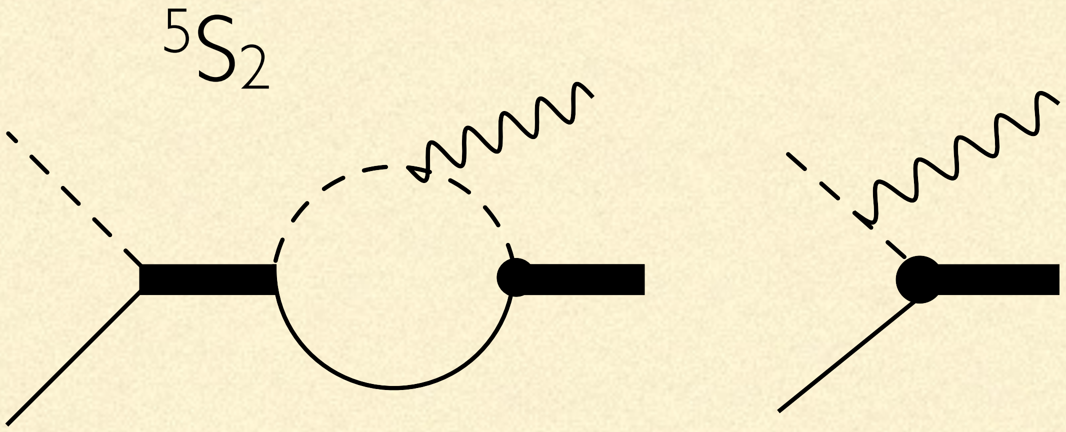
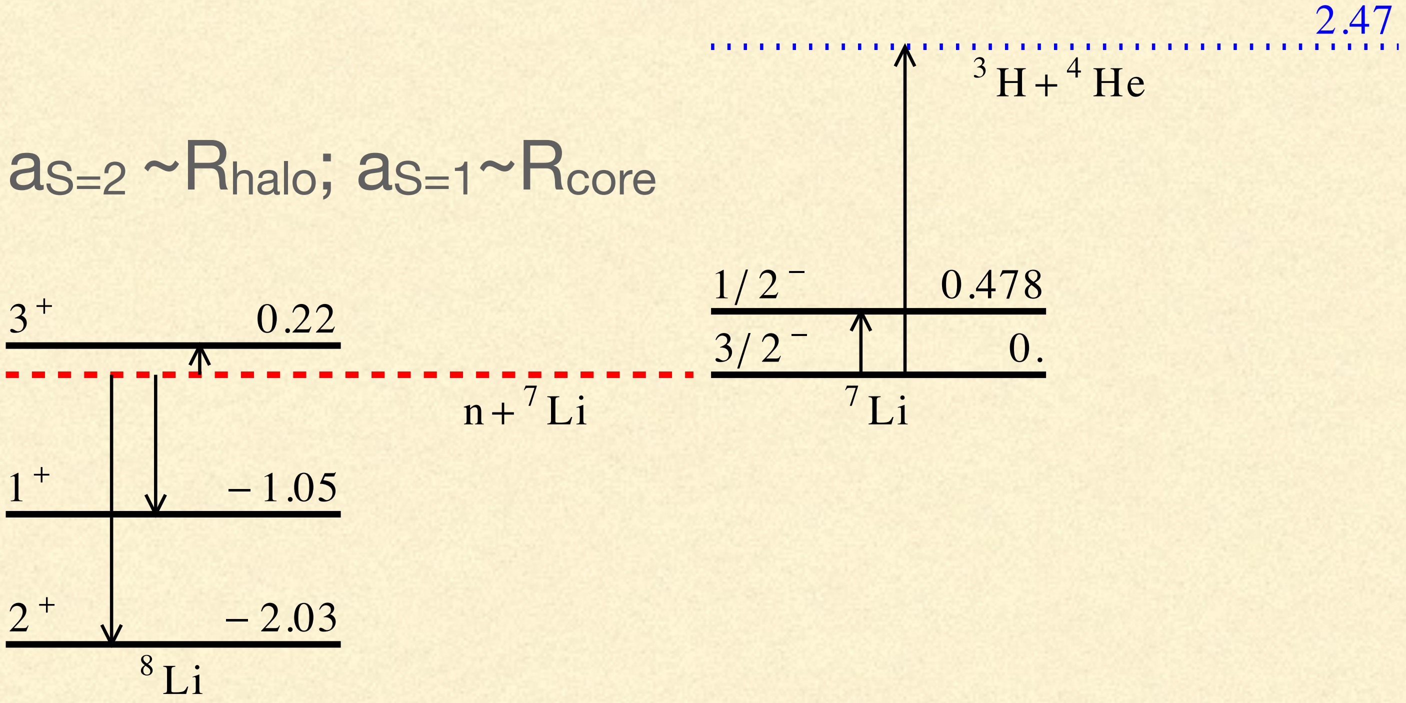




- LO calculation: S=2 (with ISI) and S=1 into P-wave bound state

$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r);$$

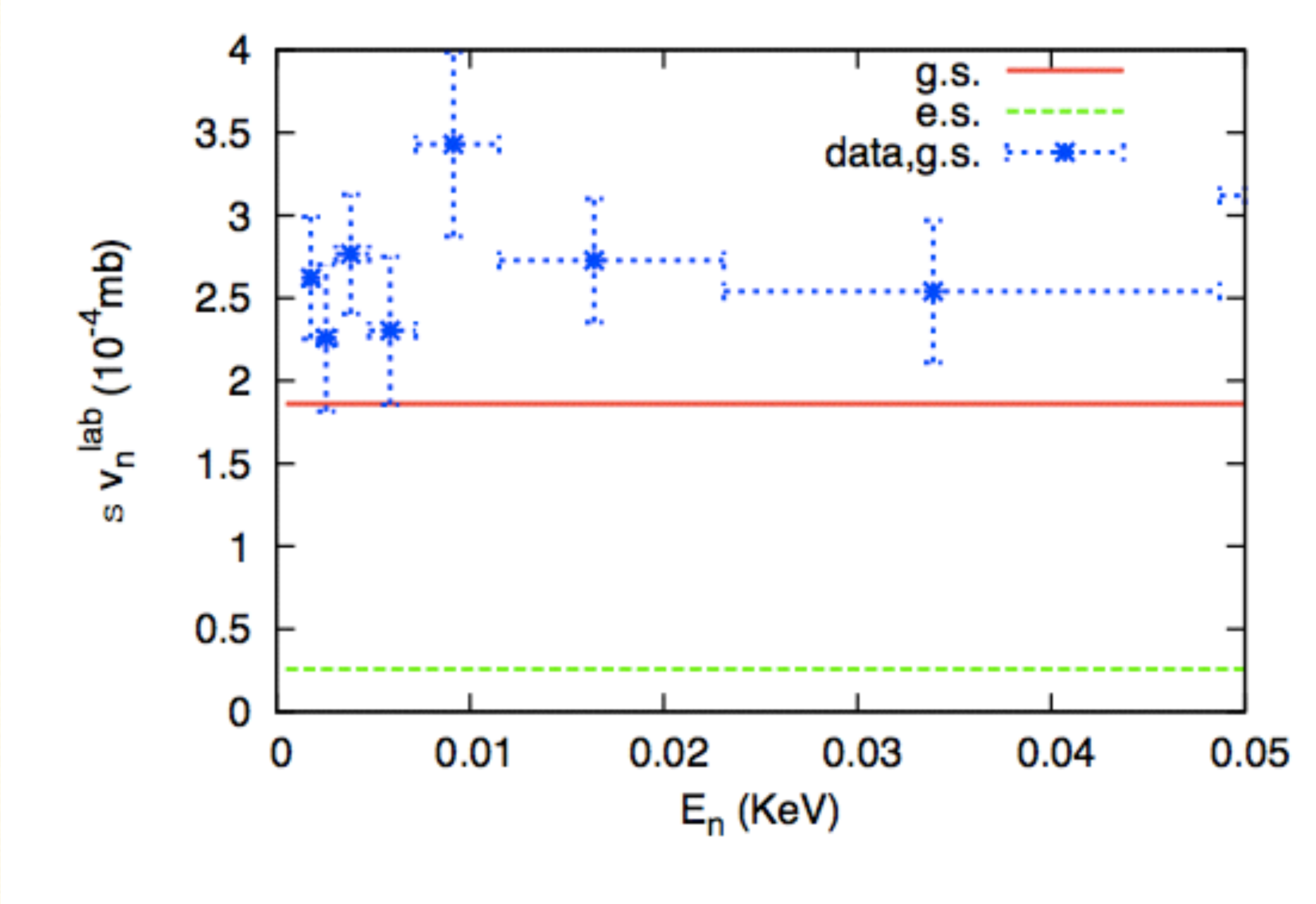
$$u_0(r) = 1 - \frac{r}{a}; u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right)$$

LO results for ^{7}Li + n→^{8}Li + γ_{E1}

Analysis: Zhang, Nollett, Phillips, PRC (2014)

Data: Barker (1996), cf. Nagai et al. (2005)

LO results for ^{7}Li + n→^{8}Li + γ_{E1}



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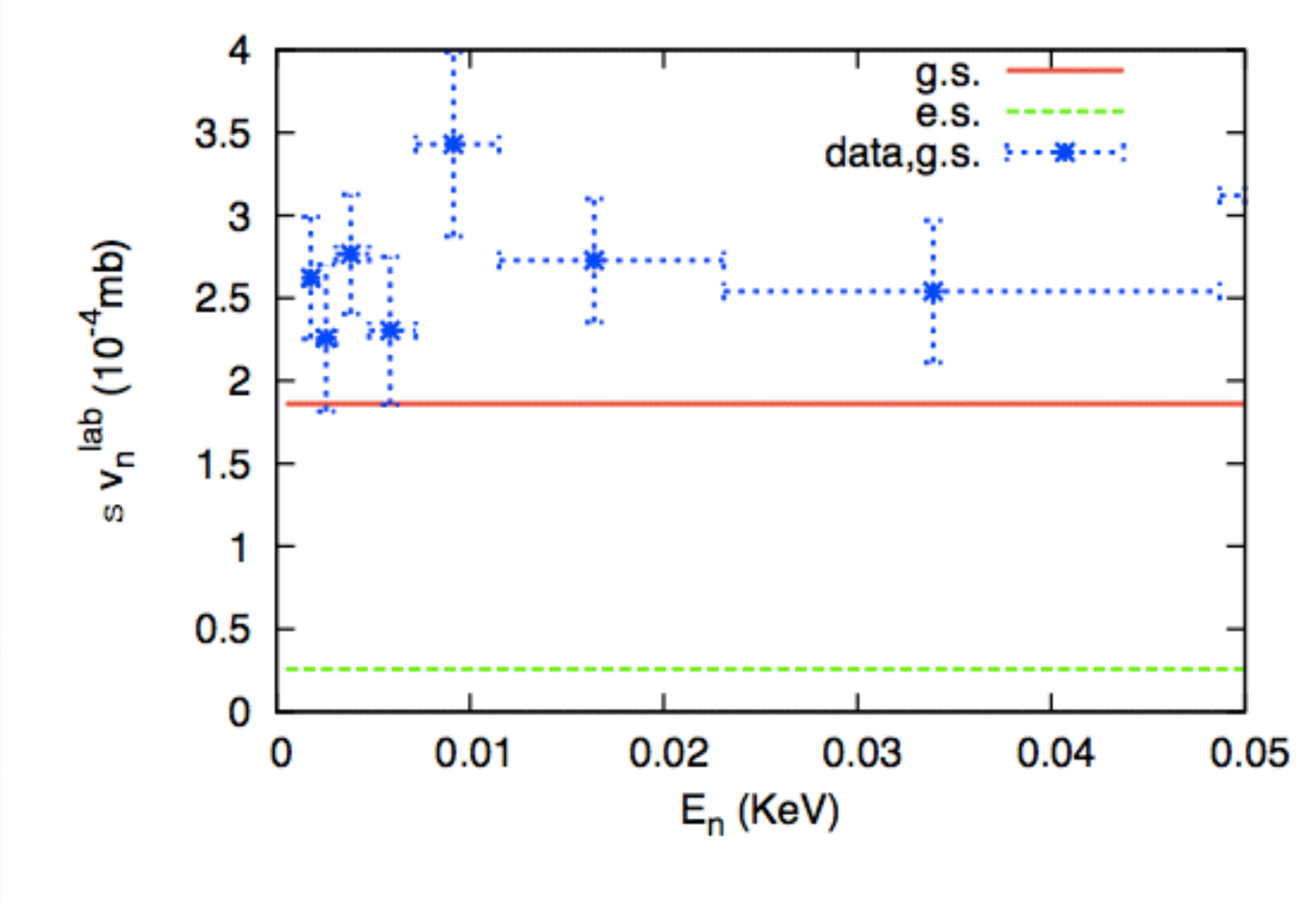

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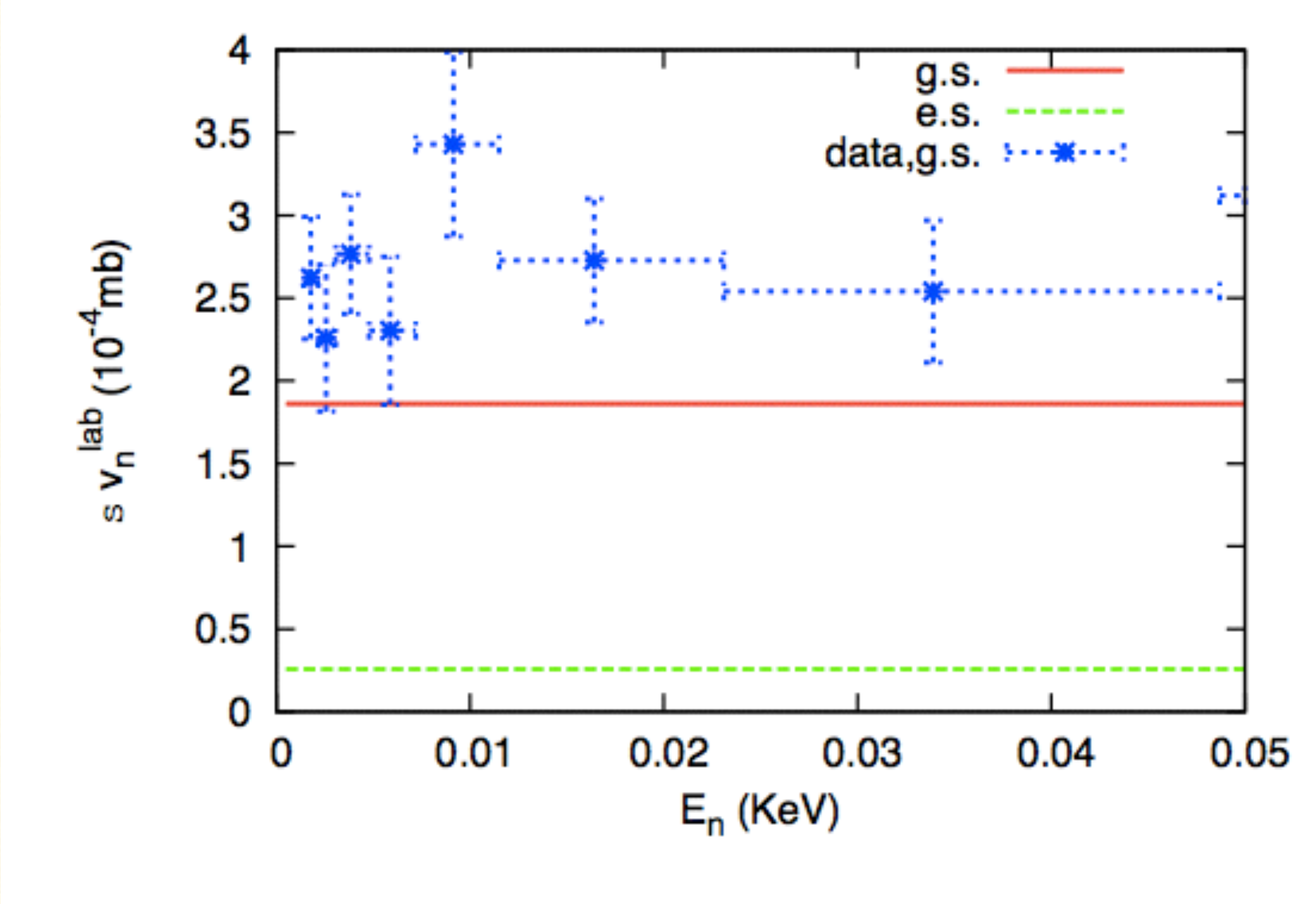

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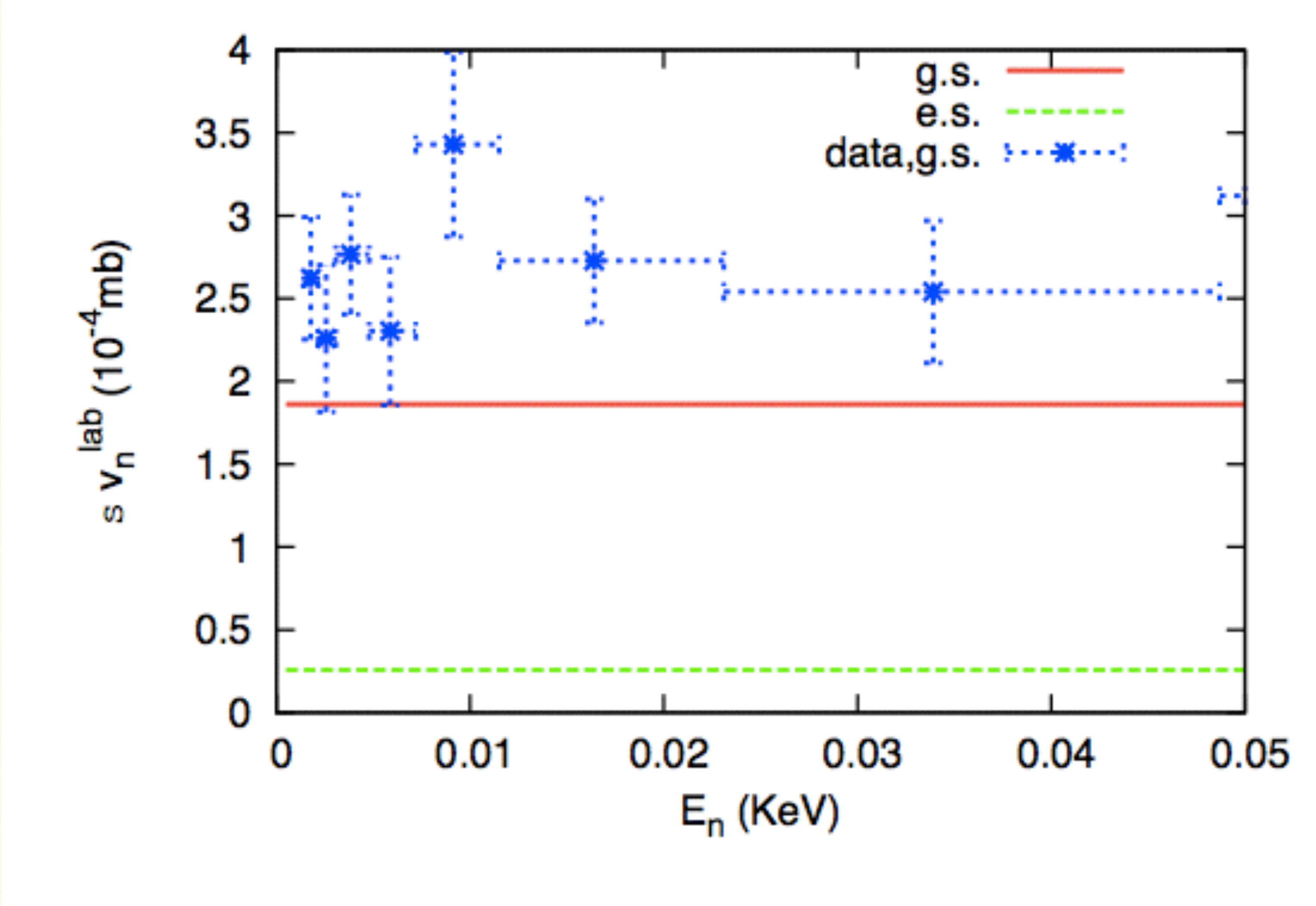

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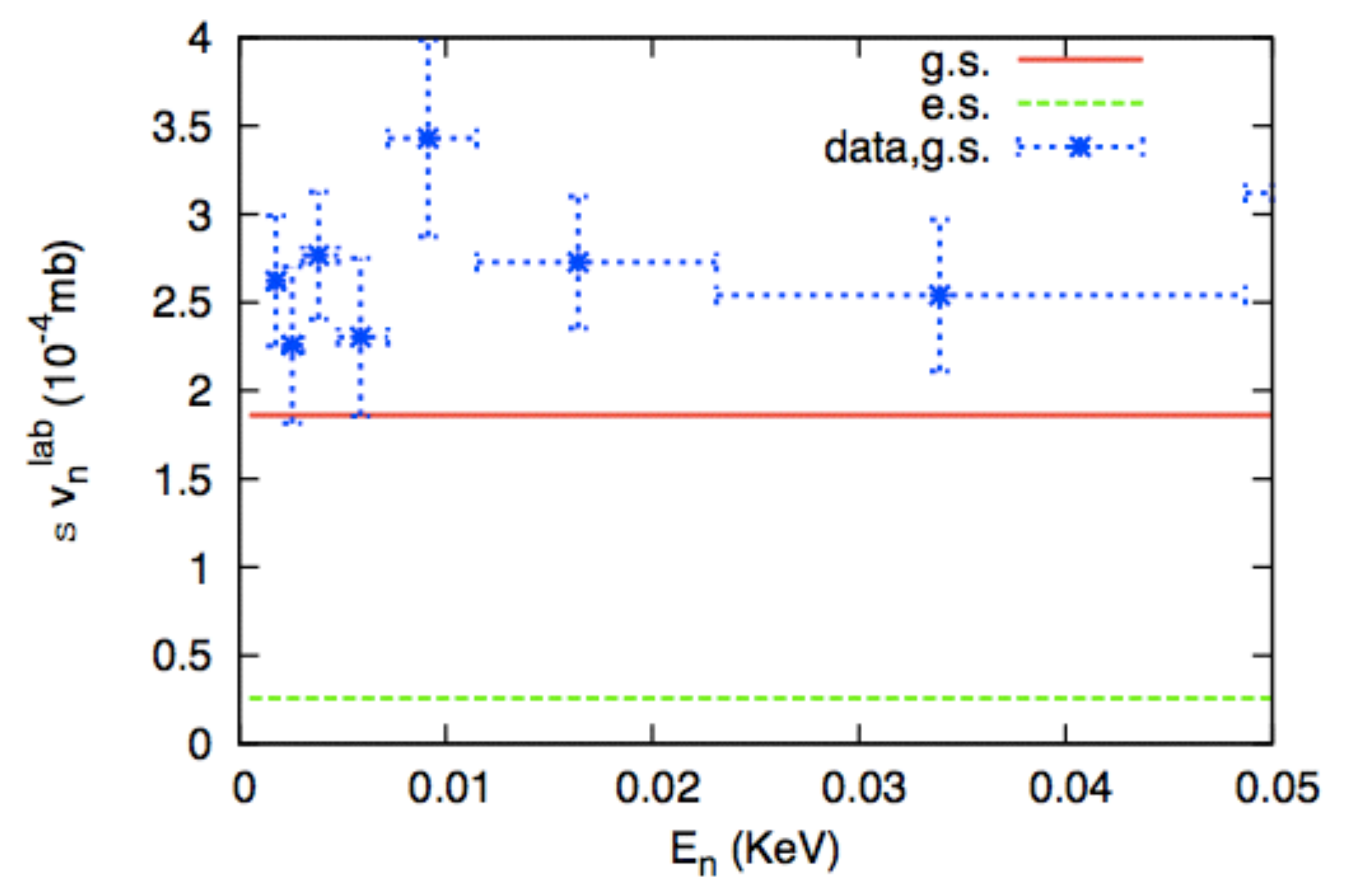

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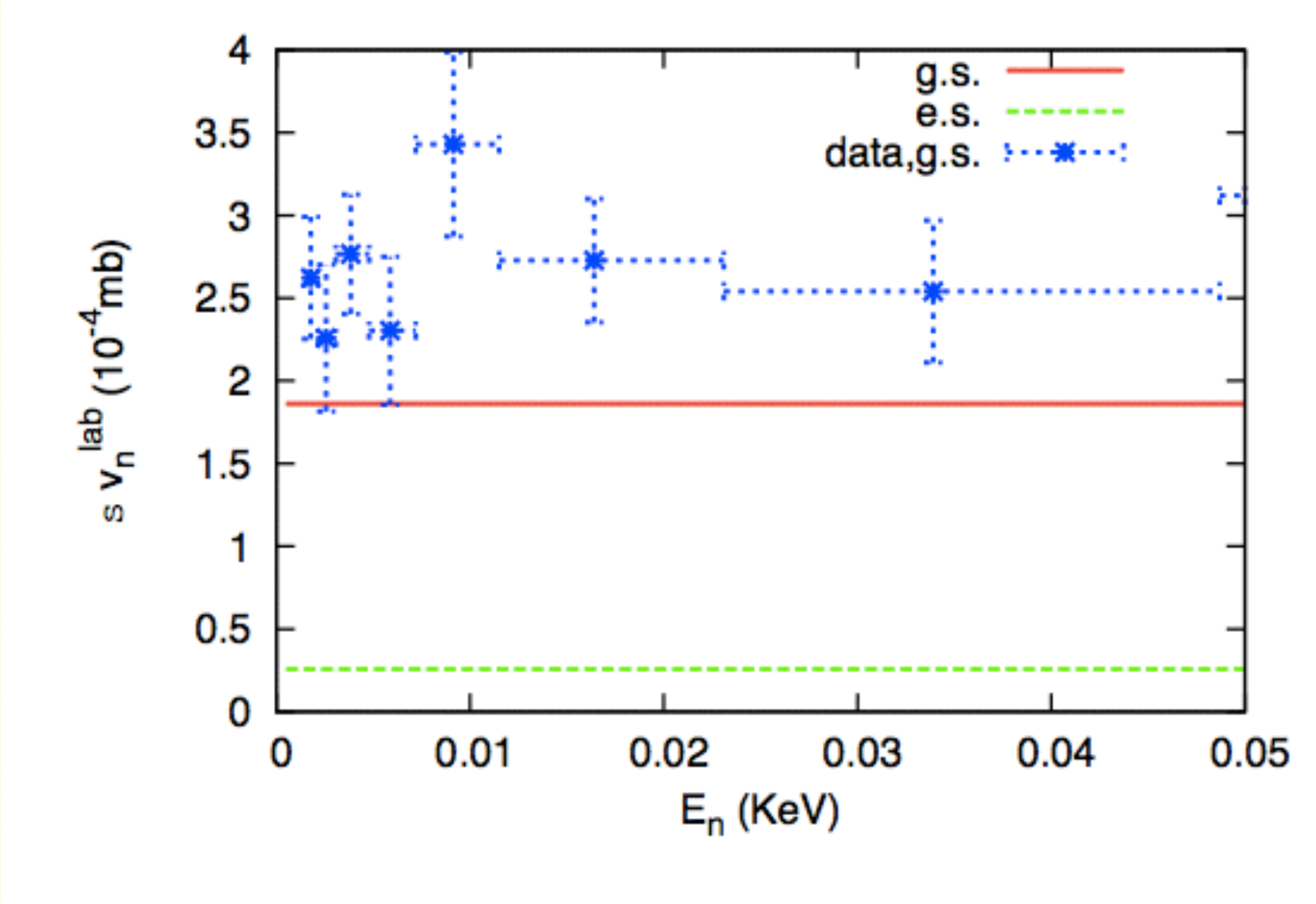

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Dynamics **predicted** through *ab initio* input

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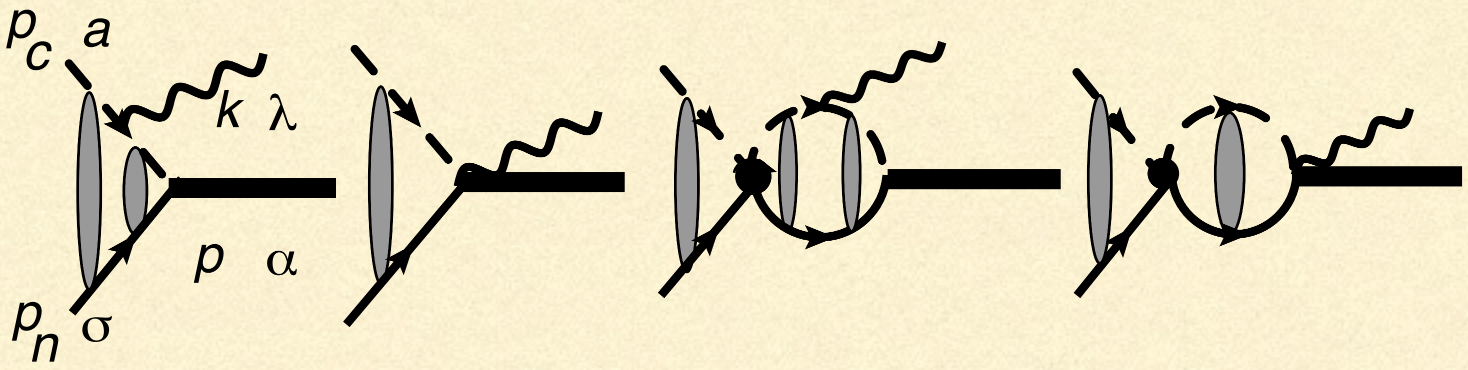

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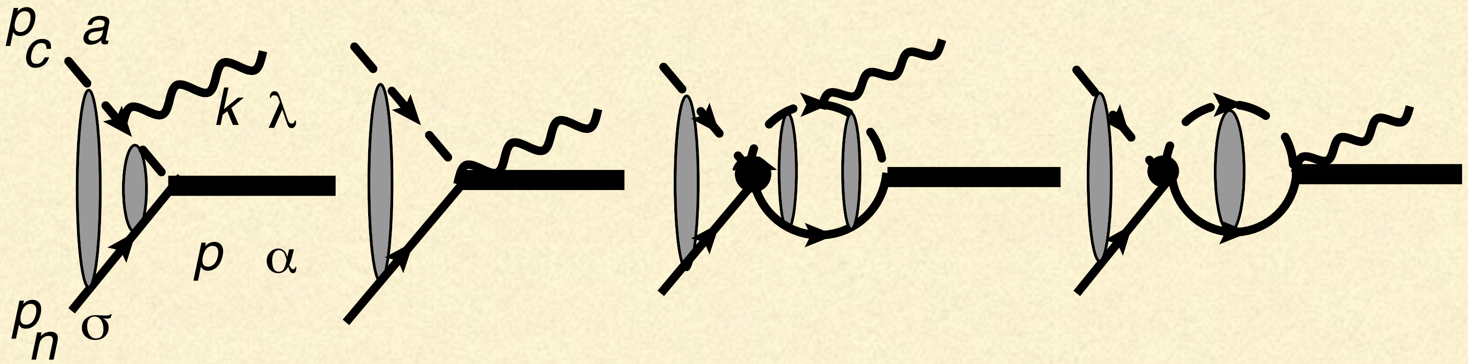

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- No use of isospin mirror symmetry

Isospin symmetry in Halo EFT?

Proton capture details

Zhang, Nollett, Phillips, PRC (2014)

cf. Ryberg, Forssen, Hammer, Platter, EPJA (2014)

$$S(E) = f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\mathrm{EC}} \left(E; \delta_s(E) \right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

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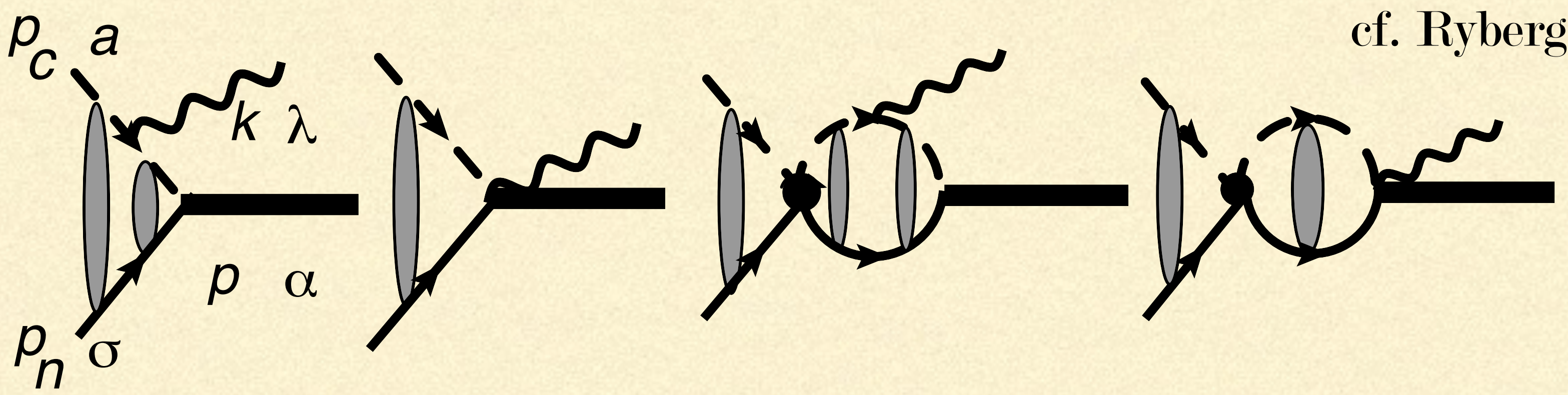

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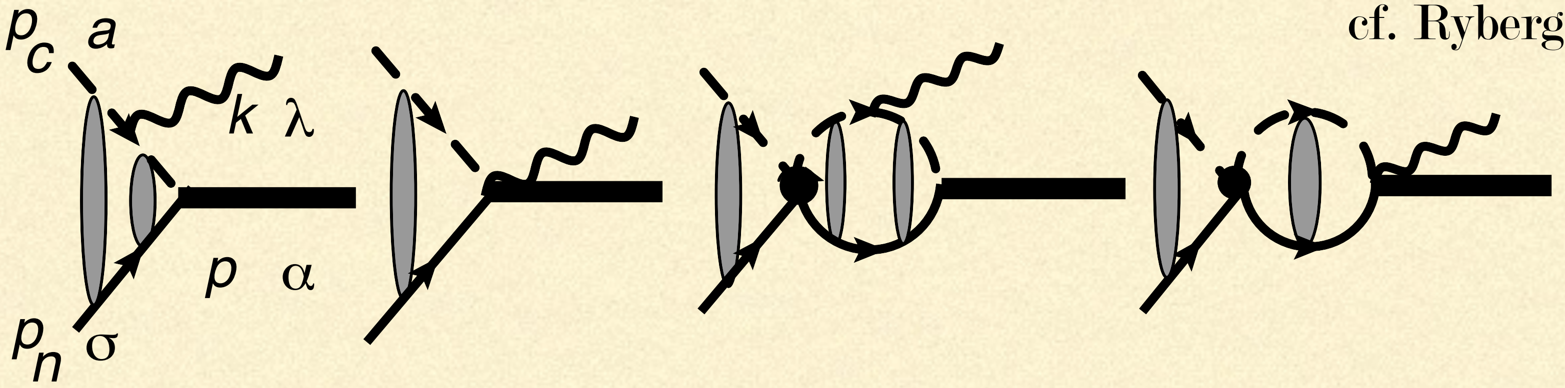

Four parameters at LO

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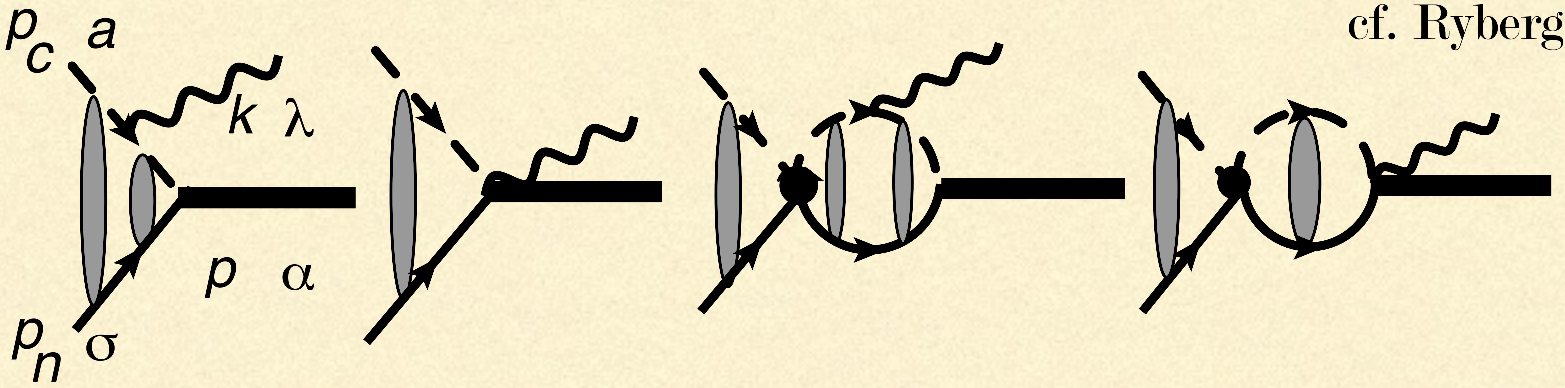

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	$A_{(3P2)}$ ($\mathrm{fm}^{-1/2}$)	$A_{(5P2)}$ ($\mathrm{fm}^{-1/2}$)	$a_{(S=1)}$ (fm)	$a_{(S=2)}$ (fm)
Nollett	-0.315(19)	-0.662(19)		
Navratil	-0.294	-0.65	-5.2	-15.3
Tabacaru	-0.294(45)	-0.615(45)		
Angulo			25(9)	-7(3)

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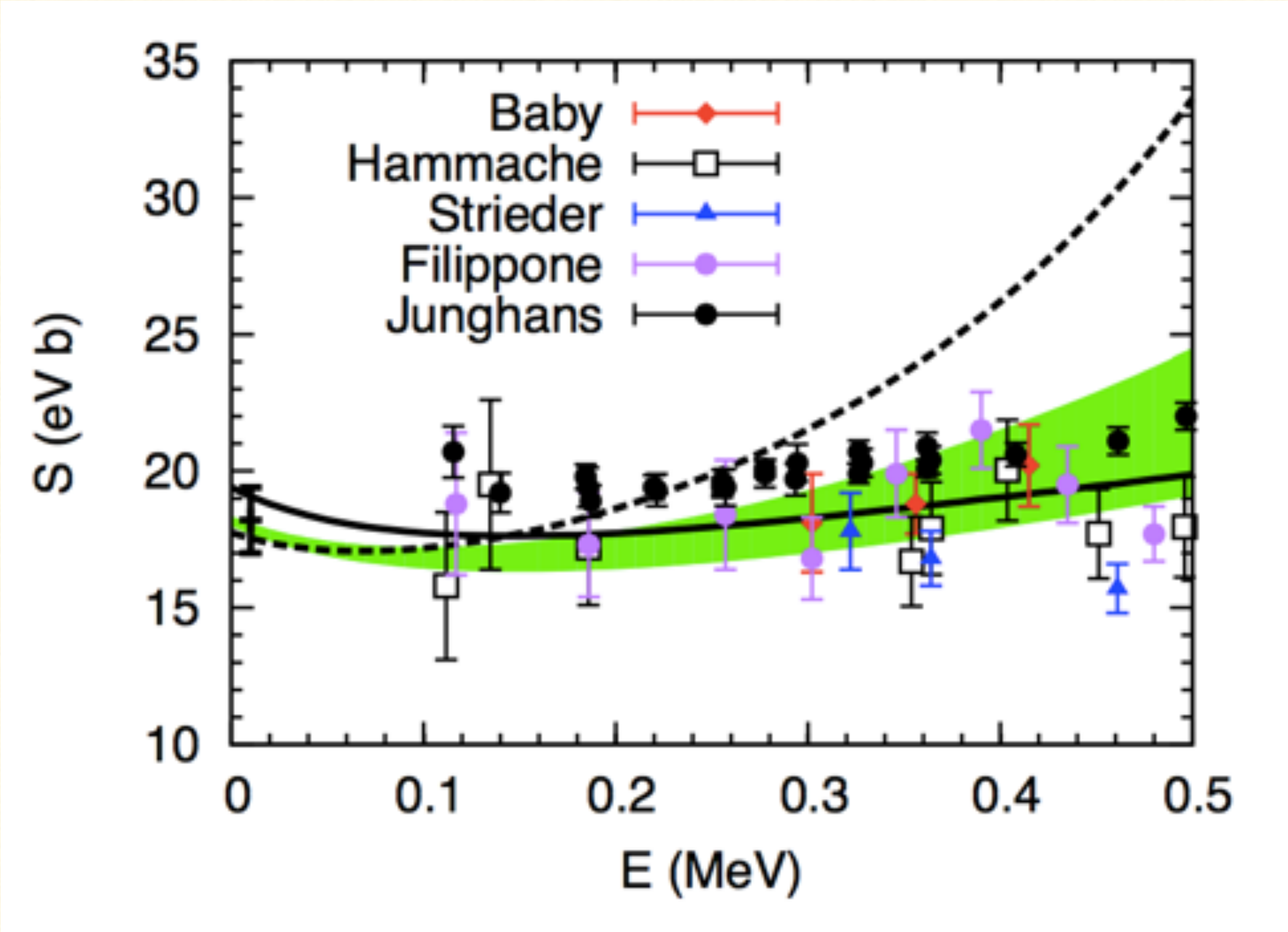

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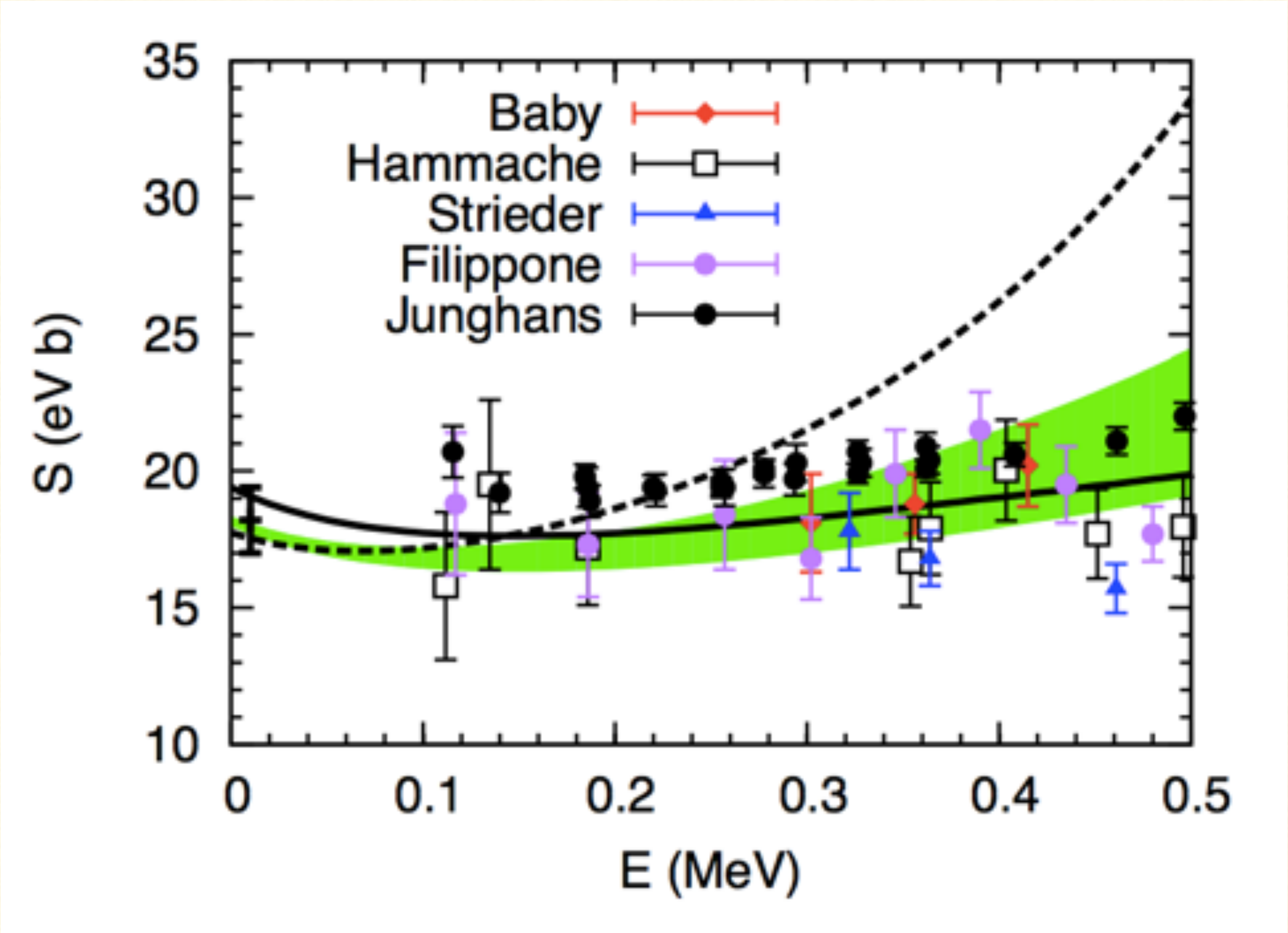

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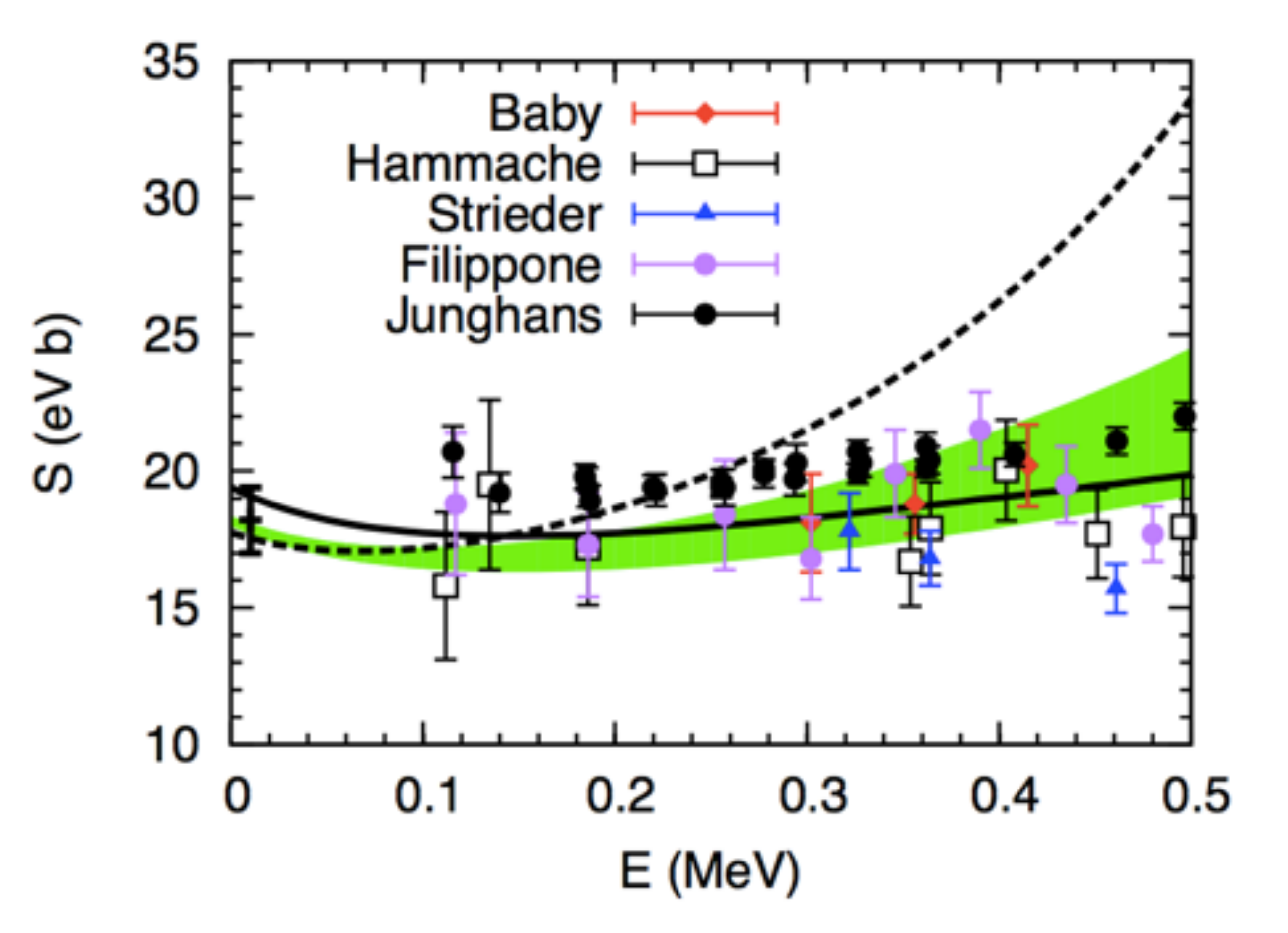

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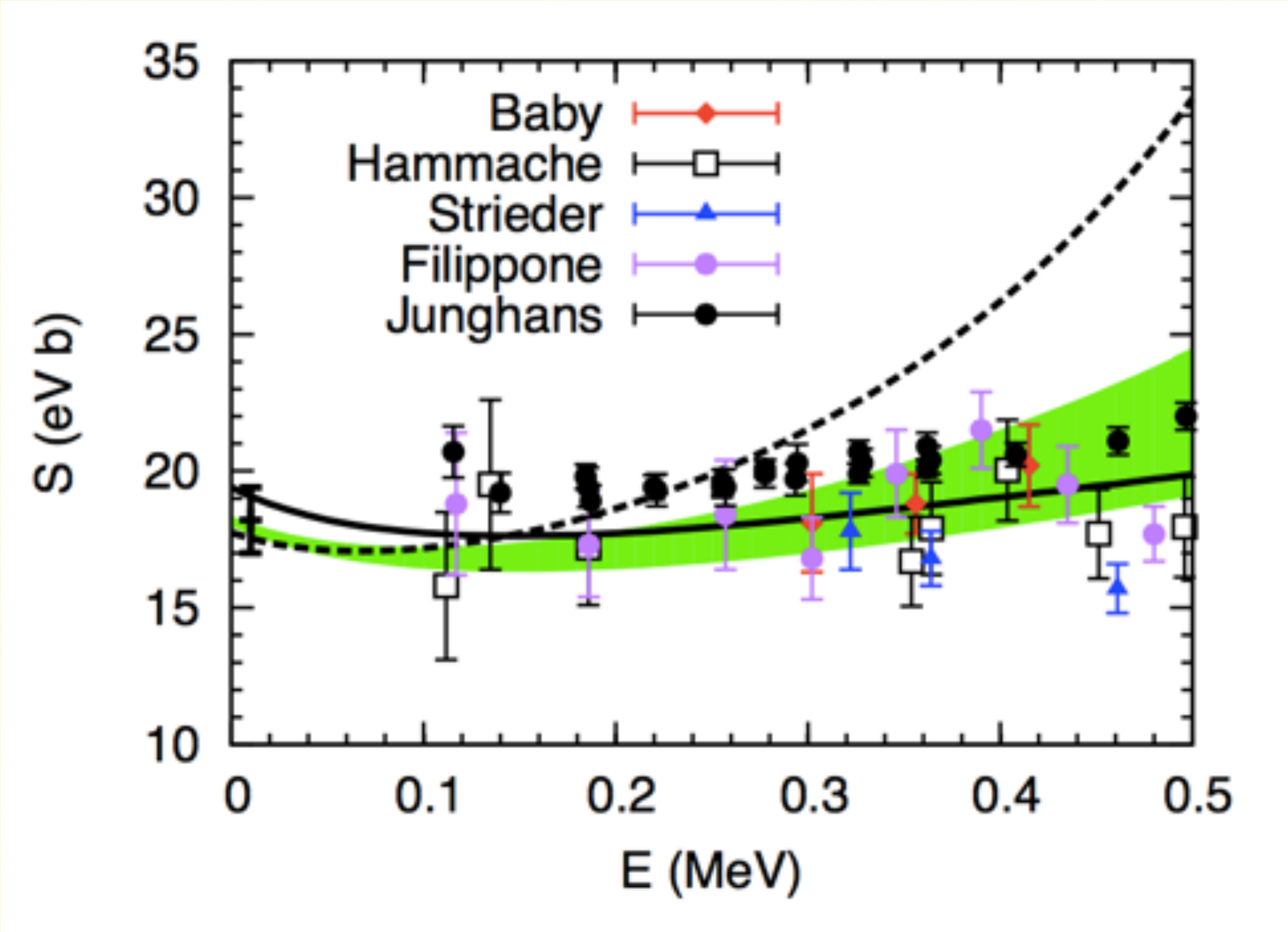
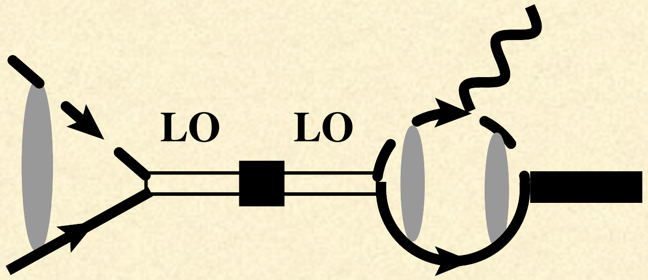


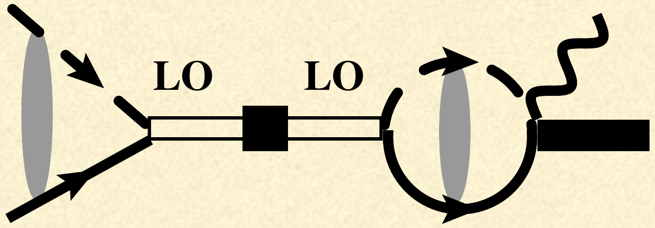
Sensitivity to input $a_{S=2}$ and $a_{S=1}$

- At solar energies it's all about the ANCs
- Use of "Navratil" input generates rapid energy dependence at higher E
- Tamed by NLO corrections: r_0 and r_c each reduce S(E) by 10% at 0.4 MeV

Additional ingredients at NLO



(V)



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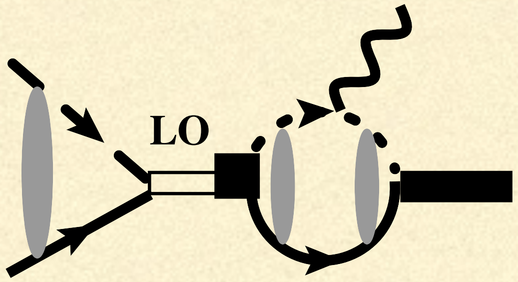

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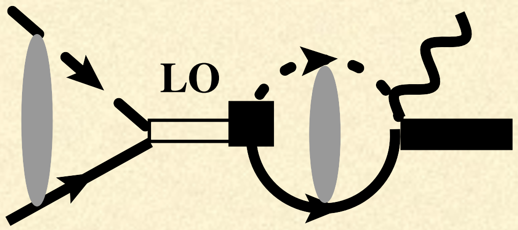

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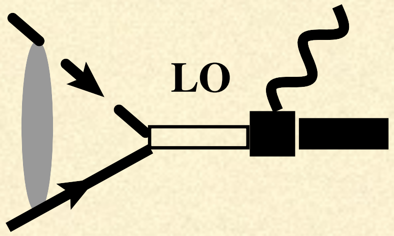

(IX)

$$S(E) = \quad f(E) \sum_s C_s^2 \left[\left| \mathcal{S}_{\text{EC}}\left(E;\delta_s(E)\right) + \overline{L}_s \mathcal{S}_{\text{SD}}\left(E;\delta_s(E)\right) + \epsilon_s \mathcal{S}_{\text{CX}}\left(E;\delta_s(E)\right) \right|^2 + |\mathcal{D}(E)|^2 \right] .$$

- Effective ranges in both $^5S_2$ and $^3S_1$: r_2 and r_1
- Core excitation: determined by ratio of ^{8}B couplings of ^{7}Be*p and ^{7}Be-p states: ϵ_1
- LECs associated with contact interaction, one each for S=1 and S=2: L_1 and L_2

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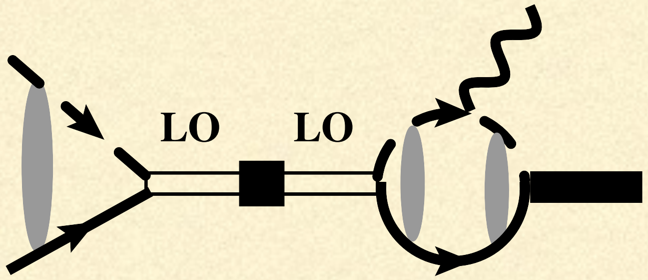

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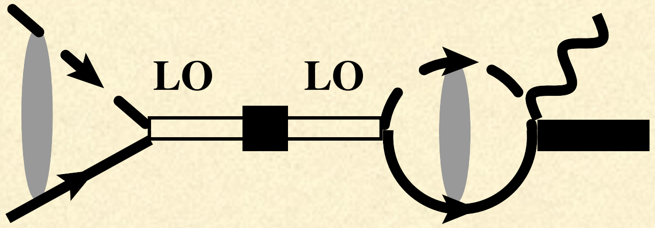

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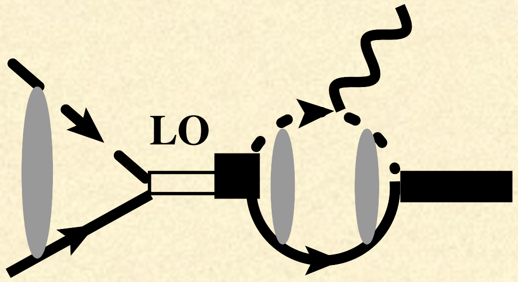

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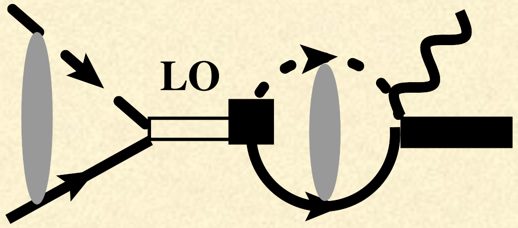

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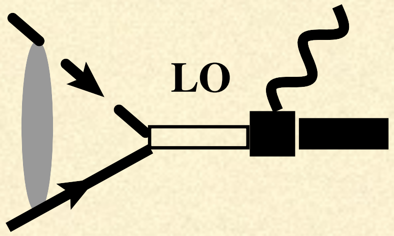

(IX)

Five more parameters
at NLO

$$S(E) = \quad f(E)\sum_s C_s^2\left[\left|\mathcal{S}_{\rm EC}\left(E;\delta_s(E)\right) + \overline{L}_s\mathcal{S}_{\rm SD}\left(E;\delta_s(E)\right) + \epsilon_s\mathcal{S}_{\rm CX}\left(E;\delta_s(E)\right)\right|^2 + |\mathcal{D}(E)|^2\right].$$

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Data situation

- 42 data points for 100 keV < $E_{c.m.}$ < 500 keV
  - Junghans (BE1 and BE3)
  - Fillipone
  - Baby
  - Hammache (1998 and 2001)

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Junghans (BE1 and BE3)	2.7% and 2.3%
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- Subtract M1 resonance: negligible impact at 500 keV and below
- Deal with CMEs by introducing five additional parameters, ξ_i

Building the pdf

$$\mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)=\mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)\mathrm{pr}\left(\vec{g},\{\xi_i\}|I\right),$$

$$\ln\mathrm{pr}\left(D|\vec{g},\{\xi_i\};T;I\right)=c-\sum_{j=1}^{N}\frac{\left[(1-\xi_j)S(\vec{g};E_j)-D_j\right]^2}{2\sigma_j^2},$$

Building the pdf

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- Second factor: priors
  - Independent gaussian priors for $\xi_{i,}$ centered at zero and with width=CME
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  - All other EFT parameters assigned flat priors, corresponding to natural ranges

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- No s-wave resonance below 600 keV

Marginalizing→pdfs

$$\mathrm{pr}\left(g_1, g_2|D;T;I\right) = \int \mathrm{pr}\left(\vec{g}, \{\xi_i\}|D;T;I\right)\, d\xi_1 \ldots d\xi_5 dg_3 \ldots dg_9$$

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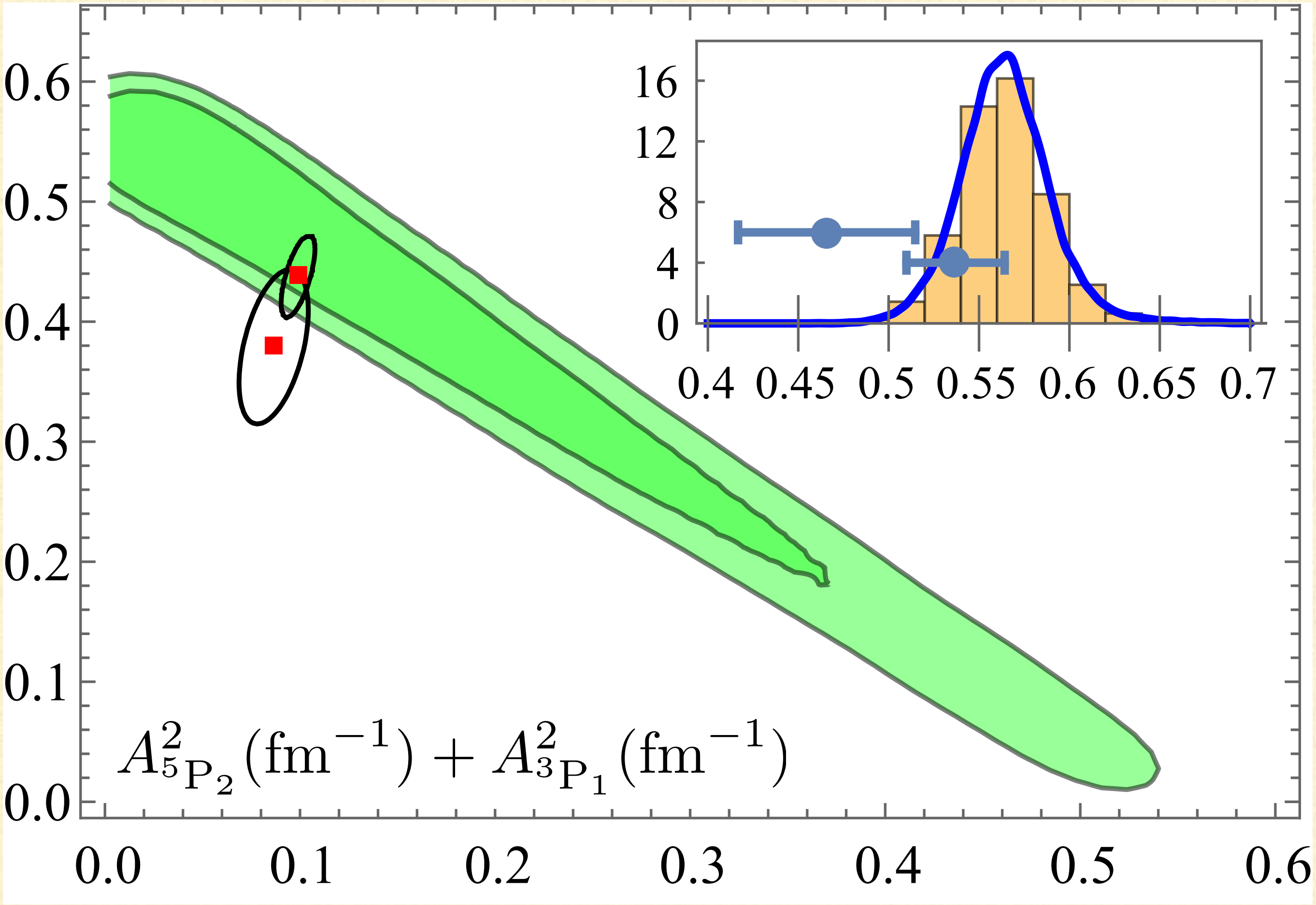

- ANCs are highly correlated but sum of squares strongly constrained

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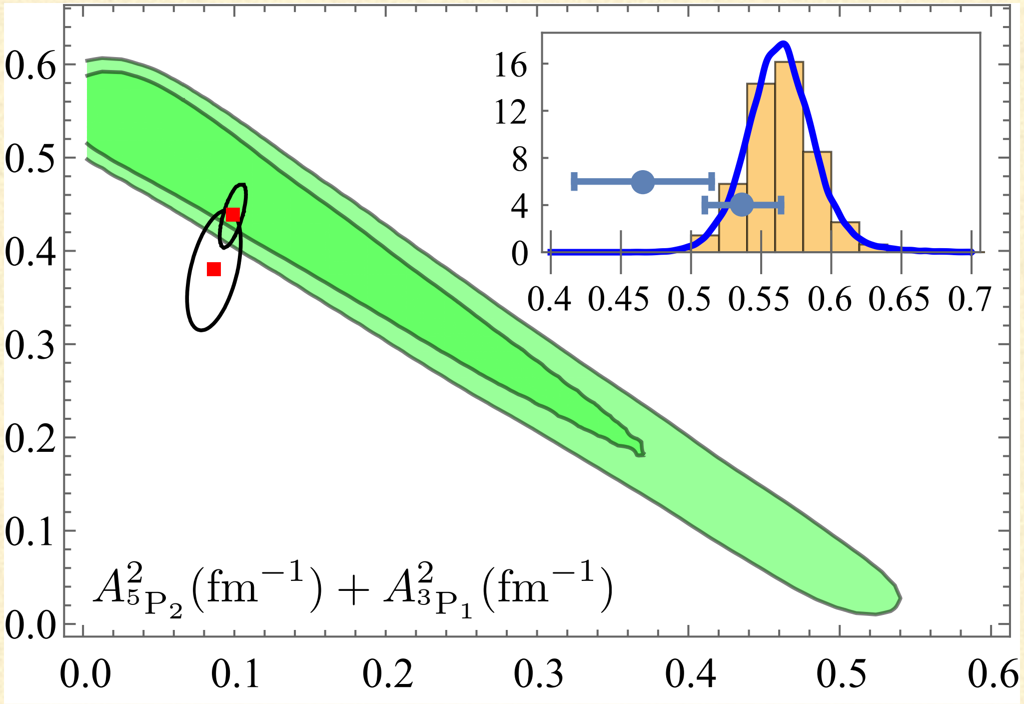

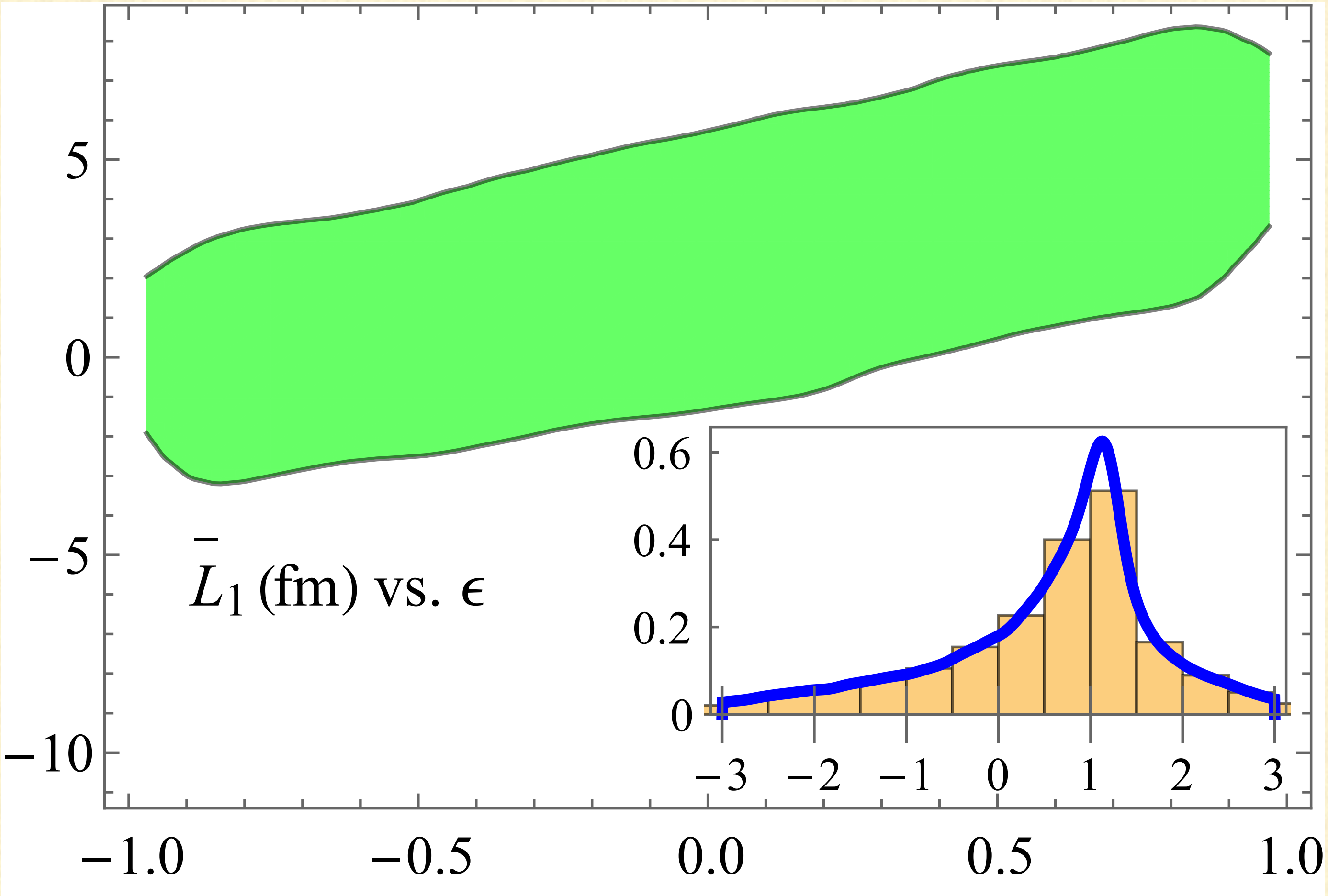

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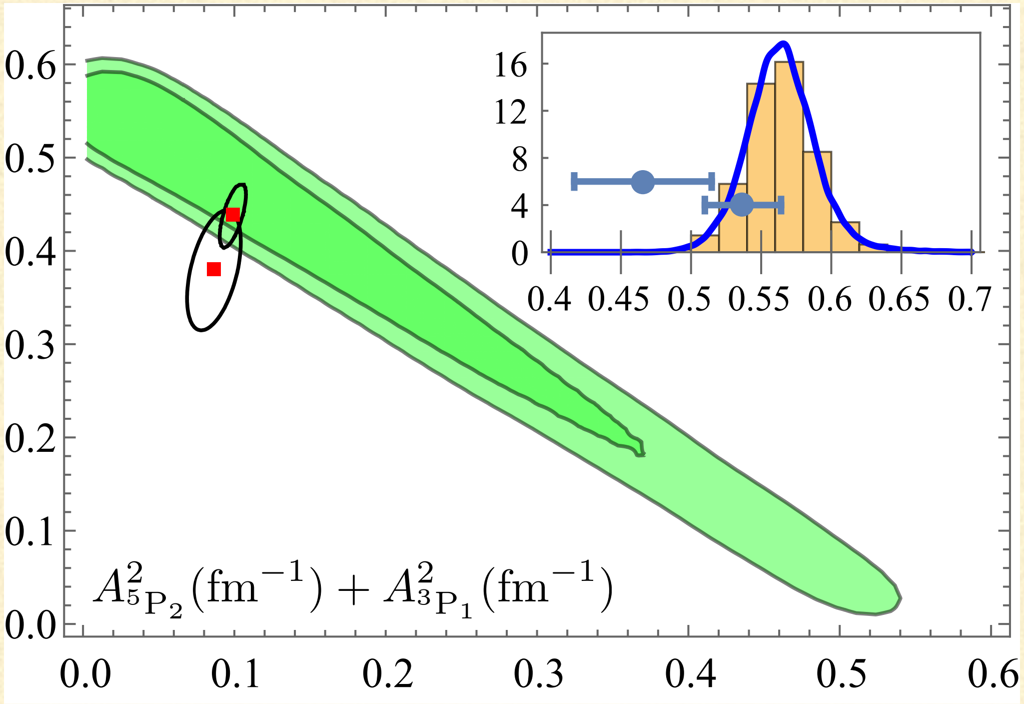

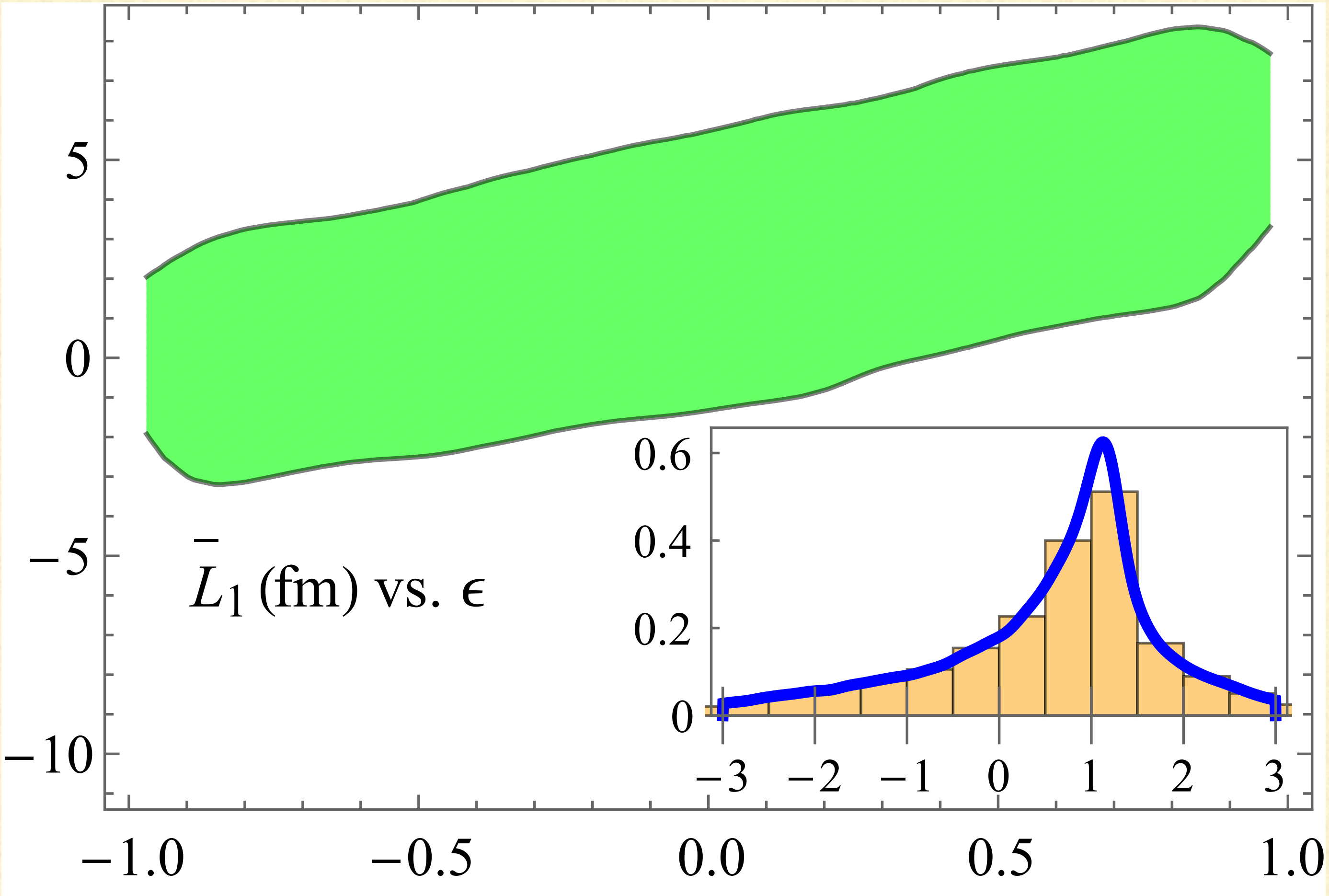

- ANCs are highly correlated but sum of squares strongly constrained
- One spin-1 short-distance parameter: $0.33\ \bar{L}_1/(\mathrm{fm}^{-1}) - \epsilon_1$

Final result

Zhang, Nollett, DP, PLB, 2015

$$\mathrm{pr}\left(\bar{F}|D;T;I\right) = \int \mathrm{pr}\left(\vec{g},\{\xi_i\}|D;T;I\right)\delta(\bar{F} - F(\vec{g}))d\xi_1 \ldots d\xi_5 d\vec{g}$$

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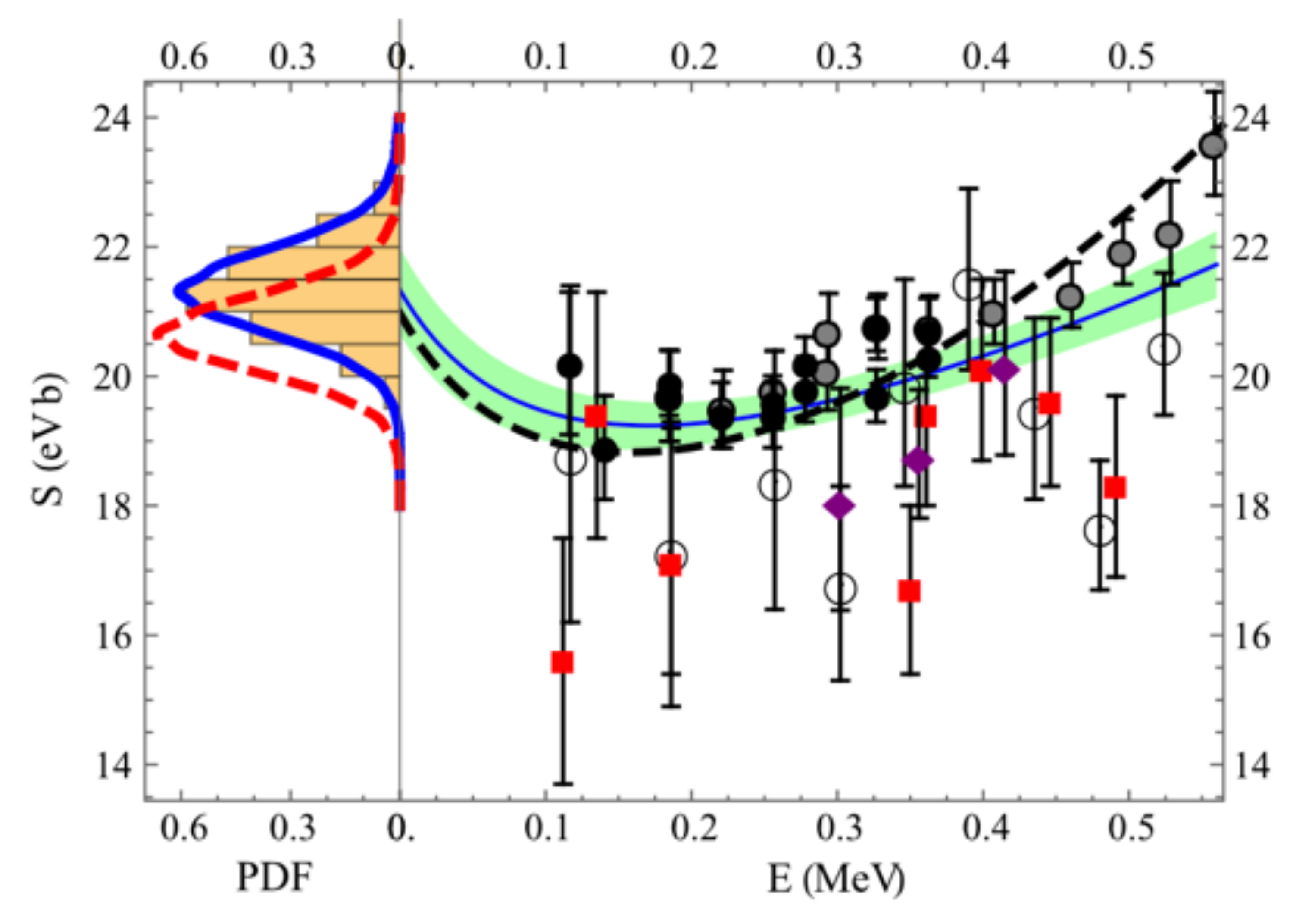

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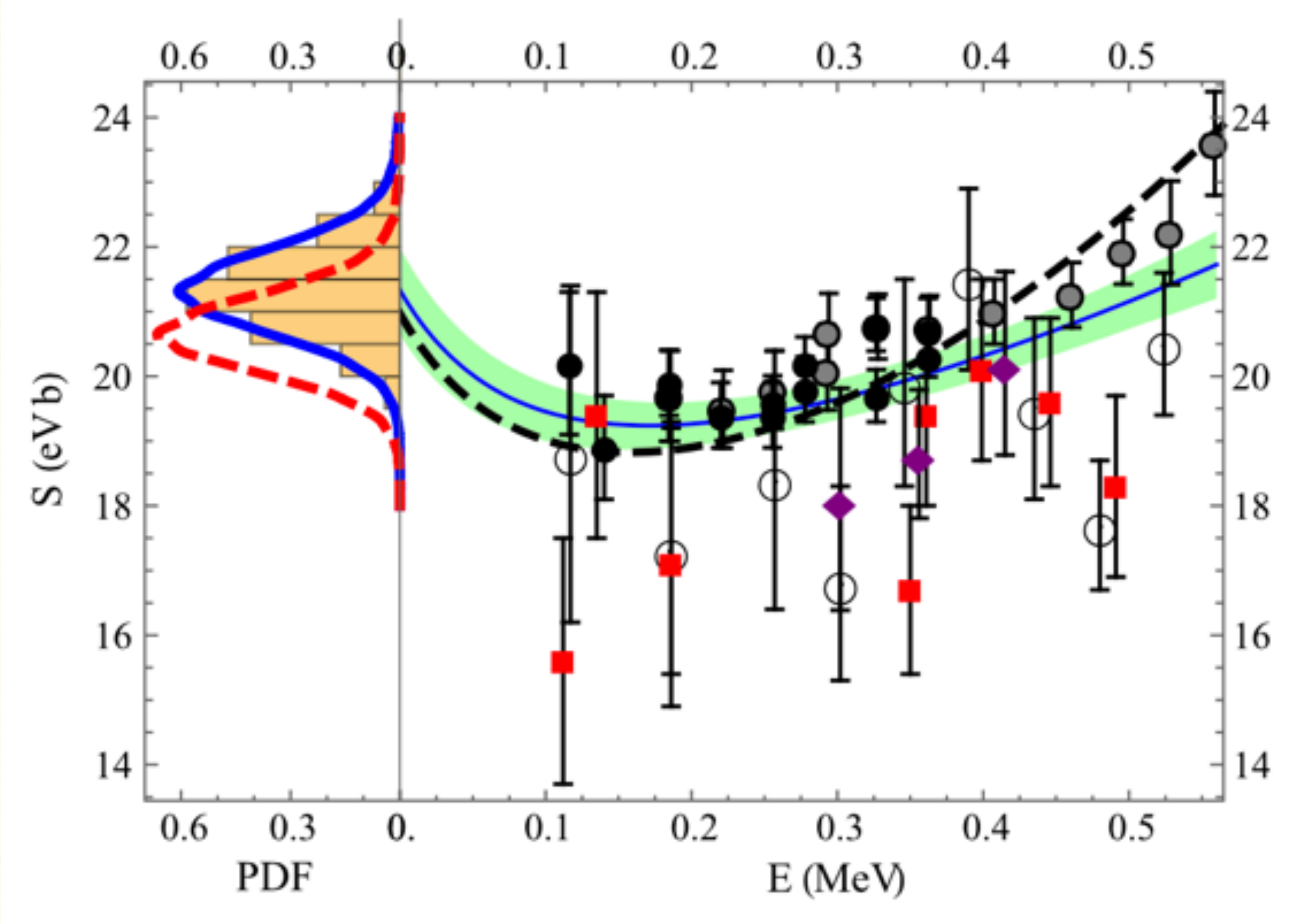

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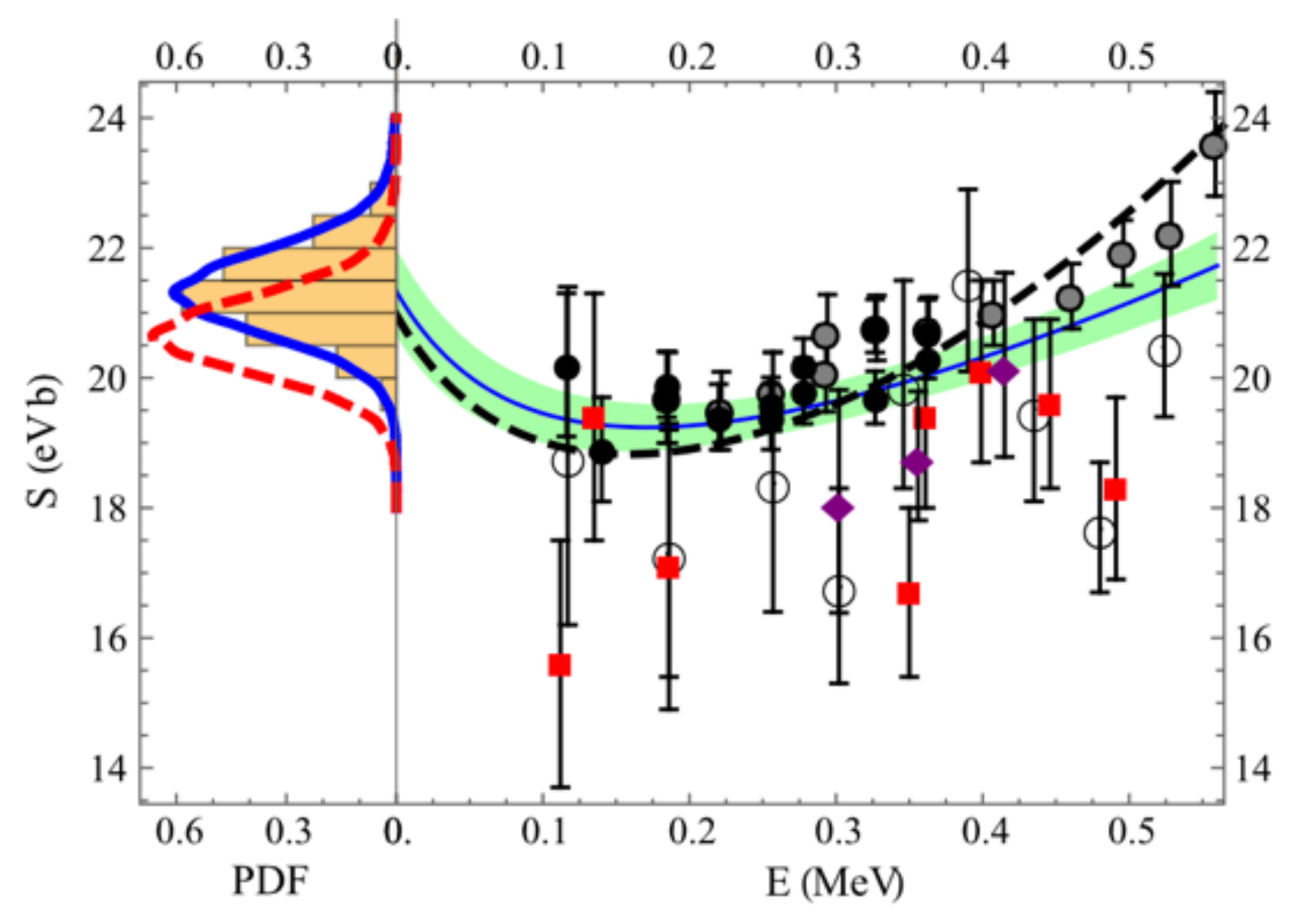

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N^{2}LO corrections?

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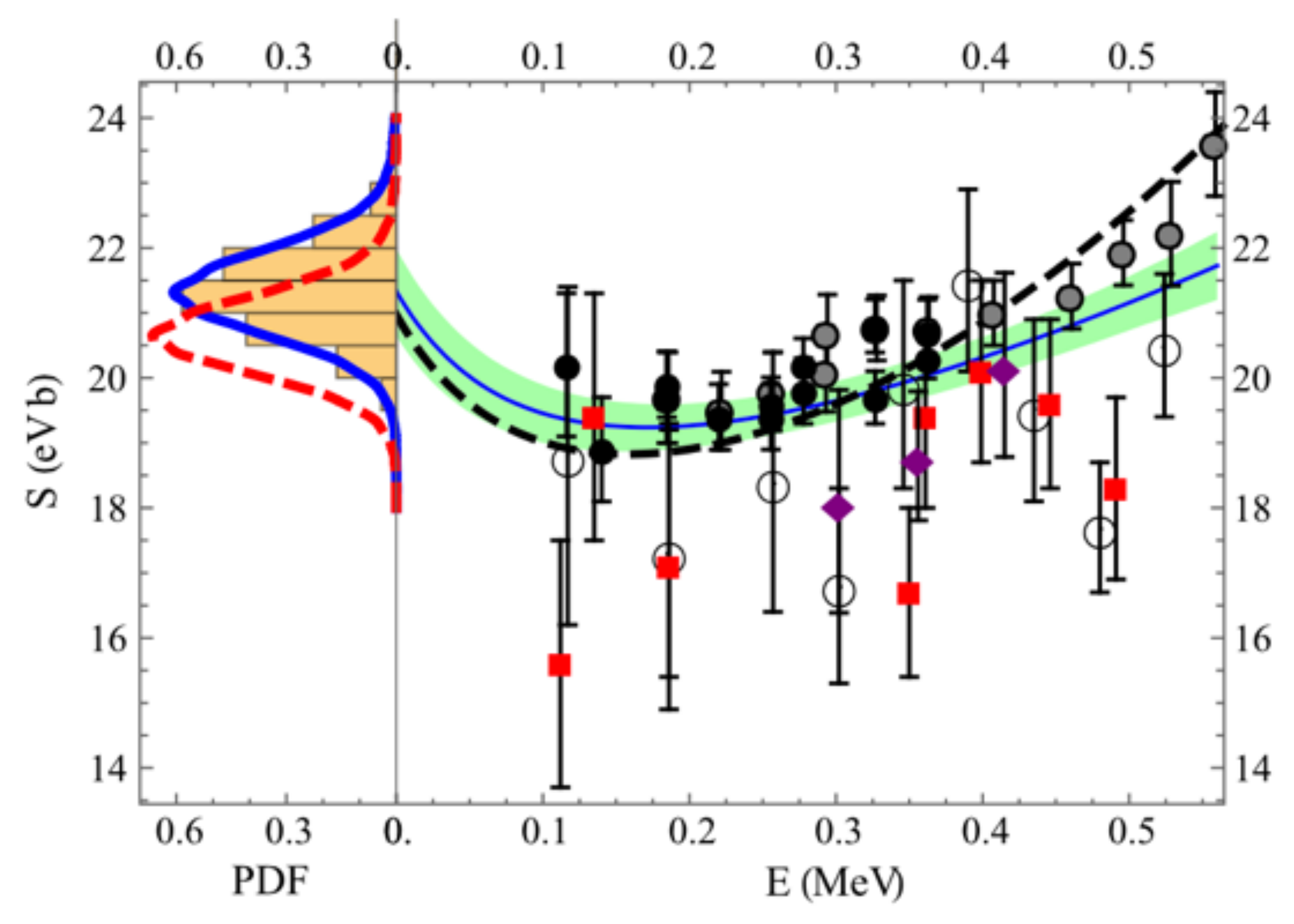

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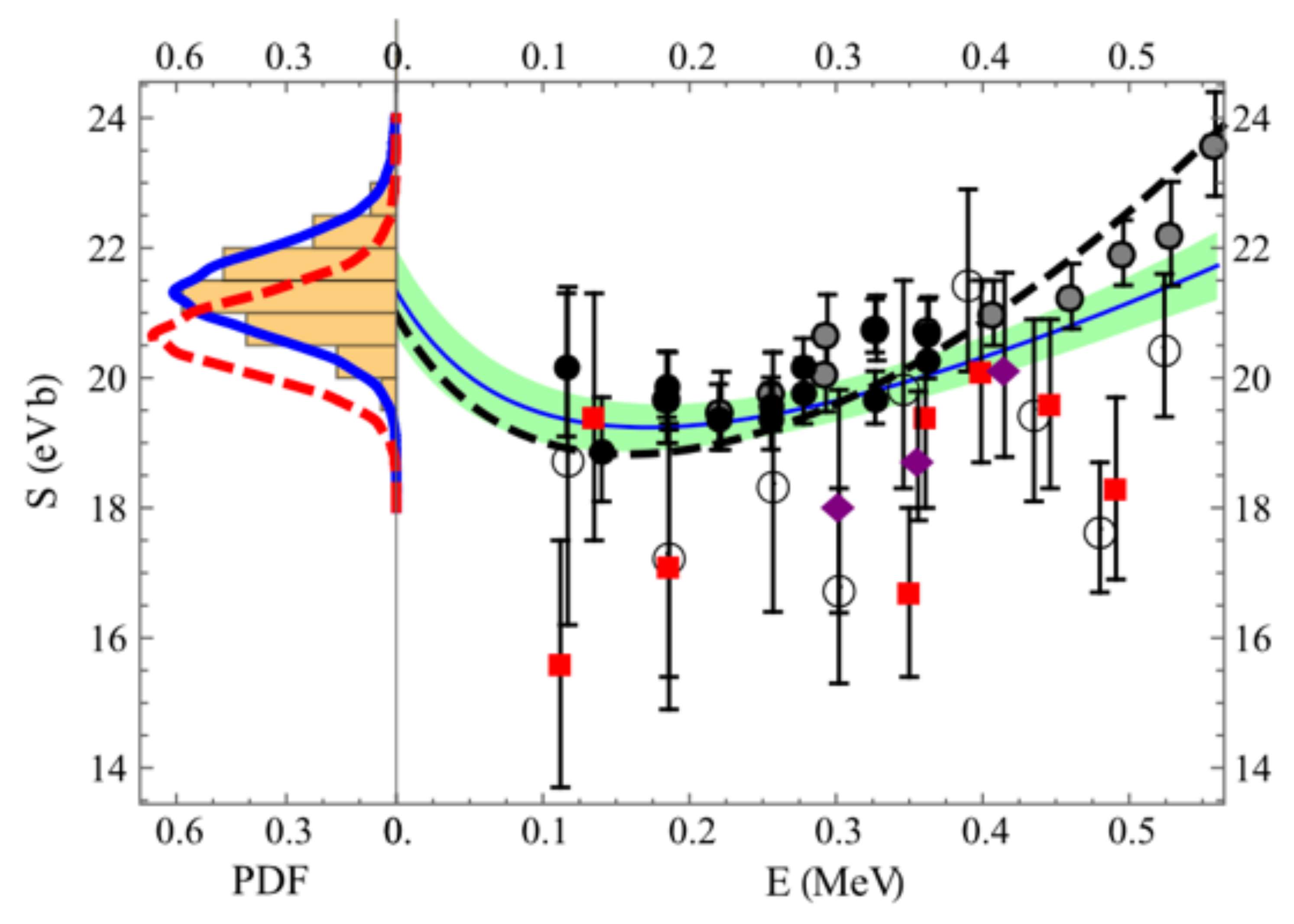

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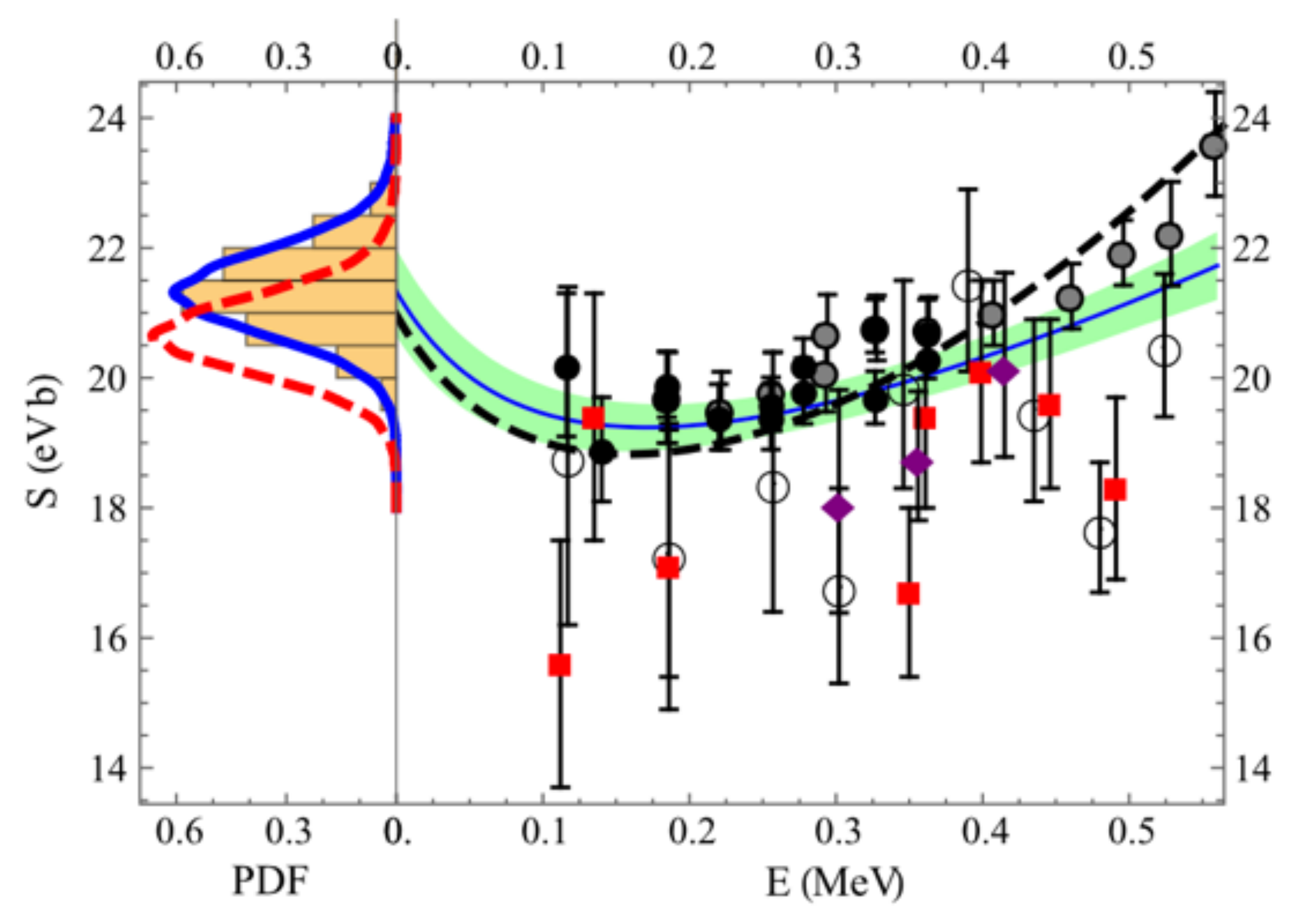

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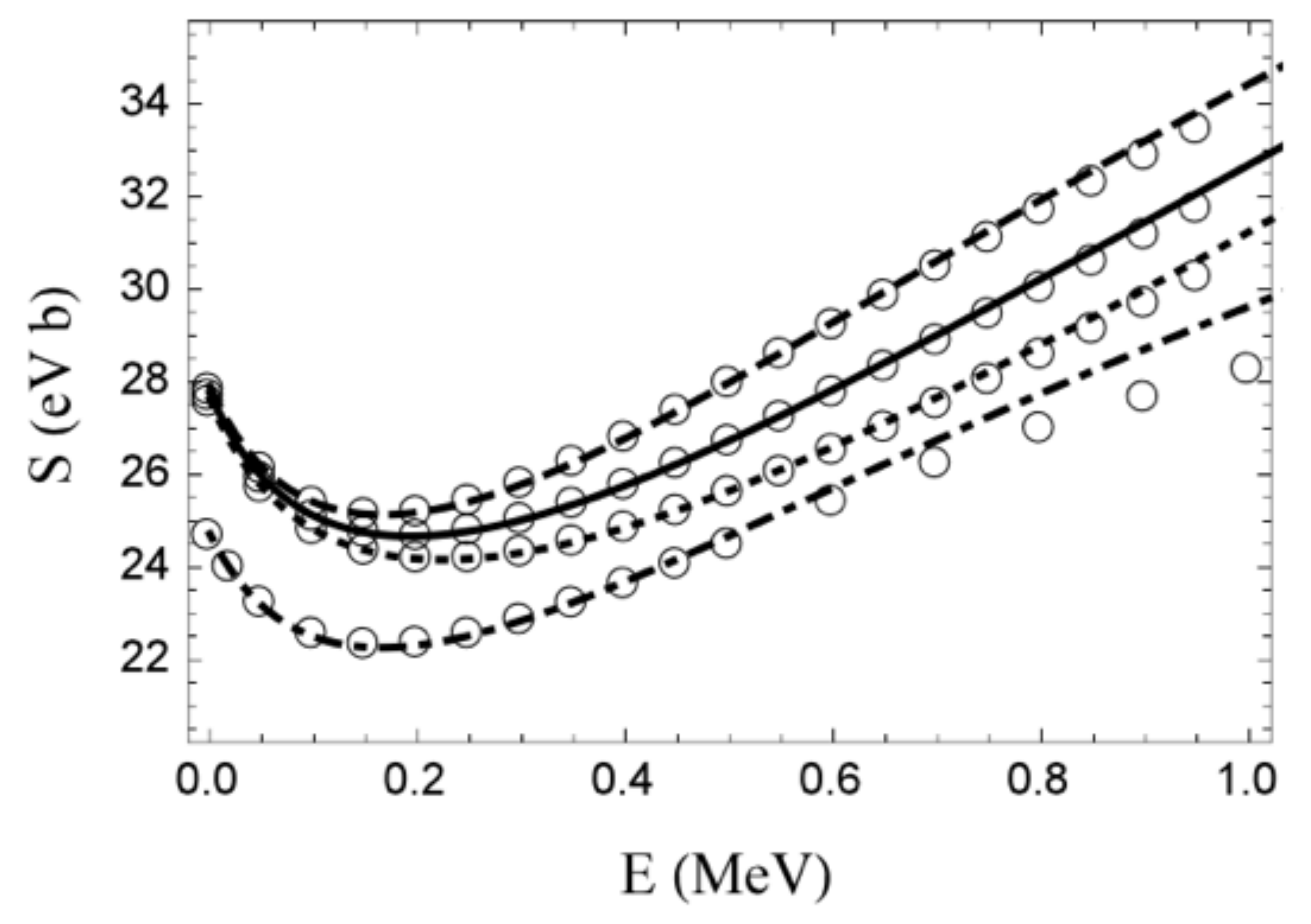

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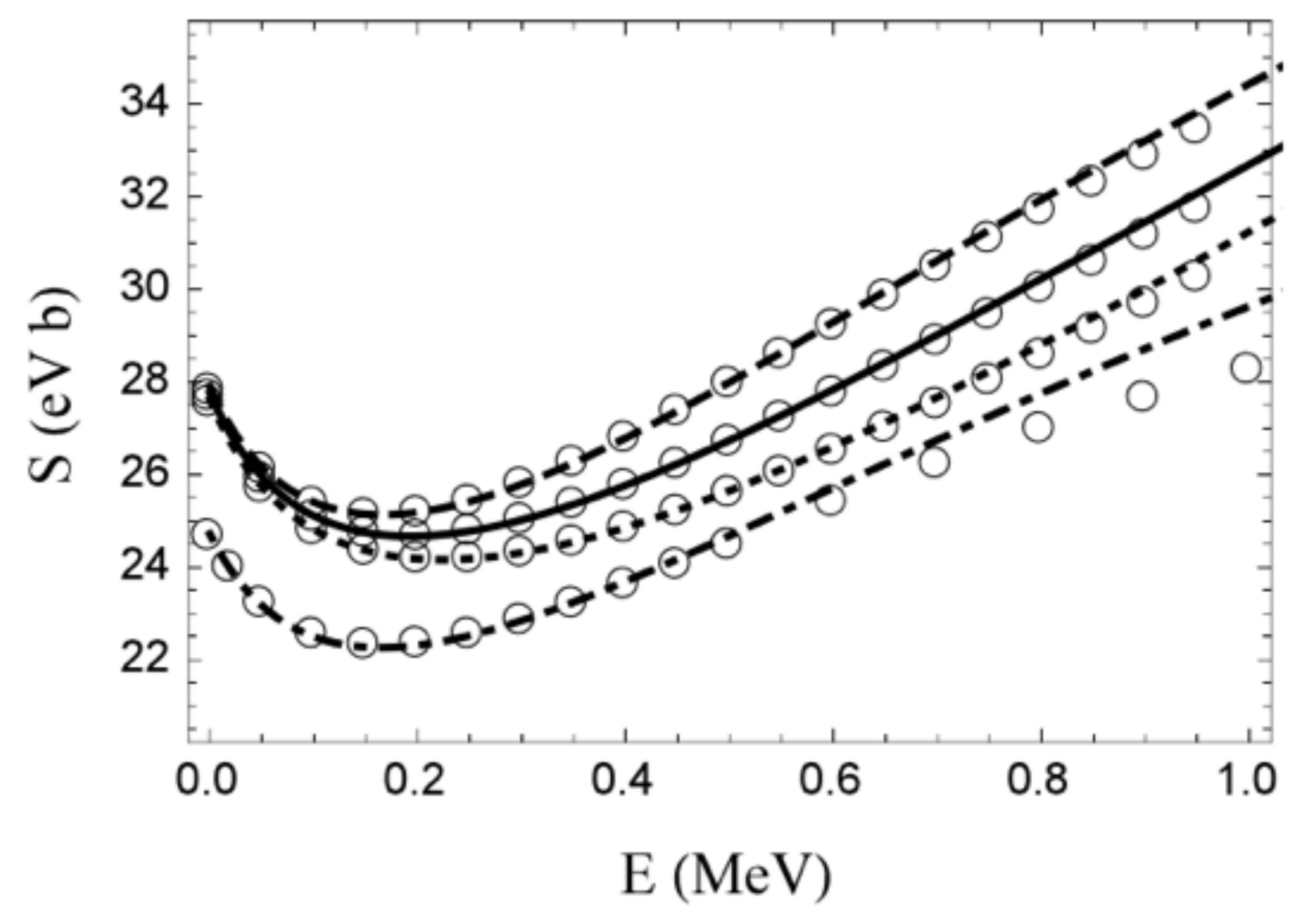

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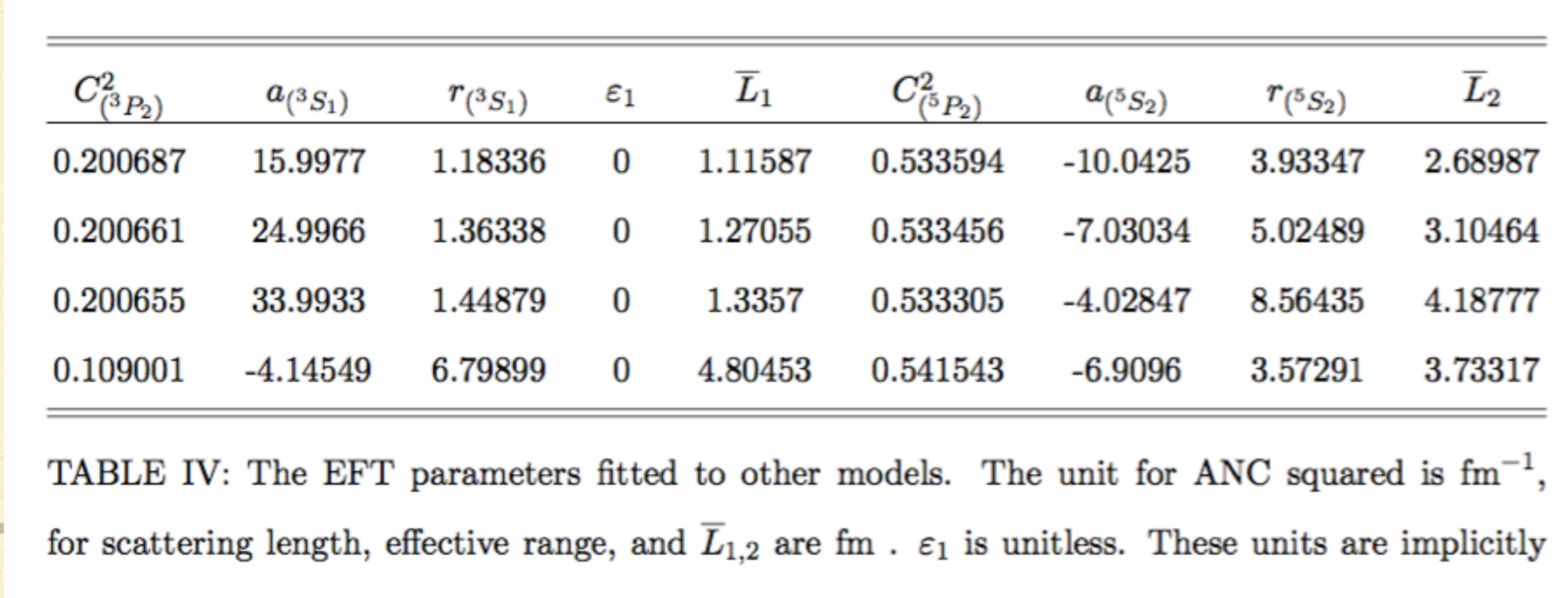
- Parameters generally obey a~1/R_{halo}, r ~R_{core}, L~R_{core}, as expected



$C^2_{(^3P_2)}$	$a_{(^3S_1)}$	$r_{(^3S_1)}$	ε_1	$\overline{L}_1$	$C^2_{(^5P_2)}$	$a_{(^5S_2)}$	$r_{(^5S_2)}$	$\overline{L}_2$
0.200687	15.9977	1.18336	0	1.11587	0.533594	-10.0425	3.93347	2.68987
0.200661	24.9966	1.36338	0	1.27055	0.533456	-7.03034	5.02489	3.10464
0.200655	33.9933	1.44879	0	1.3357	0.533305	-4.02847	8.56435	4.18777
0.109001	-4.14549	6.79899	0	4.80453	0.541543	-6.9096	3.57291	3.73317

TABLE IV: The EFT parameters fitted to other models. The unit for ANC squared is fm^{-1}, for scattering length, effective range, and $\overline{L}_{1,2}$ are fm . ε_1 is unitless. These units are implicitly

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- Taking a variety of "reasonable models" and using them to extrapolate may **over**estimate the model uncertainty

Extensions

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- Simultaneous fit to ^{7}Be p scattering data: requires inclusion of resonances
- Connection to R-matrix

Hale, Brown, Paris, PRC 89, 014623 (2014)

- Forthcoming TRIUMF experiment
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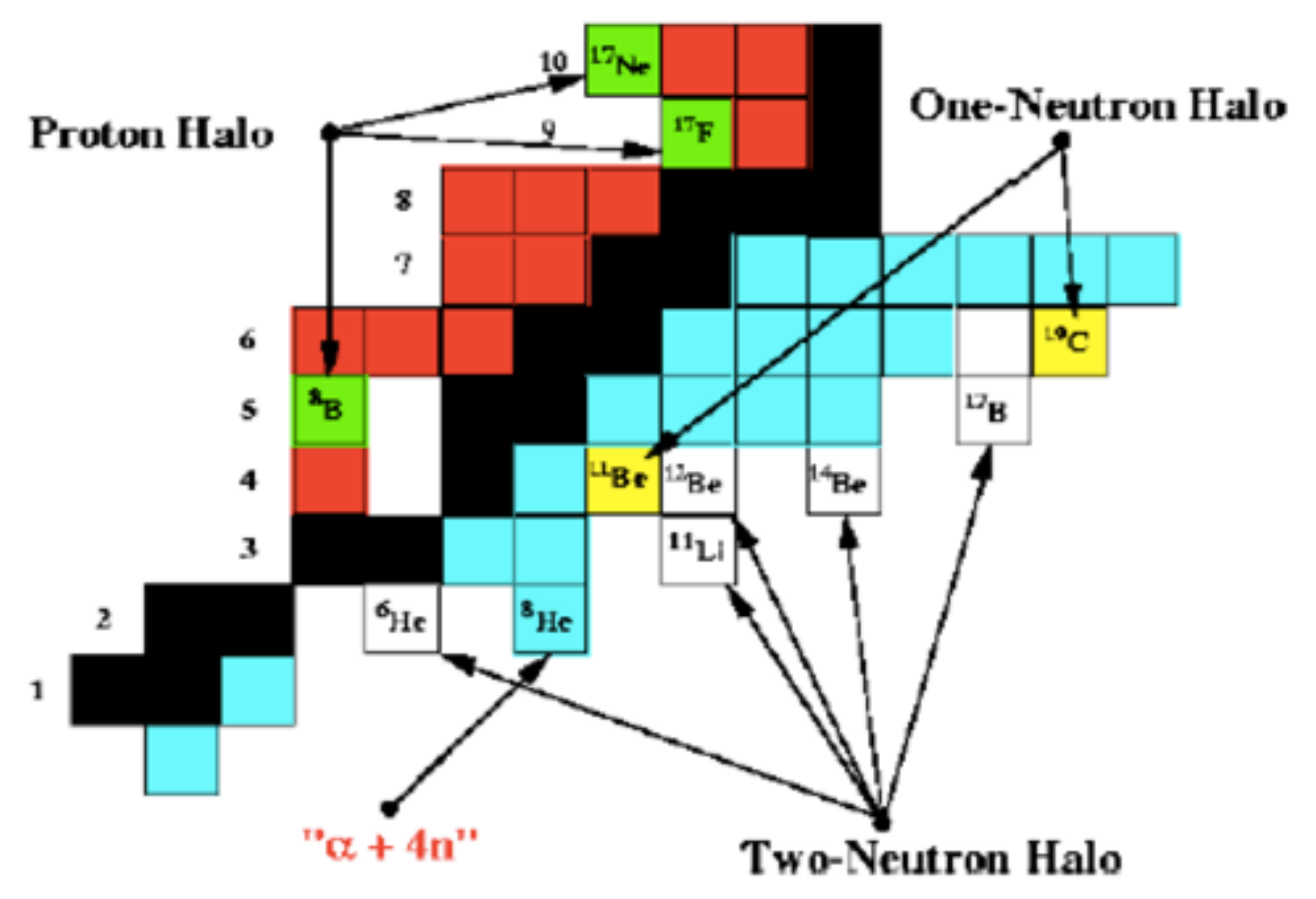
- Review of Halo EFT Hammer, Ji, DP, arXiv:1702.08605

Backup Slides

Halo nuclei

http://nupecc.org



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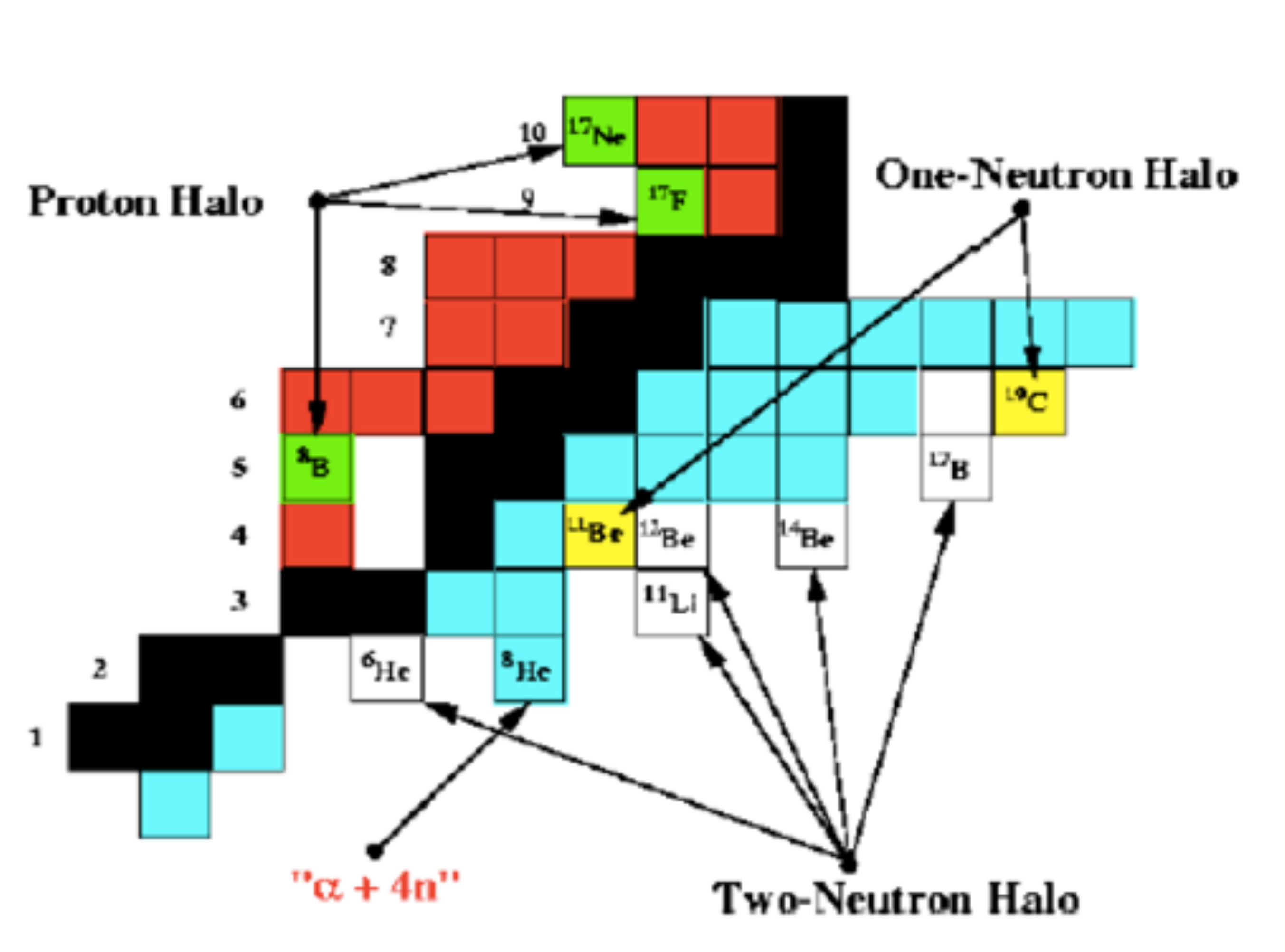

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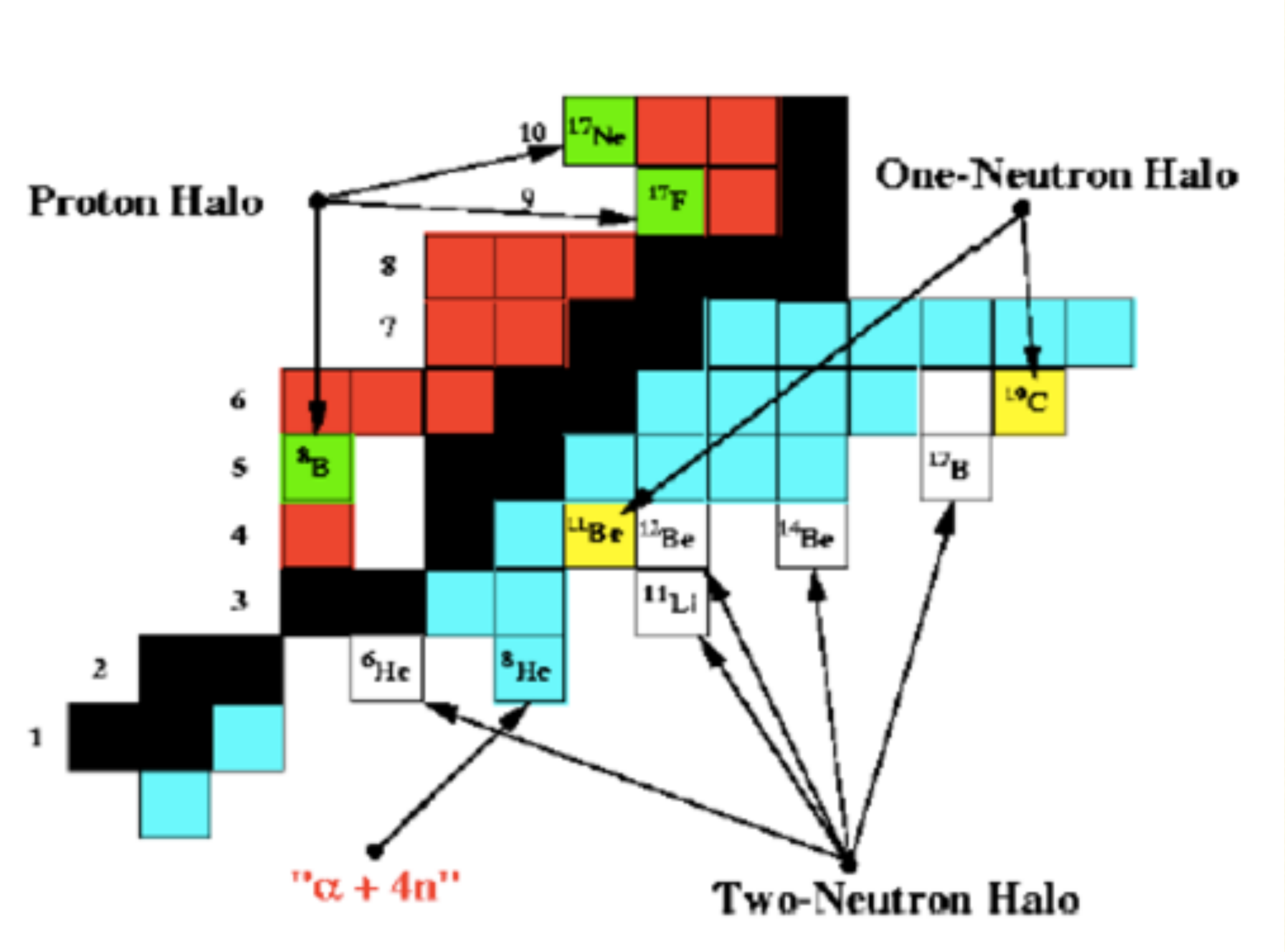

- A halo nucleus as one in which a few (1, 2, 3, 4, ...) nucleons live at a significant distance from a nuclear core.
- Halo nuclei are characterized by small nucleon binding energies, large interaction cross sections, large radii, large E1 transition strengths.

Our approach

- S-wave (and P-wave) states generated by cn contact interactions
- No discussion of nodes, details of n-core interaction, spectroscopic factors

$$u_0(r) = A_0 \exp(-\gamma_0 r)$$

- ^{19}C: input at LO: neutron separation energy of s-wave state.
- A_0 (“wave-function renormalization”) can be fit at NLO.
- P-wave states require two inputs already at LO.

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

- Straightforward computation of diagrams yields:

$$\frac{d\mathrm{B(E1)}}{dE} = e^2 Z_{eff}^2 \frac{m_R}{2\pi^2} A_0^2 \left(\frac{p'^3[2p'^3 \cot(\delta^{(1/2)}(p')) + \gamma_0^3 + 3\gamma_0 p'^2]^2}{[p'^6 + p'^6 \cot^2(\delta^{(1/2)}(p'))](p'^2+\gamma_0^2)^4} + \frac{8p'^3}{(p'^2+\gamma_0^2)^4} \right)$$

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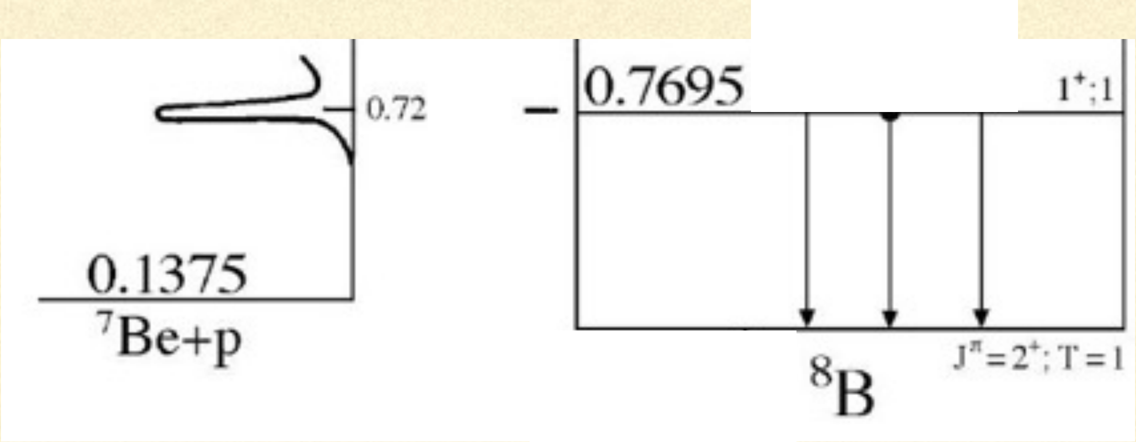
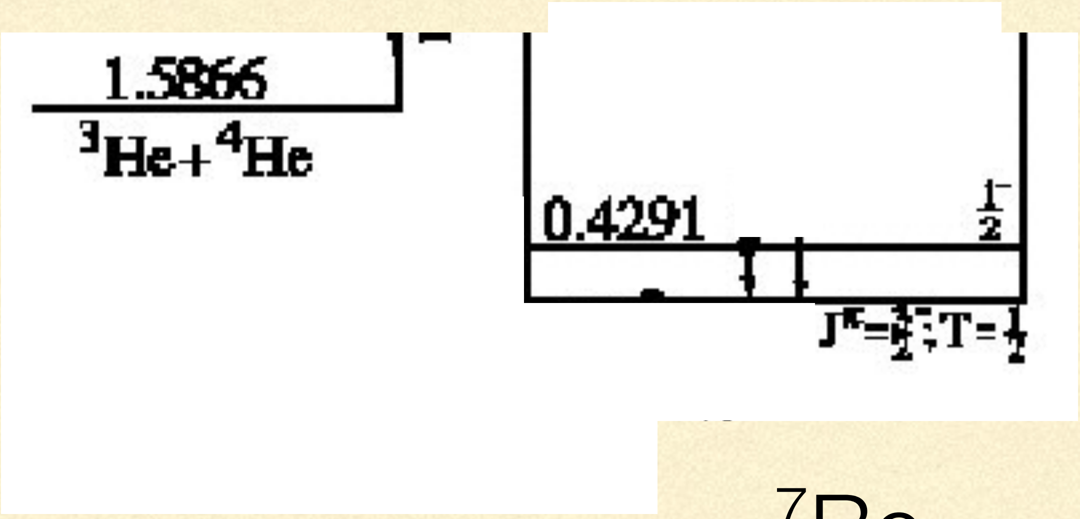
- Higher-order corrections to phase shift at NNLO. Appearance of S-to-$^2P_{1/2}$ E1 counterterm also at that order.

Scales in the ^{8}B system



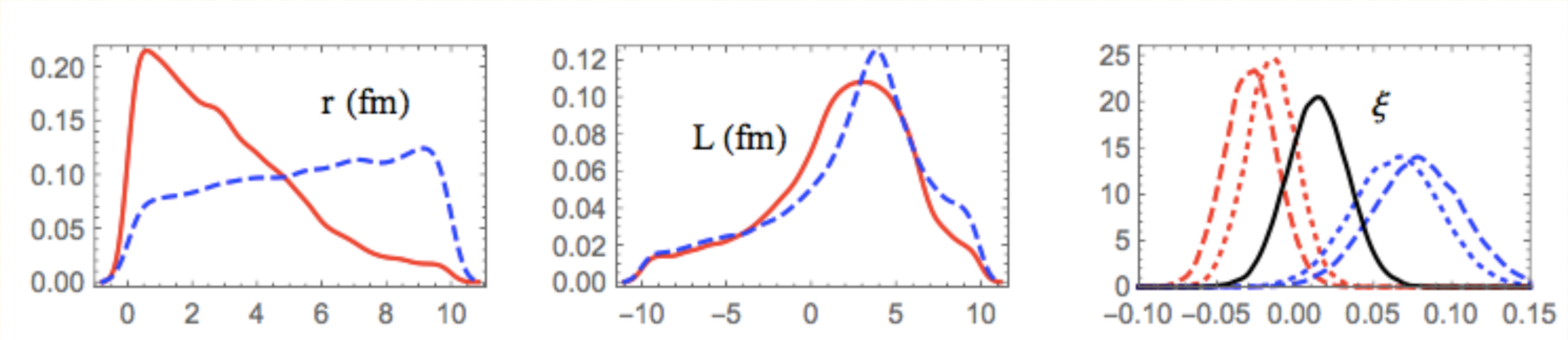
^{8}B



^{7}Be

http://www.tunl.duke.edu

More data details



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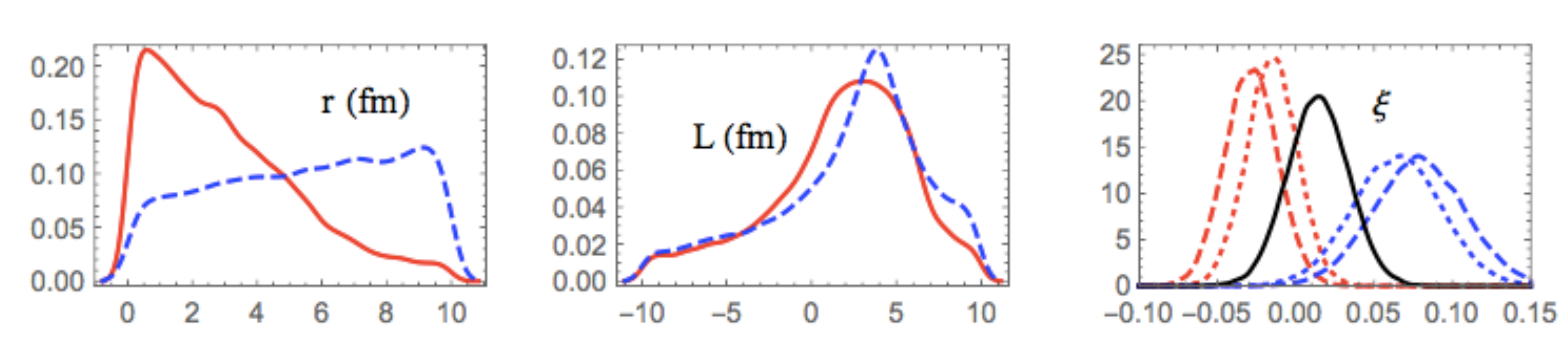

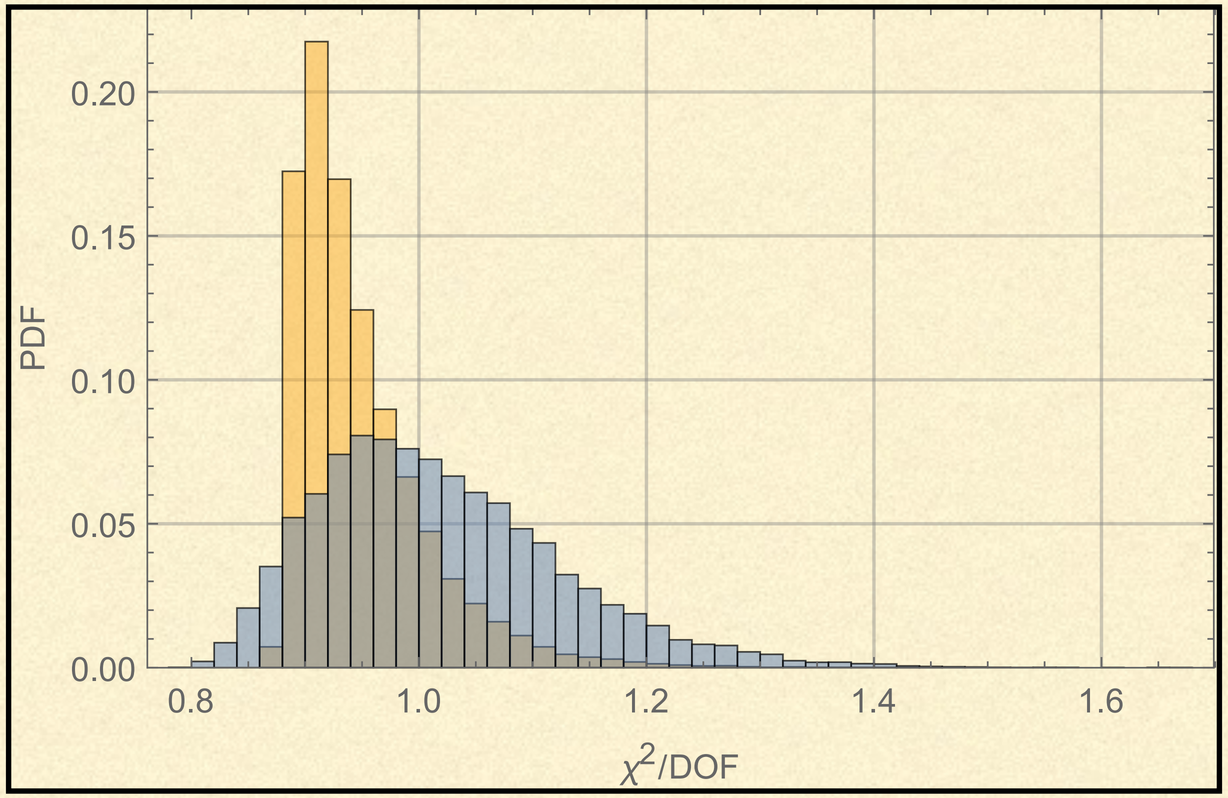

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